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Licensing interim R&D knowledge*

By

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Abstract

This paper considers three firms that engage in an R&D contest to develop a new profitable technology. For a broad range of parameters, the firm that leads the contest (i.e., has the highest probability of success) is better-off licensing or selling its superior interim knowledge to one of the two lagging firms or to both rather than holding on to its lead. Although transferring interim R&D knowledge to the lagging firms erodes the technological lead of the leading firm, it allows it to extract rents from its rivals and can possibly create value by increasing the chance that the licensee(s) will develop the new technology when the leading firm fails.

JEL Classification: D45, O32

Keywords: Interim R&D knowledge, exclusive and nonexclusive licensing, transfer of knowledge, cross-licensing.
1 Introduction

Many licensing agreements are reached at the early stages of the R&D process before the commercial success of the licensed technology has been guaranteed. Such agreements seem to be particularly common in the chemical and pharmaceutical industries where they account for over 20% of all licensing agreements (Anand and Khanna, 2000). This percentage is even higher in biotechnology. Kalamas, Pinkus, and Sachs (2002) report that about a third of all licensing deals between the top 12 pharmaceutical companies and biotechnology firms from 1991 to 2002 took place during the preclinical testing stage.\(^1\) Howard (2004) reports that there is an increasing trend towards early-stage licensing in biotechnology, with over 60% of all agreements between the top 20 pharmaceutical firms and biotechnology firms from 1997 to 2002 taking place at the discovery and the lead molecule phases, which are the earliest stages in the development process of new drugs. Moreover, over 50% of all biotechnology licensing agreements studied by Lim and Veugelers (2003) were made at very early stages of their development, before any prototypes or clinical test results were available.\(^2\) By contrast, less than 5% of the agreements in their data were at advanced stages of development. Bearing in mind that the success rate for new drugs at the preclinical testing stage is only about 20% for self-originating drugs and about 25% – 38% for acquired new drugs (see e.g., DeMasi, 2001), it is clear that early-stage licensing agreements involve interim R&D knowledge that may or may not eventually lead to a commercially successful product or process.

Despite the prevalence of early-stage technology licensing, most of the licensing literature, with only few exceptions, has studied agreements that involve commercial products or

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\(^1\)The development of new drugs consists of several, mostly sequential, phases: the discovery phase in which the targeted substance is identified and validated with a medically important function, the lead molecule phase in which the lead molecule that is supposed to interact with the targeted substance is identified and validated, the preclinical phase in which the drug is tested on animals or in vitro, the phase I, phase II, and phase III clinical trials in which the new compound is tested on human subjects, and finally, the New Drug Application (NDA) stage in which the company files an NDA with the Food and Drug Administration (FDA). For some drugs, the FDA requires additional studies (Phase IV) to evaluate long-term effects. In total, the entire development process takes on average 10 – 12 years.

\(^2\)Their data consists of over 240 U.S. biotechnology licensing contracts, dealing mainly with the health care industry, and include diagnostics, drugs, cultivation of cells, and laser imaging.
processes. This paper by contrast examines licensing of precommercial interim R&D knowledge which enhances the chances to ultimately develop a commercially profitable technology but does not guarantee it. Hence, while licensing of commercial technologies affects competition in the product market directly, licensing of interim R&D knowledge affects it only indirectly by affecting the licensees’ chances to develop a commercially successful technology. Moreover, in my model, firms have no incentives to either license or sell commercial technologies due to competition in the product market. However, as we shall see below, for a broad range of parameters, firms will have an incentive to license and/or to sell interim R&D knowledge.

Specifically, I consider an R&D contest between three firms for the development of a new technology (e.g., a new drug, a superior production process). The R&D outcome is binary: each firm either succeeds to develop the new technology or it fails. The paper focuses on the licensing decisions of the three firms at some interim stage, before the R&D contest has yet been decided. The main question then is whether at the interim stage, the leading firm with the best chance of successfully developing the new technology will prefer to hold on to its technological lead or whether it would prefer to license out its superior interim knowledge to one of the lagging firms or to both.

The decision to license interim R&D knowledge is driven in my model by the interplay between three effects. First, licensing interim R&D knowledge raises the probability that the licensee will successfully develop the new technology when the leading firm fails. This effect creates value which the leading firm can capture through a license fee. Second, the agreement raises the probability that the licensee will successfully develop the new technology when the leading firm also succeeds. This effect destroys value since competition between firms in the product market lowers their profits. Third, a licensing agreement lowers the probability that a non-licensee will be the sole developer of the new technology. Consequently, the lagging firms are willing to pay the leading firm not only for the right to obtain its superior knowledge but also in order to ensure that the remaining firm does not get access to this knowledge.³

³This effect is reminiscent of Katz and Shapiro (1986) where a licensor plays the potential licensees off against one another so that in equilibrium, the licensees are made worse off due to the innovation. This idea was also used by Anton and Yao (1994, 2002) to show that an independent inventor can extract rents from
Given these three effects, I completely characterize the optimal licensing decision of the leading firm. In particular, I show in Section 3 that for a broad range of parameters, the leading firm will prefer to license out its knowledge either exclusively to the second firm in the R&D contest or to both lagging firms. Only when it is close to successfully developing the new technology will the leading firm prefer to hold on to its technological lead. Licensing is not profitable in this case because it is highly likely that both the leading firm and the licensee(s) will successfully develop the new technology and will end up competing against each other in the product market. This implies that paradoxically, a license is not worth much when the licensor has a lot of interim R&D knowledge to transfer. It is also interesting to note that the leading firm does not issue an exclusive license to its weakest rival but rather to its strong rival who poses a greater competitive threat for the leading firm. The reason for this is that licensing knowledge to the weak rival compromises the leading firm’s chances to be the sole developer of the new technology to a larger extent as it converts a weak rival into an equally strong rival.

In Section 4, I examine how the equilibrium changes under various scenarios. In Section 4.1, I consider the case where it is feasible for the leading firm to transfer only parts of its knowledge to the lagging firms. I show that whenever the leading firm is sufficiently close to success, it will prefer to issue vacuous licences to both rivals that transfer them as little knowledge as possible. The rivals will nonetheless accept these vacuous licenses in order to ensure that the leading firm will not transfer its knowledge exclusively to their rival. In Section 4.2, I show that before being approached by the leading firm, the two lagging firms can benefit from reaching a bilateral licensing agreement in which the second firm in the contest transfers its knowledge to the last firm in the contest. The advantage of this agreement is that it strengthens the bargaining power of the two lagging firms vis-a-vis the leading firm. In Section 4.3, I examine the case in which the leading firm’s knowledge is more valuable to the lagging firms than to the leading firm. This situation is interesting because there are many cases in which relatively small innovative firms license their precommercial technologies to large firms who have better capabilities to successfully commercialize these technologies. I an exclusive buyer of his knowledge by threatening to reveal the knowledge to a rival and thereby destroy the buyer’s monopoly rents.
show that in this case, there is a broad range of parameters for which nonexclusive licenses issued. Moreover, if the leading firm’s probability of success is sufficiently high, then there is also a broad range of parameters for which the leading firm will issue an exclusive licence to the second firm in the contest. In Section 4.4, I turn to the possibility that under licensing, the success probabilities of the licensor and the licensees become positively correlated. I show that the qualitative results of Section 3 remain valid when the degree of correlation is small, but not otherwise. In particular, for moderate levels of correlation, the leading firm may prefer to issue an exclusive license to the weakest rival. And, in Section 4.5, I show that a ban on exclusive licenses will induce the leading firm to issue nonexclusive licenses to both lagging firms only if its knowledge ensures a relatively low probability of success. Otherwise, such a ban will have an unintended consequence in that it will induce the leading firm to hold on to its technological lead rather than license it out.

In Section 5, I examine the possibility that the leading firm will sell rather than license its superior interim knowledge. The difference between licensing and selling is that under licensing, the leading firm stays in the contest, whereas under selling it exits the contest after transferring its knowledge. As might be expected, selling knowledge is particularly valuable when the leading firm’s knowledge ensures a high probability of success: given that the leading firm exits the contest, the acquirer is left with a high probability of being the sole developer of the new technology. Moreover, each of the two lagging firms is eager in this case to ensure that the leading firm’s knowledge is not sold exclusively to the rival firm. By contrast, when the leading firm’s knowledge is associated with a low probability of success, the leading firm would sell its knowledge to both lagging firms. Selling knowledge, however, is not always profitable: when the knowledge of the leading firm only ensures an intermediate probability of success, the leading firm is better off holding on to its technological lead.

Finally, in Section 6, I relax the assumption that the interim knowledge of the three firms can be Blackwell ordered (i.e., the knowledge of the last firm is a subset of the knowledge of the second firm, which is in turn a subset of the knowledge of the leading firm), and assume instead that each firm has its own unique approach to R&D. In that case, the knowledge of each firm is valuable to both of its rivals. I show that in this case, the joint expected payoff of the three firms may be maximized when one of the lagging firms is the licensor, and
moreover, it may be profitable for the three firms to engage in cross-licensing agreements.

There is a sizeable literature on the licensing of commercial technologies (see Kamien, 1992, for a survey of this literature). Most of this literature considers an outside inventor (say an R&D lab) who holds a patent for a commercially profitable technology and asks what is the most profitable way for the inventor to license his technology to firms that are active in the market. By contrast, the licensor in the current paper is an active firm who engages in an R&D contest with its licensees for developing a commercially profitable technology. Licensing to rivals or potential rivals has also been studied by Gallini (1984) and Rockett (1990). Gallini shows that a firm might license its superior knowledge to a potential rival in order to lower its incentive to invent a superior product. In Rockett (1990), the licensor is an incumbent firm who licenses its technology to a weak entrant in order to deter entry by a stronger entrant. In both papers however, licensing involves commercial technologies and the main motivation is to preserve the dominant position of the licensor. In my model by contrast, licensing involves interim R&D knowledge and while it compromises the licensor’s chances to be the sole developer of the new technology, it also has the advantage of allowing the licensor to extract rents from the licensees.

Earlier papers that consider licensing of interim R&D knowledge include d’Aspremont, Bhattacharya, and Gerard-Varet (2000), Bhattacharya, Glazer, and Sappington (1992), and Bhattacharya and Guriev (2006). The focus of these papers however is different than the focus of my paper. Bhattacharya, Glazer, and Sappington (1992) study the conditions under which simple licensing schemes induce the members of a research joint venture to optimally invest in R&D and to fully disclose their private interim R&D knowledge to other members. d’Aspremont, Bhattacharya, and Gerard-Varet (2000) consider two rivals in a winners-takes-all R&D contest that bargain over the licensing of interim R&D knowledge from the leading to the lagging firm under the assumption that the leading firm has private information about the extent of its technological lead. They prove that there exists a large class of incentive compatible and individually rational direct bargaining mechanisms which induce efficient outcomes (agreement is reached without delay and the leading firm fully discloses its interim R&D knowledge). Finally, Bhattacharya and Guriev (2006) consider a research lab that wishes to license its interim R&D knowledge to one of two competing firms that can
develop this knowledge into a commercial product. The main focus of their paper is on the comparison between a patent-based licensing mode in which the research lab can commit to sell its knowledge to only one firm, and a trade-secret-based mode in which the research lab cannot make a similar commitment.

2 The model

Consider an R&D contest between three firms for developing a new commercial technology (e.g., a new drug, a superior production process). Suppose that the contest has reached some intermediate level at which the knowledge that three firms have already accumulated can be summarized by a vector \((\lambda_1, \lambda_2, \lambda_3)\), where \(\lambda_i < 1\) represents the probability that firm \(i\) will eventually succeed to develop the new technology. With probability \(1 - \lambda_i\), firm \(i\) will fail and will develop nothing. I assume without a loss of generality that \(\lambda_1 > \lambda_2 \geq \lambda_3\), so that firm 1 is the current leader in the contest, with firm 2 being second and firm 3 being last.

In what follows, I will assume that the probabilities \(\lambda_1, \lambda_2, \) and \(\lambda_3\), are independent of each other and are common knowledge.\(^4\) Moreover, I will assume that the knowledge of the three firms can be Blackwell ordered in the sense that firm 3’s knowledge is a subset of 2’s knowledge which is in turn a subset of firm 1’s knowledge. This assumption implies that firm 1 may wish to license its superior knowledge either to firm 2, or to firm 3, or to both. If it does, then the probability that the licensee(s) (firm 2 or firm 3 or both) will successfully develop the new technology at the end of the third stage increases to \(\lambda_1\), although it remains independent of the success probability of firm 1.

In Sections 4.1-4.4 below I will relax some of these assumptions and consider the possibility of a partial transfer of knowledge in Section 4.1, the possibility that firm 2 will license its knowledge to firm 3 before both firms are approached by firm 1 in Section 4.2, the

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\(^4\)The assumption that the vector \((\lambda_1, \lambda_2, \lambda_3)\) is common knowledge is clearly made for simplicity. However, given that very little is known about the licensing of interim R&D knowledge, this assumption seems like a natural starting point. Moreover, in many applications, this assumption is a reasonable approximation since firms can assess each others’ probability of success through various channels, like patent applications, results of clinical trials, public announcement, scientific publications, informal exchange of information between employees of different firms, etc.
case where firm 1’s knowledge is more valuable to firms 2 and 3 than it is to firm 1 in Section 4.3, and positive correlation between the success probabilities of firm 1 and its licensee(s) in Section 4.4.

Given the basic setup, a licensing agreement creates three main effects. First, it creates value by raising the probability that the licensee will successfully develop the new technology when firm 1 fails. Second, a licensing agreement raises the probability that the licensee will successfully develop the new technology when firm 1 also succeeds. This effect destroys value since the aggregate profits are higher when only one firm succeeds to develop the new technology. Third, a licensing agreement lowers the probability that a non-licensee will be the sole developer of the new technology. This effect allows firm 1 to extract value from its licensees by “threatening” them that should they reject its offer, it will license out its technology exclusively to the rival firm.

