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**Conference Paper**

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Beiträge zur Jahrestagung des Vereins für Socialpolitik 2010: Ökonomie der Familie - Session: Innovation and Productivity, No. E16-V1

**Provided in Cooperation with:**

Verein für Socialpolitik / German Economic Association

*Suggested Citation:* Bauer, Christian (2010) : Credit Market Imperfections, Selection, and the Distribution of Within-industry Productivity, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2010: Ökonomie der Familie - Session: Innovation and Productivity, No. E16-V1, Verein für Socialpolitik, Frankfurt a. M.

This Version is available at:

<https://hdl.handle.net/10419/37415>

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# Credit Market Imperfections, Selection, and the Distribution of Within-industry Productivity\*

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This version: February 22, 2010

## Abstract

Recent empirical research relates lower aggregate total factor productivity to more dispersed productivity levels within narrowly defined industries. This paper shows that specificity in creditor-borrower relationships will cause adverse selection in line with this evidence. It demonstrates how more severe credit market imperfections will allow less productive firms to enter and will simultaneously prevent more productive firms from entry. To this end, I introduce endogenous credit search frictions in the spirit of Diamond (1990) in a heterogenous firm model à la Hopenhayn (1992) and Melitz (2003). In a perfect credit market, financiers can cherry-pick the most profitable firms. With credit search frictions, financiers also invest in less productive firms because this makes them better off than continuing to search for a more profitable investment opportunity. Consequently, productivity dispersion increases and average productivity falls. I use the framework to assess the impact of product market competition, more efficient credit matching, and changes in the relative bargaining power of firms and banks on productivity.

*Keywords:* Credit Search, Heterogenous Firms, Productivity.

*JEL:* L11, L16, O57, G14

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\*Many thanks to Monika Schnitzer for valuable discussions and suggestions. I also thank Anna Gumpert, Dalia Marin, and Martin Watzinger for insightful comments. Comments are welcome: christian.bauer@lrz.uni-muenchen.de. I gratefully acknowledge financial support from the German Research Foundation (DfG) through GRK 801.

# 1 Introduction

Recent empirical research has attributed cross-country differences in aggregate total factor productivity (TFP) to differences in the productivity distribution across firms within narrowly defined industries. In particular, Hsieh and Klenow (2009) argue that greater within-industry productivity dispersion in China and India relative to the U.S. reveals a severe misallocation of resources in these countries.<sup>1</sup> The wider productivity dispersion may stem from distorted prices faced by individual producers – as in Restuccia and Rogerson (2008) – but also from distortions on firm entry and exit.

Shin and Buera (2009), in an attempt to structurally interpret Hsieh and Klenow's findings, attribute 25-50% of the total misallocation in less developed countries to financial frictions. Their quantitative assessment substantiates the broader view that the credit sector, by mobilizing savings, allocating resources, and scrutinizing projects, plays a crucial role in shaping the development process of new products, firms, and sectors (cf. Greenwood and Jovanovic 1990, Levine 1997, and Matsuyama 2007).<sup>2</sup> This paper shows that specificity in creditor-borrower relationships will induce adverse firm selection, thus lower average productivity, and will lead to wider productivity dispersion within a given industry.

To this end, I introduce a credit market with endogenous search frictions in the spirit of Diamond (1990) in a heterogeneous firm model à la Hopenhayn (1992a,b) and Melitz (2003). In a perfect credit market, financiers can cherry-pick the most profitable firms. With credit search frictions, financiers also invest in some less productive firms because this makes them better off than continuing to search for a more profitable investment opportunity.

The main result is that information frictions in credit markets will shift resources from more productive firms to less productive firms. Specificity in creditor-borrower relationships will cause average productivity to fall and productivity dispersion to rise.

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<sup>1</sup>Cf. Alfaro et al. (2008) and Bartelsman et al. (2009). Hsieh and Klenow (2009) measure the degree of misallocation by the size of gaps in marginal products of labor and capital across plants within narrowly defined industries. They conclude that manufacturing TFP would increase by 30-50% in China and 40-60% in India if these gaps were reduced to the observed levels in the United States.

