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Strategic Effects of Regulatory Capital Requirements in Imperfect Banking Competition

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Strategic Effects of Regulatory Capital Requirements in Imperfect Banking Competition

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Abstract
This paper analyzes the competitive effects of the Basel II Standard Approach capital requirement regulation on an oligopolistic credit market. We define the general goal of the capital regulation to be the improvement of the banking system's soundness. As in the standard literature, we assume that at least in the short-run, an adjustment of equity to the regulatory requirement may prove costly. The immediate effect of increasing the capital requirements is, thus, likely to be a reduction in the total supply of loans, and accordingly, an increase in the credit interest rate. This cost effect alone, would leave Bertrand oligopoly profits unchanged. Yet, we focus on an additional effect of a regulatory capital requirement that arises because a binding requirement changes the sequence in which the strategic pricing decisions are made. Assuming that short term recapitalization of a bank may prove to be costly we consider capital requirement regulation to temporarily constrain the bank's lending activities. The oligopolistic pricing competition is, therefore, transformed into a two stage capacity-pricing game à la Kreps and Scheinkman (1983). In a first stage banks choose the long-term structure of refinancing their assets, thereby making an imperfectly binding commitment to the loan capacity as a function of the chosen degree of capitalization and the regulatory capital requirement. In the second stage, loan price competition takes place. It is shown that capital requirement regulation may not only decrease the supply of credit through an increased marginal cost effect but also can have a spontaneous collusive effect resulting in even higher credit prices and increased profits for the banks.

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1. Introduction

The soundness of the banking system is fundamental for economic wealth and stability. Since the banking sector is particularly vulnerable to inefficient bank runs and contagion resulting in bank panics, the overall aim of banking regulation is to secure financial stability by minimizing, ex ante, the likelihood of bank runs and reduce ex post contagion, when banks fail. To reach this goal, most countries have introduced a governmental safety net including deposit insurances, lender of the last resort and bailout policies. The undesirable secondary effect of such a governmental safety net is the destruction of the market discipline, which provides strong moral hazard incentives to exploit the option value of the safety net. Greenbaum and Thakor (1995, S. 103) summaries this idea in stating: “The moral hazard engendered by one form of regulation, namely deposit insurance, creates the need for other forms of regulation such as capital requirements.” The intuition is that well capitalized banks have fewer incentives to increase asset risks. A bank endowed with more capital is less likely to exploit the option value of the deposit insurance and therefore, the probability of banking default is reduced.¹

Yet, the actual impact of a regulative capital requirement on the individual behavior of banks and the individual incentives to take excessive risk is not undisputed in banking theory literature. Berger, Herring and Szegö (1995), Santos (2001), and VanHoose (2007) offer comprehensive reviews of the theoretical literature on the impact of capital requirement regulation.

Regardless of the ambiguous theoretical predictions, there is a general consensus² that higher equity has a positive direct effect on the balance sheet structure. Banks facing a binding capital requirement have to either reduce their assets or increase their equity, thereby increasing the capital buffer for the case of asset default. Hence, a more stringent capital requirement reduces the set of states in which the bank defaults, and at the same time, it reduces the costs for the debt holder (the depositors or certain deposit insurance) given default of the bank.

In the short-run, such an adjustment of equity to the regulatory requirement may prove costly. Therefore, the immediate effect of increasing the capital requirements is likely to be a reduction in the total supply of loans, and accordingly, an increase in the credit interest rate. This effect is most often analyzed in a perfect competition environment, but would also stay valid in a Bertrand oligopoly. Thakor (1996), for instance, discusses that higher capital requirements increase the probability of each borrower being rationed by a bank competing in Bertrand competition. He argues that if the additional costs of raising equity are higher relative than other sources of raising money, then the bank may restrain from further lending and prefer to invest in marketable securities rather than in loans.

In addition to this cost effect, the introduction of a binding capital requirement regulation can have a second effect that has not yet been considered in banking literature. In fact, a binding capital requirement changes the sequence in which the strategic decisions are made, since it temporarily constraints the bank’s lending activities.³ This idea goes back to Edgeworth (1988) who emphasizes that

¹ See e.g. Furlong and Keeley (1989).
² VanHoose (2008)
³ This is in line with Brander and Lewis (1986) who analyze the strategic impact of leverage decisions on output decisions. They argue that increases in a firm’s leverage enhance the output level of the firm in a Cournot Oligopoly with random demand. In contrast, we concentrate on the effects of a strategic capital choice in a deterministic Bertrand competition and examine the impact of a capital commitment on the fierceness of the price competition.
due to exogenous capacity constraints, Bertrand oligopolists may not be able to serve the whole market demand and therefore would not undercut each other to competitive prices. Kreps and Scheinkman (1983) generalized this idea for an endogenous capacity choice. In their two stage model, the oligopolists first compete in capacities, followed by a competition in prices, which is strictly constrained by the prior capacity decision. Kreps and Scheinkman (1983) conclude that when firms precommit to a certain capacity of production before price competition takes place, the capacity and prices chosen in equilibrium are identical to the Cournot equilibrium. As a result, banks competing in a homogenous Bertrand competition can generate Cournot profits instead of zero profits. The question that arises, is can such a rigid capacity constraint be applied to the case of lending competition among banks. Freixas and Rochet (1997) even state that a capacity constraint may not be feasible as a starting point for a theoretical analysis in the context of banking.

