Betz, Regina; Calford, Evan M.; Heinzel, Christoph

Conference Paper
Initial Allocation Effects in Permit Markets with Bertrand Output Oligopoly


Provided in Cooperation with:
Verein für Socialpolitik / German Economic Association


This Version is available at:
http://hdl.handle.net/10419/37424

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Initial Allocation Effects in Permit Markets with Bertrand Output Oligopoly

Evan M. Calford    Christoph Heinzel    Regina Betz*

February 24, 2010

Abstract

We analyse the efficiency effects of the initial permit allocation given to firms with market power in both permit and output market. We examine two models: a long-run model with endogenous technology and capacity choice, and a short-run model with fixed technology and capacity. In the long run, quantity pre-commitment with Bertrand competition can yield Cournot outcomes also under emissions trading. In the short run, Bertrand output competition reproduces the effects derived under Cournot competition, but displays higher pass-through profits. In a second-best setting of overallocation, a tighter emissions target tends to improve permit-market efficiency in the short run.

Keywords: emissions trading, initial permit allocation, Bertrand competition, EU ETS, endogenous technology choice, Kreps and Scheinkman

*Calford, Betz: University of New South Wales. Heinzel: Toulouse School of Economics (LERNA). Valuable comments by Eddie Andersen, AJ Bostian, Zaida Contreras, Arghya Ghosh, Philippe Mahenc, Juan-Pablo Montero, Hodaka Morita, François Salanié, Oliver Sartor and Adeline Tubb, as well as participants of the National Honours Colloquium 2008 and the 2009 conferences of AARES and EAERE and a seminar in Toulouse are gratefully acknowledged. The usual disclaimer applies. This research was supported financially through the Environmental Economics Research Hub which is funded by the Australian Commonwealth Environmental Research Facilities (CERF) programme and the Australian Research Council (ARC) project DP0878580.
1 Introduction

Prices reflect scarcity in well-functioning markets. In its first trading phase 2005-2007, the European Union Emissions Trading Scheme (EU ETS) seemed to struggle with this principle. Following the information release that the market was long, the price of EU Allowances (EUA) after a first sharp fall began to rise again, and stayed in a range of €15–18 for a period of 6 months (Alberola et al. 2008). During this period, it was disconnected from almost all market fundamentals. Studies, implicitly assuming that all of the 11,000 covered installations were actively trading, did not find evidence of market power (e.g., Convery and Redmond 2007, Trotignon and Delbosc 2008). But only a few companies and sectors (mainly electric utilities, banks, and some hedge funds) were active in the market in the early years.

With regard to the initial allocation only the power sector was short in the first trading period (Ellerman and Buchner 2008). In addition, this sector can pass-through the costs to consumers generating so-called wind-fall profits.

We seek to explain a positive allowance price, starting from the possibility of an overall excess permit allocation received for free in a situation where two or more firms have market power in both permit and product market and trade permits. We examine two kinds of models with permit/product market interaction. In our long-run models, the choices of production technology and production capacity are endogenous, technology choice determines the marginal cost of output production and the level of emissions per unit of production. In the short-run model, technology and capacity are exogenous and thus fixed.

Our modeling is inspired by Kreps and Scheinkman (1983). We modify their framework in two ways. For the long-run analysis, we introduce, following von der Fehr (1993), an additional initial stage of technology choice and a stage of permit choice. Hence, if the permit market is interpreted as an input market, our analysis can be considered as a particular case

1The EUA spot price, from a high of almost €30 in late April 2006, plummeted 54% in only four days before unexpectedly rebounding and holding in the mentioned range from April-October 2006. Then it declined reaching some cents in 2007.

2They determine the market concentration (Herfindahl-Hirschman Index) for companies and single sectors, comparing the initial allowance allocation to the total number of permits initially allocated.

3Windfall profits accrue to a firm, when it receives inputs (such as permits) for free but incorporates their opportunity cost in its pricing decisions. Sijm et al. (2006) estimate for Germany and the Netherlands in 2005 a pass-through rate of 60-100% with windfall profits of about €6 Bn.
of market-making oligopoly, in the sense of Loertscher (2008). In the short-run analysis, we fix technology and capacity choices, and focus on the case of price competition in the product market. We thus complement the typical analysis in the literature on market power in emissions markets and related product markets by consideration of the case of Bertrand output oligopoly. Given that in many non-competitive product markets firms compete in prices rather than quantities (e.g., Loertscher 2008), and that, in particular, for electricity markets Cournot competition is typically found to deliver too few competitive results, the almost exclusive focus on Cournot output competition in this literature seems rather surprising.

The distinction between long run and short run is motivated from the different degrees of flexibility in technology and capacity choice between a mature and a newly established ETS. In a new ETS, the technology and capacity determined in the pre-ETS environment may not be optimal. Moreover, while in the long run policy makers will have better information how the allocation of permits relates to each firm's emissions level, in the short run policy makers have only limited ex-ante knowledge of how firms will react to an ETS. As a consequence, we expect in the long run no firm with market power is allocated more permits than it needs to meet its regulatory obligations. In the short run the policy makers' choice of the initial permits allocation to a firm with market power is unrestrained (other than to be positive).

For the long run, we show that the key result by Kreps and Scheinkman (1983) – that quantity pre-commitment with Bertrand competition leads to Cournot results – still holds under existence of an emissions trading scheme (ETS) with permit/product market interaction. In equilibrium, firms will hold permits such as to exactly cover their emissions. Total industry emissions and the permit price are positively related to the size of the free initial permits allocation to a firm of that sector.

While our long-run analysis was developed directly based on Kreps and Scheinkman (1983), we extend Loertscher’s analysis, for the duopoly case, by allowing for an initial endowment of the input good and analysing initial allocation effects. Moreover, we demonstrate that, under a costly capacity constraint, also Cournot-Bertrand (input-output) competition yields Cournot outcomes while Loertscher sticks to the cases of Cournot-Cournot and Bertrand-Bertrand competition.

See Montero (2009) for an overview over this literature. Two exceptions, considering Bertrand competition, constitute Requate (1993) and Montero (2002a). Requate states, in a setting with only a two-firm industry, that the number of permits issued may act as a capacity constraint, leading to a Kreps-and-Scheinkman style result. Montero compares R&D incentives from environmental-policy instruments under perfect and imperfect output and permit markets in Cournot and Bertrand output duopoly. He sticks to overall permit scarcity and does not treat initial allocation effects.

For emissions-intensive industries, technology will tend to be too dirty and capacity too high.
In the short run, we study how a marginal change in the initial permit allocation to a firm impacts on its equilibrium permit holdings, the permit price, and product market outcomes under Bertrand competition. We consider both elastic and inelastic product demand. In extension to the previous literature, we distinguish an interior and a boundary solution depending on whether, in equilibrium, at least one firm holds excess permits or all firms are constrained in their emissions. In the interior solution, we identify two kinds of distortionary effects: the Hahn (monopsony/monopoly) effects (after Hahn 1984), and the pass-through profit effect. Previously, they have implicitly been distinguished, e.g., by DiSegni Eshel (2005) in a dominant-firm setting, but not considered for Bertrand competition. Pass-through profits, the pass-through profit effect, and thus the effect on the permit price are greater under Bertrand than under Cournot competition, and increase with demand inelasticity. On the boundary, a change in the initial allocation leaves permit holdings, permit price, and product price unaffected.

A positive allowance price in the face of an overall excess permit allocation, as it occurred in the first EU-ETS phase, can, hence, also be explained if a permit-and-product-market oligopolist (whose initial permit allocation differs from its final permit holdings) competes in the product market in prices. As associated with an interior solution, this price development is particularly likely to occur under an overall excess allocation of permits and when permits are initially given for free. Conversely, in the short run, a boundary solution and thus increased permit-market efficiency are the more likely the stricter the emissions target. We do not explicitly investigate an alternative permit allocation mechanism or the implementation of the social optimum. From our study nevertheless a case for auctioning of the initial permit allocation derives, as it will help reduce pass-through profits and thus the distortionary effects of an ETS in both the short and the long run.

