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Vertical Integration, Competition and Financial Exchanges: Is there Grain in the Silo?

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Abstract

This paper aims to contribute to the ongoing discussion on horizontal vs. vertical integration as further development path in the financial security service industry, consisting of trading, clearing and settlement. Using a stylized model of the industry we investigate the incentives to vertically or horizontally integrate from the individual firm’s point of view as well as its consequences on competition and welfare. Our analysis reveals that the incentives to vertically integrate crucially depend on industry and market characteristics such as the degree of differentiation and integration of trading platforms as well as the role liquidity effects play for traders. Furthermore, we show that market forces may suffer from a coordination problem resulting from historical conditions and ending in vertically integrated structures which are not in the interest of the firms. Comparing horizontal with vertical integration, we find a potential tendency for excessive vertical integration, a problem which can be addressed by policy measures such as the Target 2 securities program.

Keywords: vertical and horizontal integration, competition, trading, settlement

\textit{JEL classification}: G15, L13, L22

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1 Introduction

Security exchanges and central security depositories (CSDS) are at the center of modern capital markets around the world. Across continents and markets we observe, however, very different industry structures. On the one hand, we find a much more fragmented structure in Europe (with more than 40 exchanges and roughly 20 CSDs; see FESE (2008)) and the Euro area as compared to the US market with only a dozen exchanges and only two CSDs. On the other hand, the degree of vertical integration differs significantly even among European countries and markets with strong vertical integration in Germany where Deutsche Börse as the main player is fully vertically integrated and much less vertical integration in other markets such as United Kingdom.

At the very same time, industry observers as well as policy makers expect further consolidation and change in the European industry in the following years to come (see Economist (2006)). While all observers agree that changes in industry is under way, it is much less clear, in which direction this change will take place and what is desirable from the point of view of industry participants and society as a whole.

Against this background our analysis aims to provide some insights into the dynamics of the industrial organizations of financial security service providers (consisting of security exchanges and CSDs), most notably into the interrelation of organizational design and market structure. Our main research questions thereby are: Under which circumstances is vertical or horizontal integration more attractive? What does this do to the industrial organization of the industry? Which organizational and industry structure is preferable from a welfare point of view?

We take up these general research questions and investigate the drivers behind vertical in comparison to horizontal integration in the security service industry. By employing a stylized model which incorporate economies of scope as well as network effects at the different levels of the value chain of the security service industry we investigate firm’s incentive to opt for vertical and also horizontal integration and how these decisions are affected by market structure characteristics. In turn, we also look into the implications of organizational structures on competition. Rather than focusing on market decision only we compare them to the social optimum from the point of view of a potential regulator.

While we focus on the security service industry our analysis resembles in many aspects the structure of other network industries and hence can be used as a benchmark for the analysis of the interrelationship between organizational design and competitive structure in many such industry, e.g. in the railroad and the electricity industry.

Our analysis reveals that the incentives to vertically integrate crucially depend on industry and market characteristics such as the degree of differentiation and integration of
trading platforms as well as the role liquidity effects play for traders. We show that the more platforms are differentiated and the more important is the liquidity effect the more pronounced is the incentive to integrate vertically. This is not only true absolutely (i.e. with respect to the decision to vertically integrate or to stay completely disintegrated) but also in comparison to the decision to integrate horizontally. Furthermore, we find that the size of the trading platform is decisive as well: larger platforms have stronger incentives to integrate vertically compared to smaller ones. At the same time, our theoretical reasoning suggests that vertical integration harms competitors. Against this background we show that financial security service providers may fall in a coordination trap. Due to, e.g. initial conditions or historical circumstances, the industry starts with a certain degree of vertical integration, leading to increased incentives for further firms to vertically integrate. Due to the negative externalities which accompany vertical integration this may lead to a bad equilibrium in which firms are in sum worse off compared to a situation in which the industry would be completely disintegrated.

When comparing vertical with horizontal integration we find that the market solution has a tendency to come up with too much vertical integration. We interpret measure such as Target 2 securities as policy instruments to provide politically-enforced horizontal integration and hence, as a potential remedy to overcome this tendency for excessive vertical integration. To derive these results we propose a stylized model which aims to depict the interrelation between the organizational design of financial security service providers and competition among them. We depict traders preferences for securities listed and traded on different exchanges by employing the Salop-model. Traders as well as exchanges are located on this circle depicting the notion of a natural affinity of certain traders for certain exchanges (e.g. due to language barriers, home bias etc). We allow for the competition of three exchanges. The securities listed on a certain exchange are settled in the associated (but yet potentially organisation-wise) independent central security depository (CSD). We are neglecting or assuming perfect competition of custodian banks and therefore provide a bare-and-bone picture of the industry and the competition therein. We view vertical integration as a measure to implement a highly specific relationship between an exchange A and the associated CSD making trades routed through this link less costly but imposing additional costs onto trades which are settled outside the associated CSD A or traded on another exchange but settled in CSD A. In that sense our notion of vertical integration is close in spirit to Grossman and Hart (1986) but also resemble the idea of vertical integration in the financial security service industry as decision for a closed rather than an open standard making external linkages partially incompatible with internal processes. Horizontal integration on the level of CSD is modeled as uniform cost-reductions displaying the notion of economies of scale and scope at this layer.
Our paper is related to several strands of the literature on securities service provider. First, our paper touches the topic of competition between trading platforms. This is analyzed in different manners by e.g. Foucault and Parlour (2004), Di Noia (2002) and Shy and Tarkka (2001) where the latter involve also on a vertical relationship namely between the brokers and stock exchanges. But they focus on the role of alliances between stock exchanges i.e. cooperation on a horizontal level whereas we focus on vertical cooperation. Second, our paper has analogies to the question of interlinking securities settlement system. A question that is analyzed by Kauko (2004) and Kauko (2007).

Third, our work is directly related to the literature on vertical integration in the securities service provider. Köppl and Monnet (2007) present a model that investigates the role of private information about costs in a merger between a stock exchange and a settlement provider. They conclude that vertical silos can prevent efficient consolidation on a horizontal level. In contrast, Holthausen and Tapking (2007) and Rochet (2005) model the vertical relationship between custodian banks and a central security depository (CSD). In the model of the former the CSD is input provider and competitor of the simultaneously. They show that the CSD leverages its monopoly power to compete for customers at the custodian level by raising it rivals costs.

Rochet (2005) however asks whether a CSD should compete directly with custodian banks or in other words, should CSDs be allowed to integrate vertically with custodian banks. He concludes that the welfare effect of such a merger hinges on the trade-off between efficiency gains and lower competition on the custodian level due to the merger. A trade-off that will be in the center of attention in our paper as well.

The most relevant paper to our analysis is Tapking and Yang (2006). They analyze different industry settings in the sense of vertical or horizontal integration in a two country model. They conclude that horizontal integration dominates vertical integration which itself is better than no consolidation. We differ from their approach in that our main interest in the integration decision is on the efficiency gain stemming from organizational restructuring that should be associated to the merger whereas Tapking and Yang (2006) take only strategic effects into account. That is, in contrast to their approach we explicitly focus on the underlying driver of organizational change and their interaction with competition. In addition, we explicitly incorporate network effects in our analysis which as we will see play an important in our model.

The paper is organized as follows. In the next section we outline the structure of the industry and turn afterwards to the basic model. On the basis of this, we discuss the incentives and consequences of vertical integration in section 4. We thereby differentiate between a starting point in which any vertical integration is absent and one in which certain financial security service provider are already vertically integrated. With this distinction
we aim to look into the potential cumulative effects as well as into situations in which history may matter. In a fourth section we compare vertical with horizontal integration. The last section discusses some extensions most notably endogenous listing decision. The final section concludes.

2 Functioning and structure of the industry

Before we turn to our model, we illustrate the basic structure of the industry by describing the functions of the securities transaction process as well as the main players in the market. The securities transaction process is basically characterized by three functions. First, the actual trading process, e.g. the matching from buyer and seller which usually takes place on an exchanges, trading platforms or via Over-The-Counter 1 to 1 trading. At this stage enormous network effects known as the liquidity effect are present. Trader favor exchanges on which other trader and therefore liquidity concentrates because it decreases the influence of their orders on the price. In addition, economies of scale and scope are associated with this process because the infrastructure can be used for many trades in the same as well as for other securities, hence leading to significant savings of fixed costs.

