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Moment Targeted Structural Innovations

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Abstract

Structural innovations are typically hidden and often identified by means of a-priori economic reasoning. Under multivariate Gaussian model innovations there is no loss measure available to distinguish between particular identifying restrictions and rotations thereof. Based on a non Gaussian copula distribution framework, this paper proposes a loss statistic that can be used to discriminate between alternative identifying assumptions on the basis of higher order moment characteristics. The merits of *Moment Targeted Structural Innovations* are illustrated by means of Monte Carlo simulations and real data applications to bivariate systems of US stock prices and total factor productivity and of international breakeven inflation rates.

Keywords: Structural innovations, copula distribution, identifying assumptions.

JEL Classification: C32, G15

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1 Introduction

Vector autoregressive (VAR) models have generated a widely adopted and rather flexible toolkit to in-sample investigate the dynamic relations of socioeconomic time series and out-of-sample forecasting (Lütkepohl 2005). Regarding the former field of theoretical and applied econometrics, VAR models are descriptive in the sense that they offer a profound understanding of reduced form features of the data. When it comes to a structural perspective on the linkage of economic variables, structural VAR models (Amisano and Giannini 1997) have to borrow from economic theory, statistical ad-hoc decompositions of reduced form covariance matrices, or simply from (economic) a-priori reasoning. Opposite to traditional simultaneous equation modeling, structural VAR models are mostly characterized by using just identifying restrictions. Generally, distinct settings of identifying restrictions might compete for the understanding of simultaneous economic relationships. For instance, in the multivariate Gaussian framework, upper and lower triangular decompositions of reduced form covariance matrices are observationally equivalent although they carry markedly distinct implications for the recursive ordering of structural innovations. Hence, it is a particular shortcoming of the Gaussian model framework that identifying restrictions cannot be tested against each other, or that there is no loss measure at hand that orders competing assumptions according to data based criteria.

Although Gaussian (quasi) log-likelihood optimization has become a powerful and often applied device for parameter estimation, further econometric tasks, interval forecasting, Value-at-Risk quantification etc., have recently raised an interest in more general frameworks to describe contemporaneous linkages among economic and financial time series. In this field copula distributions (Cherubini et al. 2004, Joe 1997, Nelsen 2006) are attracting intense interest, since a huge dimensional space of copula distributions promises most flexible response to rather general dependence patterns featuring empirical data. Given that from a statistical perspective the implementation of copula distributions is often straightforward this framework has been productively used for diverse applications with a particular focus in empirical finance. Recently, Lee and Long (2008) have introduced a copula based multivariate GARCH model to embed innovations to volatility in such a general setting. Given the plentitude of potential copula models that could be responsible for the generation of a models' stochastic characteristics, model choice and care for model misspecification are important issues in practice. Chen and Fan (2006) highlight the potential that in reality an analyst might in fact choose from two false distributional settings. Accordingly, they propose criteria for model choice among misspecified alternatives.

In this paper the framework of non Gaussian copula distributions is considered to derive loss measures associated with alternative (identifying) structural patterns. In the proposed set up, structural innovations bear the interpretation of news processes that are uncorrelated over the time dimension. In lack of multivariate normality, however, structural shocks are dependent in higher order moments. In turn, this dependence can be exploited to attach distinct losses to particular identifying assumptions. Although the proposed Moment Targeted Structural Innovations (MTSI) are not restricted to the VAR framework they contribute to a rapidly expanding literature in the field of structural modeling.

Lanne and Lütkepohl (2008, 2010) and, similarly, Rigobon (2003) propose an identification scheme distinguishing states of lower and higher variance for which the causation structure is assumed identical. The assumption of invariant structural characteristics over distinct economic states can, however, be subjected to criticism. The former approach has been further generalized towards a Markov switching model (Lanne, Lütkepohl and Maciejowska 2009) formalizing the dynamic pattern (and recurrence) of distinct variance regimes. As a main distinction, MTSI fully proceed under the presumption that available (vector valued) reduced form residuals are independent and identically distributed (iid) over the time dimension. Hence, MTSI might be seen to provide an assessment of structural assumptions in a scenario where statistical tools based on time heterogeneity cannot be applied. The MTSI concept is statistical and does not rely on a-priori reasoning with regard to some 'most likely' or 'weakest' restrictions placed on potential patterns of contemporaneous features as, for instance, in Uhlig (2005). Opposite to identifying impulse responses by means of sign restrictions (Faust 1998, see also Fry and Pagan 2007) the proposed identification scheme is analytical and does not make use of simulation techniques. Moreover, it does not restrict the long run impacts of structural innovations as outlined in Blanchard and Quah (1989).

Similar to the Markov switching approach in Lanne, Lütkepohl and Maciejowska (2009), the MTSI provide statistical loss measures that can be used to evaluate competing assumptions on the generation of contemporaneous reduced form characteristics. As a particular merit, however, MTSI loss functionals are also sensible for the contrasting of just identifying restrictions. It may associate, for instance, distinct losses to alternative recursive patterns characterizing the transmission from structural to reduced form model information. Thus, MTSI loss statistics can be used for the data based implementation of impulse response functions that make use of a Cholesky decomposition of the reduced form covariance matrix. Since MTSI are derived for iid reduced form disturbances, simplest resampling schemes, i.e. iid resampling with replacement, can be adopted to quantify the

uncertainty attached to the diagnosis of a particular direction of instantaneous causality.

The remainder of this paper is organized as follows: In the next Section structural innovations as stemming from a standardized bivariate copula distribution are introduced and the loss measure attached to competing structural assumptions is derived. Section 3 provides Monte Carlo evidence on the strength of the MTSI method to distinguish alternative directions of recursive structural models. In Section 4 two empirical applications consider the contemporaneous interaction of international breakeven inflation rates in the first place. Secondly, a bivariate system of US stock prices and total factor productivity (Beaudry and Portier 2006) is subjected to the detection of a data supported news process. Two Appendices provide the copula distribution functions used for Monte Carlo exercises and some detailed representations of third order data characteristics motivated in Section 2.2.

2 Modelling iid reduced form disturbances

In this section first, the statistical framework applied to characterize structural innovations and to attach loss measures to particular identifying assumptions is outlined. Secondly, the content of higher order moments for structural parameter identification is highlighted. Then, a strategy is sketched to evaluate the loss attached to particular identifying assumptions in a huge dimensional space of models that might generate structural innovations. Fourthly, a few remarks are made on the interpretation and implementation of impulse response functions within the non Gaussian modeling framework. Since the entire approach relies on the assumption of iid residuals, in the fifth place, a resampling scheme is outlined that supports an empirical analyst to quantify the uncertainty associated with decisions in favor of particular instantaneous causation schemes.

2.1 Copula distributed structural innovations

The statistical model embedding MTSI is in full analogy to the copula multivariate GARCH model proposed by Lee and Long (2009). Opposite to their framework, the second order moments of the serially uncorrelated, mean zero observables, denoted u_t , are time invariant, however. It is noteworthy that the origin of the observables u_t is not made explicit in this work. For instance, one might regard it as the (estimated) residual vector of a VAR model, a vector of standardized residuals determined by means of a multivariate volatility model (GARCH or stochastic volatility models, say) or it could be obtained by stacking (standardized) residuals of univariate linear regression (conditional volatility) models. With $\Phi(\bullet)$ and $\mathcal{C}(\bullet)$ denoting the Gaussian and a copula distribution

function, respectively, consider the following representation of u_t

$$u_t = De_t, \tag{1}$$

$$\text{where } e_t = \Omega^{-1/2}\xi_t, \xi_t \sim \mathcal{C}(\Phi(\xi_{1t}), \Phi(\xi_{2t}), \gamma), \text{Cov}[\xi_t] = \Omega. \tag{2}$$

As formalized in (2) both elements of ξ_t are (marginally) Gaussian distributed, i.e. $\xi_{it} \sim N(0, 1), i = 1, 2$. The joint distributional characteristics are governed by a copula distribution $\mathcal{C}(\zeta_1, \zeta_2, \gamma)$, where the random variables $\zeta_{it}, i = 1, 2$, are uniformly distributed over the interval $[0, 1]$ and γ is a parameter or vector of parameters. Thus, while their margins are Gaussian the joint distribution of the elements in ξ_t is generally not multivariate normal. In fact, the special case of joint normality is covered by the model in (2) if $\mathcal{C}(\zeta_1, \zeta_2, 0)$ is the Gaussian copula. In this case ξ_{1t} and ξ_{2t} are independent. While first and second order characteristics of e_t are identical over all possible copula choices, each of these generates a distinct dependence pattern and therefore results in model specific higher order cross moments of ξ_t, e_t and u_t .

Since the case of independence is ruled out for the elements of ξ_t , the model setup obtains for the implied second order moments

$$\text{Cov}[\xi_t] = \Omega = \begin{pmatrix} 1 & \omega \\ \omega & 1 \end{pmatrix}, \omega \neq 0.$$

By construction, the random vector $e_t = \Omega^{-1/2}\xi_t$ comprises random variables that are uncorrelated but not independent in general. The elements of e_t are considered to compose the vector of structural innovations that govern the instantaneous stochastic properties of the (reduced form) disturbances u_t . Without loss of generality and exploiting the properties of e_t it is assumed that the elements of u_t are in standardized form, i.e.

$$\text{Cov}[u_t] = D\Omega^{-1/2}\Omega\Omega^{-1/2}'D' = \begin{pmatrix} 1 & \sigma \\ \sigma & 1 \end{pmatrix} = DD' = \Sigma.$$

Notably, $\sigma, -1 < \sigma < 1$, is the empirically measurable (reduced form) correlation between u_{1t} and u_{2t} and Σ is nonsingular.

Relating reduced form and structural innovations, the matrix D is of core importance for economic and econometric modeling. For the so-called AB model, set out by Amisano and Giannini (1997) in the structural VAR framework, we have, for instance, $D = A^{-1}B$. While reduced form residuals are often easily available, the structural innovations e_t are hidden and cannot be specified without (a-priori) assumptions about the composition of D . For instance, one might presume a (lower vs. upper) triangular structure of D , such that one element of the reduced form disturbances obeys the interpretation of a structural

shock process. Alternatively, D could be a full matrix. In this case, a symmetric specification implies that observable disturbances result from some 'equal' cross weighting of two underlying structural shocks. In their context of volatility impulse response modeling, Hafner and Herwartz (2006) show that the square root matrix $D = \Sigma^{1/2}$ identifies unique structural innovations in a model with independent standardized t -distributed innovations to (multivariate GARCH) volatility. Presuming an asymmetric structure of D means that there is one dominating structural shock which is, however, not sufficient to fully explain the contemporaneous links among the reduced form disturbances. Let d_{ij} denote a typical element of D . For the purpose of identification the following additional assumptions are made:

- A1: $D \neq \Omega^{-1/2}$.
- A2: $d_{ii} > 0, i = 1, 2$.
- A3: $\mathcal{C}(\Phi(\xi_{1t}), \Phi(\xi_{2t}), \gamma)$ is not the Gaussian copula.
- A4: $E[\xi_{1t}^2 \xi_{2t}] = \theta_{21} \neq 0, E[\xi_{1t} \xi_{2t}^2] = \theta_{12} \neq 0$.

A1 is of technical nature. It can only be binding in case of a particular relation between ω and σ which is unlikely to apply in reality and which has zero probability if the correlation parameter σ is estimated from the data. In case Assumption 1 is violated, reduced form residuals are directly drawn from the copula distribution. Following Lee and Long (2009), $\Omega^{1/2}$ is set to the symmetric square root matrix of Ω such that, by assumption, both random innovations ξ_{1t} and ξ_{2t} contribute symmetrically to the generation of structural innovations e_t . Using the square root matrix for standardization appears particularly justified, since it is 'neutral' with regard to structural assumptions which are typically of recursive nature or originate from the notion of some dominating structural impulse. Alternatively, one might also think of a triangular decomposition of Ω^{-1} to formalize the transmission from ξ_t to structural innovations e_t . However, making use of rotation matrices, the space of triangular decompositions is infinite while the square root matrix $\Omega^{1/2}$ is unique up to trivial rotations (reordering of the elements in ξ_t or sign changes). Noting that the ξ_t are not jointly normally distributed each triangular decomposition is likely to result in distinct dependence patterns and, therefore, is prohibitive for a closed form distributional view at the data u_t . A2 implies that a particular element of e_{it} must impact on its own reduced form counterpart u_{it} positively. Notably, identification only requires a nonzero impact, i.e. $d_{ii} \neq 0$ is required, however, from an economic perspective it appears justified to somewhat reduce the space of admissible parameter settings. Presuming a recursive structure, i.e. having an upper or lower triangular matrix

D , A2 implies that a zero restriction prevents e_{it} to impact on $u_{jt}, j \neq i$. Assumption 3 rules out the multivariate Gaussian distribution from generating the (reduced form) stochastic properties of e_t or u_t . Since this particular distribution is known to be fully invariant under rotation of the variables in the system, two competing and distinct guesses about D could be observationally equivalent. If A3 is overly restrictive for applied work can be directly addressed by implementing the model in (1) and (2) with a Student- t copula. In case of a large estimated degrees of freedom parameter, the scope to identify the structure of D within the framework of the model in (1) and (2) is limited. Since the Gaussian case is ruled out by A3, the additional restrictions imposed by A4 appear rather mild, since dependence between ξ_{1t} and ξ_{2t} will be reflected in higher order cross moments.

In Section 2.3 a moment based distance measure is introduced that quantifies in how far a particular assumption on the structure of the matrix D is supported by empirical data. Before providing this loss measure and motivating the informational content of higher order moments for identification in Section 2.2, particular features of the model in (1) and (2) are discussed in some detail.