Section 2 introduces the model framework we use. Section 3 develops the short-run analysis. Section 4 treats the long-run analysis. We discuss our results in section 5. Section 6 concludes.
2 The Model

Consider a one-period model with four stages of an economy with polluting production and an emissions trading system in which initial permits are allocated for free. Two competing firms, $i = a, b$, with market power on both output and permit markets maximise profits

$$\Pi_i = p_i \cdot q_i - c_i(q_i, \theta_i) - (x_i - x_i^0) \cdot \phi(X) - f_i(K_i, \theta_i),$$

such that $q_i \leq K_i$,\hspace{1cm}(1)

where $p_i$ is the price charged by firm $i$, $q_i$ is the production quantity of firm $i$, $c_i$ is the marginal output production cost of firm $i$, $\theta_i \in [0, 1]$ is firm $i$’s technology parameter, $x_i$ and $x_i^0$ are the final and initial holdings of permits for firm $i$, $\phi(.)$ is the market clearing permit price, $X = x_i + x_j$ is the combined permit holding of firms $i$ and $j$, $f_i$ is the capacity installation cost of firm $i$ and $K_i$ is firm $i$’s capital.\hspace{1cm}(2)

Emissions are by-produced according to the function

$$e_i(q_i, \theta_i) = q_i \cdot (1 - \theta_i).$$

In the permit market there is, moreover, a competitive fringe of polluting firms whose product is no substitute for the output of the firms of the first two types. The competitive fringe in the permit market ensures that the permit supply (price) function, $\phi$, is increasing with respect to the permit holdings of the market power firms.\hspace{1cm}(3)

Throughout we assume a fixed overall level of permits, which may exceed the total amount of emissions from the entire economy.

We consider three formulations of this model. In the long-run models the choices of production technology and production capacity are endogenous, technology choice determines both marginal output production cost and the level of emissions per unit of production. In the short-run technology and capacity are considered to be exogenous factors.

Model 1 Long-run Bertrand model

1. Firms install a production technology.

\hspace{1cm}^7\text{We adopt the convention that the subscript } j \text{ indicates } \neq i. \text{ Our assumptions on the characteristics of the functions used are set out below.}

\hspace{1cm}^8\text{To motivate this interpretation of } \phi \text{ consider the structure of the permit market: at any given permit price, the competitive fringe will demand some proportion of the fixed level of permits, with the level of demand decreasing with permit price (von der Fehr 1993).}
2. Firms install a costly production capacity.

3. Firms purchase emissions permits.

4. Firms compete over prices in the product market.

The profit function as given generally in (1) for firm $i$ in the long-run Bertrand model (henceforth: the Bertrand model) depends on the price it sets, its output, its technology parameter, its permit holdings and its capital, so that $\Pi^B_i(p_i, q_i, \theta_i, x_i, K_i)$.

Model 2 Long-run Cournot model

1. Firms install a production technology.

2. Firms compete over quantities in the product market.

3. Firms purchase emissions permits.

The profit function in (1) for a typical firm in the long-run Cournot model (henceforth: the Cournot model) depends on its quantity produced, its technology parameter, its permit holdings and its capital, so that $\Pi^C_i(q_i, \theta_i, x_i, K_i)$. Note that in the Cournot model $p_i$ in (1) is to be replaced by the market clearing price $P$ which depends on the sum of outputs produced by the two firms.

Model 3 Short-run model

1. Firms purchase emissions permits.

2. Firms compete over prices in the product market.

As technology and capacity are exogenous, in the short-term model the last term of the profit function (1) and the capital restriction vanish and marginal output production costs will only depend on the amount of output produced, so that $\Pi_i(p_i, x_i)$ in the short run.

For the subsequent analysis we invoke the following assumptions:9

Assumption 1 The cost function $c(q, \theta): \mathbb{R}_0^+ \times [0, 1] \to \mathbb{R}_0^+$ is twice continuously differentiable, strictly convex, and satisfies $c(0) = 0$, $\frac{\partial c}{\partial q} \geq 0$ for all $q \in \mathbb{R}_0^+$ and $\frac{\partial c}{\partial \theta} < 0$ for all $\theta \in [0, 1]$.

---

9The treatment of the technology parameter $\theta$ is drawn from von der Fehr (1993).
**Assumption 2** The permit price function $\phi(X): \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$ is non-decreasing, twice continuously differentiable, convex, and satisfies $\phi(0) \geq 0$.

Following Kreps and Scheinkman (1983: 328), we further require:

**Assumption 3** The output price function $P(q)$ is strictly positive on some finite interval $(0, Q)$, on which it is twice-continuously differentiable, strictly decreasing, and concave. For $q \geq Q$, $P(q) = 0$.

**Assumption 4** The installation cost function $f(q, \theta): \mathbb{R}_0^+ \times [0, 1] \rightarrow \mathbb{R}_0^+$ is twice continuously differentiable, convex, and satisfies $f(0, \theta) = 0$, $\frac{\partial f}{\partial q} > 0$ and $\frac{\partial f}{\partial \theta} > 0$ for all $q \in \mathbb{R}_0^+$ and $\theta \in [0, 1]$.

Throughout we focus on the situation where both firms produce a positive amount in equilibrium, and the equilibrium price is strictly positive. That is, we focus only on non-trivial solutions.

Finally, the structures of the games need some discussion. We start with the short-run model. Seeing the permit market as an input market and the product market as a spot market, the given structure may seem natural: having received the initial allocation of permits at the beginning of an ETS firms first seek to cover their permit demand before engaging in the product market. The structure fits equally well if the permit market is interpreted as a forward market. For the case of electricity markets, the structure would rather be the reverse: most electricity is traded forward determining at the same time the permit amount needed to comply with the ETS. We stick to the former structure because it produces the more interesting results.\(^\text{10}\) That is, having the permit choice occurring before product market competition allows for cost raising strategies to be profitable and allows for the investigation of interior solutions in the short run model.

We adopt the same structure in the long-run Bertrand model. For the Cournot model the production decision is made before the permit holding is determined, so as to ensure our

\(^{10}\)Our structure is also considered in von der Fehr (1993: subsection 3.1) for Cournot output competition and, similarly, in Loertscher’s basic model. DiSegni Eshel (2005), von der Fehr (1993: subsection 3.3) consider simultaneous (spot) market competition for product and permits. Allaz and Vila (1993) study the forward market interpretation for a Cournot spot market, Mahenc and Salanié (2004) for a Bertrand spot market (Liski and Montero (2006) analyse the repeated context), but without focus on initial allocation effects.
main result Kreps and Scheinkman holds. The interpretation of the long run results may be applied to both the Cournot and Bertrand models. Note that the Kreps and Scheinkman result of the long-run analysis in section 4 is robust against a simultaneous inversion of stages 2 and 3 in both the Cournot and Bertrand models.\(^{11}\)

### 3 Short-Run Analysis

We solve the short-run model for sub-game perfect Nash equilibria using backwards induction beginning with the second stage. We also consider alternative assumptions that give rise to a case of inelastic product demand.\(^{12}\)

In the short-run we consider there to be two types of identical firms, rather than two individual firms. Entry and exit is not explicitly considered; we assume that the number of firms is fixed but that there is enough firms to meet demand. This ensures that the capacity constraint does not directly influence the results in the short-run - as the short-run model represents the initial stages of an ETS, during which overall production levels are likely to fall relative to business as usual baselines, it is reasonable that production constraints are not binding.

Firms of type \(a\) have lower marginal production costs but a higher emission intensity than firms of type \(b\).\(^{13}\)

\[
c_a(q_a) + d_a \cdot \phi(X) < c_b(q_b) + d_b \cdot \phi(X) \quad \text{with} \quad d_a > d_b, \tag{3}
\]

where \(d_i\) is the emission intensity per unit of production \((i = a, b)\). The requirement that type-\(a\) firms have lower carbon-inclusive marginal costs will be relaxed in subsection 3.1. In the short run, a firm faces a fixed emissions intensity. The only abatement opportunity

---

\(^{11}\)The results are unlikely to be robust with respect to simultaneous permit and product market decisions as the result of Lemma 5 will no longer hold.

\(^{12}\)The case of inelastic product demand provides the motivation for choosing Bertrand competition for the short run model. With inelastic demand the Cournot first-order conditions do not exist.

\(^{13}\)For example, type \(a\) could be coal-fired power plants, type \(b\) as gas-fired power plants or renewable generators with high fixed costs. The specification rules out the case where one plant is both cheaper to run and less emissions intensive. This case is not very interesting: the result is, intuitively, that the low-cost, low-emission firms should satisfy as much of the demand as possible at any carbon price, therefore industry emissions are independent of permit prices and initial allocations.
available to a firm is thus to reduce production. We require compliance with the ETS\textsuperscript{14} such that

\[ x_i \geq d_i q_i \quad (i = a, b). \] (4)

Moreover, there are at least two identical firms of each type, preventing monopolistic style behaviour to occur in the inelastic case when the difference in the cost structures between types is large.

