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## Competition in price and availability when availability is unobservable

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THE CENTER FOR THE STUDY  
OF INDUSTRIAL ORGANIZATION  
AT NORTHWESTERN UNIVERSITY

Working Paper #0022

Competition in Price and Availability when  
Availability is Unobservable<sup>\*</sup>

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## **Abstract**

I present a strategic model of competition in price and availability in which demand is uncertain and consumers choose where to shop given firms' observable prices and their expectations of firms' unobservable inventories. In both a single period Cournot model (inventories are chosen first) and a single period Bertrand model (prices are chosen first) I show that firms use higher prices to "signal" higher availability. This creates a floor on equilibrium prices and industry profits regardless of the number of firms. The model is useful in understanding the relationship between price and availability in the video rental industry.

## 1. Introduction

Competition in availability or service rates (the likelihood a consumer is served) is common in many retail and consumer service markets. Among the many retailers who regularly advertise availability are car dealers, video rental chains, department stores, mail order suppliers, and appliance stores. In many other industries, including movie theaters, airlines, hotels, book stores, and department stores, firms' reputations for availability clearly influence demand. But while consumers directly value high service rates because of the delays and search costs associated with stockouts, they often rely on past experience, reputations, advertising, and other signals to predict availability because inventories are typically unobservable.<sup>1</sup>

When aggregate demand is uncertain, inventory or capacity is fixed, prices are rigid, and search (for availability) is costly, then consumers, who want to avoid potential stockouts, choose where to shop on the basis of both price and the expected likelihood of being served.<sup>2</sup> Carlton (1979) derived the zero-profit competitive equilibrium of such a model under the assumption that firms simultaneously and observably commit to both price and inventory (see also Bryant, 1980 and Gould, 1978). Peters (1984) and Deneckere and Peck (1995) showed that a pure strategy equilibrium exists in the analogous oligopoly model if the number of firms is sufficiently large.<sup>3</sup>

This paper departs from this previous work on service rate competition by making the more realistic assumption that consumers do not observe firms' inventory decisions.<sup>4</sup> Asymmetric information about inventories changes the way firms compete in prices in an important and interesting way. Consumers realize that firms with high margins are more likely to stock inventory and less likely to have stockouts, so a potential price increase has two offsetting effects: first, it decreases a consumer's surplus conditional on trade taking place, and second, it

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<sup>1</sup> I use inventory and capacity interchangeably to describe a firm's output decision when it must commit to its output before its demand is known.

<sup>2</sup> Price rigidities alone introduce the possibility of industry wide stockouts (see Prescott, 1976, Eden, 1990, Deneckere, Marvel, and Peck, 1996, and Dana, 1998), but costly search causes consumers to care about individual firms' stockout rates.

<sup>3</sup> Formally, Peters (1984) and Carlton (1979) consider models in which the aggregate demand is certain but individual firm's demand is uncertain (in Peters this is formally because there are a finite number of "large" buyers) but this distinction does not change the interpretation of the results.

<sup>4</sup> This assumption also captures the unobservability of firms' expected demands.

induces the firm to hold greater inventory, which increases the likelihood that trade takes place. The net effect on a consumer's ex ante expected surplus could be either positive or negative.

One industry where one might expect to see these effects is the retail market for video rentals. I collected a small sample of prices and availability of video rentals at stores within a 4-mile radius of Northwestern University. There were 20 stores, including seven Blockbuster Video stores, one Hollywood Video outlet, and four others that were part of chains. The other eight were independent outlets. I checked the availability of four new releases, two each on two different Saturday evenings. Overall the four movie titles were available 64% of the time. At outlets with prices of \$3.50 or more they were available 78% of the time. At Blockbuster (average price of \$3.81) the titles were available 86% of the time.<sup>5</sup> At the other chains (average price of \$3.32) the titles were available 60% of the time. At the other independents (average price \$2.62) they were available 48% of the time. Although this was hardly a scientific study (neither the Northwestern University vicinity nor the movie titles were selected at random), it suggests a strong relationship between price and availability. As important, it suggests that Blockbuster's advertised claims of greater availability are valid, that consumers are aware of this relationship, and that its greater availability allows Blockbuster to charge higher prices.<sup>6</sup>

I explore two closely related single period oligopoly models. In the first model, firms commit to observable prices before they choose their inventory levels (I call this the Bertrand model). Consumers understand that firms setting higher prices have private incentives to offer greater availability. In the second model, firms commit to their inventory levels before they choose their prices (I call this the Cournot model). In this case, a firm's price acts as a "signal" of the inventory level it chose. In equilibrium consumers expect firms with higher prices to offer greater availability. Although price does not signal inventory holdings in the commonly meant

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<sup>5</sup> Two of the movies were "guaranteed" to be available at Blockbuster – the guarantee means that the consumer could rent that movie for free on a later date if it was out of stock. The guaranteed titles were available 93% of the time. The other two titles were available 79% of the time.

<sup>6</sup> This also occurs in the airline industry. Business travelers value higher priced tickets that have no restrictions on the right to change flights, subject to availability, because they realize that the higher fare increases the airline's incentives to allocate inventory to the business traveler "bucket."

sense of hidden information with a separating equilibrium (there is no hidden information in the model), I show that a firm's off-the-equilibrium-path price affects consumers' conjectures about its inventory choice.

I find that the Cournot equilibrium price is higher than the Bertrand price and that the difference depends on the number of firms. As market becomes less concentrated the equilibrium price in the Cournot model converges to the Bertrand price. In both cases, equilibrium prices are supra competitive and industry profits are strictly positive profits *even when there are arbitrarily many firms*.

The video rental industry fits the Bertrand model's stylized assumptions reasonably well. Stores adjust prices infrequently but make inventory decisions for new releases every week. Consumers do not observe inventories until after they visit the store, and even when a consumer visits a retailer multiple times so he or she might be able to recall the inventory between visits, the consumer is still uncertain about demand so may not find the physical stock of inventory on the shelves very useful in forecasting future service rates. After choosing which video rental outlet to visit, many consumers find it costly to visit a second store when their preferred movie is out-of-stock. This is particularly true when their preference is for a new release and they realize (correctly) that stockouts are correlated across firms. Of course, they may rent their second or third choice, so a multiproduct version of the model would be more appropriate. Finally it is reasonable to expect consumers to have different tastes for availability, which would if would explain the heterogeneity of retail prices and the equilibrium correlation between price and availability.

The model is related to other papers in which high prices guarantee high quality, such as Allen (1984), Klein and Leffler (1981), Shapiro (1983), and Rogerson (1983 and 1987). However each of these models is dynamic and allows high quality to be rewarded with repeat business.<sup>7</sup> The paper in the "quality guaranteeing prices" literature that is most similar is Wolinsky (1983).

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<sup>7</sup> Milgrom and Roberts (1986) show that high prices can signal high quality when the distribution of quality is exogenous, but they also rely a dynamic model with some form of repeat purchase.

He considers a static model in which competitive firms choose unobservable quality and observable prices and consumers engage in costly search for quality. Consumers choose where to shop based on price, but after visiting a firm and verifying its quality they decide whether to consume or to engage in more costly search. Wolinsky describes high prices as a signal of high quality in the same sense as I do in this paper; consumers expect that firms who offer higher prices chose higher quality. However like the other papers in this literature, Wolinsky considers competitive firms and ignores the timing of firms' price and quality decisions.