<sup>2</sup>See Banerjee and Duflo (2005) for an excellent survey on financial frictions and economic development. In addition to its direct effects, Kiyotaki and Moore 1997 and Bernanke et al. 1999 stress that financial intermediation plays an important role in amplifying macroeconomic fluctuations.

More severe credit market imperfections will allow less productive firms to enter and will simultaneously prevent more productive firms from entry. To the extent that credit conditions vary systematically across sectors and countries, this conclusion is consistent with the observed systematic variation in within-industry productivity distributions across different sectors and countries (cf. Bartelsman et al. 2009).

The model is deliberately simple. In search of profits, firms, endowed with heterogeneous projects but lacking financial resources, and financiers enter the credit market based on rational forward-looking decisions. Firm-financier pairs form randomly at a rate endogenously determined by the ratio of credit demand and supply (the credit market “tightness”). Once matched, financiers screen the project and reach a forward-looking investment decision based on the idiosyncratic project profitability, the negotiable repayment rate, and market conditions.

In equilibrium, only sufficiently productive projects are financed. I show that a less efficient credit market implies a lower cutoff productivity level. Similarly, an increase in the equilibrium credit tightness, caused e.g. by an increase in financiers’ costs of raising funds, allows comparatively less productive firms to obtain a credit. The mass of active firms may then increase or decrease depending on the properties of the underlying productivity distribution. I show, however, that utility decreases unambiguously.

I then consider how changes in the intensity of product market competition and innovations in the credit sector interact with the credit friction in the determination of the productivity distribution. I show that increasing competition and more efficient credit matching reduces the adverse selection effect of credit market imperfections. I also consider changes in the relative bargaining power of firms and financiers. Shifting bargaining power to firms entails lower rents for financiers, but also causes a decline in relative credit supply. If financiers’ bargaining weights exceed the elasticity of the credit matching function, financiers become more selective as their bargaining weight increases; otherwise, they become less selective.

These results hinge on the specificity that naturally arises, as Wasmer and Weil (2004, 945) put it, “when agents are imperfectly aware of economic opportunities, from the stochastic matching between creditors and borrowers.”<sup>3</sup>

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<sup>3</sup>In contrast, if credit markets are perfectly competitive and no match-specificity exists, but information

By summarizing credit frictions in a matching function, I build on Dell’Ariccia and Garibaldi (2005) who show that matching models fit empirically observed gross credit flows well.<sup>4</sup> In the context of start-up financing, where uncertainty and lack of information abound, the focus on credit search seems appropriate since raising funds appears to be the principal obstacle to potential entrepreneurs (Blanchflower and Oswald 1998).

The general message of the model is that information frictions in credit markets may systematically widen the productivity dispersion within industries and increase the degree of misallocation of resources. In a similar vein, in order to avoid further misallocations, policies targeted at easing credit conditions or bank lending must ensure not to raise incentives for lax selection. This is, of course, well-known: large numbers of denied credit applications may signal well-functioning credit markets.

The paper relates to three strands of literature. First, it contributes to recent research into the effects of within-industry productivity dispersion on aggregate TFP.<sup>5</sup> Restuccia and Rogerson (2008), using a growth model with heterogenous production plants calibrated to U.S. data, argue that heterogeneity in individual producer prices can decrease output and measured TFP by 30–50%. Hsieh and Klenow (2009)’s aforementioned quantification of the extent of misallocation in China and India finds substantial TFP gains from moving to “U.S. efficiency” (cf. footnote 1). Alfaro et al. (2008) reach a similar conclusion by calibrating a neoclassical growth model with monopolistically competitive, heterogenous plants to match a large sample of countries.<sup>6</sup> Using rich cross-country firm level data, Bartelsman et al. (2009) find substantial firm level variation, measured by the within-industry covariance between size and productivity, across countries.

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frictions impose a mark-up on the “frictionless repayment rate”, only the most productive credit-constrained firms will find it profitable to take out a loan.

<sup>4</sup>Similarly, Petersen and Rajan (2002) provide evidence that relates the “closeness of information” to the efficiency of credit markets. Since Diamond (1990), many authors have modeled credit friction in a search-matching framework. These include den Haan et al. (2003) on borrower–lender complementarities and the propagation of shocks; and Wasmer and Weil (2004) on interrelations between credit- and labor market imperfections.