For the inelastic case we replace Assumptions 1 and 3, respectively, with the following:

\textbf{Assumption 5} \textit{The marginal cost function } \( c(q) : \mathbb{R}_0^+ \to \mathbb{R}_0^+ \text{ is constant and finite for the closed interval } [0, q_i] \text{ and not defined for } q > q_i, \text{ so that firms have a capacity constraint at } q_i. \) The capacities, \( q_i, \) are such that \( \sum_{i \in A} q_i < Q \) \( \text{and } \sum_{i \in A, B} q_i > Q. \)

\textbf{Assumption 6} \textit{Demand is perfectly inelastic and fixed at some quantity } \( Q > 0. \)

Assumptions 5 and 6 and Assumptions 1 and 3 are alternative; Assumption 2 applies in both cases. While Assumption 6 denotes the case of perfectly inelastic demand, Assumption 3 is the standard assumption of downward sloping, hence elastic, demand. In the short run, demand will typically be relatively inelastic (\( i.e., \frac{\partial q}{\partial p} \) is small). Situations where demand may be considered perfectly inelastic in the short run include staple foods and electricity. Assumptions 1 and 2 impose the usual convexity conditions. For the inelastic case marginal costs are constant and each firm has an exogenous capacity constraint. Whilst the Assumptions 5 and 6 for inelastic demand are restrictive, they are only necessary to ensure that the prices in the inelastic demand scenario are the same as prices in the elastic demand scenario. Corresponding results for the inelastic demand scenario could be reproduced with less restrictive assumptions.

\textbf{Lemma 1} \textit{In the second-stage pricing game there is a continuum of Nash equilibria. In all equilibria each producing firm charges the same price. The continuum of Nash equilibria includes one where each firm charges a price equal to the marginal cost (including carbon costs) of the least cost effective unit: } \( p^* = c_b(q_b) + d_b \cdot \phi(X) \text{ for all firms.} \)

\textsuperscript{14}This assumption is standard in the literature as most ETS impose large penalties for non-compliance. For example, in the first EU-ETS phase the non-compliance penalty was €40 per missing permit (in phase two €100) plus a ‘make-good provision’ requiring an extra permit surrendered the next year.
The proof is given in Appendix A.1. As there are multiple equilibria for the elastic case pricing game, we require a decision rule to select a single equilibrium. We assume all firms to price so that the least cost effective producing firm receives zero profits.

In the first stage of the game, firms seek to maximise their profits (1) adapted for the short-run case over their permit holdings given the optimal choice of the output price in the second stage and the compliance condition (4). We have to distinguish the cases of (a) an interior solution, where at least one firm holds more permits than required, \( x^*_i > d_i q^*_i \), and (b) a boundary solution, where all permits are utilised to cover actual emissions, \( x^*_i = d_i q^*_i \) for all firms (see Figure 1 for an illustration).

**Figure 1:** Distinction of (a) interior and (b) boundary solution in permit-choice game.

In an interior solution, in equilibrium, we require the following first- and second-order conditions to hold for all firms of type \( i, i = a, b \)\(^{15}\)

\[
\frac{\partial \Pi_i}{\partial x_i} = \frac{\partial p}{\partial x_i} q_i + \frac{\partial q_i}{\partial x_i} p - \frac{\partial q_i}{\partial x_i} c_i - \phi(X) - (x_i - x^*_i) \frac{\partial \phi}{\partial x_i} = 0 \tag{5a}
\]

\[
\frac{\partial^2 \Pi_i}{\partial x_i^2} < 0 . \tag{5b}
\]

**Remark 1** Condition (5b) is necessary to ensure the solutions to conditions (5a) characterise a maximum. Sufficient, but not necessary, for condition (5b) to hold is that \( \phi(X) \) is linear.

The proof is given in Appendix A.2. In a boundary solution, the optimal permit holding constrains production, so that \( x^*_i = d_i q^*_i \).

\(^{15}\)For brevity, \( \frac{\partial p}{\partial x_i} \equiv \frac{\partial p}{\partial x_i} \frac{\partial \phi}{\partial x_i} \), \( \frac{\partial p}{\partial x_i} \equiv \frac{\partial p}{\partial x_i} \frac{\partial \phi}{\partial x_i} \), \( \frac{\partial q_i}{\partial x_i} \equiv \frac{\partial q_i}{\partial x_i} \frac{\partial \phi}{\partial x_i} \), and \( \frac{\partial \phi}{\partial x_i} \equiv \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_i} \).
Proposition 1 summarises the effects of an increase in the initial permit allocation to a firm on its optimal permit holdings, the permit price, and the product price.

**Proposition 1** Given an increase in the initial allocation of permits to a firm does not induce a change of the marginal unit of production in the first-stage permit-choice problem, it will,

- **a)** in an interior solution, increase
  - the number of permits the firm holds in equilibrium,
  - the permit price, and
  - the product price (i.e., increase pass-through profits),

- **b)** in a boundary solution, have no effect on
  - the number of permits the firm holds in equilibrium,
  - the permit price, or
  - the product price (i.e., there is no additional pass-through profit).

These results hold irrespective of whether demand is perfectly inelastic or elastic.

**Proof.**

- **a)** Let \( x_i^* \) be the solution to problem (5a), \( \frac{\partial \Pi_i}{\partial x_i} = 0 \). The second derivatives \( \frac{\partial^2 \Pi_i}{\partial x_i \partial x_i^2} = \frac{\partial \phi}{\partial x_i} > 0 \) and \( \frac{\partial^2 \Pi_i}{\partial x_i^2} < 0 \) (see also the proof of Remark 1) together imply both \( \frac{\partial x_i^*}{\partial x_i^0} > 0 \) and \( \frac{\partial \phi}{\partial x_i^2} > 0 \) (Figure 2). Lemma 1 implies that the product price also increases.

  - **b)** By definition, a boundary solution indicates that compliance condition (4) holds with equality. The solution to the problem is, hence, \( x_i^* = d_i q_i \) (i.e., a, b). As it is independent of \( x_i^0 \), the initial permit allocation has no bearing on the solution. □

We fix the marginal unit of production in Proposition 1 for clarity of results. As applied to power markets, for example, this assumption means that the merit order does not change. Figure 2 illustrates the proof of Proposition 1.a. As \( x_i^0 \) increases the optimal permit holding moves from \( x^* \) to \( x^{**} \). As \( \frac{\partial^2 \Pi_i}{\partial x_i \partial x_i^2} > 0 \), an increase in \( x_i^0 \) will increase the first derivative \( \frac{\partial \Pi_i}{\partial x_i} \) at all points along the profit curve. Thus, at the point where \( \frac{\partial \Pi_i}{\partial x_i} = 0 \) for the original allocation
of $x_i^0$, $\frac{\partial \Pi_i}{\partial x_i} > 0$ for the new allocation of $x_i^0$. As $\frac{\partial^2 \Pi_i}{\partial x_i^2} < 0$, there must be a value of $x$ greater than the original $x^*$ that will cause $\frac{\partial^2 \Pi_i}{\partial x_i^2} = 0$. The increase in the initial permit allocation shifts the profit function up and to the right.

The likelihood of a boundary solution depends on the strength of the overall, economy wide, emissions target.

**Corollary 1** A tightening of the emissions target reduces the optimal holding of permits for each firm, and increases the range of production technologies that produce a boundary solution.

The proof is given in Appendix A.3.

Permit-market efficiency requires all firms, in all industries, have the same marginal abatement costs in equilibrium. In our model all other industries are price takers in the permit market, and will, with Montgomery (1972), hold the efficient permit amount. To analyse permit-market efficiency we can thus focus on the firms with market power.

**Case 1** Interior solution ($x_i^* > d_i q_i^*$ for at least one firm)

It may aid intuition to rewrite the first-order conditions (5a) in the form where marginal benefits equate marginal costs. Equation (6) is for inelastic, (7) for elastic demand:

\[
\frac{\partial p}{\partial x_i} q_i = \phi(X) + (x_i - x_i^0) \frac{\partial \phi}{\partial x_i} \tag{6}
\]

\[
\frac{\partial p}{\partial x_i} q_i - \frac{\partial q_i}{\partial x_i} c_i = \phi(X) + (x_i - x_i^0) \frac{\partial \phi}{\partial x_i} - \frac{\partial q_i}{\partial x_i} p \tag{7}
\]
In the inelastic case, receiving an extra permit for free reduces a firm’s marginal cost of purchasing another permit but leaves its marginal benefit unchanged. Thus, the firm seeks to purchase additional permits, which raises the permit price. In the elastic case, the marginal benefit from permit holding includes an additional term, capturing the cost reduction associated with the decreased production due to the increased permit holding; moreover, an additional cost term captures the revenue loss associated with the decreased production. As the price will exceed a firm’s costs, marginal costs of an additional permit increase more than its marginal benefits. As purchasing an additional permit increases the product market price (reducing the quantity sold), the permit holding for each firm will be lower under elastic demand than under inelastic.

