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## Inter-temporal cost allocation and investment decisions

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THE CENTER FOR THE STUDY  
OF INDUSTRIAL ORGANIZATION  
AT NORTHWESTERN UNIVERSITY

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INTER-TEMPORAL COST ALLOCATION AND INVESTMENT DECISIONS

By

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## **ABSTRACT**

This paper considers the profit maximization problem of a firm that must make sunk investments in long-lived assets to produce output. It is shown that if per period accounting income is calculated by using a particular allocation rule for investment called the relative benefit and replacement cost (RBRC) rule, that, in a broad range of plausible circumstances, the fully optimal sequence of investments over time can be achieved simply by choosing a level of investment each period to maximize next period's accounting income. In the basic model, it is assumed that there is a single centralized decision maker so the role of the cost allocation rule is that it simplifies the seemingly-complex multi-period optimization problem by decomposing it into a series of simple single period problems. An extension to the basic model considers the case where shareholders delegate the investment decision to a better-informed manager. It is shown if accounting income based on the RBRC allocation rule is used as a performance measure for the manager, robust incentives are created for the manager to choose the profit maximizing level of investment regardless of the manager's own personal discount rate.

## INTRODUCTION

In a variety of industries, firms must make sunk investments in long-lived assets in order to produce output. Calculation of profit maximizing investment levels and evaluation of the firm's performance in such a situation is inherently complicated because of the need to consider cash flows over multiple future periods. One technique that firms routinely use to create simplified single period "snapshots" of their performance is to calculate per period accounting income using accounting measures of cost that allocate the costs of purchasing long-lived assets over the periods that the assets will be used. Firms use these single period snapshots of performance both to directly guide their investment decisions and to evaluate the performance of managers who make investment decisions. Perhaps surprisingly, there has been very little formal analysis in the economics, accounting, or finance literature that attempts to investigate whether there is any basis for these practices and, if there is, what sorts of allocation rules ought to be used to calculate accounting income when it is used to guide investment decisions. This paper provides a theory which shows that, in a broad range of plausible circumstances, a particular allocation rule called the relative benefit and replacement cost (RBRC) rule can be used both to create simple rules for calculating the fully optimal investment decision and to create robust incentives for managers to choose this level of investment when the decision is delegated to them.

The paper considers a multi-period model where a private profit-maximizing firm must make sunk investments in long-lived assets to produce output. It is shown that, in a broad range of plausible circumstances, the seemingly-complex multi-period investment problem can actually be decomposed into a series of very simple single-period problems. In particular, a

simple formula can be used to calculate a hypothetical perfectly competitive rental price for capital stock in each period, and the solution to the true multi-period investment problem can be calculated as the solution to this series of time-separable hypothetical rental problems.

Furthermore, it is shown that a particular cost allocation rule, called the relative benefit and replacement cost (RBRC) rule, has the property that the cost of purchasing a new asset that is allocated to any period using this rule is exactly equal to the hypothetical perfectly competitive rental price in that period. It follows from this that the firm can choose a fully optimal level of investment in the current period simply choosing a level of investment that maximizes next period's accounting income calculated using the RBRC allocation rule.

The above results are derived in a model where it is assumed that there is a single centralized decision-maker that owns the firm. Therefore the role of the cost allocation rule is that it simplifies the centralized decision-maker's problem of calculating the optimal level of investment. An extension of the basic model is constructed that shows that this allocation rule can also be used to create desirable investment incentives for managers when the investment decision is delegated to them. In the extension, it is assumed that shareholders delegate the investment decision to a manager because the manager has better information about current and future demand. It is shown that shareholders can use the RBRC allocation rule to create robust incentives for the manager to choose the profit-maximizing level of investment each period in the following sense. Suppose that per period accounting income is calculated using the RBRC allocation rule and the manager is paid a wage each period that depends on current and past periods' accounting income. Then, it is shown that the manager will have the incentive to choose the profit maximizing level of investment in every period for any wage function that satisfies the

property that the wage paid in any period is a weakly increasing function of current and past periods' accounting income. Furthermore, the firm does not need to know the manager's own personal rate of time preference or anything else about the manager's preferences in order to calculate the RBRC allocation rule. Therefore, the investment incentive problem is solved in a robust way and the firm is left with a considerable degree of freedom to address any other incentive problems that may exist, such as a moral hazard problem within each period, by choosing the precise functional form of the wage function each period.

In the formal model of the paper, it is assumed that assets have a known finite lifetime, that the productivity of existing assets weakly decreases at a known constant percentage rate over their lifetime, and that the purchase price of new assets also weakly decreases at a known constant percentage rate over time. The RBRC allocation rule is the unique allocation rule that satisfies the two properties that: (i) the share of cost allocated across time falls at a constant percentage rate equal to the combined effect of the rate at which the productivity of existing assets and the price of new assets falls<sup>1</sup>; and (ii) the present discounted value of the cost allocations using the firm's discount rate is equal to the initial purchase price of the asset.

Property (i) is related to the "matching principal" from accrual accounting that states that investment costs should be allocated across periods so as to match costs with benefits. In particular, in the case where the purchase price of new assets remains constant over time, the RBRC rule allocates the cost of purchasing an asset across the periods of its lifetime in proportion to its productivity which can be viewed as a version of the matching principal. Note,

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<sup>1</sup>e.g., If productivity of existing assets declines at 5% per year and the purchase price of new assets is declining at a rate of 2% per year, then the allocation shares of the RBRC rule decline at a rate of 6.9% per year ( $.95 \times .98 = .931$ ).

however, that the allocation shares under the RBRC rule are also constructed to reflect the rate of change of the replacement cost of assets, which is not suggested by the matching principal as its is normally interpreted. Property (ii) can be viewed as stating that the investment is fully allocated taking the time value of money into account. Most traditional accounting systems ignore the time value of money when allocating investment costs over time. The term “residual income” is generally used in the accounting literature to describe income measures that are calculated using an allocation rule for investments that takes the time value of money into account (Horngren and Foster 1987, pp. 873-74). Recently there has been an explosion of applied interest in using residual income both to directly guide capital budgeting decisions and as a performance measure for managers that make capital budgeting decisions. Management consulting companies have renamed this income measure “economic value added” (EVA) and very successfully marketed it as an important new technique for maximizing firm value. *Fortune*, for example, has run a cover story on EVA, extolling its virtues and listing a long string of major companies that have adopted it (Tully 1993).<sup>2</sup> This paper provides an explicit formal model which justifies the use of residual income in the capital budgeting process and also specifically identifies the particular allocation rule that should be used to calculate residual income.

There are two recent groups of papers in the literature that have attempted to provide a more formal analysis of whether or not allocation rules for investment can be identified such that the resulting measures of per period accounting income can play a useful role in the capital

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<sup>2</sup>See the roundtable discussion in the *Continental Journal of Applied Corporate Finance* (Stern and Stewart 1994) and the associated articles (Sheehan 1994, Stewart 1994).

budgeting process. One approach is due to Anctil (1996) and Anctil, Jordan and Mukherji (1998). They assume that the environment is completely stationary so that a stationary equilibrium level of capital stock exists in the sense that, if the firm begins at the stationary level of capital stock, then the firm optimally maintains a constant capital stock by incurring a constant level of investment. They also assume that there is an adjustment cost to changing the size of the capital stock and that the firm begins the first period with a capital stock that is not equal to the stationary value. The papers show that the time path of capital stock when fully optimal investments are chosen and the time path of capital stock when the firm simply attempts each period to maximize that period's residual income both converge to the stationary capital stock and thus converge to one another. This means that the policy of attempting to maximize residual income on a period-by-period basis yields a policy that converges to the optimal policy. While this is an interesting result, it only shows convergence to the optimal policy in the limit and only applies to a stationary environment.

The other approach is due to Rogerson (1997).<sup>3</sup> Rogerson (1997) considers a model where there is only a single investment opportunity at the beginning of the first period and shows that an allocation rule called the relative benefit (RB) rule has the property that the level of investment that maximizes the discounted present value of cash flows also maximizes accounting income on a period-by-period basis. This, in turn, creates robust incentives for the manager to choose the profit-maximizing investment level in the same sense as the term is used in this paper.

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<sup>3</sup>See Rogerson (1992) for an earlier, related, result. Papers that have generalized Rogerson's(1997) result and applied it in a number of different settings include Baldenius and Reichelstein (2005), Baldenius and Ziv (2003), Dutta and Reichelstein (1999, 2002), and Reichelstein (1997, 2000).



It is clear that Rogerson's(1997) result would continue to hold in a situation where the firm invests every period so long as each investment project is additively separable.<sup>4</sup> However, in the case where the firm invests to maintain and possibly increase a stock of capital that it uses to produce units of the same good over multiple periods, the investment problems are inherently not separable in this sense. That is, the marginal productivity of a new unit of investment in a given period is obviously affected by the existing level of capital stock which is in turn affected by the level of investment in previous periods. The results of this paper essentially provide a similar sort of result to the result of Rogerson (1997) for the case where a firm is engaged in ongoing investment associated with producing the same product over time.