<sup>5</sup>Other specific mechanisms that may result in resource misallocation include labor market regulations (Hopenhayn and Rogerson 1993 and Lagos 2006), managerial talent (Caselli and Gennaioli 2003 and Buera and Shin 2008) and size restrictions (Guner et al. 2008).

<sup>6</sup>Notably, Alfaro et al. (2008) match each country’s plant size distribution by an appropriate profile of output taxes and subsidies. They suggest to interpret these distortions, among other things, as favorable interest rates on loans based on non-economic factors. In my model, the information friction in the credit market has a similar effect: financiers are able to negotiate, to some extent, higher repayment rates with more productive firms; and less productive firms obtain greater market shares.

Second, this paper contributes to the analysis of aggregate consequences of financial frictions, surveyed by Matsuyama (2007). Li and Sarte (2003) provide evidence that changes in intermediation costs directly affect output. Cooley et al. (2004) show that limited financial contract enforceability amplifies the impacts of technological innovations on aggregate output.<sup>7</sup> More recently, Russ and Valderrama (2009) exploit the relative costs of bank and bond financing to explain how bank lending frictions may affect the firm size distribution through intra-industry reallocations. I exploit the specificity of creditor–borrower relationships.

Finally, the paper relates to the analysis of financially constrained firms in international trade. Following the lead of Chaney (2005) and Manova (2008), this literature explores how liquidity constraints, which may stem from the need for costly outside finance, affect export and foreign direct investment decisions of heterogeneous firms (cf. Buch et al. 2009).<sup>8</sup> In this literature, financial frictions, by raising effective capital costs, prevent less productive firms from market entry and thus imply a positive selection of active firms. Here, in contrast, less productive firms obtain a credit only because of the friction.

This paper proceeds as follows. Section 2 describes the model’s environment. Section 3 characterizes equilibrium and establishes that credit market imperfections lower average productivity and increase the productivity dispersion. Section 4 shows analytically how key parameters of the model interact with the credit friction in the determination of the productivity distribution. Section 5 explores the implications of credit search frictions for changes in product market competition, matching efficiency, and bargaining weights. Section 6 concludes.

## 2 A Parsimonious Model

*Overview.* The economy is populated by three types of agents: financiers (“banks”), firms, and worker-consumers. When firms are born, they are endowed with a project

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<sup>7</sup>Cagetti and De Nardi (2006) and Quadrini (2000) show that models with financial frictions and entrepreneurship may explain observed distributions of wealth.

<sup>8</sup>Gorodnichenkoy and Schnitzer (2009) analyze theoretically and empirically how financial constraints affect a firm’s innovation and export activities.

of random value but without financial resources. Financial resources are needed to set up production facilities.<sup>9</sup> New born firms must thus find a bank willing to invest in the firm's project. Similarly, banks must find suitable investment projects. Information imperfections and heterogeneities make this process costly and, in the spirit of Diamond (1990), give rise to a credit search friction. I follow den Haan et al. (2003) and Wasmer and Weil (2004) and introduce this friction using a credit market matching function. Much like a matching function in the labor market, this function captures the implications of the costly search process without the need to model its specific sources explicitly.<sup>10</sup>

Once a bank and a newly born firm match, the bank screens the proposed project and decides whether to finance that firm or continue searching for a more profitable firm.<sup>11</sup> If the bank decides to invest in that firm, both parties negotiate a binding loan contract, the bank pays out the loan, and the firm enters the market. The firm is active and repays its loan until the match is destroyed exogenously.<sup>12</sup> If the bank decides not to invest, the firm knows that it will not get funds anywhere and exits (it may draw a new project and start searching again).<sup>13</sup> Time is continuous (the time index is omitted throughout). I focus on equilibria with a stable industry environment.

*Credit Market Imperfections.* Let  $f$  and  $b$  denote the sets of firms and banks searching for credit and investment opportunities at a given point in time, respectively. In each short time interval of length  $dt$ ,  $m(b, f)dt$  pairs are randomly matched. The matching function  $m$  is assumed increasing in both its arguments, concave, and homogenous of degree 1. By the homogeneity of  $m$ , the matching rate depends only on the credit

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<sup>9</sup>The set-up costs may include other market entry costs and can equivalently be regarded as one-time equivalent of recurring overhead costs.

