Bredemeier, Christian

Conference Paper

Imperfect Information and the Meltzer-Richard Hypothesis


This Version is available at: http://hdl.handle.net/10419/37501

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes. You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Imperfect Information and the Meltzer-Richard Hypothesis

Christian Bredemeier
TU Dortmund University
and Ruhr Graduate School in Economics

Abstract
Standard models of voting on redistribution generate a clear-cut prediction: redistribution increases in income skewness. (the Meltzer-Richard hypothesis) Empirical evidence on this issue is mixed. Changes in income skewness are often accompanied by developments in redistribution into the opposite direction. This paper argues that it is important to distinguish between sources of changes in income skewness, polarization and upward mobility which both have the same impact on income skewness. In a model with imperfect information, these developments affect redistribution in different ways. While polarization generates a positive relation between income skewness and redistribution, upward mobility can have the opposite effect.

Keywords: Voting; Redistribution; Imperfect Information
JEL Classification No.: D72, H24, D83

1 Introduction
Standard theory predicts that democratically implemented income redistribution should always favor the receiver of the median income. Since the individually optimal degree of redistribution is a downward-sloping function of one’s income, the median income receiver is also the median voter. A clear-cut prediction that arises from such consideration is the Meltzer-Richard hypothesis (Meltzer and Richard 1981): the extent of redistribution rises when the mean-to-median ratio of the income distribution increases, since the median voter will then gain more from redistribution. Income skewness and redistribution should thus be positively related.

Empirical evidence is anything as clear as the theoretical prediction. Sometimes a positive relation between income skewness and redistribution is indeed observed, sometimes empirical studies report the opposite. At the cross section, evidence is mixed. Cross-country studies find evidence supporting the Meltzer-Richard hypothesis (Easterly and Rebelo 1993; Lindert 1996; Milanovic 2000; Mohl and Pamp 2009) as well as
contradictory results (Keefer and Knack 1995; Perotti 1996; Bassett, Burkett, and Put-terman 1999). Cross-sectional studies within one country reveal evidence in favor of the hypothesis at the municipality level (Alesina, Baqir, and Easterly 2000; Borge and Rattsø 2001) or comparing Brazilian states (Mattos and Rocha 2008) but also rejecting findings at the level of US states (Gouveia and Masia 1998; Rodríguez 1999).

Concerning time-series evidence, the study by Meltzer and Richard (1983) supports the theoretical prediction. The authors analyze US time series data of government spending and come to the conclusion that the spending level is positively related to the mean-to-median income ratio. Later studies on similar questions arrive at the contrary (Rodríguez 1999; Kenworthy and McCall 2008). These studies report situations of increasing income skewness that are accompanied by cut-backs in the welfare state. Such developments are alien to a standard majority voting model.

The present paper argues that it is not sufficient to consider the skewness of the income distribution alone. It is rather important to distinguish between sources of changes in income skewness. I discuss two developments in the income distribution which have the same effect on skewness but may affect democratically implemented redistribution in different ways: polarization and upward mobility. Under polarization I understand a development where those who are rich anyway become even richer. In contrast to this, upward mobility describes a development where initially rather poor individuals catch up to richer population groups.

Both developments increase income skewness and therefore have the same effect on redistribution in a standard voting model. However, in a model with imperfect information, these developments affect redistribution in different ways. While polarization generates the standard effect, upward mobility can result in decreasing redistribution.