The observation that a vector of hypothetical zero profit rental prices can be calculated for the case when new asset prices are changing at a constant percentage rate and the productivity of existing assets is decaying at a constant percentage rate has been made by Hausman (1997), Laffont and Tirole (2000), and Mandy and Sharkey (2003). These papers consider a model of a regulated firm subject to constant returns to scale and show that the hypothetical zero profit rental prices can be used to calculate prices for output such that the regulated firm breaks even. However, none of these papers construct what will be referred to in this paper as the “hypothetical cost function” or attempt to show that conditions exist under which the hypothetical cost function can be substituted for the true cost function for purposes of solving optimization problems. Furthermore, none of these papers consider the issue of whether or not a historical cost allocation rule can be used to calculate the hypothetical perfectly competitive

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<sup>4</sup>i.e., if the firm makes an investment each period which affects cash flows in future periods and the cash flows from each project are additively separable from the cash flows of every other project on a period-by-period basis.

rental prices. In a companion paper to this paper (Rogerson 2005), it is shown that the decomposition result can also be applied to the case of a regulated firm to calculate welfare maximizing prices. In particular, under the assumption that there are constant returns to scale within each period, it is shown that the zero-profit prices previously identified by Hausman (1997), Laffont and Tirole(2000), and Mandy and Sharkey( 2003) are also efficient because price each period is set equal to long run marginal cost.

The result of this paper is also relevant to the large literature on cost allocation in both the accounting and economics literature that generally finds that there is no economically meaningful way to allocate a joint cost between products.<sup>5</sup> The contribution of this paper is to show that this principle does not necessarily apply to the case of “over-lapping joint costs” that is naturally created when a firm engages in ongoing investment to produce output over multiple periods.<sup>6</sup> When there is a single joint cost that applies to every product, the only way that the firm can increase the output of any product is by increasing its investment in the single joint cost, and this results in increased output of all of the products. However, it will be shown that in the model of this paper where there are overlapping joint costs, it is possible to adjust the entire vector of planned investments to increase output in the current period while holding output in all other periods constant. It is this feature that creates the rather surprising result that a formula which can be interpreted as a cost allocation rule can be used to calculate the per-period cost of using long-lived assets.

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<sup>5</sup>See, for example, Demski(1981), Thomas(1978) and Young(1985).

<sup>6</sup> That is, if assets last T years, then assets purchased in period 0 are a joint cost of production for output in periods 1 through T, assets purchased in period 1 are a joint cost of production for output in periods 2 through T+1, etc.

The paper is organized as follows. Section I presents the model under the assumption there is a single central decision-maker. Section II presents the decomposition result which shows that a simple formula can be used to calculate a hypothetical perfectly competitive rental price for capital stock each period, and the solution to the true multi-period investment problem can be calculated as the solution to this series of time-separable hypothetical rental problems. Section III introduces notation to define allocation rules and shows that if accounting income is calculated using the RBRC allocation rule, that the fully optimal sequence of investments can be calculated by choosing a level of investment each period that maximizes that period's accounting income. Section IV considers the extension to the basic model in which it is assumed that shareholders delegate the investment decision to a better-informed manager and shows that robust incentives for the manager to choose the fully optimal sequence of investments are created if accounting income calculated based on the RBRC allocation rule is used as a performance measure for the manager. Finally, section V draws a brief conclusion.

## I. THE BASIC MODEL

### A. Costs and Production

Suppose that there are an infinite number of periods indexed by  $t$  where period 0 is the current period. The firm's decision problem is to choose a level of investment for every period beginning with period 0. Let  $x_t \in [0, \infty)$  denote the number of units of assets that the firm decides to purchase in period  $t$  and let  $\mathbf{x} = (x_0, x_1, \dots)$  denote the entire vector of investment decisions that the firm must make. The possibility that the firm has been in business in periods prior to period 0 and purchased assets in those periods which are still useful will be allowed for. For

negative values of  $t$ , let  $x_t^L \in [0, \infty)$  denote the number of assets that the firm purchased in period  $t$ . These assets will be referred to as legacy assets.

It will be assumed that assets purchased by the firm contribute to a stock of capital and that the capacity of the firm to produce output in any period is determined by its current stock of capital. Let  $z_t \in [0, \infty)$  denote the stock of capital in period  $t$  and let  $\mathbf{z} = (z_1, \dots)$  denote the entire vector of capital stocks from period 1 onwards.<sup>7</sup> Assumption 1 will now describe how the capital stock in each period is determined by asset purchases.

**Assumption 1:**

Suppose that the firm purchases assets according to  $\mathbf{x}$ . Then for  $t \in \{1, 2, \dots\}$  the firm's capital stock in period  $t$  is determined by

$$(1) \quad z_t = \sum_{i=1}^T x_{t-i} \beta^{i-1}$$

where  $T$  is a non-negative integer,  $\beta \in (0, 1]$  and  $x_j = x_j^L \in [0, \infty)$  for  $j \leq -1$ .

Assumption 1 can be interpreted as follows. Each asset is assumed to have a lifetime of  $T$  periods in the sense that an asset purchased in any period  $t$  contributes to the capital stock in periods  $\{t+1, t+2, \dots, t+T\}$ . One unit of the asset creates one unit of capital stock in the first

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<sup>7</sup>As will be seen, it will be assumed that assets purchased in a given period enter the capital stock beginning in the next period. Therefore, for purposes of calculating the vector of optimal asset purchases beginning with period 0, it is necessary to keep track of the level of capital stock beginning with period 1. All of the results of this paper generalize in a straightforward fashion for other lag lengths between the period in which an investment occurs and the period in which it enters the capital stock.

period of its life and the contribution weakly decays over time at a rate determined by  $\beta \in (0, 1]$ . If  $\beta$  is equal to 1, each asset remains equally useful over its entire lifetime. Smaller values of  $\beta$  correspond to higher rates of productivity decay.

It will be useful to distinguish between the capital stock that the firm would have available for use in any period due to legacy assets, which will be called the legacy capital stock, and the capital stock that the firm creates through additional asset purchases from period 0 onwards, which will be called the incremental capital stock. The level of legacy capital stock available in period  $t$  will be denoted by  $z_t^L$  and is a constant given by

$$(2) \quad z_t^L = \sum_{i=1}^T x_{t-i}^L \beta^{i-1}$$

where

$$(3) \quad x_j^L = 0 \text{ for every } j \geq 0.$$

Note for future reference that the legacy capital stock weakly decreases over time, i.e.,  $z_t^L$  is weakly decreasing in  $t$ . Let  $\psi_t(x_0, \dots, x_{t-1})$  denote the function giving the incremental capital stock available in any period  $t$  conditional on the firm's investment choices.

$$(4) \quad \psi_t(x_0, \dots, x_{t-1}) = \sum_{i=1}^T x_{t-i} \beta^{i-1}$$

where

$$(5) \quad x_j = 0 \text{ for every } j \leq -1.$$

Let  $q_t \in [0, \infty)$  denote the number of units of output that the firm produces in period  $t$  and let  $\mathbf{q} = (q_1, q_2, \dots)$  denote the vector of output levels from period 1 onwards. Let  $f(\mathbf{q})$  denote

minimum level of capital stock that the firm must have in any period to produce  $q$  units of output in that period. For simplicity, it will be assumed that there are no other inputs besides capital stock, so that a firm with a given level of capacity can produce any level of output up to its capacity at no incremental cost.<sup>8</sup> Assume that  $f(q)$  satisfies the following conditions.<sup>9</sup>

**Assumption 2:**

- (i)  $f(0) = 0$
- (ii)  $f(q)$  is twice differentiable, strictly increasing, and weakly convex for every  $q \in [0, \infty)$

Let  $a_t$  denote the purchase price of a new unit of the asset in period  $t$ . Assumption 3 will describe how asset prices are determined.

**Assumption 3:**

The purchase price of a new unit of the asset in period  $t$  is given by

$$(6) \quad a_t = \alpha^t a_0$$

where  $a_0 \in (0, \infty)$  and  $\alpha \in (0, 1]$ .

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<sup>8</sup>All of the results of this paper generalize in a completely straightforward fashion if it is assumed that there is an additional variable cost that the firm must incur when it decides to use its capacity to produce output.

<sup>9</sup>Later in the paper, when the results are proven, it will be straightforward to observe that, with the addition of one more minor assumption, the results of this paper continue to hold true when Assumption 2 is generalized to allow for the possibility that there is some fixed positive level of capital stock necessary to produce any positive level of output. The required assumption is simply that the fixed cost is not so high that the firm exits the industry.

According to Assumption 3, the price of a new unit of the asset in period 0 is a known positive number and the price either stays constant over time or declines at a constant percentage rate. Thus the possibility that technological progress is causing the price of the asset to decline over time is allowed for.<sup>10</sup> When  $\alpha$  is equal to 1, the price of new assets remains constant over time. Lower values of  $\alpha$  correspond to higher rates of price declines.

Finally, assume that the firm discounts future cash flows at some rate  $\delta \in (0,1)$ .

**Assumption 4:**

The firm discounts future cash flows at a discount rate  $\delta \in (0, 1)$ .

It will turn out to be convenient to formally view the firm as directly choosing the vector of outputs  $\mathbf{q}$  instead of as directly choosing the vector of asset purchases  $\mathbf{x}$ .<sup>11</sup> To do this, it will be necessary to describe how the firm chooses a vector of asset purchases to minimize the discounted cost of producing any vector of outputs  $\mathbf{q}$ . Since the firm discounts future cash flows and since asset prices are weakly falling over time, it is clear that the optimal policy is to simply purchase new assets as they are needed on a period-by-period basis and to never purchase new assets before they are needed. That is, beginning with period 0, the firm considers each period sequentially and plans to purchase the minimum number of assets necessary to produce the

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<sup>10</sup>The assumption that asset prices are not strictly increasing over time plays a role in generating the results of this paper. The role this assumption plays will be discussed when the results are presented.

