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Efficient Exclusion

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Abstract

In an important paper, Aghion and Bolton (1987) argue that a buyer and a seller may agree on high liquidation damages in order to extract rents from future suppliers. As this may distort future trade, it may be socially wasteful. We argue that Aghion and Bolton’s’ analysis is incomplete in some respects, as they do not model the entry of new suppliers. We construct a model where entry is costly, so that entering suppliers have to earn a quasi-rent in order to recoup the entry cost. Reducing an entrant’s profits by the help of a breach penalty then reduces the probability of entry in the first place, thus making a breach penalty less attractive for the contracting parties. We show that the initial buyer and seller only have incentives to include a breach penalty if there is excessive entry without it. Forcing the initial buyer and seller to eliminate the breach penalty reduces welfare.

**Key words:** Exclusive contracts, breach penalties, entry, efficiency  
**JEL codes:** L42
1 Introduction

In a seminal paper, Aghion and Bolton (1987) argue that a buyer and a seller may have an incentive to use partly exclusive contracts in a way that harms welfare. Aghion and Bolton analyze the contracting parties’ incentives to include liquidation damages to the seller in the event of breach by one of the parties, hereafter referred to as a breach penalty. The idea is that a breach penalty may be used to extract rents from future suppliers that enter the market at a later stage. As a by-product of rent extraction, the most efficient supplier will not always be chosen, and this harms economic efficiency. This result, that breach penalties may be anti-competitive, has been very influential, and are now referred to in leading textbooks (Church and Ware 2000, Motta 2004, Pepall et al 2002).

In this paper we argue that Aghion and Bolton’s analysis is incomplete in some respects, as they do not explicitly model the entry of new suppliers. They assume that the probability a new seller arrives, as well as his cost distribution, is exogenous. In this paper we do model the entry decision of future suppliers, and let this entry decision depend on the contract chosen by the initial buyer and seller. We find that this dramatically changes the equilibrium of the model. We show that under reasonable assumptions, the initial buyer and seller set a positive breach penalty if and only if this is (constrained) efficient. If a regulator excludes the use of breach penalty, it will reduce welfare.

We construct a model in which a large number of potential suppliers simultaneously and independently decide whether or not they will enter the market. There is a sunk cost associated with entry. The production cost for a given supplier is stochastic at the entry stage, and realized after the entry cost is incurred. In the equilibrium of the entry game, the expected quasi-rent for an entrant exactly equals the entry cost.

It follows that reducing an entrant’s quasi-profits by a breach penalty reduces the amount of entry in the first place. Thus, including a breach penalty in the initial contract is less attractive for the contracting parties when entry of suppliers is endogenous. We show that with Bertrand competition between the suppliers (if there is more than one), the optimal breach penalty is zero. If the expected quasi-rents for the entrants exceed the quasi-rent obtained under Bertrand competition, there will be excessive entry in the absence of a breach penalty, and a strictly positive breach penalty is warranted. Still, the
breach penalty is constrained efficient, and forcing the contractors to reduce the breach penalty reduces welfare. On the other hand, if the buyer has a downward-sloping demand curve, and the suppliers are restricted to use linear prices, the initial buyer and seller want to encourage entry by setting a negative breach penalty.

Our results also apply to a situation where the initial seller undertakes investments. Spier and Whinston (1995) argue the initial buyer and seller may have a common incentive to over-invest in cost-reducing technology, in order to extract rents from the entrants. With endogenous entry this is not longer the case, as overinvestments will reduce profitable entry.

We argue that a breach penalty in a contractual setting is analogous to a reservation price above the seller’s valuation in an auction. With an exogenous number of participants, it is optimal to set the reservation price above seller’s evaluation. If there is competition between auctions, it is typically optimal to set the reservation price equal to the sellers valuation, see for instance McAfee (1993) and Peters (1997).

Our results hinge on the assumption that the entrants from an ex ante perspective obtain zero expected profits. In our view, this is consistent with the assumption that the entrants are not present at the initial contracting stage, and at this point has no vested interests in the project. If they had, they would be able to approach the contracting buyer and seller and make their interests heard at the contracting stage. As shown in Bernheim and Wilson (1998), the presence of the entrant(s) at the contracting stage completely changes the environment. The quasi-rent to the entrants should therefore be attributed to investments incurred after the contract is signed. At the contracting stage it therefore seems reasonable to assume that there are several potential entrants.

There exists a literature discussing breach penalties as a remedy for rent extraction, and on how this may give rise to an inefficient allocation of resources. A seminal paper (in addition to Aghion and Bolton) is Diamond and Maskin (1977), who analyze breach penalties in a search context. Innes and Sexton (1994) argue that a breach penalty may be warranted if the buyer and the entrant collude against the initial supplier. Spier and Whinston (1995) show that breach penalties do not influence the price offered by the new supplier if the buyer and seller renegotiate their contract after getting an offer from the entrant. Their result resembles findings in the auction literature regarding the seller’s problem to commit to a reservation price (Burguet and Sakovics 1996).
To our knowledge there are no papers that explicitly model entry. Spier and Whinston argue that with perfect competition among entrants, the initial buyer and seller have no incentives to set a breach penalty. However, in their model, that is simply because there are no rents to extract from the suppliers. In our model, by contrast, there are rents to extract, but it may not be in the buyer’s and seller’s interest to do so.