The importance of informational imperfections in democratic decision making has been stressed by Downs (1957). Downs pointed out that even small information costs can lead voters to be rationally ignorant and cause pronounced uncertainty about issues important for the optimal vote. Understood broadly, imperfect information also comprises all differences between complete information and information that is reflected in behavior (Sims 2003). Such differences can arise from cognitive differences at any stage in the process between observing an information and the implementation of the appropriate response. Even with perfect information available, if voters choose not to use all information, have difficulties figuring out the appropriate response, or make mistakes while translating decisions into behavior, political decisions may appear as if voters had imperfect information in the first place. These concepts are used in increasing frequency to study voting behavior, both theoretically (Gershkov and Szentes 2009; Hansen 2005; Dhami 2003) and empirically (Mullainathan and Washington 2009; Shue and Luttmer 2009).
The present paper presents a model of direct democracy with selfish voters, perfect markets, and complete enforcement in which the relation between income skewness and redistribution depends on the drivers of changes in income skewness. The model is a version of the Romer-Roberts-Meltzer-Richard model (Romer 1975; Roberts 1977; Meltzer and Richard 1981). Agents differ with respect to their productivity and, in consequence, income. The main difference to the standard model is imperfect information about the skewness of the income distribution. Under perfect information, the extent of redistribution would be determined by the interest of the median-income earner with high-income agents wishing less redistribution and low-income agents more.

In the model presented in this paper, optimal redistribution for the middle class depends on unknown fundamentals. Due to imperfect information, there is a distribution of votes of middle-class agents around the optimal vote. Voting powers of the upper and lower classes then determine which vote from this distribution is decisive. These agents are sure to be at the bottom or the top of the distribution no matter how it is shaped. Independent of their beliefs about the shape of the distribution, these individuals will vote for either maximum redistribution or none at all.

Even though beliefs about income skewness are unbiased through the population, it is not guaranteed that the decisive median voter estimates the skewness correctly. For instance, if the lower class is large, the median voter overestimates the skewness and opts for more redistribution than would be optimal for agents of his type. In this situation, there are relatively many voters in favor of maximum redistribution such that these agents can build a majority together with middle-class agents who overestimate productivity skewness. That majority can prevent any lower tax rate, even if it would improve the situation of some of its members (who, however, are not aware of that).

Increases in income skewness have two counter-acting effects on redistribution. On the one hand, the optimal degree of redistribution for the middle class rises. On the other hand, potential shifts in voting power move the quantile of the median voter in the belief distribution towards voting for fewer redistribution. Therefore it is important to distinguish between causes of changes in the mean-to-median income ratio. If the rich become richer (polarization), only the first effect occurs and the model generates the standard prediction. If, however, the income skewness changes because more people become rich (upward mobility), then both effects are active. With pronounced informational imperfections, the second effect can dominate the first and one would observe a negative relation between income skewness and redistribution.

Previous contributions on imperfect information in models of voting on redistribution have studied related, though not identical, questions. Dhami (2003) analyzes the effects of inequality on redistribution in a model of representative democracy where voters have asymmetric information about politicians’ redistributive ambitions. Hansen (2005) and Laslier, Trannoy, and Van Der Straeten (2003) use similar models to the one presented in this paper. Hansen uses an Romer-Roberts-Meltzer-Richard type model with imperfect
information about government efficiency and studies arising biases in the level of government size. Laslier, Trannoy, and Van Der Straeten (2003) covers the topic of overtaxation in a model with uncertainty about the potential productivity of the unemployed. Both studies however do not cover the relation between income skewness and redistribution.

The remainder of the paper is organized as follows. In Section 2, the set-up of the model is described. Section 3 solves the model for individual decisions and collective choices. Section 4 contains a comparative-static analysis of a change in the mean-to-median ratio. Section 5 concludes.

2 The Model Set-up

In this section, I describe the structure of the model. It is mainly in the style of Romer (1975), Roberts (1977), and Meltzer and Richard (1981). The main difference is that agents are imperfectly informed about the productivity distribution.

Preferences and Technology. I consider an economy that consists of a mass-1 continuum of agents behaving according to the following preferences:

\[ u_i = c_i - \frac{\phi}{2} n_i^2, \]  

where \( c_i \) denotes agent \( i \)'s consumption and \( n_i \) is the amount of hours worked. If working, agents produce consumption goods \( y_i \) with linear technology

\[ y_i = a_i n_i, \]  

where \( a_i \) is an agent-specific productivity.