However, Freixas and Rochet (1997) may have overlooked that binding regulatory capital requirements can affect the nature of strategic competition among banks. In particular, when short term recapitalization is costly, capital requirements temporarily constrain the bank’s lending activities and thereby soften the price competition as already mentioned by Gehrig (1995). In a static framework, a binding capital requirement regulation can, therefore, be abused to transform the Bertrand competition into a sequential game with a Cournot outcome.

In a first stage, the capital regulated banks decide on their refunding structure consisting of equity and deposits. In the second stage, the price competition takes place while the bank’s ability to satisfy the demand resulting from the pricing decision is conditioned by the raised amount of equity and the capital requirement regulation. As recapitalization is assumed to be costly, the equity decision in the first stage is an imperfect commitment to capacity for bank loans. Notwithstanding the model of Kreps and Scheinkman (1983), the oligopolistic banks are able to extend their capacities in the second stage, but at additional costs. Such a flexible capacity constrained in Bertrand competition was already discussed by Güth (1995) and Maggi (1996) for differentiated products markets. The assumption of product differentiation thereby avoids one of the main shortcomings, for which the model of Kreps and Scheinkman (1983) has been often criticized. In their model, they assume that the firms compete in a homogenous products market, which necessitates the definition of a specific rationing rule on the customers that determines the specific demand that is addressed to each supplier. Yet, the derived results are not robust against changes in the specific rationing rule as it is formally proven by Davidson and Deneckere (1986). Assuming product differentiation is, thus, not only reasonable in a relationship bank lending context, but provides the means to well-define the demand of each firm in the second stage for any price pair and therefore avoids the dependency of the results on a specific rationing rule.

Güth (1995) and Maggi (1996) both argue that capacity constrained Bertrand competition yields a Cournot outcome for sufficiently high additional costs of the capacity extension. Applying the Maggi (1996) model to a capital regulated

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4 Following Berger et.al. (1995), we define capital requirements to be binding if the capital ratio in the presence of regulatory capital requirements is greater than the bank’s market capital requirement. Our crucial assumption in our model is thus, that regulatory capital requirements constrain a significant portion of the banks in their optimal decision on the refinancing structure.
market for loans, we analyze the effects of a capital requirement regulation on the strategic behavior of oligopolistic banks. It is shown that if costs of recapitalization are above an identified threshold, the banks would no longer have an incentive to undercut each other in the second stage price competition. The precommitment to equity, thus, changes the Bertrand price competition to a Cournot-Nash equilibrium. A comparative static analysis shows that an increase in capital requirement decreases the threshold that makes a first stage capacity decision binding. In other words, the higher the capital requirement is, the more binding a precommitment to a credit capacity made by a certain choice of capital level.

Similar to the discussed direct cost effect, the described collusive effect tends to reduce the total lending resulting from less competitive prices. However, in contrast to the cost effect, the collusive effect is likely to generate positive profits. The generated profits in turn increase the bank’s buffer against credit default risk on one hand and provide incentives against excessive risk taking by enhancing the “charter value” of the bank. An increased charter value reflects the higher expected future profits that would be lost in case of bankruptcy. The increased expected future losses in turn reduce the incentives to exploit the option value of the limited liability created by the governmental safety net, i.e. a deposit insurance. This stabilizing effect has been discussed e.g. by Keeley (1990) and Chan, Greenbaum and Thakor (1992). Based on this charter value argumentation, Allen and Gale (2004) argue that regulators face a trade-off between increasing competition and reducing stability. In the line of the charter value argumentation the identified collusive effect of the capital regulation thus further enhances stability but to the cost of decreased competition and a reduction in lending.

The rest of the paper is organized as follows. Section 2 describes a model of capacity-price competition adapted to the banking market for loans. Section 3 analyzes the two-stage game and identifies Nash equilibria. In section 4 we discuss the policy implications of our findings.

2. The Model

For simplicity we concentrate on the case of two banks, \( j = 1,2 \); \( i \neq j \) even though the results could be generalized to the case for an arbitrary number of banks.  

First, the banks decide to raise a certain proportion of equity in their refinancing structure. The amount of equity then determines, based on the minimum capital requirement regulation, a capacity to provide loans to borrowers of a certain risk class. In the second stage, the banks compete in prices that turn into loan quantities demanded by the borrowers.

2.1. The Demand for Loans

The borrowers’ demand for loans is represented based on the model of Bowley (1924) by the linear inverse demand function \( p_i(l_i, l_j) = a - bl_i - dl_j \).  

\[ p_i \]

We call this effect collusive in the sense of spontaneous tacit collusion that results from a non-cooperative Nash equilibrium and does not require any coordination among the competitors.

\[ p_i \]

See Boccard and Wauthy (2000) for a generalization of Kreps and Scheinkman (1983) to the oligopoly case with \( n \geq 2 \) competitors.

\[ p_i \]

Note that \( a, b, d \) are strictly positive and \( a > b > d \geq 1 \). If \( d \) would be negative, the goods were complements and if \( d = 0 \) the two goods would be independent in demand. If \( d = b \) the loans from each banks are perfect substitutes. The greater \( d \) the closer the varieties become substitutes.
represents the credit interest rate of bank i and $L_i$ the total lending of bank i to the borrowers. Note that $a, b, d$ are strictly positive and $a > b > d \geq 1$. If $d$ were negative, the goods would be complements and if $d = 0$ the two goods would be independent in demand. As $d \to b$ the loans become perfect substitutes. In our analysis, we will concentrate on the more general case of a heterogeneous market in order to avoid the selection of a specific customer rationing rule. Yet, since one can argue that loans are more or less homogenous goods, we will also consider the particular case $d \to b$ where loans become homogenous goods in our analysis.