The model is also related to the literature on efficiency wages. In a simple principal-agent, hidden-action model when effort is unobservable the principal would like to induce the agent to exert high effort by offering payments to the agent that are contingent on an imperfect signal of the agent's effort. But it is well known that if the agent has limited liability, it may be impossible to design an incentive compatible contract without relaxing the agent's individual rationality constraint. In this paper the absence of forward contracts plays the role of a limited liability constraint; the consumer (the principal) cannot punish the agent (the firm) when there are stockouts. The consumer, who prefers that the firm hold higher inventories, relaxes the firm's zero profit constraint in order to satisfy the firm's incentive compatibility constraint. However this analogy has several limitations. First, firms, not consumers, are setting prices. Second, prices are determined through strategic interaction among firms. And third, price may be chosen after inventory or capacity, in which case price is not acting as an incentive but as a signal of the firm's inventory level.

Daughety and Reinganum (1991) consider the determination of availability in models where firms cannot commit to price, so consumers have imperfect information about both price and output. They compare monopoly and duopoly pricing under these assumptions and show that the relative provision of availability depends on the magnitude of consumers' exogenous search costs (see also Daughety and Reinganum, 1992) For low search costs, when duopolists compete price lower than the monopoly price, firms provide less availability, but for high search

costs, when the duopoly price is the same as the monopoly price, duopolists provide more availability.

The operations literature has considered the related problem of optimal inventory choice by a single firm when stockouts adversely affect future demand, but typically treat price and consumer behavior as exogenous. These papers include Schwartz (1966 and 1970), Balcer (1980, 1983), Fergani(1976), and Robinson (1990). Conlisk and Smallwood (1979) consider competition in the probability of a breakdown when firm quality choice is unobservable, a problem which is extremely similar to competition in service rates, however they also treat price and consumer behavior as exogenous. Gans (1999a and 1999b) has recently considered a much richer dynamic model of inventory decisions and consumer choice in which consumers learn about service rates over time, but he too considers an exogenous price.

This work is also related to work on competition in inventories. Lippman and McCardle (1997) consider a model in which uncertain demand is divided in various ways among competing firms with an exogenous price (see also Parlar, 1988, and the survey by Porteus, 1990). They assume that inventories do not affect consumer choice and that price is exogenous, however inventories do affect rivals because consumers can costlessly (or with an arbitrarily small cost) buy from other firms if the first firm they visit stocks out (though the firm pays an exogenous shortage cost).

In the next section I present the basic model. In the third section I solve the model under the assumption that firms commit to prices before inventory. In Section 4, I solve the model under the assumption that firms commit to inventory or capacity before price. In Section 5, I compare the equilibria to those that arise without imperfect information. In Section 6, considers some extensions of the model and offers concluding remarks.



## 2. The model

### Consumers

I assume that there is a continuum of ex ante symmetric consumers with potential unit reservation values,  $v$ . Aggregate demand is uncertain. Firms' products are homogeneous and there is no idiosyncratic demand uncertainty. Individual consumers learn whether or not they want the good in which case their utility is  $v - p$  if they consume it and zero otherwise. Consumers who want the good choose where to shop based on firms' announced prices and their expectations of the firms' capacities which, together with consumers' aggregate behavior, determine the probability that the consumer will be served at each firm. This probability is the firm's service rate (also known as the type 2 service level or fill rate). The expected utility from visiting firm  $i$  is  $U_i = (v - p_i)s_i$ , where  $s_i$  is the expected service rate of firm  $i$  (conditional on the consumer wanting the good) and  $p_i$  is its price. Consumers visit just one firm.<sup>8</sup> I assume the cost of visiting a second firm is arbitrarily high.<sup>9</sup> When the expected utility associated with each firm is the same, consumers mix over firms. I let  $\{\phi_1, \dots, \phi_n\}$  denote the vector of probabilities with which a consumer visits each firm and require that  $\sum_{i=1}^n \phi_i = 1$ .

A firm's demand is equal to the aggregate demand times the share of consumers who choose to visit the firm. The total number of consumers who want the good is  $a$ . I assume that  $a$  has a probability density function  $f(a)$ , which is strictly positive and continuous on  $[\underline{a}, \bar{a}]$ ,  $0 \leq \underline{a} < \bar{a} < \infty$ , and a cumulative probability distribution  $F(a)$ . Expected demand for  $p \leq v$  is  $\mu = \int_{\underline{a}}^{\bar{a}} af(a)da$ .

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<sup>8</sup> I assume consumers visit one firm regardless of the probability of a stockout. See Dana and Petruzzi (2001) for a discussion of the monopoly-pricing problem with complete information when consumers choose between visiting the firm and an outside option.

<sup>9</sup> In a symmetric equilibrium when one firm stocks out they all do, so there is no reason for search ex post. This changes if search is costless. Prescott (1976) (see also Eden, 1990, and Dana, 1999) shows that when consumers search freely for the best price firms specialize in different prices and availability rates.

## Firms

I assume that  $n$  firms compete by choosing both price,  $p_i$ , and capacity,  $k_i$ . The firms' costs are  $ck_i$  which are incurred regardless of sales. The cost  $c$  represents the part of the firm's costs that are unrecoverable. The portion of costs which are recoverable (the salvage value) is normalized to zero. Prices are chosen simultaneously and are observable, but I assume that capacity or inventory choices are unobservable both to consumers and to other firms.

Below I consider two models with distinct timings. In the first case inventory (or capacity) is chosen after price, and in the second case inventory (or capacity) is chosen before (or at the same time as) price. The latter model is an adaptation of Deneckere and Peck's (1995) study of simultaneous price and capacity choice with observable capacity.

## Consumer Equilibrium

Consumers' choices of which firm to visit depend on the firms' prices and consumers' expectation of firms' service rates, which in turn depend on consumers' expectations of  $k_i$  and of other consumers' equilibrium strategies.

When  $g(a)$  is the probability distribution of  $a$ , then the probability that an individual consumer obtains the good after visiting firm  $i$  (i.e. the service rate) is clearly

$$\int_a^{\bar{a}} \frac{\min(k_i, \phi_i a)}{\phi_i a} g(a) da,$$

where  $\phi_i a$  is the demand that firm  $i$  faces in equilibrium. Following Deneckere and Peck (1995), a random customer who wants the good conditions his belief about the distribution of  $a$  on his own realized demand<sup>10</sup>. The conditional distribution of  $a$  given that a random customer wants the good is  $g(a) = af(a)/\mu$ . So firm  $i$ 's service rate, given its capacity and market share, can be written

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<sup>10</sup> This effect can be quite significant. If demand is equally likely to be 10 or 90, a consumer assigns a probability of .9 to it being the high demand state conditional on wanting the good as compared to the unconditional probability of .5.

$$s(k_i/\phi_i) = \frac{\int_{\underline{a}}^{\bar{a}} \min(k_i, \phi_i a) f(a) da}{\phi_i \mu}.$$