1. Marginals

At the first sight it appears rather restrictive to fix the marginal distribution to the Gaussian case. Owing to the standardization $e_t = \Omega^{1/2}\xi_t$, however, the actual structural form residuals belong, similar to the approach in Lanne and Lütkepohl (2010), to the family of mixed normal distributions. Given the flexible weighting scheme generated by a plentitude of possibly underlying copula distributions, the proposed model class might be sufficiently general to capture joint characteristics of empirical data.

In how far the explicit setting of the marginal distribution to the Gaussian case limits the empirical scope of the considered class of joint distributions is not clear at this stage, however. Notably, for the validity of the moment based identification scheme proposed Section 2.3 the specific marginal setting is not essential as long as particular higher order moments exist.

To get an impression of the joint distributional features implied by particular copula distributions, Figure 1 displays scatter diagrams of bivariate samples drawn from the standardized Clayton, Gumbel, Frank, the Student- t with 10 degrees of freedom and the Gaussian copula. The initial dependence level of the elements in ξ_t is small as Kendall's τ (or the linear correlation, denoted ρ , for the latter two copulae) is 0.3. While the former two copulae formalize an asymmetric joint distribution,

the Frank, the Student- t copula with 10 degrees of freedom and the Gaussian copula are symmetric around the origin. The asymmetric copulae appear to generate some upward (Clayton) and downward (Gumbel) pointing triangular pattern. The symmetric Frank and the Student t -copula distribution deviate markedly from the Gaussian benchmark. While the former appears to be more concentrated around the origin, the standardized Student- t copula is more dispersed in comparison with the Gaussian counterpart. If one would increase (reduce) the dependence parameter (τ, ρ) , the described shape features would see some strengthening (weakening). Apparently, the non Gaussian copula distributions generate a flexible space of joint distributions although, prior to standardization, all marginal distributions are fixed to the Gaussian case. Imagining an 'almost' symmetric shape even of the standardized Clayton or Gumbel distribution (with concordance parameter $\tau = 0.1$ say), it seems that the class of considered distributions is general enough to approximate cases of rather weak, medium and strong deviations from the multivariate Gaussian case. The lower left panel of Figure 1 shows a rotated version of the Clayton distribution, which appears symmetric against the unrotated Clayton distribution displayed in the lower left panel. As outlined below in Section 3 three alternative rotations of each copula distribution augment the distributional space considered in this work and, thus, provide an additional dimension to enhance the flexibility of the model class defined in (2).

2. Moment techniques vs. maximum likelihood

The copula distribution model is fully parametric, and, therefore, presuming a particular copula specification, maximum likelihood estimation of the elements in D appears straightforward. Letting aside numerical issues involved with maximum likelihood optimization, a (potentially not fully efficient) moment based identification scheme is proposed in this paper for the following two reasons: Firstly, although maximum likelihood estimation of D for given u_t and $\mathcal{C}(\zeta_1, \zeta_2, \gamma)$ appears feasible, any choice of a copula distribution (or one of its rotations) bears the risk of misspecification. In light of the plentitude of available parametric copula distributions, it appears most likely to miss 'the true' distributional setting when deciding in favor of a particular parametric model $\mathcal{C}(\zeta_1, \zeta_2, \gamma)$. Likelihood estimates might carry some content for the adequacy of particular model selections. Given the huge space of parametric copulae (Cherubini et al. 2004, Joe 1997, Nelsen 2006), however, the detection of a most convenient joint distribution model remains cumbersome for an empirical analyst and the choice between two misspecified alternatives as considered

in Chen and Fan (2006) does not resolve the issue to detect most appropriate structural innovations within a huge dimension space of distributions. Secondly, depending on the model that generates the reduced form residuals u_t (VAR, MGARCH, stacked univariate processes), it is not clear if vectors u_t fulfill the properties of maximum likelihood implied residuals. In this respect, it is noteworthy that maximization of a full copula VAR (vector error correction model) likelihood function can become rather demanding (at least) for particular copula distributions. Model estimation based on OLS is often consistent with regard to the conditional mean model parameters and therefore provides a suitable approach to extract u_t from empirical data.

2.2 The informational content of higher order moments

To verify how higher order moments can be used in the non Gaussian case to identify the elements in D consider the reduced form representation

$$u_t = D\Omega^{-1/2}\xi_t. \quad (3)$$

Noting that $\Omega^{1/2}\Omega^{1/2} = \Omega$ the typical diagonal w_1 and off diagonal elements w_2 in Ω obey the representation

$$w_1 = \sqrt{\frac{1}{2} + \frac{\sqrt{1-\omega^2}}{2}} \text{ and } w_2 = \frac{\omega}{2w_1}. \quad (4)$$

Therefore, by the model outset all elements in

$$\Omega^{-1/2} = \begin{pmatrix} w_1 & -w_2 \\ -w_2 & w_1 \end{pmatrix} \frac{1}{w_1^2 - w_2^2},$$

depend on the copula covariance ω in a well specified form. It is worthwhile pointing out that the solution given in (4) is not a unique representation for the elements w_1 and w_2 . By trivial rotations of $\Omega^{1/2}$ further 'typical elements' can be derived which all in basically the same elements to be collected in $\xi_t = \Omega^{1/2}e_t$ (except for joint sign changes, and reordering). Recall that $DD' = \Sigma$ and that the diagonal elements in Σ are unity. Therefore, the off-diagonal elements of D can be expressed in terms of their row neighbors, i.e. $d_{12} = \sqrt{1 - d_{11}^2}$, $d_{21} = \sqrt{1 - d_{22}^2}$. Moreover, the nontrivial diagonal elements, d_{11} and d_{22} , are subjected to the restriction

$$d_{11}\sqrt{1 - d_{22}^2} + d_{22}\sqrt{1 - d_{11}^2} = \sigma. \quad (5)$$

Let $\nu_{rs} = E[u_{1t}^r u_{2t}^s]$ and $\theta_{rs} = E[\xi_{1t}^r \xi_{2t}^s]$ be shorthand notations summarizing the higher order and cross moment characteristics of reduced form disturbances and of the copula

distributed random variables, respectively. The existence of all these moments follows from the setting of Gaussian marginal distributions. Notably, the moments θ_{r_0} and θ_{0_s} are known a-priori. In addition, standard results for convergence in probability imply that the moments ν_{rs} can be estimated from available data with asymptotically vanishing estimation error. From the relation in (3) a system of equations linking higher order moments is immediate to derive. For the case of third order moments, $r + s = 3$, the following representation can be shown to exist

$$\begin{pmatrix} \nu_{30} \\ \nu_{21} \\ \nu_{12} \\ \nu_{03} \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{pmatrix} \begin{pmatrix} \theta_{30} \\ \theta_{21} \\ \theta_{12} \\ \theta_{03} \end{pmatrix}. \quad (6)$$

The parameters characterizing the moment relations of the data u_t and copula distributed random variables ξ_t are rather complicated since, according to (4) and (5) the elements in $\Omega^{-1/2}$ and D obey nonlinear restrictions in the correlation level of observable data (σ) and the latent moment $\omega = \text{Cov}[\xi_{1t}, \xi_{2t}]$, respectively. In explicit terms the third order implied system (6) is given in Appendix A. For a given level of correlation σ it can be shown that the system in (6) comprises 4 unknown parameters/moments, namely $d_{11}, \omega, \theta_{12}$ and θ_{21} . The moments θ_{30} and θ_{03} are fixed by assumption of Gaussian marginals, $\theta_{30} = \theta_{03} = 0$. Note that a similar system can naturally be set out for higher order moments.

Applying straightforward algebra the four equation system (6) can be transformed into a two dimensional system by imposing the restrictions $\theta_{30} = \theta_{03} = 0$ and eliminating the unknown moments θ_{12} and θ_{21} under the assumption that these latter moments differ from zero (A4). Then, one obtains for the third order moments of u_t

$$\begin{pmatrix} \nu_{30} \\ \nu_{03} \end{pmatrix} = \begin{pmatrix} g_{12} & g_{13} \\ g_{42} & g_{43} \end{pmatrix} \begin{pmatrix} g_{22} & g_{23} \\ g_{32} & g_{33} \end{pmatrix}^{-1} \begin{pmatrix} \nu_{21} \\ \nu_{12} \end{pmatrix},$$

or, in compact notation,

$$\boldsymbol{\nu}_0 = G^{(1)} (G^{(2)})^{-1} \boldsymbol{\nu}_1 \Leftrightarrow \boldsymbol{\nu}_0 = G^{(3)} \boldsymbol{\nu}_1. \quad (7)$$

The system in (7) links the empirical third order moments of u_t in a well defined manner if the matrix $G^{(3)}$ is of rank 2. Notably, this system comprises two unknown parameters, d_{11} and ω . The parameter d_{11} is unique, i.e. identified, if there are no two distinct choices of d_{11} that give rise to the same full rank matrix $G^{(3)}$. In light of the intense complexity of the relation between $\{d_{11}, \omega\}$ and the elements in $G^{(1)}, G^{(2)}$ (and, thus, of $G^{(3)}$) analytical solutions of this identification issue are not available, though. However, numerical experiments help to clarify the identification issue. For this, the parameters

$-1 < \sigma < 1$, $-1 < \omega < 1$, $0 < d_{11} < 1$ are varied on a three dimensional grid with stepsize 0.01 within the respective support. For all combinations matrix elements d_{22} are determined from (5) and solutions that are not admissible according to A2 ($d_{22} \leq 0$) are removed from the parameter settings. To evaluate uniqueness of the elements in $G^{(3)}$ the modulus of its maximum eigenvalue is considered. Note that concentration on the first eigenvalue might be sufficient for identification while uniqueness of this modulus is not necessary, since the second eigenvalue gives another direction for matrix comparison. It turns out that for all admissible settings the rank of $G^{(3)}$ is 2. Moreover, within the relevant parameter space the modulus of the maximum eigenvalue of the matrix $G^{(3)}$ is unique over alternative settings $\{d_{11}, \omega\}$. To illustrate this to some extent, Figure 2 shows selected outcomes of the numerical treatment of the equation system (6). It displays the moduli of maximum eigenvalues for a few selections of $\sigma = \pm 0.1, \pm 0.5$ and $\omega = \pm 0.1, \pm 0.5$ as a function of d_{11} . Apparently, for given reduced form parameter σ , each d_{11} yields a distinct matrix $G^{(3)}$ for some ω . Moreover, the displayed eigenvalues differ for given d_{11} when varying the copula covariance parameter ω .

2.3 The identification scheme

Having shown in Section 2.2 that third order moments of the data carry informational content for the structural form parameters D a potential avenue for empirical modeling appears to exploit the estimated third order moment features of u_t and determine a moment estimator for d_{11} and ω on this basis. Such an approach is not adopted here for the following reasons. Firstly, the rather complex relation between the unknown parameters $\{d_{11}, \omega\}$ on the one hand and ν_{rs} , $r, s = 0, \dots, 3$, $r + s = 3$ on the other hand (see Appendix A) is likely prohibitive to derive a suitable GMM estimator. Second, the motivation of identifiability of D has been concentrated on the case of third order moment characteristics. However, with the presumption of Gaussian marginals further finite order moment conditions might be considered that promise additional strength in distinguishing alternative guesses about the structural parameters in D . Notably, (cross) moments of orders $r + s \geq 5$ are likely to suffer from substantial estimation uncertainty. Therefore power gains can be expected reasonably from joining third and fourth order moment information as also proposed in the framework of normality testing (Jarque and Bera 1980). In light of these considerations it is apparent that third order results in Section 2.2 along with assumption A4 only provide an analytical frame that is sufficient for identification. The conditions in A4 are not necessary for identification and could be replaced by other settings applying to fourth and higher order characteristics of ξ_t . Recalling the functional complexity between moments ν_{rs} and model parameters the

identification scheme proposed in this paper does not target at the moment features of u_t but, rather, at those of implied innovations ξ_t that follow from an a-priori choice of D and ω . Without presuming a particular copula model the underlying covariance structure Ω is not accessible to an analyst. Assume for the moment that the covariance/correlation parameter ω , i.e. the only unknown element in Ω is known. Conditional on ω , some guess about D , and applying the square root standardization we have

$$\xi_t = \Omega^{1/2} D^{-1} u_t, \quad \Omega^{1/2} = \begin{pmatrix} w_1 & w_2 \\ w_2 & w_1 \end{pmatrix}.$$

With weights fulfilling the restrictions in (4) and estimates $e_t = D^{-1} u_t$ implied draws from some copula distribution $\xi_t = \Omega^{1/2} e_t$ are given as

$$\xi_{1t} = w_1 e_{1t} + w_2 e_{2t}, \quad (8)$$

$$\xi_{2t} = w_2 e_{1t} + w_1 e_{2t}. \quad (9)$$

The latter estimates exhibit a Gaussian distribution if all assumptions made on D and ω hold true. Thus, in terms of (univariate) higher order moments the following characteristics apply amongst others

$$E[\xi_{it}^3] = 0, \quad E[\xi_{it}^4] = 3, \quad i = 1, 2. \quad (10)$$

The proposed identification scheme basically exploits moment characteristics of (available) estimates e_t that are related with the assumptions made for the marginal distributions. Let $e_t^{(rs)}$ be short for random variables $e_{1t}^r e_{2t}^s$. The following relations link structural innovations \underline{e}_t and underlying random vectors $\underline{\xi}_t$, where the underlining is used to indicate that these random vectors are determined from u_t conditional on a specific choice $\{D, \omega\}$:

$$\begin{aligned} \underline{\xi}_{-1}^3 &= w_1^3 e_t^{(30)} + 3w_1^2 w_2 e_t^{(21)} + 3w_1 w_2^2 e_t^{(12)} + w_2^3 e_t^{(03)}, \\ \underline{\xi}_{-2}^3 &= w_2^3 e_t^{(30)} + 3w_2^2 w_1 e_t^{(21)} + 3w_2 w_1^2 e_t^{(12)} + w_1^3 e_t^{(03)}, \\ \underline{\xi}_{-1}^4 &= w_1^4 e_t^{(40)} + 4w_1^3 w_2 e_t^{(31)} + 6w_1^2 w_2^2 e_t^{(22)} + 4w_1 w_2^3 e_t^{(13)} + w_2^4 e_t^{(04)}, \\ \underline{\xi}_{-2}^4 &= w_2^4 e_t^{(40)} + 4w_2^3 w_1 e_t^{(31)} + 6w_2^2 w_1^2 e_t^{(22)} + 4w_2 w_1^3 e_t^{(13)} + w_1^4 e_t^{(04)}. \end{aligned}$$

