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A Simple Model of Health Insurance Competition

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Abstract

This paper investigates competition between health care insurance companies under different funding regulations. We consider two alternatives advanced in recent German health care reform discussions: competition by contribution rates (health contributions) and by fees (health premia). We find that competition by contributions can yield lower company profits and higher consumer welfare than competition by premia when switching between insurance companies is costly.

JEL classifications: I 11, I 18, D43.

Keywords: Health Care Reform, Competition, Consumer Choice.

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1 Introduction

Rising expenditures due to demographic change and medical progress have put health care reform quite on top of the policy agenda in most industrialized countries. Due to the budgetary pressures, suitable strategies for cost containment are hotly debated. In that respect, the debate in Germany can be regarded as a showcase as the German system uses comparatively much resources to produce average results (OECD, 2008).

From an economic perspective, the promotion of competition is a natural recommendation. In fact, many advisors perceive the problems of health insurance as problems of an uncompetitive environment and many recent reform proposals argue in favor of a strengthening of market forces - see Wissenschaftlicher Beirat beim BMF (2004) and OECD (2008) as characteristic examples for that view.

Typically, the proposal to enhance competition goes hand in hand with the suggestion to replace income-related contributions by uniform fees, so-called health premia. It is often argued that the resulting removal of income redistribution from the health care system would kill two bird with one stone: on the one hand, both the excess burden of the implicit income tax and the overconsumption of health services due to distorted price signals would be removed. On the other hand, there would be stronger incentives for customers to switch to more advantageous offers generating more competition and hence cheaper insurance contracts (Wissenschaftlicher Beirat beim BMF, 2004; OECD, 2008).

While the first part of this argument has been discussed thoroughly in the scientific literature providing mixed findings once social compensation for poorer households is considered (Breyer & Haufler, 2000; Buchholz, 2005; Fehr & Jess, 2006; Schubert & Schnabel, 2009), the second part is often mentioned (Buchholz, 2005; Fehr & Jess, 2006) but left virtually unexamined. To the best of our knowledge, there is no theoretical analysis of the competitive virtues of contribution/tax rates versus user fees, neither in the health care context nor from a general perspective. While there exists a strand of literature addressing the choice of taxes versus fees, it focusses almost mostly on political economy aspects and/or the provision of public goods and makes no reference to competition issues (Bös, 1980; Fraser, 1996; Swope & Janeba, 2005). This is also true for Kifmann (2005) who discusses the political economy of income-related contributions in the realm of health care. From the empirical side, Cutler & Reber (1998) find a substantial competition effect by the shift from income-related to uniform health plan employer subsidies for Harvard University employees. However, removing income-related subsidies and removing income-related contributions is far from synonymous. As the paper does not provide a thorough analysis of the mechanism driving pricing decisions of insurance providers, its applicability to the present question is impaired. Frank & Lamiraud (2009) challenge the above-mentioned view that competition necessarily improves consumer
choice in health insurance. Using Swiss data, they find price differences to be persistent and switching to cheaper insurance plans to be weak. However, they do not consider how the funding mechanism itself might affect customer behavior.

This gap in the literature is somewhat surprising, as it is far from obvious that the implementation of health premia is a prerequisite for competition. Competition by contribution rates is equally conceivable and has to some extent been operated in Germany despite under tight regulations.

The present paper is a first attempt to address this gap. In a simple stylized duopoly model, we examine how consumers fare when insurance companies compete by setting either health contributions proportional to income or uniform health premia. Interestingly, our simple analysis does not provide support for the efficiency argument for funding health insurance by premia. While the question of premia versus contribution competition turns out to be virtually irrelevant when customers are very reactive to price differentials, contribution rate competition leads to lower aggregate health insurance expenditures and hence higher consumer welfare when demand inertia due to customer loyalty is introduced. This result is grounded in the fact that contribution rate competition renders richer individuals the favorite customers of insurance companies, as they pay higher prices for medical insurance. At the same time, richer people react also more sensitive to contribution rate differentials than poorer people. The attempt to attract high-income persons induces a strong incentive for insurance companies to mitigate contribution rates.

The paper is organized as follows. Section 2 presents the basic structure of the model. Section 3 analyzes both fee and contribution rate competition with an infinitely reactive demand, whereas Section 4 considers loyal consumers reacting sluggishly on price differentials. Section 5 concludes.