Note that this probability depends only on the ratio of the firm's capacity to its market share and it is equal to the firm's expected sales  $\int_{\underline{a}}^{\bar{a}} \min(k_i, \phi_i a) f(a) da$  divided by its expected demand  $\phi_i \mu$ . As in Deneckere and Peck (1995), the probability that an individual consumer is served is the same as the firm's aggregate service level (or fill rate) defined as the expected fraction of realized demand that it serves.<sup>11</sup>

Letting  $K_i = k_i/\phi_i$  simplifies the notation. In this way the firm's capacity is expressed as if it served the entire market;  $K_i$  is equal to the total industry capacity produced if every firm were to act just as firm  $i$  did. For  $K_i \in [\underline{a}, \bar{a}]$ , the service rate can be written as

$$s(K_i) = \frac{\int_{\underline{a}}^{\bar{a}} \min(K_i, a) f(a) da}{\mu} = 1 - \frac{1}{\mu} \int_{K_i}^{\bar{a}} (a - K_i) f(a) da$$

For  $K_i > \bar{a}$ , the service rate is  $s(K_i) = 1$ , and for  $K_i < \underline{a}$ , the service rate is  $s(K_i) = K_i/\mu$ . The derivative of  $s$  is  $s'(K_i) = (1 - F(K_i))/\mu$ .

So when inventory is observable the consumer's choice problem is simply to choose the firm that maximizes  $(v - p_i)s(k_i/\phi_i)$ . However inventory is unobservable. Let  $\kappa(p_i, p_{-i})$  denote the consumers' common expectations of firm  $i$ 's capacity given the vector of announced prices. It is straightforward (see Deneckere and Peck, 1995) to show that the market shares,  $\{\phi_1, \dots, \phi_n\}$ , that equate consumer surplus,  $(v - p_i)s(\kappa(p_i, p_{-i})/\phi_i)$ , across all firms are uniquely defined.

*Lemma:* (Deneckere and Peck, 1995, Propositions 4, 5, and 6) A consumer equilibrium,  $\{\phi_1, \dots, \phi_n\}$ , exists given all prices and expected capacities, and is unique whenever prices and expected capacities are restricted to be in the set of undominated firm strategies.

This lemma describes consumer strategies as a function of prices and expected inventories (all consumers have the same expectations). The next two sections characterize firms' best

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<sup>11</sup> Note that the equivalence between the aggregate fill rate and the consumers' individual expected service rate is a consequence of the assumption of homogeneous consumers.

response functions for the two different timings. The equilibria of the two static games are the simultaneous solution of the consumer strategies given in the lemma and the firms' best response functions derived below.

I assume that  $\kappa(p_i, p_{-i})$  is identity-independent and the same for all consumers, so if two firms offer the same price then all consumers expect them to have the same inventory level and the firms are visited by the same number of consumers. Similarly, any reordering of the vector of prices results in an identical reordering of the expectations.

### 3. Bertrand Timing

In this section I assume that firms first commit to prices, then observe each other's prices, and finally choose their inventories (or capacities). This assumption is clearly satisfied for firms, such as newspaper stands, magazine vendors, and video rental outlets, which make long-run price decisions and short-run inventory decisions. It also might be appropriate for firms that make short run decisions about the allocation of their capacity across segmented markets (such as movie theaters). Price commitments may arise because of advertising, costs of price adjustment, and relationship specific investments.<sup>12</sup>

Because inventory is unobservable the Bertrand game has two, not three stages. Regardless of when the inventory decision is actually made, we can model the game as a reduced-form game in which consumers' shopping decisions and firms' inventory decisions are made at the same time. Since I am interested in the subgame perfect equilibria of the game, I begin by finding the Nash equilibria of this second stage subgame.

Taking prices and consumers' subgame perfect strategies,  $\{\phi_1, \dots, \phi_n\}$ , as given, firm  $i$  solves

$$\max_{k_i} p_i \int_{\underline{a}}^{\bar{a}} \min(k_i, \phi_i a) f(a) da - ck_i,$$

or

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<sup>12</sup> Gilbert and Klemperer (2000) show that firms will use price commitments and rationing in order to induce consumers to make uncontractable investments that generate demand.

$$\max_{k_i} p_i \phi_i \mu s(k_i/\phi_i) - ck_i.$$

The first order condition for the firm  $i$  is

$$s'(k_i/\phi_i) p_i \mu = c,$$

or

$$(1 - F(k_i/\phi_i)) p_i = c,$$

which is the standard newsvendor inventory condition for a firm whose demand is  $\phi_i a$ . The firm chooses the inventory level where its service level (the probability that a stockout does not occur),  $F(k_i/\phi_i)$ , is equal to its price markup,  $(p_i - c)/p_i$ .<sup>13</sup> So

$$k(p_i, \phi_i) = \begin{cases} \phi_i F^{-1}((p_i - c)/p_i) & p_i \geq c \\ 0 & p_i < c \end{cases} \quad (1)$$

is the profit maximizing inventory strategy for firm  $i$  given  $p_i$  and  $\{\phi_1, \dots, \phi_n\}$ . Each firm's strategy depends on  $\{\phi_1, \dots, \phi_n\}$  but not on other firms' prices.<sup>14</sup> The other firms' prices only matter in so far as they affect the firms' market shares.

Equation (1) gives the firms' inventory levels as a function of consumers' strategies in any subgame after prices have been announced. So equation (1) not only defines the firms' optimal choice of inventory but also consumers' expectations of firms' inventory decisions in any subgame after prices are chosen. So the equilibrium of any subgame is obtained by solving the consumers' maximization problem subject to  $\kappa(p_i, p_{-i}) = k(p_i, \phi_i(\bar{p}))$ , where  $\phi_i(\bar{p})$  denotes consumers' equilibrium strategies as a function of all  $n$  firms' prices.

Thus firms' capacity choices and the consumers' mixed strategies are jointly determined by (1) and by the individual consumer's choice problem

$$\max_i (v - p_i) s(\kappa(p_i, p_{-i})/\phi_i)$$

which, given (1), can be written

$$\max_i (v - p_i) s(F^{-1}((p_i - c)/p_i))$$

<sup>13</sup> The function  $F(k_i/\phi_i)$  is referred to in the inventory literature as the firm's type 1 service level.

<sup>14</sup> Note that for prices less than  $c$ , the firm will choose  $k_i = 0$  (since  $k_i \geq 0$  is an omitted constraint).

which is independent of consumers' strategies. Hence competition drives price down to the price that maximizes consumer surplus.

Let  $p^* = \arg \max_{p \geq c} (v - p)s(F^{-1}((p - c)/p))$ . So  $p^*$  is the price that maximizes consumer surplus. Clearly  $p^* \in [c, v)$  and I make the additional assumption (on the distribution  $F$ ) that  $p^*$  is unique. In equilibrium, any firm that offers a price  $p^*$  in the first stage of the game will serve an equal share of the market, and any firm offering a higher or lower price will have zero share. It follows that when  $p^* > c$  every firm is active though when  $p^* = c$  firms are indifferent between being active and not.<sup>15</sup>

*Proposition 1.* In any subgame perfect equilibrium of the Bertrand timing game every active firm sets a price  $p^* = \arg \max_{p \geq c} (v - p)s(F^{-1}((p - c)/p))$  and offers a service rate  $s(F^{-1}((p^* - c)/p^*))$ . If  $p^* > c$  then each firm's output is uniquely determined; every firm is active and produces  $(1/n)F^{-1}((p^* - c)/p^*)$  units of inventory or capacity, consumers visit each firm with probability  $\phi_i = 1/n$ , and firms' earn strictly positive profits. If  $p^* = c$  then there are  $n - 1$  subgame perfect equilibria, each with a different number of active firms,  $m = 2, \dots, n$ . Each active firm chooses  $(1/m)F^{-1}((p^* - c)/p^*)$  units of inventory or capacity, consumers visit each firm with probability  $\phi_i = 1/m$ , and firms earn zero profits.