<sup>10</sup>Cf. Wasmer and Weil (2004, 947) who substantiate the close analogy between labor market imperfections, an area where matching functions have been used extensively (cf. Pissarides 2000), and credit market imperfections. Instead of being more explicit about the credit friction, I focus on the implications of the match-specificity that arises from costly search, for financiers' investment decisions.

<sup>11</sup>It will not be optimal for banks to get back at an earlier offer, so that the possibility of "recall" can be ignored.

<sup>12</sup>When active, a firm can pay its production costs without requiring further credit. This is achieved either by paying production factors after output is sold, or by assuming that output is produced and sold immediately, or by assuming that active firms, unlike newly born firms, have access to a perfect credit market.

<sup>13</sup>Together with the fact that firms cannot signal their productivity to improve the probability of matching with a bank, this ensures that the distribution of credit searching firms coincides with the underlying distribution and remains stationary over time.

market tightness as measured by the ratio of firms to banks looking for a partner to match with,<sup>14</sup>

$$\theta = \frac{f}{b}. \quad (1)$$

The instantaneous probability that a firm will match with a bank is  $p(\theta) = \frac{m}{f} = m\left(\frac{1}{\theta}, 1\right)$ ,  $p' < 0$ . An increase in the credit tightness reduces the matching probability for firms (the mean duration of credit search is  $\frac{1}{p(\theta)}$ ). From the banks' perspective, the opposite holds. For each of them, a match occurs with instantaneous probability  $\frac{m}{b} = m(\theta, 1) = \theta p(\theta) = \tilde{p}(\theta)$ ,  $\tilde{p}' > 0$ .

*Firms.* Firms are in one of three successive stages. They are either searching for credit, active in the market, or dead. While searching for credit, the firm must incur a sweat cost  $c > 0$  until it matches with a bank. At this stage, the firm has no precise knowledge about how profitable it will be in the market. Its productivity  $\varphi$ , which is constant throughout the firm's life, is unknown until the firm is screened by a bank. The underlying distribution of productivities  $G(\varphi)$ ,  $\varphi \in [0, \bar{\varphi}]$ , however, is common knowledge.

Firms, like banks, discount future income at rate  $r$ . For simplicity, screening is instantaneous and costless. Let  $\varphi^*$  denote the (yet to be determined) lowest productivity level accepted by a bank after screening. This cutoff is a sufficient statistic for the productivity distribution of active firms  $\mu(\varphi; \varphi^*) = \frac{G(\varphi)}{1-G(\varphi^*)}$ .

The value of a newly born firm  $F_0$  satisfies the Bellman equation

$$rF_0 = -c + p(\theta) \left[ G(\varphi^*) F_{out} + \int_{\varphi^*}^{\bar{\varphi}} F_1(\varphi) dG(\varphi) - F_0 \right]. \quad (2)$$

Once a match occurs and the firm is screened, it will get rejected with probability  $G(\varphi^*)$  and accepted with probability  $1 - G(\varphi^*)$ . If it is rejected, the firm dies and has value  $F_{out}$ . If it is accepted, the firm receives a loan, enters the market, and starts operating under monopolistic competition (more details below). Again for simplicity, entry occurs

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<sup>14</sup>The overall efficiency of the credit market is determined by the endogenous market tightness  $\theta$  and the exogenously given productivity of  $m$ . Throughout the paper, I loosely refer to the limiting case where the exogenous productivity goes to infinity as the "absence" of credit search frictions.



instantaneously.<sup>15</sup> The value of a firm after entry,  $F_1(\varphi)$ , solves

$$rF_1(\varphi) = \pi(\varphi; M) - \rho(\varphi) + s[F_{out} - F_1(\varphi)] \quad \forall \varphi \geq \varphi^*. \quad (3)$$

Given its productivity and the mass of active firms  $M$ , the firm earns a flow profit  $\pi(\varphi; M)$  net of loan repayments  $\rho(\varphi)$  until the match is destroyed.<sup>16</sup> This occurs at an exogenous rate  $s$ .