2 The Model

Consider an economy populated by a large number of individuals with a mass normalized to one. Persons differ with respect to gross income according to an income distribution characterized by the p.d.f. $f(y)$ with support $[y, \bar{y}]$. Let $y^A = \int_{y}^{\bar{y}} yf(y)dy$ denote aggregate, coinciding with mean income due to the normalization, while $\text{var}(y)$ is the variance of the income distribution.

Each individual faces a certain risk of illness, in which case a damage $c$ arises. We assume that this risk $p$ is the same for everyone, that is, it is uncorrelated to income. We will discuss this in the Conclusions section.

There are two health insurance companies $A$ and $B$ which compete for their customers either
by setting fees or health premia \( g_i \) or health contributions at rates \( \tau_i \) with \( i \in \{ A, B \} \).

In accordance with the regulations of the German Health Insurance System, we posit that insurance is compulsory for all persons and that insurance providers have to accept every customer, that is, there is an obligation to contract. Moreover, treatment in case of illness is standardized by state authorities. As a useful benchmark, we stipulate that the treatment has to compensate for the whole personal damage. To simplify the exposition, the treatment cost is assumed to amount to \( c \).

Due to these features, all individuals are fully insured and insurance companies face a total treatment cost of \( p \cdot c \). This implies that the sum of consumer expenditures and producer profits in the health insurance market amounts to total treatment cost \(-pc\), whatever the level of contribution rates or fees. Hence, every equilibrium in the health insurance market is Pareto-efficient: it is impossible to improve the situation of any market participant without harming another. However, these equilibria vary with respect to the division of gains between individuals and insurance companies on the one hand and among individuals on the other hand. In the following analysis, we address both issues by considering two separate indicators: consumer expenditures, the total payments by the insured and consumer welfare, the sum of individual utilities caused by these payments. Precise definitions of the measures will be formulated below.

### 3 Competition with Very Reactive Customers

This section addresses competition between insurance companies when customers are very reactive, that is, they switch immediately from one company to the other, whenever this gives them a slightest utility gain. We start with the case of fee competition, then turn to contribution rate competition and then compare the equilibria.

#### 3.1 Health Premia

Suppose that both insurance companies offer full insurance in exchange for premia \( g_A \) and \( g_B \), respectively. Then, the expected utility of a person with income \( y \) choosing company \( i \) is:

\[
p \cdot u(y - c - g_i + c) + (1 - p)u(y - g_i) = u(y - g_i),
\]

where \( u(\cdot) \) is the individual utility function displaying diminishing marginal utility of income: \( u'(\cdot) > 0, u''(\cdot) < 0 \).

Since the benefits in case of illness are regulated to be equal for both companies, preferences
over both offers are determined by cost considerations: the individual chooses A over B with certainty when \( g_A < g_B \) and chooses definitely B over A when the inequality is reversed. For equal fees, he is indifferent. Therefore, depending on the levels of the premia, the fraction of people with income \( y \) that chooses company \( i \) can be described by:

\[
d^G_i(y, g) = \begin{cases} 
1 & : \ g_i < g_j \\
0.5 & : \ g_i = g_j \\
0 & : \ g_i > g_j
\end{cases},
\]

where \( g = (g_A, g_B) \) is the vector of premia and \( j \neq i \) denotes the competing company.

The profits for insurance company \( i \) are then:

\[
\int_y^y (g_i - pc)d^G_i(y, g)f(y)dy
\]

**Proposition 1.** In the premia competition equilibrium with perfect customer mobility, both insurance companies set their premia equal to expected treatment cost: \( g_A = g_B = g^* = pc \). Company profits are zero.

**Proof.** From (3), the following best-response function of company \( i \) results:

\[
g_i(g_j) = \begin{cases} 
g_j - \varepsilon & : \ g_j > pc \\
g_j & : \ g_j = pc \\
g_i & : \ g_j < pc
\end{cases}.
\]

This gives the unique equilibrium: \( g_A = g_B = pc \), where both companies have 50% of the households as their customers and make no profits. Q.E.D.

This Bertrand-like result is driven by the infinitely high reactiveness of customers. As all insured switch immediately to the cheaper company, each company has a strong incentive to undercut the competitor’s premium whenever it exceeds the expected treatment cost. Competition drives fees down to expected treatment cost, eradicating any profits.