*Proof.* See the Appendix.

The Bertrand price announcements convey information about capacity choices and service rates because they alter the firms' incentives to provide additional capacity. By raising its price the firm increases its marginal return to holding inventory. The marginal revenue from an additional unit of inventory is  $(1 - F(k_i/\phi_i))p_i$ , so price increases are associated with a service rate increases. If a firm raises its price above its rivals' price when its rivals are all charging less than  $p^*$  then the additional likelihood of being served is more than offsets the effect of the higher

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<sup>15</sup> Without the assumptions that expectations are identity-independent the equilibrium price and service rate would still be the same but the active firms' market shares would not be determined.

price – consumer surplus is higher and all consumers will buy from the higher priced firm. However if a firm raises its price above  $p^*$  when its rivals are charging  $p^*$  the additional likelihood of being served is insufficient to offset the effect of the higher price – consumer surplus is lower and no consumers will buy from the higher priced firm.

In the next section I show that  $p^*$  is also the lower bound for prices when firms compete in capacity. But before considering that alternative timing I will first explore the determinants of the price floor,  $p^*$ . There are two important questions. First, when is  $p^* > c$ ? And second, when is  $p^*$  larger than the complete information price? I explore the first question here and the second question in Section 5

Generally speaking, as long as demand is sufficiently uncertain, the Bertrand price  $p^*$  is strictly greater than marginal cost  $c$ . If  $p^* = c$ , then  $(p^* - c)/p^* = 0$ , and  $F^{-1}((p^* - c)/p^*) = \underline{a}$ , so  $\sum_i k_i = \underline{a}$ . That is, firms' inventories are set at the minimum level, the level that guarantees that their entire inventory will always be sold. The probability of a stockout is one and the probability that firms have excess inventories is zero. Firms never regret holding too much inventory or capacity. This happens either when demand is near certain or valuations are extremely low.

*Proposition 2.* If  $s(\underline{a})\mu f(\underline{a}) < (v - c)/c$  then  $p^* > c$ , otherwise  $p^* = c$ , the firms' stockout rate is one, and inventories are equal to the demand in state  $\underline{a}$ .

*Proof.* See the Appendix.

When the minimum inventory guarantees a sufficiently high service rate, i.e., either  $\underline{a}$  is large, or potential consumer surplus is small (i.e.,  $v - c$  is small), then firms will hold only the inventory that sells with certainty. Note also that  $\underline{a} = 0$  is a sufficient condition for  $p^* > c$  since  $s(0) = 0$ ; so enough uncertainty, in the sense of the support of the distribution, is sufficient for  $p^* > c$ .

## 4. Cournot Timing

A more natural timing for many retail environments is that firms first choose inventory or capacity and then choose price. It is also more complex. In this section I assume that firms commit to their inventory or capacity before setting prices.

I actually discuss two closely related timings: a simultaneous choice game in which firms choose price and inventory (or capacity) simultaneously and then consumers decide where to shop given observed prices, and a sequential choice game in which firms first choose inventory (or capacity), then firms choose price (without observing each other's inventory), and finally consumers decide where to shop given the observed prices.<sup>16</sup> These two extensive form games have the same strategic forms (while the Bertrand game considered earlier has a different strategic form because firms observe each other's prices before choosing their inventory). However there are differences in the set of equilibrium in the two games for some refinements that depend on the extensive form. I will focus on the simultaneous move game and point out where the differences arise.

Formally this is a game of complete, but imperfect information, with infinite pure strategy sets (for firms). Firms choose price and capacity simultaneously given their expectations of each other's equilibrium price and capacity choices and of consumers' equilibrium strategies. Consumers then observe prices, form expectations of firms' capacities (and service rates), and choose a firm to visit.

Firms' capacity choices depend largely on consumers' equilibrium conjectures about firms' capacities. In order to reduce the multiplicity of outcomes I place some natural restrictions on consumer behavior. Let consumers' equilibrium expectations about a firm's capacity be represented by  $\kappa(p_i)$ . Again, I assume that the expectation function  $\kappa$  is identity-independent and the same for all consumers. Let  $\Phi(\bar{p}): \mathfrak{R}_n^+ \text{ to } \mathfrak{R}_n^+$  describe the vector of consumers' mixed strategies given the  $n$  prices. I define  $\Phi(\bar{p})$  to be symmetric if for any  $x, y$ , and

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<sup>16</sup> If firms observed each others' inventories, then a firm's price would convey information to consumers about *every* firms' inventories, not just its own.



$\hat{y}$  such that (i)  $x = \Phi(y)$ , and (ii)  $\hat{y}$  can be obtained from  $y$  by reordering the elements of  $y$ ,  $\hat{x} = \Phi(\hat{y})$  can be obtained from  $x$  by reordering the elements of  $x$  in the same way. It follows from my assumptions on  $\kappa$  that consumers' mixed strategies are symmetric.

I also restrict my attention to equilibria in which firms play symmetric pure strategies. Since firms' payoff functions are concave whenever other firms are playing pure strategies, the restriction to pure strategy best responses is without loss of generality. However I am ignoring potential asymmetric and mixed strategy equilibria of the game.

I define a symmetric pure strategy equilibrium of this game to be a set of price and inventory decisions for the firms,  $(\bar{p}, \bar{k})$ , a function of conjectures,  $\kappa(p_i)$ , describing consumers' conjectures about a firm's capacity level as a function of its announced price, and a function  $\Phi(\bar{p})$  describing consumers' mixed strategies given prices, such that (i) firms' strategies are optimal given the strategies and conjectures of consumers and the strategies of other firms, (ii) given any prices  $\bar{p}$  consumers' strategies are optimal given firms' strategies and their conjectures about capacities, and (iii) given any prices  $\bar{p}$  consumers' conjectures about firms' capacities satisfy  $\bar{k}_i = \kappa(\bar{p}_i)$ . Note that this game has no proper subgames, so this equilibrium concept is stronger than subgame perfect equilibrium. Also note that I restrict consumers to have pure strategy conjectures about firms' inventory choices even when prices are off the equilibrium path. Nevertheless I believe that this definition captures the minimum of what is rational behavior for consumers.

In contrast to Deneckere and Peck (1995), this game has many pure strategy equilibria. They show that a unique pure strategy equilibrium exists of the observable output Cournot game if  $n$  is large; no pure strategy equilibria exists otherwise. The existence problem that arises in their game is that firms can deviate by cutting price and simultaneously increasing their *observable* capacity, which enables them to capture the entire market. This can easily be seen by letting the demand uncertainty collapse to a small interval around the mean  $\mu$ , in which case a firm can lower its price a very small amount and offer enough capacity to serve the entire market, significantly increasing its profit. Indeed a useful way to think of Peters' and Deneckere and

Peck's papers is that by adding enough demand uncertainty (and enough firms) they are able to find a pure strategy equilibrium in a game that typically has none.