The composition of the economy is characterized by discrete differences in productivity and labor-force participation. Agents either have a low productivity \( a_1 \), or a high one, \( a_2 \), where \( a_2 = \sqrt{\alpha} \cdot a_1, \alpha > 1 \). Furthermore there are working agents and some who do not work. Alternatively, non-working agents can be understood as agents with zero productivity. The population can thus be split up into three groups:

(i) working agents with high productivity, who I will label with an "H" for high expected incomes before redistribution

(ii) working agents with low productivity, labeled with an "M" for medium

(iii) agents who do not work, labeled with an "L" for low

Labels refer to earnings before redistribution. Group sizes are \( s_L, s_M, \) and \( s_H \), respectively, \( s_L + s_M + s_H = 1 \). I assume that no group contains more than mass \( \frac{1}{2} \) of agents. This assumption guarantees that the median gross income falls into group M which can therefore be considered a "middle class".

4
Note that both, $\alpha$ and $\frac{s_H}{s_L}$ are determinants of the skewness of the productivity distribution which is illustrated in the upper part of Figure 1. With $\sqrt{\alpha} = \frac{s_H + s_L}{s_H}$ the distribution is symmetric. If $\sqrt{\alpha} > \frac{s_H + s_L}{s_H}$, the distribution is skewed to the right and vice versa. Skewness of the income distribution, which is key for the extent of redistribution sought by the middle class, results from the skewness of the productivity distribution together with endogenous labor supply decisions.

**Political Environment.** The economy redistributes income through a linear income tax $\tau$, the proceeds of which are to be distributed equally among the total population. Thus, an agent’s net amount of consumption is a linear combination of his own gross earnings and the average earnings in the economy,

$$c_i = (1 - \tau) \cdot y_i + \tau \cdot y,$$

where $y$ denotes the average gross income.\(^1\)

The redistribution rate $\tau \in [0, 1]$ is determined in direct democracy by pairwise votes over proposals. All agents participate in this vote. Furthermore, I assume that agents vote truthfully in the sense that they vote for their individual expected-utility maximizing $\tau$.\(^2\)

**Informational Environment.** Agents are aware of the structure of the economy and know all parameter values except for other agents’ productivities. For agents in groups M and H, this is tantamount to not knowing the productivity parameter $\alpha$. This parameter measures the difference between the middle and the upper class and is one determinant of the skewness of the productivity distribution. The parameter is drawn from a uniform distribution on $[1, \infty]$. Agents cannot observe this draw. After $\alpha$ is drawn, each agent receives an imperfect signal $\alpha_i^s$ about $\alpha$, which equals the true value in expectation. Agents’ signals are independently drawn from an identical uniform distribution on $[\alpha - \varepsilon, \alpha + \varepsilon]$.

**Time Structure.** Events happen in the following temporal ordering. First, the productivity parameter $\alpha$ is drawn. The draw is unobservable for agents. Second, agents receive signals $\{\alpha_i^s\}$ and update their beliefs. Third, the election over the redistribution parameter $\tau$ takes place, the median vote is implemented. Fourth, agents decide how much to work and produce gross income. Finally, redistribution is performed and goods are consumed.

\(^1\)The literature usually studies voting on the parameterization of some given redistribution scheme. In more general set-ups, voting equilibria may not exist (see e.g. Mueller (2003)).

\(^2\)Since any single voter has zero mass in this model, I abstain from analysing strategic voting behavior and assume "naive voting" (Feddersen and Pesendorfer 1997) instead.
3 Individual and public choices

3.1 Individual decisions

Labor supply decisions and income distribution. Working agents have to decide on how much they want to work. When taking this decisions, agents have to take the degree of income redistribution into account. Given a tax rate $\tau$, equalizing marginal benefits and costs from working results in

$$ n_i = \frac{(1 - \tau) a_i}{\phi}. $$

Redistribution reduces labor supply through a standard disincentive effect. It has the same effect on aggregate income, which is

$$ y = (1 - \tau) \cdot \frac{1}{\phi} \cdot [s_M (a_1)^2 + s_H (a_2)^2] $$

as a result of individual labor supply decisions.