### 3.2 Health Contributions

When health insurance is funded by income-related contribution rates, an individual with income \( y \) contracting with company \( i \) experiences utility:

\[
u((1 - \tau_i)y)
\]

---

1. To simplify the exposition, we abstract from any problems arising from negative incomes due to health premia exceeding personal gross income. Doing so strengthens the case for premia.
Comparing utilities from both offers, it is straightforward to see that each individual chooses the company with the lower rate whenever contribution rates differ. Otherwise, they are indifferent. As a consequence, the fraction of persons with income $y$ opting for company $i$ is given by:

$$d^T_i(y, \tau) = \begin{cases} 
1 & \text{if } \tau_i < \tau_j \\
\frac{1}{2} & \text{if } \tau_i = \tau_j \\
0 & \text{if } \tau_i > \tau_j 
\end{cases}$$  \hspace{1cm} (5)$$

with $\tau = (\tau_A, \tau_B)$ as the vector of contribution rates. Profits are:

$$\int_y (\tau_i y - pc) d^T_i(y, \tau) f(y) dy$$  \hspace{1cm} (6)$$

**Proposition 2.** In the health contribution rate equilibrium with perfect customer mobility, both insurance companies set their contribution rates equal to the ratio of expected treatment cost and average income: $\tau_A = \tau_B = \tau^* = pc/y_A$. Company profits are zero.

**Proof.** From (6), we get the best-response function:

$$\tau_i(\tau_j) = \begin{cases} 
\tau_j - \varepsilon & \text{if } \tau_j > pc/y_A \\
\tau_j & \text{if } \tau_j = pc/y_A \\
\tau_i > \tau_j & \text{if } \tau_j < pc/y_A 
\end{cases}$$  \hspace{1cm} (7)$$

Hence, there is a unique equilibrium: $\tau_A = \tau_B = pc/y_A$, where revenue equals expected treatment expenditures. Q.E.D.

Again, the fierce competition drives profits down to zero. Contribution rates leading to positive profits at least for one company can not constitute an equilibrium as each company would have an incentive to attract total demand by marginally lowering the contribution rate under the competitor’s level.

### 3.3 Comparing Equilibria

We are now in the position to compare the resulting equilibria. As mentioned above, we use two measures: consumer expenditures and consumer welfare. The first measure $CE$ is concerned with the division of the surplus between households and companies. Since consumer expenditures and company profits always add up to $-pc$, consumer expenditures equal company revenues. Hence we have:

$$CE^G = g^*, \hspace{1cm} CE^T = \tau^* y_A$$  \hspace{1cm} (8)$$
for fees and contribution rates, respectively.

Consumer welfare $CW$, however, explicitly considers the distribution of consumer expenditures among individuals. Here we have:

$$CW^G = \int y u(y - g^*) f(y) dy, \quad CW^T = \int y u((1 - \tau^*)y) f(y) dy. \quad (9)$$

**Proposition 3.** With perfect customer mobility, health premia and health contributions competition yield identical results in terms of total consumer expenditures. However, consumer welfare is higher with health contributions.

**Proof.** The first statement follows either by inserting the respective equilibrium fees and contribution rates to (8) or immediately from noting that company profits are zero in both equilibria. Hence revenues must be same and so have to be total consumer expenditures. The distribution of these expenditures among households is uniform for fee competition and progressive for contribution rate competition. Due to diminishing marginal utility of income, the sum of utilities is higher under contribution rate competition. Q.E.D.

As a first outcome, we find the current setup not to constitute an argument in favor of premia rather than contribution rates competition. Both types of competition impose the same total cost on the insured, while contribution rates are preferable from a distributional perspective. However, this preference is merely grounded in the fact that no other instruments for income redistribution are at hand. Allowing for an additional proportional income tax would easily allow fee competition to reproduce the level of consumer welfare achieved with contribution rate competition. In that case, the mode of competition would be irrelevant for all market participants and hence for society.