In a symmetric pure strategy equilibrium (where every other firm announces a common price) the firm  $i$ 's profit maximization problem is

$$\max_{p_i, k_i} p_i \phi_i \mu s(k_i / \phi_i) - c k_i = \max_{p_i, k_i} p_i \int_{\underline{a}}^{\bar{a}} \min(k_i, \phi_i a) f(a) da - c k_i$$

subject to

$$(v - p_i) s(\kappa(p_i) / \phi_i) = (v - p^C) s(k^C / \phi_j), \forall j \neq i,$$

where  $p^C$  and  $k^C$  are the other firms' common equilibrium strategies and consumers' strategies depend on the vector of prices. This set of constraints can be simplified by aggregating over all the other firms, since in equilibrium the other firms have the same market share and service rate, which yields

$$(v - p_i) s(\kappa(p_i) / \phi_i) = (v - p^C) s((n-1)k^C / (1 - \phi_i)).$$

Note that consumers' strategies depend on the firm's announced price both because price directly effects the utility offered by that firm and also because it potentially "signals" information about the firm's service rate.

Now consider what happens when Firm  $i$  lowers its price? As consumers switch from other firms to Firm  $i$  because of the more attractive terms, the remaining firms become more attractive; they have fewer customers chasing the same capacity. Consumers' strategies change until the consumer surplus from visiting each firm is again equalized. Unlike the Bertrand model, Firm  $i$  cannot capture the entire market by cutting its price because higher priced firms have already committed to offer capacity  $k^C$ .

Consumers' equilibrium strategies depend on price and their capacity expectations, but not on capacity. So the firm chooses  $k_i$  to maximize firm profits in the same way as in the Bertrand model. So given consumers' equilibrium mixed strategy

$$k(p_i, \phi_i) = \phi_i F^{-1}((p_i - c) / p_i)$$

is Firm  $i$ 's optimal capacity as a function of the vector of prices (since  $\phi_i$  depends on all firms' prices).

As before this condition not only defines the firm's choice of inventory but also consumers' expectations about  $k_i$  on the equilibrium path. The only conjectures that are consistent with optimal firm behavior are  $\kappa(p_i) = \phi_i F^{-1}((p_i - c)/p_i)$ . So in particular it must be the case that in a symmetric equilibrium consumers conjecture that firms' inventories satisfy  $k^C = (1/n)F^{-1}((p^C - c)/p^C)$ .

Without restrictions on consumers' off-the-equilibrium-path conjectures, this game has a continuum of symmetric pure strategy equilibria. For example, if consumers conjecture that any firm quoting an off-the-equilibrium-path price has zero capacity, e.g.,  $\kappa(p_i) = 0, \forall p_i \neq p^C$ , then any price  $p^C \in [c, v]$  and associated capacity  $k^C = (1/n)F^{-1}((p^C - c)/p^C)$  can be supported as a symmetric pure strategy equilibria.

However, restrictions on off-the-equilibrium path conjectures about firms' inventories seem appropriate. One natural set of conjectures is that the firm has chosen the optimal capacity given the firm's observed price and its rivals' equilibrium prices. This is especially natural when price and capacity are chosen simultaneously, since this is the least costly error the firm could have made conditional on the observed price. I also show that these are the only conjectures that satisfy a relatively simple forward induction argument discussed below.

When consumers conjecture that the firm has chosen its capacity optimally the firm's observed price and its rivals' equilibrium prices, then consumers' off-the-equilibrium-path conjectures take the same form as their on-the-equilibrium path conjectures:

$$\kappa(p_i) = \begin{cases} \phi_i(p_i, p) F^{-1}((p_i - c)/p_i) & p_i \geq c \\ 0 & p_i < c \end{cases} \quad (2)$$

where  $\phi_i(p_i, p)$  is the consumer equilibrium when all other firms charge a common price  $p$ .

*Proposition 3.* When consumers' equilibrium conjectures satisfy (2), there exists a unique symmetric pure strategy equilibrium of the Cournot timing game in which every firm offers a common price that is strictly greater than  $p^*$  and consumers' strategies satisfy  $\phi_i = 1/n$ . As  $n \rightarrow \infty$ , the equilibrium price converges to  $p^*$  where industry profits are strictly positive if  $p^* > c$  and zero otherwise.

*Proof.* See the Appendix.

Since  $\partial \phi_i(p^*, p)/\partial p_i = 0$  and  $\partial^2 \phi_i(p^*, p)/\partial p_i^2 < 0$ , it is clear that firms set prices strictly above  $p^*$ . Because capacity is chosen simultaneously (or first), every firm has a little bit of market power.

The conjectures described by (2) have an appealing forward induction quality. Consumers observe price deviations and have to attribute them to some cause. A forward induction argument says that if the firm could have benefited from the deviation by changing consumers' beliefs and actions in a rational way, then consumers should attribute the deviation to a rational forward-looking move. In this case, when consumers see an off-the-equilibrium-path price it is reasonable to suppose that they will try to reason backwards about what the firm did when it chose its inventory level. Much like reasoning forward about future play, consumers might ask themselves if there is any inventory choice such that if they believed that the firm chose that inventory level, then the firm would be better off and the firm would actually choose that inventory level (given that consumers anticipated it). If this condition is satisfied for some off-the-equilibrium-path price then the equilibrium does not satisfy a forward induction reasoning that is captured by the Never a Weak Best Response property.

*Proposition 4.* The symmetric pure strategy equilibrium satisfying (2) and characterized in Proposition 3 is the unique symmetric pure strategy equilibrium satisfying the “Never a Weak Best Response” property.

*Proof.* See the Appendix.

The NWBR property requires that an equilibrium still be an equilibrium of the modified game after any firm strategies that are never a weak best response to consumers' (and other firms') strategies are eliminated from the game. Given consumers' equilibrium strategies, it follows that every strategy for the firm in which its inventory choice does not satisfy (2) will be eliminated. This is because a strategy that fails to satisfy (2) could never be a weak best response to consumers' (and other firms') *equilibrium strategies*. The firm can always increase its profits by offering the same price and changing its inventory level to the level satisfying (2). The equilibrium of the modified game (in which (2) is now satisfied) will be the equilibrium described in Proposition 3.

## 5. Observable versus unobservable output

How does the equilibrium when output is unobservable compare to the equilibrium when output is observable, and in particular to the equilibrium derived by Carlton (1979) and Deneckere and Peck (1995)? This is clearly the first step toward measuring the welfare loss associated with imperfect information. If output is observable, then the firm's optimal output given price is

$$(1 - F(k_i/\phi_i))v = c$$

or

$$k(p_i, \phi_i) = \begin{cases} \phi_i F^{-1}((v - c)/v) & p_i \geq c \\ 0 & p_i < c \end{cases}$$

Comparing this to (2) it is immediately apparent that holding price fixed, output and service rates are lower under incomplete information (except in the case of a monopolist).