Let a vector $\boldsymbol{\xi}_t$ collect higher order powers of marginals implied by the assumptions on D and ω , $\boldsymbol{\xi}_t = (\underline{\xi}_{-1t}^3, \underline{\xi}_{-2t}^3, (\underline{\xi}_{-1t}^4 - 3), (\underline{\xi}_{-2t}^4 - 3))'$ and $\boldsymbol{\xi} = (\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \dots, \boldsymbol{\xi}_T)'$. The following loss measures are attached to $\{D, \omega\}$ and $\{D\}$, the singled out a-priori guess about the structural pattern, respectively,

$$L(D, \omega) = \bar{\boldsymbol{\xi}}' \left(\frac{\tilde{\boldsymbol{\xi}} \tilde{\boldsymbol{\xi}}}{T} \right)^{-1} \bar{\boldsymbol{\xi}} \quad \text{and} \quad \mathcal{L}_D = \min_{\omega} L(D, \omega). \quad (11)$$

In (11) $\bar{\xi}$ denotes the unconditional mean of the elements in ξ , and $\tilde{\xi}$ is the matrix of centered variables ξ_t . In practice, the \mathcal{L}_D loss can be found by means of a grid search for the unknown parameter ω , $|\omega| < 1$. As given in (11) the model selection criterion exploits rigorously the features of the presumed marginal Gaussian distribution up to moments of order 4 and is similar to the Jarque-Bera statistic (Jarque and Bera 1980) for testing the normality assumption. In its exact form the statistic differs from the Jarque-Bera statistic, since the ξ_t are not jointly normal even if the choice $\{D, \omega\}$ coincides with the true model parameters. In case that the elements of ξ_t are Gaussian, standard central limit theorems apply to confirm that the $L(D, \omega)$ statistic in (11) is asymptotically χ^2 distributed with 4 degrees of freedom.

2.4 Impulse response functions

Impulse response functions (IRFs) are often applied for the analysis of dynamic relationships characterizing a multiple time series with reduced form disturbances u_t . Notably, the MTSI approach can assist an analyst when determining an upper or lower triangular covariance decomposition for the outset of IRFs. By definition, IRFs trace the effects of a unit shock hitting a particular variable on all variables of a system. Implementing IRFs by means of triangular decompositions proceeds from the perspective that a system contains a variable specific shock. Assume without loss of generality that e_{1t} is such a shock process, meaning that an analyst applies a lower triangular decomposition of the empirical reduced form covariance to determine a systems' IRFs. In the framework of the Gaussian model the 'on-impact' effect on u_{2t} of a unit shock hitting e_{1t} is σ . Given that in the Gaussian model $E[e_{2t}|e_{1t} = 1] = 0$ the instantaneous reaction of the u_{2t} process does not entail any response to own innovations which facilitates the derivation of IRFs and allows for a global interpretation of the functional patterns. Moreover, the effects of positive and negative shocks of the same size are identical (except for the sign impacts). This changes if the elements in e_t lack independence. In the MTSI framework it is not justified to presume that the instantaneous response of u_{2t} to a unit shock in e_{1t} is fully driven by the shock process, since $E[e_{2t}|e_{1t} = 1] \neq 0$, in general. If the marginal processes in u_t are not jointly normally distributed, implied IRFs are local in the sense that $E[e_{2t}|e_{1t} = 1] = m(e_{1t})$. In addition, also the sign of the shock might matter since for asymmetric copula distributions $E[e_{2t}|e_{1t} = e] \neq (-1)E[e_{2t}|e_{1t} = -e]$. Hence, IRFs implied by the framework in (1) and (2) are similar to conditional moment profiles introduced in Gallant et al. (1993). However, opposite to the more general concept of conditional moment profiles, locality in the MTSI framework does not refer to states defined in terms of the reduced form residuals u_t . In the copula model and applying a lower

triangular covariance decomposition, the 'on impact' effect of an isolated unit shock in e_{1t} could be given as

$$E[u_{2t}|e_{1t} = 1] - E[u_{2t}|e_{1t} = 0, e_{2t} = 0] = \sigma + \sqrt{1 - \sigma^2}E[e_{2t}|e_{1t} = 1].$$

Depending on the underlying distributional model, this quantity can differ markedly from σ , its counterpart in the Gaussian setting. In relative terms the 'on impact' approximation error involved with standard IRFs applied in non Gaussian cases is most severe if σ is small.

Accordingly, typical IRFs derived under the assumption of joint normality suffer from a risk of misspecification and a spurious global interpretation, whenever the data do not accord with the presumption of joint normality. Given the large class of distributional models spanned by the framework in (2) one may expect that misspecification or locality of IRFs is more the rule rather than exceptional. Moreover, since $E[e_{2t}|e_{1t} = 1] \neq 0$ one may regard the shock variable e_{1t} to instantaneously cause e_{2t} and, therefore, refer to a triangular matrix D as formalizing some pattern of instantaneous causation (rather than merely a direction of recursion).

2.5 A resampling scheme for empirical data

Since the MTSI loss functional in (11) is derived within an iid setting, simplest resampling with replacement techniques are natural candidates to quantify the uncertainty attached to the decision in favor of a particular pattern of instantaneous causation. Suppose an analyst wants to distinguish between two presumptions made for the D matrix. For instance, she could be interested in a comparison of a lower vs. an upper triangular scheme, denoted D_l and D_u , respectively. The respective loss statistics are in shorthand notation \mathcal{L}_l and \mathcal{L}_u . The following resampling scheme is designed to quantify the particular merits of these alternative assumptions:

1. Determine \mathcal{L}_l and \mathcal{L}_u from the data.
2. Draw sufficiently often, H times say, reduced form disturbances $\{u_t^*\}_{t=1}^T$ from $\{u_t\}_{t=1}^T$ with replacement and determine bootstrap losses $\mathcal{L}_{h,l}^*$ and $\mathcal{L}_{h,u}^*$, $h = 1, \dots, H$. In the empirical section the number of bootstrap replications is fixed to $H = 1000$.

From the set of bootstrap statistics $\{\mathcal{L}_{h,l}^*, \mathcal{L}_{h,u}^*\}_{h=1}^H$ the following empirical frequencies carry informational content for the data supported scheme of instantaneous linkage of the news processes in e_t

$$\hat{P}_{lu} = \frac{1}{H} \sum_{h=1}^H I(\mathcal{L}_{h,l}^* < \mathcal{L}_{h,u}^*), \hat{Q}_{ul} = \frac{1}{H} \sum_{h=1}^H I(\mathcal{L}_u < \mathcal{L}_{h,l}^*), \hat{Q}_{lu} = \frac{1}{H} \sum_{h=1}^H I(\mathcal{L}_l < \mathcal{L}_{h,u}^*), \quad (12)$$

where $I(\cdot)$ is an indicator function. All these frequencies are informative for the robustness at which a particular guess about D is stronger supported by empirical data in comparison with a competing view. Since one would expect empirical frequencies close to unconditional quotes of 50% under a multivariate Gaussian framework, the resampling scheme is also informative in how far the data generating model can be distinguished from the multivariate Gaussian case. Since the MTSI loss statistics are, similar to model selection criteria, not designed for inferential purposes, frequencies as defined in (12) do not obey the interpretation of formal significance levels. To assess if particular statistics deviate from an unconditional success frequency one might compare the \widehat{P}_\bullet and \widehat{Q}_\bullet statistics with approximate confidence bands $0.5 \pm 2\sqrt{0.25/H}$. Alternatively and for the case of the \widehat{P}_\bullet statistics, it is also sensible to adopt the Monte Carlo test principle (Dufour 1989) and simulate the distribution of \widehat{P}_\bullet statistics under multivariate normality. As a particular merit, Monte Carlo critical values (opposite to ad-hoc bounds $0.5 \pm 2\sqrt{0.25/H}$) naturally adapt to the sample size T .

3 Monte Carlo analysis

To investigate the scope of the MTSI distance measure in (11) to detect recursive structures in serially uncorrelated random vectors u_t , a rather simple simulation experiment is provided in this section. First the simulation design and implementation issues are sketched before simulation results are discussed in detail.

3.1 The simulation design

Monte Carlo experiments consist of the generation of structural innovations entering reduced form disturbances according to a particular recursive direction. Then, MTSI loss statistics \mathcal{L}_\bullet are compared to uncover the shock process that underlies the data. Lower and upper triangular matrices are used alternatively for data generation, and the underlying recursion is determined from the loss statistics \mathcal{L}_l and \mathcal{L}_u as described in Section 2.3.

Seven distinct copulae (plus the Gaussian) are employed for generating random variables ξ_t , namely the Clayton, Farlie-Gumbel-Morgenstern (FGM), Frank, Gumbel, Plackett, Ali-Mikhail-Haq (AMH) and Student- t copula with 10 degrees of freedom. Formal representations of the copula functions are given in Appendix B. For further reference and an intensified discussion of distributional features generated by the considered copulae the reader may consult Cherubini et al. (2004) or Nelsen (2006). Moreover, these references provide suitable algorithms for drawing random vectors from the copula distributions. In addition, for each employed copula $C^{(1)} = C(\zeta_1, \zeta_2, \gamma)$ three rotations are used for data

generation, namely,

$$C^{(2)} = C((1 - \zeta_1), \zeta_2, \gamma), C^{(3)} = C(\zeta_1, (1 - \zeta_2), \gamma), C^{(4)} = C((1 - \zeta_1), (1 - \zeta_2), \gamma). \quad (13)$$

While the so-called survival copula $C^{(4)}$ in (13) does not change the general direction of dependence between ξ_{1t} and ξ_{2t} the two other (partial rotations) invert the dependence direction as already illustrated for the case of the (rotated) Clayton copula in Figure 1 (upper and lower left hand side panel). Thus, leaving out the Gaussian, 28 distinct copula distributions are employed in the Monte Carlo study 20 of which (Frank, Plackett, FGM, Student- t , AMH and their rotations) imply a symmetric contemporaneous relation between the elements of ξ_t . Similarly, 8 copulae (derived from the Clayton and Gumbel model) generate asymmetric dependence.

The underlying copula distributions are characterized by distinct degrees of contemporaneous dependence measured in terms of Kendall's τ (Clayton, Gumbel, Frank, Plackett, FGM, AMH) or the linear correlation coefficient (Student- t , Gaussian). To be explicit, alternative settings $\tau, \rho = 0.1, 0.2, \dots, 0.8$ are considered for the Clayton, Gumbel, Frank, Plackett and the Student- t copula with 10 degrees of freedom. Regarding the FGM and AMH copula it is known that these models support only a limited range of dependence in terms of Kendall's τ . Accordingly, FGM and AMH copulae are parameterized to have measures of concordance $\tau = 0.1$ and $\tau = 0.2$. For the purpose of benchmarking, the Gaussian copula is also used for data generation. In this setting, however, the average identification success should be identical to the unconditional probability of predicting the correct recursive direction scheme, i.e. 50%.

For a given draw $\{e_t = \Omega^{-1/2}\xi_t\}_{t=1}^T$ reduced form residuals are determined as

$$u_t = D_l e_t \text{ or } u_t = D_u e_t, t = 1, \dots, T,$$

where

$$D_l = \begin{pmatrix} 1 & 0 \\ \sigma & \sqrt{1 - \sigma^2} \end{pmatrix} \text{ and } D_u = \begin{pmatrix} \sqrt{1 - \sigma^2} & \sigma \\ 0 & 1 \end{pmatrix}. \quad (14)$$

Overall, 23 values of σ are chosen, namely,

$$\sigma = -.95, -.90, .-.80, \dots, -.10, -.05, 0, .05, .10, .20, \dots, .90, .95.$$

After generation of u_t the correlation parameter is estimated from standardized data, i.e.

$$\hat{\sigma} = \frac{1/T \sum_t \hat{u}_{1t} \hat{u}_{2t}}{\sqrt{(1/T \sum_t \hat{u}_{1t}^2)(1/T \sum_t \hat{u}_{2t}^2)}},$$

$$\text{with } \hat{u}_{it} = (u_{it} - \bar{u}_i)/s(u_i), \bar{u}_i = \frac{1}{T} \sum_{t=1}^T u_{it}, s(u_i)^2 = \frac{1}{T-1} \sum_{t=1}^T (u_{it} - \bar{u}_i)^2, i = 1, 2.$$

Then, the correlation estimate $\hat{\sigma}$ is used to determine alternative structural estimates

$$\hat{e}_t = \hat{D}_l^{-1} \hat{u}_t \text{ and } \check{e}_t = \hat{D}_u^{-1} \hat{u}_t.$$

For the translation of estimated structural disturbances (\hat{e}_t or \check{e}_t) to underlying random variables ξ_t the latter's covariance must be known. Since ξ_t is latent, loss measures in (11) are determined by means of a grid search for the copula covariance parameter $\omega = 0.01j$, $j = 0, \pm 1, \pm 2, \dots, \pm 99$. With either $\{\hat{e}_t, \omega\}$ or $\{\check{e}_t, \omega\}$ alternative loss statistics \mathcal{L}_l and \mathcal{L}_u are determined, respectively, and a lower (upper) triangular structure is diagnosed for the transmission of e_t to u_t if $\mathcal{L}_l < \mathcal{L}_u$ ($\mathcal{L}_u < \mathcal{L}_l$).