*Banks.* A bank can also be in one of three stages. It is either searching for an investment project (a newly born firm), receiving repayments from an earlier investment, or dead.<sup>17</sup> While searching, the bank incurs a flow cost  $k > 0$ . It will stop searching if it has matched with a firm that, after screening, turns out to be sufficiently profitable (=productive). For projects with  $\varphi \geq \varphi^*$ , the firm and the bank negotiate a binding loan contract  $(\ell, \rho; \varphi)$  which requires the bank to instantaneously provide a loan of size  $\ell$  and the firm to repay a flow  $\rho$  until the match is destroyed.<sup>18</sup> The value of a searching bank  $B_0$  solves the Bellman equation

$$rB_0 = -k + \tilde{p}(\theta) \int_0^{\bar{\varphi}} \max[B_1(\varphi) - B_0 - \ell, 0] dG(\varphi). \quad (4)$$

$B_1(\varphi)$  is the bank's value after signing a loan contract with a firm with productivity  $\varphi$  and paying out the loan.

The unique cutoff productivity is determined by the value of those screened firms for which the bank is indifferent between financing and continuing to search:<sup>19</sup>

$$B_1(\varphi^*) = B_0 + \ell. \quad (5)$$

If the bank agrees to finance the firm, it pays out the loan and subsequently earns a

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<sup>15</sup>I do not allow financiers and firms to optimally time implementation. Francois and Lloyd-Ellis (2003) show how clustering of implementation may lead to cycles.

<sup>16</sup>In this sense, as in Wasmer and Weil (2004), the "loan" is much like equity.

<sup>17</sup>Banks need not die. If all specificity is lost once the firm dies, banks can equivalently go back to searching for new investment opportunities.

<sup>18</sup>The contract specifies that the firm must use the loan instantaneously to finance its start-up costs.

<sup>19</sup>I show later that  $B'_1(\varphi) > 0$  and that a binding  $\varphi^*$  exists.

flow  $rB_1(\varphi)$  until the match is destroyed:

$$rB_1(\varphi) = \rho(\varphi) + s[B_{out} - B_1(\varphi)] \quad \forall \varphi \geq \varphi^*. \quad (6)$$

*Loan Contract.* The specific repayment  $\rho$  is the outcome of bilateral generalized Nash bargaining over the rent generated by a match. The threat point of both parties is to stop negotiating and start looking for another partner to match with.<sup>20</sup> The “excess utility” of a bank is  $B_1(\varphi) - \ell - B_0$ . The excess utility of a firm with productivity  $\varphi$  ( $\geq \varphi^*$ ) is  $F_1(\varphi) - F_0$ . Banks and firms have bargaining weight  $\beta \in (0, 1)$  and  $1 - \beta$ , respectively. The repayment flow solves

$$\rho = \arg \max [B_1(\varphi) - \ell - B_0]^\beta [F_1(\varphi) - F_0]^{1-\beta}.$$

Since  $\frac{\partial [B_1(\varphi) - \ell - B_0]}{\partial \rho} = -\frac{\partial [F_1(\varphi) - F_0]}{\partial \rho}$ , the equilibrium repayment supports

$$\beta [F_1(\varphi) - F_0] = (1 - \beta) [B_1(\varphi) - \ell - B_0] \quad \forall \varphi \in [\varphi^*, \infty). \quad (7)$$

*Isoelastic demand and linear production.* Consumers have standard C.E.S. preferences over the continuum of available goods. The goods will be substitutes, implying that the elasticity of substitution between any two goods is a constant  $\varepsilon > 1$ . Aggregate demand is exogenous and equal to  $E$ .

Normalizing wages to unity, the unit cost of a firm with productivity  $\varphi$  is  $\frac{1}{\varphi}$ . Since the elasticity of demand is constant and homogenous across firms, prices are uniform constant mark-ups over the respective marginal costs. Equilibrium profits (before repayment of credit) thus amount to a constant fraction of revenues  $r(\varphi) = (\varphi P)^{\varepsilon-1} E$  (see Appendix A.1), where  $P \equiv \left[ \int p(j)^{1-\varepsilon} dj \right]^{-\frac{1}{\varepsilon-1}}$  is the aggregate price index (Dixit and Stiglitz 1977).<sup>21</sup> This implies that the profit ratio of any two active firms is given by the

<sup>20</sup>This will of course not occur in equilibrium.