### 4 Competition with Loyal Customers

It is well known that immediate switching is rather a useful theoretical benchmark than a good description of actual customer behavior. Instead, substantial price differences seem to go hand in hand with a low intensity of changing providers - see Frank & Lamiraud (2009) for respective evidence for the Swiss Health Care System. Therefore, we revisit the issue of fee and contribution rate competition in a setting where consumers are imperfectly reactive to price differentials. For this purpose, we introduce customer loyalty into the analysis. Apart from that, we proceed like in the precedent section: considering premia competition first, we then turn to contribution rate competition and finally compare.
4.1 Health Premia

We assume now that individuals differ not only with respect to income, but also along a second dimension, which we call customer loyalty. This loyalty or attachment is modelled as a personal cost experienced from not being insured by the favorite company.

As this loyalty is meant to reflect personal characteristics unrelated to income, we assume the distributions of loyalty and income to be uncorrelated. To fix ideas, define the cost \( \mu \) as the loyalty for company A over company B and take it to be uniformly distributed in the interval \([-\bar{\mu}, \bar{\mu}]\), \(\bar{\mu} \geq 0\) for each income level. Hence the probability that a person with income \( y \) has the cost \( \mu \) is \( \frac{1}{2\bar{\mu}} \). The higher \( \bar{\mu} \), the more dispersed customer loyalties. As the distribution has a zero mean, loyalties are symmetrically distributed among the two companies: all other things equal, one half of individuals prefers A over B whereas the other half prefers B over A.

Obviously, loyalty affects demand also in the presence of fee differences: A person with income \( y \) chooses insurer A when \( u(y - g_A + \mu) > u(y - g_B) \), which is tantamount to:

\[
\mu > g_A - g_B,
\]

that is, his loyalty is stronger than the premium differential. Taking the support of the cost distribution into account, we arrive at the following expression for the fraction of people with income \( y \) choosing company A as a function of offered fees:

\[
\tilde{d}_A^G(y, g) = \begin{cases} 
1 & : g_A < g_B - \bar{\mu} \\
\frac{\bar{\mu} - (g_A - g_B)}{2\bar{\mu}} & : g_A \in [g_B - \bar{\mu}, g_B + \bar{\mu}] \\
0 & : g_A > g_B + \bar{\mu}
\end{cases},
\]

and \( \tilde{d}_B^G(y) = 1 - \tilde{d}_A^G(y) \). The profit of company \( i \) is given by:

\[
\int_y^\infty (g_i - pc)\tilde{d}_i^G(y)f(y)dy \tag{11}
\]

Maximizing this expression with respect to \( g_i \) yields the first-order condition:

\[
\int_y^\infty \tilde{d}_i^G(y)f(y)dy + \int_y^\infty (g_i - pc)\frac{\partial \tilde{d}_i^G(y)}{\partial g_i}f(y)dy = 0 \tag{12}
\]

In what follows, we concentrate on symmetric equilibria.

**Proposition 4.** In the symmetric premia competition equilibrium with switching costs, both insurance companies set their premia equal to a markup on expected treatment cost which
depends positively on the dispersion of switching costs:

\[ g_A = g_B = \tilde{g} = pc + \tilde{\mu} \]  

**Proof.** In the symmetric equilibrium, we have \( \tilde{\mu} = 1/2 \) and \( \frac{\partial \tilde{G}(y)}{\partial g_i} = -\frac{1}{2\tilde{\mu}} \). Using these expressions in (12) gives:

\[
\frac{1}{2} - (g_i - pc) \int_y^{\tilde{y}} \frac{1}{2\tilde{\mu}} f(y) dy = 0.
\]

As \( \int_y^{\tilde{y}} f(y) dy = 1 \), that condition becomes: \( 1 - (g_i - pc)/2\tilde{\mu} = 0 \), which is solved by \( g_i = pc + \tilde{\mu} \).

Q.E.D.

Condition (12) shows the tradeoff between positive and negative marginal effects of raising the fee level. The positive effect - called funding effect in the sequel - accrues because higher premia generate more revenue from the customers. The negative effect - called erosion effect in the sequel - is the reduction in profits due to the loss of customers. This erosion effect is the weaker, the more dispersed switching cost are. Thus, equilibrium premia depend positively on \( \tilde{\mu} \).

### 4.2 Contribution rates

Consider now the case where insurance companies compete by health contributions. The preference of a person with income \( y \) of A over B is now reflected in the condition:

\[
\mu > (\tau_A - \tau_B)y
\]

As above, A is preferred when the loyalty exceeds the cost differential between both providers. However, this cost differential now depends positively on personal gross income. Payments being proportional to income, any given contribution rate differential affects high income earners stronger than low income households.