Provided that each bank only chooses prices or quantities, which secure strictly positive profits ($b p_i - d p_i \leq a (b - d)$) and ($d L_i \leq a - b L_i$) the inverse demand function can be reversed\(^{10}\) to obtain the direct demand curve.

$$L_i = a / (b + d) - b p_i / (b^2 - d^2) + d p_i / (b^2 - d^2) \quad (1)$$

2.2. The Banks

For simplicity it is assumed that banks invest only in loans demanded by the representative borrowers of one risk class. Hence, all assets have identical external ratings. In line with the Standard Approach to regulatory capital requirement under the current Basel II banking regulation accord, where unrated assets are accounted with 100\%, we assume a risk-weight equal to unity. The intuition is that most corporate borrowers, especially small and medium sized firms, do not have external ratings. The loans are financed by deposits (D) and equity (E). This implies the balance sheet constraint:

$$L \leq E + D. \quad (2)$$

Depositors are fully insured against default at a premium normalized to zero. Hence, depositors are insensitive to the bank’s exposure to risk and they are willing to supply any amount of deposits at a deposit rate $r_D$. This allows us to isolate the effect of competition in the loan market. We further assume that a profit maximizing bank would voluntarily raise the equity proportion, which balances the opportunity costs of raising capital and the marginal expected costs of bankruptcy. We define such a market capital requirement as the capital ratio $\delta_{MR}$ that maximizes the value of the bank in the absence of a binding capital requirement but in the presence of a regulatory safety net that can be exploited by the bank.\(^{11}\) A binding capital requirement, hence a capital requirement above the voluntarily raised capital rate, translates into the condition that the opportunity costs of equity prevail the cost of deposit financing from the perspective of the individual bank.\(^{12}\) This local comparative advantage of deposit financing over equity financing is reflected in costs of equity above the deposit rate: \(^{13}\) $r_c = r_D + \epsilon$. 

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8 The heterogeneity could emerge from the reputation of the bank, the specific service offered to the borrower or relationship banking combined with switching costs to the borrower.

9 This implies a certain restriction on the parameters of the model that is discussed in 2.2.

10 The inversion is only allowed under the assumption that both firms always satisfy their demand, otherwise, the quantity demanded of one firm is rather a function of the residual demand left by the rationing opponent. See Boccard and Wauthy (1998) for an analysis of the robustness of the Güth-Maggi results when firms are allowed to ration their customers. Further, these equations are only valid for $b \neq d$.

11 See Berger, Herring and Siegö (1995) for an extensive discussion of market capital requirements.

12 Alternatively, we could have assumed that bank’s refund with long term deposits, as a part of TIER II capital, which is also accounted for bank’s minimum capital requirement under the cur-
Additionally, we assume an important imperfection concerning banks’ equity financing: In the first stage banks can raise equity capital at \( r_E \) in an unrestricted manner. However, recapitalization in the second stage (e.g. seasoned equity offerings) is only possible at higher costs\(^{14}\). These costs represent the dilution costs that a bank might face when organizing an urgent equity raise. The additional costs of recapitalization are reflected in \( \theta > 0 \) such that recapitalization would produce costs equal to \( \theta = r_E + \theta \).\(^{13}\) To secure strictly positive profits we assume \( a > r_E \).

A bank’s capacity to provide loans is indirectly defined by the exogenous regulatory capital obligation \( \delta \) that requires banks to refinance their assets by an amount of equity equal to: \( E_i \geq \delta L_i \). The precommitment to a specific amount of equity therefore, gives an imperfect capacity constraint defined by:

\[
K_i = E_i / \delta
\]

By normalizing operating costs to zero, the long run variable costs of a bank become:

\[
VC = r_D D + r_E E + r_E \delta (\max(L - K, 0)).
\]

To minimize costs, banks try to increase the proportion of deposit refunding i.e., reduce the proportion of equity. The balance sheet constraint (2), thus, becomes binding and we can express \( D \) as a function of loans and equity: \( D = L - E \). After substituting the balance sheet constraint as well as the capacity constraint (3) into equation (4), mathematical simplification yields:

\[
VC(L_i) = \begin{cases} 
  r_E L_i & \text{if} \quad L_i \leq E_i, \\
  r_D L_i + cE_i & \text{if} \quad E_i < L_i \leq E_i / \delta \\
  r_D L_i + c\delta L_i + \theta (\delta L_i - E_i) & \text{if} \quad L_i > E_i / \delta
\end{cases}
\]

As long as the bank’s total lending is (smaller or) equal to the bank’s equity, the bank will refund its assets solely with equity. If the bank wants to lend more than its equity stock, it needs to raise additional debt in the form of deposits to refund its assets. Supplying loans up to the regulatory capital constraint then generates the costs of the deposit rate plus the premium per equity unit. For lending above the capacity, the recapitalization costs arise only for additionally raised capital, which is reflected in the subtraction of equity in the last term of the third piece of the function. Figure 1 illustrates the cost structure for such a capital constrained bank.