The paper is organized as follows. In section 2, a simple version of the model is presented. We replicate the result of Aghion and Bolton with exogenous entry, and then show how it vanishes with endogenous entry. We also show that the contracting agents’ investment decisions are optimal. In section 3 we give conditions as to when a breach penalty that differs from zero is optimal. In section 4 we present a generalized version of the model and show that our results are true for a wide class of entry games. The last section concludes.

2 The model

We consider a buyer $B$ that demands one unit of an indivisible good, and has a willingness to pay for this good equal to 1. The buyer and an initial seller (supplier) $S$ agree on a contract, hereafter referred to as the initial contract. The seller has production cost equal to $c^* < 1$, which is common knowledge. We refer to the buyer $B$ and the seller $S$ as the incumbent agents. Before trade takes place, new suppliers may enter the market. If a new supplier offers a sufficiently attractive deal to the buyer, the buyer may change his trading partner and pay $S$ a breach penalty.

We consider simple contracts of the form $(P^0, P^*, B)$, where $P^0$ denotes a "sign-on fee" paid by the buyer to the seller, $P^*$ denotes payment from the buyer to the seller at delivery, and $B$ denotes a breach penalty paid by the buyer to the seller if the buyer switches to a new supplier. The initial buyer and seller write a contract that maximizes their joint expected surplus. Without loss of generality we assume that $P^* = c^*$. The up-front payment $P^0$ can be used to share the surplus between the buyer and the seller so that $P^*$ is superfluous.

The entrants are not present at the initial contracting stage. One interpretation of this is that at the date when the initial contract is signed, no firm or entrepreneur considers it likely that they will enter this market later on. If they did, they would have approached the initial buyer and seller at
the earliest possible stage. In order to obtain a reasonably high probability that at least one firm enter it follows that the number of potential entrants must be high.

If a new supplier enters the market, its production cost $c$ is drawn from a continuous distribution function $F$, with density $f$. The timing is as follows:

1. $B$ and $S$ agree on the contract.

2. A large number $N$ of potential entrants independently and simultaneously consider whether they will enter the market. There is a sunk cost $k$ associated with entering the market.

3. After the sunk cost is incurred, production cost $c$ is realized.

4. The entrants (if any) make price offers to the buyer. The buyer chooses the supplier that offers the lowest price (included the breach penalty).

5. Trade takes place.

Let us now consider a symmetric equilibrium in which all firms enter the market with equal probability $q$ (for a more general formulation we refer to section 4). The number of entrants is thus approximately Poisson distributed with parameter $\lambda = Nq$.\footnote{Note the similarity between this model and so-called directed search models in labour economics, see Montgomery (1991).}

If there is only one entrant, he obtains a profit of $\max[c^* - B - c_i, 0]$, where $c_i$ is the realized cost. If more than one firm enter, the profit of entrant $i$ is only strictly positive if it has lower cost than the other entrants, and in addition $c^* - B > c_i$. In this event, the firm’s profit is $\min(c^* - B, c_{-i}) - c_i$, where $c_{-i}$ is the lowest cost among the other entrants.\footnote{Note is that it is not crucial that the entrants observe each other’s costs, as long as the distribution of costs are drawn from the same distribution for all the entrants. Suppose firms have private information about their costs, and submit bids as in a first price auction. Then we know from the revenue equivalence theorem that the allocation and expected profits will be the same as with Bertrand competition.}

Let $P(c)$ denote the probability that an entrant with costs lower than $c$ appears. For a given firm that enters, this is also the probability that there exists another entrant with costs below $c$. The number of entrants with costs less than or equal to $c$ is Poisson distributed with rate $\lambda F(c)$. It follows that
$P(c) = 1 - e^{-\lambda F(c)}$. The associated density is given by $p(c) = \lambda f(c)e^{-\lambda F(c)}$. The expected profit of an entrant with cost $c_i$ is thus

$$\pi(c_i) = \int_{c_i}^{c^*-B} \left( (z - c_i)p(z)dz + (1 - P(c^* - B))(c^* - B - c_i) \right)$$

In the appendix we show that the expected profits $\Pi = E\pi(c)$ can be written as

$$\Pi = \int_0^{c^*-B} e^{-\lambda F(c)}F(c)dc \quad (1)$$

In equilibrium of the entry game we must have that $\Pi = k$, that is, the expected quasi-profits when entering the market must be equal to the entry cost $k$.

### 2.1 Exogenous entry

We first study the model when entry, represented by the parameter $\lambda$, is considered exogenous by $B$ and $S$. This is analogous with the assumptions in Aghion and Bolton (1987) that the distribution of the entrant’s costs is exogenous.