Equations (6) and (7) show two distortionary effects which arise in relation to the oligopolists’ market power in permit and product market. As qualitatively the same for elastic and inelastic demand, we only discuss the inelastic case. Note, first, that the marginal costs of an additional permit held in equilibrium are not equal to the permit price, \( \phi(X) \). They also comprise a term for the extra amount to be paid for all other permits purchased, \( (x_i - x_i^0) \frac{\partial \phi}{\partial x_i} \). It decreases in the initial allocation, increases with the permits held, and vanishes if the initial allocation exactly equals the firm’s permit holdings in equilibrium. It is positive if and only if the firm is a net seller \( (x_i - x_i^0 > 0) \), causing lower permit holdings than under perfect competition (as discussed in detail in Hahn 1984). We call this effect the *Hahn monopsony* effect. For a net buyer \( (x_i - x_i^0 < 0) \), it is negative, causing higher permit holdings than in the perfectly competitive case – the *Hahn monopoly* effect. Second, purchasing an extra permit implies a benefit due to the increase in revenue, \( \frac{\partial p}{\partial x_i} q_i \). Compared to a situation without pass-through profits, profits and permit holdings are increased. We refer to this as the *pass-through profit* effect. Note that in the inelastic case, marginal changes in the initial permit allocation have no effect on production: in an internal solution the output is already at a maximum and is not affected by permit holdings.

Given the emissions target is set such that marginal social benefits of pollution reduction are equal to its marginal social costs, the permit market cannot be operating efficiently if a firm’s permit holdings exceed its emissions in equilibrium; non-utilised permits imply
emissions below their socially efficient level. The two kinds of initial allocation effects described have opposing impacts on market efficiency. When firms are net permit buyers, market efficiency is improved by the Hahn monopsony effect, but declines by the pass-through profit effect, as the initial allocation increases. When firms are net permit sellers, market efficiency declines due to both Hahn monopoly and pass-through profit effect, as the initial allocation increases. Thus, we would expect the efficient initial allocation here to be smaller than in the efficient case discussed by Hahn (1984).

If the permit market were perfectly competitive, then Hahn’s results for efficiency hold irrespective of the choice of Bertrand or Cournot competition. Using Bertrand competition here allows us to model pass-through profits in a more intuitive manner: changes in permit allocation directly affect prices in a Bertrand framework, rather than as a secondary flow on effect from quantity choices under Cournot competition. The Hahn effects will exist in a formulation with Cournot output competition, too, as will the pass-through profit effect. The size of the pass-through profit effect, however, is smaller with Cournot than with Bertrand. The purchase of an additional permit by any firm will raise the permit price, raising the carbon-inclusive marginal costs of the least cost-effective firm. The extent of the pass-through profit effect is determined by the rate of transmission of this increase in marginal costs to the product market price. In our Bertrand formulation, where we have chosen to focus on the unique equilibrium where the price equals the marginal cost of the least cost-effective firm, this transmission rate is 100 per cent; under Cournot competition it will be less than 100 per cent. Pass-through profits cause, hence, greater distortions under Bertrand competition.

**Case 2** Boundary solution \( x_i^* = d_iq_i^* \) for all firms

In a boundary solution to the first-stage permit-choice problem, the results are independent of whether demand is inelastic or elastic. In both cases a small change in the initial permit allocation will not affect the solution to that problem; it will still lie on the boundary. The proof of Proposition 1 implies that with an increase in the initial allocation of permits the profit function shifts up and to the right. Hence, a marginal increase in the initial allocation

---

16In a similar vein Smith and Yates (2003) show that non-emitting members of the economy purchasing and retiring permits (e.g., environmental groups) is a signal of an inefficient market, although in their model the inefficiency stems from uncertainty over the optimal size of the emissions target.

17The stationary point of the profit function occurs at a higher level of permit holdings, and the initial
here simply raises the firm’s profits. Thus, the pass-through profit effect still exists in the sense that the existence of a positive permit price implies that the product market price is higher than it would be if the price of permits was 0. But the initial permit allocation to a firm no longer affects the pass-through profit effect. As on the boundary the equilibrium permit holdings do not change, marginal changes in the initial permit allocation have no effect on production; permits are expensive enough that firms simply wish to minimise their holdings for a given quantity of production. All permits are utilised to cover actual emissions. For a boundary solution, the initial allocation of permits has no impact on efficiency because the solution is independent of the initial allocation of permits. In a more long-run view, however, the initial allocation of permits may affect the entry/exit decision of firms as a high initial allocation creates higher profits. Thus, at least one initial allocation of permits will lead to efficiency of the permit trading scheme. In practice, it would be extremely difficult to give the correct initial allocation to all firms, implying that least-cost emissions reductions are unlikely to be achieved.

Hence, in the short run, when capacity and technology choice is fixed, the efficiency of an ETS depends much on whether firms with market power are constrained by the requirement to hold permits. (It is clear that when the potential for pass-through profits exists, the pattern of the initial allocation of permits may affect both the final distribution of permits amongst firms and the permit price, through the pass-through profit effect and Hahn effects.) If firms are constrained by the requirement to hold permits the initial allocation of permits affects firms’ profits, but not their optimal permit holdings. In no circumstances does a marginal change in the initial allocation of permits affect the level of production in the short run when demand is perfectly inelastic. In the case of elastic demand (where the comparison is possible), the effect of permit allocations on permit prices is greater in Bertrand competition than Cournot competition.

3.1 Technology Switching

In this subsection we extend the analysis to consider the potential for technology switching. As each firm has a fixed production technology, technology switching may only be observed wealth of the firm has increased (Figure 2).
at the industry level. We relax the assumption of equation (3), to allow type-\(b\) firms to have lower marginal costs for some values of \(\phi(X)\). As \(c_a < c_b\) and \(d_a > d_b\), type-\(a\) firms will be cheaper at low values of \(\phi\) (and type-\(b\) firms cheaper at high values). We define the technology-switching carbon price, \(\bar{\phi}\), as the value of \(\phi\) such that

\[
c_a(q_a) + d_a \cdot \phi(X) = c_b(q_b) + d_b \cdot \phi(X) .
\]  

(8)

The impact of \(\bar{\phi}\) will depend on the elasticity of the function \(\phi(X)\).

The inclusion of a switching price introduces a discontinuity in equation (5a). The assumption that firms price such that the least cost effective firm receives zero profits implies that the discontinuity is a downward (upward) shift in profit for type-\(a\) (\(b\)) firms as a marginal change in permit holdings induces a shift in \(\phi(X)\) from \(\bar{\phi} - \epsilon\) to \(\bar{\phi} + \epsilon\) for small \(\epsilon\). Therefore, a type-\(a\) firm would prefer the carbon price to be slightly below (rather than slightly above) the switching price. At an industry level, overall production from type-\(a\) firms would be larger when the carbon price is slightly below the switching price (as type-\(a\) firms are relatively cheaper). This relatively higher production from type-\(a\) firms may be viewed as industry-level technology switching.

As in order to produce a firm must hold a positive quantity of permits, the ability for type-\(a\) firms to place downward pressure on \(\phi\) is constrained, whereas the ability for type-\(b\) firms to place upward pressure on \(\phi\) is not. Thus when \(\phi(X)\) is relatively elastic, type-\(b\) firms will purchase sufficient permits such that \(\phi(X) > \bar{\phi}\). When \(\phi(X)\) is relatively inelastic, purchasing sufficient permits such that \(\phi(X) > \bar{\phi}\) may be prohibitively expensive for type-\(b\) firms, and the carbon price may be below the switching point. Hence, the carbon price is more likely to be below (above) the switching price when type-\(a\) (\(b\)) firms have greater market power.\(^{18}\) In particular, if type-\(a\) firms have more market power than type-\(b\) firms, and \(\phi(X)\) is relatively inelastic, then the switching price may act as an artificial ceiling for the carbon price.

\(^{18}\)Factors that determine market power include: market share on the output market, the initial allocation of permits and the comparative costs of production.
4 Long-Run Analysis

We now turn to the long-run models, 1 and 2, of section 2. We solve the long-run models by first exogenously fixing technology choice and demonstrating the Kreps and Scheinkman result. To do this, we solve the Cournot sub-game for sub-game perfect Nash equilibria (SPNE) using backward induction. We show that the solution for the Cournot game is identical to the equivalent Bertrand formulation. We then endogenise technology choice and conduct comparative statics on the initial permit allocation. Again, we require full compliance with the emissions trading scheme, so that \( x^*_i \geq e^*_i \forall i \).

