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The impact of unskilled immigration on local labour markets*

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In a search-theoretic two region model, we analyse the impact of unskilled immigration and skill biased technological change on native wages and unemployment in local labour markets. We show that a specific combination of number and skills of immigrants generates a technology adoption of firms leading to insignificant wage changes. Thus our models explains the observed phenomenon of skill biased technological change, wage inequality and high inflows of immigrants in local labour markets. Our analysis also considers possible compensating or magnifying effects of native out-migration.

Keywords: search theory, immigration, wage inequality

JEL–Code: F22, J31, J60

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1 Introduction

During the 1980’s, wage inequality in the US has increased sharply: the college premium rose by 0.92 per cent per year from 1979 to 1995 (see among others Autor et al., 1998) while from the 1990’s onward overall wage inequality slowed down but was still growing in the upper tail of the wage distribution (cf. Autor et al 2003). This pattern of increase in wage inequality was almost identical for all local labour markets within the US. In a recent overview, Machin stated on the reasons and characteristics of wage inequality that “the wage distribution has been characterized by longrun growth in the relative demand for skills driven by technological change (rather than trade) and that changes in skill supply and institutional changes have affected the timing of how [skill biased technological change] sbtc impacts on the wage structure in different contexts.” (Machin (2008))

A theoretical explanation for a labour supply driven wage inequality gives Acemoglu (2007, 2003, 2002) who characterises the impact of differing skill supplies on the technology choice of firms. Acemoglu argues that the increase of relative labour demand is a consequence of the increase in relative labour supply. According to this so-called Acemoglu effect, an exogenous supply shock like the massive increase of educated workers from the baby boomer generation lead firms to adopt skill intensive technologies. Apparently, the increase of the relative labour demand is overproportional so that overall wage inequality rises.

Following the Acemoglu-effect, Lewis (2004, 2003) tries to reconcile a different empirical puzzle: some local labour markets in the US face strong inflows of mostly unskilled immigrants, but empirical studies find only a negligible effect on native wages (cf. Card, 2001).\(^1\) The empirical studies on local labour markets typically control for associated

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\(^1\)Immigrants tend to concentrate to certain geographical areas: in Dade County 45 per cent of the total population are foreign-born (cf. Chiswick and Sullivan, 2005). Similar developments can be observed in other countries like the UK: London contains more than 40 per cent of all immigrants (Dustmann et al. (2003)).
absorbing effects like (i) local demand shocks that attract immigration, (ii) a changing industry structure (à la the Rybczynski theorem) and (iii) native out-migration which offsets the inflows of immigrants into local labour markets. Card (2005) argues that there is only a negligible impact of immigration on native wages in local labour markets. According to Lewis (2004, 2003), an exogenous supply shock like massive inflows of unskilled immigrants leads firms to adopt unskilled intensive technologies. He finds first evidence of unskilled biased technological change (usbtc) in local labour markets with high immigration rates which leads to wage neutrality of immigration.

However, it is hardly possible to reconcile these empirical findings – the rise in wage inequality induced by increases in skilled labour and the negligible effect on wage inequality from increases in unskilled immigration – with one theoretical explanation. The reasons are twofold. First, the economy-wide increase in wage inequality in favour of skilled labour and unskilled biased technological change in local labour markets cannot be simultaneously explained by an Acemoglu effect (at least, in the sense of his original formulation). For local labour markets with immigration inflows, the rise of skilled labour would just be compensated by unskilled immigration resulting in no change in wage inequality at all. On the other hand, the rise in relative skilled labour supply in local labour markets with no immigration would generate an Acemoglu effect. Thus, we must draw on different explanations of the economy-wide increase in wage inequality as e.g., outsourcing (cf. Feenstra and Hanson, 1999), trade with developing countries (cf. Wood, 1998) or demand changes (cf. Berman et al., 1998; Krugman, 1994). Second, the Acemoglu effect must exactly neutralise the wage effects of immigration meaning that the technology adoption of firms leads to constant unskilled native wages. A rigorous application of the Acemoglu effect would imply a decrease in wage inequality in the local labour market that experienced unskilled immigration. Thus, from a theoretical viewpoint, one cannot explain the economy-wide increase in wage inequality by the Acemoglu
effect and the wage neutrality of immigration by the same argument.

This paper introduces a search-theoretical model that is compatible with most of the empirical facts of the impact of immigration on local labour markets. We give a rationale for the kind of immigration that may have led to the proposed effect of Lewis (2004, 2003) including a possible technology adoption effect of immigration (Acemoglu effect). We show that only a specific combination of quantity and skills of immigrants may replicate the empirical observations of Lewis. We combine this *usbtc* with an exogenous *sbtc* and show how wage inequality evolved in the same way over local labour markets – despite of massive inflows of immigrants in certain local labour marktets. Therefore we give a comprehensive rationale for the wage inequality development for the US including *sbtc* and immigration.2

Our paper has two additional features. First, we give an alternative indicator for *usbtc* (Lewis uses the on-the-job computer use). In our case, the number of vacancies in the unskilled sector is higher in local labour markets with high immigration rates. Thus, the number of vacancies (easily observable) can be used as an indicator for *usbtc*. Second, we analyse possible out-migration of native workers from one to other local labour markets. This is one of the aforementioned absorbing effects. Lewis abstains from including native out-migration in his analysis why we provide an indicator for a further compensating or magnifiyiing effect due to native out-migration.