The buyer and the seller choose the breach penalty $B$ so as to maximize expected joint profits. Let $W^{BS}$ denote the initial agents’ expected joint profits, $W^E$ the expected gross profit for all the entrants, and $W = W^{BS} + W^E$ the sum of all the firms’ expected gross profits. Then

$$W = 1 - c^* + \int_0^{c^*-B} (c^* - c)p(c)dc$$

Since $W^E = \lambda \Pi$, where $\Pi$ is given by (1), it follows that

$$W^{BS} = 1 - c^* + \int_0^{c^*-B} (c^* - c)p(c)dc - \lambda \int_0^{c^*-B} e^{-\lambda F(c)}F(c)dc \quad (2)$$

The first integral denotes the gross social value associated with entry (for a given $\lambda$). The social value is maximized when $B = 0$. The last term reflects rent extraction: by increasing $B$, the incumbents reduce the entrants’ expected profits. The first order condition for $B$ is given by
\[
B = \frac{e^{-\lambda F(c)} F(c)}{p(c)} = \frac{F(c^* - B)}{f(c^* - B)} \tag{3}
\]

The second order conditions are satisfied if the rate \( F/f \) is increasing in \( c \), and this corresponds to the standard hazard rate assumptions in the literature on optimal contracts. See for instance Laffont and Tirole (1993).

### 2.2 Endogenous entry

We now endogenize entry. For a given \( \lambda \), suppliers that enter the market obtain an expected profit \( \Pi \) given by (1). Equilibrium in the entry game requires that \( \Pi = k \). Thus, for any given \( B \), the free entry condition determines \( \lambda \).

It follows from (2) that \( \lambda \) is decreasing in \( B \). On the other hand, the surplus \( S \) of the incumbent supplier and buyer is increasing in \( \lambda \). It follows that the incumbents are more reluctant to increase the breach penalty when they realize that this will influence the entry decisions of suppliers. We will therefore expect that the optimal value of \( B \) is lower when we allow for entry of suppliers. We will actually show a stronger result, that the optimal breach penalty is zero.

To this end, write \( W \) and \( W^{BS} \) and \( W^E \) as functions of \( \lambda \). Free entry implies that \( W^E(\lambda) = k\lambda \). The surplus of the initial agents is thus \( W(\lambda) - k\lambda \). For a given \( \lambda \), the highest aggregate profit \( \tilde{W}(\lambda) \) is obtained when the most efficient firm carries out production. It follows that

\[
\tilde{W}(\lambda) = 1 - c^* + \int_0^{c^*} (c^* - c)p(c)dc = 1 - c^* + \int_0^{c^*} e^{-\lambda F(c)} dc.
\]

Let \( W^* \) denote the maximum of the relaxed program of maximizing \( \tilde{W}(\lambda) - \lambda k \). It follows that the buyer and the seller can never do better than \( W^* \).

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\(^3\)Differentiating 2 with respect to \( \lambda \) yields:

\[
\frac{\partial W^{BS}}{\partial \lambda} = BF(c^* - B)e^{-\lambda F(c^* - B)} + \lambda \int_0^{c^* - B} e^{-\lambda F(c)} F(c)dc \tag{4}
\]

which is strictly positive.
The first order condition to this relaxed program with respect to $\lambda$ is given by

$$\int_{0}^{c^*} \lambda f(c) e^{-\lambda F(c)} dc = k$$

From (1) it follows that this is the solution to the entry game provided that $B = 0$. Since the ex post efficient production decision is realized when $B = 0$, it follows that the incumbent agents can achieve $W^*$ by setting $B = 0$.

A social planner maximizes total profits less of entry costs, that is, $W(\lambda) - k\lambda$. Since this is equal to $W^{BE}$, it follows that $B = 0$ is socially optimal as well. We have thus shown the following proposition:

**Proposition 1** With free entry of firms, the optimal breach penalty for the incumbent agents is zero. This is also the socially optimal breach penalty.

To understand why it is optimal to set $B = 0$, note the following. When the breach penalty is zero, an entrant is paid exactly his marginal contribution to aggregate profits. That is, the entire cost saving $c^* - c_i$ if he is the only firm that enters, his cost advantage if he is the most efficient firm that enters, and zero otherwise. This ensures that the optimal number of suppliers enter the market. Furthermore, as all the entrants are on their participation constraint, all profits less of entrance costs are allocated to the incumbents.\(^4\)

### 2.3 Investments by the incumbent seller

Spier and Whinston (1995) show that in the presence of renegotiation between the incumbent buyer and seller, breach penalties have no bite. They further argue that cost-reducing investments by the initial seller may act as a substitute for breach-penalties, as lower costs reduce the price the buyer has to pay if a more efficient supplier enters. The initial seller will therefore overinvest.

Suppose the sellers’ costs $c^*$ depend on investments $I$ undertaken by the seller, so that $c^* = c^*(I)$. In the absence of coordination problems between the buyer and the seller, we assume that $I$ is chosen so as to maximize joint profits. We follow Spier and Whinston and rule out breach penalties.