When output is observable the firms' capacity choice is independent of its price, so consumers always prefer to shop at firms with lower prices. Moreover firms choose the socially optimal capacity. To see this note that social welfare is

$$\max_K v \int_a^{\bar{a}} \min(K, a) f(a) da - ck,$$

where  $K$  is the sum of firms' individual capacities, so the socially optimal capacity is

$$(1 - F(K))v = c,$$

or

$$K = F^{-1}((v - c)/v).$$

Prices do not affect welfare because demand is assumed to be inelastic. However the equilibrium price level depends on the timing. Under perfect information and Bertrand timing (when price is chosen first), with two or more firms, price is driven down to the zero profit level.

Profits are zero when

$$p \int_{\underline{a}}^{\bar{a}} \min(F^{-1}((v - c)/v), a) f(a) da - c F^{-1}((v - c)/v) = 0$$

or

$$p \mu s(F^{-1}((v - c)/v)) - c F^{-1}((v - c)/v) = 0.$$

So

$$p = \frac{F^{-1}((v - c)/v)}{\mu s(F^{-1}((v - c)/v))} c,$$

and since,

$$s(F^{-1}((v - c)/v)) = \frac{\int_{\underline{a}}^{\bar{a}} \min(F^{-1}((v - c)/v), a) f(a) da}{\mu},$$

the perfect information Bertrand price is

$$p_{PIB} = \frac{F^{-1}((v - c)/v)}{\int_{\underline{a}}^{\bar{a}} \min(F^{-1}((v - c)/v), a) f(a) da} c.$$

The price is equal to cost times the ratio of equilibrium capacity to expected sales. The perfect information Bertrand price is strictly greater than unit marginal cost, though exactly equal to the expected cost of a unit sold. Under perfect information, capacity is strictly greater than expected sales unless there is no demand uncertainty, i.e.,  $\underline{a} = \bar{a}$ .

A natural question is which is higher,  $p_{PIB}$  or  $p^*$ ? Either may be higher. In particular, when  $s(\underline{a})\mu f(\underline{a}) > (v - c)/c$  then  $p^* = c$  and clearly  $p_{PIB} > p^*$ . But if there is a lot of demand uncertainty and valuations are high  $p^* > p_{PIB}$ . For example if demand is uniformly distributed on

$[0,100]$ , cost is 10, and consumers' valuation is 50, then the symmetric information price is 16.67 (with an equilibrium service rate of 96%) and the Bertrand imperfect information service equilibrium price is 20 (with an equilibrium service rate of 75%).

There are two effects here. Asymmetric information lowers the equilibrium service rates which lowers the expected costs of unsold inventory and pushes prices down. However, asymmetric information raises the price that firms charge because higher prices lead to greater availability. Prices may be higher or lower, but expected profits are higher. In general the more uncertain is demand and the greater are consumer valuations, the more likely it is that the imperfect information price will be higher.

This result has an analogy in efficiency wage literature. Efficiency wages always exceed the reservation wage of the worker but they still may be higher or lower than symmetric information wages. The reason they can be lower is that it may be optimal for the worker to work so much harder when effort is contractible that his or her reservation wage becomes very large.

The above comparison is for the Bertrand timing game. Under Cournot timing the comparison will depend on whether price and quantity are chosen sequentially or simultaneously. If firms can simultaneously deviate in both price and quantity, as assumed by Deneckere and Peck (1995), then for small  $n$  no pure strategy equilibria exists. For sufficiently large  $n$  the Bertrand timing equilibrium price becomes a lower bound for the price in the Cournot timing model; as the number of firms goes to infinity, the equilibrium price approaches this lower bound. However if firms choose output first and then price, and output is observable to other firms when they set price, then firms' prices can be functions of other firms' inventories or capacities. An analysis of this case has not been done and is beyond the scope of this paper.

## **6. Extensions and conclusion**

I have shown that when consumers care about expected availability but availability is unobservable, then price can act as a signal of availability, even in a single period model. Like

some efficiency wage models, where workers' expected pay exceeds their reservation wage, firms here earn efficiency profits that induce them to hold more inventories. In the Bertrand (price first) model announced prices directly induced firms to hold more inventories, while in the Cournot (inventory first) model prices transmitted information about firms' past, but unobservable, inventory decisions.

The Cournot model also served as an example of an important class of problems for game theory. In the Cournot game firms take observable actions (price) that signal other previously taken hidden actions (inventory). Consumers must use price, and in particular off-the-equilibrium-path prices, to make inferences about past unobservable inventory choices. I argued that the Never-a-Weak-Best-Response property is the right way to capture the forward-induction-like reasoning that parties use when forming expectations about firms' past inventory decisions. The model argues for a larger role for using forward-induction-like arguments when reasoning about past play.

Daughety and Reinganum (1995) consider a similar information problem in a model where general product liability induces firms to use price to signal product safety. Unobserved product safety affects the firm's marginal cost (in part by affecting the size or number of liability claims against it), so price can signal product safety. They consider a two-stage model in which the firm invests in safety-related R&D, then learn the stochastic outcome of its investment, and finally sets price. However consumers in their paper do not change the conjectures about past play given the observed prices; they only update their beliefs about the stochastic realization of the firm's quality. In other words their equilibrium conjectures about the firm's R&D strategy, and the resulting distribution of product safety, are independent of the price offered. This is partly because in response to on-the-equilibrium-path prices consumers assign zero probability to off-the-equilibrium-path play, and partly because the authors assume that off-the-equilibrium-path prices do not alter consumers' equilibrium conjectures about past play (although relaxing this assumption in plausible ways would not alter their results). Another paper with related qualities is Bagwell (1995). He shows that when a Stackelberg leader's output is observed with noise, the



follower will not use his observations to alter his conjecture about the firm's capacity. As a consequence the Stackelberg leader has no commitment power and the only pure strategy equilibrium is the Cournot equilibrium.<sup>17</sup>

The most striking characteristic of the video rental industry absent from the model is the heterogeneity of prices and availabilities. One possible explanation is that when consumers are heterogeneous and place different values on availability firms may position themselves at different price and availability points. Also, even with homogeneous consumers, if consumers' search costs are finite then firms may have an incentive to specialize in high price and availability and serve only customers who are rationed at less expensive stores.

But another important source of heterogeneous prices is reputations for availability. In an earlier version of the paper, I showed that a repeated game extension of the single period model had equilibria in which some firms have reputations for providing greater availability. Formally modeling reputations for availability provides greater insight into the marketing, pricing, and inventory strategies of Blockbuster Video, and the pricing and availability strategies of major US airlines, such as American and United.

Several other simplifying assumptions might be relaxed. For example inventory decisions are usually analysed in dynamic models with periodic replenishment. Also, the assumption that there is no idiosyncratic demand uncertainty is unrealistic, but not difficult to relax. I also consider only one way that firms could signal availability. Advertising, product selection, and product quality, are other firm decisions that might influence consumers' perceptions of availability. In particular one could relax the assumption that the firms sell only one product. When firms sell multiple products then the consumer's decision about which firm to visit depends on the expected availability of all of the firm's products and more importantly on how the demands are related. With two products, firms have an incentive to hold more inventories when they sell complementary products because the cost of a stock out is the lost margin on the

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<sup>17</sup> Van Damme and Hurkens (1997) derives a mixed strategy equilibrium in which the leader's commitment power is partially restored. Maggi (1999) shows that commitment power is also restored when the observational uncertainty is small relative to uncertainty about the leader's private information.

understocked item as well as the margin on any complementary product that did not stock out.