From (14), the fraction of people with income \( y \) opting for company A becomes:

\[
\bar{d}_A^T(y, \tau) = \begin{cases} 
1 & : \tau_A < \tau_B - \frac{\tilde{\mu}}{y} \\
\frac{\bar{\mu} - (\tau_A - \tau_B)y}{2\bar{\mu}} & : \tau_A \in [\tau_B - \frac{\tilde{\mu}}{y}, \tau_B + \frac{\tilde{\mu}}{y}] \\
0 & : \tau_A > \tau_B - \frac{\tilde{\mu}}{y}
\end{cases}
\]

Inspection of (15) shows that that fraction is income-dependent for moderate contribution rate differentials: Among people with equal income, the fraction choosing A decreases in \( y \) when \( \tau_A > \tau_B \) and increases when \( \tau_A < \tau_B \). Intuitively, the higher income, the lower the
expressed loyalty towards the more expensive insurance company, as this involves a higher financial sacrifice. By the same token, richer people are more responsive to contribution rate increases:

$$\frac{\partial \tilde{d}_T(y, \tau)}{\partial \tau_A} = \begin{cases} -\frac{y}{2\bar{\mu}} : \tau_A \in [\tau_B - \frac{\bar{\mu}}{y}, \tau_B + \frac{\bar{\mu}}{y}] \\ 0 : \text{otherwise} \end{cases}$$ (16)

Taking this behavior into account, companies maximize profits:

$$\int_y y \tilde{d}_T(y) (y) dy$$

with respect to their health contribution rate $\tau_i$. This leads to the first-order condition:

$$\int_y y \tilde{d}_T(y) f(y) dy + \int_y (\tau_i y - pc) \frac{\partial \tilde{d}_T}{\partial \tau_i} f(y) dy = 0$$ (17)

This expression can be interpreted analogous to (12). The first term denotes the positive funding effect, the higher revenue generated from the customer base. The second term measures the erosion effect, that is the reduction of profits due to the reduction of the customer base. Obviously, the profit reduction can be decomposed in a reduction of revenue and cost, respectively.

**Proposition 5.** *In the contribution rate equilibrium with switching costs, both insurance companies set their contribution rates as a markup on expected treatment cost which depends both on the dispersion of switching cost and the ratio of the first and the second moment of the income distribution:*

$$\tau_A = \tau_B = \bar{\tau} = (pc + \bar{\mu}) \cdot \frac{y^A}{\int_y y^2 f(y) dy}$$ (18)

**Proof.** In a symmetric equilibrium, we have: $\tilde{d}_T = 1/2$ and $\frac{\partial \tilde{d}_T}{\partial \tau_i} = -\frac{y}{2\bar{\mu}}$ from (16). Hence (17) becomes:

$$\int_y \frac{y}{2} dy - \int_y (\tau y - pc) \frac{y}{2\bar{\mu}} f(y) dy = 0$$

$$\Leftrightarrow y^A - \int_y \frac{\tau y^2}{\bar{\mu}} f(y) dy + \frac{pc}{\bar{\mu}} y^A = 0$$

Solving for $\tau$, this expression immediately yields (18) Q.E.D.
The basic reason for the positive dependency of contribution rates on the dispersion of switching costs is the same as in the case of fee competition: the higher $\bar{\mu}$, the less sensible demand reacts on a contribution rate increase and the less important is the erosion effect. Additionally, the distribution of income becomes contentious by affecting both the funding and the erosion effect: the funding effect is now proportional to average income for a marginal increase in the contribution rate collects a infinitesimally higher income share from all customers. The erosion effect is influenced by the higher sensitivity of richer people to contribution rate increases. Here we have to disentangle the impacts on revenue and cost. On the one hand, the cost savings due to the shrinking number of clients are proportional to average income in our setup. On the other hand, the loss in revenues is more than proportional, because not only the reduction of the customer base but also the revenues per customer increase in income. For that reason, the second moment of the income distribution is involved.

4.3 Comparing equilibria

How do the two equilibria compare regarding consumer expenditures and consumer welfare? The following proposition gives a clear-cut answer.

**Proposition 6.** Whenever there is income inequality, health contribution competition leads to lower consumer expenditures and higher consumer welfare than premia competition.