Relying on a combination of the frameworks of Albrecht and Vroman (2002) and Lumpe and Weigert (2009), we model an economy with two local labour markets. For expository purpose, we call this two labour markets Los Angeles and Pittsburgh. Los Angeles is characterised unskilled and skilled native workers, high inflows of immigrants, exogenous skill biased technological change and unemployment. Pittsburgh has the same

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2There are mainly three results of the empirical literature: (i) the existing immigration has a negative impact on native wages/employment (cf. Borjas, 2005) and wage inequality (cf. Borjas et al., 1997), (ii) the existing immigration has a negligible impact on wages/employment (cf. Card, 2005) and wage inequality (cf. Lewis, 2004, 2003) and (iii) immigration has a positive impact on wages/employment (cf. Peri, 2007; Ottaviano and Peri, 2005).
characteristics as Los Angeles except for immigration. Accounting for typical characteristics of the immigrant population of the US, we model immigrants as unskilled having a positive probability of returning home (cf. Borjas, 1994). Due to this last feature, the job destruction rate of immigrants is higher implying that immigrants both earn lower wages and have higher unemployment rates than unskilled natives. In both local labour markets, skilled natives may apply for skilled or unskilled jobs. Unskilled native workers and immigrants are limited to unskilled jobs. As Albrecht and Vroman (2002), we get two kind of equilibria: a cross-skill matching equilibrium where skilled worker also take unskilled jobs, and an ex post segmentation equilibrium which has separated labour markets. We concentrate in general on the cross-skill matching equilibrium as we find it more realistic (cf. Albrecht and Vroman (2002)). In this setting, we may replicate the results of Lewis which indicate an offsetting effect of unskilled immigration for the technology choice of firms. An increasing number of unskilled (native and immigrant) workers increases the incentives of firms to create more vacancies resulting in higher wages for unskilled workers. Once immigrants are lower skilled than native unskilled workers (and thus the average skill endowment of an unskilled worker falls), incentives to create more vacancies falls resulting in lower wages of unskilled workers. Lewis results then hold in a special case of our model where both effects exactly compensate each other.

The remainder of the paper is structured as follows: in section 2 we present the basic

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3This kind of discrimination is adapted from Müller (2003).

4We rely on the model of Albrecht and Vroman (2002) as we use as benchmark labour market (meaning without immigration) because it offers the possibility to examine changes in the skill mix of labour supply caused by sbtc or unskilled immigration.

5Depending on the substitutability or complementarity of immigrants, most of the theoretical models would predict a negative wage effect on unskilled native worker (cf. Borjas, 1999). These findings are usually robust to different labour market institutions (cf. among others Schmidt et al. (1994); Fuest and Thum (2000)). Our model differs from these models in (i) the introduction of local labour markets, (ii) the analysis of out-migration and (iii) the introduction of search frictions. Some articles have already analysed migration in a search theoretic context. In difference to our approach, these models focus mainly on migration patterns and welfare effects in a two-country setting (cf. Sasaki, 2007; Ortega, 2000).
structure of the model. In section 3 we derive both the cross skill matching and the ex post segmentation equilibrium. In section 4 we analyse the comparative static effects of the model and show the results of a numerical simulation of the model. Section 5 concludes.

2 Basic Model

In our model, we have two local labour markets: Pittsburgh has no immigration but sbtc and is modelled according to Albrecht and Vroman (2002). Los Angeles has inflows of unskilled immigrants and is a combination of the frameworks Albrecht and Vroman (2002) and Lumpe and Weigert (2009).

2.1 Households

We develop an equilibrium matching model of the Diamond-Mortensen-Pissarides type. We have an economy with two cities – Los Angeles (LA) and Pittsburgh. Both cities are populated by a mass one of identical risk-neutral native workers $N = 1$. Los Angeles also experiences immigration $I > 0$ implying that the total population of the Los Angeles labour market is $1 + I$. There is no immigration in Pittsburgh. We have two types of native workers in both labour markets: low-skilled workers with labour market productivity $z_L$, and high-skilled workers with labour market productivity $z_H$. We assume throughout the paper that $z_H > z_L$. Immigrants enter the LA labour market with skill level $z_I \leq z_L$. In LA, low-skilled native workers constitute part $\pi$ of the native population and high-skilled workers $1 - \pi$, respectively. Immigrants enter the labour market at rate $\mu > 0$ and leave the labour market due to return migration to their home country at rate $r > 0$. The steady-state number of immigrants in the LA labour market is $I = \mu/r$.

$^6$Throughout the paper subscript $H$ denotes high-skilled natives, $L$ low-skilled natives and subscript $I$ denotes immigrants.
Both, native and immigrant workers start their working life in the unemployment pool. We abstract from on-the-job search.

2.2 Matching

We follow Albrecht and Vroman (2002), and assume that high-skilled jobs can only be done by high-skilled natives but low-skilled jobs can be done by all three types of workers. Thus, labour markets are not fully separated. Matching is the same in both labour markets. We denote the number of unemployed workers by $u$ and the number of vacancies by $v$. Labour market tightness is measured by $\theta = v/u$. The random process by which vacancies and unemployed workers find each other is represented by a matching function: $m(u, v) > 0$ with $u, v > 0$. The matching function denotes the number of matched vacancies and workers per unit of time.\footnote{The matching function $m(u, v)$ is assumed to be twice continuously differentiable, homogeneous of degree one and exhibits the following properties: $m(0, v) = m(u, 0) = 0$, $\partial m/\partial u$, $\partial m/\partial v > 0$, $\partial^2 m/\partial u^2$, $\partial^2 m/\partial v^2 < 0$ and $\partial^2 m/\partial u\partial v > 0$. Relying on empirical estimations (as, e.g., Petrongolo (2001)), the constant returns to scale assumption of the matching function is plausible.} The application arrival rate for vacant jobs $q(\theta)$ can then be written as: $q(\theta) = m(u, v)/v = m(1/\theta, 1)$ with $q'(\theta) < 0$ and $\lim_{\theta \to 0} q(\theta) = \infty$. The fraction of unemployed low-skilled workers is $\gamma$ and for high-skilled workers $1 - \gamma$. The effective arrival rate for high-skilled vacancies is therefore $(1 - \gamma)q(\theta)$. In low-skilled unemployment, there is another probability $\eta$ by which low-skilled workers are either immigrants or natives. Thus, the final application rate in the low-skilled sector is $\eta \gamma q(\theta)$ for immigrants and $(1 - \eta)\gamma q(\theta)$ for native low-skilled workers.

An unemployed worker meets a vacant job at the rate $p(\theta) = m(u, v)/u = \theta q(\theta)$ with $p'(\theta) > 0$ and $\lim_{\theta \to 0} p(\theta) = 0$. The fraction of low-skilled vacancies is denoted by $\phi$ giving the effective arrival rate for low-skilled workers as $\phi p(\theta)$. The effective arrival rate for high-skilled jobs is $(1 - \phi)p(\theta)$. Native low-skilled workers and immigrants meet a vacant job at the same rate. Potential employers cannot directly search either a native
low-skilled or immigrant worker. Therefore we have not included ex ante discrimination.