Labor supply decisions as described by equation (4) imply that income is a quadratic function of productivity. Inequalities are thereby magnified as can be seen from Figure 1. The mean-to-median income ratio is $s_M a_1 + s_H a_2 / a_1 = s_M + s_H \alpha$. The income distribution is skewed to the right and the mean-to-median ratio greater than 1 if $\alpha > \frac{s_H + s_L}{s_H}$ and vice versa.
Preferred tax rates. When agents vote for a certain tax rate $\tau$, they form rational expectations about the disincentive effects redistribution has. Rationally anticipating subsequent labor supply decisions of all agents, an agent votes for that tax rate that maximizes her expected indirect utility.

For a non-working agent, $i \in L$, transfers are the only source of income. Since she does not work, expected indirect utility is

$$E_iu_i = \tau \cdot (1 - \tau)^2 \cdot \frac{1}{\phi} \cdot [s_M E_i(a_1)^2 + s_H E_i(a_2)^2] \forall i \in L. \quad (5)$$

The tax rate that maximizes expected indirect utility for this agent is independent of expected productivities and

$$\tau_i = \frac{1}{2} \quad \forall i \in L,$$

which is the Laffer-curve maximizer in this model. Agents in this group do not need to know the skewness of the distribution in order to determine their optimal rate of redistribution. They are obviously transfer receivers independent of the exact shape of the distribution.

In contrast to this, the skewness of the distribution matters for the preferred tax rate of agents in the middle class. Agents in this group receive their own net income as well as transfers and incur utility losses from working. Their expected indirect utility is

$$E_iu_i = (1 - \tau)^2 \cdot \frac{(a_1)^2}{\phi} + \tau \cdot (1 - \tau)^2 \cdot \frac{s_M E_i(a_1)^2 + s_H E_i(a_2)^2}{\phi}$$

$$- \frac{\phi}{2} \cdot (1 - \tau)^2 \cdot \frac{(a_1)^2}{\phi^2} \forall i \in M, \quad (6)$$

which is maximized by

$$\tau_i = \max \left[ \frac{1 - (s_M + s_H E_i \alpha)}{1 - 2(s_M + s_H E_i \alpha)}, 0 \right] \forall i \in M. \quad (7)$$

The term in the round brackets is the expected mean-to-median income ratio. The preferred tax rate of an agent in the middle class is a (weakly) upward sloping function of her expectation of income skewness. When $E_i \alpha$ is high, the agent believes that income differences between the upper and the middle class are pronounced and that she can gain much from taxing the members of the upper class. In the opposite case, when $E_i \alpha$ is low, the agents believes to pay much taxes in order to finance transfers to the lower class. A middle-class agent $i$ wishes positive redistribution only if the expected mean-to-median ratio is above 1, i.e. if she believes the income distribution is skewed to the right.³

³The expression on the right hand side of equation (7) is also positive if the expected mean-to-median ratio is less than one half and would then describe a minimizer. However, this is prohibited due to assumed restrictions to group sizes.
Finally, members of the upper class can only lose from redistribution. Their expected indirect utility,

\[ E_i u_i = (1 - \tau)^2 \frac{(a_2)^2}{\phi} + \tau \cdot (1 - \tau) \cdot s_M E_i \left( \alpha^{-1/2} a_2 \right)^2 + s_H (a_2)^2 \]

is a strictly downward sloping function of the tax rate \( \tau \), since it is sure that \( \alpha > 0 \). Agents in group H therefore vote for

\[ \tau_i = 0 \quad \forall i \in H. \]

Considering the expected indirect utility functions (5), (6), and (8), one can see that they all have a unique maximizer on \([0, 1]\). Thus, preferences over \( \tau \) are single-peaked for all agents. When determining election outcomes, the median-voter theorem is therefore applicable.

### 3.2 Belief formation and belief distribution

To determine the median voter, the distribution of votes has to be considered. Since some agents vote based on subjective beliefs, the distribution of beliefs needs to be determined first. This distribution arises as a result of agents’ belief updating based on their individual signals.