Second, trading is followed by the clearing process. In this process the bi-/multilateral obligations are calculated by the Clearing House, in recent years more and more involving a Central Counterparty (CCP). The CCP takes the legal position of everyone’s counterparts and therefore bears the risk of all these participants. Usually it is able to net the trades with a participants and therefore bears less risk than the sum of risk the original counterparts otherwise would have to. Hence, again economies of scale and scope (the CCP facing lower net risks if different securities or more of the same security is cleared in the particular CCP) are present at this stage. Usually the clearing house is owned by the exchange.

The final stage takes the settlement process where the transactions are completed, the cash and securities transferred. This service is offered originally by central security depositories (CSDs) which holds the security and allows transactions by book entry. Again systems may be used for different securities and cash settlements may be netted which implies the presence of economies of scope at this stage. Beside CSDs there are also custodian banks offering these services and take a role of an intermediary. They usually have an account at the main CSDs allowing their customers trading securities kept at different CSDs (usually different countries) via one account. They are also able to net trades on their customers accounts which makes them not only a buyer of an input from the CSDs but also at least to a certain degree a competitor.
Furthermore the CSDs offers safe-keeping of securities, e.g. the distribution of information by the security issuer, dividend flow etc. The safe-keeping is needed to perform transactions but it is not compared to the other processes not necessarily involved in every transaction.

If an entity owns the provider of all three transaction services we refer to this as a vertical integrated exchange or a silo.

### 3 The basic model

We consider a setting in which three exchanges or trading platforms (A, B, and C) compete with each other. Besides the three exchanges there exist three central security depositories (A, B, and C). Central security depositories may or may not be vertically integrated with the exchanges. Clearing services are provided by the trading platform and therefore are not considered separately. The costs to trade one unit of a security are identical across all three exchanges and denoted by $c^T$, the cost to settle for CSD $i$ by $c^S_i$. A security which is listed on a particular exchange is kept in the respective CSD implying that a given security can be traded on different exchanges but is settled in one CSD only, giving this CSD monopoly power in this process. We assume perfect competition between custodian banks and neglect them therefore in our analysis. The total number of securities is normalized to 1. The number of securities listed on either exchange is denoted by $n_i$ ($i = A, B, C$).

Traders are uniformly distributed on the perimeter of a circle of length equal to one with density equal to one. All consumers demand inelastically one unit of each security listed on either of the three exchanges. The reservation price of all traders for trading and settlement services is denoted by $V$. This reservation price excludes the price of the security traded which we normalize for matters of simplicity to zero. Since we are only interested in the overall number of trades rather than the bilateral relation between seller and buyer this reservation price is assumed to be identical for all traders. The three exchanges and the corresponding CSDs are symmetrically located on the perimeter of the circle at 0, 1/3 and 2/3. Whereas CSDs can price discriminate between trades originated at different exchanges, exchanges cannot price discriminate between securities kept at different CSDs. We denote the price of CSD $j$ ($j = A, B, C$) for trades taking place on exchange $i$ by $p^S_{ij}$ while $p^T_i$ stands for the price of exchange $i$ charged for prices taking place on exchange $i$. Traders who have to pay both prices are assumed to expect the exchange in which the security is listed as being the more liquid one, hence, increasing utility of traders trading on this platform by $k$. This is in line with the empirical observation that liquidity of a stock is usually concentrated on the stock exchange where the company got
its primary listing (see Halling, Pagano, Randl, and Zechner (2008)). In the following, we refer to $k$ as the liquidity parameter.

The further away a trader is located from the exchange she is actually trading on, the higher is the disutility she is realizing from this trade. Suppose a trader is located at $x$ and trades on exchange $i$. Let the closest distance between the trader and the exchange be defined as $d^i_x$. Then, she realizes a disutility of $td_x$ with $t$ denoting the degree of differentiation across the exchanges. This disutility term reflects the notion that there are differences across exchanges which merely stem from locational differences, such as language, regulation and the like. The more pronounced these differences are the larger is $t$. We are aware that these differences usually take the form of discrete steps but this will only complicate the analysis without influencing the outcomes and their implications.

Hence, we can state the utility of a trader being located at point $x$ on the perimeter of the circle who considers buying one unit of a security which is listed, say on exchange $A$ as follows

$$U^A_x = \begin{cases} 
V - p^T_A - p^S_{AA} + k - td^A_x & \text{if trading takes place on exchange } A \\
V - p^T_j - p^S_{Aj} - td^j_x & \text{if trading takes place on exchange } j \ (j = B, C)
\end{cases}$$

(1)

In cases in which securities are listed on exchange $B$ or $C$ the corresponding utility functions apply.

Our analysis rests on the notion that the market is not fully covered, hence, leaving room for market coverage effects of vertical integration. Thereby, we also avoid that the CSD face a price-inelastic demand with all the special feature of such a specific demand curve. This is in line with the clearly observed home bias (see e.g. Tesar and Werner (2008)) by which investors focus more heavily on local securities reflecting e.g. (perceived) costs of price dispersion as well as (perceived) informational advantages when buying local assets. Furthermore, we impose a regularity assumption which states that the competition between exchanges takes places for traders being located between them, a standard assumptions in Salop-type model. Whereas, the first notion requires that transport costs are sufficiently large, the second one demands that the liquidity effect is not too large avoiding to make the exchange on which the security is listed too strong.

More precisely, we impose:

**Assumption 1**

$$t > k > \frac{1}{3}t$$

**Assumption 2**

$$\frac{11}{12}t > v > \frac{9}{8}t - \frac{5}{8}k$$
with $v = V - c_S - c_T$ being the net social reservation price.

Therefore, we can derive the total demand of the trading platform on which the security is listed (say A) as the sum of the two marginal traders ($x^A_1$ and $1 - x^A_2$, see figure 3) being located between this platform and the two trading platform with which it competes (B and C). Total demand for the two remaining platforms stems from the sum of the respective demand accruing to these platforms when competing with platform A ($\frac{1}{3} - x^A_1$ for platform B and $x^A_2 - \frac{2}{3}$ for platform C) as well as the respective demand arising from the marginal trader on platforms B and C who is just indifferent between buying or not buying at all ($x^A_3 - \frac{1}{3}$ for platform B as well as $\frac{2}{3} - x^A_4$ for platform B).

Our assumptions stated above ensure that $0 < x^A_1 < 1/3$, $2/3 < x^A_2 < 1$ as well as $x^A_3 < x^A_4$, i.e. the marginal traders for which platforms A and B as well as A and B compete is located strictly between them. The last inequality implies that the market is not fully covered.

Deriving the marginal traders from the indifference conditions (of buying from a compet-
ing platform or buying not at all) yields the following demand functions

\begin{align*}
d_{AB}^A &= x_1^A = \frac{p_T^B - p_T^A + p_s^A - p_s^{AB} + k + \frac{1}{3}t}{2t} \\
d_{AC}^A &= 1 - x_2^A = \frac{p_T^C - p_T^A + p_s^C - p_s^{AC} + p_s^{AA} + k + \frac{1}{3}t}{2t} \\
d_{BA}^A &= \frac{1}{3} - x_1^A = \frac{p_T^A - p_T^B - p_s^{AB} + p_s^{AA} - k + \frac{1}{3}t}{2t} \\
d_{CA}^A &= x_2^A - \frac{2}{3} = \frac{p_T^A - p_T^C - p_s^{AC} + p_s^{AA} - k + \frac{1}{3}t}{2t} \\
d_{BB}^A &= x_3^A - \frac{1}{3} = \frac{V - p_T^B - p_s^{AB}}{t} \\
d_{CC}^A &= \frac{2}{3} - x_4^A = \frac{V - p_T^C - p_s^{AC}}{t},
\end{align*}

with e.g. \(d_{iB}^A\) denoting the demand for trades of platform \(i\) of securities listed on platform A when competing with platform B.