To study small and larger sample properties of the MTSI identification scheme alternative sample sizes are $T = 100, 200, 400, 1000$. The number of replications of each Monte Carlo experiment is $R = 5000$. All simulations are performed in Gauss 6.0. Simulation results are throughout provided in terms of relative frequencies of correct directional identification, i.e.

$$\hat{f}_l = \frac{1}{R} \sum_{r=1}^R I(\mathcal{L}_{r,l} < \mathcal{L}_{r,u} | D_l) \text{ and } \hat{f}_u = \frac{1}{R} \sum_{r=1}^R I(\mathcal{L}_{r,u} < \mathcal{L}_{r,l} | D_u), \quad r = 1, \dots, R. \quad (15)$$

Given that the unconditional success frequency is 0.5 and that 5000 Monte Carlo replications are run by experiment, significant identification success can be diagnosed if estimates \hat{f}_\bullet , $\bullet = l, u$, exceed an upper threshold of $0.5 + 2\sqrt{0.25/5000} = 0.514$. Similarly, $\hat{f}_\bullet < 0.486$ hints at a systematic failure of the MTSI based identification strategy.

3.2 Monte Carlo results

Table 1 documents frequency estimates \hat{f}_l and \hat{f}_u for time series dimensions $T = 200$, $T = 400$ and $T = 1000$ and small to medium levels of dependence of copula distributed innovations ξ_t . For the Clayton, Gumbel, Frank, Student- t and Plackett copula Kendall's τ (the linear correlation for the Student- t) is either $\tau, \rho = 0.2$ or $\tau, \rho = 0.4$. For the FGM and AMH copula simulation results are provided for a parameter choice implying $\tau = 0.2$. Notably, the case $\tau, \rho = 0.2$ formalizes a rather mild state of dependence such that the joint distribution of ξ_{1t} and ξ_{2t} is (in small samples) rather difficult to distinguish from the uninformative case of Gaussian innovations. Simulation results are tabulated for alternative degrees of contemporaneous correlation, $\sigma = 0.1, 0.3, 0.5, 0.7, 0.9$. Moreover, simulation results are provided for the case where a rotated copula distribution (namely (C^2)) has been used for the generation of structural innovations. Before discussing simulation results in more detail it is worthwhile pointing out that \hat{f}_l and \hat{f}_u (lower vs. upper triangular recursion) are rather similar and also the patterns of correct identification do

not differ markedly if structural innovations e_t are generated from a raw ($C^{(1)}$) or rotated copula ($C^{(2)}$).

With regard to finding the actual direction of causality, notably, no frequency estimate \hat{f}_l or \hat{f}_u is significantly below 0.5 and, moreover, these frequencies increase with the sample size, i.e. they are uniformly higher for $T = 400$ in comparison with $T = 200$. Only a few estimates \hat{f}_\bullet are below the upper 95% confidence bound constructed around the unconditional success ratio of 50%. Maximum rates of correct identification are listed throughout for $T = 1000$. Thus, the proposed identification scheme does not suffer from systematic or eventual biases towards the likelihood of suggesting a false direction of instantaneous causation. Moreover, the MTSI based identification scheme appears consistent. For the case of an underlying Gaussian distribution, it turns out that, as expected, \hat{f}_l and \hat{f}_u are insignificantly close to the unconditional success frequency of 50% throughout.

Conditional on the small sample size $T = 200$, smallest and highest success frequencies \hat{f}_l are, respectively, 76.0% (Plackett, $\tau = 0.4$, $\sigma = 0.7$) and 50.4% (Student- t , $\rho = 0.4$, $\sigma = 0.1$). Generally, rates of correct identification are relatively low for the Student- t copula (with 10 degrees of freedom). The relatively weak content for identification of the Student- t model cannot be addressed to the symmetry of this distribution. Other symmetric copula specifications yield estimates \hat{f}_l that markedly exceed the identification rates achieved under the Student- t model. Generally the estimates \hat{f}_\bullet are lower for boundary levels of correlation ($\sigma = 0.1$ or $\sigma = 0.9$) than for more medium levels ($\sigma = 0.3$ or $\sigma = 0.7$). Intuitively, while scenarios with $\sigma \approx 0$ are characterized with rather weak contemporaneous linkage at all, at the opposite end, $|\sigma| \approx 1$ formalizes some states of coincidence. In both cases the identification of a recursive pattern from observed data is rather demanding at least in small samples.

For the asymmetric distributions (Clayton and Gumbel) we obtain the interesting result that the identification success appears to shrink for $\sigma = 0.5$ if the dependence of the elements in ξ_t is weakened, i.e. if one compares results for $\tau = 0.4$ and $\tau = 0.2$. In fact, particular levels of concordance (in combination with selected values of σ) eventually require large dimensional sample information to take advantage of the overall consistency of the MTSI approach. In the documented large sample cases ($T = 1000$) the frequencies of finding the correct recursion are almost throughout rather high with identification under the Student- t model being the least favorable. Since the Student- t copula is specified with 10 degrees of freedom, this result is not critical for the MTSI method as it is clear that with increasing the degrees of freedom parameter the Student- t model converges to the Gaussian case and, therefore, lacks identifiability in the limit.

Further simulation results are displayed graphically for the Clayton, Gumbel and Frank copula in Figure 3 and the remaining distributions in Figure 4. In the left (right) hand side panels of these Figures simulation results are displayed for a given level of dependence (for a given sample size) with varying sample sizes (degrees of dependence). In analogy with the \hat{f}_\bullet statistics documented in Table 1 these results allow the following general remarks:

1. In case of uncorrelated reduced form data ($\sigma = 0$ or Σ being the identity matrix) the MTSI implied frequency of detecting a lower triangular recursion coincides with an unconditional success frequency of 50%. In the asymptotic case $T = 1000$ this theoretical probability is matched rather closely for all copulae.
2. For the given level of moderate dependence characterizing the underlying copula innovations ξ_{1t} and ξ_{2t} ($\tau, \rho = 0.2$) the MTSI identification scheme is consistent as the frequency of correctly detecting the lower triangular scheme is increasing in the sample size and is (close to) unity for numerous simulation experiments with large sample size ($T = 1000$) and medium (positive or negative) level of correlation.
3. Looking at the identification success as a function of the empirical correlation σ , it appears that for most copulae the detection of the correct lower triangular recursion pattern is more likely if σ is of moderate to medium size in absolute value. Generally, patterns of successful identification appear symmetric around $\sigma = 0$.

The double humped pattern of successful identification is more characteristic for the Gumbel, Frank, Plackett, FGM and AMH copula as it is for the Clayton and Student- t model. For the latter two distributions the pattern of successful identification exhibits marked local minima for reduced form correlation of $|\sigma = 0.4|$ (Clayton) and $|\sigma = 0.6|$ (Student- t) which are most evident in the large sample case ($T = 1000$).

4. Conditional on the rather low copula dependence level ($\tau, \rho = 0.2$) the MTSI scheme is unlikely to detect the true recursive pattern with considerable nontrivial probability in small samples. In the weak dependence scenario and for $T = 100$ highest success frequencies of up to 65% ($|\sigma| \approx 0.4$) are obtained if random variables ξ_t are drawn from the FGM distribution.
5. For a given sample size the degree of dependence between ξ_{1t} and ξ_{2t} stimulates the identification success for all considered symmetric copula distributions. For the Clayton copula the identification rates achieved if $\tau = 0.6$ are rather similar (and eventually superior) to those obtained if $\tau = 0.8$. Similarly, for the Gumbel model

the identification success deteriorates for higher levels of dependence τ and states of relatively high correlation between reduced form model disturbances ($|\sigma \approx 0.7|$).

3.3 Summary

Overall, the simulation based evidence is supportive for the MTSI approach as providing a powerful tool for comparing alternative structural assumptions that might underly reduced form disturbances of empirical models. As the approach is fully based on an assumption of iid residuals it is clear that particular scenarios might be difficult to distinguish in finite samples. Potential cases where MTSI loss measures are likely close to each other are (very) small sample cases or scenarios in which the correlation of reduced form disturbances is either small or close to unity. In any case, with the resampling scheme described in Section 2.5 at hand, an analyst using the MTSI loss statistics for model choice can suitably control the degree of uncertainty attached to a particular model choice.

4 Empirical applications

To further illustrate the scope of MTSI implied loss statistics two prominent issues of empirical macroeconometrics are considered in this section. Firstly, MTSI loss statistics are used to uncover potential patterns of monetary dependence in four systems of international breakeven inflation rates. Secondly, raised recently by Beaudry and Portier (2006) loss statistics $\mathcal{L}(\bullet)$ are determined to rank competing notions of news processes in a bivariate system of US stock prices and total factor productivity.

4.1 International linkage of breakeven inflation

4.1.1 Monetary dependence

A core concern of monetary policy is long term price stability. Since the 1990s inflation targeting has become a widely followed strategy to implement a moderate and predictable evolution of prices. The essential element in such a framework is to anchor long run inflation expectations. Hence, the extent to which central banks are able to implement stable, unique and definite beliefs is crucial. The ability of monetary authorities in small economies for the targeting of inflation expectations could be threatened by the neighborhood to or intense trading relationships with leading economies such as the US or the European Monetary Union (EMU). Particular central banks that might be subjected to cross market monetary transmission in this respect are the Bank of Canada or the Bank of England. In this section monetary linkage is empirically assessed in

terms of the degree to which ex-ante inflation rates are determined on international markets comprising a subset of the G7, namely Canada, France, the UK and the US. Inflation compensation as implied by the (liquidity adjusted, Shen 2006) difference between yields of long term 'Treasury Inflation Protected Securities' and conventional bonds is regarded to measure future inflation prospects. Daily price quotes cover the time period 4/2/2001- 9/30/2008. Breakeven inflation rates as analyzed below are available from the net (<http://www.bepress.com/snede/vol13/iss4/art5/>, see also Herwartz and Roestel 2009). Liquidity adjusted breakeven inflation rates are displayed in Figure 5. By graphical inspection adjusted breakeven rate dynamics appear consistent in the sense that all respective monetary authorities have been communicating inflation targets around 2% to 3% over the last decade.

4.1.2 Reduced form estimates

In total we consider four bivariate systems of innovations governing daily break even inflation rate changes. Below, these systems are labeled with roman numbers and comprise the following combinations of bond markets

I	II	III	IV
CA,US	FR,US	UK,US	UK,FR

The systems I, III and IV are ordered such that a presumably dominating market is listed after a potentially dominated market. For the second system one might also a-priori consider the US bond market to informationally dominate the European counterpart. Regarding recent empirical evidence (Ehrmann and Fratzscher 2004), it is, however, conceivable that the notion of US dominance (over the Euro Area) has seen some weakening over more recent time periods.

Modeling first and second order features of the ex-ante inflation expectations we find that changes of liquidity adjusted breakeven inflation rates show at most rather weak serial correlation features but instead marked patterns of volatility clustering. Quantifying second order characteristics by means of GARCH(1,1) specifications turns out to approximate higher order features of the Canadian and UK breakeven rate changes accurately. With regard to the remaining two series diagnostic results indicate that GARCH(1,1) implied standardized residuals might show some (mild) remaining conditional heterogeneity. An expansion of the empirical model towards a GARCH(2,1) or GARCH(1,2), however, failed to provide reasonable (i.e.) positive GARCH parameters and/or to improve the diagnostic features of standardized residuals. Therefore, underlying market innovations are extracted from breakeven inflation rate changes by means of univariate GARCH(1,1)

models. GARCH(1,1) implied standardized residual processes are stacked to obtain bivariate systems of reduced form bond market innovations. Estimation and diagnostic results and, moreover, the cross correlation matrix of univariate GARCH residuals are shown in Table 2. From the theoretical outset of the MTSI approach it is clear that its identification potential vanishes in case that reduced form residual processes are jointly Gaussian. It well established that residuals of volatility models, though being iid distributed, often fail to exhibit a Gaussian distribution. For the considered systems of breakeven inflation innovations explicit tests on joint normality are not provided in detail. In fact, Jarque-Bera test statistics (Jarque and Bera 1980) determined for the four breakeven systems vary between 211.11 (CA/US) and 2589.23 (UK/US).

4.1.3 Structural analysis

From the empirical results in Table 2 we deduce that the correlation estimates obtained in the four dimensional system vary between 0.16 (UK/US) and 0.44 (CA/US). Noting that almost 2000 time series observations enter the analysis for each system one might expect that if the true contemporaneous market relation is of a recursive type the MTSI identification scheme delivers clear cut identification results. In the case that both markets of a system contribute to reduced form disturbances, the true scheme of instantaneous causality is not of a recursive structure. A-priori, given that monetary policy in the Euro area and the US are both likely to issue unrelated news one might expect the second system to deliver some mixed evidence on recursive patterns of processing structural innovations. Consequently, for this system a symmetric square root decomposition of the reduced form covariance might be most convenient according to a-priori considerations. Empirical MTSI loss measures for systems of (standardized) breakeven inflation rates are shown in the upper part of Table 3.