<sup>21</sup>Like in Manova (2006, Sec. 3.3)’s baseline model, an active firm’s operative decisions do not depend on the financial friction.

(weighted) ratio of their productivity levels. In particular,

$$\pi(\varphi) = \left[ \frac{\varphi}{\varphi^*} \right]^{\varepsilon-1} \pi(\varphi^*). \quad (8)$$

Making the substitution from firms to productivity levels in the price index yields  $p^{1-\varepsilon} = M \left( \frac{\varepsilon-1}{\varepsilon} \tilde{\varphi} \right)^{\varepsilon-1}$ . Here,  $\tilde{\varphi}$  is the implied average productivity of active firms  $\tilde{\varphi}(\varphi^*) \equiv \left[ \int_{\varphi^*}^{\tilde{\varphi}} \varphi^{\varepsilon-1} d\mu(\varphi) \right]^{\frac{1}{\varepsilon-1}}$ ,  $\tilde{\varphi}' > 0$ . With this and  $\varepsilon \equiv \varepsilon \left( \frac{\varepsilon-1}{\varepsilon} \right)^{\varepsilon-1}$ , average profits can be written as<sup>22</sup>

$$\bar{\pi} \equiv \pi(\tilde{\varphi}(\varphi^*); M) = \int_0^{\tilde{\varphi}} \pi(\varphi; M) d\mu(\varphi) = \frac{E}{\varepsilon M}. \quad (9)$$

*Interpretation of key parameters.* Following Wasmer and Weil (2004), I refer to banks' search costs as a measure of credit conditions: tightening credit conditions raise  $k$ . The firms' search costs  $c$  capture administrative costs during the fund raising stage, which are e.g. determined by the degree of red tape.<sup>23</sup>  $\beta$  and  $1 - \beta$  can literally be interpreted as bargaining weights. However, we can also think of them as the relative degree of patience in a Rubinstein game with alternating offers.  $\beta$  thus includes the financiers' ability to delay the loan approval decision (e.g. by blaming regulatory requirements that need to be verified or because of unfavorable economic conditions, which, inter alia, may be affected by monetary policy or the strategic dissemination of relevant news).<sup>24</sup>

### 3 Characterization of Equilibrium

In this section, I define equilibrium, solve the model, and show how consumption utility is affected by credit market imperfections. To begin with, I add the final equilibrium conditions.

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<sup>22</sup>In equilibrium,  $\mu' = 0$  for  $\varphi < \varphi^*$ .

<sup>23</sup> $c$  and  $k$  can equivalently be regarded as flow equivalents of one-time costs.

<sup>24</sup>Instead of subsuming these strategic interactions in  $\beta$ , we could try and model them explicitly. Alternatively also, we could alter the banks' discount rate at the fund raising stage. Both routes will lead to qualitatively similar results.

*Outside values.* For simplicity, dying firms and banks are left with zero value:

$$F_{out} = 0, \quad (10)$$

$$B_{out} = 0. \quad (11)$$

*Free entry.* Following Melitz (2003) and Wasmer and Weil (2004), I assume that setting up a firm and a bank is costless at the initial stage. Free entry in equilibrium then ensures

$$B_0 = 0, \quad (12)$$

$$F_0 = 0. \quad (13)$$

*Stationarity.* In a stable industry environment, the mass of dying matches must equal the mass of newly formed matches:

$$sM = fp(\theta) [1 - G(\varphi^*)]. \quad (14)$$

Equations (1)–(14) comprise a system of 14 equations in 14 unknowns.

**Definition 1 (Equilibrium).** An “anonymous” steady state equilibrium is a vector of (constant)  $B_0, B_1, F_0, F_1, \rho(\varphi), \theta, \varphi^*, \pi(\varphi), \bar{\pi}, M, B_{out}, F_{out}, f$ , and  $b$  that solves (1)–(14).