Total demand of the trading platforms from a security listed on platform A emerge as \(d_A^A = d_{AB}^A + d_{AC}^A\), \(d_B^A = d_{BA}^A + d_{BB}^A\), and \(d_C^A = d_{CA}^A + d_{CC}^A\). In case trading for a security takes place on platforms B or C demand functions can be derived be simply replacing A with the respective platform on which the security is listed.

Hence we can state the profit function of the trading platforms:

\[ \pi_T^i = (n_i d_i^i + \sum_{i \neq j} n_j d_j^i) (p_i^T - c^T) \]

as well as of the settlement platform:

\[ \pi_S^i = n_i \sum_j (p_{ij} - c^S) d_j^i \]

### 4 Vertical integration and competition: the symmetric case

We now turn to the analysis of vertical integration. Thereby, we start with a setting in which there is not vertical integration at all and one of the entities, say A considers integrating trading and settlement. Later on, we contrast this with the decision to vertically integrate trading and settlement in A given that the other two entities are already vertically integrated. This comparison allows us to investigate potential cumulative effects of vertical integration: is vertical integration more or less likely if the other exchanges are already vertically integrated.
How do we depict vertical integration? We interpret vertical integration as a process which allows specific adjustments between the respective trading and settlement processes (e.g. establishing more efficient straight-through-processing) as well as faster coordination in the vertically integrated organization as compared to arm length’s transactions. Vertical integration allows for specific investments between trading and settlement, most notably in the area of software and IT processes. In the absence of vertical integration, such specific investment might lead to severe hold-up problems between the two parties involved. Hence, our interpretation of vertical integration is on the one hand in line with the information we got from many industry experts (which we received in the course of a number of interviews and discussions) and on the one hand conforms with the basic arguments of the theory of the firm literature in the tradition of Grossman and Hart (1986). These specific investments tie trading platform A and settlement A together. This closer link between the two, however, comes at a cost: it makes interaction of the trading platform A with the other two settlement organizations as well as the interaction of settlement in A with the two other trading platforms more difficult because for these it becomes more difficult to route trades of securities not listed on that platform more costly. Hence, vertical integration resembles a closed standard (with basically a (partially) incomplete technology). The efficiency of the standard increases but the interaction with agents outside the standard becomes more difficult (see e.g. Shy (2001)). We depict this notion in a simple manner.

With vertical integration of settlement and trading in A, trades on A are settled at lower costs in A \( (c_{AA}^S = c^S - y) \) but all cross-routings become more costly \( (c_{AC}^S = c_{AB}^S = c_{CA}^S = c_{BA}^S = c^S + y) \), with \( y \) depicting the efficiency parameter associated with vertical integration. This entire process of vertical integration which creates a more efficient link between settlement in A and trading in A but higher costs for the other links is depicted in figure 4.
We focus our analysis on these changes in efficiency in the interaction between exchanges and settlement organization. Settlement and trading prices in the vertically integrated organization are undertaken separately. That is, we neglect one benefit of vertical integration in our set-up in which settlement providers exert market power: the internalization of the external effect of the pricing decision of the trading entity on settlement (double marginalization effect) as well as the other way round (settlement in A could charge prices in order to strategically affect the competition between the trading platforms). This is, from our point of view justified by two arguments. First, the implementation of an integrated decision process requires a proper transfer pricing system, which is often quite cumbersome. Second, the effects of the internationalization process are quite obvious and very well investigated (see e.g. Tirole (1988), p. 174ff): they clearly favor vertical integration. Thereby, by neglecting this effect we distort against vertical integration, a fact which should be kept in mind when interpreting our results.

In order to avoid a too cumbersome technical discussion, we proceed as follows. We concentrate first on the symmetric case in which an equal number of securities is listed on the three exchanges. We investigate vertical integration and ask for the comparative static effects. Does an increase, say in the liquidity parameter $k$ increase or decrease the incentives for vertical integration? In a subsequent step, we ask for the consequences of a more asymmetric size of the trading platforms on the incentives to integrate vertically.
For the symmetric case we derive the profit-maximizing trading and settlement prices for A (the prices for B and C can be stated correspondingly). This gives us the subsequent reaction functions for $i, j, l \in \{A, B, C\}$ and $i \neq j \neq l$ (see the appendix)

$$p_i^T = \frac{1}{2} c_i^T + \frac{1}{12} t + \frac{1}{4} V + \frac{1}{8} (p_j^T + p_i^T) - \frac{1}{8} p_i^S - \frac{3}{16} (p_{ij}^S + p_i^T) + \frac{1}{16} (p_{ij}^S + p_i^S + p_{ij}^S + p_i^T) \ (10)$$

and

$$p_{ii}^S = \frac{1}{2} c_{ii}^T + \frac{1}{2} k + \frac{1}{6} t - \frac{1}{2} p_i^T + \frac{1}{4} (p_j^T + p_i^T) + \frac{1}{2} (p_{ij}^S + p_i^S) - \frac{1}{4} (c_{ij}^S + c_i^S) \ (11)$$

$$p_{ij}^S = \frac{1}{2} c_{ij}^S - \frac{1}{6} k + \frac{1}{18} t + \frac{1}{6} p_j^T - \frac{1}{2} p_i^T + \frac{1}{3} p_i^S - \frac{1}{6} c_{ii}^S + \frac{1}{3} V \ (12)$$

It is important to note while direct trading and settlement activities are complements, they are strategic substitutes (see Bulow, Geanakoplos, and Klemperer (1985) for the concept), i.e. $\partial p_i^T / \partial p_J^A < 0 \forall j$ and $\partial p_i^S / \partial p_i^T < 0 \forall i$. Hence, price increases by the settlement provider (to either trading platform) induce the trading platform to lower its price strategically. This mechanism will turn out to be important in our further analysis.

While price of up- or downstream firms are strategic substitutes, the prices of the respective competitors, that is among trading platforms as well as among settlement providers are strategic complements. Increases in prices of the immediate competitor lead to strategic price increases, i.e. reaction functions are upward sloping. This depicts the conventional feature of the Salop model. Finally, we should note that changes of the prices charged by settlement providers B and C lead to a price reduction of trading platform A. All these interaction are decisive in our analysis of the vertical integration process.

Overall, we have twelve prices (3 trading prices and 3 settlement prices for each trading platform) which we need to solve simultaneously. By doing this, we find

$$p_A^T = \frac{1}{5} V - \frac{1}{5} c^s + \frac{4}{5} c^T + \frac{1}{15} t - \frac{7}{65} y \ (13)$$

$$p_B^T = p_C^T = \frac{1}{5} V - \frac{1}{5} c^s + \frac{4}{5} c^T + \frac{1}{15} t - \frac{19}{130} y \ (14)$$

$$p_{AA}^S = \frac{2}{5} V + \frac{3}{5} c^s - \frac{2}{5} c^T + \frac{3}{10} t + \frac{1}{2} k - \frac{29}{65} y \ (15)$$

$$p_{BB}^S = p_{CC}^S = \frac{2}{5} V + \frac{3}{5} c^s - \frac{2}{5} c^T + \frac{3}{5} t + \frac{1}{2} k + \frac{19}{260} y \ (16)$$

$$p_{AB}^S = p_{AC}^S = \frac{2}{5} V + \frac{3}{5} c^s - \frac{2}{5} c^T + \frac{2}{15} t + \frac{149}{260} y \ (17)$$

$$p_{BA}^S = p_{CA}^S = \frac{2}{5} V + \frac{3}{5} c^s - \frac{2}{5} c^T + \frac{2}{15} t + \frac{36}{65} y \ (18)$$

$$p_{BC}^S = p_{CB}^S = \frac{2}{5} V + \frac{3}{5} c^s - \frac{2}{5} c^T + \frac{2}{15} t + \frac{19}{260} y \ (19)$$

Vertical integration leads to a decrease of all trading prices. This effect is less pronounced in the integrated exchange A as compared to the non-integrated exchanges B and C. With
settlement, only the services provided via the direct, more efficient link becomes cheaper, while all other settlement services become more expensive. The somewhat surprising result of the effect of vertical integration on relative trading prices stems from the fact that direct trading and settlement prices are strategic substitutes (see Eq.(10)-(12)): higher settlement prices lead trading platforms to reduce their trading prices. Hence, platforms B and C which face higher settlement prices for securities listed on A have an incentive to reduce their price. Given that trading prices are strategic complements this triggers a reduction in A’s trading price. This is reinforced by the marginal weighted increase in settlement prices which leads, given that they are strategic substitutes to A’s trading price, to a decrease in A’s trading price as well.