<sup>11</sup>Since the choice of a vector of outputs and a vector of investments is formally equivalent, the firm's decision will be interchangeably referred to in the text as an output decision or an investment decision.

desired amount of output next period given its existing assets. If sufficient capital stock will be available without any new investment, then the firm purchases no assets. Therefore the number of assets that the firm purchases in period  $t$  will depend on output levels up to and including period  $t+1$ . Let  $\varphi_t(q_1, \dots, q_{t+1})$  denote the function giving the number of units of assets the firm purchases in period  $t$  to produce the output vector  $(q_1, \dots, q_{t+1})$ .<sup>12</sup>

Let  $C(\mathbf{q})$  denote the present discounted cost of producing the vector of outputs  $\mathbf{q}$ . It is equal to the discounted cost of purchasing the required assets.

$$(7) \quad C(\mathbf{q}) = \sum_{t=0}^{\infty} \delta^t \varphi_t(q_1, \dots, q_{t+1}) a_t.$$

In the following analysis, whether or not a vector of outputs satisfies the property the firm fully utilizes all of the new investment required to produce it will turn out to be important. The remainder of this subsection will formally define this property and then identify a sufficient condition for it to hold. Consider a vector of outputs  $\mathbf{q}$ . Let  $g_t(q_t)$  denote the incremental amount of capital stock that is required in period  $t$  in order to produce  $q_t$ ,

$$(8) \quad g_t(q_t) = \text{Max} \{ f(q_t) - z_t^L, 0 \},$$

and let  $\xi_t(q_1, \dots, q_t)$  denote the incremental amount of capital stock that is actually available for use in period  $t$  given that the firm invests efficiently in order to produce  $(q_1, \dots, q_t)$ ,

$$(9) \quad \xi_t(q_1, \dots, q_t) = \psi_t(\varphi_0(q_1), \dots, \varphi_{t-1}(q_1, \dots, q_t)).$$

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<sup>12</sup>The manner in which the  $\varphi_t$  functions are constructed is formally described by equations (A.1)-(A.2) in the appendix.



A vector of outputs  $\mathbf{q}$  will be said to satisfy the fully utilized new investment (FUNI) property if, in every period, the firm uses all of the incremental capital stock that it creates in order to produce  $\mathbf{q}$ . Formally,  $\mathbf{q}$  will be said to satisfy FUNI if

$$(10) \quad \xi_t(q_1, \dots, q_t) = g_t(q_t) \quad \text{for every } t \in \{1, 2, \dots\}.$$

For any vector of outputs  $\mathbf{q}$  let  $\tau(\mathbf{q})$  denote the first period in which the firm must purchase new assets in order to produce  $\mathbf{q}$ .

$$(11) \quad \tau(\mathbf{q}) = \min \{t: f(q_t) > z_t^L\}.$$

A vector of outputs  $\mathbf{q}$  will be said to satisfy the monotonicity above legacy levels (MALL) property if output is weakly increasing from period  $\tau(\mathbf{q})$  onwards. Formally,  $\mathbf{q}$  satisfies MALL if

$$(12) \quad t \geq \tau(\mathbf{q}) \Rightarrow q_t \leq q_{t+1}.$$

It is straightforward to show that a sufficient condition for  $\mathbf{q}$  to satisfy FUNI is that it satisfies MALL. This result is very intuitive. If  $\mathbf{q}$  satisfies MALL, this means that once the firm begins to invest in new capital stock, that its future requirements for capital will always be at least as large as its current requirements. Therefore, whenever the firm purchases a new asset, there will continue to be a need for the asset in future periods of the asset's life.

**Lemma 1:**

If  $\mathbf{q}$  satisfies MALL then it satisfies FUNI.

**Proof:**

See Appendix.

QED

**B. Demand**

Assume that demand in period  $t$  depends only on the price charged in period  $t$ . Let  $P_t(q_t)$  denote the inverse demand curve in period  $t$  and let  $R_t(q_t) = P_t(q_t)q_t$  denote the period  $t$  revenue function. Let  $R(\mathbf{q})$  denote the present discounted value of revenue for the vector of outputs  $\mathbf{q}$ .

$$(13) \quad R(\mathbf{q}) = \sum_{t=1}^{\infty} R_t(\mathbf{q}) \delta^t$$

Assumption 5 states the properties that the revenue functions will be assumed to satisfy.

**Assumption 5:**

For every  $t \in \{1, 2, \dots\}$

- (i)  $R_t(q_t)$  is a twice differentiable function defined over  $[0, \infty)$ .
- (ii)  $R_t(0) = 0$
- (iii)  $R_t'(q_t)$  is strictly decreasing and is negative for large enough values of  $q_t$
- (iv) For every  $t \in \{1, 2, \dots\}$  and every  $q \in [0, \infty)$

$$(14) \quad R_t'(q) \leq R_{t+1}'(q)$$

Conditions (i)-(iii) are normal regularity conditions. However condition (iv) has more

economic content. It states that marginal revenue is weakly increasing over time. This is a condition that we would normally expect to see satisfied if demand for a product was growing over time but that we would not expect to see satisfied if demand was declining over time. The role that this assumption plays in generating the results of this paper will be discussed below when the results are presented.

### **C. The True Profit Maximization Problem**

The firm's profit maximization problem is to choose a vector of outputs,  $\mathbf{q}$ , to maximize the present discounted value of its cash flows which is given by

$$(15) \quad R(\mathbf{q}) - C(\mathbf{q})$$

This will sometimes be referred to as the "true" profit maximization problem to distinguish it from a purely hypothetical profit maximization problem that will be described below.

## **II. THE HYPOTHETICAL PROFIT MAXIMIZATION PROBLEM**

The firm's decision-making problem would be considerably simpler if it was able to rent capital stock on a period-by-period basis, instead of having to purchase assets, since the multi-period investment problem would then decompose into a series of simple single-period problems. Of course, in the model of this paper it is assumed that no such rental market for assets exists. This assumption is satisfied in many real economic situations because it can be very expensive to install or move assets and/or because the assets used by individual firms must be specialized in ways that make the assets less useful for other firms. This paper will show, that, it is still

possible to simplify the firm's decision-making process by calculating the hypothetical perfectly competitive rental prices that would exist in the absence of these problems, even though, in reality, no such rental markets exists. In particular, it will be shown that a very simple formula exists to calculate hypothetical perfectly competitive rental prices for capital stock. Then it will be shown that the vector of capital stocks that would be profit maximizing for the firm to choose if it was able to rent capital at these hypothetical prices turns out to be the solution to the true profit maximization problem that the firm actually faces (given that it must purchase assets), so long as the solution to the former problem satisfies the FUNI property. The proof is completed by showing that sufficient conditions have been assumed to guarantee that the solution to the hypothetical profit maximization problem does, in fact, satisfy the FUNI property.

### A. The Hypothetical Perfectly Competitive Rental Rates

Let  $w_t$  denote the price of renting one unit of capital stock in period  $t$  and let  $\mathbf{w} = (w_1, w_2, \dots)$  denote an entire vector of rental prices. Suppose that a hypothetical supplier of rental services can enter the market in any period by purchasing one unit of the asset and then renting out the available capital stock over the asset's life. Then under the assumptions that suppliers incur no extra costs besides the cost of purchasing the asset, they can rent the asset to a customer every period, and they have a discount rate equal to  $\delta$ , the zero profit condition that must be satisfied by a perfectly competitive equilibrium is

$$(16) \quad a_t = \sum_{i=1}^T w_{t+i} \beta^{i-1} \delta^i \quad \text{for every } t \in \{0, 1, 2, \dots\}$$

Let  $w_t^*$  denote the period  $t$  rental price given by

$$(17) \quad w_t^* = (k^*/\alpha) a_t$$

where the constant  $k^*$  is defined by

$$(18) \quad k^* = \{1 - \beta\alpha\delta\} / \{\delta [1 - (\beta\alpha\delta)^T]\}$$

and let  $\mathbf{w}^* = (w_1^*, \dots)$  denote the entire vector of these values for rental prices. It is straightforward to verify that these rental prices satisfy the zero profit condition (16).

**Lemma 2:**

The vector of rental prices  $\mathbf{w}^*$  satisfies

$$(19) \quad a_t = \sum_{i=1}^T w_{t+i}^* \beta^{i-1} \delta^i \quad \text{for every } t \in \{0, 1, 2, \dots\}$$

Proof:

Straightforward algebra.

QED

The vector  $\mathbf{w}^*$  will therefore be called the vector of perfectly competitive rental prices.

**B. The Hypothetical Profit Maximization Problem**

The hypothetical profit maximization problem is defined to be the problem of choosing a vector of outputs  $\mathbf{q}$  to maximize the present discounted value of cash flows under the assumption

that the firm can use its legacy capital stock in each period at no incremental cost, and that it can augment its capital stock in each period by renting capital stock at the perfectly competitive rental rate. Let  $H_t(q_t)$  denote the hypothetical cost of producing  $q_t$  units of output in period  $t$ .

$$(20) \quad H_t(q_t) = \begin{cases} 0, & f(q_t) \leq z_t^L \\ w_t^*[f(q_t) - z_t^L], & f(q_t) \geq z_t^L \end{cases}$$

Then let  $H(\mathbf{q})$  denote the total discounted hypothetical cost of producing the vector of outputs  $\mathbf{q}$ .