For the systems comprising Canadian and French breakeven rates jointly with US inflation expectations an upper triangular scheme is characterized by smaller loss statistics as the lower triangular counterpart. Thus, according to these statistics shocks in the US rate are more likely to impact on the remaining rate in comparison with a recursion where innovations of the US rate cannot be interpreted as a news process. Over all bootstrap replications this direction of instantaneous causation is confirmed in $\approx 80\%$ (CA/US) and $\approx 70\%$ (FR/US) of all replications. Table 3 also lists 90% and 95% Monte Carlo confidence intervals for the latter frequencies supporting the upper triangular recursion for both systems at the 5% significance level. For both bivariate systems the upper triangular scheme is also found to offer smaller loss in comparison with a matrix formalizing symmetric contemporaneous causality. In 71% (CA/US) and 64% (FR/US) of all bootstrap

replications loss statistics are smaller for the upper triangular recursion in comparison with the symmetric response matrix. Regarding the French breakeven rate to represent EMU inflation expectation the diagnosed upper triangular scheme is at odds with recent empirical evidence for a weakened impact of the US market on European interest rates (Ehrmann and Fratzscher 2004), and more in line with traditional views on interest rate transmission from the US to the German and smaller European markets (Katsimbris and Miller 1993, Hassapis, Pittis and Prodromidis 1999).

The third system comprising standardized innovations of US and UK inflation compensation is closest to the notion of symmetric instantaneous causality, i.e. with regard to inflation expectations particular recursive transmission cannot be retrieved from these markets. On the one hand this result might be surprising in light of the literature available on US dominance, however, for the informational content of breakeven inflation rates it is noteworthy that the trading of inflation protected securities has a by far longer tradition in the UK in comparison with all remaining markets. Owing to weakened liquidity in US markets at the beginning of the sample period one might attribute particular informational content to UK inflation expectations. From a statistical perspective also the relatively low level of correlation between reduced form information for these two markets might be a reason for the mixed evidence with regard to potential patterns of instantaneous causation. A liquidity based argument might also apply when considering the UK/FR system which does not reflect a dominance of French/EMU breakeven rates. In fact, one diagnoses some evidence of a lower triangular relation characterizing this particular system. From resampling it turns out that in almost 68% of all bootstrap replications the lower triangular scheme is confirmed which is outside the confidence band obtained when imitating the bootstrap outcomes in a bivariate Gaussian system.

Summarizing the results on monetary dependence for the two smaller economies considered, it turns out that in comparison with the Bank of Canada, the Bank of England is likely better able to target domestic long run inflation.

4.2 Is US total factor productivity a shock process ?

4.2.1 Reduced form modeling

Recently, Beaudry and Portier (2006) have raised the issue if in a bivariate system comprising a technology measure (TFP) and stock prices (SP) as an indicator of future expectations about the business cycle, surprises i.e. news are released in expectations (i.e. stock prices) or technology. Interestingly, the former case would suggest that technological change is to some extent 'foreseen' or processed in stock prices. The quarterly data on US

total factor productivity and stock prices spans the period 1947 to 2000 and can be drawn from the net (<http://www.aeaweb.org/articles.php?doi=10.1257/aer.96.4.1293>). To clarify the origin of news Beaudry and Portier (2006) also employ higher dimensional systems comprising consumption and/or hours worked which is beyond the scope of the MTSI approach as introduced in this work. For the bivariate system two alternative identification schemes are applied in Beaudry and Portier (2006) one of which relies on long term identifying restrictions that allow a non vanishing response of the news shock on total factor productivity. Alternatively, Beaudry and Portier (2006) use a Cholesky decomposition excluding on impact dynamics operating from stock prices on total factor productivity. It turns out that these two identification schemes obtain highly correlated news shocks. The Cholesky type lower diagonal identification scheme used by Beaudry and Portier (2006) can be subjected to loss comparison with an alternative upper triangular decomposition scheme. It is this particular aspect of the relationship between technology and stock prices that can be subjected to a loss assessment in the MTSI framework.

To investigate the empirical linkage of stock prices and factor productivity four alternative VAR specifications are used to extract reduced form residuals. According to standard model selection criteria (AIC, BIC, HQ) applied to level data with deterministic trend, lag order 2 is broadly supported. Accordingly, the four systems analyzed are a VAR(2) with trend for level data, a VAR(2) for level data without trend and a VAR(2) and VAR(1) model for first differences of stock prices and total factor productivity both excluding a deterministic trend. Reduced form residuals extracted from these model specifications are throughout rather similar and the empirical residual correlation in these bivariate systems is around $\hat{\sigma} \approx 0.16$. VAR estimates are provided in Table 4. Noting an only weak correlation of reduced form disturbances in the TFP/SP system and the relatively small time series dimension the discriminatory content of the MTSI loss statistics is likely limited on the one hand. On the other hand, documented statistics testing the presumption of joint reduced form normality are significant at any reasonable level such that MTSI loss statistics naturally apply for a comparison of distinct structural assumptions.

4.2.2 Structural analysis

Triangular schemes MTSI loss statistics for the US system are reported in the lower part of Table 3. All estimated VAR systems deliver smallest loss statistics for the lower triangular decomposition ($\mathcal{L}_l < \mathcal{L}_u$) implying that innovations in stock prices have no contemporaneous effect on reduced form innovations featuring total factor productivity. Thus, total factor productivity is confirmed to bear the interpretation of a news process.

Applying resampling techniques the decision in favor of the lower triangular scheme is supported in about 60% of all replications. These frequencies are, however, within 95% confidence intervals for such a statistic under inconclusive normality. Moreover, for the level systems the data based \mathcal{L}_l loss is in about 80% less than the \mathcal{L}_u^* statistics obtained by resampling. The evidence in favor of symmetric instantaneous causation is weaker as it is for the most likely (i.e. the lower triangular) recursive scheme. Interestingly, Beaudry and Portier (2006) rely on this identifying assumption on the basis of a-priori reasoning.

Impulse response functions Recalling the marked evidence against joint normality featuring the TFP/SP system and the discussion in Section 2.4 it is noteworthy that the interpretation of common IRFs has to be given a local perspective, and, moreover, the selection of $(0, 1)$ or $(1, 0)$ shocks could be subjected to criticism.

To illustrate in some more detail the issues arising from non Gaussian structural innovations for impulse response modeling, suppose one is interested in the effects of a unit shock hitting TFP on both variables of the TFP/SP system. Adopting a lower triangular decomposition of the reduced form covariance a scatter diagram of the implied structural innovations (VAR 2, level data, with trend) is shown in Figure 6. Confirming the test on joint normality for the reduced form disturbances the structural innovations appear to differ markedly from an elliptical shape. Using simple kernel techniques to evaluate the expectations of e_{2t} conditional on $e_{1t} = \pm 1$ underpin the case of local contemporaneous characteristics, i.e. $E[e_{2t}|e_{1t} = 1] = -0.144$ and $E[e_{2t}|e_{1t} = -1] = -0.016$. Accordingly, Figure 7 shows the IRFs for two distinct shock scenarios, $(1, 0)$ and $(1, -0.144)$, along with 95% bootstrap based confidence intervals. As a result, both estimates IRFs agree in their overall shape. In the short run a positive technology shock stimulates stock prices. For the 'more realistic' right hand side response of stock prices, however, the initial stimulation is weaker and, moreover, significance of the effect is not diagnosed at all.

5 Conclusions

In this paper a loss functional is introduced that carries informational content to discriminate between competing structural relations in a non Gaussian framework. Although *Moment Targeted Structural Innovations* (MTSI) are determined within a fully parametric class of copula distributions the approach is nonparametric in the sense that it does not rely on the a-priori choice of a particular copula distribution. Providing a ranking of alternative just identifying structural data representations, MTSI assist in determining a data supported structural view at the economy. Put differently, MTSI highlight to

which extent particular identifying restrictions are not supported by empirical processes. The MTSI concept fully relies on the notion of data being independent and identically distributed over the time dimension such that resampling with replacement supports an analyst to resolve inferential issues with regard to competing a-priori settings of structural data relations.

For the purpose of simplicity and computational tractability the outline of MTSI in this paper has addressed the bivariate case exclusively. The generalization towards higher order systems is straightforward and feasible in principle. As an alternative, however, one might also consider for a $K > 2$ dimensional system all possible bivariate subsystems and integrate partial evidence to an overall system perspective. The determination of MTSI in higher dimensional systems is an interesting direction of future research. Moreover, the proposed loss functional to evaluate competing MTSI processes exploits only third and fourth order moments of a prespecified marginal distribution. Identifying restrictions to be drawn from even higher moments are not exploited. Using higher order data characteristics might easily suffer from rather volatile empirical moments. In how far higher order information can be used to enhance the informational content of the loss measure proposed in this study might also be addressed in future work.

Throughout, the derivation of MTSI relied on the presumption of underlying Gaussian innovations generating the mixed normal structural innovations. Utilizing a more general framework, mixed symmetric distributions say, appears as an interesting avenue for generalizing the scope of structural moment targeting in empirical practice. The derivation of powerful loss statistics under more general than mixed normal settings is regarded a further direction of future research.

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Tables and Figures

			$D_l, C^{(1)}$					$D_l, C^{(2)}$			$D_u, C^{(1)}$		
	τ, ρ	$T; \sigma$.1	.3	.5	.7	.9	.1	.3	.5	.1	.3	.5
CLA	2	200	.546	.545	.540	.646	.512	.552	.550	.523	.552	.550	.521
CLA	4	200	.719	.755	.519	.733	.544	.713	.751	.510	.713	.751	.509
GUM	2	200	.528	.537	.532	.544	.515	.520	.529	.518	.520	.529	.519
GUM	4	200	.559	.563	.492	.484	.512	.557	.563	.490	.557	.563	.491
FRA	2	200	.514	.548	.574	.594	.566	.504	.539	.575	.504	.539	.574
FRA	4	200	.545	.621	.672	.718	.649	.533	.605	.664	.533	.606	.663
STT	2	200	.510	.528	.526	.512	.525	.508	.522	.525	.508	.524	.524
STT	4	200	.504	.527	.529	.520	.530	.509	.530	.530	.509	.531	.529
PLA	2	200	.511	.544	.572	.595	.558	.508	.541	.574	.507	.540	.573
PLA	4	200	.547	.653	.726	.760	.699	.546	.640	.725	.546	.639	.725
FGM	8	200	.562	.671	.679	.606	.688	.517	.618	.696	.516	.618	.695
AMH	2	200	.497	.527	.560	.594	.551	.526	.537	.561	.526	.536	.559
GAU	2	200	.509	.502	.503	.504	.508	.497	.494	.512	.497	.494	.512
CLA	2	400	.625	.620	.564	.784	.527	.621	.613	.559	.621	.613	.558
GUM	2	400	.539	.558	.557	.596	.545	.550	.563	.554	.550	.562	.553
FRA	2	400	.537	.595	.649	.674	.623	.517	.577	.640	.517	.578	.639
STT	2	400	.514	.527	.528	.519	.539	.499	.534	.536	.500	.533	.538
PLA	2	400	.541	.597	.653	.682	.630	.515	.585	.651	.515	.585	.650
FGM	8	400	.562	.682	.732	.665	.723	.531	.620	.753	.530	.620	.753
AMH	2	400	.527	.577	.629	.689	.600	.540	.576	.623	.540	.575	.623
GAU	2	400	.505	.502	.504	.499	.496	.506	.496	.498	.506	.495	.497
CLA	2	1000	.742	.779	.657	.965	.590	.746	.779	.661	.747	.779	.661
GUM	2	1000	.628	.679	.670	.805	.645	.630	.670	.656	.630	.669	.656
FRA	2	1000	.579	.697	.779	.835	.753	.562	.687	.779	.562	.686	.778
STT	2	1000	.521	.595	.624	.557	.639	.535	.600	.634	.534	.600	.634
PLA	2	1000	.571	.703	.799	.843	.770	.564	.695	.799	.564	.695	.799
FGM	8	1000	.603	.670	.812	.776	.742	.552	.636	.809	.551	.636	.808
AMH	2	1000	.580	.674	.743	.863	.698	.580	.669	.742	.580	.669	.742
GAU	2	1000	.491	.504	.504	.503	.508	.508	.513	.508	.509	.513	.508

Table 1: Simulation results: Frequencies of detecting the correct direction of instantaneous causation. Underlying random variables ξ_t are drawn from alternative copulae, namely Clayton (CLA), Gumbel (GUM), Frank (FRA), Student- t with 10 degrees of freedom (STT), Plackett, FGM, AMH, and the Gaussian (GAU). D_l (lower triangular) and D_u (upper) indicate the underlying true recursion while τ, ρ indicate the concordance or the linear correlation featuring (CLA, GUM, FRA, PLA, FGM, ALH) or (STT, GAU), respectively. Rotated copulae are obtained by simulating from $C(\zeta_1, (1 - \zeta_2))$. Simulation results for underlying rotated copulae $C((1 - \zeta_1), \zeta_2)$ and $C((1 - \zeta_1), (1 - \zeta_2))$ are in full analogy to those listed in the table for the cases of negative and positive dependence.

co.	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	LM1	LM5	corrs ($\hat{\sigma}$)		
CA	2.95E-06 (1.51)	.030 (3.92)	.968 (120.)	0.040 (0.84)	3.907 (0.56)	CA	FR	UK
FR	4.84E-06 (2.25)	0.069 (4.24)	0.922 (55.5)	5.387 (0.02)	12.46 (0.03)	.230		
UK	1.47E-06 (1.03)	0.024 (3.29)	0.973 (117.)	0.299 (0.58)	2.833 (0.73)	.207	.293	
US	2.19E-06 (1.10)	0.047 (2.61)	0.954 (60.1)	5.033 (0.02)	25.70 (0.00)	.440	.214	.159

Table 2: GARCH(1,1) parameter estimates and model diagnostics for changes of breakeven inflation rates in Canada (CA), France (FR) the UK and the US, denoted ε_t . For a particular variance process the conditional variance $\sigma_t^2 = E[\varepsilon_t^2 | \mathcal{F}_{t-1}]$ characterizing the time series ε_t is, $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$. Values in parentheses are either t -ratios (for parameter estimates) or p -values (for the LM-statistic testing against conditional heteroskedasticity in GARCH implied standardized residuals). The right hand side panel shows unconditional correlations for the standardized GARCH(1,1) residuals.