Furthermore, vertical integration influences the market shares of the three platforms. For securities listed on trading platform A, traders which remain inactive for those securities increase to:

$$\frac{1}{3} - d_{BB} - d_{CC} = \frac{286t - 312v + 333y}{390t}$$

(20)

The decreased coverage of the market stems from the fact that the sum of trading and settlement prices, traders located between B and C have to pay.

Platforms B and C lose not only traders for securities listed on A and located between B and C but also when competing with platform A for securities listed on A:

$$d_{AB} + d_{AC} = \frac{78k + 26t + 153y}{156t}$$

(21)

The absolute amount of trades on platform A in securities listed on platform A increases with vertical integration. Hence, platform B and C lose trades in this segment as well.

The corresponding effects for securities listed on B and C with \(p, q \in \{B, C\}\) and \(p \neq q\) are:

$$1 - d^I_{AA} + d^I_{pp} = \frac{572t - 624v + 291y}{780t}$$

(22)

for the size of the inactive traders and

$$d^I_{qA} + d^I_{qp} = \frac{156k + 52t + 81y}{312t}.$$  

(23)

for trades taking place on the platforms on which the securities are actually listed.

Plugging all this into the demand function and Eq. (8) as well Eq. (9) allows us to derive the profit difference of the sum of profits in trading and settlement in A:

$$\Delta(\pi^S_A + \pi^T_A) = y \frac{198900k + 8476t - 173472v + 278409y}{608400t}$$

(24)

The decisive question we first address is whether vertical integration pays. The answer to this is in the affirmative if the term in brackets in the numerator of Eq. (24) is positive.
We thus define $\Gamma = 198900k + 8476t - 173472v + 278409y$ and use this term to investigate comparative static effect. Since $\partial \Gamma / \partial t > 0$ this implies that vertical integration is more likely to pay off if the services of trading platforms are perceived to be more differentiated (higher $t$). The intuition behind this is as follows. With a higher degree of platform differentiation relative more trades taking place on platform A are settled via the efficient link. This is due to the fact that with a higher $t$ relative fewer trades of securities listed on platforms B and C are traded on A (or in other words, the number of traders located far away from the platform on which the securities are listed increase). This effect overcompensates the fact that with platform differentiated platform A loses partially its competitive advantage vis-a-vis platforms B and C stemming from the liquidity effect.

Furthermore, due to $\partial \Gamma / \partial k > 0$, vertical integration is more likely to increase the profits of the vertically integrated organization if liquidity matters a lot. The more liquidity matters, the higher the share of trades executed on platform A being settled via the efficient link. The more important the liquidity effect is the larger the share of trades of a particular security taking place on the platform on which this particular security is listed. Hence, the absolute and relative cost advantage of vertical integration is most pronounced.

In addition, we find $\partial \Gamma / \partial v < 0$. A higher net social reservation price leads to more trades on A of securities listed on B and C. These trades are settled through the inefficient link after vertical integration. Hence, since these trades increase absolutely and relatively with a higher net social reservation price, this makes vertical integration less attractive.

Obviously, the larger is the efficiency gain of vertical integration, the more likely it is that vertical integration pays ($\partial \Gamma / \partial y > 0$). This is, given the circumstance that trades on platform A take place relatively more often in securities listed on platform A relative to those listed on platforms B and C. Hence, absolutely more trades are settled via the efficient link in our symmetric setting. If this link become even more efficient (larger $y$), this makes vertical integration more attractive.

We can summarize our findings in

**Proposition 1** Vertical integration is more likely to pay off if platforms are more differentiated, demand for liquidity is high, efficiency gains via vertical integration is pronounced and the social net reservation price is low.

Our comparative statistics do not answer the question whether a positive $\Gamma$ is indeed feasible. We turn to this next. Furthermore, we address how vertical integration affects the profits of the other platform, consumer surplus as well as overall welfare in a subsequent step. Figure 3 displays Eq. (24) as well as our assumptions 1 and 2 on parameters (the grey area is not compatible with these assumptions) this effect in $k - t$ space (our third parameter $v$ is normalized to 1). Besides reflecting the results of Proposition 1 once again,
it clearly shows that vertical integration may indeed pay off. The white range in the figure displays the parameter combinations which are not only feasible but also increase the sum of profits of A with vertical integration even if the marginal efficiency effect is evaluated at $y = 0$.

![Figure 3: Effect of vertical integration on profit of A](image)

The corresponding profit difference for platforms B and C reads as:

$$\Delta(\pi_q^S + \pi_q^T) = y \frac{105300k - 36868t - 215904v + 172233y}{1216800t}$$

Given our assumptions for the feasible parameter range (which imply that $v > 0.5t > 0.5k$) it immediately becomes obvious that the profit differential is always negative for sufficiently small efficiency gains (e.g. $y$ approaching zero). Despite the fact that B and C gain via less intense competition, they loose market shares and traders with respect to securities listed on platform A to either trading platform A and to the non-trading camp. Overall this leads to a decrease in profits.

Computing the difference effect of vertical integration on the well-being of traders yields for consumer surplus (see the appendix for details of the computation)

$$\Delta ConsumerSurplus = y \frac{152100k + 7436t - 129792v + 203289y}{608400t}$$

Pretty much the same mechanisms apply as with Proposition 1. A more pronounced liquidity effect and more perceived differentiation of platforms lead to relative more trades
via the efficient link. Securities listed on exchange A are more often traded on this platform. There traders gain via lower prices through which settlement providers channel efficiency gains to traders. Figure 4 reveals that traders – in sum – may indeed gain from vertical integration absolutely. This implies that in our symmetric setting the gains of the traders who benefit from the standardization of the vertically integrated exchange (i.e. those having a preference to trade on exchange A) outweigh the losses traders who have a preference for other exchanges.

Figure 4: Effect of vertical integration on consumer surplus

The overall difference of welfare (i.e. taking the sum of all profits and consumer surplus) in the symmetric case amounts to

$$\Delta Welfare = y \frac{456300k - 20956t - 519168v + 653931y}{608400t}$$

The corresponding figure (5) clearly shows that vertical integration may improve overall welfare in our symmetric setting. Given that the liquidity effect is relatively large (relative to the net social reservation price) and platforms are sufficiently differentiated a sufficiently large share of trades takes place on the exchanges on which securities are indeed listed. Rather little trades take place on the other exchange. Hence, the cost-benefit trade-off of vertical integration is such that even in our symmetric framework vertical integration increases overall welfare.