4.1 Cournot Game

We solve the Cournot game, as presented in Model 2, beginning with the third stage, where each firm simultaneously maximises its profit (1) (with \( P(q_i + q_j) \) substituted for \( p_i \)) such that the full-compliance requirement holds (\( x^*_i \geq e^*_i \)).

**Lemma 2** For an initial permit allocation \( x^0_i \leq e_i \), the solution of the second-stage game is \( x^*_i = e^*_i = q^*_i(1 - \theta_i) \) for \( i = a, b \).

**Proof.** For all \( x_i \geq x^0_i \), \( \frac{\partial \Pi}{\partial x_i} = -(x_i - x^0_i) \frac{\partial \phi}{\partial x_i} - \phi(X) < 0 \). The solution follows. □

Thus, firms will only hold permits to cover their emissions exactly. The assumption that \( x^0_i \leq e_i \) is sufficient but not necessary; the solution given in Lemma 2 above will still hold as long as the excess allocation of permits is not too large (\( \frac{\partial \Pi}{\partial x_i} \) still negative). The intuition is: the model requires that a firm cannot hold less permits than it produces emissions. This can, for example, be ensured by high non-compliance penalties. Also, it is not profitable for a firm to hold more permits than is necessary. Doing so involves an additional cost for no additional benefits as production and revenues have already been realised in the second stage of the game.

\[ ^{19} \text{Note that this is different from our short-run model, where it may be profitable to hold excess permits to raise a rival's costs, or to manipulate the market.} \]
By substituting $x^*_i = e^*_i = q^*_i(1 - \theta_i)$ into equation (1), the first-stage quantity maximisation problem of firm $i$, $i = a, b$, may be written as

$$\max_{q_i} \Pi^C_i = q_i \cdot P(q_i + q_j) - q_i \cdot c_i(\theta_i) - [q_i(1 - \theta_i) - x^0_i] \cdot \phi[.] - f_i(q_i, \theta_i), \quad (9)$$

where $\phi[.] = \phi[q_i(1 - \theta_i) + q_j(1 - \theta_j)]$.

**Lemma 3** The SPNE for the two-stage game where firms compete under Cournot competition and then purchase emissions permits can be found by solving problem (9) simultaneously for all firms.

**Proof.** Existence and uniqueness of the solution to problem (9) follow from Tirole (1988: 225n). □

### 4.2 Bertrand Game with Cournot Outcomes

Now turn to a Bertrand version of the game, with quantity precommitment, as given by Model 1. Again we solve for sub-game perfect Nash equilibria. We first analyse the stage-4 pricing game, for given technologies (stage 1) capacity constraints (stage 2) and permit holdings (stage 3). Kreps and Scheinkman (1983: 335) summarize the required results for the stage-4 pricing subgame in the following proposition which we reproduce adapted to our notation and setting for convenience. Let $q_i$ denote the installed capacity of firm $i$, $q_i^*$ the solution to problem (9), and $q_i(\overline{q}_j)$ the optimal solution to problem (9) for a firm, when its competing firm installs capacity $\overline{q}_j$.\(^{20}\)

**Proposition 2** (adapted from Kreps and Scheinkman (1983)).

1. If $\overline{q}_i \leq q_i^*$ for the two firms, $i = a, b$, both firms name price $p_a = p_b = p = P(\overline{q}_a + \overline{q}_b)$, where $\overline{q}_a$, $\overline{q}_b$ are the capacity constraints installed in stage 1. Thus each firm has revenue $\overline{q}_i P(\overline{q}_a + \overline{q}_b)$.

\(^{20}\)Note that the proof in Kreps and Scheinkman (1983) is for the case when marginal costs are 0. Here we have positive marginal costs. But this does not change the nature of the results for situations where both firms produce positive quantities (ibid.: 337).
2. If $q_a \geq q_b$ and $q_a > q_a^*$, firm $a$ has expected revenue $q_a(q_b)P(q_a(q_b)+q_b)$ whilst firm $b$ has a uniquely determined expected revenue between $q_a(q_b)P(q_a(q_b)+q_b)$ and $q_a(q_b^*)P(q_a(q_b)+q_b)$.

3. If $q_b \geq q_a$ and $q_b > q_b^*$, firm $b$ has expected revenue $q_b(q_a)P(q_b(q_a)+q_a)$ whilst firm $a$ has a uniquely determined expected revenue between $q_b(q_a)P(q_b(q_a)+q_a)$ and $q_b(q_a^*)P(q_b(q_a)+q_a)$.

We now determine the optimal permit holdings in the stage-3 game, noting that the permit choice does not directly affect the results of Proposition 2. The only impact permit holdings have on a firm’s pricing decision is that they may act as an artificial capacity constraint, if $x_i < q_i(1-\theta_i)$. When $x_i \geq q_i(1-\theta_i)$, there is no impact on the pricing game as the cost of purchasing permits is, by assumption, a sunk cost during the pricing game.

**Lemma 4** Let $\overline{q}_i$ be firm $i$’s installed capacity, $i = a, b$. Then, its optimal permit holding is $x_i^* = \overline{c}_i = \overline{q}(1-\theta_i)$.

The proof is given in Appendix A.4. The intuition is probably more instructive. A firm has no incentive to purchase more than $\overline{c}_i$ permits. For, to do so would incur a cost that cannot be recovered, and has no strategic advantage – the rival firm will still be able to purchase the full amount of permits required, and regard this cost as sunk in the pricing game (stage 3). A firm also will never purchase less than $\overline{c}_i$ in a consistent equilibrium. To do so would imply there was an over-investment in capacity. Clearly, then, a firm must purchase $x_i^* = \overline{c}_i$ emissions permits.

As might be expected in a game with perfect information, the firms’ choice of $x_i$, and the associated unit costs, can be calculated following the firm’s capacity choice. Thus, the permit costs may be incorporated into installation costs at the stage-2 decision. As they are sunk during the stage-4 pricing game, permit costs are then indistinguishable from installation costs, which obviously are also considered as sunk. To state the next lemma we introduce the following function $F$, it may be considered a ‘sunk cost function.’

$$F(q_i, \theta_i, q_j, \theta_j) \equiv f_i(q_i, \theta_i) + [q_i(1-\theta_i) - x_i^0] \cdot \phi[q_i(1-\theta_i) + q_j(1-\theta_j)] .$$ (10)
Lemma 5 Assume $0 \leq x_i^0 \leq q_i(1 - \theta_i)$.\footnote{This assumption is again not strictly necessary. The permit allocation must not be ‘too’ high, however.} The cost function $F(q_i, \theta_i, q_j, \theta_j) : \mathbb{R}_0^+ \times [0, 1] \times \mathbb{R}_0^+ \times [0, 1] \rightarrow \mathbb{R}_0^+$ is twice continuously differentiable, and satisfies $F(0, \theta_i, q_j, \theta_j) = 0$, \(\frac{\partial F(q_i, \theta_i, q_j, \theta_j)}{\partial q_i}\bigg|_{q_i = 0} > 0\) and \(\frac{\partial^2 F(q_i, \theta_i, q_j, \theta_j)}{\partial q_i^2}\) $> 0$ for all $q_i, q_j \in \mathbb{R}_0^+$ and $\theta_i, \theta_j \in [0, 1]$.

The proof is given in Appendix A.5.

Stages 2 and 3 of model 1 may be interpreted as a single-stage capacity installation process, where firm $i$’s profit, $\Pi_i$, can be written as

$$\Pi_i^B = q_i \cdot P(q_i + q_j) - q_i \cdot c_i(\theta_i) - F_i(q_i, \theta_i, q_j, \theta_j).$$

(11)

Thus, our game has the same form as the game presented in Kreps and Scheinkman (1983), with the exception that our installation cost function depends on the capacity of the rival firm. Lemma 4 demonstrates, however, that the choice of permit holdings is not affected by the rival firm’s choice of capacity. Hence, our model presented in equation (11) is equivalent to the model presented in Kreps and Scheinkman (1983).