The structure of the economy is common knowledge. However, before receiving the signal, agents only know that \( \alpha \) is not less than 1. All values above 1 are equally likely from the perspective of agents. An agent’s prior belief about the productivity difference \( \alpha \) is thus a uniform distribution on \([1, \infty]\).

The signal \( \alpha_i^S \) then carries the information that further values for \( \alpha \) are impossible. Specifically, after receiving her signal an agent knows, that \( \alpha \) cannot be above \( \alpha_i^S + \varepsilon \) or below \( \alpha_i^S - \varepsilon \). Otherwise the signal would not have been possible. The second information is only new if \( \alpha_i^S - \varepsilon > 1 \) since the agent already knew that value below 1 are not possible. The support of the posterior distribution is thus

\[ \left[ \max \left(1, \alpha_i^S - \varepsilon\right), \alpha_i^S + \varepsilon \right]. \]

Since the signal is uniformly distributed, the agent can not differentiate between values in the support. All values \( \alpha \in \left[ \max \left(1, \alpha_i^S - \varepsilon\right), \alpha_i^S + \varepsilon \right] \) are associated with the same density for the received signal \( \alpha_i^S \). The posterior distribution is thus uniform on above support.

Correspondingly, the posterior expectation of agent \( i \) is the arithmetic mean of the upper and the lower bound of the support:

\[ \mu_i = E_i [\alpha] = \begin{cases} \frac{1}{2} + \frac{\alpha_i^S + \varepsilon}{2}, & \alpha_i^S < 1 + \varepsilon \\ \alpha_i^S, & \text{else} \end{cases}. \]
Since there is a continuum of agents, each possible signal realization on \([\alpha - \varepsilon, \alpha + \varepsilon]\) is drawn by an equal mass of agents. For productivity differences pronounced enough \((\alpha \geq 1 + 2\varepsilon)\), the distribution of subjective expectations about \(\alpha\) can be written as a uniform distribution,\(^4\)

\[
g(\mu) = \begin{cases} 
\frac{1}{2\varepsilon}, & \alpha - \varepsilon \leq \mu \leq \alpha + \varepsilon \\
0, & \text{else} 
\end{cases} 
\]

The distribution of beliefs is the same across all groups and equal to the economy-wide distribution described by equation (10).

For \(\alpha \geq 1 + 2\varepsilon\), beliefs are unbiased, i.e. the economy-wide mean belief is true, \(\bar{\mu} = \int \mu g(\mu) d\mu = \alpha\). Nevertheless, almost every agent misestimates \(\alpha\) in one or the other direction. For \(\alpha < 1 + 2\varepsilon\), the distribution of beliefs is biased away from \(\alpha\), therefore I will restrict the analysis to the former case.

### 3.3 Election Outcomes

Since preferences over the tax rate \(\tau\) are single peaked, the median vote is the unique Condorcet winner. The cumulative density \(F\) of votes and the determination of the median voter are illustrated in Figure 2. The distribution of votes in the economy is as follows: On the one hand, there is a mass of people at both extremes. Fraction \(s_H\) of agents (the upper class \(H\)) vote for zero redistribution, \(F(0) = s_H\). Fraction \(s_L\) of agents (the lower class \(L\)) vote for the Laffer-curve maximizing tax rate, \(F(\frac{1}{2}) = 1 - s_L\). On the other hand, there is non-degenerate distribution of votes between these two extremes. Since beliefs about \(\alpha\) differ across agents, votes of agents in the \(M\) group differ from one another. Beliefs are unbiased within groups, therefore the optimal tax rate for the middle class, \(\tau_{\text{opt}}^M\), is the median of the vote distribution within the \(M\) group.\(^5\)