\(^{13}\) The assumption that equity funding is more costly than deposit funding is standard in the literature. See \{Diamond 2000 #152\}, \{Hellmann 2000 #135\} and \{Rime 2005 #185\}.

\(^{14}\) The cost of issuing new equity may be quite substantial. Besides the costs of an additional general meeting including the preparation of the registration statement and prospectus, registration fees, printing and mailing costs, underwriting fees or the costs of an issue being underpriced as argued by Berger, Herring and Szegö (1995). In addition an extension of equity in the short term may be interpreted as a signal of financial distress combined with reputational losses. See e.g. Myers and Majluf (1984) and Repullo and Suarez (2008) for an application to banking.

\(^{15}\) A rigid capacity constraint as discussed by Kreps und Scheinkman (1983) would imply \( \theta \to \infty \), such that a supply of loans beyond \( L_i > E_i / \delta \) is not possible. Yet, an imperfect capacity commitment is reflected by finite costs of “production” above capacity \( \theta > 0 \).
Figure 1: The cost of refunding assets for lending above the capacity

Note that during the second stage of the Bertrand competition, the costs of the first stage equity choice are sunk. The marginal costs determining the second stage price decision therefore can be summarized as follows, given the capacity choice of the first stage.

**Lemma 1:** The marginal costs function of a capital constrained bank is the piecewise defined function:

$$MC(L_i) = \begin{cases} 
0 & \text{if } L_i \leq E_i, \\
r_D & \text{if } E_i < L_i \leq E_i/\delta \\
 r_D + (c + \theta)\delta & \text{if } L_i > E_i/\delta 
\end{cases}$$

Like in Dixit and Norman (1980) it will be assumed in our analysis that a firm is willing to meet any level of demand beyond its installed capacity provided that the price is above the additional costs of extending the capacity, thus rationing is excluded in our analysis. For simplicity, we further assume that the principal-agent problem between the management and the bank owner is solved and the manager responsible for the pricing decision acts in the best interest of the bank (equity) owners.

2.3. **Benchmark**

The capital requirement has two effects that both reduce the total lending amount, while increasing the credit interest rate. In order to distinguish the two effects it is useful to compare the results of our analysis to a Bertrand competition without precommitment to a lending capacity. The rise in the interest rate due to the cost effect can then be identified by comparing the interest rate without any capital requirement (very high leverage) with the Bertrand prices to regulated equity ratio. The collusive effect is then found by the comparison of the Bertrand price with higher marginal costs to the prices and referring profits of the Cournot outcome.
Unregulated Bertrand Duopoly Benchmark

In an unregulated environment, the banks facing the described cost structure would maximize their profits by raising the market capital requirement \( \delta^{MR} \). At this rate the marginal costs of deposit funding equals the marginal cost of capital funding. This assumption aims to reflect the higher probability of bankruptcy for high leveraged banks and the referred expected costs of financial distress. Below the market capital requirement rate the banks would like to raise more capital in order to reduce the expected costs of financial distress. Above the rate, the banks find it profitable to reduce the proportion of equity in order to reduce their costs of refunding. The profit maximizing bank would, thus, try to maximize the following function:

\[
\Pi_{iu} = p_i L_i(p_i, p_j) - r_0 (L_i(p_i, p_j) - E_i) - r_0 E_i = (p_i - r_0) L_i(p_i, p_j)
\]  

(6)

Each bank chooses the credit price as a best response function \( R_{iu}(p_j) \) to the other bank’s loan price

\[
R_{iu}(p_j) = \frac{a(b-d) + br_0 + dp_j}{2b}.
\]  

(7)

The symmetric Bertrand equilibrium price is, therefore:

\[
p^*_u = \frac{a(b-d) + br_0}{2b-d}.
\]  

(8)

and the corresponding lending quantity is

\[
L^*_u = \frac{(a-r_0)b}{(b+d)(2b-d)}.
\]  

(9)

In an unregulated differentiated Bertrand oligopoly the opponents each gain the following profits:

\[
\Pi^*_u = \frac{b(b-d)(a-r_0)^2}{(b+d)(2b-d)^2}.
\]  

(10)

For the particular case, where \( d \to b \) we get \( p^*_u = r_0 \) and \( L^*_u = \frac{a-r_0}{2b} \) resulting in zero profits, which is the original Bertrand result.

Regulated Bertrand competition with Simultaneous Pricing and Equity Decision

To pattern a capital regulated Bertrand competition without the opportunity to commit strategically to a certain capacity; we assume for the moment that recapitalization is not costly. When \( \theta \to 0 \), the banks can adjust their equity continuously to their price choice, such that they participate in unconditioned Bertrand competition. The only constraint banks face is the minimum capital requirement. In such a case the banks will try to maximize the following profit function:

\[
\Pi_{ir} = (p_i - r_0 - \delta c)L_i(p_i, p_j).
\]  

(11)

The reaction function for each bank is:

\[
R_{ir}(p_j) = \frac{a(b-d) + br_0 + \delta c + dp_j}{2b}.
\]  

(12)

The intersection of the price reaction curves then gives the symmetric Bertrand equilibrium prices and quantities.
Strategic Effects of Capital Requirements in Bertrand Competition

\[ p^*_c = \frac{a(b - d) + b(r_0 + \xi c)}{2b - d} \quad (13) \]

and

\[ l^*_c = \frac{(a - (r_0 + \xi c))b}{(b + d)(2b - d)}, \quad (14) \]

respectively.