	\mathcal{L}_l	\mathcal{L}_u	\mathcal{L}_s	\hat{P}_{lu}	\hat{P}_{ls}	\hat{P}_{us}	\hat{Q}_{lu}	\hat{Q}_{ul}	\hat{Q}_{ls}	\hat{Q}_{us}
GARCH(1,1) residuals from breakeven inflation rates										
CA/US	5.805	4.799	5.760	.202	.328	.710	.422	.930	.628	.834
FR/US	6.667	5.605	6.105	.325	.341	.645	.440	.821	.500	.769
UK/US	4.326	4.697	4.377	.460	.472	.550	.897	.820	.899	.833
UK/FR	5.933	7.268	7.737	.676	.772	.537	.927	.632	.950	.796
US total factor productivity and stock prices (VAR residuals)										
VAR2, t, c	0.972	1.221	1.084	.641	.594	.340	.784	.618	.747	.620
VAR2, c	0.926	1.188	1.051	.613	.562	.351	.756	.604	.733	.607
Δ VAR2	2.653	3.700	3.210	.600	.468	.279	.604	.389	.576	.383
Δ VAR1	2.315	3.043	2.678	.592	.469	.289	.591	.420	.565	.410

Table 3: Empirical MTSI loss measures for bivariate systems of international breakeven inflation rates (upper panel) and US total factor productivity vs. stock prices (lower panel). In the left hand side of the table loss statistics $\mathcal{L}_l, \mathcal{L}_u, \mathcal{L}_s$ are given, respectively, for the lower and upper triangular Cholesky and the square root decomposition of the reduced form covariance matrix. Smallest loss measures \mathcal{L} are in bold face. Bootstrap based frequencies of particular relations are documented in the right hand side. The bootstrap based frequency estimates are defined in (12). The number of bootstrap replications is $H = 1000$, and accordingly and ad-hoc significance bound is $0.5 \pm 2\sqrt{0.25/1000} = [0.468, 0.532]$. Determining critical values for the \hat{P}_{lu} statistics by means of simulations from the Gaussian obtains [33.7, 68.9] and [36.5, 63.0] ([34.3, 63.4] and [39.2, 60.6]) as 95% and 90% confidence regions for sample size $T = 200$ ($T = 2000$), respectively.

	TFP	SP	TFP	SP	Δ TFP	Δ SP	Δ TFP	Δ SP
TFP(-1)	0.944 (13.7)	0.165 (0.42)	0.945 (13.8)	0.227 (0.57)	-0.036 (-0.51)	0.295 (0.73)	-0.024 (-0.35)	0.217 (0.55)
TFP(-2)	0.040 (0.58)	-0.414 (-1.06)	0.045 (0.66)	-0.225 (-0.57)	0.085 (1.24)	-0.603 (-1.54)	-	-
SP(-1)	0.057 (4.66)	1.283 (18.6)	0.058 (4.99)	1.346 (19.8)	0.054 (4.35)	0.367 (5.17)	0.058 (4.87)	0.345 (5.09)
SP(-2)	-0.055 (-4.47)	-0.288 (-4.12)	-0.057 (-4.83)	-0.354 (-5.15)	0.007 (0.53)	-0.042 (-0.56)	-	-
c	0.021 (1.20)	-0.030 (-0.31)	0.019 (1.14)	-0.072 (-0.73)	0.003 (3.71)	0.008 (1.82)	0.003 (4.33)	0.005 (1.36)
t	2.37E-05 (0.53)	0.001 (3.33)	-	-	-	-	-	-
LM 10	3.180 (0.53)		2.620 (0.62)		3.512 (0.48)		3.152 (0.53)	
JB	95.12		80.03		89.85		90.03	

Table 4: VAR parameter estimates for bivariate systems of US total factor productivity and stock prices (Beaudry and Portier 2006). Model selection criteria (AIC, BIC, HQ, not documented) are in favor of a VAR order two if the model is specified in levels and contains a linear trend). VAR parameter estimates are documented with t -ratios in parentheses. LM is the Lagrange Multiplier test (p -value in parentheses) for multivariate serial correlation up to lag order 10. JB is the Jarque-Bera statistic on joint normality of both reduced form residual processes. Under the null hypothesis of normality JB is χ^2 distributed with 4 degrees of freedom, p -values are not provided. Dependent variables are listed in the top row, where Δ is short for the first difference operator. The left hand side column lists the conditioning variables from which TFP and SP are either in levels or in first differences. Moreover, c and t signify a constant and a trend, respectively, entering the VAR. Estimation and diagnostic results are obtained from Eviews 6.0.

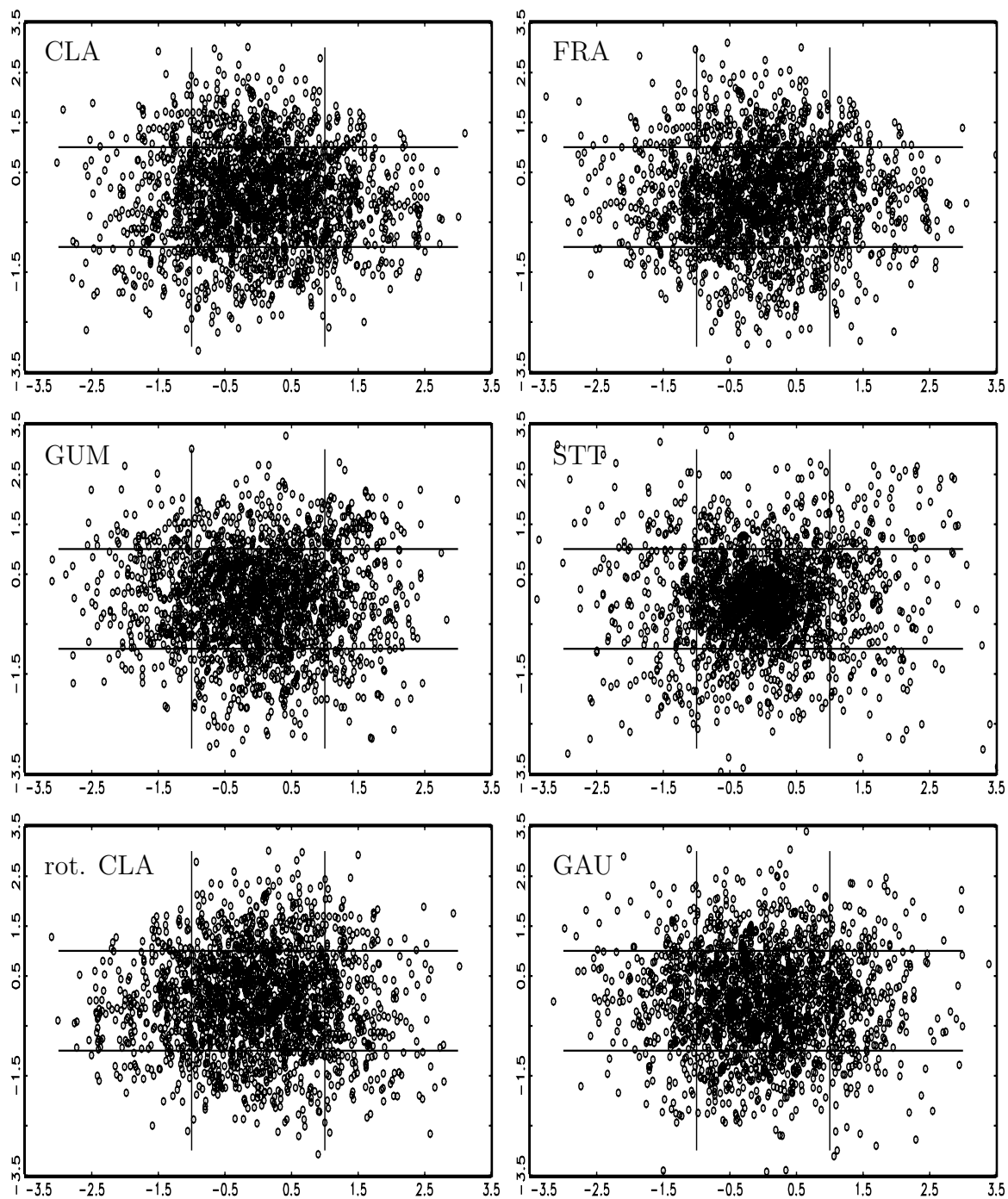


Figure 1: Simulated iid uncorrelated innovation processes e_t ($T = 2000$) from 6 alternative copula distributions, Clayton (CLA, upper left), Gumbel (GUM, medium left), rotated Clayton (lower left), Frank (FRA, upper right), Student- t with 10 degrees of freedom (STT, medium right) and Gaussian (GAU, lower right). Kendall's τ (or the linear correlation for STT and GAU) is $\tau, \rho = 0.3$. Horizontal and vertical lines indicate absolute unit levels.

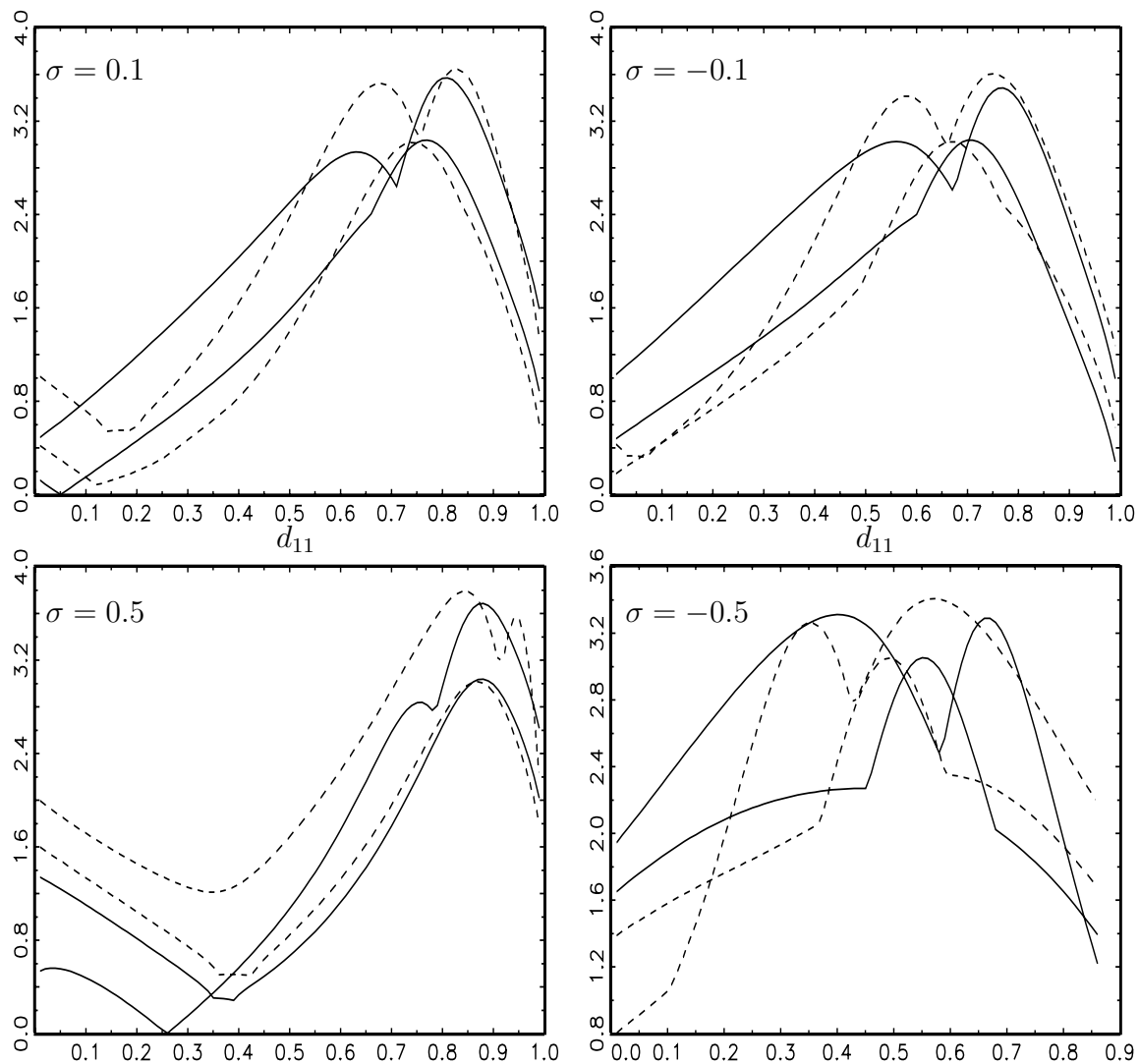


Figure 2: Largest moduli (y -axis) of the matrix $G^{(3)}$ as implied by distinct levels of reduced correlation (σ), copula covariance (ω) and alternative upper left elements of D , d_{11} (x -axis). Solid curve give results for positive choices $\omega = 0.1, 0.5$ and dashed curves are for negative copula covariance, $\omega = -0.1, -0.5$.