An obvious limitation of our analysis so far is our symmetry assumption. We argue in the following, however, that relaxing this assumption has straightforward consequences,
which do not affect our positive, qualitative analysis so far. We do this by proving that locally (i.e. at the symmetric setting) the profit difference is always strictly increasing in \( n_A \). Taking the first order derivative of the profit difference with respect to \( n_A \) gives us (see the appendix for a derivation of this):

\[
\frac{\partial \Delta (\pi_A^S + \pi_A^T)_{n_A=1/3}}{\partial n_A} = y \frac{2405520k + 785876t - 47892v + 2965353y}{5272800t} > 0
\]  

The positive sign of this expression follows from our assumptions stated above.
Investigating this relationship not only locally but for different $n_A$ we were unable – due to the very lengthy terms – to provide a general proof. We therefore evaded this by running a large number of numerical exercises. Thereby, we can show that the sign of 4 when mapped onto the size of platform A (i.e. $n_A$) is indeed positive implying a rising incentive to vertically integrate with size. Figure 6 depicts an example of the fraction of (24) for $y = 0$ for a given parameter combination. We have rerun this graph for a very large set of parameter constellations which all yielded the same increasing pattern (which implies () being positive for all $n_A$). Hence, we think that we have enough evidence to argue that larger platforms have, ceteris paribus, a more pronounced incentive to vertically integrate. This is quite intuitive. The larger a trading platform is, i.e. the more securities are listed on it, the more (less) trades are settled via the (in-) efficient link, making hence vertical integration more attractive. (One has to be careful with the figure because extreme values, large or small, of $n_A$ will violate assumptions 1 and/or 2)

With respect to welfare matters are much less clear-cut. If we differentiate the welfare surplus generated by vertical integration and evaluate around the symmetric setting, we find:

$$\frac{\partial \Delta (\text{Welfare})_{n_A=1/3}}{\partial n_A} = \frac{11501100k + 1636388t - 6591936v + 16150599y}{10545600t} \quad (29)$$

This expression is positive if $k$ is sufficiently large. Then, the efficiency gains via the efficient link are large and dominate the welfare losses by the the loss of traders and trades via the inefficient links. An even larger integrated exchange strengthens this effect. If $k$ is very small the welfare effect might become negative because efficiency gains will not offset the negative effects.

It can also be observed that for all parameters for which Eq. (4) is positive eq. 29 will be positive as well while if Eq. (4) is negative the sign of Eq. (29) is not clear. This means that there are situation where welfare decreases in symmetric setting but would increase if security service provider of a relatively large country decides to integrate.

5 Vertical integration if everybody else is

One of the main aim of this paper is to shed light on industry dynamics and the evolution of the trading and settlement industry. Against this background it is important to investigate to which extent the incentives to integrate vertically hinge on the existing industry structure, i.e. on the degree of vertical integration in the industry. Our in a nutshell: is vertical integration more or less attractive if competitors are already integrated or not?

We start out by looking into the incentives of A to vertically integrate if one of the other platforms (say B) is already vertically integrated. This implies that the already
vertically integrated platform not only has established an efficient link in B but has build up inefficient links with A and C. Hence, the vertical integration process only adds an efficient link in A and an inefficient with C (rather than with B and C as in our previous). Somewhat surprisingly, the incentives to integrate vertically increase! This can be seen as follows. When computing our profit difference for A we get:

$$\Delta(\pi_A^S + \pi_A^T)_{A+B} = \frac{36075k + 6097t - 17784v + 47160y}{152100t}$$ (30)

The denominator of this expression is lowest, if we, against the background of our assumptions allow for the lowest $k$ ($k = \frac{1}{3}t$) and $y$ ($y = 0$) as well as for the highest feasible $v$ ($v = \frac{11}{12}t$). With these parameter combinations we get for the term in brackets in the denominator of Eq. 30

$$(12025t + 6097t - 16302t) = 1820t > 0$$

The intuitive explanation can be best understood by noting that the integration of one platform (say B) implies that the cost benefits for A stemming from its own integration increase. After the integration of B, A faces a competitor with an efficient link, hence, making it more attractive to establish an efficient link on its own as well. Furthermore, after B-integration, there is already an inefficient link between A and B thereby eliminating these additional costs of A-integration. Taking these two effects together makes A-integration more attractive after B-integration compared to the situation in which A goes for a head-start with respect to vertical integration.

Obviously given A and B decided to integrate C will follow. Therefore, we observe a bandwagon-effect. If one exchange decides to integrate, the others will follow. This also means, that say for historical reasons, exchange and respective CSD are integrated in country A, the probability that the security service providers in country B and C decide to do the same is high.

Taking the bandwagon-effect into account makes our analysis so far incomplete because we should observe no integration or full-scale integration. Therefore one should compare situation without or full-scale integration from a welfare as well as from a private point-of-view.

Comparing welfare in these two situation gives us

$$\Delta Welfare = y\frac{225k + 11t - 192v + 321y}{150t}$$ (31)

This show us, that compared to situation of A-integration for a larger set of parameters welfare increases (see Eq. (7)).
More interesting is now the comparison of private profits of the securities service providers. The difference of A’s profit amounts to

$$\Delta \pi_A = y \frac{75k + t - 72V + 111y}{225t}$$

(32)

Comparing this with (24) shows that now the Delta is positive for a smaller set of parameters. In other words, we find situations where it is advantageous for one exchange to integrate but then the others will follow and in the end they are all worse off. Hence, we find a coordination for a set of parameters, see figure 8 where the dark grey shaded area is the parameter region where we observe a prisoner’s dilemma.

6 Horizontal cooperation

Given network economies as well as economies of scale and scope, horizontal integration is an alternative to vertical integration which is intensively and hotly discussed in the financial press. Therefore, we consider horizontal integration for its own sake but also in comparison to vertical integration. In our framework horizontal integration of trading platforms for matters of exploitation of liquidity and network effects would lead to a joint trading platform which would eliminate the limited access of certain traders to certain stocks and hence, the coverage gap in our base model. In a nutshell, horizontal integration would change the market structure at the trading level and hence, the entire...
model structure, making a comparison with vertical integration infeasible. In order to avoid this problem, we consider horizontal integration at the settlement level. There, horizontal integration leaves market structure unchanged, the settlement providers stay in their monopolistic situation.

To capture the notion of network effects as well as of economies of scope at the clearing and settlement level we model horizontal integration as cost savings by $(0 < b < 1)$ of all settlement providers. Hence, while trading platforms’ costs amount to $c^T$, the costs of all settlements are $c^S - by$.

Given that the monopolistic settlement providers all experience identical cost savings, their monopolistic profits increase. Since there are only gains but no costs to horizontal integration, this is quite obvious. Rather than introducing costs of integration as a countervailing balance we pursue what we consider a much more interesting alternative: to compare the incentives to integrate horizontally with the ones of vertical integration.

In a first step we compare, from the viewpoint of individual firms, the gains from horizontal integration with the potential gains from vertical integration of one platform given that all other platforms are vertically disintegrated. This basically depicts a situation with the very same starting point in which a settlement provider can either decide to vertically integrate with the corresponding trading platform or join forces with the other settlement providers to integrate horizontally (since there is no reason in our model to stop short of integrating all three settlement providers we equate horizontal integration with the integration of
all three rather than of only two settlement providers, a procedure which affects our comparison quantitatively but not qualitatively). Furthermore, given that our previous discussion has revealed that stand-alone vertical integration requires higher incentives than the vertical integration decision with others already being vertically integrated, this comparison allows a prediction on the final equilibrium starting from a situation with complete disintegration.

In a second step we take this to another level and compare the private incentives to integrate horizontally or vertically with the optimal decision rule in which the central planner compares horizontal integration with full-size vertical integration. With this second step, which also takes place against the background of our previous findings on positive feedback effects of vertical integration we ask whether it may be feasible that the market solution may end up in a bad equilibrium due to a coordination problem of settlement and trading platforms.

6.1 Horizontal vs. Vertical Integration

With horizontal integration we derive from Eqs. (10)-(12) the following prices in equilibrium:

\[ p_i^T = \frac{1}{5}V - \frac{1}{5}c^S + \frac{12}{15}c^T + \frac{1}{15}t + \frac{1}{5}by \]  
\[ p_{si}^S = \frac{2}{5}V + \frac{3}{5}c^S - \frac{2}{5}c^T + 2k + \frac{3}{10}t - \frac{3}{5}by \]  
\[ p_{ij}^S = \frac{2}{5} + V\frac{3}{5}c^S - \frac{2}{5}c^T + \frac{2}{15}t - \frac{3}{5}by \]

Plugging these prices into the demand functions of the trading platforms allows us to derive the following differential profit expression for all settlement provider (i.e. the sum of settlement providers’ profits after horizontal integration minus the one before integration):

\[ \Delta(\pi_A + \pi_B + \pi_C) = 8by\frac{2t + 6v + 3by}{75t} \]  
\[ \Sigma_{profits} = y\frac{198900k + 8476t - 173472v + 278409y - 64896b(2t + 6v + 3by)}{608400t} \]  

We interpret this expression as the differential incentives of one settlement provider to choose horizontal rather than vertical integration.