The Bertrand prices and quantities result in the profits of:

\[ \Pi^*_c = \frac{b(b - d)(a - (r_0 + \xi c))^2}{(b + d)(2b - d)^3}. \quad (15) \]

Comparing (8) and (13) shows that the cost effect increases the price by \( \frac{\xi c}{2b - d} \). The comparison of (9) and (14) confirms the resulting decrease in the demand that is addressed to the oligopoly banks \( \left( \frac{\xi c}{(b + d)(2b - d)} \right) \). It is easy to verify, that the cost effect matches the increase in marginal costs due to the capital requirement \( \xi c \).

Here again, when \( d \to b \) we get the original Bertrand result that the price equals the marginal costs \( p^*_0 = r_0 + \xi c \) and the demand equals \( l^*_c = \frac{a - (r_0 + \xi c)}{2b} \) which results in zero profits.

3. The Two-Stage Game

In contrast to the benchmark, we now assume that \( \xi \) is significantly greater than zero. This is the essential assumption that allows for strategic capacity precommitment. The oligopolistic banks choose a binding constraint on the loan supply and compete in a second stage on the prices for loans. We solve the two-stage game by backwards induction and, therefore, we first analyze the best responses of each bank in the price stage and the corresponding payoffs.

3.1. The Second Stage Price Game

In the second stage, the equity decision has already taken place. The respective capacities \( K_i = E_i/\xi \), defined by the first stage capacity decision, thus, exogenously condition the pricing decisions. The imperfect capacity constrained banks try to maximize their profits by the optimal pricing decision as a function of the opponent’s price choice and the installed capacities. The equilibrium prices of both duopolists then determine the equilibrium demand, which can be up to, or above the respective capacities. Thereby, the banks (by assumption) always satisfy the demand addressed to them.

Hence, the bank has the option to choose a relatively high price resulting in a demand up to the banks capacity or a low price resulting in a demand above the capacity. Applying Lemma 1 we define the total profit function as follows:

\[ \Pi_i(p_i, p_j) = \begin{cases} 
  p_i l_i - r_0 E_i & \text{if } l_i \leq E_i, \\
  (p_i - r_0) l_i - c E_i & \text{if } E_i < l_i \leq E_i/\xi \\
  (p_i - r_0 - \xi(c + \xi) l_i + \xi E_i & \text{if } l_i > E_i/\xi 
\end{cases} \quad (16) \]

The positive last term in the third piece of the profit function reflects the saved costs from raising capital in the first stage. Note that the profit function is not differentiable at the points \( l_i = E_i \) and \( l_i = E_i/\xi \).

In the two stage game the price response function of each bank is a function of the opponent’s price with different segments determined by the capacity
levels chosen in the first round and the parameters of the model. Hence, the best reaction function is characterized by different branches. Since the capacity levels are fixed, the branch with the best response depends on the price chosen by the opponent.

**Lemma 2:** In the second stage price game, the best reaction function $R^i(p_j)$ consists of six branches that depend on the model parameters and the opponent’s price choice:

\[
R^i_0(p_j) = p^i_{l,SE} \quad \text{for} \quad p_j \leq r_D, \quad R^i_1(p_j) = p^i_l \quad \text{for} \quad p_j > p^i_l, \\
R^i_2(p_j) = p^i_H \quad \text{for} \quad p_j \leq p^i_l, \quad R^i_3(p_j) = p^i_{l,a} = 0 \quad \text{for} \quad p_j > p^i_{l,a}, \\
R^i_4(p_j) = p^i_{l,a} \quad \text{for} \quad p^i_{l,a} < p_j \leq p^i_H, \quad R^i_5(p_j) = p^i_{l,a} = 0 \quad \text{for} \quad p_j > p^i_{l,a}.
\]

**Proof:** If bank $j$ chooses a very low price, even below the refunding costs of deposits $p_j < r_D$, the bank $i$ faces a very low demand. The best reaction is to choose a price:

\[
p^i_{l,SE} = \frac{(b-d)a + dp_j}{2b}.
\]

Denote this best reaction as branch 0 of the reaction function of bank $i$.

A relatively low price\footnote{The opponent’s price is $r_D \leq p_j < p^i_l$ where $p^i_l$ is defined implicitly by the point where branch 1 equals branch 2. The residual demand for bank $i$ was so low that it would not be profitable to produce up to the capacity level and to leave certain capacity unused.} of the opponent induces a high demand for loans from the opponent bank and only few loans demanded from bank $i$ such that the demand is smaller than the capacity. Therefore, the optimal response $p^i_l$ is the Bertrand price for producing below the capacity constraint:

\[
p^i_l = \frac{(b-d)a + dp_j + br_D}{2b}.
\]

This part of the best reaction function is denoted as branch 1 of the reaction function.

As the rival bank increases the price for loans, the demand addressed to bank $i$ increases until the demand equals capacity. For such a price of the opponent, the optimal response function is defined by the capacity clearing price:

\[
p^i_H = \frac{(b-d)a + dp_j + br_D}{2b}.
\]

Define this as branch 2 of the reaction function.