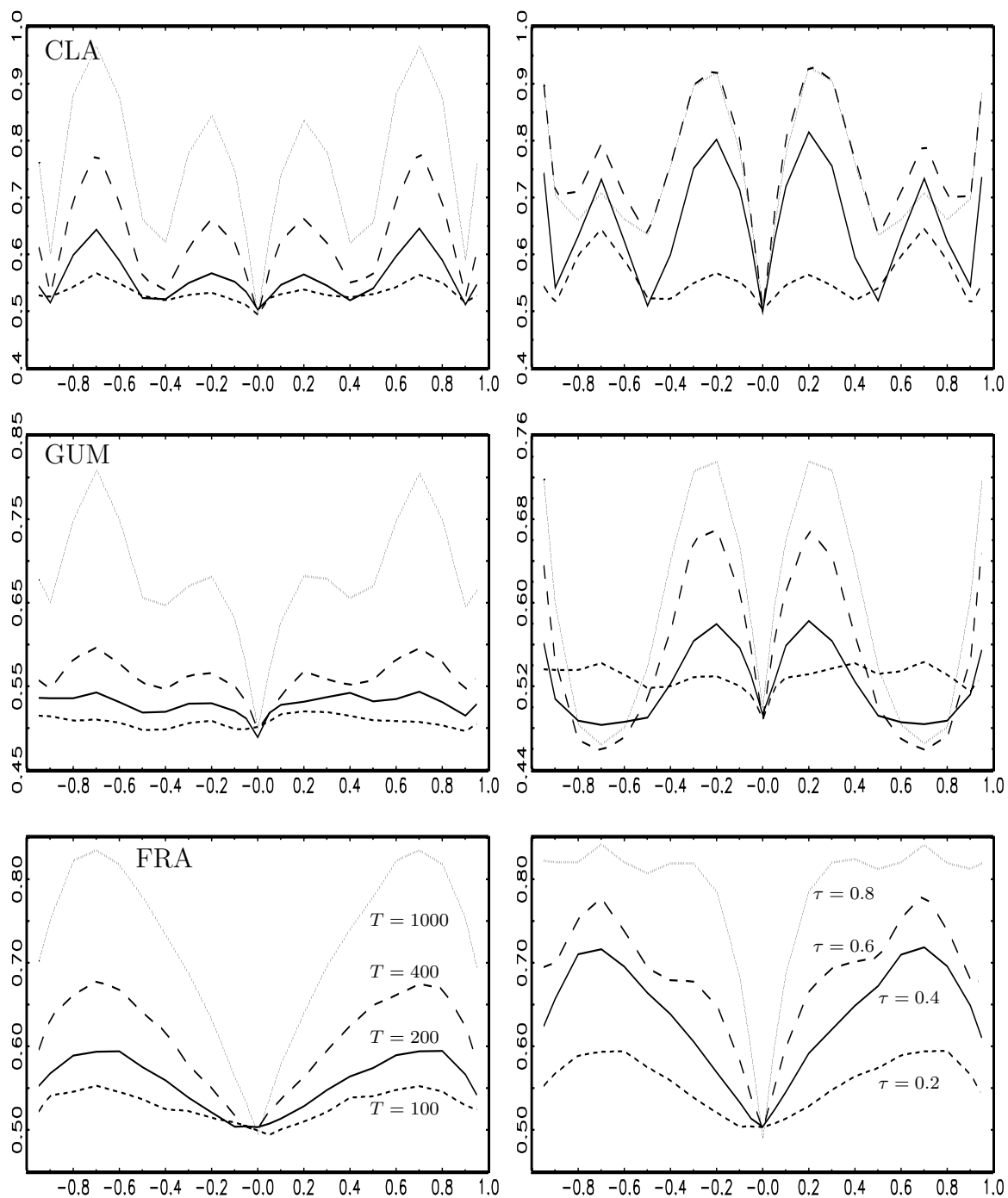


Figure 3: Simulation results 1: Frequencies of correct detection of a lower triangular recursive pattern, \hat{f}_l , for 3 copulae. Estimates \hat{f}_l are shown for given $\tau = 0.2$ and sample sizes $T = 100, 200, 400$ and $T = 1000$ (left hand side) and for given $T = 200$ and concordance measures for the elements of ξ_t being $\tau = 0.2, 0.4, 0.6$ and $\tau = 0.8$ (right hand side). For identification of single figures see the bottom panels.

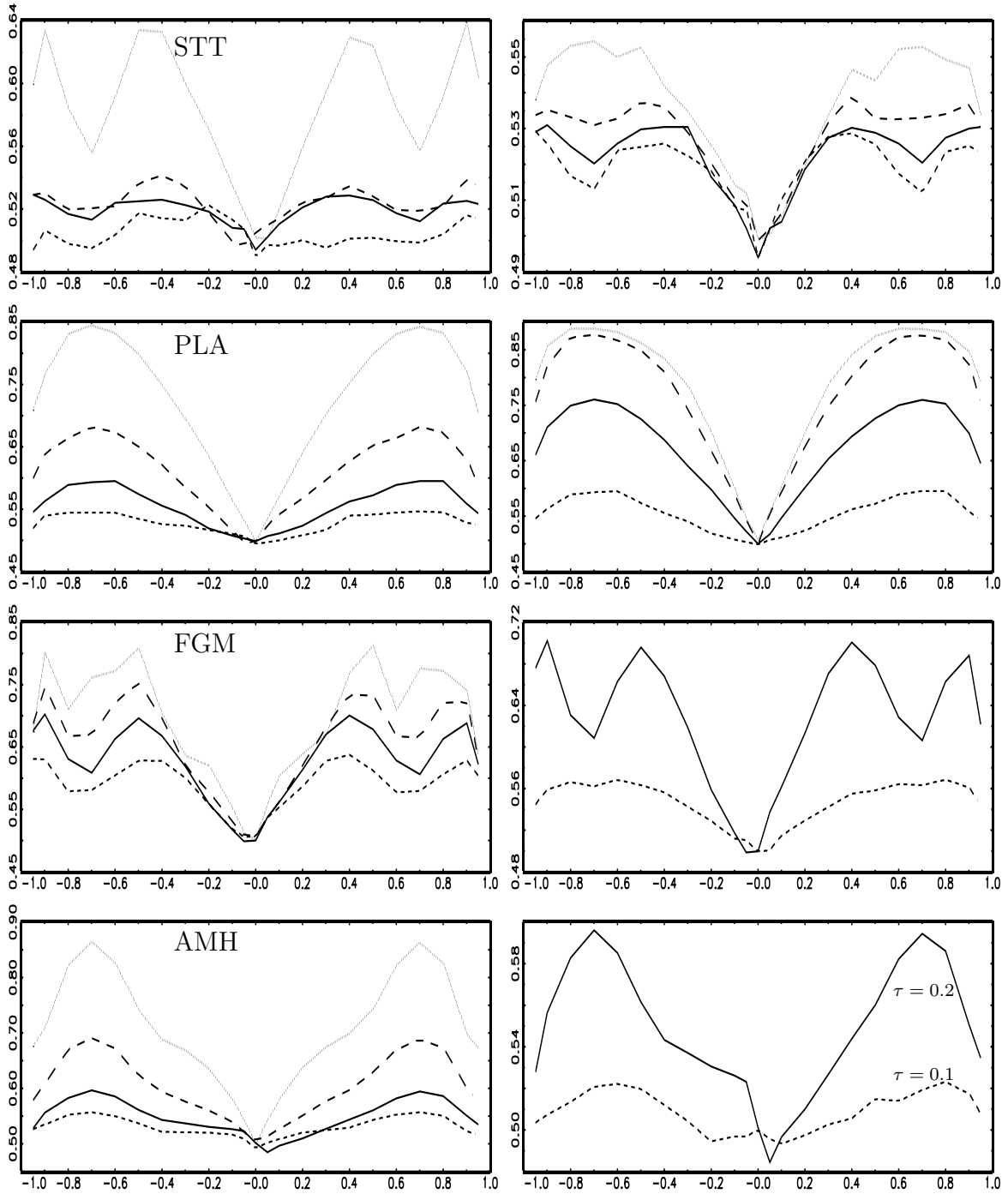


Figure 4: Simulation results 2: Frequencies of correct detection of a lower triangular recursive pattern, \hat{f}_l for 4 copulae. For the FGM and AMH copula distinct levels of concordance are used for simulation $\tau = 0.1$ and $\tau = 0.2$. For further reference see Figure 3.

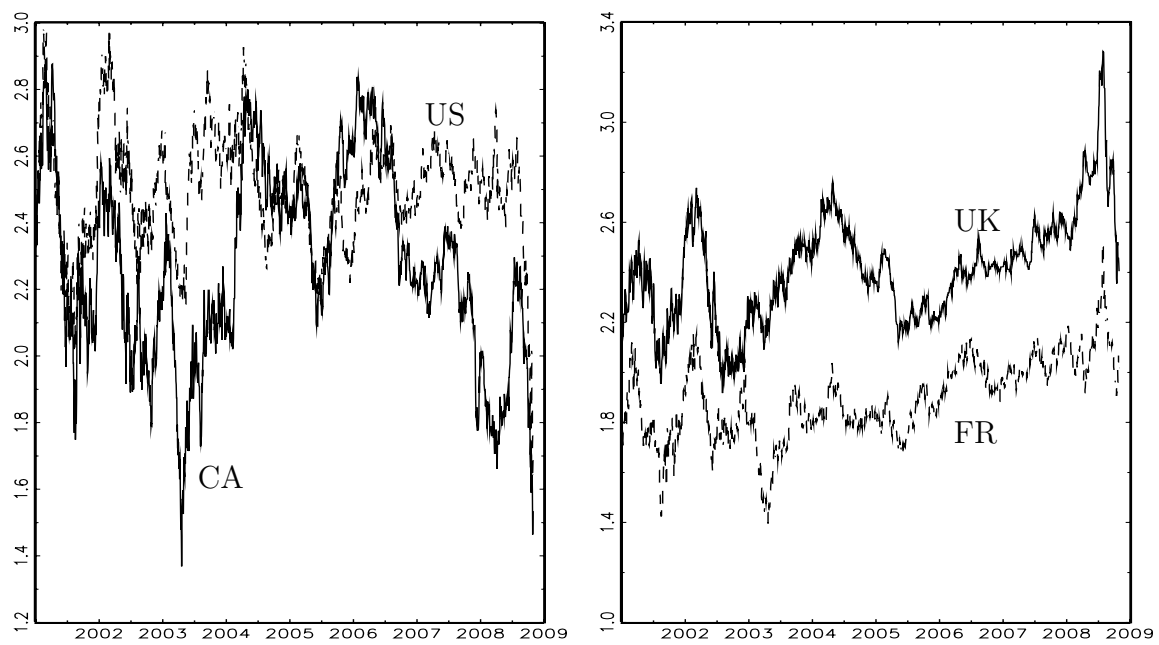


Figure 5: Liquidity adjusted breakeven inflation rates for Canada and the US (left hand side) and France and the UK (right hand side).

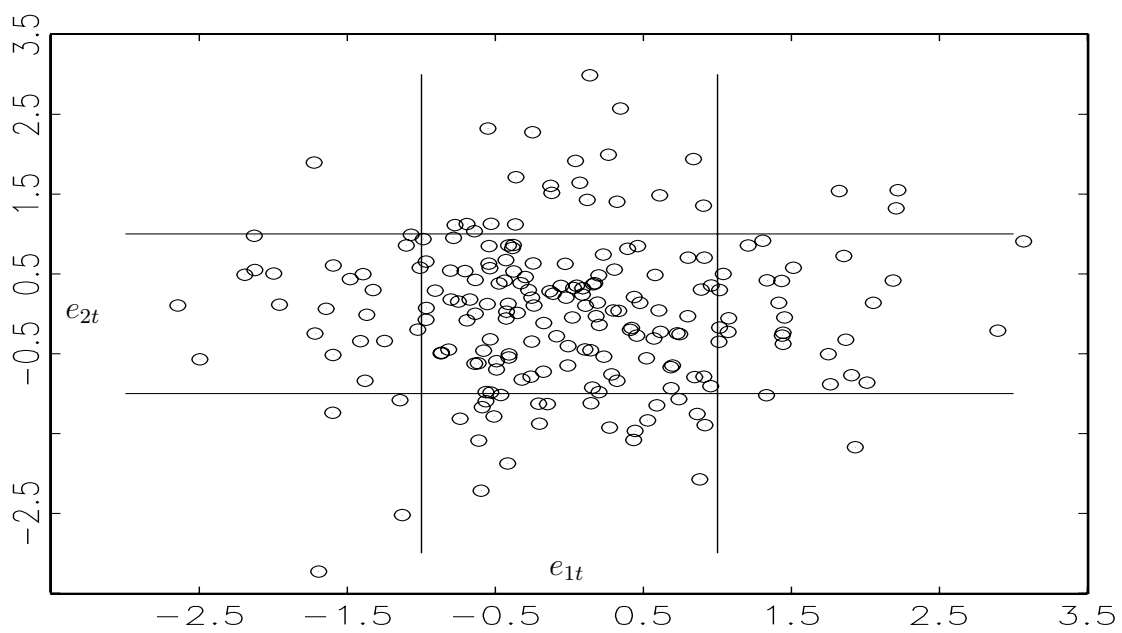


Figure 6: Lower triangular implied structural innovations for the US TFP/SP system as implied by a VAR model of order 2 estimated for level data and including a deterministic trend.