The equilibrium can be characterized as follows. Combining (5), (6), (11), and (12) shows that the cutoff condition requires the equilibrium repayment flow of the marginal firm to equal the periodized loan size:

$$\rho(\varphi^*) = (r + s)\ell. \quad (15)$$

More generally, the credit contract implements

$$\rho(\varphi) = \beta\pi(\varphi; \varphi^*, M) + (1 - \beta)(r + s)\ell \quad \forall \varphi \in [\varphi^*, \infty). \quad (16)$$

This “sharing rule” depends on the idiosyncratic productivity of the firm and on

market conditions. The former is a direct implication of the match specificity resulting from the costly search process.

Applying (10)-(13) to the firms' and banks' value functions yields two free entry conditions (see Appendix A.2):

$$\frac{c}{p(\theta)} = \int_{\varphi^*}^{\bar{\varphi}} \frac{\pi(\varphi) - \rho(\varphi)}{r+s} dG(\varphi), \quad (17)$$

$$\frac{k}{\bar{p}(\theta)} + [1 - G(\varphi^*)] \ell = \int_{\varphi^*}^{\bar{\varphi}} \frac{\rho(\varphi)}{r+s} dG(\varphi). \quad (18)$$

A firm on average “draws  $\frac{1}{p(\theta)}$  times” until it finds a bank. If it is matched, conditional on  $\varphi \geq \varphi^*$ , the firm receives the expected flow of net profits until the match is destroyed, and nothing otherwise. A bank on average draws  $\frac{1}{\bar{p}(\theta)}$  times until it matches with a firm. With probability  $1 - G(\varphi^*)$ , the match is sufficiently profitable, in which case the bank pays out the loan and receives match-specific repayments until the match is destroyed; and nothing otherwise. In equilibrium, expected costs equal expected benefits for both firms and banks.

Combining the equilibrium repayment rate (16) in turn with (17) and (18) yields

$$\frac{c}{(1-\beta)p(\theta)} = \int_{\varphi^*}^{\bar{\varphi}} \frac{\pi(\varphi)}{r+s} dG(\varphi) - \ell[1 - G(\varphi^*)], \quad (19)$$

$$\frac{k}{\beta\bar{p}(\theta)} = \int_{\varphi^*}^{\bar{\varphi}} \frac{\pi(\varphi)}{r+s} dG(\varphi) - \ell[1 - G(\varphi^*)]. \quad (20)$$

Under free entry, the equilibrium sharing rule naturally equates the “share of the pie” to firms' and banks' expected search costs.

By construction, the entry conditions (19) and (20) allow me to solve for the credit market tightness without simultaneously having to determine the productivity cutoff. In particular, due to the recursive structure, I obtain Wasmer and Weil (2004)'s solution, derived in a representative firm framework:

$$\theta^* = \frac{1 - \beta k}{\beta c}. \quad (21)$$

The tightness of the credit market, and hence the firms' expected duration of credit search, is increasing in  $k$  and decreasing in  $c$  and  $\beta$  (and vice versa from the bank's

perspective).

Combining the equilibrium credit tightness (21) with (9) and (19) yields average profits consistent with free entry and the equilibrium entry costs:

$$\bar{\pi} = \int_{\varphi^*}^{\bar{\varphi}} \pi(\varphi) d\mu(\varphi) = \left\{ \frac{c}{(1-\beta)p(\theta^*)[1-G(\varphi^*)]} + \ell \right\} (r+s). \quad (\text{FE})$$

Combining the equilibrium repayment in (15) and (16) shows that banks extend credit to firms who at least earn the periodized loan size:<sup>25</sup>

$$\pi(\varphi^*) = (r+s)\ell. \quad (22)$$

Note that the tightness of the credit market will affect the equilibrium cutoff as  $\pi(\varphi^*)$  is determined by the cutoff condition (22) and the free entry condition (FE). Combining (8) and (22) for firms with the average productivity yields the cutoff condition in terms of average profits:

$$\bar{\pi} = \left[ \frac{\tilde{\varphi}(\varphi^*)}{\varphi^*} \right]^{\varepsilon-1} (r+s)\ell. \quad (\text{CC})$$

By construction of the model, (CC) resembles the “Zero Cutoff Profit Condition” in Melitz (2003). The determination of the cutoff is illustrated in Figure 1. (FE) and (CC) uniquely pin down the equilibrium  $\bar{\pi}$  and  $\varphi^*$ .<sup>26</sup> The cutoff productivity fully summarizes the impact of credit search frictions on the productivity distribution.