**Proof.** See Appendix.

The intuition behind this result is simple: compared to premia competition, contribution rate competition shifts the focus of companies towards richer individuals, as they are the more lucrative clients. However, these households react more sensibly on contribution rate increases. This renders health contribution competition fiercer: both companies moderate their claims in order not to put the high-income people off.

This intuition is easily substantiated by considering the relative strengths of funding and erosion effects. Take the case of a degenerate income distribution ($\text{var}(y) = 0$) as the starting point. For such a distribution, the distinction between premia and contribution competition is meaningless. So both types of competition yield identical results, as can be seen from comparing (13) and (18) for $\text{var}(y) = 0$ implying $\int_{\frac{y}{2}}^{y^2} f(y) dy = (y^4)^2$.

Now introduce income inequality by a mean-preserving spread of the income distribution. This does not affect premia competition as neither funding nor erosion effect are income-dependent. However, things are different for contribution rate competition. While the funding effect is proportional to mean income, the erosion effect is strictly convex in income. Hence income inequality emphasizes the negative erosion effect relative to the positive funding effect. Therefore company profits must be lower under contribution rate competition whenever the income distribution is not degenerate. These lower profits translate into lower consumer
expenditures. As contribution rate competition also reliefs poorer households, it also creates higher consumer welfare.

5 Conclusion

The analysis has shown that health insurance competition via health contributions can be fiercer than via health premia. This result stands in contrast to popular conjectures in the literature and in public debate.

Simple as it is, the model should not be misinterpreted such that contribution rates are definitely preferable to premia. A number of aspects, which may tilt the balance in favor of premia, like the eradication of implicit income taxation have not been incorporated to the analysis. However, as many studies find those aspects to be of minor significance, the decision between contribution rates and fees as funding instruments for health care appears more delicate than suggested in the literature.

Obviously, the model stands open to a number of extensions some of which we would like to discuss briefly. First, we could introduce income-dependent risks of illness, in order to account for the salient finding of the literature that income and health are positively correlated (Deaton, 2003). Allowing for this correlation would strengthen our results because it makes richer clients even more attractive for companies. Second, we could allow for public health insurance to be limited to subsets of the population as is the case in Germany where only persons with income below the so-called insurance ceiling are mandatory members in public health insurance. As spillovers in company pricing decisions for persons above and below that threshold are unlikely, this would amount to reprising the above analysis with a truncated income distribution. As the above results are not dependant on the shape of the distribution, they are left unaffected by such a modification. Third, one could consider a combination of both funding schemes like it is in action in Germany at the moment. Finally, we could abandon the assumption of full public insurance and allow for private coinsurance. In such a model, which would incorporate essential features of the analysis in Breyer and Haufler (2000), we expect the degree of risk aversion and its dependency on income to have an important bearing on the findings. While we conjecture that our results go through when risk aversion is sufficiently low and not too decreasing in income, we leave a fully-fledged analysis for future research.
Proof of Proposition 6

In order to prove that contribution rate competition implies lower consumer expenditures, a comparison of company revenues is sufficient. These revenues amount to $\tilde{\tau}y^A$ and $\tilde{g}$ respectively.

Contribution rate competition is superior to premia competition if and only if:

$$\frac{\tilde{\tau}y^A}{\int_y y^2 f(y)dy} < \frac{pc + \bar{\mu} (y^A)^2}{\int_y y^2 f(y)dy} < pc + \bar{\mu}$$

(19)

The second moment of a distribution equals the sum of the variance and the squared mean:

$$\int_y y^2 f(y)dy = var(y) + (y^A)^2,$$

as the following calculation reveals:

$$var(y) = \int_y (y - y^A)^2 f(y)dy$$

$$= \int_y (y^2 - 2yy^A + (y^A)^2) f(y)dy$$

$$= \int_y y^2 f(y)dy + \int_y ((y^A)^2 - 2yy^A) f(y)dy$$

$$= \int_y y^2 f(y)dy - (y^A)^2$$

This property renders (19) correct whenever there is income heterogeneity:

$$(y^A)^2 < var(y) + (y^A)^2 \iff var(y) > 0$$

Consumer welfare is higher under contribution rate competition as consumer expenditures are lower in total and more progressively distributed than under premia competition.
References