\(^4\)This efficiency result corresponds to the so-called Mortensen rule for efficiency in matching models, see Mortensen (1982) and Jullien, Kennes and King (2004).
Suppose first that the initial buyer and seller treat the amount of entry, defined by $\lambda$, as exogenous. From (2) it follows that the ex post gross profits (net of investment costs) for the initial buyer and the seller is

$$W_{BS}(I) = (1 - c^*)(1 - P(c^*))$$

(5)

$$+ \int_0^{c^*} (1 - c)p(c)dc - \lambda \int_0^{c^*} e^{-\lambda F(c)}F(c)dc$$

(6)

$$= 1 - c^* + \int_0^{c^*} P(c)dc - \lambda \int_0^{c^*} e^{-\lambda F(c)}F(c)dc$$

(7)

The incumbent seller chooses $c^*$ so as to maximize $W_{BS}(I) - I$. The first order condition is given by

$$\frac{dW_{BS}(I)}{dI} = -(1 - P(c^*))c^*(I) - \lambda e^{-\lambda F(c^*)}F(c^*)c^*(I) = 1$$

The last term reflects rent extraction, which represents a private gain but not a social gain. (The last factor in the last term, $\lambda e^{-\lambda F(c^*)}F(c^*)$, denotes the probability that exactly 1 supplier with costs less than $c^*$ enters.) It is trivial to show that the incumbent agents overinvest.

Suppose then instead that the initial agents take into account that $\lambda$ depends on $c^*$ in such a way that the zero profit condition holds. It follows from the free entry condition $\Pi = k$ that $\int_0^{c^*} e^{-\lambda F(c)}F(c)dc = k$. The incumbent agents thus maximize

$$W_{BS}(I) - I = 1 - c^* + \int_0^{c^*} P(c)dc - \lambda \int_0^{c^*} e^{-\lambda F(c)}F(c)dc - I$$

For a given $c = c^*$, let $\lambda(c^*)$ denote the corresponding equilibrium value of $\lambda$. We know that for any given $c^*$, $\lambda$ maximizes $W_{BS}$. Due to the envelope theorem, we know that the effect of $I$ on $\lambda^*$ can be neglected, and it follows that

$$-c^*(I)(1 - P(c^*)) = 1$$

This is also the first order condition for the social optimum. We have thus shown the following result:
Proposition 2 Suppose the initial supplier can undertake cost-reducing investments. With free entry, the incumbent buyer and seller will invest the socially optimal (first best) level.

The intuition is exactly the same as for our earlier efficiency results. The initial agents have the opportunity to extract rents from the entrants. However, they do not have an incentive to do so, as this will reduce the amount of entry.

In Bernheim an Whinston (1998), a more complicated situation is modelled. One buyer and two sellers are present at the contracting stage. Later on, a new seller may arrive. Bernheim and Whinston show that the initial agents’ joint profit may be maximized if one of the sellers is excluded from the market, as this will reduce the competition for delivery to the entering buyer. With endogenous entry of new buyers, such rent extraction will reduce the probability of entry. We conjecture that when the incumbent buyer and seller take entry into account, the incentive to exclude one of the sellers will be eliminated.

3 Strictly positive (or negative) breach penalties

In the previous section, optimal entry was realized if the breach penalty was set equal to zero. However, under slightly different assumptions the entry decision of suppliers may not be optimal, neither from a social perspective nor from the perspective of the incumbent firms. This may call for breach penalties that are different from zero.

3.1 Excessive profits to entrants

In the previous section, prices were determined by Bertrand competition. Other forms of competition may give larger profit shares to the entrants, and thereby increase entry. This may call for positive breach penalties.

As an example, suppose an entrant, when observing the cost of its competitors (if any), withdraws from the market without submitting any bids, if one of the competitors have lower costs. This is a weakly dominant strategy for the entrant. At the same time, it increases the profits of entering the
market dramatically: without a breach penalty, the entire surplus created by entry is allocated to the entrants. Since the entrants obtain zero profit, the entire surplus from entry is spent on entry costs. Aggregate net profits (entry costs subtracted) is thus reduced to \(1 - c^*\), i.e., the same as if there were no entry at all. This is clearly not optimal.

More generally, less competition *ex post* yields higher quasi-rents to the entrants. Let \(\Pi(\lambda; B)\) be a reduced-form expected profit function to an entrant, showing expected quasi-rent when entering the market. As above, let \(\Pi(\lambda, B)\) denote the expected quasi-rent to an entrant under Bertrand competition (given by equation 1). We assume that \(\Pi(\lambda; B) > \Pi(\lambda, B)\) for all \(\lambda, B\). To simplify, we assume that both competition regimes give rise to the same *ex post* allocation: The entrant with the lowest cost is chosen provided that his costs are lower than \(c^* - B\).

As before, let \(W(\lambda, B)\) denote aggregate gross profits of all the firms. As above, the incumbent firms thus want to maximize \(W(\lambda(B), B) - \lambda(B)k\). Under Bertrand competition, this was achieved by setting \(B = 0\). This simultaneously gave rise to efficient trade *ex post* and an optimal entry level, which we denoted by \(\lambda^*\). With the profit function \(\Pi(\lambda; B)\), the level of entry when \(B = 0\), exceeds \(\lambda^*\).