For simplicity, I assume that once the R&D contest ends, the three firms engage in Bertrand competition in the product market. Consequently, if more than one firm successfully develops the new technology (whether this technology leads to a new product or to marginal cost reduction), competition in the product market drives the profits of all firms to 0. If only one firm is successful, then this firm monopolizes the product market and earns a profit that I normalize to 1. This modelling approach allows me to study the above three effects of licensing in as simple manner as possible. It should be noted however that the same three effects will also be present under alternative models of product market competition, albeit the last two effects discussed above will be less extreme.5 Another advantage of this modelling approach is that firms never wish to engage in licensing when λ1 is close to 1 (i.e., when the technology is close to being commercial) because then there is a high chance that firm 1 and its licensee(s) will end up competing in the product market and will earn 0 profits. Moreover, firms have no incentive to sell fully commercial technologies since a commercial technology is worth as much to the licensor as it is worth to the licensee. Therefore the in-

5The complication will arise from the fact that there will be 6 possible payoffs for each firm associated with the following 6 cases: (i) neither firm succeeds, (ii) the firm and one rival fail while the other rival succeeds, (iii) the firm fails while its two rivals succeed, (iv) the firm succeeds while its two rivals fail, (v) the firm and one other rival succeed, (vi) and all three firms succeed. In the present model, there are only 2 payoffs: 1 when the firm succeeds while the other two firms fail, and 0 otherwise.
centive to license or sell R&D knowledge arises in my model precisely because the knowledge is interim and is not associated with a fully developed commercial technology.

3 Exclusive and nonexclusive licenses

In the second stage of the game, firm 1 decides whether or not to license out its superior knowledge to firms 2 and 3. Absent licensing, the expected payoff of firm 1 is given by:

$$\pi_1(n, n) = \lambda_1(1 - \lambda_2)(1 - \lambda_3),$$

where \((n, n)\) indicates that firm 1 did not license its knowledge to neither firm 2 nor firm 3. The expected payoffs of firms 2 and 3 are analogous. Equation (1) shows that absent licensing, a firm earns a monopoly profit (normalized to 1) only if it succeeds to develop the new technology while its two rivals fail.

If firm 1 decides to license out its knowledge, it can either issue an exclusive license to only one rival or issue nonexclusive licenses to both rivals.\(^6\) In both cases, the licensees fully obtain firm 1’s knowledge and hence their probability to successfully develop the new technology jumps to \(\lambda_1\).

In order to find out whether firm 1 will issue licenses and whether these licenses will be exclusive or nonexclusive, suppose that at the beginning of the licensing stage, firm 1 can make a pair of take-it-or-leave-it offers to firms 2 and 3 at fees \(T_2\) and \(T_3\), respectively. If both firms reject their respective offers, then none of them gets a license and no payments are made. If only one firm accepts firm 1’s offer, then this firm obtains an exclusive license and pays the associated fee to firm 1. The rejecting firm pays nothing and does not get access to firm 1’s knowledge. If both firms accept, then a tie-breaking rule determines whether firm 2 gets an exclusive license, or firm 3 gets an exclusive license, or both firms get licenses. The precise type of the tie-breaking rule is chosen by firm 1 along with the fees \(T_2\) and \(T_3\) and will be specified in Lemma 1 below.

If firm 1 licenses its knowledge exclusively to firm 2 for a license fee \(T_2\), then the

\(^6\)When firm 1 issues an exclusive license to firm \(j = 2, 3\), it commits not to transfer its knowledge to firm \(k \neq j\). Firm \(j\) on its part, also commits not to transfer the licensed knowledge to firm \(k\).
expected payoffs of the three firms are given by:

\begin{align}
\pi_1(y, n) &= \lambda_1(1 - \lambda_1)(1 - \lambda_3) + T_2, \\
\pi_2(y, n) &= \lambda_1(1 - \lambda_1)(1 - \lambda_3) - T_2,
\end{align}

and

\[ \pi_3(y, n) = \lambda_3(1 - \lambda_1)^2. \]

Apart from \( T_2 \), these equations differ from equation (1) in that firm 2’s probability of success is now \( \lambda_1 \) instead of \( \lambda_2 \). The expected payoffs of the three firms are completely analogous when firm 1 licenses its knowledge exclusively to firm 3 instead of firm 2.

If firm 1 issues nonexclusive licenses to both firms 2 and 3, then the expected payoffs of the three firms become:

\[ \pi_1(y, y) = \lambda_1(1 - \lambda_1)^2 + T_2 + T_3, \]

and

\[ \pi_j(y, y) = \lambda_1(1 - \lambda_1)^2 - T_j, \quad j = 2, 3. \]

That is, both firms 2 and 3 pay fees to firm 1 and all three firms have the same probability, \( \lambda_1 \), of developing the new technology.

**Lemma 1:** Suppose that firm 1 wishes to issue an exclusive license to firm \( j = 2, 3 \). Then, the optimal scheme from its perspective is to make take-it-or-leave-it offers with

\[ T_j^* = (1 - \lambda_1) (\lambda_1(1 - \lambda_k) - \lambda_j(1 - \lambda_1)), \quad k \neq j, \]

and

\[ T_k^* = 0, \]

and set a tie-breaking rule that specifies that only firm \( j \) gets a license if both firms accept their respective offers. If firm 1 wishes to issue nonexclusive licenses to both firms 2 and 3, then the optimal scheme from its perspective is to set

\[ \hat{T}_j^* = (\lambda_1 - \lambda_j) (1 - \lambda_1)^2, \quad j = 2, 3, \]
and set a tie-breaking rule that specifies that both firms get licenses if both accept their respective offers.

**Proof:** First, suppose that firm 1 wishes to issue an exclusive license to firm 2, and suppose that it makes simultaneous take-it-or-leave-it offers to firms 2 and 3 with licensing fees $T_2^*$ and $T_3^*$ and a tie-breaking rule that specifies that firm 2 will receive an exclusive license if both firms accept their respective offers. The licensing fees specified in the Lemma are the solutions of $\pi_2(y, n) = \pi_2(n, y)$ and $\pi_3(y, n) = \pi_3(y, n)$ and hence ensure that (accept, accept) is a Nash equilibrium (recall that due to the tie-breaking rule, firm 2 obtains an exclusive license when both firms 2 and 3 accept their respective offers). To see why, notice that if firm 3 accepts the offer, then $T_2^*$ leaves firm 2 indifferent between accepting and rejecting. Likewise, if firm 2 accepts the offer, then $T_3^*$ leaves firm 3 indifferent between accepting and rejecting. The tie-breaking rule then determines that firm 2 is the exclusive licensee. Similar arguments apply when firm 1 wishes to issue an exclusive license to firm 3.

Next, suppose that firm 1 wishes to issue nonexclusive licenses to firms 2 and 3 and makes them simultaneous take-it-or-leave-it offers with licensing fees $\hat{T}_2^*$ and $\hat{T}_3^*$ and a tie-breaking rule that specifies that both firms will receive a license if both accept their respective offers. Since $\hat{T}_2^*$ and $\hat{T}_3^*$ specified in the lemma are the solutions of $\pi_2(y, y) = \pi_2(n, y)$ and $\pi_3(y, y) = \pi_3(y, n)$, it is clear that (accept, accept) is a Nash equilibrium.

Finally, note that the licensing fees stated in the lemma are the highest that firms 2 and 3 will agree to pay for licenses since they represent for each firm the difference between its expected payoff when it gets a license (the “best” outcome that the firm can hope for) and its expected payoff when the rival firm gets an exclusive license (the “worst” outcome from the firm’s perspective).

Lemma 1 shows that whenever firm 1 wants to issue licenses, it can do so by making take-it-or-leave-it offers to firms 2 and 3 that both firms accept. The tie-breaking rule then specifies whether both firms or only one of them will get a license. The optimal licensing fees

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7Of course, firm 1 can always break the indifference of firms 2 and 3 and make (accept, accept) a strict Nash equilibrium by lowering the license fees slightly. Since this point is trivial, I will not mention it in the sequel.
are designed such that firm 1 extracts not only the entire surplus that the licensees receive from getting access to firm 1’s knowledge but also the surplus from preventing the rival firm from getting an exclusive access to this knowledge. In a sense then, firm 1 plays firms 2 and 3 off against one another since if a licensee rejects his offer, the rival firm receives an exclusive license. This situation is of course the worst case scenario for each firm since then it is left behind in the R&D contest and its chances to win the contest are diminished. Hence, the licensing fees can be viewed as reflecting a payment for firm 1’s knowledge, as well as for preventing the rival firm from being the exclusive licensee of this knowledge.

Interestingly, $T_2^* > T_3^*$ (firm 2 pays a higher fee for an exclusive license) when $\lambda_1 < 1/2$ and vice versa when $\lambda_1 > 1/2$. The reason for this is that the exclusive license fees are equal to the difference between the expected payoff when the firm is an exclusive licensee and its expected payoff when its rival is the exclusive licensee. Since $\lambda_2 \geq \lambda_3$, the expected payoff of firm 2 as an exclusive licenses exceeds the expected payoff of firm 3 as an exclusive licensee because firm 2 faces a “weak” non-licensee whose probability of success is $\lambda_3$, whereas in the opposite case, firm 3 faces a “strong” non-licensee whose probability of success is $\lambda_2$. On the other hand, firm 2 has a higher probability to succeed without a license, so its “disagreement payoff” (i.e., its expected payoff when firm 3 is the exclusive licensee) is higher than the “disagreement payoff” of firm 3. It turns out that the first consideration dominates when $\lambda_1 < 1/2$ so $T_2 > T_3$, whereas the second consideration dominates when $\lambda_1 > 1/2$ so $T_3 > T_2$. In the case of nonexclusive licenses, only the second consideration is relevant since the probability of being the sole developer of the new technology is equal to $\lambda_1(1 - \lambda_1)^2$ for all three firms. Hence, firm 1 can extract more money from firm 3 so $\hat{T}_3^* \geq \hat{T}_2^*$.

When firm 1 issues licenses it has to trade off the fees that it receives against the erosion in its chance to be the sole developer of the new technology. The next proposition studies this trade-off and fully characterizes firm 1’s licensing decisions.

**Proposition 1:** In equilibrium, firm 1 will

(i) issue nonexclusive licenses to both firms 2 and 3 if $\lambda_1 < 1/3$;

(ii) issue an exclusive license to firm 2 if $1/3 \leq \lambda_1 < \lambda_1^*$, where $\lambda_1^* \in \left(\frac{1}{2\lambda_3}, 1\right)$ is defined
implicitly by

\[ B(\lambda_1, \lambda_2, \lambda_3) \equiv \pi_1^*(y, n) - \pi_1(n, n) \]
\[ = \lambda_1 (1 - \lambda_3) (1 - 2\lambda_1 + \lambda_2) - \lambda_2 (1 - \lambda_1)^2 = 0, \]  

and is increasing with \( \lambda_2 \) and decreasing with \( \lambda_3 \); and

(iii) not issue any licenses if \( \lambda_1 \geq \lambda_1^* \).

**Proof:** If firm 1 issues an exclusive license to firm 2 at \( T_2^* \), then its expected payoff is

\[ \pi_1^*(y, n) = \lambda_1(1 - \lambda_1)(1 - \lambda_3) + T_2^* \]
\[ = (1 - \lambda_1) [2\lambda_1(1 - \lambda_3) - \lambda_2(1 - \lambda_1)]. \]  

If firm 1 issues an exclusive license to firm 3 at \( T_3^* \), then its expected payoff is

\[ \pi_1^*(n, y) = \lambda_1(1 - \lambda_2)(1 - \lambda_1) + T_3^* \]
\[ = (1 - \lambda_1) [2\lambda_1(1 - \lambda_2) - \lambda_3(1 - \lambda_1)]. \]  

If firm 1 issues nonexclusive licenses at \( \hat{T}_2^* \) and \( \hat{T}_3^* \), then its expected payoff is

\[ \pi_1^*(y, y) = \lambda_1(1 - \lambda_1)^2 + \hat{T}_2^* + \hat{T}_3^* \]
\[ = (1 - \lambda_1)^2 [3\lambda_1 - \lambda_2 - \lambda_3]. \]  

And, if firm 1 does not issue any licenses, then its expected payoff is given by (1).

Comparing equations (8)-(10) reveals that since \( \lambda_1 > \lambda_2 \geq \lambda_3 \), then \( \pi_1^*(y, y) > \max\{\pi_1^*(y, n), \pi_1^*(n, y)\} \) for all \( \lambda_1 < 1/3 \), and \( \pi_1^*(y, n) > \max\{\pi_1^*(y, y), \pi_1^*(n, y)\} \) for all \( 1/3 < \lambda_1 < 1 \). That is, if firm 1 wishes to issue licenses at all, it will issue nonexclusive licenses to both firms 2 and 3 if \( \lambda_1 < 1/3 \) and will issue an exclusive license to firm 2 if \( 1/3 < \lambda_1 < 1 \).

To examine whether firm 1 will issue licenses at all, suppose that \( \lambda_1 < 1/3 \). In that case, if firm 1 issues licenses at all, it will issue nonexclusive licenses to both firms 2 and 3. Using equations (10) and (1) yields,

\[ \pi_1^*(y, y) - \pi_1(n, n) = (1 - \lambda_1)^2 [3\lambda_1 - \lambda_2 - \lambda_3] - \lambda_1(1 - \lambda_2)(1 - \lambda_3) \]
\[ = (2\lambda_1 - \lambda_2 - \lambda_3)(1 - 3\lambda_1 + \lambda_1^2) + \lambda_1(\lambda_1^2 - \lambda_2\lambda_3) > 0, \]
Hence, whenever \( \lambda_1 < 1/3 \), firm 1 is better off issuing nonexclusive licenses than not issuing any licenses.