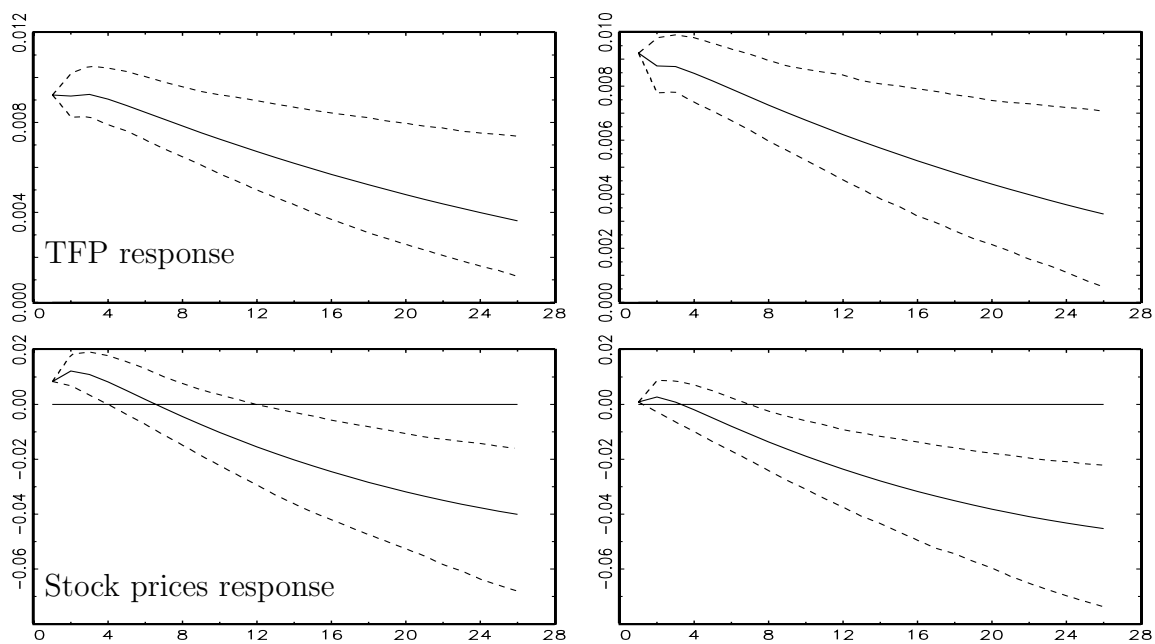


Figure 7: Estimated responses to a unit TFP shock in the US TFP/SP system. In the left hand side panels the unit shock is $(1, 0)$ while in the right hand side the considered shock is $(1, -0.144)$. 95% confidence bands are determined by resampling with replacement from reduced form disturbances u_t and reestimating the VAR.

Appendix A - Third order moment linkage

In this appendix explicit representations of the moment system in (6) is given, for d_{11} denoting the upper left element of D and \underline{w}_1 and \underline{w}_2 being the typical diagonal and off-diagonal elements of $\Omega^{-1/2}$, respectively. Recall also the restrictions/representation in (5) and (4). The system in (6) is, in specific,

$$\begin{aligned}
\nu_{30} &= \{d_{11}^3 \underline{w}_1^3 + 3d_{11}^2(1 - d_{11}^2)^{1/2} \underline{w}_1 \underline{w}_2 + 3d_{11}(1 - d_{11}^2) \underline{w}_1 \underline{w}_2^2 + (1 - d_{11}^2)^{3/2} \underline{w}_2^3\} \theta_{30} \\
&+ \{3((1 - d_{11}^2)^{1/2} \underline{w}_1 + d_{11} \underline{w}_2)(d_{11} \underline{w}_1 + (1 - d_{11}^2)^{1/2} \underline{w}_2)^2\} \theta_{21} \\
&+ \{3((1 - d_{11}^2)^{1/2} \underline{w}_1 + d_{11} \underline{w}_2)^2(d_{11} \underline{w}_1 + (1 - d_{11}^2)^{1/2} \underline{w}_2)\} \theta_{12} \\
&+ \{(1 - d_{11}^2)^{3/2} \underline{w}_1^3 + 3d_{11}(1 - d_{11}^2) \underline{w}_1 \underline{w}_2 + 3d_{11}^2(1 - d_{11}^2)^{1/2} \underline{w}_1 \underline{w}_2^2 + d_{11}^3 \underline{w}_2^3\} \theta_{03} \\
\nu_{21} &= \{d_{11}^2(1 - d_{22}^2)^{1/2} \underline{w}_1^3 + 2d_{11}(1 - d_{11}^2)^{1/2}(1 - d_{22}^2)^{1/2} \underline{w}_1 \underline{w}_2 + d_{11}^2 d_{22} \underline{w}_1^2 \underline{w}_2 \\
&+ (1 - d_{11}^2)(1 - d_{22}^2)^{1/2} \underline{w}_1 \underline{w}_2^2 + 2d_{11}(1 - d_{11}^2)^{1/2} d_{22} \underline{w}_1 \underline{w}_2^2 + (1 - d_{11}^2) d_{22} \underline{w}_2^3\} \theta_{30} \\
&+ \{(d_{11} \underline{w}_1 + (1 - d_{11}^2)^{1/2} \underline{w}_2)^2(d_{22} \underline{w}_1 + (1 - d_{22}^2)^{1/2} \underline{w}_2) \\
&+ 2((1 - d_{11}^2)^{1/2} \underline{w}_1 + d_{11} \underline{w}_2)(d_{11} \underline{w}_1 + (1 - d_{11}^2)^{1/2} \underline{w}_2)((1 - d_{22}^2)^{1/2} \underline{w}_1 + d_{22} \underline{w}_2)\} \theta_{21} \\
&+ \{2((1 - d_{11}^2)^{1/2} \underline{w}_1 + d_{11} \underline{w}_2)(d_{11} \underline{w}_1 + (1 - d_{11}^2)^{1/2} \underline{w}_2)(d_{22} \underline{w}_1 + (1 - d_{22}^2)^{1/2} \underline{w}_2) \\
&+ ((1 - d_{11}^2)^{1/2} \underline{w}_1 + d_{11} \underline{w}_2)^2((1 - d_{22}^2)^{1/2} \underline{w}_1 + d_{22} \underline{w}_2)\} \theta_{12} \\
&+ \{((1 - d_{11}^2) d_{22} \underline{w}_1^3 + (1 - d_{11}^2)(1 - d_{22}^2)^{1/2} \underline{w}_1^2 \underline{w}_2 + 2d_{11}(1 - d_{11}^2)^{1/2} d_{22} \underline{w}_1^2 \underline{w}_2 \\
&+ 2d_{11}(1 - d_{11}^2)^{1/2}(1 - d_{22}^2)^{1/2} \underline{w}_1 \underline{w}_2^2 + d_{11}^2 d_{22} \underline{w}_1 \underline{w}_2^2 + d_{11}^2(1 - d_{22}^2)^{1/2} \underline{w}_2^3\} \theta_{03} \\
\nu_{12} &= \{d_{11}(1 - d_{22}^2) \underline{w}_1^3 + (1 - d_{11}^2)^{1/2}(1 - d_{22}^2) \underline{w}_1^2 \underline{w}_2 + 2d_{11}(1 - d_{22}^2)^{1/2} d_{22} \underline{w}_1^2 \underline{w}_2 \\
&+ 2(1 - d_{11}^2)^{1/2}(1 - d_{22}^2)^{1/2} d_{22} \underline{w}_1 \underline{w}_2^2 + d_{11} d_{22}^2 \underline{w}_1 \underline{w}_2^2 + (1 - d_{11}^2)^{1/2} d_{22}^2 \underline{w}_2^3\} \theta_{30} \\
&+ \{2(d_{11} \underline{w}_1 + (1 - d_{11}^2)^{1/2} \underline{w}_2)(d_{22} \underline{w}_1 + (1 - d_{22}^2)^{1/2} \underline{w}_2)((1 - d_{22}^2)^{1/2} \underline{w}_1 + d_{22} \underline{w}_2) \\
&+ ((1 - d_{11}^2)^{1/2} \underline{w}_1 + d_{11} \underline{w}_2)((1 - d_{22}^2)^{1/2} \underline{w}_1 + d_{22} \underline{w}_2)^2\} \theta_{21} \\
&+ \{(d_{11} \underline{w}_1 + (1 - d_{11}^2)^{1/2} \underline{w}_2)(d_{22} \underline{w}_1 + (1 - d_{22}^2)^{1/2} \underline{w}_2)^2 \\
&+ 2((1 - d_{11}^2)^{1/2} \underline{w}_1 + d_{11} \underline{w}_2)(d_{22} \underline{w}_1 + (1 - d_{22}^2)^{1/2} \underline{w}_2)((1 - d_{22}^2)^{1/2} \underline{w}_1 + d_{22} \underline{w}_2)\} \theta_{12} \\
&+ \{(1 - d_{11}^2)^{1/2} d_{22}^2 \underline{w}_1^3 + 2(1 - d_{11}^2)^{1/2}(1 - d_{22}^2)^{1/2} d_{22} \underline{w}_1^2 \underline{w}_2 + d_{11} d_{22}^2 \underline{w}_1^2 \underline{w}_2 \\
&+ (1 - d_{11}^2)^{1/2}(1 - d_{22}^2) \underline{w}_1 \underline{w}_2^2 + 2d_{11}(1 - d_{22}^2)^{1/2} d_{22} \underline{w}_1 \underline{w}_2^2 + d_{11}(1 - d_{22}^2) \underline{w}_2^3\} \theta_{03} \\
\nu_{03} &= \{(1 - d_{22}^2)^{3/2} \underline{w}_1^3 + 3(1 - d_{22}^2) d_{22} \underline{w}_1^2 \underline{w}_2 + 3(1 - d_{22}^2)^{1/2} d_{22}^2 \underline{w}_1 \underline{w}_2^2 + d_{22}^3 \underline{w}_2^3\} \theta_{30} \\
&+ \{3(d_{22} \underline{w}_1 + (1 - d_{22}^2)^{1/2} \underline{w}_2)((1 - d_{22}^2)^{1/2} \underline{w}_1 + d_{22} \underline{w}_2)^2\} \theta_{21} \\
&+ \{3(d_{22} \underline{w}_1 + (1 - d_{22}^2)^{1/2} \underline{w}_2)^2((1 - d_{22}^2)^{1/2} \underline{w}_1 + d_{22} \underline{w}_2)\} \theta_{12} \\
&+ \{d_{22}^3 \underline{w}_1^3 + 3(1 - d_{22}^2)^{1/2} d_{22}^2 \underline{w}_1^2 \underline{w}_2 + 3(1 - d_{22}^2) d_{22} \underline{w}_1 \underline{w}_2^2 + (1 - d_{22}^2)^{3/2} \underline{w}_2^3\} \theta_{03}
\end{aligned}$$

Appendix B - Copula distribution functions

This appendix lists briefly the distribution functions of the copulae, $C^\bullet = C(\zeta_1, \zeta_2; \gamma)$ used in the Monte Carlo exercises described in Section 3. Recall that ζ_1 and ζ_2 are uniform distributed random variables. For a detailed discussion of these copulae (and others) see Cherubini et al. (2004), Joe (1997) or Nelsen (2006).

1. AMH

$$C^{\text{AMH}} = \frac{\zeta_1 \zeta_2}{1 - \gamma(1 - \zeta_1)(1 - \zeta_2)}, \gamma \in [-1, +1].$$

2. Clayton copula

$$C^{\text{CLA}}(\zeta_1, \zeta_2; \gamma) = (\zeta_1^{-\gamma} + \zeta_2^{-\gamma} - 1)^{-1/\gamma}, \gamma \in [-1, \infty), \gamma \neq 0.$$

3. Frank copula

$$C^{\text{FRA}}(\zeta_1, \zeta_2; \gamma) = -\frac{1}{\gamma} \ln \left[1 - \frac{(1 - e^{-\gamma \zeta_1})(1 - e^{-\gamma \zeta_2})}{(1 - e^{-\gamma})} \right], \gamma \in \mathbb{R}/\{0\}$$

4. FGM

$$C^{\text{FGM}} = \zeta_1 \zeta_2 + \gamma \zeta_1 \zeta_2 (1 - \zeta_1)(1 - \zeta_2), \gamma \in [-1, +1]$$

5. Gaussian

$$C^{\text{GAU}} = \Phi_R(\Phi^{-1}(\zeta_1), \Phi^{-1}(\zeta_2)),$$

where $\Phi_R(\cdot, \cdot)$ is the bivariate Gaussian distribution with correlation matrix R .

6. Gumbel copula

$$C^{\text{GUM}} = \exp \left\{ - [(-\ln \zeta_1)^\gamma + (-\ln \zeta_2)^\gamma]^{1/\gamma} \right\}, 1 \leq \gamma < \infty.$$

7. Plackett

$$C^{\text{PLA}} = \frac{[1 + (\gamma - 1)(\zeta_1 + \zeta_2)] - \sqrt{[1 + (\gamma - 1)(\zeta_1 + \zeta_2)]^2 - 4\zeta_1 \zeta_2 \gamma (1 - \gamma)}}{2(\gamma - 1)}, \gamma > 0, \gamma \neq 1.$$

8. Student- t

$$C^{\text{STT}} = T_{R, \kappa}(t_\kappa^{-1}(\zeta_1), t_\kappa^{-1}(\zeta_2)),$$

where κ is the degrees of freedom parameter, and $T_{R, \kappa}(\cdot)$ be the bivariate Student distribution with κ degrees of freedom and correlation matrix R . Moreover, $t_\kappa(\cdot)$ is the univariate Student- t CDF with κ degrees of freedom.