And firms have an incentive to hold fewer inventories when they sell substitute products because the cost of a stock out is the lost margin on the understocked item less the expected margin on the sale of a substitute product that did not stock out.

## Appendix

### *Proof of Proposition 1.*

First I show that the price  $p^*$  is an equilibrium price. If a firm deviates and offers a price  $\hat{p}$  less than  $p^*$ , then consumers will not visit that firm because in equilibrium consumers anticipate that the firm will have zero inventory. To see this suppose instead that consumers thought the firm would choose some strictly positive capacity,  $\hat{k}_i$ . Then in equilibrium consumers will choose to visit that firm with the strictly positive probability that equates the expected surplus from the firm to the expected surplus of the firm or firms offering  $p^*$ . However, given these expectations, the firm's optimal capacity choice is  $k(\hat{p}, \phi_i)$  as given in (1), which is strictly lower than consumers expectations,  $\hat{k}_i$ , since by the definition of  $p^*$ ,

$$(v - \hat{p})s\left(F^{-1}\left((\hat{p} - c)/\hat{p}\right)\right) < (v - p^*)s\left(F^{-1}\left((p^* - c)/p^*\right)\right)$$

and in a consumer equilibrium,

$$(v - \hat{p})s\left(\hat{k}_i/\phi_i\right) = (v - p^*)s\left(F^{-1}\left((p^* - c)/p^*\right)\right),$$

which implies that

$$F^{-1}\left((\hat{p} - c)/\hat{p}\right) < \hat{k}_i/\phi_i < F^{-1}\left((p^* - c)/p^*\right)$$

since  $\hat{p} < p^*$  and  $s$  is increasing. So

$$k(\hat{p}, \phi_i) = \phi_i F^{-1}\left((\hat{p} - c)/\hat{p}\right) < \hat{k}_i,$$

and consumers will not visit the firm and in the subgame they expect  $\hat{k}_i = 0$ .

If a firm offers a price above  $p^*$ , then again, given rational expectations about the firm's capacity, consumers will realize that the firm is offering an expected consumer surplus strictly lower than  $(v - p^*)s\left(F^{-1}\left((p^* - c)/p^*\right)\right)$  and will not buy from the firm, so the subgame equilibrium expectations are that the firm will not produce.

Suppose  $p^* = c$ . Now I prove that no equilibria exist with active firms offering any other prices. Then the reasoning is identical to the Bertrand reasoning. If the lowest priced firm offers a price greater than  $c$  then any other firm can increase its profits and capture the entire market by

undercutting that firm's price, because in any subgame after prices have been announced the unique equilibrium is for the lowest priced firms to be the only active firms. So in equilibrium at least one firm offers  $p^* = c$ . Since profits are zero, firms are indifferent between being active and inactive. So firms may be inactive in equilibrium, however at least two firms must be active in equilibrium, otherwise the only active firm could raise its price without losing any market share. Since  $p^* = c$ , there is no incentive to hold speculative inventory, and the industry capacity is simply  $\underline{a}$ . With  $m$  active firms, the market is split evenly by assumption.

Suppose  $p^* > c$ . I now show that  $p^*$  is the unique equilibrium price for all firms. Suppose all firms offered other prices in equilibrium. Then any firm could deviate to  $p^*$  and offer  $(v - p^*)s(F^{-1}((p^* - c)/p^*))$  in equilibrium surplus to consumers in the subsequent subgame and capture the entire market. And they would always strictly want to do so, because profits are strictly positive when  $p^* > c$ . Let  $\phi_i \pi(k_i/\phi_i)$  be firm  $i$ 's profits if it offers capacity  $k_i$  and charges equilibrium price  $p^*$ , where

$$\pi(k_i/\phi_i) = p^* \mu s(k_i/\phi_i) - c k_i/\phi_i.$$

Since firms choose inventory to maximize profit,  $\max_{k_i} \pi(k_i/\phi_i) \geq \pi(\underline{a})$ , and  $\pi(\underline{a}) > 0$  since

$$\pi(\underline{a}) = p^* \mu s(\underline{a}) - c \underline{a} = (p^* - c) \underline{a}.$$

So  $\max_{k_i} \pi(k_i/\phi_i) > 0$ . So industry profits at  $p^*$  are strictly positive, and firm profits are strictly positive for any firm with strictly positive market share. *Q.E.D.*

*Proof of Proposition 2.*

Let  $U(p) = (v - p)s(F^{-1}((p - c)/p))$ . So  $p^* = \arg \max_{p \geq c} U(p)$  and  $p^*$  either is the solution to  $U'(p) = 0$ , or  $p^* = c$ . Since

$$U'(p) = -s(F^{-1}((p - c)/p)) + (v - p) \frac{c^2}{f(F^{-1}((p - c)/p))p^3 \mu},$$

if

$$U'(c) = -s(\underline{a}) + \frac{(v-c)}{c\mu f(\underline{a})} \leq 0,$$

then the equilibrium price will satisfy  $p^* = c$  and firms produce only to meet demand at  $\underline{a}$ . They hold no speculative inventory. So if  $s(\underline{a})\mu f(\underline{a}) \leq (v-c)/c$  then  $p^* > c$ . *Q.E.D.*

*Proof of Proposition 3.*

Suppose that consumers' beliefs satisfy (2). We can substitute (2) into the firm's profit maximization constraint as follows

$$(v - p_i)s\left(F^{-1}\left((p_i - c)/p_i\right)\right) = (v - p^C)s\left((n-1)k^C/(1 - \phi_i)\right), \quad (\text{A1})$$

which uniquely defines consumers' equilibrium strategies  $\phi_i$  in terms of  $p_i$ , the other firms' common price  $p^C$ , and the other firms' common capacity  $k^C$ . Here after I will suppress the superscript C and denote the symmetric equilibrium price and capacity of each firm by  $p$  and  $k$ .

Clearly

$$k = \frac{1}{n} F^{-1}\left(\frac{p-c}{p}\right),$$

so the constraint can be rewritten as

$$(v - p_i)s\left(F^{-1}\left((p_i - c)/p_i\right)\right) = (v - p^C)s\left(((n-1)/n)F^{-1}\left((p^C - c)/p^C\right)/(1 - \phi_i)\right). \quad (\text{A2})$$

which defines consumers' strategies  $\phi_i(p_i, p)$  as a function of firm i's own price and other firms' common price. The left-hand side of (A2) is maximized at  $p_i = p^*$  and since  $s$  is an increasing function, it is clear that for all  $p$  the function  $\phi_i(p_i, p)$  is also maximized at  $p_i = p^*$ . So

$$\partial\phi_i(p_i, p)/\partial p_i|_{p_i=p^*} = 0.$$

Differentiating (A1) with respect to  $p_i$  yields

$$\frac{\partial\phi_i(p_i, p)}{\partial p_i} = \frac{U'(p_i)\mu(1 - \phi_i)^2}{(v - p)\left(1 - F\left(\left(\frac{n-1}{n}\right)F^{-1}\left(\frac{p-c}{p}\right)/(1 - \phi_i)\right)\right)\left(\frac{n-1}{n}\right)F^{-1}\left(\frac{p-c}{p}\right)}, \quad (\text{A3})$$

where  $U(p_i) = (v - p_i)s\left(F^{-1}\left((p_i - c)/p_i\right)\right)$  and

$$U'(p_i) = -s\left(F^{-1}\left((p_i - c)/p_i\right)\right) + (v - p_i) \frac{c}{\mu p_i} \frac{c}{f\left(F^{-1}\left((p_i - c)/p_i\right)\right) p_i^2}.$$

Since  $U'(p^*) = 0$  by definition and  $U''(p^*) < 0$  by assumption,  $\partial\phi_i(p^*, p)/\partial p_i = 0$  and  $\partial^2\phi_i(p^*, p)/\partial p_i^2 < 0$ . Note that  $\partial\phi_i(p_i, p)/\partial p_i$  is continuously differentiable.