Next, suppose that \( \lambda_1 \geq 1/3 \). Then, if firm 1 issues licenses at all, it will issue an exclusive license to firm 2. The difference between the expected payoff of firm 1 when it issues an exclusive license to firm 2 and when it issues no licenses is given by \( B(\lambda_1, \lambda_2, \lambda_3) \), which is defined in the proposition. Note that \( B(\lambda_1, \lambda_2, \lambda_3) \) is concave in \( \lambda_1 \) and that evaluated at \( \lambda_1 = 1/3 \),

\[
B\left(\frac{1}{3}, \lambda_2, \lambda_3\right) = \frac{(1 - \lambda_2)(1 - \lambda_3) - 4\lambda_2\lambda_3}{9} > 0, \tag{12}
\]

where the inequality follows because, given that \( 1/3 = \lambda_1 > \lambda_2 \geq \lambda_3 \), the first term in the numerator is bounded from below by \( 4/9 \), whereas the second term is bounded from above by \( 4/9 \). On the other hand, as \( \lambda_1 \) approaches 1,

\[
\lim_{\lambda_1 \to 1} B(\lambda_1, \lambda_2, \lambda_3) = -(1 - \lambda_2)(1 - \lambda_3) < 0. \tag{13}
\]

Since \( B(\lambda_1, \lambda_2, \lambda_3) \) is an inverse U-shaped function of \( \lambda_1 \), (12) and (13) ensure the existence of a unique value of \( \lambda_1 \), denoted \( \lambda_1^* \), such that \( B(\lambda_1, \lambda_2, \lambda_3) > 0 \) for all \( 1/3 < \lambda_1 < \lambda_1^* \) and \( B(\lambda_1, \lambda_2, \lambda_3) < 0 \) for all \( \lambda_1^* < \lambda_1 < 1 \). The value of \( \lambda_1^* \) is implicitly defined by \( B(\lambda_1, \lambda_2, \lambda_3) = 0 \). Notice for later use that evaluated at \( \lambda_1^* \), \( B(\lambda_1, \lambda_2, \lambda_3) \) is decreasing with \( \lambda_1 \). Also, notice that evaluated at \( \lambda_1 = \frac{1}{2 - \lambda_3} \),

\[
B\left(\frac{1}{2 - \lambda_3}, \lambda_2, \lambda_3\right) = \frac{(1 - \lambda_3)(\lambda_2 - \lambda_3)}{(2 - \lambda_3)^2} > 0.
\]

Hence, \( \lambda_1^* > \frac{1}{2 - \lambda_3} \).

To examine how \( \lambda_1^* \) varies with \( \lambda_2 \) and \( \lambda_3 \), note that \( \frac{\partial \lambda_1^*}{\partial \lambda_j} = -\frac{\partial B(\lambda_1^*, \lambda_2, \lambda_3)}{\partial \lambda_j} \), \( j = 2, 3 \). Since evaluated at \( \lambda_1^* \), \( B(\lambda_1, \lambda_2, \lambda_3) \) is decreasing with \( \lambda_1 \), it follows that \( \frac{\partial \lambda_1^*}{\partial \lambda_j} = \text{sign}\left[ \frac{\partial B(\lambda_1^*, \lambda_2, \lambda_3)}{\partial \lambda_2}\right] \), \( j = 2, 3 \). The result follows by noting that

\[
\frac{\partial B(\lambda_1^*, \lambda_2, \lambda_3)}{\partial \lambda_2} = \lambda_1^* (1 - \lambda_3) - (1 - \lambda_1^*)^2
= \lambda_1^* \left(\frac{1 - \lambda_3}{\lambda_2}\right)[\lambda_2 - (1 - 2\lambda_1^* + \lambda_2)]
= \lambda_1^* \left(\frac{1 - \lambda_3}{\lambda_2}\right)[2\lambda_1^* - 1] > 0,
\]
where the second equality follows by substituting for \((1 - \lambda_1^*)^2\) from (7) and the inequality follows since \(\lambda_1^* > 1/2\). Likewise,
\[
\frac{\partial B(\lambda_1^*, \lambda_2, \lambda_3)}{\partial \lambda_3} = -\lambda_1^* (1 - 2\lambda_1^* + \lambda_2) = -\frac{\lambda_2 (1 - \lambda_1^*)^2}{1 - \lambda_3} < 0,
\]
where the second equality follows by substituting for \((1 - 2\lambda_1^* + \lambda_2)\) from (7).

Proposition 1 shows that when \(\lambda_1\) is close to 1, firm 1 is better off holding on to its technological lead and not issuing any licenses. To see why, note from Lemma 1 that as \(\lambda_1\) approaches 1, the fees that firm 1 can get by licensing its knowledge approach 0. The reason for this is simple: whenever \(\lambda_1\) is close to 1, it is highly likely that after firm 1’s knowledge has been licensed, more than one firm will ultimately develop the new technology, in which case competition in the product market will drive profits to 0. Therefore, firm 1 has little to gain from licensing its knowledge. At the same time, firm 1 stands to lose significantly from giving up its technological lead when \(\lambda_1\) is large; hence it is clear that firm 1 is better off not issuing any licenses.

By contrast, when \(\lambda_1\) is intermediate (between 1/3 and \(\lambda_1^*\)), firm 1 prefers to issue an exclusive license to its closest rival, firm 2. Although Lemma 1 shows that firm 3 is willing to pay more than firm 2 for an exclusive license when \(\lambda_1 < 1/2\), licensing knowledge exclusively to firm 2 also implies a smaller erosion of firm 1’s technological lead, as firm 1’s chance to be the sole developer of the new technology is higher when the non-licensee is firm 3, whose probability of success, \(\lambda_3\), is smaller than that of firm 2. Notice that since \(\lambda_1^*\) is increasing with \(\lambda_2\) and decreasing with \(\lambda_3\), the range of parameters for which firm 1 will issue an exclusive license to firm 2 (of which \(\lambda_1^*\) is the upper bound) expands as firms 2 becomes a stronger competitor and as firm 3 becomes a weaker competitor.

When \(\lambda_1 < 1/3\), firm 1 is better off licensing its knowledge to both firms 2 and 3. Now, firm 1’s chance to develop the new technology is relatively small, so licensing involves only a small loss of technological advantage. Although the licensing fees that firm 2 and 3 are willing to pay in this case to ensure that they are not left behind in the R&D contest are also small, they are sufficiently large to more than compensate firm 1 for this loss.

Finally, note that Proposition 1 implies that firm 1 never wishes to issue an exclusive license to firm 3 which is lagging behind the firms 1 and 2 in the R&D contest. Such an
option is dominated by issuing nonexclusive licenses to firms 2 and 3 when $\lambda_1 < 1/3$ and by issuing an exclusive license to firm 2 when $\lambda_1 > 1/3$.\footnote{The result that firm 1 never issues an exclusive license to the “weak” rival (firm 3) stands in contrast to Rockett (1990), where the dominant firm prefers to license out its technology to a “weak” rival in order deter entry by a “strong” rival. The difference arises because the motivation for licensing in Rockett’s paper is to preserve the dominant position of the licensor, whereas in my paper, the main motivation is to increase the overall probability of success and extract surplus from the licensees.}

4 Extensions

4.1 Partial transfer of knowledge

Thus far I have assumed that when firm 1 licenses its knowledge to firms 2 and 3, it transfers it fully. The question is what happens when firm 1 can transfer only parts of its superior knowledge: will it have an incentive to transfer only limited amounts of its superior knowledge or transfer all of it?

To examine this question, suppose that firm 1 can control how much of its superior knowledge it transfers to firms 2 and 3, and let $\Delta_2 \leq \lambda_1 - \lambda_2$ and $\Delta_3 \leq \lambda_1 - \lambda_3$ be the amounts of knowledge transferred to firms 2 and 3, respectively. Moreover, suppose that as before, firm 1 can make take-it-or-leave-it offers to firms 2 and 3, which, are designed such that if a firm rejects its respective offer, then firm 1 will transfer its entire knowledge to the rival firm.\footnote{To illustrate, suppose that firm 1 wishes to issue an exclusive license to firm 2. Firm 1 can then offer firms 2 and 3 to obtain $\Delta_2$ and $\Delta_3$ at fees $T_2$ and $T_3$, respectively, such that firm i’s expected payoff if it accepts is equal to its payoff if it rejects the offer and firm j receives an exclusive license. In addition, firm 1 sets a tie-breaking rule that stipulates that when both firms accept, firm 2 receives an exclusive license with $\Delta_2 = \lambda_1 - \lambda_2$.}

The resulting expected payoff of firm 1, as a function of $\Delta_2$ and $\Delta_3$, is given by

$$
\pi_1(\Delta_2, \Delta_3) = \lambda_1(1 - \lambda_2 - \Delta_2)(1 - \lambda_3 - \Delta_3) \\
+ [(\lambda_2 + \Delta_2)(1 - \lambda_1)(1 - \lambda_3 - \Delta_3) - \lambda_2(1 - \lambda_1)^2] \\
+ [(\lambda_3 + \Delta_3)(1 - \lambda_1)(1 - \lambda_2 - \Delta_2) - \lambda_3(1 - \lambda_1)^2].
$$

(14)

Since $\pi_1(\Delta_2, \Delta_3)$ is a linear function of $\Delta_2$ and $\Delta_3$, there are 4 possibilities in equilibrium:
(i) $\Delta_2 = \lambda_1 - \lambda_2$ and $\Delta_3 = \lambda_1 - \lambda_3$ (firm 1 transfers its entire knowledge to both firms), (ii) $\Delta_2 = \lambda_1 - \lambda_2$ and $\Delta_3 = 0$ (firm 1 transfers its entire knowledge exclusively to firm 2), (iii) $\Delta_2 = 0$ and $\Delta_3 = \lambda_1 - \lambda_3$ (firm 1 transfers its entire knowledge exclusively to firm 3), or (iv) $\Delta_2 = \Delta_3 = 0$ (firm 1 transfers none of its knowledge). It is easy to check that in the latter case, $\pi_1(0, 0) > \pi_1(n, n)$: if firm 1 prefers to transfer none of its knowledge, then it will issue firms 2 and 3 vacuous licenses that transfer them virtually no knowledge rather than not issue any licenses. Firms 2 and 3 will nonetheless accept these vacuous licenses because of firm 1’s “threat” to transfer its entire knowledge to the rival firm if its offer is rejected. Essentially then, firm 1 plays firms 2 and 3 off against one another and extracts money from them in exchange for its guarantee not to transfer its entire knowledge exclusively to the rival firm.

I now establish the following result:

**Proposition 2:** Suppose that firm 1 can transfer partial amounts of knowledge to firms 2 and 3. Then in equilibrium, firm 1 will

(i) issue nonexclusive licenses to firms 2 and 3 and will transfer them its entire knowledge if $\lambda_1 \leq 1/3$;

(ii) issue an exclusive license to firm 2 and transfer it its entire knowledge if $1/3 < \lambda_1 < \frac{1-2\lambda_3}{2-3\lambda_3}$, where $\frac{1-2\lambda_3}{2-3\lambda_3} \leq 1/2$;

(iii) issue nonexclusive licenses to firms 2 and 3 but will transfer them as little knowledge as possible if $\lambda_1 \geq \frac{1-2\lambda_3}{2-3\lambda_3}$.

**Proof:** See the Appendix.

Proposition 2 is illustrated in Figure 1. As the figure shows, firm 1’s ability to license partial knowledge matters only when $\lambda_1 \geq \frac{1-2\lambda_3}{2-3\lambda_3}$. In this range, firm 1 is issuing nonexclusive “vacuous” licenses to firms 2 and 3 and transfers them virtually no knowledge rather than issuing an exclusive license to firm 2 or not issuing licenses at all. As argued above, firms 2 and 3 accept the vacuous licenses because they wish to ensure that firm 1 will not transfer its entire knowledge exclusively to the rival firm. A similar scheme was implicitly ruled out
Figure 1: Partial transfers of knowledge
in the previous subsection by the assumption that when firm 1 signs a licensing agreement, it transfers its entire knowledge to the licensee.

One may wonder why firm 1 does not wish to sign similar vacuous agreements in which it transfers no knowledge to its rivals when \( \lambda_1 < \frac{1-2\lambda_3}{2-3\lambda_3} \). After all, such agreements allow firm 1 to extract money from its rivals without compromising its technological lead. However, while transferring knowledge raises the probability of a tie between firm 1 and its licensees, it also raises the probability that the licensees will successfully develop the new technology when firm 1 fails. Hence, transferring knowledge has the potential of creating value that firm 1 can capture through the license fees. This value-creating effect is particularly large when \( \lambda_1 \) is small, because then the probability of a tie is small.

### 4.2 Transfer of knowledge between firms 2 and 3

The fact that firm 1 plays firms 2 and 3 off against one another when it makes them take-it-or-leave-it offers, suggests that before firms 2 and 3 are approached by firm 1, the two firms may wish to engage in a licensing agreement, according to which firm 2 will transfer its knowledge to firm 3. This agreement can then favorably affect the terms of the licensing agreements that firm 1 will eventually offer firms 2 and 3.