Kreps and Scheinkman (1983) show that both firms will select quantities such that $q_i = q_i^*$, and, hence, such that the equilibrium price is determined by the first condition in Proposition 2.\footnote{Much of Kreps and Scheinkman (1983) is in fact dedicated to dismiss conditions two and three in Proposition 2 as off-equilibrium paths: when firms install excess capacity they intensify the price competition in the third stage of the game and reduce their profits.} We may therefore use the rewritten equation (11) to state:

$$\max_{q_i} \Pi_i^B = q_i \cdot P(q_i + q_j) - q_i \cdot c_i(\theta_i) - [q_i(1 - \theta_i) - x_i^0] \cdot \phi[\cdot] - f_i(q_i, \theta_i), \ i = a, b, \quad (12)$$

where $\phi[\cdot] = \phi[q_i(1 - \theta_i) + q_j(1 - \theta_j)]$. Problem (12) is identical to problem (9), the equivalent problem for the Cournot case. We conclude:

**Proposition 3** The two-stage game where firms compete under Cournot competition and then purchase emissions permits yields the same results as the three-stage game where each firm installs costly production capacity, purchases emissions permits and then competes under Bertrand competition. The equilibrium of both games can be found by solving problem (12) simultaneously for all firms.
The Kreps-and-Scheinkman framework has received much attention over the past two decades. For our application of Kreps and Scheinkman the structure of the game is important. The key parameter is the timing of the permit-choice and quantity-competition stages (the capacity-installation stage in the Bertrand formulation). If capacity installation occurred after the purchase of permits, our function $F$ would not exhibit the correct shape to allow the Kreps-and-Scheinkman result to hold. Note the symmetry between the Cournot and Bertrand formulations. In both formulations the quantity competition stage occurs before the purchase of permits. Changing the order of events in only one of the games invalidates the results. A similar Kreps-and-Scheinkman result could, however, be generated, if the first stage of both games was the purchase of permits.\textsuperscript{23}

### 4.3 Effects of the Initial Permit Allocation on Permit-Market Efficiency

We now consider the technology choice stage of games 2 and 1. Both of these models are equivalent to the two-stage game where firms maximise the profit function as (identically) contained in problems (9) and (12) by first making a choice of technology, $\theta_i$, and then choosing a quantity of production. We use the two-stage formulation of the game for analytical ease and focus on the choice of technology.

The optimal strategy for a firm in the first stage is to choose $\theta_i$ such that the following conditions of, respectively, first and second order hold:

$$\frac{d\Pi_i}{d\theta_i} = \frac{\partial \Pi_i}{\partial \theta_i} + \frac{\partial \Pi_i}{\partial q_i} \frac{dq_i}{d\theta_i} + \frac{\partial \Pi_i}{\partial \theta_j} \frac{d\theta_j}{d\theta_i} + \frac{\partial \Pi_i}{\partial q_j} \frac{dq_j}{d\theta_i} = 0 \quad (13a)$$

$$\frac{d^2\Pi_i}{d\theta_i^2} < 0. \quad (13b)$$

Optimality of the second-stage production decision implies $\frac{d\Pi_i}{dq_i} = 0$, and the simultaneity of

\textsuperscript{23}That varied situation may generate different results in the following subsection 4.3. We do not consider the case further as there is no clear economic intuition to support a firm purchasing emissions permits before it installs production capacity.

21
the permit choice that $\frac{d\theta_j}{d\theta_i} = 0$. So equation (13a) can be rewritten as

$$d\Pi_i = \frac{\partial \Pi_i}{\partial \theta_i} d\theta_i + \frac{\partial \Pi_i}{\partial q_j} dq_j = 0.$$  

(14)

**Remark 2** Condition (13b) is required to ensure that the value of $\theta_i$, found by solving either condition (13a) or condition (14), is a maximum.

For the rest of this section, we assume that condition (13b) holds.

Again, we study how a marginal change in the initial permit allocation to a firm affects the equilibrium. Unfortunately, its effect on technology and production choices of each firm is ambiguous. For, an increase in the initial permit allocation may lead to the installation of either cleaner or dirtier technology, and may either increase or decrease a firm’s total cost function, depending on the relative slopes of the product demand, permit price and cost functions.\(^{24}\) Moreover, the long-run model, unlike the short-run model, does not explicitly include a variable for permit holdings – they are uniquely determined by technology installed and quantity produced. Therefore, Hahn or pass-through profit effects cannot be isolated.

Proposition 4 summarises these effects. The results hold for all initial allocations of permits, and in particular irrespective of the sign of $(x^* - x_0)$.\(^{25}\)

**Proposition 4** An increase in the initial allocation of permits to a firm with market power in the permit and product market will:

- increase total industry emissions,
- increase the price of permits.

The proof is given in Appendix A.6.

\(^{24}\)Consider, for example, the case where an increase in the initial allocation promotes the installation of cleaner technology, which increases the firms’ total cost of production. (As cleaner technology has lower installation costs, but higher marginal costs, it may either increase or decrease total costs.) Becoming less competitive the firm reduces production, causing a loss in profits from production. The reduction in production and the cleaner technology make it require fewer emissions permits (it is less polluting), causing a rise in profits from permit sale. The competing firm will, however, increase production levels, because it faces weaker competition. The competitor may also install dirtier technology, increasing its permit holdings and emissions levels. The net effect of the increase in the initial permit allocation will be an increase in total industry emissions.

\(^{25}\)The results are, of course, subject to Assumptions 3 and 4 and the structure of the games.
Again, the result is quite pessimistic when considering the ability of an ETS to provide least-cost emissions reductions. For least-cost abatement, all firms must have an equal MAC in equilibrium. The fact that emissions depend on the initial allocation implies that efficiency in the permit market can only occur if and only if regulators choose the ‘correct’ initial allocation of permits. Specifically, as the emissions, and therefore the MAC, of both firms depend on the free allocation of permits to both firms, the ‘correct’ initial allocations ensuring MAC is equalised across both firms is likely to be impossible to determine in a realistic situation.

Note, moreover, that permit-market efficiency does not imply an optimal level of investment in technology; it only implies that the MAC is equal for all firms, for a given level of technology. Optimal technology investment would imply a given level of production is produced at least total cost, which comprise both monetary and environmental cost. Permit-market efficiency is a necessary condition for optimal investment: without it a firm will not internalise the environmental externalities associated with their technology choice. Without defining specific technologies we cannot speculate on how the initial allocation of permits might be used to promote social efficiency.

In the long run, it cannot be profitable for a firm to hold more permits than required for compliance. This makes intuitive sense, as any commitment to holding excess permits would not be expected to be sustainable in the long term. As a consequence, permit-market efficiency is more likely in the long than the short run.

5 Discussion

We discuss the argumentative application of our analysis to the EU ETS and the power sector in subsection 5.1. Subsection 5.2 considers our contribution to the literature on permit-market efficiency, the literature with a (more) long-run view in environmental economics, and to general industrial organisation. Subsection 5.3 contains policy implications.
5.1 Argumentative Application to EU ETS and Power Sector

As set out in the introduction, our analysis has a particular argumentative application to the EU ETS and the price anomaly which occurred in its first trading phase 2005-2007. Our short-run analysis suggests that it may well have been possible for some firms to hold more permits than they had emissions. For example, if electricity generators (who face inelastic demand in the short run) were allocated permits such that their optimal permit choice is an interior solution to problem (5), then their actions would have directly contributed to the high permit prices. The possibility of individual excess permit holdings in equilibrium in the situation of an overall permit over-allocation with positive permit price has been treated in the literature, but only for the case of perfect or Cournot competition and elastic demand in the product market (e.g., Maeda 2003, DiSegni Eshel 2005). We show that the Hahn monopsony and monopoly effects and a pass-through profit effect also occur under Bertrand competition and elastic or inelastic product demand (Proposition 1). In particular, in the interior solution, the pass-through profit effect becomes the bigger the more inelastic product demand, reaching 100 per cent for inelastic demand; it is always bigger than under Cournot. Such very high pass-through rates of (opportunity) costs to consumers – hence, windfall profits – are in line with empirical evidence for the power sector (Sijm et al. 2006). This makes a case for the realism of considering Bertrand output competition in this setting.

Our analysis in section 3 approximates the conditions in the short run or beginning of an ETS, where decisions regarding technology and plant size have been made beforehand and product demand is relatively inelastic. In particular, the analysis in section 3.1 provides a potential explanation of the price movements in the EU ETS from April - October 2006. Sijm et al. (2006) estimate the carbon switching price (from coal to gas generation) as being approximately €18.5. Given that the carbon price is likely to have been relatively inelastic with respect to an individual firms’ permit holdings, and that coal generators were awarded a large quantity of grandfathered permits (engendering them with potential market power), the observed prices during this period are consistent with the switching price acting as a price ceiling.

The long-run analysis in section 4 rather looks at the case where sufficient time has elapsed
for firms to update and reinstall their production technology, as it may be expected in an established ETS. The extended Kreps-and-Scheinkman model then predicts that an increase in the free permit allocation to a sector with market power in permit and product market – such as conceivably the electricity industry – will increase the permit price. In the long run, firms do not have an incentive to hold excess permits anymore because they can adjust their technology. This long-run effect will, however, hardly have played a role for the price decline later in the first EU-ETS phase. The allowances rather lost their value because they could not be banked for the second phase.