We say that the breach penalty \(B^*\) set by the incumbent agents is constrained socially optimal if the following holds: Suppose a planner could chose the breach penalty, but nothing else. Then he would choose the same breach penalty as the incumbent agents. We can then show the following proposition:

**Proposition 3** Suppose the competition regime, for any given \(\lambda\) and \(B\), gives rise to higher quasi-rents for the entrants than Bertrand competition. Then the optimal breach penalty, \(B^*\) is strictly positive. Furthermore, the breach penalty is constrained socially optimal.

Proof: Due to the envelope theorem, \(\frac{\partial W(\lambda, B)}{\partial B} = 0\) evaluated at \(B = 0\). It follows that \(\frac{\partial W(\lambda, B)}{\partial B} = \left[\frac{\partial W(\lambda, B)}{\partial B} - k\right] \frac{\partial \lambda}{\partial B} > 0\). To see why this is constrained efficient, note that the incumbent agents choose \(B\) so as to maximize aggregate profits net of entry costs, just as the planner would do. QED.

Note that although the breach penalty enhances efficiency, first best as defined in the previous section cannot be achieved. This is because reducing entry by a breach penalty comes at a cost, as it distorts *ex post* efficiency. In order to obtain first best efficiency, the breach penalty should be an increasing
function of the cost difference \( c^* - c \), and be equal to zero at \( c = c^* \). However, such a breach penalty may be hard to implement.

To understand why the breach penalty is constrained efficient, note that there are no externalities in the model. Thus, increasing \( B \) does not reduce the \textit{ex ante} profit of the entrants, which is zero anyway. It follows that the interests of the incumbent agents and of the planner are aligned.

### 3.2 Downward sloping demand curve

Suppose now that the buyer has downward sloping demand curve, given by \( q = D(p) \), with \( D'(p) < 0 \). We will study two cases: when firms compete by submitting two-part tariffs \((p_i, T)\) (which give rise to an efficient volume of trade) and when they only submit a per-unit price \( p_i \). We retain our assumption that firms compete in a Bertrand fashion.

If the firms submit price-quantity pairs, the entrant with the lowest cost \( c_i \) offers a contract \((c_i, T)\) and thereby wins the contract provided that \( c_i \) is less than \( c^* - B \). The constant \( T \) is given by

\[
T(c_i, \bar{c}) = \int_{c_i}^{\bar{c}} D(p) dp
\]

where \( \bar{c} = \min\{c_{-i}, c^* - B\} \). Expected profit can thus be written as

\[
\Pi = \int_0^{c^* - B} D(c)e^{-\lambda F(c)} F(c) dc
\]  

(8)

See the appendix for a proof. The only difference between this expression and (1) is the new factor \( D(c) \). By using the same logic as when we derived proposition 1 we get the following result:

**Proposition 4** Suppose the buyer’s demand function is given by \( D(p) \). If the entrants offer two-part tariffs, then the optimal breach penalty is equal to 0. This is also the socially optimal value of \( B \).

Suppose then that only linear prices are allowed. The equilibrium of the bidding game is then again that the entrant (if any) with the lowest cost is chosen as the supplier, provided that his costs are below \( c^* - B \). Let \( p^M(c_i) \)
denote the monopoly price of a supplier with cost $c_i$, and let $c_{-i}$ denote the lowest cost among the other entrants. The price the supplier charges is then given by

$$p = \min\{p^M(c_i), c_{-i}, c^*-M\}$$

We can now show the following proposition:

**Proposition 5** Suppose the entrants only set linear prices. Then the optimal breach penalty for the incumbent agents is strictly negative. Furthermore, the breach penalty is constrained efficient as defined above.

The proof is given in the appendix. Intuition for the result can be obtained by understanding that the social surplus associated with entry exceeds the profits to the entrant. Suppose $p^M(c_{\text{min}}) \geq c^*$ and $B = 0$. If an entrant is not the most cost-efficient agent, he receives zero profits, which is also his contribution to aggregate profits (social contribution). If he is the most efficient agent, he receives the saved costs associated with his entry. However, this is less than the increase in aggregate profits. As the price falls, the demand by the buyer increases. The gain to the buyer therefore exceeds the loss in profits for the second most productive entrant. Entry thus gives rise to a positive externality to the buyer that exceeds the negative externality for the other entrants, and as a result there is too little entry. If $p^M(c_{\text{min}}) < c^*$, this strengthens the positive externality even further.

## 4 Generalizations

In the previous sections we studied optimal breach penalties for one particular entry game. In this section we will show that our results hold for a wide class of entry games provided that the entrants earn zero profits.

Let the demand function for the incumbent buyer be given by a function $D(p)$, which may be stochastic at the point at which the entrants sunk their cost $k$.