To explore this possibility, suppose that firms 2 and 3 expect that firm 1 will offer them licenses, and recall from Lemma 1 that firm 1 sets its licensing fees such that the expected payoffs of firms 2 and 3 under both exclusive and nonexclusive licenses will be equal to their respective expected payoffs when firm 1 issues an exclusive license to the rival firm. These expected payoffs are equal to \( \lambda_2(1 - \lambda_1)^2 \) and \( \lambda_3(1 - \lambda_1)^2 \), respectively. Since the joint expected payoff of firms 2 and 3, \( (\lambda_2 + \lambda_3)(1 - \lambda_1)^2 \), increases with \( \lambda_3 \), it is clear that the two firms benefit from transferring firm 2’s knowledge to firm 3 and thereby raising \( \lambda_3 \) to \( \lambda_2 \). As Proposition 1 shows, such an agreement does not affect firm 1’s choice between exclusive and nonexclusive licenses since this choice depends only on whether \( \lambda_1 \) is above or below \( 1/3 \). On the other hand, since \( \lambda_1^* \) decreases with \( \lambda_3 \), the agreement between firms 2 and 3 narrows the range of \( \lambda_1 \) for which firm 1 issues an exclusive license to firm 2.

Even when firm 2 and 3 do not expect that firm 1 will offer them licenses, i.e., whenever \( \lambda_1 > \lambda_1^* \), they can still benefit from reaching a licensing agreement provided that

18
\( \lambda_2 < 1/2 \). Such an agreement raises their joint expected payoff from \((1 - \lambda_1)(\lambda_2(1 - \lambda_3) + \lambda_3(1 - \lambda_2))\) to \(2\lambda_2(1 - \lambda_2)(1 - \lambda_1)\). The reason why \( \lambda_2 \) needs to be below \(1/2\) for such an agreement to be jointly profitable is that it must raise firm 3’s chances to be the sole developer of the new technology by more than it lowers firm 2’s chance.

**Proposition 3:** Suppose that firms 2 and 3 expect that firm 1 will issue licenses (either exclusive or noneclusive). Then, they will benefit from licensing firm 2’s knowledge to firm 3 before they are approached by firm 1. This agreement will narrow the set of parameters for which firm 1 issues an exclusive license to firm 2. If firms 2 and 3 expect that firm 1 will not issue any licenses, then they still benefit from a licensing agreement between themselves provided that \( \lambda_2 < 1/2 \).

### 4.3 Firm 1’s knowledge is worth more to firms 2 and 3

There are many cases in which relatively small firms license out their interim R&D knowledge to large corporations. This situation is quite common for example in the software industry or in biotechnology.\(^{10}\) An important feature of such agreements is that the licensees have more resources and better capabilities to successfully develop and commercialize the licensed technology. Consequently, the R&D knowledge is more “valuable” to the licensee than it is to the licensor. A natural question then is how would firm 1’s incentive to license its technology change when its knowledge is worth more to firms 2 and 3. This question is addressed in the next proposition.

**Proposition 4:** Suppose that when firm 1’s knowledge is \( \lambda_1 \), firm 1’s probability of developing the new technology is merely \( \phi \lambda_1 \), where \( \phi \in [0, 1] \). That is, whenever \( \phi < 1 \), firm 1’s technology is worth more to firms 2 and 3 than to firm 1. Then, in equilibrium, firm 1 will

(i) issue nonexclusive licenses to both firms 2 and 3 if \( \lambda_1 < \lambda_1(\phi) \equiv \frac{1+\phi-\sqrt{1-\phi+\phi^2}}{3\phi} \), where \( \lambda_1(\phi) \) falls from 1/2 when \( \phi = 0 \) to 1/3 when \( \phi = 1 \),

\(^{10}\)For an excellent source on biotechnology licensing agreements (many of which are between small biotechnology firms and large pharmaceutical firms), see http://www.jameshatton.com/public/
(ii) issue an exclusive license to firm 2 if \(\lambda_1(\phi) \leq \lambda_1 < \lambda_1^{*}\), where \(\lambda_1^{*}\) is defined in Proposition 1, or if \(\lambda_1 \geq \lambda_1^{*}\) and \(\phi < \phi^{*}\), where \(\phi^{*} \in (0, 1)\) is defined by

\[
\phi^{*} = 1 + \frac{B(\lambda_1, \lambda_2, \lambda_3)}{\lambda_1 [(2 - \lambda_3) (\lambda_1 - \lambda_2) + \lambda_1 (\lambda_2 - \lambda_3)]},
\]

(15)

with \(B(\lambda_1, \lambda_2, \lambda_3)\) being defined in (7); and

(iii) not issue any licenses if \(\lambda_1 \geq \lambda_1^{*}\) and \(\phi > \phi^{*}\).

\[\text{Proof:}\] See the Appendix.

Proposition 4 is illustrated in Figure 2. Noting that Proposition 1 refers to the case where \(\phi = 1\) (the upper edge of the square), it follows that a decrease in \(\phi\), which lowers the probability that firm 1 can successfully develop the new technology on its own, induces firm 1 to issue nonexclusive licenses to both firms 2 and 3 for a broader range of \(\lambda_1\). In particular, this range expands from \((0, 1/3]\) when \(\phi = 1\) to \((0, 1/2]\) when \(\phi = 0\) (notice that \(\phi = 0\) corresponds to the case where firm 1 is an outside research lab). Similarly, a decrease in \(\phi\) expands the range of \(\lambda_1\) values for which firm 1 prefers to issue an exclusive license to firm 2 rather than not issue any licenses. When \(\phi\) is sufficiently small, firm 1 never holds on to its technological lead and either issues nonexclusive licenses when \(\lambda_1\) is relatively small or issues an exclusive license to firm 2 when \(\lambda_1\) is relatively large.

4.4 Correlation

So far I have assumed that the success probabilities of the three firms are independent of each other and remain so even following licensing agreements. Now, I relax this assumption and assume that once licensing takes place, the success probabilities of the licensor and the licensee(s) become positively correlated.

To capture this correlation in as simple manner as possible, I will assume that if licensing takes place, the success probabilities of firms 1 and its licensee(s) are perfectly correlated with probability \(\rho\), but are completely independent with probability \(1 - \rho\).\(^{11}\) With

\(^{11}\)For an alternative modelling approach to correlation between the success probabilities of firms that engage in an R&D contest, see for example Dasgupta and Maskin (1987).
Figure 2: Firm 1’s technology is worth more to firms 2 and 3
this assumption in place, the expected payoffs of the three firms when firm 1 issues an exclusive license to firm 2 for a license fee $T_2$ are given by:

$$\pi_1(y, n, \rho) = (1 - \rho) \lambda_1 (1 - \lambda_1) (1 - \lambda_3) + T_2, \quad (16)$$

$$\pi_2(y, n, \rho) = (1 - \rho) \lambda_1 (1 - \lambda_1) (1 - \lambda_3) - T_2, \quad (17)$$

and

$$\pi_3(y, n, \rho) = \lambda_3 \left[ \rho (1 - \lambda_1) + (1 - \rho) (1 - \lambda_1)^2 \right]. \quad (18)$$

The expected payoffs when firm 1 licenses its knowledge exclusively to firm 3 are completely analogous. Equations (16)-(18) reflect the fact that firms 1 and 2 can be the sole developers of the new technology only if their success probabilities are independent and only if each of their rivals fail. Firm 3 can be the sole developer of the new technology either when the success probability of firms 1 and 2 is perfectly correlated and both fail, or when the success probabilities of firms 1 and 2 are independent and both fail.

When firm 1 issues nonexclusive licenses to both firms 2 and 3, the expected payoffs of the three firms become:

$$\pi_1(y, y, \rho) = (1 - \rho) \lambda_1 (1 - \lambda_1)^2 + T_2 + T_3, \quad (19)$$

and

$$\pi_j(y, y, \rho) = (1 - \rho) \lambda_1 (1 - \lambda_1)^2 - T_j, \quad j = 2, 3. \quad (20)$$

That is, a firm can be the sole developer of the new technology only if the success probabilities of all three firms are independent and the firm’s two rivals fail. When firm 1 does not issue any licenses, the expected payoff of each firm $i$ is given by equation (1). Noting that the firms’ payoffs decrease with $\rho$, it is clear that under correlation, firm 1 will issue licenses for a smaller set of parameters than under independence. However, following the same steps as in the proof of Proposition 1, it is easy to verify that the qualitative results of Proposition 1 remain valid so long as the degree of correlation, $\rho$, is not too large.

Things are different however when $\rho$ is relatively large. In the next proposition, I establish two important differences:

**Proposition 5:** In the presence of correlation between the success probabilities of firm 1 and its licensees, the following is true:
(i) Firm 1 will never issue nonexclusive licenses to both firms 2 and 3 whenever

\[ \rho > \bar{\rho} \equiv \frac{(1 - 3\lambda_1)(\lambda_1 - \lambda_2)}{\lambda_1(1 - 3\lambda_1 + 3\lambda_2)}, \]

where \( \bar{\rho} \in [0, 1] \) for all \( \lambda_1 < 1/3. \)

(ii) Firm 1 will issue an exclusive license to firm 3 whenever \( \lambda_1 < 1/3 \) and \( \bar{\rho} < \rho < \hat{\rho}, \)

where

\[ \hat{\rho} \equiv 1 - \frac{\lambda_1(1 - \lambda_2) + \lambda_3(1 - 2\lambda_1 + \lambda_1\lambda_2)}{\lambda_1(1 - \lambda_1)(2 - 2\lambda_2 + \lambda_3)}, \]

with \( \hat{\rho} \in [0, 1] \) for all \( \lambda_1 < 1/3. \)

**Proof:** See the Appendix.

Proposition 5 shows that under correlation between the success probabilities of firm 1 and its licensee(s), the pattern of licensing is different than it is under independence. In particular, unlike the case of independence, firm 1 will not issue nonexclusive licenses at all whenever \( \rho \) is sufficiently large. Moreover, whenever \( \lambda_1 < 1/3 \) and \( \rho \) is large but not too large, firm 1 will issue an exclusive license to firm 3, which it never does under independence. To illustrate, suppose that \( \lambda_1 = 0.3 \) (i.e., below 1/3), \( \lambda_2 = 0.07, \) and \( \lambda_3 = 0. \) Then \( \bar{\rho} = 0.247 \) and \( \hat{\rho} = 0.285; \) hence, whenever \( 0.247 < \rho < 0.285, \) firm 1 will not issue nonexclusive licenses and will instead issue an exclusive license to firm 3. And, if \( \rho > 0.285, \) then firm 1 will not issue any licenses. It should be noted however that for some values of \( \lambda_1, \lambda_2, \) and \( \lambda_3, \bar{\rho} > \hat{\rho}, \) in which case the set of \( \rho \) values for which firm 1 will issue an exclusive license to firm 3 is empty. For example, holding \( \lambda_2 = 0.07 \) and \( \lambda_3 = 0 \) fixed, \( \hat{\rho} > \bar{\rho} \) only when \( \lambda_1 \in (0.07, 0.162) \cup (0.288, 0.33). \) Otherwise, if \( \lambda_1 \in (0.162, 0.288), \) then \( \hat{\rho} > \bar{\rho}. \)

### 4.5 Bans on exclusive licenses

In this section I consider the consequences of bans on exclusive licensing agreements.\(^{12}\) At first blush it might be thought that such bans are a good idea since they forces firm 1 to

\(^{12}\)In the U.S., exclusive licensing is treated under the “rule of reason,” see *Morraine Products v. ICI America Inc.*, 538 F.2d.134 (7th Cir) cert denied, 429 U.S. 941 (1976) (see also Section 3.4 in the 1995 Department of Justice and Federal Trade Commission Antitrust Guidelines for the Licensing of Intellectual
license its knowledge to both firms 2 and 3 and thereby, not only raise the likelihood that the new technology will be developed, but also raise the likelihood that it will be developed by more than one firm (in which case there will be competition in the product market instead of monopoly). However, the next proposition shows that such bans may backfire in the sense that if firm 1 cannot issue an exclusive license to firm 2, it may prefer to simply hold on to its technological lead and not issue any licenses.

**Proposition 6:** If firm 1 is not allowed to issue exclusive licenses then in equilibrium, it will issue nonexclusive licenses to both firms 2 and 3 if \( \lambda_1 < \lambda_1^{**} \), where \( 1/3 < \lambda_1^{**} < \lambda_1^* \), and will not issue any licenses otherwise.

**Proof:** See the Appendix.

Proposition 6 implies that a ban on exclusive licenses has the intended effect only when \( 1/3 < \lambda_1 < \lambda_1^{**} \): in that case, firm 1 licenses its knowledge to both firms 2 and 3 instead of licensing it exclusively to firm 2. However, when \( \lambda_1^{**} < \lambda_1 < \lambda_1^* \), a ban on exclusive licenses backfires because it induces firm 1 to stop issuing licenses instead of issuing an exclusive license to firm 2. As a result, there will be less dissemination of knowledge in this range rather than more.

## 5 Acquisition of knowledge

In this section I consider the possibility that firm 1 will sell its superior knowledge rather than license it out. The difference between selling and licensing is that when firm 1 sells its knowledge, it exits the R&D contest altogether, whereas under licensing it stays in the contest. For instance, an exclusive sale of knowledge to firm \( j = 2, 3 \) could correspond to a situation in which firm \( j \) acquires firm 1 or acquires the relevant R&D lab or division of firm 1.\(^{13}\) A nonexclusive sale of knowledge could correspond to the case where firm 1 transfers

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\(^{13}\)In the U.S., an outright sale of intellectual property rights by their owner or a license that precludes all other persons, including the licensor, from using the licensed intellectual property “are most appropri-
its knowledge to firms 2 and 3 and commits to exit the R&D contest. In Section 5.1, I study the pattern of selling agreements that arise in equilibrium. In Section 5.2, I examine firm 1’s choice between licensing and selling its knowledge when both options are available.