Where the economy-wide median voter is located depends on relative group sizes. As neither group \(L\) nor group \(H\) contains at least 50% of the population, the median voter is surely a member of the \(M\) group. The economy-wide median voter, \(m\), is the agent whose preferred tax rate \(\tau_m\) is such that \(M\)-group voters of mass \(\frac{1}{2} - s_H\) opt for less redistribution than herself since mass \(s_H\) of voters vote for zero redistribution anyway. Thus the median voter is at the lower \(\frac{1}{2} - \frac{s_H}{s_M}\) quantile of the vote distribution within the \(M\) group. Since, for agents in the middle class, \(\tau_i\) is an upward sloping function of \(E_i\alpha\), the median voter is also at the lower \(\frac{1}{2} - \frac{s_H}{s_M}\) quantile of subjective expectations of income differences.

---

\(^4\)Under this restriction, the smallest signal is \(S_{\text{min}} = 1 + \varepsilon\). The agent that receives this signal determines her mean belief according to the second case in equation (9), i.e. \(\mu_{\text{min}} = S_{\text{min}}\). Since no agents receive a smaller signal, all agents build mean belief equal to their received signals. The distribution of mean belief is thus identical to the distribution of signals and given by (10).

\(^5\)Such distribution of votes would also occur if middle-class voters had perfect information but made random mistakes in determining the optimal tax rate or, in the terminology of e.g. Shue and Luttmer (2009), misvoted.
The distribution of subjective expectations is characterized by equation (10). Since the distribution is uniform, the lower \( s_{H} + \frac{s_{M}}{2} \) quantile can be calculated as the corresponding linear combination of upper and lower bound, \( \left( 1 - \frac{s_{H}}{s_{M}} \right) \cdot (\alpha - \varepsilon) + \frac{s_{H}}{s_{M}} \cdot (\alpha + \varepsilon) \). The median voter’s expectation of \( \alpha \) is thus

\[
\mu_{m} = \alpha + \frac{s_{L} - s_{H}}{s_{M}} \varepsilon. \tag{11}
\]

If and in which direction the median voter’s belief differs from the truth depends on the relative sizes of the upper and lower class.\(^6\)

Applying the belief-vote mapping of the middle class (7) to this belief, the preferred redistribution rate of the median voter and thus the implemented rate of redistribution is

\[
\tau^{*} = \max \left[ \frac{1 - (s_{M} + s_{H} \mu_{m})}{1 - 2 (s_{M} + s_{H} \mu_{m})}, 0 \right]. \tag{12}
\]

Positive redistribution occurs when

\[
\alpha + \frac{s_{L} - s_{H}}{s_{M}} \varepsilon > \frac{s_{L}}{s_{H}} + 1, \tag{13}
\]

i.e. when the median voter believes the income distribution to be right skewed and, equivalently, the mean-to-median income ratio to exceed 1.

\(^6\)To obtain the expression for \( \mu_{m} \) in equation (11), use that \( s_{L} + s_{M} + s_{H} = 1 \).
3.4 Suboptimal redistribution for the middle class

Other than in standard models, implemented redistribution does not necessarily have to be optimal for the median-income receiver. As a comparison to the implemented tax rate as given by equation (12), the optimal tax rate for agents in the middle class is

\[ \tau_{opt}^M = \max \left[ \frac{1 - (s_M + s_H \alpha)}{1 - 2(s_M + s_H \alpha)}, 0 \right]. \]

Therefore, with positive redistribution, its extent is suboptimal for the middle class whenever \( \mu_m \neq \alpha \) or, equivalently, the implemented tax rate, if it differs from 0, is optimal for agents in group M only if

\[ s_L = s_H, \tag{14} \]

i.e. only if the lower and the upper class are of equal size.

Equation (14) describes a special parameter constellation. When the upper and the lower class are of equal size, then the median voter will be exactly in the center of the belief distribution of the middle class. Then, and only then, the decisive median voter will have unbiased beliefs about the productivity distribution. In Figure 2, that would imply that \( \tau_{opt}^M \) would not only be the median of the vote distribution of the middle class, but, by coincidence, also the median of the economy-wide vote distribution, \( \tau^* = \tau_{opt}^M \).