We do not specify the price-setting mechanism. However, we require the mechanism to allocate production to the entrant with the lowest cost if his cost is below $c^*-B$, otherwise the incumbent seller produces the good. Furthermore, we require that the expected profit of an entrant can be written
Π(B, n), strictly decreasing in both arguments (as long as c* − B is in the support of c), where n is the number of entrants.

Our most important generalization regards the entry game. We consider the same model as above, but allow the entrants to coordinate their actions to some extent. Let \( \bar{q}(Z) = (q_1(Z), \ldots, q_N(Z)) \) denote any equilibrium strategy profile in the game as a function of the parameters of the model, where \( q_i \) is the probability that firm \( i \) enters. We let the numbering of the firms reflect their probability of entry, so that \( q_i \geq q_j \) if \( i < j \). Let \( \Omega(Z) \) denote the set of equilibrium strategy profiles which is such that all potential entrants are indifferent as to their entrance probability \( q_i \). (In what follows we write \( Q \) and \( \Omega \) as a function of \( B \) only.) For any equilibrium strategy profile, let \( Q(B) = \sum_{i=1}^{N} q_i(B) \). Then \( Q(B) \) is the expected number of entrants. Note that in the previous sections, \( Q = \lambda \).

Note that we require that any equilibrium strategy profile, for any given vector of parameter values, assigns exactly one equilibrium vector of entry probabilities. Thus, a given strategy profile \( \bar{q}(B) \) assigns exactly one vector of entry probabilities for each \( B \).

The equilibrium analyzed earlier is an element in \( \Omega \) such that \( q_i(B) = q_j(B) \) for all \( i, j \) and all \( B \). Another equilibrium may be the following: Let \( n \) denote the highest numbers of firms that can enter the market and obtain a strictly positive profit. An equilibrium strategy profile is then that \( q_i = 1 \) for all \( i < n \), and \( q_i = 0 \) for all \( i > n + 1 \), while agent \( n \) and \( n + 1 \) randomize.

We assume that both the incumbent agents know which equilibrium strategy profile \( \bar{q}(B) \) will be realized. Furthermore, we assume that both the incumbent agents and the planner treat \( \bar{q}(B) \) as exogenous. A social planner’s objective is to maximize total profits net of entry cost. Total gross profit can be written as \( W(\bar{q}(B), B) \). A breach penalty is (constrained) socially efficient if a planner would choose the same breach penalty if he could set the breach penalty but nothing else (in particular, the planner has to take the equilibrium in the entry game as given). The following result obtains:

**Proposition 6** For any equilibrium strategy profile \( \bar{q}(B) \in \Omega \) the incumbents will set the (constrained) socially optimal breach penalty.

Proof: The incumbent agents set \( B \) so as to maximize \( W(\bar{q}(B), B) - W^E = W(\bar{q}(B), B) - kQ(B) \). This is the same as maximizing aggregate net profits. The proposition thus follows.

We also want to derive some general statement about the optimal value of the breach penalty. In order to obtain this, we have to assume some structure
on the equilibrium set. More specifically, we require that \( q(B) \) can be written as \( q(B) = H(Q(B)) \), where the correspondence \( H \) is monotone, increasing, and hemicontinuous. Or, less abstract, we require that \( \frac{dq}{dB} \leq 0 \) for all \( B \). In what follows, we define \( W(Q(B)) \equiv W(\bar{q}(B)) \).

We say that the price mechanism is efficient if the following holds: for any given \( B \), any strategy profile \( \bar{q} \), all entrants obtain an expected profit by entering which is equal to his contribution to aggregate profits in a hypoeutectic situation in which the cost of the incumbent is equal to \( c^* - B \). To be more precise, \( \tilde{W}(\bar{q}(B)) \) is the hypothetical aggregate gross profits when the incumbent’s cost is equal to \( q^* - B \). We then require that \( \tilde{W}(\bar{q}_{-i}(B), a) = \Pi \) for all \( i \), where \( \tilde{W}(\bar{q}_{-i}(B), a) \) denotes expected aggregate profits when all firms but \( i \) follow the profile \( \bar{q} \) while \( i \) enters with probability \( i \). Since \( \tilde{W}(\bar{q}(B)) \) is linear in \( q_i \), this is equivalent to the requirement that \( \frac{\partial \tilde{W}(\bar{q}(B))}{\partial q_i} = \Pi \) for all \( i \).

**Proposition 7** Suppose that the price setting mechanism is efficient and the equilibrium strategy profile satisfies the above requirements. Then the optimal breach penalty is zero. If the pay-off to an entrant is higher (lower) than his contribution to aggregate profits, a strictly positive (negative) breach penalty is warranted.

To prove that \( B = 0 \) when the price mechanism is efficient, suppose the opposite is true. Denote the optimal \( B \) by \( B^* \), and suppose without loss of generality that \( B^* > 0 \). Consider the expression

\[
\frac{d}{dB}[W(Q(B), B) - Q(B)k] = Q'(B)(W_Q - k) + W_B
\]

As the last expression is negative, it is sufficient to show that \( W_Q - k \geq 0 \). To see this, note that \( W_Q > W_Q \), as the former includes payments of breach penalties to the incumbents. Since the pricing mechanism is efficient, we have that \( W_Q = k \). Hence the proposition follows. QED.