This will often be referred to as the hypothetical cost function. It is defined by

$$(21) \quad H(\mathbf{q}) = \sum_{t=1}^{\infty} \delta^t H_t(q_t).$$

The hypothetical profit maximization problem is then to choose  $\mathbf{q}$  to maximize

$$(22) \quad R(\mathbf{q}) - H(\mathbf{q}).$$

Substitution of (13) and (21) into (22) yields

$$(23) \quad \sum_{t=1}^{\infty} [R_t(q_t) - w_t^* H_t(q_t)] \delta^t$$

Note that  $q_t$  appears only in the  $t^{\text{th}}$  term of equation (23). Therefore the solution to the hypothetical problem decomposes into a series of single period problems. Namely,  $q_t$  is chosen to maximize

$$(24) \quad R_t(q_t) - w_t^* H_t(q_t).$$

To describe the solution to this problem it will be useful to introduce some additional notation. Consider the simplified period  $t$  rental problem that the firm would face if it entered period  $t$  with no legacy capital stock. The firm's profit in period  $t$  would then be given by

$$(25) \quad R_t(q_t) - w_t^* f(q_t).$$

Sufficient assumptions have been made to guarantee that there is a unique value of  $q_t$  that maximizes (25) over  $q_t \in [0, \infty)$ . Let  $q_t^{\min}$  denote this level of output. Let  $q_t^{\max}$  denote the level of output at which period  $t$  marginal revenue becomes zero. Now define the level of output  $q_t^*$  by

$$(26) \quad q_t^* = \begin{cases} q_t^{\min}, & z_t^L \leq f(q_t^{\min}) \\ f(z_t^L), & f(q_t^{\min}) \leq z_t^L \leq f(q_t^{\max}) \\ q_t^{\max}, & z_t^L \geq f(q_t^{\max}) \end{cases}$$

and let  $\mathbf{q}^* = (q_1^*, q_2^*, \dots)$  denote the entire vector of these output levels. It is straightforward to see that  $q_t^*$  is the unique level of output that maximizes (24). That is, if the firm's legacy capital stock is smaller than the level of capital stock required to produce  $q_t^{\min}$ , then the firm rents sufficient capital to produce  $q_t^{\min}$ . If the legacy level of capital stock is greater than this amount but less than the level of capital stock required to produce  $q_t^{\max}$ , the firm uses all of its legacy capital stock to produce output but does not rent any new capital stock. Finally, if the legacy of capital stock is so large that the firm could produce more output than  $q_t^{\max}$ , the firm only uses enough of its legacy capital stock to produce  $q_t^{\max}$ .

**Lemma 3:**

The vector of outputs  $\mathbf{q}^*$  is the unique solution to the hypothetical profit maximization problem.

Proof:

Straightforward.

QED

**C. Comparing the Hypothetical Cost Function and the True Cost Function**

Lemma 4 shows that the true cost function is equal to the hypothetical cost function over the set of vectors of outputs satisfying FUNI and that the true cost is strictly greater than the hypothetical cost for all other vectors of outputs.

**Lemma 4:**

(27)  $C(\mathbf{q}) = H(\mathbf{q})$  if and only if  $\mathbf{q}$  satisfies FUNI

(28)  $C(\mathbf{q}) > H(\mathbf{q})$  for every  $\mathbf{q}$  which does not satisfy FUNI

**proof:**

See Appendix.

QED

The intuition for Lemma 4 is as follows. The rental prices are constructed to have the property that the cost of purchasing an asset is exactly equal to the cost of renting the same asset if the asset is rented for every period of the asset's life. By definition, when  $\mathbf{q}$  satisfies FUNI, the rental services the firm would choose in the hypothetical case where it can rent assets could be produced by purchasing assets and fully utilizing them. Therefore the cost should be the same. When  $\mathbf{q}$  does not satisfy FUNI, the firm would take advantage of a rental market to use fewer



assets in some periods than would be possible in the case where it had to purchase its own assets. This means that the hypothetical cost is then lower than the true cost. Intuitively, the fact that a firm must purchase its own assets and cannot resell them places limits on the extent to which it can reduce its costs by reducing asset usage from period to period. However, when FUNI is satisfied, this means that the constraint is not binding, and the true cost of production is therefore equal to the hypothetical cost of production.

#### **D. Directly Calculating the Marginal Cost of Increasing Capital Stock in a Single Period**

The result of Lemma 4 that the true cost function is separable over a broad range of outputs might seem somewhat surprising in light of the fact that each asset represents a joint cost of producing output across multiple periods. A widely accepted general principle in both the economics and managerial accounting literatures that study cost allocation is that, there is generally no economically meaningful way to assign a joint cost to individual products.<sup>13</sup> Thus we might expect that the cost of producing a vector of joint products would not be additively separable in each product. The resolution to this apparent conflict lies in the fact that there are “multiple overlapping” joint costs in the model of this paper instead of a single joint cost. That is, assets purchased in period 0 are a joint cost of production for output in periods  $\{1, 2, \dots, T\}$ , assets purchased in period 1 are a joint cost of production for output in periods  $\{2, 3, \dots, T+1\}$ , etc. When there is a single joint cost for all products, the only way to increase the output of a single product is to increase investment in the joint cost and this results in increased output of all products. Thus increasing the production of one good necessarily results in increases

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<sup>13</sup>See, for example, Demski(1981), Thomas(1978), and Young(1985).

in the production of all goods. However, in the model of this paper, where there are multiple overlapping joint costs, this is not necessarily true.

An illuminating way to see this point is to directly calculate the marginal cost of increasing the capital stock in a given period while holding the capital stock in all other periods fixed by directly determining the adjustments in asset purchases that are necessary to do this. The present discounted value of these adjustments is, by definition the marginal cost of providing a unit of capital stock in the current period. It will be seen that the marginal cost of increasing capital usage in period  $t$  is equal to the hypothetical perfectly competitive rental rate  $w_t^*$ .

Suppose that the firm plans to purchase at least one unit of the asset every period.<sup>14</sup> Also, for simplicity, assume that the productivity of existing assets does not decay during their lifetime so that  $\beta = 1$ .<sup>15</sup> Then the firm can increase its capital stock in period 1 by one unit while holding the capital stock in all other periods constant by implementing the following series of adjustments to its investment plans. The firm must purchase an additional unit of the asset in period 0 to increase the capital stock by one unit in period 1. However, it will now be able to reduce its asset purchases by one unit in period 1. Now when period  $T$  arrives, the extra asset that the firm purchased in period 0 will no longer be available in the next period, so the firm will have to purchase an extra unit of the asset in period  $T$  to maintain its level of capital stock at the

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<sup>14</sup>The assumption that the firm is purchasing at least one unit of the asset every period will be made use of to calculate the cost of increasing the capital stock by one unit. Correspondingly smaller increases in the capital stock require correspondingly smaller levels of investment every period. The assumption that there is a positive level of investment every period implies that FUNI is satisfied and is actually slightly stronger than the assumption that FUNI is satisfied.

<sup>15</sup>A slightly more complicated version of the same calculation can be done for any value of  $\beta$ .

previously planned level. However, as before, it will now be able to reduce its asset purchases by one unit in period  $T+1$ . This process continues indefinitely. That is, the firm can increase its capital stock in period 1 by exactly one unit and hold the capital stock in all other periods constant by shifting the purchase of one unit of the asset forward in time from period 1 to 0,  $T+1$  to  $T$ ,  $2T+1$  to  $2T$ , etc. The present discounted value of the cost of these adjustments calculated in period 1 dollars is, by definition, the marginal cost of increasing the capital stock by one unit in period 1. It is straightforward to directly calculate this value and show that it is  $(k^*/\alpha) a_1$ , which is  $w_1^*$ , the hypothetical perfectly competitive rental rate in period 1. A similar calculation obviously applies for any period  $t$ .

#### **E. Sufficient Conditions for the Solution to the Hypothetical Problem to Also be the Solution to the True Problem**

Proposition 1, stated below, follows immediately from Lemma 4.

##### **Proposition 1:**

- (i) Suppose that  $\mathbf{q}$  is a solution to the hypothetical profit maximization problem. Then if  $\mathbf{q}$  satisfies FUNI, it is also a solution to the true profit maximization problem.
- (ii) If a unique solution exists to the hypothetical profit maximization problem and satisfies FUNI, then it is also the unique solution to the true profit maximization problem.

##### **proof:**

See Appendix

QED

Lemma 3 has already established that  $\mathbf{q}^*$  defined by equation (26) is the unique solution to the hypothetical profit maximization problem. Lemma 5 shows that  $\mathbf{q}^*$  satisfies MALL.

**Lemma 5:**

The vector of outputs,  $\mathbf{q}^*$  satisfies the MALL property.

**Proof:**

See appendix.

QED

Recall that Lemma 1 has already established that if  $\mathbf{q}$  satisfies the MALL property, then it also satisfies the FUNI property. Therefore it follows immediately from Propositions 1 and Lemmas 1, 3, and 5 that the solution to the hypothetical profit maximization problem is also the unique solution to the true profit maximization problem. This result is stated as Proposition 2.

**Proposition 2:**

The vector of outputs  $\mathbf{q}^*$  is the unique solution to the true profit maximization problem.

Proof:

As above.