Filled jobs cost the wage \( w(z,y) \) and a fixed cost \( k \). They are destroyed by some exogenous rate \( s \). Vacant jobs incur search costs \( k \) at each instant of time and have a minimum skill requirement. When the vacancy will be filled, it will produce output according to the following production technology:

\[
f(z_i, y) = \begin{cases} y & \text{if } z_i \geq y, \\ 0 & \text{if } z_i < y. \end{cases} \quad i = (H).
\]

Firms take only investments in high-skilled jobs whereas the filling of a low-skilled jobs is independent of the arriving worker (native or immigrant).

2.3 Value Functions

All individuals and firms discount future payments at the common rate \( \rho \). We denote the expected value of unemployment by \( U(z_i) \), \( i = (H, L, I) \) and the expected present value of employment by \( W(z_i, y) \), \( (i = H, L, I) \). An unemployed worker receives the instantaneous unemployment benefit \( b \), and will meet a vacant job at rate \( \phi p(\theta) \), thereby swapping the value of unemployment \( U(\bullet) \) with the value of employment \( W(\bullet) \). Thus, the flow value of low-skilled unemployment is given by:

\[
\rho U(z_i) = b + \phi p(\theta)[W(z_i, z_L) - U(z_i)], \quad i = (L, I). \tag{1}
\]

The high-skilled worker may occupy either a high-skilled job (with rate \( (1-\phi)p(\theta) \)) or a low-skilled job (with rate \( \phi p(\theta) \)). For low-skilled workers arrive only low-skilled jobs. The flow value of native high-skilled unemployment is given by:

\[
\rho U(z_H) = b + p(\theta)\{\phi \max[W(z_H; z_L) - U(z_H), 0] + (1-\phi)[W(z_H, z_H) - U(z_H)]\}. \tag{2}
\]
While being employed a worker receives instantaneously wage $w(z_i, y)$. The job is expected to be closed at rate $s$ and the worker enters the unemployment pool at that rate. The flow value of an employed worker can be written as:

$$\rho W(z_i, y) = w(z_i, y) + s[U(z_i) - W(z_i, y)], \quad i = (H, L, I).$$ (3)

We look at the expected present value of firms, which are either producing or searching for a worker. Let $V(z_i)$ and $J(z_i, y)$ be the expected present value of a vacant and a filled job, respectively. The flow value of a producing firm is given by:

$$\rho J(z_i, y) = y - w(z_i, y) - k + s[V(y) - J(z_i, y)], \quad i = (L, I),$$ (4)

and consists of the flow profits of a match $y - w(z, y) - k$ and the potential loss caused by the destruction of the job. The flow value of a high-skilled native vacancy can be written as:

$$\rho V(z_H) = -k + (1 - \gamma)q(\theta)[J(z_H, z_H) - V(z_H)],$$ (5)

compromising the costs of searching $k$ and the potential change from a vacant to a productive job. For the derivation of the flow value of a low-skilled vacancy, $\rho V(z_L)$, it is important that ex ante a firm does not know whether it will produce with a native or an immigrant worker. The flow value of a vacant low-skilled job can then be written as:

$$\rho V(z_L) = -k + q(\theta)\{\eta(\theta)\gamma [J(z_I, z_L) - V(z_L)]
\quad + (1 - \eta(\theta))\gamma [J(z_L, z_L) - V(z_L)]
\quad + (1 - \gamma) \max [J(z_H, z_L) - V(z_H), 0]\},$$ (6)

where the first and the second part of (6) show the probability that the low-skilled vacancy is filled by an immigrant or a low-skilled native and the third part shows the arrival of a high-skilled native.
The present model contains Albrecht and Vroman (2002) and Lumpe and Weigert (2009) as special cases. If we abstract from immigration ($\mu = 0$), we are back in the model of Albrecht and Vroman. This case will serve as Pittsburgh labour market and thus is as benchmark to the Los Angeles labour market. If we abstract from high-skilled labour ($\gamma = 1$), we get the model of Lumpe and Weigert with an exogenous and constant educational attainment of native workers.

2.4 Wage setting

The wage setting between workers and firms follows a Nash bargaining process. Free entry of firms generates an asset value of a vacancy of zero: $V(z_H) = V(z_L) = 0$. The wage is then a solution for the following optimisation problem:

$$w(z_i, y) = \arg \max (W(w) - U)^\beta (J(w) - V)^{1-\beta}, \quad i = (H, L, I),$$

(7)

where $\beta$ is interpreted as the bargaining power of workers.\(^8\) The wage setting function can be derived from (7) by inserting the free entry condition, (3) and (4):

$$w(z_i, y) = \beta(y - k) + (1 - \beta)\rho U(z_i), \quad i = H, L, I.$$

(8)

There are four different wages for Los Angeles. First, a low-skilled native worker on a low-skilled job earns wage $w_{LL} = w(z_L, z_L) = \beta(z_L - k) + (1 - \beta)\rho U(z_L)$. Second, a high-skilled native worker on a low-skilled vacancy: $w_{HL} = w(z_H, z_L) = \beta(z_L - k) + (1 - \beta)\rho U(z_H)$. Third, a high-skilled native worker on a high-skilled job: $w_{HH} = w(z_H, z_H) = \beta(z_H - k) + (1 - \beta)\rho U(z_H)$. These three wage setting equations apply for the Pittsburgh as well as for the Los Angeles labour market. But for the LA labour market, there is a fourth wage setting equation for immigrants on low-skilled jobs: $w_{IL} = w(z_I, z_L) =$

\(^8\)Taking this into account of differing bargaining powers between immigrants and natives would not qualitatively alter the results.
\[ \beta(a_L - k) + (1 - \beta)(\rho + r)U(z_I). \] We can therefore summarise in