**Sequential entry**

Let us now consider sequential entry. Quasi-rents may then occur in this model due to the integer problem. Suppose for instance that it is profitable for one entrant to enter the market, but not for a second entrant. The firm
that enters may then earn positive profits, and the incumbent firms may have an incentive to extract some of this rent by using a breach penalty. This has the same flavor as results from the literature on auctions, where it is shown that the integer problem may make it optimal to set a positive breach penalty, see for instance Engelbrecht-Wiggans (1993). It may then be argued that the breach penalty reduces welfare, as it distorts trade \textit{ex post} and may even lead to too little entry.

However, an unattractive feature of this equilibrium is that an important ingredient of the model, the sequence at which firms are supposed to enter, is unmodelled. Furthermore, \textit{ex ante} identical suppliers obtain different \textit{ex ante} profits, as the suppliers that enter may obtain a net profit by entering while those who do not enter obtain zero profits. Presumably the suppliers would engage in some activities that would enhance their prospects of being the first firm to enter.

Suppose therefore that the entrants, by incurring a cost (effort) $r$, may improve their chances of entering first. This may for instance reflect that an entrant may speed up the entry process. We assume that $r$ does not create social value, and is thus a complete waste. The cost $r$ is incurred before $k$. Suppose the sequence at which the firms enter is equal to the ranking of their effort $r$. Suppose further that the potential entrants choose $r$ simultaneously and independently. Finally, suppose it is only optimal for one supplier to enter the market. The pre-entry game is thus a tournament (all pay auction), see Fudenberg and Tirole (1986), or Klemperer (2004) for a survey.

There is no pure strategy equilibrium in this game. However, one can easily show that in any equilibrium, all participants obtain zero profit. Thus, the aggregate effort is exactly equal to the net expected profit when entering the market. As a result, (\textit{ex ante}) profit to the firm that enters has no social value, as it will be dissipated in the pre-entry game anyway.

**Proposition 8** Suppose the pre-entry game is as described above. Then (\textit{ex ante}) rents to the incumbent has no social value. It follows that the incumbent buyer and seller set the breach penalty at the (constrained) socially optimal level.

When the incumbent agents set the breach penalty, they trade off rent extraction to the entrant and \textit{ex post} efficient trade, and when doing this
he does not put any weight on rent to the entrant\textsuperscript{5}. However, rent to the
entrant has no social value, as it is dissipated in the pre-entry game. The
planner thus faces exactly the same trade-off as the incumbent agents when
setting $B$.

\section{Conclusion}

We have analyzed the extent to which buyers and sellers may have incentives
to agree on breach penalties so as to extract a surplus from future suppliers
and thereby reduce overall welfare. We argue that in order to adequately
address this issue, the entry decision of potential suppliers must be explicitly
modelled. We then construct a model where entry is costly, so that entering
firms have to earn a quasi-rent in order to recoup the entry cost. Reducing
the entrants’ quasi-rent by agreeing on a breach penalty then reduces the
amount of entry, making breach penalties less attractive to the initial buyer
and seller. We have shown that if entrants compete in a Bertrand fashion,
the incumbent buyer and seller set the breach penalty equal to zero. If the
profits to the entrant is higher (lower) than under Bertrand competition, this
induces too much (too little) entry, and the incumbent agents set a positive
(negative) breach penalty. However, the breach penalty is still (constrained)
socially optimal, in the sense that forcing the firms to change or eliminate
the breach penalty will reduce welfare.

\section*{Appendix}

\textbf{Derivation of equation (1)}

By using integration by parts we find that

\footnote{\textsuperscript{5}We assume that the constraint that the supplier must earn at least $k$ in expected
terms to be willing to enter does not bind.}
\[
\pi(c_i) = \int_{c_i}^{c^*-B} (z - c_i)p(z)dz + (1 - P(c^* - B))(c^* - B - c_i) \\
= -[c_i^{c^*-B}(z - c_i)(1 - P(z)) + \int_{c_i}^{c^*-B} (1 - P(z))dz + (1 - P(c^* - B))(c^* - B - c_i) \\
= \int_{c_i}^{c^*-B} (1 - P(z))dz
\]

Note that \(\pi'(c) = -e^{-\lambda F(c)}\). The expected profit of an entrant is thus

\[
E\pi = \int_{0}^{c^*-B} \pi(c)f(c)dc \\
= -\int_{0}^{c^*-B} \pi'(c)F(c)dc \\
= \int_{0}^{c^*-B} e^{-\lambda F(c)}F(c)dc
\]

where we again have used integration by parts.