That our long-run results are independent of whether oligopolistic competition in the product market is modelled in quantities or prices depends on two assumptions: efficient demand rationing, and the relative structure of the Cournot and Bertrand formulations of the games. We discuss the first assumption more generally in the next subsection. Here, we want to note its possible immediate validity in the case of power markets: having received the utilities’ price bids, there is, in fact, typically a centralised market authority which may clear the electricity market in intervals of several minutes according to the efficient rationing rule.\(^{26}\)

On the other hand, to cover their emissions, firms may purchase pollution permits before, during, or after production (as long as they acquire them before the compliance deadline). Evidence from Europe suggests that the internal risk management procedures of electricity generators require them to purchase their emissions permits before they produce the associated emissions. In a simplified model of electricity markets, the natural order of actions is thus: install capacity; purchase permits; compete over prices. We adopt this order in both models.

Whether the predictions of our short-run analyses apply in the case of the first EU-ETS phase, \textit{i.e.}, whether certain companies did exercise permit market power, hinges on the availability of firm-level data, especially of permit holdings and permit-market transactions. This data is currently still confidential. Observations relating to our long-run analysis still only have to occur in the course of a high-volume permit market, such as the EU ETS.

\(^{26}\)Moreover, as mentioned below, the Kreps-and-Scheinkman result is particularly likely to hold even independent from the rationing rule used if capacity cost is high (Davidson and Deneckere 1986).
5.2 Bertrand versus Cournot and Long-Run Considerations in the Literature

Studies on permit-market efficiency have been following two modelling strategies. Either they have focused on the permit market, assuming there is no feedback from the output market structure; or, if they include the product market, competition on it has typically been modelled in quantities.\textsuperscript{27} Clearly, from a technical viewpoint, it is much easier to work with Cournot rather than Bertrand competition. Bertrand, however, is more general in that it can also treat inelastic product demand. Moreover, for the short-run case, it may provide results that are, \textit{e.g.}, with respect to pass-through profit rates, closer to empirical findings than under Cournot competition. Whilst it is intuitively more natural for many markets to use Bertrand competition under an ETS, the fact that, under certain conditions, equivalence between the Bertrand and Cournot versions of the game can be established also under an ETS (Proposition 3), implies that we may use a Cournot game to approximate these cases.\textsuperscript{28}

The long-run aspect with endogenous technology and capacity choice (as developed in section 4) has in the literature on permit-market efficiency thus far only been considered by von der Fehr (1993: subsection 3.3), but not with Bertrand competition or a focus of initial allocation effects. It adds in particular to contributions which compare R&D incentives from different environmental-policy instruments under different degrees of perfection of output and permit markets and, hence, also go beyond the question of static short-run efficiency (\textit{e.g.}, Montero 2002b,a, Bruneau 2004). They show that under market power, contrary to perfect competition, command-and-control instruments may generate stronger incentives to innovate

\textsuperscript{27}Studies of the first type include, \textit{e.g.}, Hahn (1984), Westskog (1996), Maeda (2003), Malueg and Yates (2009), Lange (2008), Wirl (2009); of the second type Misiolek and Elder (1989), von der Fehr (1993), Requate (1993), Sartzetakis (1997, 2004), Montero (2002a,b), DiSegni Eshel (2005). (Exceptions with respect to Cournot output competition include Requate (1993) and Montero (2002a), cf. footnote 5.) As a firm’s main concern is to ensure it has enough permits to cover its emissions, market power in the permit market has typically been modelled with competition in quantities (only Lange 2008, Malueg and Yates 2009 and Wirl 2009 have recently also considered supply function equilibria).

\textsuperscript{28}It is interesting to note that also the literature on power market regulation has been concentrating on Cournot competition and supply function equilibria (\textit{e.g.}, Borenstein \textit{et al.} 2000, Joskow and Tirole 2000, Gilbert \textit{et al.} 2004, Willems \textit{et al.} 2009) – although Wolfram (1999), for example, shows that both models predict markedly excessive mark-ups for British electricity spot prices in the 1990s. Also Bushnell \textit{et al.} (2008) can approximate actual spot prices in several US states in their static Cournot framework only by taking into account (the frequently used) long-term contracts between utilities and retailers. Whether this softening effect of forward commitments on competition also holds in a dynamic setting is, however, doubted in the theoretical literature (Liski and Montero 2006).
than market-based instruments. Moreover, their results for Cournot and Bertrand output competition do not in general coincide. This underlines the importance of both market-power issues and the distinct analysis of different forms of imperfect competition also in the more long-run view.29

Our long-run analysis was developed directly based on Kreps and Scheinkman (1983), who focus only on a single output market. Loertscher (2008) has recently summarized and extended the literature dealing with their framework to cover the situation where firms compete, as market-making oligopolists, on both input and output markets. Interpreting the permit market as the input market, our model may be considered a variation of Loertscher’s model. We extend his analysis, for the duopoly case, by allowing for an initial endowment of the input good and analysing initial allocation effects. While we interpret this endowment as an initial allocation of emissions permits, it could also be seen as an initial inventory of stock. A second extension concerns the combination of forms of competition in input and output market. While Loertscher treats the cases of Cournot-Cournot and Bertrand-Bertrand (input-output) competition, we demonstrate that, under a costly capacity constraint, also Cournot-Bertrand competition yields Cournot outcomes.

Our analysis, like Loertscher, uses efficient demand rationing and does not consider uncertainty. According to the efficient rationing rule, customers with the highest reservation prices are serviced by the firm with the lowest prices – a situation which cannot be expected generally in real-world markets. The use of alternative rationing rules can lead to different results (including mixed strategy equilibria) in the Kreps-and-Scheinkman second-stage pricing game over some range of the parameter (capacity) space (Davidson and Deneckere 1986). Efficient and proportional rationing lead to the benchmark results between which equilibrium behaviour is likely to be more aggressive than under Cournot competition, but less competitive than in the Bertrand case (Loertscher 2008). Moreover, the Kreps-and-Scheinkman result has been shown to be robust even under proportional rationing if capacity cost is sufficiently high (Davidson and Deneckere 1986) and independently of the prevailing rationing rule if all costs are sunk at the first stage and demand is uniformly elastic (Madden

29 Interesting is also the idea to compare the option values of permits purchase and abatement measures in view of their (ir-)reversibility (e.g., Chao and Wilson 1993).
Reynolds and Wilson (2000) find that, under uncertain demand at capacity installation, in many cases a symmetric equilibrium does not exist, even for identical firms. Our framework does not focus on symmetric firms, so we do not expect symmetric solutions. The introduction of uncertainty may be an interesting extension to our analysis (e.g., Baldursson and von der Fehr 2004).

### 5.3 Policy Implications

The ETS design implications in the face of firms with market power in permit and product market one may derive from our analysis are independent of the kind of output competition, but differ to some extent for short and long run. In both cases, the substitution of auctioning for free allocation of the initial permit allocation will eliminate pass-through profits, and is likely to lead to further efficiency increases. In the short run, in an interior solution, it is, however, not clear whether full permit auctioning decreases the impact of the Hahn effects. For, permits are normally auctioned in conjunction to a secondary permit market. A firm with market power may purchase some permits at auction and some in the secondary market, leading to less efficiency gains than possible if it got all of its permits from the auction. (We assume an efficient auction.) A particular contribution of this paper is to explicitly treat the boundary solutions in the short-run analysis, where permit holdings, permit price and product price in equilibrium are independent from the initial permit allocation given to the firms. We show that a boundary solution is the more likely the more stringent the emissions target (Corollary 1). Again, also with a sufficiently stringent target permit-market efficiency is more easily achieved with auctioning of the initial permits than if they are given for free.

In the long run, in our case without banking of permits, holding more permits than required for compliance cannot be profitable for a firm. For equalisation of the firms’ MAC under the ETS, and hence least-cost abatement, auctioning of initial permits remains advantageous over free allocation. Indeed, due to the unsustainability of holding excess permits in the long term, permit-market efficiency is more likely in the long than the short run. However, the stringency of the emissions target does not impact on the likelihood of permit-market

---

30 In line with this conclusion, in relation to the earlier US SO$_2$ markets (with wide free allocation but stringent targets) market power did not occur as an issue (Joskow et al. 1998, Stavins 1998).
efficiency in the long run.