5.1 Exclusive and nonexclusive sales of knowledge

Assume that after the vector \((\lambda_1, \lambda_2, \lambda_3)\) is realized, firm 1 can make a pair of take-it-or-leave-it offers to firms 2 and 3 in which it offers to sell them its knowledge for fees \(T_2\) and \(T_3\). If both offers are rejected, then the expected payoff of firm 1 is given by equation (1) and the payoffs of firms 2 and 3 are analogous. If only one firm accepts firm 1’s offer, then this firm acquires firm 1’s knowledge exclusively. If both firms accept, then a tie-breaking rule that will be specified in Lemma 2 below, determines which firm will acquire firm 1’s knowledge.

The resulting expected payoffs of the three firms when firm 1’s knowledge is acquired exclusively by firm 2 are given by:

\[
\pi_1^*(y, n) = T_2, \tag{21}
\]

\[
\pi_2^*(y, n) = \lambda_1(1 - \lambda_3) - T_2, \tag{22}
\]

and

\[
\pi_3^*(y, n) = \lambda_3(1 - \lambda_1). \tag{23}
\]

The expected payoffs when firm 1 sells its knowledge exclusively to firm 3 are analogous. If firm 1 sells its knowledge to both firms 2 and 3, then the expected payoffs are:

\[
\pi_1^*(y, y) = T_2 + T_3, \tag{24}
\]

and

\[
\pi_j^*(y, y) = \lambda_1(1 - \lambda_1) - T_j, \quad j = 2, 3. \tag{25}
\]

The following lemma is the analog of Lemma 1 for the case of sales of knowledge:
**Lemma 2:** Suppose that firm 1 wishes to sell its knowledge exclusively to firm $j = 2, 3$. Then, the optimal scheme from its perspective is to make each take-it-or-leave-it offers with

$$T_j^s = \lambda_1(1 - \lambda_k) - \lambda_j(1 - \lambda_1), \quad k \neq j,$$

and

$$T_k^s = 0,$$

and set a tie-breaking rule that specifies that firm 1’s knowledge will be sold exclusively to firm $j$ if both firms accept their respective offers. If firm 1 wishes to sell its knowledge to both firms 2 and 3, then the optimal scheme from its perspective is to set

$$\hat{T}_j^s = (\lambda_1 - \lambda_j)(1 - \lambda_1), \quad j = 2, 3,$$

and set a tie-breaking rule that specifies that firm 1’s knowledge will be sold to both firms if both accept their respective offers.

The proof is completely analogous to the proof of Lemma 1 and hence is omitted. Note that the fees that firm 1 gets when it sells its knowledge are $1/(1 - \lambda_1)$ times the corresponding fees when firm 1 licenses its knowledge. This reflects the fact that when firm 1 sells it knowledge it exits the contest and hence does not pose a competitive threat to firms 2 and 3.

The next result characterizes firm 1’s decision when it can only sell its knowledge to rival but not license it.

**Proposition 7:** In equilibrium, firm 1 will

(i) sell its knowledge to both firms 2 and 3 if $\lambda_1 < 1/2$ and $(\lambda_1 - \lambda_2 - \lambda_3)(1 - 2\lambda_1) > \lambda_1\lambda_2\lambda_3$;

(ii) not sell its knowledge if $\lambda_1 < 1/2$ and $(\lambda_1 - \lambda_2 - \lambda_3)(1 - 2\lambda_1) < \lambda_1\lambda_2\lambda_3$ or if $1/2 \leq \lambda_1 < 1/(2 - \lambda_3)$, where $1/(2 - \lambda_3) < \lambda_1^*$;

(iii) sell its knowledge exclusively to firm 2 if $\lambda_1 > 1/(2 - \lambda_3)$.
Proof: If firm 1 sells its knowledge exclusively to firm 2 at $T_2^s$, then its expected payoff is

$$\pi_1^{s*}(y, n) = \lambda_1(1 - \lambda_3) - \lambda_2(1 - \lambda_1). \quad (26)$$

If it sells its knowledge exclusively to firm 3 at $T_3^s$, then its expected payoff is

$$\pi_1^{s*}(n, y) = \lambda_1(1 - \lambda_2) - \lambda_2(1 - \lambda_1). \quad (27)$$

If firm 1 sells its knowledge to both firms 2 and 3 for $\tilde{T}_2^s$ and $\tilde{T}_3^s$, then its expected payoff is

$$\pi_1^{s*}(y, y) = (1 - \lambda_1)(2\lambda_1 - \lambda_2 - \lambda_3). \quad (28)$$

And, if firm 1 holds on to its knowledge, then its expected payoff is given by equation (??).

Comparing equations (26)-(28) reveals that since $\lambda_1 > \lambda_2 \geq \lambda_3$, then $\pi_1^{s*}(y, y) > \max\{\pi_1^{s*}(y, n), \pi_1^{s*}(n, y)\}$ for all $\lambda_1 < 1/2$, and $\pi_1^{s*}(y, y) > \max\{\pi_1^{s*}(y, y), \pi_1^{s*}(n, y)\}$ for all $\lambda_1 > 1/2$. That is, if firm 1 wishes to sell its knowledge, then it will sell it either to both firms if $\lambda_1 < 1/2$, or exclusively to firm 2 if $\lambda_1 > 1/2$.

To examine whether firm 1 will sell its knowledge at all, suppose that $\lambda_1 \leq 1/2$. Then, firm 1 needs to decide between selling to both firms 2 or 3 or not selling it all. Using equations (28) and (1) yields,

$$\pi_1^{s*}(y, y) - \pi_1(n, n) = (1 - \lambda_1)(2\lambda_1 - \lambda_2 - \lambda_3) - \lambda_1(1 - \lambda_2)(1 - \lambda_3)$$

$$= (\lambda_1 - \lambda_2 - \lambda_3)(1 - 2\lambda_1) - \lambda_1\lambda_2\lambda_3.$$  

Hence, the expression in the proposition.

Next, suppose that $\lambda_1 > 1/2$, so that if firm 1 sells its knowledge at all, it will sell it exclusively to firms 2. Using equations (26) and (1), yields

$$\pi_1^{s*}(y, n) - \pi_1(n, n) = \lambda_2(2 - \lambda_3) \left(\lambda_1 - \frac{1}{2 - \lambda_3}\right).$$

Hence, firm 1 will sell its knowledge exclusively to firm 2 if $\lambda_1 > 1/(2 - \lambda_3)$, and will not sell it at all if $1/2 \leq \lambda_1 < 1/(2 - \lambda_3)$, where by Proposition 1, $1/(2 - \lambda_3) < \lambda_1^*$.  

Proposition 7 shows that when $\lambda_1$ is sufficiently large, i.e., above $1/(2 - \lambda_3)$, firm 1 would prefer to sell its knowledge exclusively to firm 2. On the other hand, when $\lambda_1$ is
intermediate, i.e., above 1/2 and below 1/(2−λ_3), firm 1 prefers to hold on to its technological lead. Note that the higher λ_3 is, the larger λ_1 has to be to ensure that firm 1 finds it optimal to sell its knowledge to firm 2. Intuitively, both T_2^s and T_3^s decreases with λ_3, since the more likely firm 3 is to develop the new technology on its own, the less keen firm 2 is on acquiring firm 1’s knowledge (there is a high probability that it will not be the sole developer of the new technology), and the less valuable firm 1’s knowledge is to firm 3. This renders a sale of knowledge less attractive to firm 1.

As λ_1 drops below 1/2, firm 1 may wish to sell its knowledge to both firms 2 and 3 instead of holding on to its technological lead, provided that (λ_1 − λ_2 − λ_3)(1 − 2λ_1) > λ_1λ_2λ_3. Notice that this condition surely fails when λ_1 = 1/2 or when λ_1 ≤ λ_2 + λ_3. By continuity, firm 1 would not wish to sell its knowledge unless λ_1 is sufficiently below 1/2 and sufficiently above λ_2 + λ_3. On the other hand, the condition surely holds when λ_3 = 0 (firm 3 is far behind firm 1), and by continuity, when λ_3 is close to 0.

5.2 Sell or license?

The next step is to examine whether firm 1 would wish to license its knowledge or sell it if it can choose between the two alternatives.

**Proposition 8:** Suppose that firm 1 can either license its knowledge, sell its knowledge, or hold on to its technological lead. Then, firm 1 will

(i) license its knowledge to both firms 2 and 3 if 1/3 ≤ λ_1 < λ_1^*,

(ii) license its knowledge exclusively to firm 2 if 1/3 ≤ λ_1 < λ_1^*, and

(iii) sell its knowledge exclusively to firm 2 if λ_1 ≥ λ_1^*.

**Proof:** See the Appendix.

Proposition 8 shows that whenever firm 1 can choose between licensing and selling, then it will always choose one of these options rather than hold on to its technological lead. The option of selling is preferred when firm 1’s chances to develop the new technology are
particularly large. Licensing is not attractive in this case because it raises the likelihood that more than one firm will develop the new technology in which case competition in the product market will drive profits to 0. By contrast, selling is attractive because firm 1 exits the context after selling its knowledge.

6 Non-Blackwell ordered knowledge

So far I have assumed that the knowledge of the three firms can be Blackwell ordered: firm 3’s knowledge is a subset of firm 2’s knowledge, which is in turn a subset of firm 1’s knowledge. In this section I relax this assumption and assume instead that while the success probabilities at the interim stage are still such that $\lambda_1 > \lambda_2 \geq \lambda_3$, these probabilities correspond to three different approaches to R&D. Hence, if firm $i$ licenses its knowledge to firm $j$, then firm $j$ has two different approaches to R&D: its own original approach, and firm $i$’s approach. Consequently, the success probability of firm $j$ becomes $\tilde{\lambda}_{ji} = 1 - (1 - \lambda_i)(1 - \lambda_j)$; that is, firm $j$ succeeds unless both its own approach and firm $i$’s approach fail. The situation differs from the one considered earlier in two ways. First, the knowledge of firm 3 is now useful to firms 1 and 2 and the knowledge of firm 2 is useful to firm 1. Second, if firm $j = 2, 3$ licenses in firm 1’s knowledge, then its success probability increases by $\tilde{\lambda}_{j1} - \lambda_j = \lambda_1 - \lambda_1\lambda_j$, whereas earlier it has increased only by $\lambda_1 - \lambda_j$. That is, firm 1’s knowledge is now more useful to firms 2 and 3 than it is when knowledge is Blackwell ordered. As before, the expected payoff of firm 1 absent licensing agreements is given by equation (1) and the expected payoffs of firms 2 and 3 are analogous.

6.1 Unilateral licensing agreements

I begin the analysis by considering the case where only firm $i$ can issue licenses. This case is a natural extension of the situation considered in Proposition 1. The only difference is that now, the licensor does not have to be firm 1 since the knowledge of every firm is useful for the other two firms.

14I will retain however the assumption that the success probabilities of the firms are uncorrelated even if one firm licenses out its interim R&D knowledge to a rival firm.
Proposition 9: Suppose that only firm $i$ can license out its interim R&D knowledge. Then firm $i$ will issue nonexclusive licenses to both rivals if $\lambda_i < \tilde{\lambda}_i^*$, where $\tilde{\lambda}_i^*$ is defined implicitly by the equation
\[
\frac{2 - \lambda_j - \lambda_k}{(1 - \lambda_j)(1 - \lambda_k)} = \frac{\lambda_i (2 - \lambda_i)}{1 - \lambda_i},
\]
and will not issue any licenses otherwise.

Proof: First, note that if firm $i$ issues an exclusive license to firm $j$, while “threatening” to transfer its entire knowledge to firm $k \neq i, j$ if its offer is rejected, then firm $i$’s expected payoff is
\[
\tilde{\pi}_{ij} = \lambda_i \left( 1 - \tilde{\lambda}_{ji} \right) (1 - \lambda_k) + \left[ \tilde{\lambda}_{ji}(1 - \lambda_i)(1 - \lambda_k) - \lambda_j(1 - \lambda_i)(1 - \tilde{\lambda}_{ki}) \right]
\]
\[
= \lambda_i (1 - \lambda_i) (2 - \lambda_j)(1 - \lambda_k),
\]
where $\lambda_i \left( 1 - \tilde{\lambda}_{ji} \right) (1 - \lambda_k)$ is the probability that firm $i$ will be the sole developer of the new technology when it licenses out its knowledge exclusively to firm $j$, and the square bracketed term is the difference between the probability that firm $j$ will be the sole developer of the new technology when it gets an exclusive license from firm $i$ and when firm $k \neq j$ gets such a license. On the other hand, if firm $i$ issues nonexclusive licenses to both firms $j$ and $k$, then its expected payoff is
\[
\tilde{\pi}_i(y, y) = \lambda_i \left( 1 - \tilde{\lambda}_{ji} \right) \left( 1 - \tilde{\lambda}_{ki} \right) + \left[ \tilde{\lambda}_{ji}(1 - \lambda_i)(1 - \lambda_k) - \lambda_j(1 - \lambda_i)(1 - \tilde{\lambda}_{ki}) \right]
\]
\[
+ \left[ \tilde{\lambda}_{ki}(1 - \lambda_i)(1 - \lambda_j) - \lambda_k(1 - \lambda_i)(1 - \tilde{\lambda}_{ji}) \right]
\]
\[
= \lambda_i (1 - \lambda_i) [(1 - \lambda_i)(1 - \lambda_j)(1 - \lambda_k) + \lambda_i (2 - \lambda_j)(1 - \lambda_k)].
\]

Now, notice that
\[
\tilde{\pi}_i(y, y) - \tilde{\pi}_{ij} = \lambda_i (1 - \lambda_i) (1 - \lambda_j)(1 - \lambda_i + \lambda_i \lambda_k) > 0.
\]

Hence, nonexclusive licenses to both rivals dominate exclusive licenses to just one rival. Moreover,
\[
\tilde{\pi}_i(y, y) - \pi_i(n, n) = \lambda_i (1 - \lambda_i) (1 - \lambda_j)(1 - \lambda_k) \left[ \frac{2 - \lambda_j - \lambda_k}{(1 - \lambda_j)(1 - \lambda_k)} - \frac{\lambda_i (2 - \lambda_i)}{1 - \lambda_i} \right].
\]

To determine the sign of this expression, note that $\frac{2 - \lambda_j - \lambda_k}{(1 - \lambda_j)(1 - \lambda_k)} > 0$ is positive and independent of $\lambda_i$, while $\frac{\lambda_i (2 - \lambda_i)}{1 - \lambda_i}$ is a monotonically increasing function of $\lambda_i$, which increases from 0 when
\( \lambda_i = 0 \) to infinity when \( \lambda_i = 1 \). Hence, there exists a unique value of \( \lambda_i \), denoted \( \tilde{\lambda}_i^* \), such that \( \bar{\pi}_i(y, y) > \pi_i(n, n) \) for \( \lambda_i < \tilde{\lambda}_i^* \) and \( \bar{\pi}_i(y, y) < \pi_i(n, n) \) for \( \lambda_i > \tilde{\lambda}_i^* \), where \( \tilde{\lambda}_i^* \) is the value of \( \lambda_i \) at which the bracketed term vanishes. ■

Proposition 9 indicates that firm \( i \) will never issue an exclusive license to only one of its rivals: either it issues nonexclusive licenses to both rivals if \( \lambda_i \) is below some threshold, or else it does not issue licenses at all. Therefore, when the licensor is firm \( 1 \), the situation is very different than in the case where knowledge is Blackwell ordered in which case firm \( 1 \) licenses out its knowledge exclusively to firm \( 2 \) (its “strong” rival) when \( \lambda_1 \) is intermediate. The situation in the two cases is qualitatively similar however when \( \lambda_1 \) is either small (in both cases firm \( 1 \) issues nonexclusive licenses to firms \( 2 \) and \( 3 \)) or large (in both cases firm \( 1 \) does not issue any licenses).