Solving for the critical \( b \) which just leads to indifference between vertical and horizontal integration (at \( y = 0 \) yields

\[ b^* = \frac{198900k + 8476t - 173472v}{64896(2t + 6v)} \]
Given our assumptions, $b^*$ is always smaller than 1. This is rather intuitive. With $b = 1$ all settlement providers face efficient links with horizontal integration while with vertical integration, the integrating platform has to trade-off the direct efficient link with the indirect inefficient link. These cost disadvantage of these inefficient link overcompensates the competitive advantage that the other settlement providers have to rely on inefficient links as well. Overall this implies that with $b = 1$ horizontal integration dominates vertical integration.

Looking into the comparative statics of $b^*$ reveals that a more pronounce liquidity effect (larger $k$) makes vertical integration relatively more attractive (leads to an increase of $b^*$). With horizontal integration all settlement links are becoming cheaper to the same extent. Hence, a change in the market shares of the different trading platforms does not affect profits in the case of horizontal integration. In contrast, with vertical integration a more pronounced liquidity effect, shifts more trades into the settlement via the efficient link. This has a positive effect on the profits of the integrated platform making vertical integration relative to horizontal integration more attractive.

Pretty much the same mechanisms applies with social net willingness-to-pay (larger $v$). This parameter affects the incentives to integrate, however, just in the opposite direction: it makes vertical integration relatively less attractive. The larger is $v$ the more trades are settled via the inefficient link in the case of vertical integration leading to a deterioration of integration profits. The effect of the platform differentiation parameter $t$ is ambiguous and reflects the fact that it has an impact on both, the profit differential with vertical as well as with horizontal integration.

We now turn to the evaluation of different industry equilibrium (with either full horizontal or full vertical integration) from a welfare point of view and the private incentives to reach these equilibria (remember: if security service provider in one country decides to integrate the others will follow). We especially aim to ask whether there is a potential coordination effect in the sense that it is socially feasible to have full-scale vertical integration but private incentives stand in the way of a headstart such that initially horizontal integration is preferred over vertical integration despite the fact that in the end the latter is socially dominant.

The private incentives to reach either of the two industry equilibria has been investigated above (see Eq. (38) and note once again that due to the bandwagon effect positive initial incentive to vertical integration lead finally to an industry equilibrium with full vertical integration). Hence, we need to evaluate full-scale vertical integration (embracing all three platforms) and horizontal integration from a welfare point of view. Computing the relative
gains in welfare accruing from full-scale vertical versus horizontal integration gives us:

\[
\Sigma_{\text{Welfare}} = \frac{225k + 11t - 192v + 321y - 32b(2t + 6v + 3by)}{150t}
\]

which, if positive, implies that vertical integration is dominating horizontal integration from a welfare point of view.

Evaluated at \( y = 0 \) this yields the critical social \( b_{sf}^* \):

\[
b_{sf}^* = \frac{456300k + 22308t - 389376v}{64896(2t + 6v)}
\]

As a next step, we compare private incentives to reach either equilibrium with the relative advantageousness of either equilibrium from a social point of view. When comparing \( b^* \) with \( b_{sf}^* \) we find

\[
\text{sign} \ (b_{sf}^* - b^*) = \text{sign} \ (257400k + 13832t - 215905v)
\]

Hence, with pronounced liquidity effects, the industry may indeed end up in a bad equilibrium. While full scale vertical integration is socially optimal decentralized decision making leads to an equilibrium with horizontally integrated settlement providers. We summarize this in

**Proposition 2** With strong (weak) liquidity effects and weak (strong) social net reservation prices, the industry will end up in a bad equilibrium in which settlement providers are horizontally (vertically) integrated.

The economic intuition behind this is the following. Strong liquidity effects imply that a relative large share of trades take place via the efficient link while relatively few trades take place via the inefficient link. Hence, traders benefit relatively much, while the other firms loose relatively little. Hence, the vertical integration decision of the individual exchange and settlement provider exerts a positive net externalities leading to too little vertical and comparatively too much horizontal integration. The opposite argument holds for weak liquidity effects. There, relatively many traders are executed via the inefficient links thereby hurting competing firms relatively much and benefitting traders relatively little. The deciding firms exerts negative net externalities and decides too often for vertical integration. Pretty much the same arguments holds true for the net social reservation price. This parameter being high (low) implies relatively many (few) trades take place via the inefficient link leading to a negative (positive) net externality of the vertical integration decision and hence to too much (little) vertical and too little (much) horizontal integration.
7 Endogenous Listings

Our analysis so far was undertaken against the background of a given distribution of listings of securities across the trading platforms. We proceed by thinking about endogenizing the listings of securities. In order to endogenize the listing decision it is crucial to think about the objective function behind this listing decision. There are two obvious factors determining the listing decision, the first one being the costs of the listing, direct (paid to the trading platform) and indirect (costs of reporting and disclosing information and the like). The second factor, which is potentially much more importance for listing decision is the degree of access to capital markets. This latter effect is strongly affected by the size and liquidity on the respective trading platform. With listings it is especially important to reach a wide range of potential investors. In our model, the numbers of investors reached equals the market coverage. We thus consider the listing decision as the outcome of a cost-benefit trade-off in which the price paid is contrasted with the gain which accrues from the investors the company/organization listing its security reaches. We therefore depict the objective function as

\[ L = a \times \text{coverage} - p^L \]  

(41)

with \text{coverage} denoting the market reach of the security and \( p^L \) the price to be paid to the trading platform. The parameter \( a \) stands for the weight of the coverage effect.

Since integration affects the market coverage, the trading platform has to reduce \( p^L \) in order to avoid losing listed securities. We aim to analyse this additional effect of integration on firms’ profits stemming from changes in market coverage and hence, necessary adjustments in listing prices. Obviously, this additional effect does not only hinge on the precise nature of the change in market coverage but also on the weight of the coverage effect, i.e. on \( a \). Whereas it is safe to conjecture that this additional effect does reduce the incentive to vertically integrate the precise nature of this reduction in incentive for vertical integration requires further analysis (TBC).

8 Conclusion

We consider the main contribution of this paper to shed light on the interrelationship between the choice of organizational design and competition in the financial security service industry. The main empirical predictions of our analysis are that we should expect to see less rather than more vertical integration if further real integration e.g. in the European Union prevails and cross-border trades become more attractive for investors. Furthermore, our analysis lets us expect that larger exchanges have a stronger incentive to
vertically integrate as compared to smaller ones. Despite the fact that we have focused our analysis on the financial security service providers we think a number of insights take over to other network industries such as the railway industry, the gas industry etc. Adopting our framework to other industries should allow us to approach questions of organizational design such as the question whether the network infrastructure should be closely aligned with one upstream firm or be operated independently. Obviously, our analysis overlooks a number of potential factors which may be the route of further industry. The most important aspect which is not included in the analysis so far is the market entry of new players. The open issue in our context is whether changes in organizational design on the one hand reflect entry threat and on the other hand how and what kind of changes in organizational design may erect market entry barriers. Furthermore, it would be interested to see, where, given certain prevailing organizational structure and competitive situations, market entry is most likely to occur and what implications this has on competition and organizational structure. We leave this to a further step in our research.