6 Conclusion

We study how the initial allocation of permits given to companies at the beginning of an emissions trading phase impacts on their behaviour and market outcomes, when firms have market power in permit and product market. We develop a long-run model with endogenous technology and capacity choice, and a short-run model with fixed technology and capacity. The latter extends the standard models of market power in ETS (e.g., Hahn 1984, von der Fehr 1993, Westskog 1996) by consideration of the output market with Bertrand competition. The long-run analysis introduces permit trading in capacity-choice models, such as that of Kreps and Scheinkman (1983). We show that, in the short run, initial allocation effects derived with Cournot competition carry over to Bertrand output competition, with two extensions. The Bertrand case is more general in that it can also treat inelastic product demand; and it generates higher pass-through profit rates (similar to those which could be observed for the electricity industry under the EU ETS). In the long run, under endogenous technology and capacity choice, Bertrand competition yields Cournot outcomes also under an ETS, if corresponding structures of the two versions of the game are chosen. Qualifications in the literature with respect to the demand rationing rule apply. A crucial assumption in this paper is that the initial permit allocation is given to the firms for free. We show that in the short run (also under free allocation), permit-market efficiency is likely to increase with the strength of the emissions target. Further efficiency increases could be achieved by auctioning the permits.

Our analysis is based on static games under certainty. Whether the results are robust in a dynamic setting, as may be more realistic especially in an established ETS, and how they may change if uncertainty is taken into account constitute interesting extensions for future research.
Appendix

A.1 Proof of Lemma 1

\((a)\) It will clearly be profitable for type-\(a\) firms to produce to their capacities, and to charge the highest price possible. Type-\(b\) firms compete over the residual demand under Bertrand competition with constant marginal costs. As is well known, the solution for this problem is for type-\(b\) firms to charge price equal to marginal cost. Type-\(a\) firms want to price as high as possible: given efficient rationing, they can charge equal to the marginal cost of type-\(b\) firms and still sell their full capacity. Thus the solution is for all firms to charge equal to the marginal cost of type-\(b\) firms.

\((b)\) The case of elastic demand is covered in Dastidar (1995), see in particular Lemma 9 and Proposition 2. □

A.2 Proof of Remark 1

\[
\frac{\partial^2 \Pi_i}{\partial x_i^2} = \frac{\partial^2 p}{\partial x_i^2} \cdot q_i - q_i \frac{\partial^2 c_i}{\partial x_i^2} - 2 \frac{\partial \phi}{\partial x} - (x_i - x_i^0) \frac{\partial^2 \phi}{\partial x_i^2} < 0 \text{ if } \frac{\partial^2 c_i}{\partial x_i^2}, \frac{\partial \phi}{\partial x}, (x_i - x_i^0) \frac{\partial^2 \phi}{\partial x_i^2} \geq 0 \text{ and } \frac{\partial^2 p}{\partial x_i^2} \leq 0.
\]

\[
\frac{\partial^2 c_i}{\partial x_i^2}, \frac{\partial \phi}{\partial x}, \frac{\partial^2 \phi}{\partial x_i^2} \geq 0 \text{ hold from Assumptions 5 and 2.}
\]

\(\phi(X)\) linear implies that \(\frac{\partial^2 \phi}{\partial x_i^2} = 0\). For firms of type \(a\), \(\phi(X)\) linear also implies that \(\frac{\partial^2 p}{\partial x_i^2} = 0\), and we are done. For firms of type \(b\), some more work is needed.

\[
\frac{\partial^2 \Pi_b}{\partial x_b^2} = \left(\frac{\partial^2 p}{\partial x_b^2} \cdot q_b - q_b \frac{\partial^2 c_b}{\partial x_b^2} - 2 \frac{\partial \phi}{\partial x} - (x_b - x_b^0) \frac{\partial^2 \phi}{\partial x_b^2}\right) \cdot q_b - q_b \frac{\partial^2 c_b}{\partial x_b^2} - 2 \frac{\partial \phi}{\partial x} - (x_b - x_b^0) \frac{\partial^2 \phi}{\partial x_b^2}
\]

\[
= \left(d_b \cdot \frac{\partial^2 \phi}{\partial x_b^2} + 1 \cdot \frac{\partial^2 c_i}{\partial x_i^2}\right) \cdot q_b - q_b \frac{\partial^2 c_b}{\partial x_b^2} - 2 \frac{\partial \phi}{\partial x} - (x_b - x_b^0) \frac{\partial^2 \phi}{\partial x_b^2}
\]

from Lemma 1. So,

\[
\frac{\partial^2 \Pi_b}{\partial x_b^2} = d_b \cdot \frac{\partial^2 \phi}{\partial x_b^2} - 2 \frac{\partial \phi}{\partial x} - (x_i - x_i^0) \frac{\partial^2 \phi}{\partial x_i^2},
\]

which is negative when \(\phi(X)\) is linear. □
A.3 Proof of Corollary 1

A tightening of the emissions target can be introduced to the model as an exogenous increase in the permit price function, \( \phi \). An increase in \( \phi \) decreases \( \frac{\partial \Pi_i}{\partial x_i} \) and, by the same logic as presented in the proof of Proposition 1.a, reduces the equilibrium value of \( x_i^* \) for any given production technology (as represented by the continuous variable \( \theta \) in the long-run models, but exogenous and hidden in the short run model). All production technologies that previously produced a boundary solution will still produce a boundary solution. Also, there will be at least 1 new production technology that will produce a boundary solution (e.g. the production technology that previously gave the solution \( x_i^* = d_i q_i^* + \epsilon \) for small \( \epsilon \)). □

A.4 Proof of Lemma 4

Let \( \Pi(x_i, q_i) \) denote the profit of firm \( i \) as in equation (1) when it installs capacity \( q_i \), purchases permits \( x_i \) and prices are determined as in Proposition 2, for a given choice of permits by the opposing firm. We begin by using forwards induction to eliminate \( x_i < x_i^* \). We denote the capacity constraint implied by the permit holding \( x_i < x_i^* \) by \( \hat{q}_i = \frac{x_i}{1 - \theta_i} \). Now, \( \Pi(x_i, \hat{q}_i) > \Pi(x_i, q_i) \). This implies that the firm has ‘over-installed’ capacity in the first stage of the game, and we reject this situation as an equilibrium using the concept of forward equilibrium. For \( x_i > x_i^* \) the capacity constraint is \( q_i \). Now, \( \Pi(x_i^*, q_i) > \Pi(x_i, q_i) \). Hence, a firm will not install permits such that \( x_i > x_i^* \). Therefore, firms will install permits such that \( x_i = x_i^* \) for \( i = a, b \). □

A.5 Proof of Lemma 5

The double continuous differentiability follows from Assumption 4. At \( q_i = 0, \frac{\partial E}{\partial q_i} = \frac{\partial f_i}{\partial q_i} + (1 - \theta_i) \phi[q_i(1 - \theta_i) + q_j(1 - \theta_j)] + [q_i(1 - \theta_i) - x_i^0] \frac{\partial \phi}{\partial q_i} > 0 \). For all \( q_i \) holds:
\[
\frac{\partial^2 E}{\partial q_i^2} = \frac{\partial^2 f_i}{\partial q_i^2} + 2(1 - \theta_i) \frac{\partial \phi}{\partial q_i} + [q_i(1 - \theta_i) - x_i^0] \frac{\partial^2 \phi}{\partial q_i^2} > 0.
\]
□

A.6 Proof of Proposition 4

Be \( E \equiv q_i(1 - \theta_i) + q_j(1 - \theta_j) \) the total emissions of the industry and note that \( \frac{\partial^2 \Pi_i}{\partial \theta_i \partial \theta_j} = \frac{\partial \phi}{\partial \theta_i} + \frac{\partial \phi}{\partial \theta_j} \frac{\partial q_i}{\partial \theta_i} + \frac{\partial \phi}{\partial \theta_j} \frac{\partial q_i}{\partial \theta_j} + \frac{\partial \phi}{\partial \theta_i} \frac{\partial q_j}{\partial \theta_j} = \frac{\partial \phi}{\partial \theta_i} \). For \( \frac{\partial \phi}{\partial \theta_i} > 0 \), \( \frac{\partial^2 \Pi_i}{\partial \theta_i \partial \theta_j} > 0 \), and, by condition (13b), \( \frac{\partial \theta_i}{\partial x_i} > 0 \),
such that $\frac{d\phi}{dx_i} > 0$. For $\frac{d\phi}{dx_i} < 0$, $\frac{d^2\Pi_i}{dx_i dx_j} < 0$, and, by condition (13b), $\frac{d\phi}{dx_i} < 0$, such that $\frac{d\phi}{dx_i} > 0$. Then, with Assumption 2 and Lemma 2, $\frac{dF}{dx_i} > 0$. □

References