If the opponent raises the price even further, such that the demand for loans from bank $i$ becomes very high\footnote{The opponent’s price is $p_j > p^i_H$, which is defined implicitly by the point where branch 2 equals branch 3. Here, bank $i$ faces high residual demand that makes it even profitable to expand the capital at higher costs, thereby allowing producing above the capacity.} and above the installed capacity, the optimal response is to expand capacity. The best reaction is a Bertrand price taking $\theta$ into account:

\[
p^i_{l,a} = \frac{(b-d)a + dp_j + b(r_D + (c+\theta)\delta)}{2b}.
\]

This branch we denote as branch 3 of the reaction function.

A further increase in the opponent’s price beats down the opponent’s demand to zero. In this case the non-negativity constraints of the optimization...
problem become binding. Therefore, the best response of bank \( i \) facing a very high price of its opponent and zero demand for the opponent’s loans, responds optimally with the equality version of the inequality constraint:

\[
p_{i}^{L} = (b - d)a - \frac{b}{d}p_{j}.
\]

This branch is denoted as branch 4.

An additional increase in the price for loans by bank \( j \) raises the loans demanded from bank \( i \) and, thus, allows for further increases in \( p_{i} \). Yet, Bank \( i \) would only increase its price up to a demand equal to the monopoly output with marginal cost \((c + \theta)\delta\). For any further increase in the opponent’s price, it would not be optimal to further increase \( p_{i} \) since the monopoly price is by definition, the profit maximizing price. The best response function thus becomes vertical, and no derivation from the monopoly price would be profitable.

\[
p_{i}^{M} = \frac{1}{2}(a + r_{d} + \delta(c + \theta)).
\]

Define the monopoly price section of the reaction function as branch 5.

The entire best reaction function of bank \( i \) denoted as \( R^{i}(p_{j}) \) is depicted in Figure 2.

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**Proposition 1**: If \( d \neq b \) there exists a unique price vector \( p^{*} = (p_{i}^{∗}, p_{j}^{∗}) \) that qualifies as a Nash equilibrium of the second stage. This equilibrium is defined by the intersection of either branch 1, 2 or 3.

**Proof**: Branch 0 is not a feasible candidate for a Nash equilibrium, since each player can strictly increase his profits of the two stage game by increasing the leverage. Similarly, branches four and five are not feasible candidates for Nash equilibria, since the firm earning zero profits could strictly increase its profits by offer-
ing a lower credit price and, thus, would be strictly better off by deviating from the high price decision. Therefore, only branches one, two, and three are reasonable candidates to describe a Nash equilibrium price choice in the second stage. The reaction function that consists solely of branches 1, 2 and 3 is continuous by definition and monotone. For branches 1 and 3, the slope equals \( d/2b \) and for branch 2 the slope is \( d/b \) with \( b > d > 0 \). Hence, the slope of the reaction functions is between 0 and 1. For the particular case where \( b = d \) the slope of branch 2 would be \( \frac{\partial R(p_i)}{\partial p_j} = 1 \), such that the intersection of the reaction functions gives a continuum of equilibria as illustrated in Figure 3.

![Figure 3: Continuum of equilibria for perfect homogenous loans](image)

Yet, even a slight degree of differentiation results in a unique equilibrium solution. Since this is true for both agents, whenever \( d \neq b \) there must be a unique intersection of the best response functions that defines a pure strategy equilibrium in the price subgame for any pair of capacities installed in the first stage. Since the equilibrium is determined by the possible intersection of 3 branches there are three possible second stage equilibrium types in our symmetric model. Proposition 1 is summarized in Figure 4.

![Figure 4: Branches of the reaction function](image)

If we know at which branch the reaction functions intersect, it is possible to determine the first stage payoffs as a function of the respective capacity choices. For these payoffs there are 3 different possible results which depend on the parameters of the model e.g. the level of additional costs associated with an expansion of the capacity.

In general, the optimal price choice is implicitly defined with the point where marginal benefit of higher demand generated by lowering the price equals the marginal costs of expanding the supply of loans. For very low additional costs \( (\theta \to 0) \) the capacity choice in the first stage would not be binding and the equilibrium of the two stage game would be the Bertrand equilibrium in quantities and prices as calculated in the benchmark.
If the costs of adjusting equity are significantly different from zero \( (\theta > 0) \) the highest price that can be sustained as equilibrium is, thus:

\[
p^\theta = \frac{\left( (b - d) a + b (r_0 + (c + \theta) \delta) \right)}{(2b - d)}. \tag{23}
\]

With an increasing cost factor \( \theta \) the highest attainable price, the symmetric Bertrand equilibrium price with recapitalization costs, also increases. If the recapitalization costs are equal to or higher than the critical value \( \theta^H \) the highest attainable Bertrand equilibrium price (with recapitalization costs) equals to the Cournot price for a supply within the capacity constraint. This critical value is implicitly defined at the point where the Bertrand best price response with marginal costs including the cost or recapitalization \( r_0 + (c + \theta) \delta \) equals the Cournot best response price with marginal costs equal to \( r_0 + c\delta \):

\[
\theta^H = \frac{d^2 (a - r_0 - c\delta)}{(2b - d b \delta)}. \tag{24}
\]

Hence, if the short term expansion costs are above this critical level, it would not be profitable to provide loans above the capacity. In other words, the capacity commitment becomes strictly binding. As argued by Maggi (1996) \( \theta \) determines, thus, the irrevocability of the precommitment. The higher the \( \theta \), the more effective the capacity commitment device is. The interesting implication for the regulation of banks is the impact of the regulatory requirement on the critical level of recapitalization costs.