QED

In summary, the proof that  $\mathbf{q}^*$  is the solution to the true profit maximization problem occurs in two major steps. The first step is to show that if the solution to the hypothetical profit maximization problem satisfies FUNI, that it is also the solution to the true profit maximization problem. The second step is to identify sufficient conditions for the solution to the hypothetical

problem to satisfy FUNI. Since Lemma 1 shows that a sufficient condition for a vector of outputs to satisfy FUNI is that it satisfy MALL, the second step reduces to the problem of finding sufficient conditions for the solution to the hypothetical problem to satisfy MALL. Recall that the MALL property is a slightly weakened version of the property that output is weakly increasing over time. Specifically, if the firm begins period 0 with enough legacy capacity that it engages in no new investment for some number of initial periods, then the MALL property does not require that output be weakly increasing over these time periods. Rather, it requires that once the firm reaches a period where it engages in new investment, that output be weakly increasing from that point on. Assumption 5-(iv) (that demand is weakly increasing over time in the sense that marginal cost is weakly increasing over time) and Assumption 3 (that the purchase price of new assets is weakly decreasing over time which in turn implies that the hypothetical rental prices are weakly decreasing over time) are sufficient to guarantee this. In a sense, the basic economic requirement for the investment problem to decompose into a series of separable single period problem is that the market be one in which the firm is planning on staying the same size or growing larger as measured by its capacity. While this requirement will not always be satisfied, there are obviously many situations in which it will be satisfied.

## **F. Discussion**

The above results show that the optimal number of additional assets for the firm to purchase in period 0 is equal to the number of additional assets it would be optimal to rent in period 1 at the rental price  $w_1^*$ . Substitution of  $a_1 = \alpha a_0$  into (17) shows that  $w_1^*$  is equal to  $k^* a_0$ . Therefore in order to calculate the optimal investment level in the current period, the firm only

needs to calculate the number of assets that it would be profit maximizing to rent next period if the rental rate was equal to the constant  $k^*$  multiplied by the current purchase of assets,  $a_0$ .

In addition to being very simple, this problem also has low informational requirements about the nature of future demand in the sense that the precise functional forms of future demand curves beyond period 1 do not need to be known. That is, so long as the firm knows that future demand will exhibit weakly increasing marginal revenue over time, there is no need for the firm to know the precise functional forms of the future demand curves beyond the immediate next period in order to calculate the fully optimal investment decision in the current period. Of course the optimal level of investment in the current period depends on the rate at which future asset prices will change,  $\alpha$ , the rate at which the productivity of existing assets decays,  $\beta$ , and the length of the asset lifetime,  $T$ , because  $k^*$  depends on all of these factors. The result of this section, is that the effects of all of these factors can be captured by analyzing a simple single period problem directly involving only the next period demand curve where the effects of all of these other factors are captured by the parameter  $k^*$ .

### **III. USING COST ALLOCATION RULES TO CALCULATE OPTIMAL INVESTMENT LEVELS**

The previous section has shown that the optimal number of assets for the firm to purchase in the current period (period 0) is equal to the number of additional assets that it would be optimal to rent in the next period (period 1) if additional assets could be rented at a rental rate equal to the constant  $k^*$  multiplied by the current purchase price of assets,  $a_0$ . As will be explained below, any cost allocation rule for assets can be characterized as a vector of real numbers  $\mathbf{s} = (s_1, \dots, s_T)$  where  $s_1$  is interpreted as the share of the original purchase price of the

asset allocated to the  $i^{\text{th}}$  period of the asset's life. It therefore follows immediately that if accounting income is calculated using an allocation rule such that  $s_1 = k^*$ , a level of investment in period 0 will be optimal if and only if it maximizes period 1 accounting income. This section will introduce definitions and notation to formally describe allocation rules and formally demonstrate this point. It will also identify one particularly natural allocation rule that has the property that  $s_1 = k^*$ . This will be called the relative benefit and replacement cost (RBRC) allocation rule.

Define a depreciation rule to be a vector  $\mathbf{d} = (d_1, d_1, \dots, d_T)$  such that  $d_i \geq 0$  for every  $i$  and

$$(29) \quad \sum_{i=1}^T d_i = 1$$

where  $d_i$  is interpreted as the share of depreciation allocated to the  $i^{\text{th}}$  period of the asset's life. Let  $D$  denote the set of all depreciation rules. Define an allocation rule to be a vector  $\mathbf{s} = (s_1, s_2, \dots, s_T)$  that satisfies  $s_i \geq 0$  where  $s_i$  is interpreted as the share of the asset's purchase cost that is allocated to the  $i^{\text{th}}$  period of the asset's life. Let  $S$  denote the set of all allocation rules. For any  $\gamma \in [0, \infty)$  an allocation rule will be said to be "complete given  $\gamma$ " if the discounted sum of the allocation shares using  $\gamma$  as a discount rate is equal to 1, i.e., if

$$(30) \quad \sum_{i=1}^T s_i \gamma^i = 1$$

It is clear that for any  $\mathbf{s} \in S$  there exists a unique value of  $\gamma \in [0, \infty)$  such that  $\mathbf{s}$  is complete given  $\gamma$ . Let  $\Gamma(\mathbf{s})$  denote the unique value of  $\gamma$  that  $\mathbf{s}$  is complete with respect to.

Firms generally think of themselves as directly choosing a depreciation rule and a discount rate instead of as directly choosing an allocation rule. The cost allocated to each period is then calculated as the sum of the depreciation allocated to that period plus imputed interest on the remaining (non-depreciated) book value of the asset. Formally, for any depreciation rule,  $\mathbf{d}$ , and discount rate  $\gamma$  the corresponding allocation rule is given by

$$(31) \quad s_i = d_i + \{(1-\gamma)/\gamma\} \sum_{j=i}^T d_j$$

It is straightforward to verify that the resulting allocation rule determined by (31) is complete given  $\gamma$ . It is also straightforward to verify that for any  $\mathbf{s} \in S$ , there is a unique  $(\mathbf{d}, \gamma) \in D_x[0, \infty)$  such that (31) maps  $(\mathbf{d}, \gamma)$  into  $\mathbf{s}$ . It is defined by  $\gamma = \Gamma(\mathbf{s})$  and

$$(32) \quad d_i = \sum_{j=i}^T \gamma^{j-i} s_j - \sum_{j=i+1}^T \gamma^{j-i-1} s_j$$

Therefore one can equivalently think of the firm as choosing either a depreciation rule and a discount rate or as choosing an allocation rule. For the purposes of this paper, it will be more convenient to view the firm as directly choosing an allocation rule.<sup>16</sup>

It will be useful to define one particular allocation rule, denoted by  $\mathbf{s}^* = (s_1^*, \dots, s_T^*)$ , which will be called the relative benefit and replacement cost (RBRC) allocation rule.

$$(33) \quad s_i^* = (\alpha\beta)^{i-1} k^*.$$

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<sup>16</sup>See Rogerson (1992) for a fuller discussion of the relationship between depreciation and allocation rules and their properties.



The name “relative benefit and replacement cost” stems from the following interpretation. Since the contribution of an existing asset to the firm’s capital stock declines over the life of the asset at a rate determined by the parameter  $\beta$ , an allocation rule  $\mathbf{s}$  can be interpreted as allocating the cost of an asset over its lifetime in proportion to the relative benefits it creates if the allocation rule satisfies  $s_{i+1} = \beta s_i$ . Similarly, since the cost of purchasing a new replacement asset declines over time at a rate determined by the parameter  $\alpha$ , an allocation rule  $\mathbf{s}$  can be interpreted as allocating the cost of an asset over its lifetime in proportion to the relative cost of replacing the asset if it satisfies  $s_{i+1} = \alpha s_i$ . Therefore, a cost allocation rule can be interpreted as reflecting both the relative benefits that the asset produces and the relative cost of replacing the asset if it satisfies

$$(34) \quad s_{i+1} = \alpha\beta s_i.$$

It is straightforward to show the RCB allocation rule satisfies (34) and is complete with respect to  $\delta$  and, furthermore, that it is the unique allocation rule that satisfies these two properties.

Let  $A_t(q_1, \dots, q_t, \mathbf{s})$  denote the total cost allocated to period  $t$  if the firm produces the output vector  $(q_1, \dots, q_t)$  and uses the allocation rule  $\mathbf{s}$ . This will be called the period  $t$  accounting cost and is given by

$$(35) \quad A_t(q_0, \dots, q_t, \mathbf{s}) = \sum_{i=1}^T x_{t-i} a_{t-i} s_i$$

where

$$(36) \quad x_j = \begin{cases} \varphi_j(q_1, \dots, q_{j+1}), & j \geq 0 \\ x_j^L, & j \leq -1. \end{cases}$$

Equations (35)-(36) can be explained as follows. For any period  $t$ , part of the cost of purchasing

assets in periods  $t-i$  for  $i \in \{1, 2, \dots, T\}$  is allocated to period  $t$ . The total cost of purchasing assets in period  $t-i$  is equal to  $x_{t-i} a_{t-i}$  and the share  $s_i$  of this cost is allocated to period  $t$ . The total cost allocated to period  $t$  is the sum over all values of  $i \in \{1, 2, \dots, T\}$ .