References


Appendix

Derivation of profit-maximizing trading and settlement prices

In order to derive the profit-maximizing trading and settlement prices we plug in the demand functions in Eqs. (8) as well as in (9) and take first-order derivatives with respect to prices. This procedure yields 12 reaction functions, for trading platform $i, j, l \in \{A, B, C\}$ we find:

$$p^T_i = \frac{1}{2n_i + 3n_j + 3n_l} \left[ \frac{t}{6}(2n_i + n_j + n_l) + \frac{k}{2}(2n_i - n_j - n_l) + V(n_j + n_l) + \frac{c^T}{2}(2n_i + 3n_j + 3n_l) + \frac{p^T_j}{2}(n_i + n_j) + \frac{p^T_i}{2}(n_j + n_l) - n_ip^S_{ii} + \frac{1}{2}(n_ip^S_{ij} + n_ip^S_{ik} + n_jp^S_{jj} + n_lp^S_{ll}) \right]$$

The corresponding settlement prices for $i \neq j \neq l$ are:

$$p^S_{ii} = \frac{1}{2}c^S_{ii} + \frac{1}{2}k + \frac{1}{6}t - \frac{1}{2}p^T_i + \frac{1}{4}(p^T_i + p^T_j) + \frac{1}{2}(p^S_{ij} + p^S_{il}) - \frac{1}{4}(c^S_{ij} + c^S_{il})$$

$$p^S_{ij} = \frac{1}{2}c^S_{ij} - \frac{1}{6}k + \frac{1}{18}t + \frac{1}{6}p^T_i - \frac{1}{2}p^T_j + \frac{1}{2}p^S_{ii} - \frac{1}{6}c^S_{ii} + \frac{1}{3}V$$
Combining these prices yields for $n_B = n_C = \frac{1}{2} n_A$ (when we are analyzing asymmetries, we analyze them in one dimension keeping the others symmetric):

$$ p^T_A = \frac{1}{12(29 + n_A - 4n_A^2)} \left[ 24c^T(11 + 2n_A - n_A^2) + 12k(3n_A + 8n_A + 3) + 4l(n_A + 5n_A + 4) + 24k(7 - 3n_A - 2n_A^2) - 24n_Ac^A_B(n_A - 3) + 6n_A(c^A_B + c^C_B)(n_A + 5) + 6(c^A_B + c^C_B)(3 - 2n_A - n_A^2) \right] $$

$$ (45) $$

$$ p^T_B = \frac{1}{12(493 + 46n_A - 67n_A^2 - 4n_A^3)} \left[ 24c^T(204 - 5n_A - 18n_A - n_A^2) + 12k(34 - 117n_A + 44n_A + 3n_A^2) + 4l(119 - 61n_A + 13n_A + n_A^2) + 12k(85 + 56n_A - 31n_A^2 - 2n_A^3) + 24n_Ac^A_B(34 - 15n_A - n_A^2) - 6n_Ac^A_B(395 - 66n_A + 29n_A^2) - 6n_Ac^C_B(47 - 78n_A + 19n_A^2) - 6c^B_B(121 - 133n_A - n_A^2 + 13n_A^2) + 6c^C_C(53 - 35n_A - 29n_A^2 + 11n_A^3) + 3c^C_C(79 - 29n_A - 79n_A^2 + 29n_A^3) - 3c^B_B(385 - 419n_A + 35n_A^2 + 35n_A^3) + 3c^A_A(109 - 155n_A + 35n_A^2 + 11n_A^3) - c^B_B(211 + 129n_A - 165n_A^2 + 15n_A^3) \right] $$

$$ (46) $$

$$ p^T_C = \frac{1}{12(493 + 46n_A - 67n_A^2 - 4n_A^3)} \left[ 24c^T(204 - 5n_A - 18n_A - n_A^2) + 12k(34 - 117n_A + 44n_A + 3n_A^2) + 4l(119 - 61n_A + 13n_A + n_A^2) + 12k(85 + 56n_A - 31n_A^2 - 2n_A^3) + 24n_Ac^A_B(34 - 15n_A - n_A^2) - 6n_Ac^A_B(395 - 66n_A + 29n_A^2) - 6n_Ac^C_B(47 - 78n_A + 19n_A^2) - 6c^B_B(121 - 133n_A - n_A^2 + 13n_A^2) + 6c^C_C(53 - 35n_A - 29n_A^2 + 11n_A^3) + 3c^C_C(79 - 29n_A - 79n_A^2 + 29n_A^3) - 3c^B_B(385 - 419n_A + 35n_A^2 + 35n_A^3) + 3c^A_A(109 - 155n_A + 35n_A^2 + 11n_A^3) - c^B_B(211 + 129n_A - 165n_A^2 + 15n_A^3) \right] $$

$$ (47) $$

$$ p^S_A = \frac{1}{8(29 + n_A - 4n_A^2)} \left[ (1 - c^T)(88 + 16n_A - 8n_A^2) + k(128 - 28n_A - 28n_A^2) + t(72 - 4n_A - 12n_A^2) + 4c^A_A(29 + 7n_A - 2n_A^2) - 2n_A(c^A_B + c^C_B)(5 + n_A) \right] $$

$$ (48) $$

$$ p^S_B = \frac{1}{24(493 + 46n_A - 67n_A^2 - 4n_A^3)} \left[ (1 - c^T)(204 - 5n_A - 18n_A - n_A^2) - 6k(68 - 234n_A + 88n_A^2 + 6n_A^3) - 4l(374 - 107n_A + 80n_A^2 + 5n_A^3) - 24n_Ac^A_B(34 - 15n_A - n_A^2) + 6c^A_A(986 + 487n_A - 200n_A^2 - 37n_A^3) + 6c^A_B(47 - 78n_A + 19n_A^2) + 6c^B_B(121 - 133n_A - n_A^2 + 13n_A^2) + 6c^C_C(53 - 35n_A - 29n_A^2 + 11n_A^3) + 3c^C_C(79 - 29n_A - 79n_A^2 + 29n_A^3) + 3c^C_C (109 - 155n_A + 35n_A^2 + 11n_A^3) + 3c^C_C(7 + 43n_A - 55n_A^2 + 5n_A^3) \right] $$

$$ (49) $$

$$ p^S_C = \frac{1}{8(493 + 46n_A - 67n_A^2 - 4n_A^3)} \left[ (1 - c^T)(204 - 5n_A - 18n_A - n_A^2) - 4k(459 + 163n_A - 11n_A^2 - 7n_A^3) + 2n_Ac^A_B(395 - 66n_A + 29n_A^2) + 2n_Ac^A_B(47 - 78n_A + 19n_A^2) + 2n_Ac^A_B(1107 - 41n_A - 135n_A^2 + 5n_A^3) - 2n_Ac^A_B(53 - 35n_A - 29n_A^2 + 11n_A^3) - c^A_B(79 - 29n_A - 79n_A^2 + 29n_A^3) + 2n_Ac^A_B(385 - 419n_A - n_A^2 + 35n_A^3) - c^A_B(109 - 155n_A + 35n_A^2 + 11n_A^3) + c^A_B(7 + 43n_A - 55n_A^2 + 5n_A^3) \right] $$

$$ (50) $$

$$ p^F_A = \frac{1}{24(29 + n_A - 4n_A^2)} \left[ 24c^T(11 + 2n_A + n_A^2) + 6k(68 - 16n_A + 6n_A^3) + 4l(374 - 107n_A - 5n_A^3) + 24n_Ac^A_B(3 + n_A) - 6n_Ac^B_B(5 + n_A) + 6c^C_C(3 - 2n_A - n_A^2) + 3c^C_C(n_A + 5) \right] $$

$$ (51) $$

$$ p^F_B = \frac{1}{24(493 + 46n_A - 67n_A^2 - 4n_A^3)} \left[ 24c^T(204 - 5n_A - 18n_A - n_A^2) - 12k(34 - 117n_A + 44n_A^2 + 3n_A^3) + 4l(374 - 77n_A + 80n_A^2 + 5n_A^3) + 24c^T(204 - 5n_A - 18n_A - n_A^2) - 24n_Ac^A_B(34 - 15n_A - n_A^2) - 6n_Ac^A_B(47 - 78n_A + 19n_A^2) + 6c^A_B(395 - 66n_A - 29n_A^2 - 6n_Ac^A_B(53 - 35n_A - 29n_A^2 + 11n_A^3) + 6c^B_B(121 - 133n_A - n_A^2 + 13n_A^2) + 3c^C_C(2357 - 235n_A - 269n_A^2 + 19n_A^3) - 3c^A_B(79 - 29n_A - 79n_A^2 + 29n_A^3) + 3c^A_B(7 + 43n_A - 55n_A^2 + 5n_A^3) + 3c^A_B(109 - 155n_A + 35n_A^2 + 11n_A^3) \right] $$