**Lemma 3:** A higher minimum capital requirement ratio \( \delta \) reduces \( \theta^H \).

**Proof:** Differentiating the critical cost level with respect to the capital requirement gives:

\[
\frac{\partial \theta^H}{\partial \delta} = -\frac{d^2 (a - r_0)}{(2b - d) b \delta^2} \tag{25}
\]

Since \( a > r_0 \) and \( b > d^{18} \), the partial derivatives clearly indicate a negative relationship between the capital requirement and the critical recapitalization cost level that changes the Bertrand competition into Cournot equilibrium outcomes.

### 3.2. The First Stage Capacity Game

Anticipating the best reaction of the second stage, bank \( i \) will choose, in the first stage, an optimal level of equity that results in a capacity equal to the equilibrium demand of the second stage. It would not be profit enhancing to deviate from a capacity choice that equals demand in the second stage since equity is expensive. A profit-maximizing bank therefore takes no more equity than it is required by regulation to satisfy the loans demanded in the second stage. Reducing the equity to the amount the required loans demand, in the second stage, would not affect the demand and prices but saves costs. Similarly, a capacity below the anticipated equilibrium demand would not be optimal because raising additional equity in the first stage would save costs of recapitalization in the second stage without influencing the equilibrium prices or demand in the second stage. Hence, in the first stage

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18 As \( d \) approaches \( b \) (the goods are nearly homogenous) this value approaches \( -\frac{(a - r_0)}{\theta^H} \), which is clearly negative.
equilibrium, a profit-maximizing bank will raise the exact amount of equity that satisfies the equilibrium demand in the second stage $E_i/\delta = L_i$.

Applying this argumentation to both agents it becomes clear, that only the interceptions of branches 2 of the best response function, qualify for a Nash-equilibrium of the whole game. This interception defines a capacity clearing equilibrium where the prices chosen in the second stage guarantee a demand that just clears the capacity defined by the equity raised in the first stage.

**Lemma 4:** When $0 < \theta < \theta^H$, the equilibrium equity choice is characterized by $E_i = L_i(p^\delta_0)\delta$ and prices equal to $p^\delta_0$.

**Proof:** For intermediate recapitalization costs, the banks would still have incentives to revoke the capacity commitment. When installing Cournot capacities they would have the incentives to undercut the prices of the opponent. Thus, the Nash equilibrium for moderate costs is exactly the capacity that allows to meet the demanded quantity of Bertrand prices for marginal costs equal to $(\bar{r}_D + \delta(c + \theta))$, which was defined above as $p^\delta_0$. For a price that exactly equals the marginal costs with recapitalization, the banks have no incentives to undercut the opponent in prices and thus, no bank will deviate from the strategy. Anticipating this price pair the optimal capacity chosen is, thus, $E_i/\delta = K_i = L_i(p^\delta_0)^{19}$.

In the second stage, the optimal prices chosen then generate a demand that exactly clears the capacity. Without any excess demand above capacity, no recapitalization costs occur to the bank. Hence, the imperfect capacity commitment in the first stage allows banks to set their prices higher (by $\delta\theta$) than the marginal costs that actually occur to the bank and therefore raise profits.

Now consider the case where costs are $\theta \geq \theta^H$ to the critical level. Given that in equilibrium the capacity decision will equal the anticipated demand, firms know that the optimal price in the second stage will be the intersection of branch two of both agents. The symmetric anticipated equilibrium price will be defined by

$$p_i^{\theta \geq \theta^H} = a - E_i/\delta - E_j/\delta.$$  

Both banks simultaneously maximize the objective function with respect to the constraint of the optimal equity level.

$$\Pi_i = \left(p_i^{\theta \geq \theta^H} - r_0\right) E_i/\delta - cE_i$$  

The First Order Condition then gives the optimal equity choice

$$E_i^* = \frac{a - (r_D + \delta c)}{(2b + d)\delta}$$  

The second stage best reply to the installed capacity is then the Cournot prices:

$$p^* = \frac{(ab + (b + d)(r_D + \delta c))}{2b + d}$$  

The optimal symmetric capacity and the according prices result in the following symmetric profits:

---

19 Formally the optimal capacity is the quantity equal to the demand for the price pair that maximizes $\max(p_i - r_0 + c\delta)D(p_i, p_j)$ subject to $p_j = R_j(p_i, E_i/\delta)$ and $p^\delta_0 \leq p^* \leq p^\delta_0$. 
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Lemma 5: For recapitalization costs $\theta \geq \theta^H$ the equilibrium of the two stage game is characterized by the Cournot equilibrium prices and quantities.

Proof: If banks would compete in a Cournot competition on quantities with symmetric costs, they would select their optimal output as a function of the other bank’s optimal loan supply. Consequently, bank $i$ maximizes the profit function with respect to the optimal loan amount $\pi_i = (p_i(L_i, L_j) - (r_0 + \delta c))L_i$. The First Order Condition for a Maximum determines the best response function of bank $i$ to any amount of loans supplied by bank $j$ that is:

$$R^C(L_j) = \frac{a - dL_j - (r_0 + \delta c)}{b}.$$  \hfill (31)

The symmetric Cournot equilibrium quantity is given by the intersections of the reaction functions. The Cournot quantity is

$$L^C = \frac{a - (r_0 + \delta c)}{2b + d}$$  \hfill (32)

and the respective Cournot equilibrium price is

$$p^C = \frac{ab + (b + d)(r_0 + \delta c)}{2b + d}.$$  \hfill (33)

The symmetric Cournot equilibrium payoff is:

$$\pi^C = \frac{b(a - (r_0 + \delta c))^2}{(2b + d)^2}.$$  \hfill (34)

Substituting (2) into (28) shows that (28) equals (32). Obviously, also (29) and (30) equal each with (33)and (34).