High-skilled native workers earn always more than low-skilled native workers. Low-skilled native workers always earn more than immigrant workers (with less or the same amount of human capital):

\[ w_{HH} > w_{HL} > w_{LL} > w_{IL}. \]

For future reference, we will give the closed form values for a filled job. These can be derived by substituting for the wage setting in (4) taking account of the free entry condition:

\[ J_{ij} = \frac{(1 - \beta)(z_i - k - \rho U(z_j))}{(\rho + s)}, \quad i = j = H, L, \]
\[ J_{ij} = \frac{(1 - \beta)(z_i - k - \rho U(z_j))}{(\rho + s)}, \quad i = L, \quad j = H, \]
\[ J_{ij} = \frac{(1 - \beta)(z_i - k - \rho U(z_j))}{(\rho + s + r)}, \quad i = L, \quad j = I. \] (9)

3 Equilibrium

We can differentiate two types of equilibria: first, the equilibrium where high-skilled workers take also low-skilled jobs. Following Albrecht and Vroman (2002), we call this equilibrium cross-skill matching equilibrium. The other type of equilibrium will be called ex post segmentation equilibrium as high-skilled workers do not take low-skilled jobs. Still, there exists the possibility of multiple equilibria.

3.1 Cross-skill matching

For a match to be formed in the cross-skill matching equilibrium, we maximise the Nash-product (cf. 7) and derive the following condition:

\[ W(z, y) + J(z, y) \geq U(z) + V(y). \] (10)
By inserting the expressions from (3) and (4) for $W(z, y)$ and $J(z, y)$ and using the free entry condition in (10), a match will be formed if

$$y - k \geq \rho U(z).$$

3.1.1 Stocks

The flow equation $\dot{u}$ is the difference between inflows into unemployment and outflows from unemployment. With immigrants, low and high-skilled natives being in the pool of unemployed workers, we have three different flow equations. The flow equation for low-skilled natives is $\dot{u}_L = s(\pi - (1 - \eta)\gamma u) - \phi p(\theta)(1 - \eta)\gamma u$, and the flow equation for high-skilled natives is $\dot{u}_H = s(1 - \pi - (1 - \gamma)u) - p(\theta)(1 - \gamma)u$. For immigrants, the flow equation is $\dot{u}_I = \mu + s(I - \eta \gamma u) - \phi p(\theta)\eta \gamma u - r\eta \gamma u$. In steady state $\dot{u}_i = 0$, we obtain

that inflows equal outflows:

$$\mu + s(I - \eta \gamma u) = \phi p(\theta)\eta \gamma u + r\eta \gamma u, \quad (11)$$

$$s(\pi - (1 - \eta)\gamma u) = \phi p(\theta)(1 - \eta)\gamma u, \quad (12)$$

$$s(1 - \pi - (1 - \gamma)u) = p(\theta)(1 - \gamma)u. \quad (13)$$

The steady state conditions can be solved for the unemployment rate $u$, the fraction of low-skilled vacancies $\phi$ and the rate of unemployed immigrants $\eta$:

$$u = \frac{s(1 - \pi)}{(1 - \gamma)(s + p(\theta))}, \quad (14)$$

9The condition holds if $z \geq y$. 

12
\[
\phi = \frac{\pi(1 - \gamma)(s + p(\theta)) - s(1 - \eta)\gamma(1 - \pi)}{p(\theta)(1 - \eta)\gamma(1 - \pi)}, \tag{15}
\]

\[
\eta = \frac{(\mu/r)(s + r)(s + \phi p(\theta))}{\pi s(s + \phi p(\theta) + r) + (\mu/r)(s + r)(s + \phi p(\theta))}. \tag{16}
\]

The properties for the Pittsburgh labour market are easily obtained by setting \(\mu = 0\), which gives the same results as Albrecht and Vroman (2002).

### 3.1.2 Job creation condition

From the free-entry condition and equations (4), (5) and (6) and including (9), we can derive the job creation conditions (or equal-value conditions) for either a low-skilled job or a high-skilled job:

\[
G_L(\theta, \gamma) \equiv \eta \gamma J_{IL} + (1 - \eta)\gamma J_{LL} + (1 - \gamma)J_{HL} = \frac{k}{q(\theta)}, \tag{17}
\]

\[
G_H(\theta, \gamma) \equiv (1 - \gamma)J_{HH} = \frac{k}{q(\theta)}. \tag{18}
\]

Equation (17) implies that the value of a low-skilled job taken by a high-skilled worker is strictly positive: \(J_{HL} > 0\). Finally, we derive the closed form solution for the expected present value of unemployment from (3), (1) and (2):

\[
(\rho + r)U(z_I) = \frac{b(\rho + s + r) + \phi p(\theta)\beta(z_I - k)}{\rho + s + \phi p(\theta)\beta}, \tag{19}
\]

\[
\rho U(z_L) = \frac{b(\rho + s) + \phi p(\theta)\beta(z_L - k)}{\rho + s + \phi p(\theta)\beta}, \tag{20}
\]

\[
\rho U(z_H) = \frac{b(\rho + s) + [\phi z_L + (1 - \phi)z_H - k]p(\theta)\beta}{\rho + s + p(\theta)\beta}. \tag{21}
\]

As in the standard matching models, the asset value of unemployment equals the weighted average of the asset values of being unemployed (asset value of unemployment benefits) and the asset value of being employed. Introducing these closed form
values for unemployment into the job creation conditions gives us the equilibrium values of the market tightness $\theta$ and the rate of unemployed low-skilled workers $\gamma$. Thus, these equilibrium values solve the flow equations for the stocks: the unemployment rate $u$, the rate of low-skilled vacancies $\phi$ and the number of immigrants in the low-skilled labour market $\eta$.

The cross-skill matching equilibrium is obtained by $\theta$ and $\gamma$ and a vector of stock variables $u$, $\phi$, and $\eta$ that satisfy the job creation conditions $G_L(\theta, \gamma)$ and $G_H(\theta, \gamma)$, Nash bargaining, and the flow conditions for the stocks.