**Derivation of equation 8**

The expected profit for a firm with cost \(c_i\) is

\[
\pi(c_i) = \int_{c_i}^{c^*-B} T(c_i, c)p(c)dc + T(c_i, C^* - B)(1 - P(c^* - B))
\]

Using integration by parts as \(i\) gives

\[
\pi(c_i) = \int_{c_i}^{c^*-B} D(c)(1 - P(c))dc
\]

Expected profit can thus be written as (see equation 1)

\[
\Pi = \int_{0}^{c^*-B} D(c)e^{-\lambda F(c)}F(c)dc
\]

which is what we wanted to show.
Proof of proposition 5

Denote by \( q_2(c_2, c) = \min[q^M(c), c^* - B, c_2] \) the price charged in the situation where at least two firms enter, and by \( q(c) = \min[q^M(c), c^* - B] \) the price if only one entrant appears. Clearly \( q(c) \geq q_2(c_2, c) \). Gross social surplus can now be written

\[
W = \int_{c^*}^{\infty} D(q) dq + \int_{0}^{c_2} \left[ \int_{q_2(c_2, c)}^{e^*} D(q) dq + [q_2(c_2, c) - c] D(q_2(c_2, c)) \right] f(c)\lambda^2 e^{-\lambda F(c_2)} f(c_2) dc dc_2 \\
+ \int_{c^* - B}^{c^*} \int_{0}^{c_2} \left[ \int_{q_2(c_2, c)}^{e^*} D(q) dq + [q_2(c_2, c) - c] D(q_2(c_2, c)) \right] f(c)\lambda^2 e^{-\lambda F(c_2)} f(c_2) dc dc_2 \\
+ \int_{0}^{c^*} \left[ \int_{q(c)}^{e^*} D(q) dq + [q(c) - c] D(q(c)) \right] e^{-\lambda} f(c) dc
\]

where the first term is incumbent surplus in case no firm enters. The second and third term capture the expected surplus conditioned that at least two firms enter, whereas the last term expresses the surplus in the case where exactly one firm enters.

Since \( q(c) \geq q_2(c_2, c) \) it follows that
\[
W \geq \tilde{W} \equiv \int_{c^*}^{\infty} D(q) dq \\
+ \int_0^{c^*} \int_0^{c^*} \left[ \int_0^{q(c)} D(q) dq + [q(c) - c] D(q(c)) \right] f(c) \lambda^2 e^{-\lambda F(c_2)} f(c_2) dc \, dc_2 \\
+ \int_0^{c^*} \int_0^{c^*} \left[ \int_0^{q(c)} D(q) dq + [q(c) - c] D(q(c)) \right] f(c) \lambda^2 e^{-\lambda F(c_2)} f(c_2) \, dc \, dc_2 \\
+ \int_0^{c^*} \int_0^{c^*} \left[ \int_0^{q(c)} D(q) dq + [q(c) - c] D(q(c)) \right] e^{-\lambda F(c)} f(c) \, dc \\
= \int_0^{c^*} \int_0^{c^*} \left[ \int_0^{q(c)} D(q) dq + [q(c) - c] D(q(c)) \right] e^{-\lambda F(c)} f(c) \, dc
\]

Differentiating with respect to \( \lambda \) yields

\[
\frac{\partial (\tilde{W} - \lambda k)}{\partial \lambda} = \int_0^{c^*} \int_0^{c^*} \left[ \int_0^{q(c)} D(q) dq + [q(c) - c] D(q(c)) \right] e^{-\lambda F(c)} f(c) (1 - \lambda F(c)) \, dc
\]

Furthermore (since each term is increasing in \( \lambda \))

\[
\frac{\partial (\tilde{W} - \lambda k)}{\partial \lambda} \leq \frac{\partial (W - \lambda k)}{\partial \lambda}
\]

The expected profit of an entrant with cost \( c_i \) is

\[
\pi(c_i) = \int_{c_i}^{\infty} [q_2(c_2, c_i) - c_i] D(q_2(c_2, c_i)) p(c_2) dc_2
\]
hence the expected profit is

\[
\Pi = \int_0^{c^* - B} \int_c^{\infty} [q_2(c_2, c) - c] D(q_2(c_2, c)) p(c_2) f(c) dc_2 dc \\
= \int_0^{\infty} \int_0^{\min\{c^* - B, c_2\}} [q_2(c_2, c) - c] D(q_2(c_2, c)) p(c_2) f(c) dc_2 dc \\
= \int_0^{\infty} \int_0^{c^*} [q_2(c_2, c) - c] D(q_2(c_2, c)) \lambda e^{-\lambda F(c)} f(c_2) f(c) dc_2 dc \\
+ \int_{c^* - B}^{c^*} \int_0^{\infty} [q_2(c_2, c) - c] D(q_2(c_2, c)) \lambda e^{-\lambda F(c)} f(c_2) f(c) dc_2 dc \\
\leq \int_0^{\infty} [q(c) - c] D(q(c)) e^{-\lambda F(c)} f(c)(1 - \lambda F(c)) dc
\]

As \( \frac{\partial(W - \lambda k)}{\partial \lambda} \geq \Pi \) the social value of a higher entry rate exceeds the expected profit from entering, hence the social optimal \( B \) is negative.

References


Church, J and Ware, R (2000), Industrial Organization, McGraw-Hill.