Unfortunately, there is no obvious way to determine which unilateral licensing agreements will emerge if the three firms can freely bargain with each other. However, to the extent that these agreements will be efficient (i.e., generate the highest joint expected profit), it is interesting to examine which nonexclusive licensing arrangements maximize the joint expected payoff of the three firms.

**Proposition 10:** Suppose that only firm \( i \) can license out its interim R&D knowledge, and let \( \delta_j \equiv 1 - \lambda_j \) denote the probability that firm \( j = 1, 2, 3 \) fails. Then the joint expected payoff of the three firms is maximized when the licensor is

(i) firm 1 if \( \delta_1 + \delta_2 > H \),

(ii) firm 2 if \( \delta_1 + \delta_2 < H < \delta_2 + \delta_3 \), and

(iii) firm 3 if \( \delta_2 + \delta_3 < H \),

where \( H \equiv \frac{1}{3} \left( 1 + \frac{\delta_1 \delta_2 + \delta_1 \delta_3 + \delta_2 \delta_3}{\delta_1 \delta_2 \delta_3} \right) \).

**Proof:** See the Appendix.

Proposition 10 implies that in general, there is no reason to expect that in equilibrium, firm \( 1 \) will be the licensor. Whenever the sum of the success probabilities of firms \( 1 \) and \( 2 \)
is relatively large (i.e., the sum of their probabilities of failure, \( \delta_1 + \delta_2 \), is sufficiently small), then it is efficient for the three firms to let firms 2 or 3 issue nonexclusive licenses.

### 6.2 Cross-licensing agreements

When interim R&D knowledge is non-Blackwell ordered, the knowledge of each firm is useful to each of its two rivals. Hence, the three firms may wish to engage in cross licensing, whereby they get access to each other’s interim R&D knowledge. For instance, the three firms may engage in a three-way cross-licensing agreement in which they all share each other’s knowledge. Alternatively, firms \( i \) and \( j \) may engage in a bilateral cross-licensing agreement, with firm \( k \) either engaging in a unilateral licensing agreement with firm \( i \), or with firm \( j \), or with both, or not engaging in any licensing agreement. Moreover, firm \( k \) can either be the licensor or the licensee in such unilateral licensing agreements. Given the large number of possibilities, it is obvious that a full-blown analysis of cross-licensing agreements would require a separate paper. In what follows, I will simply show that the joint expected payoff of the three firms can be higher with cross-licensing agreements than with unilateral licensing agreements. This suggests in turn that in equilibrium, firms will engage in some sort of cross-licensing agreements.

To this end, consider first a three-way cross-licensing agreement, in which all three firms share their interim R&D knowledge. Recalling from Proposition 10 that \( \delta_i \) is the probability that firm \( i \) fails, the success probability of each firm under a three-way cross-licensing agreement is \( 1 - \delta_1 \delta_2 \delta_3 \). That is, each firm succeeds unless the R&D approaches of all three firms fail. Hence, the joint expected payoff of the three firms becomes

\[
V_{123} = 3 (1 - \delta_1 \delta_2 \delta_3) (\delta_1 \delta_2 \delta_3)^2,
\]

where \((1 - \delta_1 \delta_2 \delta_3) (\delta_1 \delta_2 \delta_3)^2\) is the probability that one of the three firms succeeds while the other two fail.

Now, suppose that \( \delta_1 + \delta_2 > H \). Then, the most profitable unilateral agreement is the one in which firm 1 licenses out its interim R&D knowledge to firms 2 and 3. To show
that a three-way cross-licensing agreement can be even more profitable, note that

\[ V_{123} - \tilde{\Pi}_1 (y, y) = 3 (1 - \delta_1 \delta_2 \delta_3) (\delta_1 \delta_2 \delta_3)^2 - \delta_1^2 \left[ \delta_2 + \delta_3 + \delta_2 \delta_3 (1 - 3 \delta_1) \right] \]

\[ = 3 \delta_1^2 \delta_2 \delta_3 \left( 1 - \delta_2 \delta_3 \right) \left[ \delta_1 - L (\delta_2, \delta_3) \right], \]

where \( L (\delta_2, \delta_3) \equiv \frac{1 + \frac{\delta_1}{\delta_2 + \delta_3} - 3 \delta_2 \delta_3}{3 (1 - \delta_2 \delta_3)^2} \). This expression is positive if and only if \( \delta_1 > L (\delta_2, \delta_3) \). Since \( \delta_1 < \delta_2 \), it follows that a necessary condition for finding \( \delta_1 \) such that \( L (\delta_2, \delta_3) < \delta_1 < \delta_2 \) is that \( \delta_2 > L (\delta_2, \delta_3) \). To illustrate, suppose that \( \delta_2 = \delta_3 \). Then \( \delta_2 = L (\delta_2, \delta_2) \) for all \( \delta_2 > 0.84 \). For example, if \( \delta_2 = \delta_3 = 0.9 \), then \( L (\delta_2, \delta_2) = 0.768 \). Hence, \( V_{123} > \tilde{\Pi}_1 (y, y) \) whenever \( 0.768 < \delta_1 < 0.9 \). It is straightforward to verify that with \( \delta_2 = \delta_3 = 0.9 \), \( \tilde{\Pi}_1 (y, y) > \pi_1 (n, n) \), so indeed firm 1 would rather issue nonexclusive licenses to firms 2 and 3 than not issue any licenses. Moreover, it is straightforward to verify that whenever \( \delta_2 = \delta_3 = 0.9 \) and \( 0.768 < \delta_1 < 0.9 \), \( H < 1.508 \) while \( \delta_1 + \delta_2 > 1.668 \). Hence, Proposition 10 implies that the joint expected payoff of the three firms is higher when firm 1 issues nonexclusive licenses than when either firms 2 or 3 issue nonexclusive licenses.

Now, let me show that bilateral cross-licensing agreements could also yield a higher joint expected payoff for the three firms than unilateral nonexclusive licenses. To this end, note that if firms \( i \) and \( j \) reach a bilateral cross-licensing agreement, then the success probability of each of them becomes \( 1 - \delta_i \delta_j \), while the success probability of firm \( k \neq i, j \) remains \( \lambda_k \equiv 1 - \delta_k \). Hence, the aggregate expected payoff of the three firms is

\[ V_{ij} = 2 (1 - \delta_i \delta_j) \delta_1 \delta_2 \delta_3 + (1 - \delta_k) (\delta_i \delta_j)^2, \]

where \( 2 (1 - \delta_i \delta_j) \delta_1 \delta_2 \delta_3 \) is the probability that either firm \( i \) or firm \( j \) succeeds while the other two firms fail, and \( (1 - \delta_k) (\delta_i \delta_j)^2 \) is the probability that firm \( k \) succeeds while firms \( i \) and \( j \) fail.

Assuming one again that \( \delta_1 + \delta_2 > H \), so the highest joint expected payoff of the three firms is attained when firm 1 issues nonexclusive licenses to firms 2 and 3, let me compare \( \tilde{\Pi}_1 (y, y) \) with the joint expected payoff when firms 1 and 2 reach a bilateral cross-licensing agreement:

\[ V_{12} - \tilde{\Pi}_1 (y, y) = 2 (1 - \delta_1 \delta_2) \delta_1 \delta_2 \delta_3 + (1 - \delta_3) (\delta_1 \delta_2)^2 - \delta_1^2 \left[ \delta_2 + \delta_3 + \delta_2 \delta_3 (1 - 3 \delta_1) \right] \]

\[ = \delta_1 \left[ 2 \delta_2 \delta_3 - \delta_1 (\delta_2 + \delta_3 + \delta_2 \delta_3 (1 - 3 \delta_1) - \delta_2 \delta_3 (1 - 3 \delta_3)) \right]. \]
To show that this expression could be either positive or negative, let $\delta_2 = \delta_3 = \delta$. Then

$$V_{12} - \tilde{\Pi}_1 (y, y) = 3\delta_1 \delta (\delta - \delta_1) \left( \frac{2}{3} - \delta \delta_1 \right).$$

Since $\delta_1 < \delta$, it follows that $V_{12} > \tilde{\Pi}_1 (y, y)$ if and only if $\delta \delta_1 < \frac{2}{3}$. For example if $\delta_2 = \delta_3 = 0.9$, then $V_{12} > \tilde{\Pi}_1 (y, y)$ provided that $\delta_1 < 0.741$.

## 7 Conclusion

In this paper I have examined the incentives of firms to engage in licensing of precommercial, interim R&D knowledge. This knowledge boosts the success probability of the licensees but does not guarantee it. I have shown that in a broad range of cases, the leading firm in the R&D contest will prefer to license its superior knowledge to one of the lagging rivals or to both rather than hold on to its technological lead. Such licensing agreements have two main advantages from the leading firm’s point of view: First, they have the potential to create value by increasing the chance that the licensee(s) will develop the new technology when the leading firm fails. The leading firm in turn can capture this value through the license fee(s) that it charges. Value creation is not guaranteed however since the licensing agreements also raise the probability that both the licensor and the licensee(s) will develop the new technology and will end up competing in the product market. Second, licensing agreement(s) allow the leading firm to extract surplus from the lagging firms as each licensee(s) pay(s) not only for access to the leading firm’s superior knowledge, but also in order to ensure that the remaining firm will not obtain exclusive access to this knowledge.

There clearly remain a number of interesting extensions that must be addressed before we have a good understanding of the incentives to engage in licensing of interim R&D knowledge and their implication. I will now mention just a few of these extensions. First, in this paper I have treated the success probabilities of the three firms as exogenous parameters and did not consider the implications of licensing for the incentives to invest in R&D. A natural extension of the current analysis would be to add an initial stage to the model at which the three firms choose how much to invest in R&D; these investments in turn determine the vector $(\lambda_1, \lambda_2, \lambda_3)$. Moreover, one can also add a second investment stage which takes
place after the licensing agreements are reached. For example, one can assume that the overall success probability of firm $i$ is given by $p(\lambda_i, \tau_i)$, where $\lambda_i$ is the amount of firm $i$’s interim R&D knowledge, and $\tau_i$ is the additional investment level that firm $i$ chooses after licensing agreements are reached, and $p(\cdot, \cdot)$ increases in both arguments. In such a model, licensing will also have an added advantage of allowing the licensee(s) to obtain the leading firm’s knowledge without having to costly develop it. Moreover, such a model will also make it possible to explore in more detail the competitive effect that bans on exclusive licensing since these bans will not only affect the incentives to license interim R&D knowledge but will also affected the incentives to invest in R&D.

Second, in this paper I have assumed that there is Bertrand competition in the product market. As mentioned earlier, this assumption has the advantage that firms make a positive profit only if they are the sole developers of the new technology. This payoff structure not only simplifies the analysis considerably but also implies that licensing is worthwhile precisely because the licensed knowledge is interim - firms will never license fully developed technologies as competition between them will drive their post licensing profits to 0. However, in future research it will be interesting to examine the pattern of licensing agreements that emerges under alternative types of competition in the product market.

Third, throughout the paper I have assumed that the vector of success probabilities, $(\lambda_1, \lambda_2, \lambda_3)$, is common knowledge. While this assumption is a natural starting point and while it establishes an important benchmark, it would be interesting in future research to relax this assumption and examine the case where $\lambda_i$ is a private information for firm $i$. This extension is obviously much harder than the first two due to the complexity of analyzing multilateral bargaining under asymmetric information. As mentioned in the Introduction, two papers that make a progress in this direction are d’Aspremont, Bhattacharya, and Gerard-Varet (2000) and Bhattacharya and Guriev (2006). These papers however consider somewhat simpler situations than the one considered here: the first paper considers a model with only two firms, while in the second paper, the licensor is an outside research lab that cannot develop the final product.