### 3.2 Ex Post Segmentation Equilibrium

Now, high-skilled workers take only high-skilled vacancies $z_L - k < \rho U(z_H)$ and thus we have segmented labour markets of high- and low-skilled jobs. We derive corresponding steady state conditions to the cross-skill matching equilibrium: there will be again three flow equations determining the unemployment rate, the rate of low-skilled vacancies and the number of immigrants. Finally, two job creation conditions determine the labour market tightness and the rate of unemployed low-skilled workers.

#### 3.2.1 Stocks

For low-skilled natives and immigrants, the steady state condition for the inflows and outflows of unemployment (cf. (11) and (12)) stays the same but for high-skilled natives changes the condition as follows:

$$(1 - \phi)p(\theta)(1 - \gamma)u = s(1 - \pi - (1 - \gamma)u.$$  \hfill (22)

The equilibrium conditions for number of unemployed worker, the low-skilled vacancy
rate and the rate of unemployed immigrants are as follows:

\[
u = \frac{s(\gamma - \gamma \eta - 2\pi \gamma + \pi \gamma \eta + \pi)}{(1 - \gamma) \gamma (p(\theta)(1 - \eta) + 2s(1 - \eta))}, \tag{23}
\]

\[
\phi = \frac{s\pi((1 - \gamma) \gamma(1 - \eta)(p(\theta) + 2s))}{p(\theta)(1 - \eta) \gamma s(\gamma - \gamma \eta - 2\pi \gamma + \pi \gamma \eta + \pi)} - \frac{s}{p(\theta)}, \tag{24}
\]

\[
\eta = \frac{\left(\frac{\mu}{r}\right)(r + s)(s + \phi p(\theta))}{\pi s(s + \phi p(\theta) + r) + \left(\frac{\mu}{r}\right)(s + r)(s + \phi p(\theta))}. \tag{25}
\]

### 3.2.2 Job creation condition

The asset value of unemployment for low-skilled natives does not change (cf. 1). Obviously, the asset value of unemployment for high-skilled workers changes because they match only with high-skilled vacancies:

\[
\rho U(z_H) = \frac{b(\rho + s) + (1 - \phi)p(\theta)\beta(z_H - k)}{\rho + s + (1 - \phi)p(\theta)\beta}. \tag{26}
\]

The job creation condition will be the same for the high-skilled filled jobs:

\[
F_H(\theta, \gamma) \equiv (1 - \gamma) J_{HH} = \frac{k}{q(\theta)}, \tag{27}
\]

but it will change for the low-skilled filled jobs because of the absence of high-skilled workers:

\[
F_L(\theta, \gamma) \equiv \eta \gamma J_{IL} + (1 - \eta) \gamma J_{LL} = \frac{k}{q(\theta)}. \tag{28}
\]

The ex post segmentation matching equilibrium is obtained by \(\theta\) and \(\gamma\) and a vector of stock variables \(u, \phi,\) and \(\eta\) that satisfy the job creation conditions \(F_L(\theta, \gamma)\) and \(F_H(\theta, \gamma),\) Nash bargaining, and the flow conditions for the stocks.
4 Comparative Statics and Simulation

4.1 Comparative Statics

The comparative static results depend on the existing equilibria: we can examine effects of the movement from the cross-skill matching to the ex post segmentation equilibrium and vice versa or from the existing equilibrium to the same equilibrium. To follow the intuition of Lewis, we include the same kind of technological change which exists at Pittsburgh and at Los Angeles. Thus, we consider the following technology:

\[ f(z, y) = \begin{cases} \alpha y & \text{if } z \geq y \\ 0 & \text{if } z < y \end{cases} \]

By augmenting \( \alpha \) for \( z_H = y \), we spread the productivity gap between both type of jobs (low and high-skilled) which can be considered as exogenous \( sbtc \). The effects of this increase in \( z_H \) are twofold: first, high-skilled jobs will produce more output and the value of unemployment for high-skilled native workers increases: it is worthwhile to wait longer for a high-skilled vacancy. Therefore firms which offer low-skilled jobs are worse off as they have to pay more for a high-skilled worker. The cross-skill matching equilibrium will move to a ex post segmentation equilibrium because native worker will not take any low-skilled jobs. With a constant \( \theta \) and a rising \( \gamma \) \((\partial \theta / \partial z_H = 0, \partial \gamma / \partial z_H > 0)\), the overall effects are an increasing number of unemployed workers \( \partial u / \partial z_H > 0 \) (because high-skilled workers wait longer for a job) and a decreasing rate of vacancies \( \partial \phi / \partial z_H < 0 \) (see (14) and (15)). This kind of technological change applies to both local labour markets – Pittsburgh and Los Angeles – because \( sbtc \) has taken place over all local labour markets in the US.

Suppose now that we have at the same time low-skilled immigrants at Los Angeles. If these immigrants are even lower skilled than native low-skilled workers, the average
productivity of a low-skilled job will decrease. This would spread the productivity gap between both jobs even more. Overall unemployment should rise more because low-skilled vacancies are even worse for firms and high-skilled workers will receive increasing wages. Focusing on mass migration of low-skilled workers (more low-skilled immigrants instead of lower skilled immigrants and thus an increasing \( \mu \)), the effects will be rather different. Low-skilled vacancies will be more attractive due to the higher number of immigrants in the market which enlarges the unemployment value for immigrants and low-skilled natives and leads to higher wages of immigrants and low-skilled natives. The labour market is therefore characterised by higher low-skilled vacancies, lower low-skilled unemployment and a declining wage inequality between and low and high-skilled natives.

Changing the type of equilibrium – from cross-skill matching to ex post segmentation equilibrium – the indirect effect of lower skilled immigrants and technological change on the value of a low-skilled vacancy decreases as we have not any high-skilled workers who take low-skilled jobs. This leads to rising \( \theta \) and \( \gamma \), but the effects on \( u \) and \( \phi \) are ambiguous. By the movement from the cross-skill matching to the ex post segmentation equilibrium, we expect a rise in unemployment because high-skilled workers have more high-skilled vacancies but losing low-skilled vacancies.