Let  $I_t(q_1, \dots, q_t, \mathbf{s})$  denote the total accounting income in period  $t$  if the firm produces the output vector  $(q_1, \dots, q_t)$  and the allocation rule  $\mathbf{s}$  is used to calculate accounting cost. It is given by

$$(37) \quad I_t(q_1, \dots, q_t, \mathbf{s}) = R_t(q_t) - A_t(q_0, \dots, q_t, \mathbf{s})$$

Proposition 3 now states the desired result.

**Proposition 3:**

Suppose that  $\mathbf{s}$  is an allocation rule such that the first period allocation share,  $s_1$ , is equal to  $k^*$ . Then the fully optimal period 1 output level,  $q_1^*$ , is also the unique value of output that maximizes period 1 accounting income calculated using  $\mathbf{s}$ .

**Proof:**

See Appendix.

QED

From the definition of the RBRC allocation rule in equation (33), it is clear that  $s_1^*$  is equal to  $k^*$ . Therefore, the fully optimal output level is the unique value of output that maximizes period 1 accounting income calculated using the RBRC allocation rule.

Since the requirement that  $s_1$  be equal to  $k^*$  only determines the first period allocation share, there are of course a continuum of other allocation rules that also satisfy this requirement. However, the RBRC allocation rule is a particularly natural and simple rule meeting this

requirement and it is not clear that other equally simple or natural rules satisfying this requirement can be identified. Furthermore, as will be shown in the next section, the RBRC allocation rule is actually the unique allocation rule satisfying a more stringent set of conditions that create additional desirable properties.

#### **IV. MANAGERIAL INVESTMENT INCENTIVES**

The above results are derived in a model where it is assumed that there is a single centralized decision-maker who owns the firm. Therefore the role of the cost allocation rule is that it simplifies the calculation of the optimal investment level. This section will consider a situation where shareholders delegate the investment decision to the manager of the firm and show that there is a sense in which using accounting income based on the RBRC allocation rule as a performance measure can create robust incentives for the manager to choose the profit maximizing investment level. Specifically, it will be assumed that shareholders create a compensation scheme for the manager by choosing an allocation rule to calculate accounting income, and then base the manager's wage each period on current and possibly historic values of accounting income. It will be shown that, if the RBRC allocation rule is used to calculate accounting income, that so long as the wage function each period is weakly increasing in current and past periods' accounting income, that the manager will have the incentive to choose the profit maximizing investment plan regardless of the manager's own personal rate of time preference or any other aspect of the manager's own preferences. Therefore, by restricting themselves to using a compensation scheme where the RBRC allocation rule is used to calculate accounting income and the manager's wage each period is determined by some weakly increasing

function of current and past periods accounting incomes, shareholders can guarantee in a robust way that the manager will choose the profit maximizing investment level. Furthermore, shareholders are left with considerable degrees of freedom to address other managerial incentive issues by choosing the particular form of the wage function each period. For example, if there was a moral hazard problem within each period so that the manager must be given incentives to exert effort each period, the wage function could be tailored to address this problem while use of the RBRC allocation rule simultaneously guaranteed that the manager chose the profit maximizing investment level.

One remark should be noted about this result. The result itself does not establish that a compensation scheme of this sort (i.e., a compensation scheme created by using the RBRC allocation rule to calculate accounting income and making wage each period a weakly increasing function of current and past periods' accounting income) is the fully optimal solution to a completely specified multi-period principal agent problem. However, as will be discussed below, calculation of fully optimal contractual solutions to these sorts of multi-period principal agent models is generally difficult or impossible and often yields solutions that depend in extreme ways on aspects of the environment that the principal is unlikely to have precise knowledge about. In such situations, it seems likely that real firms must of necessity adopt simpler approaches to identifying contracts that appear to work well. The result of this paper can be interpreted as offering such an approach. This issue will be discussed further after the result is presented.

## A. The Basic Result

Assume that the production/demand environment is as described in the previous sections. Assume that shareholders know the parameters of model necessary to calculate the RBRC allocation rule, i.e., assume that they know the length of lifetime for assets,  $T$ , the extent to which the productivity of an asset decreases during its lifetime,  $\beta$ , and the rate at which the purchase price of new assets is dropping over time,  $\alpha$ . However, assume that they do not have complete information about other aspects of the environment that would be necessary to calculate the optimal decision. In particular, assume that they do not have complete information about the future demand curves,  $P_t(q_t)$ . Assume that the manager knows all of the functions and parameters in the model so that the manager is able to calculate the optimal investment/production plan.

Suppose that shareholders delegate the production decision to the manager in order to take advantage of his private information and that they create a compensation scheme for the manager by choosing an allocation rule and wage function. The allocation rule is used to calculate each period's accounting income. The wage function determines the wage the manager receives each period as a function of the accounting income in the current period and past periods. Assume that the manager has a utility function defined over vectors of wage payments over time. For any given wage function, one can therefore define an indirect utility function of the manager over vectors of accounting income. It will turn out to be useful to employ notation that suppresses the manager's utility function over wage payments and the wage function chosen by shareholders and instead focuses on the indirect utility function over income created by the composition of these two functions. Let  $y_t$  denote period  $t$  accounting income and let  $\mathbf{y} = (y_1, y_2, \dots)$  denote an entire vector of accounting incomes. Let  $U(\mathbf{y})$  denote the manager's indirect

utility function over vectors of accounting incomes that is created by the choice of a wage function and the manager's own direct utility function over wage payments.

The accounting income in period  $t$  given the allocation rule  $\mathbf{s}$  and the vector of outputs  $(q_1, q_2, \dots, q_t)$  has already been defined by the function  $I_t(q_1, \dots, q_t, \mathbf{s})$  in equation (37). Then if  $U(\mathbf{y})$  is the manager's indirect utility function over vectors of accounting income created by some wage function, the manager will choose the vector of outputs  $\mathbf{q}$  to maximize

$$(38) \quad U(I_1(q_1, \mathbf{s}), I_2(q_1, q_2, \mathbf{s}), \dots).$$

In general this would appear to be a very complex problem whose solution depends on the precise functional form of  $U$  which in turn depends on both the wage function and manager's own underlying preferences over vectors of wage payments, including the manager's own personal discount rate. Therefore, even if it was possible to identify a wage function and allocation rule that induced the manager to choose the efficient level of investment each period, it appears that it would be necessary to have detailed information about the manager's preferences, including his own personal discount rate, in order to perform such a calculation.

This section will show that almost all of this apparent complexity can be avoided by the appropriate choice of an allocation rule. In particular, most of the apparent complexity is created by the fact that, in general, the manager faces trade-offs between maximizing accounting income in different periods. That is, an investment plan that would maximize accounting income in any given period is unlikely to also maximize accounting income in all other periods. Therefore, selecting the optimal investment plan requires the manager to make complex tradeoffs between periods. The wage function and the manager's own underlying utility function over wage

payments both will have significant and potentially complex effects on how the manager trades off accounting income between periods. However, suppose that there was a vector of outputs that simultaneously maximized the accounting income in *every* period. Then, so long as the indirect utility function  $U$  was weakly increasing in all of its arguments, this vector of outputs would also obviously maximize the managers's utility regardless of the precise functional form of  $U$ . The fairly modest condition that each period's wage is weakly increasing in current and past periods' accounting income is clearly sufficient to guarantee that  $U$  is weakly increasing in each of its arguments.<sup>17</sup> Therefore if there was a vector of outputs that simultaneously maximized the accounting income in every period, this vector of outputs would maximize the manager's utility for the entire class of wage functions satisfying the modest condition that each period's wage is a weakly increasing function of current and past periods' accounting income. The result of this section is that the RBRC allocation rule has this property with respect to the profit maximizing vector of outputs  $\mathbf{q}^*$ .

Formally, an allocation rule  $\mathbf{s}$  will be said to create robust incentives for the manager to choose the output vector  $\mathbf{q}$  if  $\mathbf{q}$  maximizes each period's accounting income calculated using  $\mathbf{s}$ .

**Definition:**

An allocation rule  $\mathbf{s}' = (s_1', s_2', \dots, s_T')$  will be said to create robust incentives for the manager to choose the output vector  $\mathbf{q}' = (q_1', q_2', \dots)$  if

$$(39) \quad (q_1', q_2', \dots, q_t') \in \underset{(q_1, \dots, q_t)}{\operatorname{argmax}} I_t(q_1, \dots, q_t, \mathbf{s}') \quad \text{for every } t \in \{1, 2, \dots\}.$$

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<sup>17</sup>Of course, it must also be assumed that the agent's direct utility function over wages is weakly increasing in each period's wage.

As discussed above, if an allocation rule creates robust incentives for the manager to choose an output vector, this means that the manager will find it optimal to choose this output vector when his wage each period is a weakly increasing function of current and past periods' accounting income calculated using this allocation rule.

The main step in showing that the RBRC allocation rule creates robust incentives for the manager to choose the profit maximizing output vector,  $\mathbf{q}^*$ , is presented in Lemma 6. One additional piece of notation must be introduced to state this lemma. In any period  $t$ , the firm will own and operate units of capital that were originally purchased  $i$  periods earlier for every  $i \in \{1, 2, \dots, T\}$ . An allocation rule  $\mathbf{s}$  will determine a potentially different accounting cost for each vintage of capital. Let  $c(t, i, \mathbf{s})$  denote the accounting cost of a unit of capital in period  $t$  calculated using the allocation rule  $\mathbf{s}$ , given that it was purchased  $i$  periods earlier.

$$(40) \quad c(t, i, \mathbf{s}) = \beta^{1-i} \alpha^{-i} s_i a_t.$$

This formula can be explained as follows. The price of purchasing a unit of the asset in period  $t-i$  is equal to  $\alpha^{-i} a_t$ . In order to provide one unit of capital stock in period  $t$  the firm must purchase  $\beta^{1-i}$  units of the asset in period  $t-i$ . Therefore the total cost of purchasing sufficient assets in period  $t-i$  to create one unit of capital in period  $t$  is equal to  $\beta^{1-i} \alpha^{-i} a_t$ . The share  $s_i$  of this cost is allocated to period  $t$ .