4. Policy Implications

A minimum capital requirement can turn a Bertrand competition in the loans market into a Cournot equilibrium resulting in lower credit output, higher credit prices and higher profits for oligopolistic banks. In contradiction to the literature arguing that regulatory capital requirements increase the cost of producing loans and thereby reducing the profits of the banks, this paper shows that the regulation can also increase the bank’s profits, and thus, the charter value of the bank in excess of the increased cost of equity. Following the argumentation of the charter value hypothesis, such increased profits reduce the bank’s risk taking incentives. This decrease in the bank’s exposure to risk may enhance the stability in banking, however, to the social costs of an additional reduction in lending. These two effects may alter the decision on the optimal requirement rate by the regulator in a stage zero. In such a zero stage, the regulator has to balance, on one hand, the aim of stabilizing the banking sector against the aim of enhancing competition and efficiency.

Proposition 2: There exists a critical value of the minimum capital requirement $\delta^H$. Such that, whenever $\delta \geq \delta^H$, ceteris paribus, then $L_i^c < L_i^r < L_i^*$ result-
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ing from \( p_i^c = p_i^c > p_i^r > p_i^u \). This results in equilibrium profits equal to \( \Pi_i^r = \Pi_i^u \geq \Pi_i^c \).

**Proof:** Holding the recapitalization costs fixed, equation (24) implicitly defines \( \tilde{\delta} \) which is the critical level of a minimum capital requirement that may be chosen by the regulator at a stage zero. This value gives the critical regulatory capital requirement that induces collusive behavior among price competing banks. A requirement equal to or above this level, allows banks to gain Cournot profits by further raising prices and thereby decreasing lending. Comparing the benchmarks with Lemma 5 then gives the proof of Proposition 2.

First, compare the total lending of bank \( i \) in equilibrium with and without a collusive effect (The regulated Bertrand benchmark): \( l^c < l^r \), the difference in the equilibrium total lending amounts to: \( \frac{d(a - \tau_0 - 5c)}{(b + d)(4b^2 - d^2)} \), which is clearly negative for \( a > \tau_0 \) and \( b > d \). It is also easy to show that \( p^c > p_i^r \), since the difference is \( \frac{d(a - (\tau_0 + 5c))}{(b + d)(4b^2 - d^2)} \), which is strictly positive. The Cournot profits are greater than the regulated Bertrand profit, which can be seen from the positive difference: \( \frac{b^2 d^2 (a - \tau_0 - 5c)^2}{(2a + d)^2 (2a - d)^2 (b + d)} \). While in the differentiated Bertrand competition the banks can also earn profits (which are strictly lower than the Cournot profits), these profits decline, the less heterogeneous the banks are. As \( d \to b \), the Bertrand profits in the regulated and unregulated case both approach to zero, while the capacity constrained Cournot profits approach the strictly positive profits equal to \( \frac{b^2 (a - \tau_0 - 5c)^2}{b} \).

When raising the minimum capital requirement for banks, the regulator, thus, not only needs to balance the cost effect but also the identified collusive effect against enhanced stability, to find an optimal regulatory capital requirement.

5. Discussion

The analysis has shown that a binding regulatory capital requirement reduces the incentives of competing banks to undercut in prices. This collusive effect results from the strategic complementarity of prices. The intensification of capital requirements restricts the market volume and thus results in higher interest rates. The banks have fewer incentives to engage in fierce Bertrand competition. The intensity of the price competition in the second stage thereby depends on the cost of recapitalization and the level of capital requirement. The higher the costs of recapitalization \( \tilde{\delta} \), the higher the mark-up on the Bertrand price and the lower the loans demanded in the second stage. This holds until the costs of recapitalization reach the critical level \( p^c_\tilde{\delta} = p^c \). For any costs equal or above the critical level it will be optimal to install Cournot capacities in the first stage and ask for Cournot prices in the second stage, which maximizes the non-cooperative equilibrium profits. This paper therefore offers a justification for the usage of the Cournot model in the context of banking.

In contrast to the literature on the impact of capital regulation, this analysis suggests that banks in fierce Bertrand competition may benefit from the introduction of a binding capital constraint due to regulatory capital requirements. This suggests a certain demand for regulation on the side of banks.

From the point of view of the regulator this result has ambiguous implications. On one hand, the regulator might prefer Bertrand competition among banks with lower prices and higher loan supply for the macroeconomic benefits of efficiency as described for example by Smith (1998). On the other hand, the high-
The strategic effects of capital requirements in Bertrand competition may further stabilize the banking sector. Assuming that the regulator aims at an optimal trade-off between incentives for competitiveness of bank services and the solvency and stability of the industry, the effects on the strategic interaction among banks may be minor compared to other incentive effects of the requirement that influence the stability of banks. Nevertheless, they should be taken into account in the design of prudential banking regulation. The collusive effect should also be considered, especially in the discussion of an increase of capital requirements. Since banking regulation also tries to reduce the bank’s exposure to risk, it will be important to analyze the capital constraint effects on a bank’s portfolio risk decision.

6. References


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