4.2 Numerical example

4.2.1 Pittsburgh

The following numerical example shall illustrate the theoretical discussion and will be calibrated for the US economy. The matching function is assumed to be of Cobb-Douglas type \( m(u, v) = 2\sqrt{uv} \) which gives an arrival rate of \( p(\theta) = 2\sqrt{\theta} \). Thus, the elasticity of the matching function is 0.5 (cf. Petrongolo and Pissarides (2001)). The bargaining power \( \beta \) is also set to 0.5 for the US which is in line with the estimates of Abowd and Allain (1996). For the values of the other exogenous variables, we choose values close to
those applied by Albrecht and Vroman (2002): $s = 0.2$, $\rho = 0.05$, $b = 0.1$, $k = 0.3$. The job destruction rate is lower in our case because we have not included on-the-job search in our model.\footnote{Shimer (2005) gives an annual job separation rate of 0.408. Petrongolo and Pissarides (2001) show that on-the-job changes represented about 20 per cent in the US to up to 40 per cent in the UK of total hires.}

<table>
<thead>
<tr>
<th>$z_H$</th>
<th>$\theta$</th>
<th>$p(\theta)$</th>
<th>$u$</th>
<th>$\gamma$</th>
<th>$\phi$</th>
<th>$w_{LL}$</th>
<th>$w_{LH}$</th>
<th>$w_{HH}$</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>1.676</td>
<td>2.59</td>
<td>0.080</td>
<td>0.639</td>
<td>0.834</td>
<td>0.643</td>
<td>0.665</td>
<td>0.765</td>
<td>0.122</td>
</tr>
<tr>
<td>1.25</td>
<td>1.676</td>
<td>2.59</td>
<td>0.086</td>
<td>0.666</td>
<td>0.732</td>
<td>0.637</td>
<td>0.680</td>
<td>0.804</td>
<td>0.167</td>
</tr>
<tr>
<td>1.3</td>
<td>1.676</td>
<td>2.59</td>
<td>0.092</td>
<td>0.688</td>
<td>0.657</td>
<td>0.632</td>
<td>0.695</td>
<td>0.845</td>
<td>0.213</td>
</tr>
<tr>
<td>1.35</td>
<td>1.790</td>
<td>2.68</td>
<td>0.144</td>
<td>0.607</td>
<td>0.391</td>
<td>0.601</td>
<td>–</td>
<td>0.936</td>
<td>0.335</td>
</tr>
</tbody>
</table>

Table 1: Skill biased technological change in Pittsburgh

$(z_L = 1$, $\pi = 0.60$, $b = 0.1$, $\beta = 0.5$, $s = 0.2$, $k = 0.3$, $\rho = 0.05)$

The baseline scenario has a cross-skill matching equilibrium. The overall unemployment rate is 0.086 per cent and about 70 per cent of these unemployed workers are low-skilled worker (see row 1 in table 1). This implies that most of the jobs in Pittsburgh are low-skilled jobs. Thus, the wages for low-skilled workers and high-skilled workers on low-skilled jobs are nearly the same. Most of the vacancies are issued for the low-skilled sector due to relatively higher number of low-skilled workers in the labour market. For the labour market tightness, we get $\theta = 1.676$ implying a vacancy rate $v = 0.133$. The average duration of native unemployment is about 5 months ($12 \times (1/2.59) = 4.63$) whereas the average duration of a vacancy is about 8 months ($12 \times (1.676/2.59) = 7.76$).

With $sbtc$ (an increase in $z_H$), firms begin to offer more vacancies for high-skilled workers which shows the decrease of $\phi$ (the overall vacancy rate increases to $v = 0.1538$). With this increasing number of high-skilled vacancies, the value of unemployment for high-skilled workers increases. Therefore the unemployment rate of low-skilled workers increases and their wages fall while wages for high-skilled workers increase. The increase
of high-skilled wages comes either from the better bargaining position of high-skilled workers (through the higher value of unemployment: $U_H$ increases from 12.61 to 13.78) and the higher output of a match (through the higher educational attainment). Still, the overall unemployment rate increases slightly due to the worsening of the vacancy rate for low-skilled workers. Finally we get increasing wage inequality which corresponds to the analysis of Acemoglu (1998). In the last row of table 1, we show the case of a change in the type of equilibrium.

With an increase to $z_H = 1.35$, the cross-skill matching equilibrium changes into an ex post segmentation equilibrium. Now, the labour markets for low- and high-skilled workers are separated and thus the overall unemployment rate increases sharply through more unemployment of high-skilled workers. They wait now longer for a high-skilled vacancy to come because their value of unemployment increases largely with their higher wage on high-skilled vacancies ($U_H = 16.54$). The vacancy rate also increases sharply ($v = 0.241$) but mostly for high-skilled workers as the decline in $\phi$ shows.

4.2.2 Los Angeles

We will start with a relatively modest number of immigrants in the LA labour market. The steady-state stock of immigrants $I = \mu/\delta_I = 0.3$ meaning that $\sim 23\%$ of the total population are immigrants. This is a comparably low number of immigrants: in Dade county in the US, 45 per cent of the total population are immigrants. We then raise the number of immigrants in the labour market up to 28 per cent of the total population. In the last row of table 2 and table 3, we decrease the labour market productivity of immigrants from $z_I = 0.95$ to $z_I = 0.9$. This is line with the empirical studies which show that the educational endowment of immigrants has decreased for the last decades (cf. Borjas (1994)). Most of values are choosen to replicate the empirical result of the same wage inequality in both local labour markets. All the remaining exogenous parameters are set to the same values as for Pittsburgh.
Table 2: Skill biased technological change and immigration in Los Angeles
\( z_L = 1, \pi = 0.4, b = 0.1, \beta = 0.5, s = 0.2, k = 0.3, \rho = 0.05 \)

Table 3: Skill biased technological change and immigration in Los Angeles
\( z_L = 1, \pi = 0.4, b = 0.1, \beta = 0.5, s = 0.2, k = 0.3, \rho = 0.05 \)

The first table shows the values for \( \eta, \theta, u, \gamma, \) and \( \phi \); while the second table shows the different wages and the wage premium. In the baseline scenario, the labour market tightness is \( \theta^E = 1.811 \) which is slightly higher to the labour market tightness of Pittsburgh. The vacancy rate is also slightly higher with \( v = 0.19 \). The average duration of native unemployment is about 5 months \( (12 \times (1/2.69) = 4.46) \), which is also close to the baseline scenario of the Pittsburgh labour market. The average duration of a vacancy is about 8 months \( (12 \times (1.81/2.69) = 8.07) \). As for Pittsburgh, including on-the-job search would provide us with a lower average duration of a vacancy.