To find the symmetric equilibrium price, I implicitly substituting the constraint, (A1), into the objective function and rewrite it as

$$\max_{p_i, k_i} p_i \left[ \phi_i \mu - \int_{k_i/\phi_i}^{\bar{a}} (\phi_i a - k_i) f(a) da \right] - c k_i.$$

The first order condition with respect to  $k_i$  is

$$\frac{\partial \pi}{\partial k_i}(p_i, p, k_i) = 0$$

or

$$k_i = \phi_i F^{-1}\left(\frac{p_i - c}{p_i}\right)$$

and the first order condition with respect to price is

$$\begin{aligned} \frac{\partial \pi}{\partial p_i}(p_i, p, k_i) &= \left[ \phi_i(p_i, p) \mu - \int_{k_i/\phi_i(p_i, p)}^{\bar{a}} (a \phi_i(p_i, p) - k_i) f(a) da \right] \\ &\quad + p_i \frac{d\phi_i(p_i, p)}{dp_i} \left[ \mu - \int_{k_i/\phi_i(p_i, p)}^{\bar{a}} a f(a) da \right] \\ &= 0 \end{aligned} \tag{A4}$$

so  $\pi_{p_i}(p_i, p, k_i^*(p_i, p)) = 0$  implicitly defines firm  $i$ 's best response function  $p_i(p)$ . This best response relationship is increasing. First note that by the implicit function theorem and the envelope theorem

$$\frac{dp_i}{dp} = - \frac{\frac{d}{dp} g(p_i, p, k_i^*(p_i, p))}{\frac{d}{dp_i} g(p_i, p, k_i^*(p_i, p))} = - \frac{\frac{\partial \pi}{\partial p \partial p_i}(p_i, p, k_i^*(p_i, p))}{\frac{\partial^2 \pi}{\partial p_i^2}(p_i, p, k_i^*(p_i, p))}.$$

Then note that

$$\frac{\partial^2 \pi}{\partial p_i^2}(p_i, p, k_i^*(p_i, p)) < 0$$

because  $\left[ \phi_i \mu - \int_{k_i/\phi_i}^{\bar{a}} (\phi_i(p_i, p)a - k_i) f(a) da \right]$  is decreasing in  $p_i$ ,  $d\phi_i(p_i, p)/dp_i$  is negative and decreasing in  $p_i$ , and  $p_i \left[ \mu - \int_{k_i/\phi_i}^{\bar{a}} af(a) da \right] = p_i \int_{\underline{a}}^{k_i/\phi_i} af(a) da$  is positive and increasing in  $p_i$ , Finally note that

$$\frac{\partial \pi}{\partial p \partial p_i}(p_i, p, k_i^*(p_i, p)) > 0$$

because  $\left[ \phi_i \mu - \int_{k_i/\phi_i}^{\bar{a}} (\phi_i(p_i, p)a - k_i) f(a) da \right]$  and  $d\phi_i(p_i, p)/dp_i$  are both increasing in  $p$ . So  $p_i(p)$  is strictly increasing.

This implies that the equilibrium is unique. To prove that it exists, since  $p_i(p)$  is continuous, it suffices to observe that  $p_i(p^*) > p^*$ , which follows immediately from (A4), and that  $\lim_{x \rightarrow v} p_i(x) < v$ , since  $\lim_{p \rightarrow v} d\phi_i(p_i, p)/dp_i = -\infty$  (at a price lower than  $v$  the firm captures the entire market), so  $p_i(p)$  has unique fixed point.

The only remaining part of the proof is convergence. First I solve implicitly for the symmetric equilibrium.

$$\frac{\partial \phi_i(p, p)}{\partial p_i} = \frac{U'(p) \mu (1 - \phi_i)^2}{(v - p)(1 - F(nk))(n - 1)k},$$

and

$$k = \frac{1}{n} F^{-1}((p - c)/p)$$

so

$$\frac{\partial \phi_i(p, p)}{\partial p_i} = \frac{U'(p) \mu (1 - \phi_i)^2}{(v - p) \left( \frac{c}{p} \right) (n - 1)k}.$$

So  $\frac{\partial \phi_i(p, p)}{\partial p_i}$  is decreasing in  $n$ , which implies that  $p$  is decreasing in  $n$ . As  $n$  approaches infinity,

$\frac{\partial \phi_i(p, p)}{\partial p_i}$  goes to zero for price greater than  $p^*$ , so price must be converging to  $p^*$ . *Q.E.D.*

*Proof of Proposition 4.*

An equilibrium of the strategic-form game is a price and quantity for each firm and, for consumers, a probability distribution over visiting each firm for each possible realization of

prices. Suppose there exists a symmetric Bayesian equilibrium that has different payoffs than the equilibrium described in Proposition 3 and that does not satisfy (2) off the equilibrium path.

Then that equilibrium cannot satisfy NWBR; all the off-the-equilibrium path strategies that do not satisfy (2) are never a weak best response and will be eliminated, so the candidate equilibrium will not be an equilibrium of the modified game. Recall that given (2), a firm's service rate is uniquely defined by prices, and is independent of consumers' strategies. Also recall that it is also possible to eliminate, by strict dominance, any firm strategies outside of the set  $(p_i, k_i) \in [0, v) \times (0, \bar{a}]$ . So consumers' off-the-equilibrium path conjectures in the modified game all satisfy (2) and the unique equilibrium is the one Proposition 3.

Finally it is still necessary to prove that the symmetric Bayesian equilibrium in Proposition 3 satisfies the NWBR property. This requires that we decide how to apply NWBR to infinitesimal consumers. Since consumers remain indifferent over pure strategies, it seems that NWBR cannot eliminate consumer strategies at all. Alternatively, imagine a representative consumer who maximizes his own utility and imagine every consumer adopts that strategy. Then the consumer equilibrium is a dominant strategy equilibrium. It doesn't matter which approach we take. In either case the set of equilibria of the pruned game will stay the same.

The set of strategies for firms can also be pruned. In particular extremely low and extremely high prices on the interval are strictly dominated. Iterative dominance will shrink the set of prices, but as long as the range of prices includes the equilibrium prices, the equilibrium will remain unchanged, and as long as the equilibrium is unchanged the equilibrium strategies will never be pruned. So the equilibrium does satisfy the NWBR property. *Q.E.D.*



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