$$ (52) $$

Plugging in the respective costs yields (13)-(19) and (33)-(35).
By inserting the prices into the demand functions we get

\[
d_A^t = \frac{1}{12(29 + n_A - 4n_A^2)} \left[ (V - c^T)(36n_A - 12) + h(204 - 84n_A - 24n_A^2) + t(64 - 16n_A - 8n_A^2) - 6c_{AP}^s(29 - 9n_A - 4n_A^2) + 3(c_{AB}^s + c_{AC}^s)(29 - 17n_A) - 15(c_{AB}^s + c_{AC}^s)(1 - n_A) + 3(c_{AB}^s + c_{AC}^s)(11 - 9n_A - 2n_A^2) - 6(c_{B}^s + c_{C}^s)(2 - 3n_A + n_A^2) \right]
\]

\[
d_A^d = \frac{1}{8(493 + 46n_A - 67n_A^2 - 4n_A^3)} \left[ 4(V - c^T)(357 + 140n_A - 27n_A^2 - 2n_A^3) - 4k(153 + 281n_A + 33n_A^2 + n_A^3) - 4(68 + 47n_A + 208n_A^2 + n_A^3) + 4n_A c_{AP}^s(187 + 90n_A + n_A^2) - 2n_A c_{Ap}^s(325 - 13n_A) - 2(c_{AP}^s)(151 - 6n_A + 11n_A^2) + 2(c_{pq}^s)(356 + 159n_A - 38n_A^2 - 9n_A^3) - 2(c_{pq}^s)(50 - 29n_A - 14n_A^2 - 7n_A^3) + 13(c_{pq}^s)(5 - n_A - 5n_A^2 + n_A^3) - c_{pq}^s(2419 + 631n_A - 223n_A^2 - 19n_A^3) + 13(c_{pq}^s)(37 - 23n_A - 13n_A^2 + n_A^3) - c_{pq}^s(167 - 211n_A + 25n_A^2 + 19n_A^3) \right]
\]

\[
d_A^d = \frac{1}{8(493 + 46n_A - 67n_A^2 - 4n_A^3)} \left[ 4(V - c^T)(425 - 60n_A - 39n_A^2 - 2n_A^3) - 4k(323 - 219n_A + 3n_A^2 + n_A^3) + 4(34 + 53n_A - 14n_A^2 + n_A^3) + 2(c_{AP}^s)(448 - 260n_A - 17n_A^2) - 2(c_{AP}^s)(1479 - 497n_A - 113n_A^2 + 31n_A^3) + 2(c_{AP}^s)(113 - 108n_A + 291n_A^2) + 2(c_{pq}^s)(207 - 215n_A + 11n_A^2 + 19n_A^3) - 2(c_{pq}^s)(54 - 37n_A - 34n_A^2 + 17n_A^3) - c_{pq}^s(127 - 43n_A - 127n_A^2 + 43n_A^3) - c_{pq}^s(151 - 157n_A + 17n_A^2 - 11n_A^3) - 13(c_{pq}^s)(25 - 25n_A - n_A^2 + n_A^3) + c_{pq}^s(569 - 629n_A + 7n_A^2 + 53n_A^3) \right]
\]

\[
d_A^d = \frac{1}{12(29 + n_A - 4n_A^2)(17 + n_A)} \left[ 2(V - c^T)(51 - 150n_A - 9n_A^2) + 3k(901 - 342n_A - 119n_A^2 + 6n_A^3) - t(935 - 242n_A + 125n_A^2 + 8n_A^3) + 30n_A c_{AP}^s(17 + 10n_A) + 2n_A c_{Ap}^s(621 - 24n_A - 67n_A^2) - 6n_A c_{AP}^s(54 + 17n_A - 17n_A^2) - c_{pq}^s(2439 + 774n_A - 327n_A^2 - 78n_A^3) - 6c_{pq}^s(44 - 43n_A - 10n_A^2 + 9n_A^3) + 3c_{pq}^s(353 + 165n_A - 23n_A^2 - 27n_A^3) - 3c_{pq}^s(50 - 29n_A - 14n_A^2 - 7n_A^3) + c_{pq}^s(656 + 159n_A - 38n_A^2 - 9n_A^3) + c_{pq}^s(624 - 651n_A - 48n_A^2 + 75n_A^3) \right]
\]

\[
d_A^d = \frac{1}{8(493 + 46n_A - 67n_A^2 - 4n_A^3)} \left[ 8(V - c^T)(204 - 5n_A - 18n_A^2 - n_A^3) - 2k(361 + 188n_A - 21n_A^2 - 2n_A^3) + 2(85 + 66n_A - 31n_A^2 - 2n_A^3) + 8n_A c_{AP}^s(34 - 15n_A - n_A^2) + 2n_A c_{Ap}^s(669 - 60n_A - 53n_A^2) - 2(c_{AP}^s)(127 + 84n_A - 43n_A^2 + 2n_A^3 + 2(c_{pq}^s)(208 - 217n_A - 16n_A^2 + 25n_A^3) + 2(c_{pq}^s)(353 + 165n_A - 23n_A^2 - 27n_A^3) - c_{pq}^s(311 - 253n_A - 119n_A^2 + 61n_A^3) - 13(c_{pq}^s)(5 - n_A - 5n_A^2 - n_A^3) + c_{pq}^s(167 + 211n_A + 25n_A^2 + 19n_A^3) + c_{pq}^s(2341 + 919n_A - 361n_A^2 - 91n_A^3) \right]
\]
Derivation of consumer surplus

We can express consumer surplus as the sum of consumer surplus of all the different segments:

\[ CS = n_1\int_0^{x_1^A} (V - p_A^T - p_{AA}^S + k - tx)dx + \int_{x_1^A}^{\frac{1}{3}} (V - p_B^T - p_{AB}^S - t*\frac{1}{3} - x)dx + \]

\[ n_2\int_0^{x_1^B} (V - p_A^T - p_{BA}^S - tx)dx + \int_{x_1^B}^{1/3} (V - p_B^T - p_{BB}^S + k - t((1/3) - x))dx + \]

\[ n_3\int_0^{x_1^C} (V - p_A^T - p_{CA}^S - tx)dx + \int_{x_1^C}^{1/3} (V - p_B^T - p_{CB}^S - t((1/3) - x))dx + \]

\[ \int_{x_1^{2/3}}^{x_1^{3/2}} (V - p_C^T - p_{BC}^S - t(x - (2/3)))dx + \int_{x_1^{3/2}}^{1} (V - p_A^T - p_{BA}^S - t(1 - x))dx + \]

\[ \int_{x_1^{3/2}}^{x_1^{4/3}} (V - p_C^T - p_{BC}^S - t(x - (2/3)))dx + \int_{x_1^{4/3}}^{1} (V - p_A^T - p_{BA}^S - t(1 - x))dx + \]

\[ \int_{x_1^{4/3}}^{x_1^{5/2}} (V - p_C^T - p_{CC}^S + k - t(x - (2/3)))dx + \int_{x_1^{5/2}}^{1} (V - p_A^T - p_{CA}^S - t(1 - x))dx + \]

Plugging in the respective equilibrium prices for the symmetric case with vertical integration of only A yields after some tedious calculations:

\[ CS = \frac{(64896v(\frac{3}{2}v + t - 2y) + 50700k(\frac{3}{2}k + t + 3y) - 115934t^2 + 7436ty + 203289y^2)}{608400t} \]  

(59)

For the symmetric case with vertical integration of all security service provider we get

\[ CS = \frac{(192v(\frac{3}{2}v + t - 2y) + 150k(\frac{3}{2}k + t + 6y) - 343t^2 + 108ty + 1188y^2)}{1800t} \]  

(60)

and for horizontal integration

\[ CS = \frac{(192(\frac{3}{2}v + t + 3by) + 150k(\frac{3}{2}k + t) - 343t^2 + 192ty + 288b^2y^2)}{1800t} \]  

(61)

Welfare is then calculated as the sum of consumer surplus and the sum of profits.