We can clearly show that, even with massive inflows of low-skilled immigrants, we may have the same wage premium \( (\omega = w_{HH} - w_{LL}) \) in Los Angeles and Pittsburgh. In the second row of table 2 and 3, we have only augmented the number of immigrants in Los Angeles (from 23 per cent to 28 per cent); while in the third row of table 2 and 3, we have decreased the skill endowment of immigrants \( z_I \) (from 0.95 to 0.90). However, the
wage premium ($\omega$) remains the same over both local labour markets: for example, 0.127 in Pittsburgh versus 0.121 in Los Angeles. This is possible because there are conflicting incentives for firms to offer low-skilled vacancies. On the one hand, a higher number of low-skilled workers in the labour markets induces firms to invest more in low-skilled vacancies due to the lower vacancy costs. On the other hand, lower skill endowments of immigrants decreases the productivity of a low-skilled job and thus decreases incentives to invest in low-skilled vacancies. Therefore, wages of low-skilled native workers are slightly lower in Los Angeles than in Pittsburgh. Obviously, immigrant wages are even lower because of the higher probability to leave the match. And finally, $sbtc$ leads to nearly the same increases in wages for high-skilled workers in both labour markets. Interestingly, only low-skilled immigration in combination with high inflows of immigrants will result in a constant wage premium over local labour markets. If we would admit fewer low-skilled immigrants, wages of low-skilled natives would decrease. Furthermore, only admitting high-skilled immigrants would lead to rising wage inequality. The higher number of low-skilled vacancies $\varphi$ in Los Angeles shows the $usbtc$ in this local labour market.

Lewis (2003) measures indirectly the technology adoption of firms through the use of computers in the different local labour markets. We may now propose different indirect measures of this technology adoption. For example, the measure of low-skilled vacancies $\phi$ should be higher and decrease slower as $z_H$ increases in Los Angeles than in Pittsburgh.

### 4.3 Extension: native out-migration

Most of the empirical studies control for native out-migration from local labour markets which may offset the wage effects of immigration. Interestingly, these empirical studies are also controversial with respect to the existence of out-migration: Card and DiNardo (2000) and Card (2001) find no out-migration of natives caused by immigration but
Borjas (2005) explains an important part of the non-existing effect of immigration on local labour markets with native out-migration. However, we will discuss if native out-migration may reverse our results from the previous sections.

Native workers would leave their local labour market if their value of unemployment differ over both local labour markets. The indifference condition for both skill groups is: $U_{i}^{LA} = U_{i}^{PI}$, $i = L, H$. Thus, we will compare the values of unemployment for high- and low-skilled native workers in both local labour markets (for comparable scenarios of table 1 and table 3):

<table>
<thead>
<tr>
<th>Pittsburgh</th>
<th>Los Angeles</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{H}$</td>
<td>$U_{H}$</td>
</tr>
<tr>
<td>12.61</td>
<td>12.80</td>
</tr>
<tr>
<td>13.18</td>
<td>13.11</td>
</tr>
<tr>
<td>13.78</td>
<td>13.61</td>
</tr>
</tbody>
</table>

Table 4: Values of unemployment for high- and low skilled native workers

The values of unemployment of low skilled workers are nearly similar over both labour markets so that there is no native out-migration of low-skilled worker. The value of unemployment for high-skilled native workers is higher in Los Angeles than in Pittsburgh. If therefore high-skilled natives would move from Los Angeles to Pittsburgh, wage inequality would increase (decrease) in Pittsburgh (Los Angeles). Thus, in Pittsburgh, we would get segmented labour markets whereas in Los Angeles, we would have cross-skill matching and high rate of low-skilled jobs. However, the difference of the values of unemployment in both labour markets would vanish if we consider moving costs.

5 Conclusion

We model two local labour markets (Pittsburgh and Los Angeles) where both are characterised by $sbtc$, equilibrium unemployment and not perfectly segmented labour markets.
of high- and low skilled workers. Only one of these labour markets has massive inflows of immigrants (Los Angeles). This allows us to introduce \textit{usbtc} in the local labour market with immigration. Due to a positive probability of returning to their home countries, immigrants receive lower wages and have a higher unemployment rate compared to natives. We can show that both local labour markets will have the same relative wage although only Los Angeles had massive inflows of immigration. This gives a theoretical explanation of the empirical results of Lewis (2004, 2003). The effects of an increase in the number of immigrants and a decrease in the skill endowments of immigrants leads to \textit{usbtc} but on the rounds of endogenous creation of vacancies does not result in a change in relative wages in Los Angeles compared to Pittsburgh. Furthermore, we discuss the implications of our results for native out-migration.

6 Appendix

6.1 Derivation of (10) and (8)

Introducing the equations for \(W(\bullet)\) and \(J(\bullet)\) and the free-entry condition in (10) gives:

\[
W(z, y) + J(z, y) \geq U(z) + V(y),
\]

\[
\frac{w(z, y) + sU(a)}{\rho + s} + \frac{y - w(z, y) - k}{\rho + s} \geq U(z),
\]

\[
y - k \geq \rho U(z).
\]

For immigrants, the depreciation rate is \(\rho + \delta_I\) instead of \(\rho\). Introducing the equations for \(W(\bullet)\) and \(J(\bullet)\) and the free-entry condition in (7) gives:

\[
(1 - \beta)W(z, y) = \beta J(z, y) + (1 - \beta)U(z),
\]
\[(1 - \beta) \left( \frac{w(z, y) + sU(z)}{\rho + s} \right) = \beta \left( \frac{y - w(z, y) - k}{\rho + s} \right) + (1 - \beta)U(a), \]
\[(1 - \beta)(w(z, y) + sU(z)) = \beta(y - w(z, y) - k) + (1 - \beta)(\rho + s)U(z),\]
\[\beta(y - w(z, y) - k) + (1 - \beta)[(\rho + s)U(z) - w(z, y) - sU(z)] = 0,\]
\[w(z, y) = \beta(y - k) + (1 - \beta)\rho U(z).\]