If condition (14) is not fulfilled, the median voter will be someone who misestimates the skewness of the productivity distribution and therefore votes for a potentially suboptimal redistribution rate. For instance, if \( s_L > s_M \), the median voter overestimates the skewness and opts for more redistribution than would be optimal for agents of his type. In this situation, there are relatively many voters in favor of maximum redistribution such that these agents can build a majority together with middle-class agents who overestimate productivity skewness. That majority can prevent any lower tax rate, even if it would improve the situation of some of its members (who, however, are not aware of this).

4 Changes in Income Skewness

Standard models of voting on redistribution predict that the extent of redistribution increases in the mean-to-median income ratio. The empirical evidence on this prediction is mixed (see Section 1). In this section, I analyze how changes in the mean-to-median income ratio affect election outcomes in this paper’s model.

In this model, changes in mean-to-median income ratio can occur in two ways. They can be caused either by polarization, i.e. income growth of the rich relative to the middle class (the rich becoming richer, captured by an increase in the parameter \( \alpha \)) or upward mobility, i.e. by growth of the middle or upper class in size relative to the lower class (more agents become rich, captured by an increase in \( s_H \)). While these two scenarios
Figure 3: Income Distribution, mean (thick dashed line), and median (thin dashed line) income before and after polarization \((\tilde{\alpha} > \bar{\alpha}, \kappa = \frac{1-\tau}{\phi})\)

have similar impact on the skewness of the income distribution, their effect on the vote distribution differs. The reason is that while the first scenario simply moves earnings shares to the right of the distribution, the second also moves voting power.

4.1 Polarization

Consider first the case of polarization, an increase in the relative productivity of the upper class, so assume the parameter \(\alpha\) rises, say from \(\tilde{\alpha}\) to \(\bar{\alpha}\), with \(\tilde{\alpha} > \bar{\alpha}\). Figure ?? illustrates the effects of polarization on distributional skewness. The left panel shows a symmetric distribution where mean and median income are identical.\(^7\) The right part shows the income distribution after polarization. Due to income growth of the upper class, mean income (thick dashed line) has risen while median income (thin dashed line) has remained constant. The mean-to-median ration has thus increased through polarization.

The effects of this change on the implemented redistribution rate are illustrated graphically in Figure 4. The thin dashed line represents the initial vote distribution with \(\alpha = \tilde{\alpha}\) whereas the thick solid line stands for the new vote distribution associated with \(\alpha = \bar{\alpha}\).

Since group sizes are constant, the median voter’s quantile in the belief distribution remains unchanged. As the economy-wide mean belief shifts to \(\bar{\alpha}\), the median voter’s belief (as given by 11) increases as well,

\[
\tilde{\mu}_m = \tilde{\alpha} + \frac{s_L - s_H}{s_M} \varepsilon > \bar{\mu}_m. \tag{15}
\]

According to the belief-vote mapping in the middle class (7), the implemented redistribution rate is now

\[
\tilde{\tau}^* = \max \left[ \frac{1 - (s_M + s_H\tilde{\mu}_m)}{1 - 2(s_M + s_H\tilde{\mu}_m)}, 0 \right] \geq \bar{\tau}^*
\]

\(^7\)A symmetric distribution is chosen only for illustrational purposes in the figure, subsequent results do not require symmetry.
and either larger or equal than with the lower difference in productivities $\tilde{\alpha}$. Thus, the model predicts that, in reaction to income growth of the rich, one observes indeed a positive correlation between the mean-to-median income ratio and redistribution. This prediction is equivalent to the standard model’s one.

### 4.2 Upward mobility

Effects are not as clear if the mean-to-median income ratio changes due to changes in relative group sizes. Consider a scenario where the upper class grows at the expense of the lower class (for simplicity with constant size of the middle class). Assume that group sizes change from $\tilde{s}_L$, $s_M$, $\tilde{s}_H$ to $\tilde{s}_L$, $s_M$, $\tilde{s}_H$, with $\tilde{s}_L < \tilde{s}_L$ and $\tilde{s}_H > s_H$. Figure ?? illustrates the effects of upward mobility on distributional skewness. Starting from a symmetric distribution (left panel), upward mobility increases mean income (thick dashed line) while median income (thin dashed line) is not affected. Thus the mean-to-median ratio is larger after upward mobility (right panel).