Lemma 6 now observes that the RBRC allocation rule has the property that for any period  $t$ , the accounting cost of a unit of capital for any vintage of capital is the same and is equal to the hypothetical perfectly competitive rental rate for that period  $w_t^*$ . Furthermore the RBRC allocation rule is the only allocation rule satisfying this property.



**Lemma 6:**

For every  $t \in \{1, 2, \dots\}$  and  $i \in \{1, \dots, n\}$ , the RBRC allocation rule,  $\mathbf{s}^*$ , satisfies

$$(41) \quad c(t, i, \mathbf{s}^*) = w_t^*.$$

Furthermore, the RBRC allocation rule is the unique allocation rule that satisfies this property.

**Proof:**

See Appendix.

QED

Lemma 6 states that the RBRC allocation rule calculates cost by imputing a cost of  $w_t^*$  to every unit of capital that the firm owns in that period. However, this is precisely how cost is calculated in the hypothetical profit maximization problem. The result that  $\mathbf{q}^*$  maximizes each period's accounting income therefore follows directly.

**Proposition 4:**

The RBRC allocation rule,  $\mathbf{s}^*$ , creates robust incentives for the manager to choose the profit maximizing vector of outputs,  $\mathbf{q}^*$ .

**Proof:**

See Appendix.

QED

**B. Discussion**

As has already been mentioned, the above result does not show that a contract using the RBRC allocation rule to calculate accounting income is a fully optimal solution to a completely

specified principal agent problem. It is clear that such a result would be straightforward to prove in a model where it was assumed that the only incentive/information problem is that the manager is better informed than shareholders about some information necessary to calculate the fully optimal investment/production plan. However, there would be no need in such a model to base the manager's wage on any measure of the firm's performance. That is, one fully optimal contract would be for shareholders to simply pay the manager a constant wage each period sufficient to induce the manager to accept the job. Then the manager would be (weakly) willing to choose the profit maximizing investment/output plan.

Therefore, in reality, the result of this paper will only be useful in situations where there is some additional incentive problem which requires shareholders to base the manager's wage on some measure of the firm's performance. A natural candidate would be to assume that there is a moral hazard problem within each period, i.e., that each period the manager can exert unobservable effort which affects the firm's cash flow that period. The modeling problem this creates is that calculating optimal contracts that are fully optimal solutions to multi-period principal agent problems is exceedingly complex. Furthermore, an essential aspect of the situation being considered in this paper is that the manager must have better information than shareholders about some aspect of the environment in order to justify the assumption that shareholders delegate the investment/production decision to management in the first place. Adding the problem of incomplete information to a multi-period moral hazard problem increases the complexity and difficulty of the problem by another order of magnitude. It is generally impossible to solve such problems using existing techniques unless extreme simplifying assumptions are made about the nature of the incomplete information. Furthermore, even when

sufficient assumptions are made to allow calculation of a solution, it often appears that the nature of the solution depends in extreme ways on particular aspects of the environment such as particular aspects of the agent's preferences that the principal is unlikely to have reliable information about. Thus, it is not clear that such contracts would be suitable for use in the real world where robustness to small changes in the environment is likely to be a desirable and important feature.

In light of these difficulties, the result of this paper can be interpreted as offering a useful alternative approach. In particular, this paper shows that, by restricting themselves to choosing a compensation scheme where accounting income is calculated using the RBRC allocation rule and each period's wage is a weakly increasing function of current and past periods' accounting income, shareholders can guarantee in a robust way that the investment incentive problem will be completely solved and still leave themselves considerable degrees of freedom to address remaining incentive issues. By using accounting income based on the RBRC allocation rule as a performance measure, shareholders could thereby guarantee that the investment incentive problem was completely solved and then use a "trial and error" process over time to identify a wage function that appeared to create the appropriate level of effort incentives.

Note that in cases where it is possible to calculate a fully optimal contract it may be that the fully optimal contract will not necessarily induce the agent to choose the profit maximizing level of investment. A general lesson from the incentives literature is that when one calculates the fully optimal contractual solution to a situation involving multiple interacting incentive problems, it is often the case that it is optimal to purposely distort the solution to one problem away from the first best in order to get extra leverage on the other problem. However, it

precisely these sorts of calculations that are exceedingly complex and that are unlikely to be robust to small changes in the contracting environment. Therefore, if shareholders have an opportunity to guarantee that a first best solution is created to one of the two incentive problems while still leaving themselves considerable degrees of freedom to address the second incentive problem, this may be a very attractive alternative in the real world.

This paper's approach of showing that a certain allocation rule induces the manager to make first-best investment decisions so long as each period's wage is weakly increasing in current and past periods' accounting income was first used in Rogerson (1997) to analyze a model where it is assumed that investment only occurs once, in the initial period. The current paper essentially provides a similar sort of result for the more complex case where investment occurs every period. The reader is referred to Rogerson(1997) for a more detailed discussion of the rationale for this approach. Dutta and Riechelstein (2002) have shown that Rogerson's (1997) allocation rule can be made part of a fully optimal contractual solution to a fully specified principal agent model that includes a moral hazard component in what they refer to as a LEN model - which means that contracts are assumed to be linear, utility is assumed to be exponential, and noise is assumed to be normal. An interesting project for future research would be to determine if the Dutta/Reichelstein(2002) approach could be adapted to the model of this paper where investment occurs every period, and, more generally, whether the RBRC allocation rule could be shown to be part of a fully optimal contract in more general circumstances.

## V. CONCLUSION

This paper shows that, in a surprisingly broad range of plausible circumstances, the seemingly complex multi-period profit maximization problem faced by a firm that must invest in long-lived assets to produce output can be decomposed into a series of much simpler single period problems. Namely, it is shown that a simple formula exists to calculate hypothetical perfectly competitive rental prices for assets and that the capital stock that the firm would choose to employ each period if it could rent assets at the hypothetical perfectly competitive rental rate turns out to also be the capital stock that maximizes the firm's profits given that it must purchase assets. Furthermore, it is shown that a simple cost allocation rule called the relative benefit and replacement cost (RBRC) rule has the property that the cost of purchasing an asset allocated to any period is exactly equal to the hypothetical perfectly competitive rental price for that period. This implies that accounting income calculated using the RBRC allocation rule has two desirable properties. First, it can be used to simplify the calculation of the optimal investment level, because in any given period the firm can calculate the fully optimal level of investment for that period simply by calculating the level of investment that maximizes accounting income in the first period of the investment's life. Second, robust incentives for management to choose the profit maximizing level of investment can be created by using this measure of accounting income as a performance measure.

**APPENDIX  
PROOFS OF PROPOSITIONS**

**Proof of Lemma 1:**

**Step #1:** If  $\mathbf{q}$  satisfies MALL then  $g_t(q_t)$  is weakly increasing in  $t$ .

For every  $t < \tau(\mathbf{q})$ , the required incremental capital stock is equal to zero and is thus weakly increasing. The fact that total output is weakly increasing for  $t \geq \tau(\mathbf{q})$  implies that the total required capital stock is weakly increasing. Inspection of equation (2) and (3) shows that the legacy capital stock is weakly decreasing. Since the total required capital stock is weakly increasing and the amount of available legacy capital stock is weakly decreasing, the required incremental capital stock must be weakly increasing. QED

**Step #2:** If  $g_t(q_t)$  is weakly increasing in  $t$ , then  $\mathbf{q}$  satisfies FUNI.

Let  $x_t = \varphi_t(q_1, \dots, q_{t+1})$  denote the number of assets that the firm purchases in period  $t$  to produce  $\mathbf{q}$ . The values of  $x_t$  are defined by sequentially defining the vector of variables  $(e_0, x_0, e_1, x_1, \dots)$  beginning with period 0 as follows.<sup>18</sup> Define  $e_t$  to be the incremental capital stock that the firm will have available in period  $t+1$  if the firm makes no new asset purchases in period  $t$ , conditional on all of the new asset purchases it has made in periods before period  $t$ . It is formally defined by

$$(A.1) \quad e_t = \sum_{i=1}^{T-1} x_{t-i} \beta^i$$

where  $x_j = 0$  for every  $j \leq -1$ . The required incremental capital stock in period  $t+1$  is given by  $g_{t+1}(q_{t+1})$ . The rule the firm follows in period  $t$  is to purchase only as many new assets as are

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<sup>18</sup>i.e., First  $e_0$  is defined. This allows  $x_0$  to be defined which in turn allows  $e_1$  to be defined, etc.

needed to create the required incremental capital stock in period  $t+1$ . Therefore  $x_t$  is defined by

$$(A.2) \quad x_t = \text{Max} \{ g_{t+1}(q_{t+1}) - e_t, 0 \}$$

The level of incremental capital stock available in period  $t+1$  is given by  $e_t + x_t$ . The FUNI condition requires that all of the incremental capital stock be used which means that

$$(A.3) \quad x_t + e_t = g_{t+1}(q_{t+1})$$

Inspection of (A.2) shows that (A.3) will be true if and only if

$$(A.4) \quad g_{t+1}(q_{t+1}) \geq e_t.$$

Therefore, to prove that the FUNI property is satisfied it is sufficient to prove that (A.4) is true for every  $t \in \{0, 1, 2, \dots\}$ . This can be proven by induction. It is clear that (A.4) is true for  $t = 0$  because  $e_0$  is equal to zero and the incremental capital stock in any period must be non-negative by definition. For the induction step, suppose that (A.4) is true for  $t-1$ , i.e.,

$$(A.5) \quad g_t(q_t) \geq e_{t-1}.$$

In period  $t-1$  the firm therefore purchases sufficient assets so its incremental capital stock is equal to  $g_t(q_t)$ . The amount of incremental capital stock that will be available in period  $t+1$  if no new assets are purchased in period  $t$  must be less than or equal to this amount. Therefore

$$(A.6) \quad e_t \leq g_t(q_t)$$

Since  $g_t(q_t)$  is weakly increasing in  $t$ , (A.6) implies that

$$(A.7) \quad e_t \leq g_{t+1}(q_{t+1})$$

which is the desired result.