6.2 Derivation of the steady-state values of \( u, \phi, \) and \( \eta \) for cross-skill matching

The steady-state value of \( u \) can be derived as follows:

\[p(\theta)(1 - \gamma)u = s(1 - \pi - (1 - \gamma)u),\]
\[p(\theta)(1 - \gamma)u + s(1 - \gamma)u = s(1 - \pi),\]
\[(1 - \gamma)(p(\theta) + s)u = s(1 - \pi),\]
\[u = \frac{s(1 - \pi)}{(1 - \gamma)(p(\theta) + s)}.\]

The steady-state value of \( \phi \) can be derived by \((12)/(13)\):

\[\phi \frac{(1 - \eta)\gamma}{(1 - \gamma)} = \frac{\pi - \gamma(1 - \eta)u}{(1 - \pi) - (1 - \gamma)u},\]
\[\phi = \frac{(1 - \gamma)(\pi - \gamma(1 - \eta)u)}{(1 - \eta)\gamma((1 - \pi) - (1 - \gamma)u)}.\]

By inserting of \((14)\), we get:

\[\phi = \frac{(1 - \gamma)\pi - \gamma(1 - \eta)(s(1 - \pi))/(p(\theta) + s)}{(1 - \eta)\gamma(1 - \pi) - \gamma(1 - \eta)(s(1 - \pi))/(p(\theta) + s)}.\]
\[\phi = \frac{(1 - \gamma)\pi(p(\theta) + s) - \gamma(1 - \eta)(s(1 - \pi))}{(1 - \eta)\gamma(1 - \pi)(p(\theta))}.\]

The steady-state value of \(\eta\) can be derived from (12) and (11). By converting (12) and (11), we get:

\[(1 - \eta)\gamma u = s\pi/(\phi p(\theta) + s),\]

and

\[\eta\gamma u = I(r + s)/(\phi p(\theta) + s + r).\]

Dividing both expressions gives:

\[\frac{1 - \eta}{\eta} = \frac{s\pi(\phi p(\theta) + s + r)}{I(r + s)(\phi p(\theta) + s)},\]

\[\eta = \frac{I(r + s)(\phi p(\theta) + s)}{s\pi(\phi p(\theta) + s + r) + I(r + s)(\phi p(\theta) + s)}.

### 6.3 Rewriting the job creation condition

Taking (18) and including (21):

\[
\frac{z_H - k - \left(\frac{b(\rho + s) + p(\theta)\beta(\phi z_L + (1 - \phi)z_H - k)}{\rho + s + p(\theta)\beta}\right)}{\rho + s} = \frac{k}{(1 - \beta)(1 - \gamma)q(\theta)},
\]

\[
\left(\frac{\rho + s + p(\theta)\beta)[z_H - k] - b(\rho + s) - p(\theta)\beta\phi z_L + (1 - \phi)z_H - k]}{\rho + s + p(\theta)\beta}\right)
\]

\[= \frac{k(\rho + s)}{(1 - \beta)(1 - \gamma)q(\theta)},\]

\[
\left(\frac{[(\rho + s)(z_H - k - b) - p(\theta)\beta(z_H - k) - p(\theta)\beta\phi(z_L - z_H) + p(\theta)\beta(z_H - k)]}{\rho + s + p(\theta)\beta}\right)
\]

\[= \frac{k(\rho + s)}{(1 - \beta)(1 - \gamma)q(\theta)},\]

\[
(1 - \beta)\left(\frac{[(\rho + s)(z_H - k - b) + p(\theta)\beta\phi(z_H - z_L)]}{\rho + s + p(\theta)\beta}\right) = \frac{k(\rho + s)}{(1 - \gamma)q(\theta)}.
\]
\[
q(\theta) \frac{(1-\beta)(1-\gamma)[(\rho+s)(z_H - k - b) + p(\theta)\beta\phi(z_H - z_L)]}{(\rho + s + p(\theta)\beta)} = k.
\]

6.4 Derivation of the steady-state values of \(u\) and \(\phi\) for ex post segmentation

From (12), we get \(\phi = s(\pi - \gamma(1 - \eta)u)/p(\theta)(1 - \eta)\gamma u\) and insert this expression into (22):

\[
(1 - \left(\frac{s(\pi - \gamma(1 - \eta)u)}{p(\theta)(1 - \eta)\gamma u}\right))p(\theta)(1 - \gamma)u = s(1 - \pi - (1 - \gamma)u),
\]

\[
[(p(\theta)(1 - \eta)\gamma u + s(1 - \eta)\gamma u - s\pi](1 - \gamma) + s(1 - \eta)\gamma(1 - \gamma)u = (s - s\pi)(1 - \eta)\gamma,
\]

\[
[p(\theta)(1 - \eta)\gamma u + 2s(1 - \eta)\gamma u](1 - \gamma) = (s - s\pi)(1 - \eta)\gamma + s\pi(1 - \gamma),
\]

\[
u = \frac{s(\gamma - \gamma\eta - 2\pi\gamma + \pi\gamma\eta + \pi)}{[p(\theta) + 2s](1 - \eta)\gamma(1 - \gamma)}.
\]

Inserting of (23) in \(\phi = s(\pi - \gamma(1 - \eta)u)/p(\theta)(1 - \eta)\gamma u\):

\[
\phi = \frac{s(\pi - \gamma(1 - \eta)u)}{p(\theta)(1 - \eta)\gamma u} = \frac{s\pi}{p(\theta)(1 - \eta)\gamma u} - \frac{s}{p(\theta)},
\]

\[
\phi = \frac{s\pi((1 - \gamma)\gamma(1 - \eta)(p(\theta) + 2s))}{p(\theta)(1 - \eta)\gamma s(\gamma - \gamma\eta - 2\pi\gamma + \pi\gamma\eta + \pi)} - \frac{s}{p(\theta)}.
\]

The derivation of the expression for \(\eta\) is the same as for the cross-skill matching case.
References