The consequences of this scenario on redistribution are illustrated in Figure 6. Again, the thin dashed line stands for the initial vote distribution and the thick solid line represents the vote distribution after the change.

Since the compositional change affects the skewness of the income distribution, it alters the belief-vote mapping of the middle class. Agent $i \in M$ with belief $\mu_i$ now votes for

$$\tau_i = \max \left[ \frac{1 - (s_M + \tilde{s}_H \mu_i)}{1 - 2 (s_M + \tilde{s}_H \mu_i)}, 0 \right] \geq \tau_i.$$

For a given belief about $\alpha$, middle-class agents now vote for more redistribution. In the figure, this effect is manifested in the movement of the non-degenerate part of the
distribution to the right. This increase in redistribution sought by the middle class does, however, not imply that implemented redistribution necessarily increases as well.

When the upper class increases in size, voting power shifts towards this group as well. In the figure, this is associated with an upward movement of the non-degenerate part of the distribution, since more mass lies at zero redistribution. As a consequence, the economy-wide median voter’s position within the belief distribution moves to the left. The median voter’s belief about productivity differences (as given by 11) is now

\[ \bar{\mu}_m = \alpha + \frac{s_L - s_H}{s_M} \varepsilon < \bar{\mu}_m. \]

These two developments result in an ambiguous effect on the implemented redistribution rate. While the increase in redistribution sought by the middle class tends to increase implemented redistribution, the shift in voting power has the opposite effect. Which effect is dominant depends on the degree of informational imperfections as measured by the inverse precision of the signal, \( \varepsilon \).

If informational imperfections are pronounced enough, precisely if

\[ \varepsilon > \alpha \cdot \frac{s_M}{s_M + 4s_H - 1}, \tag{16} \]

the second effect is dominant and the prediction of the standard model is turned upside-down.\(^8\) Then, caused by an increase in \( s_H \), one would observe the mean-to-median income ratio and redistribution move into opposite directions. The model’s prediction concerning the consequences of a rise in income skewness is therefore in general not clear. This ambiguity can be seen as a reason for why, in reality, one sometimes observes positive relationships between redistribution and inequality and sometimes the opposite.

\(^8\)Condition (16) is obtained by plugging equation (11) into equation (12), taking of the derivative with respect to \( s_H \), and checking the sign.
Figure 6: Vote distribution before (thin dashed line) and after (thick solid line) upward mobility ($\bar{s}_L < \tilde{s}_L$, $\bar{s}_H > \tilde{s}_H$).

5 Conclusion

Despite a sharp theoretical prediction, empirical evidence on the relationship between the mean-to-median income ratio and redistribution is mixed. Some empirical studies find a positive relationship, some a negative. Changes in income skewness are often accompanied by developments in redistribution into the opposite direction.

This paper has argued that it is important to distinguish between sources of changes in income skewness. In a model with imperfect information, polarization and upward mobility, though having the same effect on income skewness, affect redistribution in different ways.

I presented a model of direct democracy under imperfect information in which the relation between the mean-to-median income ratio and redistribution depends on the sources of changes in income skewness. While polarization generates a positive relation between income skewness and redistribution, upward mobility can have the opposite effect.

The mechanism leading to this non-standard results model works through the existence of extreme voter groups that can lead to a median voter with biased beliefs. Increases in income skewness lead to stronger redistribution sought by the middle class. However, if voting power is shifted to the rich group, the position of the median voter moves towards voting for fewer redistribution. If informational imperfections are strong enough, the second effect dominates. Then, the model generates a relationship between the mean-to-median income ratio and the extent of redistribution that would seem anomalous in light of standard voting models.
References