Finally, in Section 6 of the paper I have only briefly considered the case where the knowledge of the three firms is non-Blackwell ordered. In future research it would be useful to
study a full-blown model of multilateral bargaining between three or more firms over licensing of interim R&D knowledge which cannot be Blackwell ordered and examine the conditions under which firms reach unilateral licensing agreements, cross-licensing agreements, or a mixture of both.

8 Appendix

Following are the proofs of Propositions 2, 4-6, 8, and 10.

Proof of Proposition 2: Differentiating $\pi_1(\Delta_2, \Delta_3)$ with respect to $\Delta_j$, $j = 2, 3$, yields

$$
\frac{\partial \pi_1(\Delta_2, \Delta_3)}{\partial \Delta_j} = (1 - 3\lambda_1)(1 - \lambda_k - \Delta_k) + (\lambda_1 - \lambda_k - \Delta_k)
$$

where $k \neq j$. From the first line of (29) it is clear that $\frac{\partial \pi_1(\Delta_2, \Delta_3)}{\partial \Delta_j} > 0$, $j = 2, 3$ when $\lambda_1 \leq 1/3$, whereas from the second line it is clear that $\frac{\partial \pi_1(\Delta_2, \Delta_3)}{\partial \Delta_j} < 0$, $j = 2, 3$, when $1/2 \leq \lambda_1 \leq 1$. Hence, firm 1 will license its entire knowledge to firms 2 and 3 if $\lambda_1 \leq 1/3$, but will prefer to transfer them as little knowledge as possible if $\lambda_1 \geq 1/2$. Assuming that it is possible to sign licensing agreements in which virtually no knowledge is transferred, and recalling that $\pi_1(0, 0) > \pi_1(n, n)$, it follows that whenever $\lambda_1 \geq 1/2$, firm 1 will issue nonexclusive licenses to firms 2 and 3 and will transfer them virtually no knowledge.

The remaining question is what happens when $1/3 < \lambda_1 < 1/2$. To address this question, suppose that $\Delta_k = \lambda_1 - \lambda_k$ (firm 1 licenses its entire knowledge to firm k). Then,

$$
\frac{\partial \pi_1(\Delta_2, \Delta_3)}{\partial \Delta_j} = (1 - 3\lambda_1)(1 - \lambda_1) < 0
$$

where the inequality follows since $\lambda_1 > 1/3$. Hence, firm 1 would like to set $\Delta_j = 0$. Consequently, firm 1 will never license its entire knowledge to both firms 2 and 3. Rather, it will either license its entire knowledge exclusively to firm 2, or license its entire knowledge exclusively to firm 3, or issue nonexclusive licenses to both firms and transfers them virtually no knowledge. However, Proposition 1 shows that whenever $\lambda_1 > 1/3$, an exclusive license with firm 2 dominates an exclusive license with firm 3. Hence, if firm 1 issues an exclusive license, it will issue it to firm 2. This implies in turn that in
equilibrium, it must be the case that $\Delta_3 = 0$. Substituting $\Delta_3 = 0$ in (29) reveals that
\[
\frac{\partial \pi_1(\Delta_2,0)}{\partial \Delta_2} = 1 - 2\lambda_3 - \lambda_1 (2 - 3\lambda_3)
= (2 - 3\lambda_3) \left[ \frac{1 - 2\lambda_3}{2 - 3\lambda_3} - \lambda_1 \right].
\]
Therefore, firm 1 will transfer its entire knowledge exclusively to firm 2 if $\lambda_1 < \frac{1 - 2\lambda_3}{2 - 3\lambda_3}$ and will issue nonexclusive licenses to both firms if $\lambda_1 \geq \frac{1 - 2\lambda_3}{2 - 3\lambda_3}$, where $\frac{1 - 2\lambda_3}{2 - 3\lambda_3}$ decreases from 1/2 when $\lambda_3 = 0$ to 0 when $\lambda_3 = 1/2$ (recall that $\lambda_3 < \lambda_1 < 1/2$).

**Proof of Proposition 4:** Given that firm 1’s probability to develop the new technology is $\phi \lambda_1$, equations (8)-(10) and (1) become
\[
\pi_1^*(y, n, \phi) = \phi \lambda_1 (1 - \lambda_1)(1 - \lambda_3) + [\lambda_1 (1 - \phi \lambda_1) (1 - \lambda_3) - \lambda_2 (1 - \phi \lambda_1) (1 - \lambda_1)], \quad (30)
\]
\[
\pi_1^*(n, y, \phi) = \phi \lambda_1 (1 - \lambda_2)(1 - \lambda_1) + [\lambda_1 (1 - \phi \lambda_1) (1 - \lambda_2) - \lambda_3 (1 - \phi \lambda_1) (1 - \lambda_1)], \quad (31)
\]
\[
\pi_1^*(y, y, \phi) = \phi \lambda_1 (1 - \lambda_1)^2 + [\lambda_1 (1 - \phi \lambda_1) (1 - \lambda_1) - \lambda_2 (1 - \phi \lambda_1) (1 - \lambda_1)]
+ [\lambda_1 (1 - \phi \lambda_1) (1 - \lambda_1) - \lambda_3 (1 - \phi \lambda_1) (1 - \lambda_1)], \quad (32)
\]
and
\[
\pi_1(n, n, \phi) = \phi \lambda_1 (1 - \lambda_2)(1 - \lambda_3). \quad (33)
\]
Comparing equations (30)-(32) reveals that since $\lambda_1 > \lambda_2 \geq \lambda_3$, then $\pi_1^*(y, y) > \max\{\pi_1^*(y, n), \pi_1^*(n, y)\}$ for all $\lambda_1 < \lambda_1(\phi) \equiv 1 + \phi - \sqrt{1 - \phi + \phi^2}$, and $\pi_1^*(y, y, \phi) > \max\{\pi_1^*(y, y, \phi), \pi_1^*(n, y, \phi)\}$ for all $\lambda_1(\phi) < \lambda_1 < 1$, where $\lambda_1(\phi)$ falls from 1/2 when $\phi \to 0$ to 1/3 when $\phi \to 1$. Hence, if firm 1 issues licenses at all, it will issue nonexclusive licenses to both firms 2 and 3 if $\lambda_1 < \lambda_1(\phi)$ and will issue an exclusive license to firm 2 if $\lambda_1(\phi) < \lambda_1 < 1$.

Now suppose that $\lambda_1 < \lambda_1(\phi)$ and let $H(\phi) \equiv \pi_1^*(y, y, \phi) - \pi_1(n, n, \phi)$. Using equations (32) and (33),
\[
H(\phi) \equiv (1 - \lambda_1)(2\lambda_1 - \lambda_2 - \lambda_3)
+ \phi \lambda_1 [(1 - \lambda_1)(1 - 3\lambda_1 + \lambda_2 + \lambda_3) - (1 - \lambda_2)(1 - \lambda_3)].
\]
Clearly, firm 1 will issue nonexclusive licenses to both firms 2 and 3 rather than not issue any licenses provided that $H(\phi) > 0$. Since $(1 - \lambda_1) < (1 - \lambda_2)$ and $(1 - 3\lambda_1 + \lambda_2 + \lambda_3) < (1 - \lambda_3)$,
it follows that \( H'(\phi) < 0 \). Moreover, evaluated at \( \phi = 1 \),

\[
H(1) = (1 - \lambda_1)^2(3\lambda_1 - \lambda_2 - \lambda_3) - \lambda_1(1 - \lambda_2)(1 - \lambda_3).
\]

Notice that evaluated at \( \phi = 1 \), \( \lambda_1(\phi) \) which is the upper bound on \( \lambda_1 \), is equal to \( 1/3 \). It is easy to verify that \( H(1) \) is an inverse U-shaped function of \( \lambda_1 \) for all \( \lambda_1 \leq 1/3 \). Hence, \( H(1) \) attains its lowest value either when \( \lambda_1 = 0 \) or when \( \lambda_1 = 1/3 \). But, when \( \lambda_1 \to 0 \), then \( H(1) \to 0 \) since \( \lambda_1 > \lambda_2 \geq \lambda_3 \), and when \( \lambda_1 = 1/3 \), then \( H(1) = (1 - \lambda_2 - \lambda_3 - 3\lambda_2\lambda_3)/9 > 0 \), where the inequality follows because \( 1/3 = \lambda_1 > \lambda_2 \geq \lambda_3 \). Hence, \( H(\phi) > 0 \) for all \( \phi \in [0, 1] \), implying that whenever \( \lambda_1 < \lambda_1(\phi) \), firm 1 is better off issuing nonexclusive licenses to firms 2 and 3 than not issuing any licenses.

Next, suppose that \( \lambda_1 \geq \lambda_1(\phi) \). Then, if firm 1 issues licenses at all, it issues an exclusive license to firm 2. To determine if issuing an exclusive license to firm 2 dominates issuing no licenses at all, let \( M(\phi) \equiv \pi_1^i(y, n, \phi) - \pi_1(n, n, \phi) \). Using equations (30) and (33),

\[
M(\phi) \equiv B(\lambda_1, \lambda_2, \lambda_3) + (1 - \phi) \lambda_1 [(2 - \lambda_3)(\lambda_1 - \lambda_2) + \lambda_1(\lambda_2 - \lambda_3)]. \tag{34}
\]

Noting that the square brackets term in \( M(\phi) \) is strictly positive, it follows that \( M'(\phi) < 0 \). Moreover, note that \( M(0) = (\lambda_1 - \lambda_2) + \lambda_1(\lambda_2 - \lambda_3) > 0 \) and \( M(1) = B(\lambda_1, \lambda_2, \lambda_3) \). Since Proposition 1 implies that \( B(\lambda_1, \lambda_2, \lambda_3) > 0 \) for all \( \lambda_1 < \lambda_1^* \), it follows that whenever \( \lambda_1 < \lambda_1^* \), then \( M(\phi) > 0 \) for all \( \phi \in [0, 1] \). Consequently, whenever \( \lambda_1(\phi) \leq \lambda_1 < \lambda_1^* \) (this interval exists since \( \lambda_1(\phi) \leq 1/2 \) and since Proposition 1 implies that \( \lambda_1^* > 1/2 \)), it is optimal for firm 1 to issue an exclusive license to firm 2, irrespective of the value of \( \phi \).

On the other hand, when \( \lambda_1 \geq \lambda_1^* \), Proposition 1 implies that \( B(\lambda_1, \lambda_2, \lambda_3) < 0 \). Hence, for each \( \lambda_1 \geq \lambda_1^* \), there exists a unique value of \( \phi \in (0, 1) \), denoted \( \phi^* \), such that \( M(\phi) > (\leq)0 \) for all \( \phi < (>)\phi^* \). The value of \( \phi^* \) reported in (15) is given by the solution to \( M(\phi) = 0 \). Consequently, whenever \( \lambda_1 \geq \lambda_1^* \), it is optimal for firm 1 to issue an exclusive license to firm 2 if \( \phi < \phi^* \) and issue no licenses at all if \( \phi > \phi^* \).

**Proof of Proposition 5:** First, note that at the optimum, the license fees under exclusive licenses, \( T_2^* \) and \( T_3^* \), are given by the solutions to \( \pi_2(y, n, \rho) = \pi_2(n, y, \rho) \) and \( \pi_3(n, y, \rho) = \pi_3(y, n, \rho) \), while the license fees under nonexclusive licenses, \( T_2^* \) and \( T_3^* \), are given by the solutions to \( \pi_2(y, y, \rho) = \pi_2(n, y, \rho) \) and \( \pi_3(y, y, \rho) = \pi_3(y, n, \rho) \). That is, the license fees are
set such that each licensee receives the same payoff as in the case where firm 1 issues an exclusive license to the rival firm. Given $T_2^*, T_3^*, \hat{T}_2^*$, and $\hat{T}_3^*$, the expected payoff of firm 1 when it issues an exclusive license to firm 2 is given by:

$$
\pi_1^*(y, n, \rho) = \pi_1^*(y, n) - \rho \lambda_1 (1 - \lambda_1) (Z - 3\lambda_3),
$$

(35)

its expected payoff when it issues an exclusive license to firm 3 is given by

$$
\pi_1^*(n, y, \rho) = \pi_1^*(n, y) - \rho \lambda_1 (1 - \lambda_1) (Z - 3\lambda_2),
$$

(36)

and its expected payoff when it issues a nonexclusive license to both firms 2 and 3 is given by

$$
\pi_1^*(y, y, \rho) = \pi_1^*(y, y) - \rho \lambda_1 (1 - \lambda_1) (Z + 1 - 3\lambda_1),
$$

(37)

where $\pi_1^*(y, n), \pi_1^*(n, y),$ and $\pi_1^*(y, y)$ are given by (8), (9), and (10) and $Z \equiv 2 + \lambda_2 + \lambda_3$. When firm 1 does not issue any licenses, its expected payoff is given by equation (1).

(i) To prove that firm 1 will never issue nonexclusive licenses, it is sufficient to show that issuing an exclusive license to firm 3 dominates issuing nonexclusive licenses whenever $\rho > \rho$. To this end, note that

$$
\pi_1^*(n, y, \rho) - \pi_1^*(y, y, \rho) = (1 - \lambda_1) [(3\lambda_1 - 1) (\lambda_1 - \lambda_2) + \rho \lambda_1 (1 - 3\lambda_1 + 3\lambda_2)].
$$

If $\lambda_1 \geq 1/3$, then the first term inside the square brackets is positive; if $1 - 3\lambda_1 + 3\lambda_2 > 0$, then the second term is positive as well so $\pi_1^*(n, y, \rho) > \pi_1^*(y, y, \rho)$. If $1 - 3\lambda_1 + 3\lambda_2 < 0$, then $\pi_1^*(n, y, \rho) - \pi_1^*(y, y, \rho)$ is decreasing with $\rho$ and equal to $(1 - \lambda_1)\lambda_2$ when $\rho = 1$. Hence, $\pi_1^*(n, y, \rho) > \pi_1^*(y, y, \rho)$ for all $\rho \leq 1$. If $\lambda_1 < 1/3$, then the first term inside the square brackets is negative, while the second term is positive and increasing with $\rho$. Hence, $\pi_1^*(n, y, \rho) > \pi_1^*(y, y, \rho)$ for all $\rho > \rho$, where $\rho > 0$ since $\lambda_1 < 1/3$ and $\rho < 1$ since $\lambda_1 - \lambda_2 < \lambda_1$ and since $1 - 3\lambda_1 < 1 - 3\lambda_1 + 3\lambda_2$.