QED

**Proof of Lemma 4:**

Consider any vector of outputs  $\mathbf{q}$ . By construction and by the definition of FUNI,  $g_t(q_t)$  and  $\xi_t(q_1, \dots, q_t)$  satisfy the following two conditions.

$$(A.8) \quad \xi_t(q_1, \dots, q_t) \geq g_t(q_t) \quad \text{for every } t \in \{1, 2, \dots\}$$

$$(A.9) \quad \xi_t(q_1, \dots, q_t) = g_t(q_t) \quad \text{for every } t \in \{1, 2, \dots\} \text{ if and only if } \mathbf{q} \text{ satisfies FUNI.}$$

Substitute equation (19) into equation (7) and reorder the summation to yield

$$(A.10) \quad C(\mathbf{q}) = \sum_{t=1}^{\infty} w_t^* \delta^t \left\{ \sum_{i=1}^T \beta^{i-1} \varphi_{t-i}(q_1, \dots, q_{t-i+1}) \right\}$$

where  $\varphi_j(q_1, \dots, q_{j+1}) = 0$  for every  $j \leq -1$ . Substitution of (4) and (9) into (A.10) yields

$$(A.11) \quad C(\mathbf{q}) = \sum_{t=1}^{\infty} w_t^* \delta^t \xi_t(q_1, \dots, q_t)$$

Substitution of (8) and (20) into (21) shows that the hypothetical cost function can be written as

$$(A.12) \quad H(\mathbf{q}) = \sum_{t=1}^{\infty} w_t^* \delta^t g_t(q_t)$$

The result then follows by comparing equation (A.11) to (A.12) in light of (A.8)-(A.9). QED



**Proof of Proposition 1:**

**Part (i):**

Suppose that  $\mathbf{q}'$  is a solution to the hypothetical problem and satisfies FUNI. Now suppose for contradiction that  $\mathbf{q}'$  is not a solution to the true profit maximization problem. Then there exists a vector of quantities  $\mathbf{q}''$  such that

$$(A.13) \quad R(\mathbf{q}'') - C(\mathbf{q}'') > R(\mathbf{q}') - C(\mathbf{q}').$$

Since  $\mathbf{q}'$  satisfies FUNI,

$$(A.14) \quad C(\mathbf{q}') = H(\mathbf{q}').$$

All output vectors, including  $\mathbf{q}''$  must satisfy

$$(A.15) \quad C(\mathbf{q}'') \geq H(\mathbf{q}'')$$

Substitution of (A.14) and (A.15) into (A.13) yields

$$(A.16) \quad R(\mathbf{q}'') - H(\mathbf{q}'') > R(\mathbf{q}') - H(\mathbf{q}')$$

which contradicts the assumption that  $\mathbf{q}'$  is a solution to the hypothetical problem. QED

**Part (ii):**

Suppose that  $\mathbf{q}'$  is the unique solution to the hypothetical profit maximization problem and satisfies FUNI. By the result of part (i),  $\mathbf{q}'$  is also a solution to true profit maximization problem. Now suppose for contradiction that  $\mathbf{q}'$  is not the unique solution to the true profit maximization problem. Then there exists a  $\mathbf{q}'' \neq \mathbf{q}'$  such that

$$(A.17) \quad R(\mathbf{q}'') - C(\mathbf{q}'') = R(\mathbf{q}') - C(\mathbf{q}').$$

Substitution of (A.14) and (A.15) into (A.17) yields

$$(A.18) \quad R(\mathbf{q}'') - H(\mathbf{q}'') \geq R(\mathbf{q}') - H(\mathbf{q}')$$

which implies that  $\mathbf{q}''$  is a solution to the hypothetical problem, which is a contradiction. QED

**Proof of Lemma 5:**

Inspection of equation (17) shows that Assumption 3 implies that  $w_t^*$  is weakly decreasing in  $t$ .

This and Assumptions 2 and 5-(iv) imply that  $q_t^{\min}$  is weakly increasing in  $t$ . Inspection of (2)-(3)

shows that  $z_t^L$  is weakly decreasing in  $t$ . Therefore there exists a  $t^* \in \{1, 2, \dots\} \cup \infty$  such that

$$(A.19) \quad q_t^{\min} > f(z_t^L) \Leftrightarrow t \geq t^*$$

From the definition of  $\mathbf{q}^*$  in equation (26) and (A.19) it follows that

$$(A.20) \quad f(q_t^*) > z_t^L \Leftrightarrow t \geq t^*$$

and

$$(A.21) \quad q_t^* = q_t^{\min} \quad \text{for } t \geq t^*.$$

Equation (A.20) implies that  $\tau(\mathbf{q}^*) = t^*$  and equation (A.21) together with the fact that  $q_t^{\min}$  is

weakly increasing in  $t$  then imply that  $q_t^*$  is weakly increasing for every  $t \geq t^*$ . QED

**Proof of Proposition 3:**

The function  $\phi_0(q_1)$  denotes the number of assets that the firm must purchase in period 0 in order to produce  $q_1$  units of output in period 1 and is given by

$$(A.22) \quad \phi_0(q_1) = \begin{cases} 0, & f(q_1) \leq z_t^L \\ f(q_1) - z_t^L, & f(q_1) \geq z_t^L. \end{cases}$$

Substitution of (A.22) into (35)-(36) shows that period 1 accounting cost is determined by

$$(A.23) \quad A_1(q_1) = \begin{cases} A_1^L & f(q_1) \leq z_t^L \\ A_1^L + s_1 a_0 [f(q_1) - z_t^L], & f(q_1) \geq z_t^L \end{cases}$$

where  $A_1^L$  is the accounting cost of legacy assets allocated to period 1 given by

$$(A.24) \quad A_1^L = \sum_{i=2}^T x_{1-i}^L s_i$$

In the hypothetical period 1 rental problem it is assumed that the firm can use its legacy assets at no cost and then must rent additional assets as required at a price of  $w^*_1$  which, as shown in the last section, is equal to  $k^* a_0$ . Therefore the hypothetical cost of production is given by

$$(A.25) \quad H_1(q_1) = \begin{cases} 0 & f(q_1) \leq z_t^L \\ k^* a_0 [f(q_1) - z_t^L], & f(q_1) \geq z_t^L \end{cases}$$

A comparison of (A.23) and (A.25) shows that if  $s_1$  is equal to  $k^*$ , then period 1 accounting cost is equal to a constant plus period 1 hypothetical cost. QED

**Proof of Lemma 6:**

The RHS of equation (40) gives the definition of  $c(t, i, \mathbf{s})$  and the RHS of equation (17) gives the definition of  $w_t^*$ . Therefore an allocation rule  $\mathbf{s}$  satisfies  $c(t, i, \mathbf{s}) = w_t^*$  if and only if these two expressions are equal, i, e, if and only if

$$(A.26) \quad \beta^{1-i} \alpha^{-i} s_i a_t = (k^*/\alpha) a_t$$

Equation (A.26) can be rewritten as

$$(A.27) \quad s_i = (\beta\alpha)^{i-1} k^*$$

By equation (33), the RHS of (A.27) is the definition of  $s_i^*$ . QED

**Proof of Proposition 4:**

Lemma 6 states that each unit of capital stock will create an accounting cost of  $w_t^*$ .

Therefore the firm will maximize its period  $t$  accounting income by choosing  $q_t$  to maximize

$$(A.28) \quad \eta(q_t) = \begin{array}{ll} R_t(q_t) - w_t^* z_t^L, & f(q_t) \leq z_t^L \\ R_t(q_t) - w_t^* f(q_t), & f(q_t) \geq z_t^L \end{array}$$

and by choosing the entire vector  $\mathbf{q}$  to make sure that the incremental capital stock in period  $t$  is fully utilized. However, inspection of (A.28) and (20) shows that  $\eta(q_t)$  differs from  $H(q_t)$  only by a constant. Since  $q_t^*$  maximizes  $H_t(q_t)$  it must therefore also maximize  $\eta(q_t)$ . Since the vector  $\mathbf{q}^*$  satisfies FUNI the firm fully utilizes all incremental capital stock in every period. In particular, then, the firm must fully utilize its incremental capital stock in period  $t$ . QED

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