(ii) To prove that firm 1 will issue an exclusive license to firm 3, I need to show that this option yields a higher expected payoff than all other options. To this end, note that from part (i) of the proof, that $\pi_1^*(n, y, \rho) - \pi_1^*(y, y, \rho)$ if $\lambda_1 < 1/3$ and $\rho > \rho$. Moreover, whenever $\lambda_1 < 1/3$,

$$
\pi_1^*(n, y, \rho) - \pi_1^*(y, n, \rho) = (1 - \lambda_1) (\lambda_2 - \lambda_3) (1 - 3 (1 - \rho) \lambda_1) > 0.
$$

38
Finally, note that $\pi^*_1(n, y, \rho) - \pi_1(n, n)$ is decreasing with $\rho$ and positive if $\rho < \hat{\rho}$. □

**Proof of Proposition 6:** Absent exclusive licenses, firm 1 faces a choice between issuing nonexclusive licenses and not issuing licences at all. Hence, it is enough to compare $\pi^*_1(y, y)$ and $\pi_1(n, n)$. To this end, note from equations (1) and (10) that $\pi_1(n, n)$ is increasing with $\lambda_1$ for all $\lambda_1$, while $\pi^*_1(y, y)$ is first increasing with $\lambda_1$ when $\lambda_1 < 1/3 + 2/(3 + \lambda_3) / 9$ but then decreasing with $\lambda_1$ when $1/3 + 2/(3 + \lambda_3) / 9 < \lambda_1 < 1$. Moreover, $\pi^*_1(y, y) = \pi_1(n, n) \rightarrow 0$ when $\lambda_1 \rightarrow 0$, $\pi^*_1(y, y) = 0 < \pi_1(n, n)$ when $\lambda_1 \rightarrow 1$, and $\frac{\partial \pi_1(n, n)}{\partial \lambda_1} > 0$ for $\lambda_1$ close to 0. Hence, there exists a unique value of $\lambda_1$, denoted $\lambda_1^{**}$, such that $\pi^*_1(y, y) > \pi_1(n, n)$ for all $\lambda_1 < \lambda_1^{**}$ and $\pi^*_1(y, y) < \pi_1(n, n)$ for all $\lambda_1^{**} < \lambda_1 < 1$.

To establish that $\lambda_1^{**} > 1/3$, note that evaluated at $\lambda_1 = 1/3$,

$$\pi^*_1(y, y) - \pi_1(n, n) = \frac{(1 - \lambda_2)(1 - \lambda_3) - 4\lambda_2\lambda_3}{9} > 0,$$

where the inequality follows because $1/3 = \lambda_1 > \lambda_2 \geq \lambda_3$, so $(1 - \lambda_2)(1 - \lambda_3) > 4/9$ while $4\lambda_2\lambda_3 < 4/9$. Hence, $\lambda_1^{**}$ which is attained at the intersection of $\pi^*_1(y, y)$ and $\pi_1(n, n)$ exceeds $1/3$. To compare $\lambda_1^{**}$ with $\lambda_1^*$, recall from Proposition 1 that $\lambda_1^*$ is defined implicitly by the solution to $\pi^*_1(y, n) = \pi_1(n, n)$. Since $\pi^*_1(y, n) > y^*_1(y, y)$ for all $1/3 < \lambda_1 < 1$ and since both $\lambda_1^*$ and $\lambda_1^{**}$ exceed $1/3$, it follows that $\lambda_1^{**} < \lambda_1^*$. □

**Proof of Proposition 8:** Suppose that $\lambda_1 < 1/3$. Propositions 1 and 7 imply that under licensing, firm 1 will license its knowledge to both firms 2 and 3 and its expected payoff will be $\pi^*_1(y, y)$, while under selling it will either sell its knowledge to both firms 2 and 3 or will not sell it at all, so its expected payoff will be $\max\{\pi^{**}(y, y), \pi_1(n, n)\}$. The proof of Proposition 1 shows that $\pi^*_1(y, y) > \pi_1(n, n)$ for all $\lambda_1 < 1/3$. Moreover, since $\lambda_1 < 1/3$, it follows that

$$\pi^*_1(y, y) - \pi^{**}_1(y, y) = (1 - \lambda_1)^2(3\lambda_1 - \lambda_2 - \lambda_3) - (1 - \lambda_1)(2\lambda_1 - \lambda_2 - \lambda_3) = \lambda_1(1 - \lambda_1)(1 - 3\lambda_1 + \lambda_2 + \lambda_3) > 0.$$

Hence, $\pi^*_1(y, y) > \max\{\pi^{**}(y, y), \pi_1(n, n)\}$ for all $\lambda_1 < 1/3$, implying that in this range, the best option from firm 1’s perspective is to license its knowledge to both firms 2 and 3.

Next, suppose that $1/3 \leq \lambda_1 < 1/2$. Then, Proposition 1 implies that firm 1 will license its knowledge exclusively to firm 2 under licensing and will obtain an expected payoff of
\[ \pi_1^*(y, n) \]. Under selling, Proposition 7 implies that firm 1 will either sell its knowledge to both firms 2 and 3 or will not sell it at all, so its expected payoff will be \( \max\{\pi_1^*(y, y), \pi_1(n, n)\} \).

The proof of Proposition 1 shows that \( \pi_1^*(y, n) > \pi_1(n, n) \) for all \( 1/3 \leq \lambda_1 < \lambda_1^* \). Since \( \lambda_1^* > 1/(2 - \lambda_3) \geq 1/2 \), it follows that \( \pi_1^*(y, n) > \pi_1(n, n) \) for all \( \lambda_1 < 1/2 \). Moreover, since \( \lambda_1 < 1/2 \),

\[
\pi_1^*(y, n) - \pi_1^*(y, y) = (1 - \lambda_1)(2\lambda_1(1 - \lambda_3) - \lambda_2(1 - \lambda_1)) - (1 - \lambda_1)(2\lambda_1 - \lambda_2 - \lambda_3) \\
= (1 - \lambda_1)[\lambda_3(1 - 2\lambda_1) + \lambda_1 \lambda_1] > 0.
\]

Hence, \( \pi_1^*(y, n) > \max\{\pi_1^*(y, y), \pi_1(n, n)\} \) for all \( \lambda_1 < 1/2 \), implying that in this range, firm 1 will prefer to license its knowledge exclusively to firm 2.

Now let \( 1/2 \leq \lambda_1 < 1/(2 - \lambda_3) \). Since \( \lambda_1^* \geq 1/(2 - \lambda_3) \), Proposition 1 implies that under licensing, firm 1 will licenses its knowledge exclusively to firm 2 and will get an expected payoff of \( \pi_1^*(y, n) \). Proposition 7 shows that under selling, firm 1 will prefer to hold on to its technological lead and will get an expected payoff of \( \pi_1(n, n) \). The proof of Proposition 1 shows however that \( \pi_1^*(y, n) > \pi_1(n, n) \) for all \( 1/3 < \lambda_1 < \lambda_1^* \). Since \( \lambda_1^* \geq 1/(2 - \lambda_3) \), it follows that when \( 1/2 \leq \lambda_1 < 1/(2 - \lambda_3) \), firm 1 will prefer to licenses its knowledge exclusively to firm 2.

If \( 1/(2 - \lambda_3) \leq \lambda_1 < \lambda_1^* \), then Proposition 1 shows that under licensing, firm 1 will license its knowledge exclusively to firm 2 and its expected payoff will be \( \pi_1(y, n) \). Proposition 7 shows that under selling, firm 1 will sell its knowledge exclusively to firm 2 and will get an expected payoff \( \pi_1^*(y, n) \). Now,

\[
\pi_1^*(y, n) - \pi_1^*(y, n) = (1 - \lambda_1)(2\lambda_1(1 - \lambda_3) - \lambda_2(1 - \lambda_1)) - (\lambda_1(1 - \lambda_3) - \lambda_2(1 - \lambda_1)) \\
= \lambda_1[1 - 2\lambda_1)(1 - \lambda_3) + \lambda_2(1 - \lambda_1)] .
\]

The sign of this expression depends on the square bracketed term. This term decreases with \( \lambda_1 \) so it is minimized at \( 1/(2 - \lambda_3) \). Evaluated at \( \lambda_1 = 1/(2 - \lambda_3) \), the square bracketed term becomes \( \frac{(\lambda_2 - \lambda_3)(1 - \lambda_3)}{2 - \lambda_3} > 0 \). Hence, \( \pi_1^*(y, n) > \pi_1^*(y, n) \) for all \( \lambda_1 \geq 1/(2 - \lambda_3) \), implying that whenever \( 1/(2 - \lambda_3) \leq \lambda_1 < \lambda_1^* \), firm 1 will prefer to license its knowledge exclusively to firm 2.

Finally, suppose that \( \lambda_1 \geq \lambda_1^* \). Proposition 1 shows that under licensing, firm 1 will prefer to hold on to its technological lead, so its expected payoff will be \( \pi_1(n, n) \). Proposition
7 shows that under selling, firm 1 will prefer to sell its knowledge exclusively to firm 2, so its expected payoff will be $\pi^{**}(y, n)$. Proposition 7 reveals that $\pi^{**}(y, n) > \pi_1(n, n)$ for all $\lambda_1 > 1/(2 - \lambda_3)$. Since $\lambda_1^* > 1/(2 - \lambda_3)$, it follows that $\pi^{**}(y, n) > \pi_1(n, n)$, for all $\lambda_1 \geq \lambda_1^*$, implying that firm 1 will prefer to sell its knowledge exclusively to firm 2.

**Proof of Proposition 10:** To determine which unilateral licensing agreements maximize the joint payoff of the three firms, note that the joint expected payoffs of the three firms when firm 1 issues nonexclusive licenses to firms 2 and 3 is given by:

$$
\tilde{\Pi}_1(y, y) = \lambda_1 \left(1 - \tilde{\lambda}_{21}\right) \left(1 - \tilde{\lambda}_{31}\right) + \tilde{\lambda}_{21} \left(1 - \lambda_1\right) \left(1 - \tilde{\lambda}_{31}\right) + \tilde{\lambda}_{31} \left(1 - \lambda_1\right) \left(1 - \tilde{\lambda}_{21}\right)
$$

$$
= \delta_1^2 [\delta_2 + \delta_3 + \delta_2 \delta_3 (1 - 3\delta_1)].
$$

The joint expected payoffs when firms 2 and 3 issues nonexclusive licenses, $\tilde{\Pi}_2(y, y)$ and $\tilde{\Pi}_3(y, y)$, are analogous.

Now note that

$$
\tilde{\Pi}_1(y, y) - \tilde{\Pi}_2(y, y) = 3 (\delta_2 - \delta_1) \delta_1 \delta_2 \delta_3 [\delta_1 + \delta_2 - H],
$$

$$
\tilde{\Pi}_1(y, y) - \tilde{\Pi}_3(y, y) = 3 (\delta_3 - \delta_1) \delta_1 \delta_2 \delta_3 [\delta_1 + \delta_3 - H],
$$

and

$$
\tilde{\Pi}_2(y, y) - \tilde{\Pi}_3(y, y) = 3 (\delta_3 - \delta_2) \delta_1 \delta_2 \delta_3 [\delta_2 + \delta_3 - H],
$$

where $H$ is defined in the proposition. Recalling that $\lambda_1 > \lambda_2 \geq \lambda_3$, it follows that $\delta_1 < \delta_2 \leq \delta_3$. Hence, $\delta_1 + \delta_2 < \delta_2 + \delta_3 \leq \delta_2 + \delta_3$. Moreover, $H$ is a decreasing function of $\delta_1$, $\delta_2$, and $\delta_3$ and is equal to 4/3 when $\delta_1, \delta_2$ and $\delta_3$ approach 1. Hence, there are three possible cases:

(i) $\delta_1 + \delta_2 > H$. Then $\tilde{\Pi}_1(y, y) > \tilde{\Pi}_2(y, y)$. Moreover, since $\delta_3 \geq \delta_2$, then $\tilde{\Pi}_1(y, y) > \tilde{\Pi}_3(y, y)$. Hence, the aggregate expected payoffs are largest when firm 1 issues nonexclusive licenses.

(ii) $\delta_1 + \delta_2 < H < \delta_2 + \delta_3$. Then $\tilde{\Pi}_2(y, y) > \tilde{\Pi}_1(y, y)$ and $\tilde{\Pi}_2(y, y) > \tilde{\Pi}_3(y, y)$, so the aggregate expected payoffs are largest when firm 2 issues nonexclusive licenses.

(iii) $\delta_2 + \delta_3 < H$. Then $\tilde{\Pi}_3(y, y) > \tilde{\Pi}_2(y, y)$. Moreover, since $\delta_2 \geq \delta_1$, then $\tilde{\Pi}_3(y, y) > \tilde{\Pi}_1(y, y)$. Hence, the aggregate expected payoffs are largest when firm 3 issues nonexclusive licenses.

\[ \blacksquare \]
9 References