**Result 1.** *In the absence of credit search frictions ( $p \rightarrow \infty$ ), banks cherry-pick the most profitable firms and finance firms with productivity  $\bar{\varphi}$  only. Credit search frictions relax the minimum productivity requirement ( $\varphi^* < \bar{\varphi}$ ).*

*Proof.* Combining (21), (FE), and (CC) provides an implicit expression for  $\varphi^*$ :

$$\frac{c}{(1-\beta)p(\theta^*)\ell} = \left\{ \left[ \frac{\tilde{\varphi}(\varphi^*)}{\varphi^*} \right]^{\varepsilon-1} - 1 \right\} [1-G(\varphi^*)]. \quad (23)$$

<sup>25</sup>Nash bargaining naturally ensures that firms always want to continue once they are financed ( $rF_1(\varphi) = \pi(\varphi; M) - \rho(\varphi) - sF_1(\varphi) = \frac{r}{r+s} [\pi(\varphi; M) - \rho(\varphi)] = (1-\beta)r \left[ \frac{\pi(\varphi; M)}{r+s} - \ell \right] \geq 0$ ).

<sup>26</sup>As in Melitz (2003), the (CC) curve need not be decreasing everywhere as in the figure. Existence and uniqueness of equilibrium do not hinge on this property (cf. Appendix A.4).

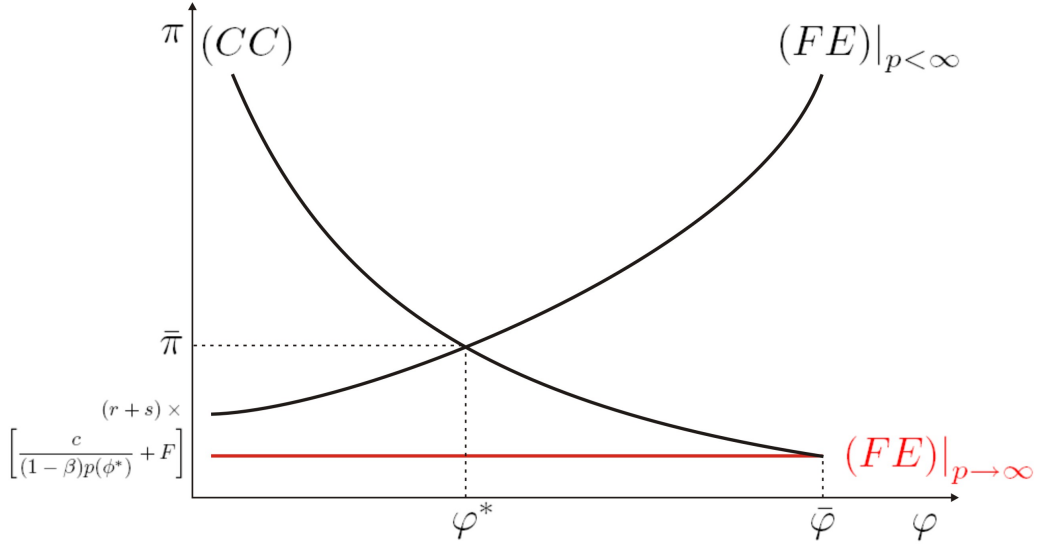


Figure 1: Cutoff and average profits with credit search frictions.

Suppose credit market imperfections are negligibly small, i.e.  $p \rightarrow \infty$  (because the productivity of  $m$  is infinitely large). Then, (23) implies  $\varphi^* \rightarrow \bar{\varphi}$ . Appendix A.4 shows that the right-hand side of (23) is decreasing in  $\varphi^*$ . Hence,  $\varphi^* < \bar{\varphi}$  if  $p < \infty$ .  $\square$

The intuition for this result is straightforward: if switching from one firm to another is costless (and screening is costless as well), the bank will continue searching until a firm with productivity  $\bar{\varphi}$  is found. As becomes clear below, the result that only  $\bar{\varphi}$ -firms are financed as  $p \rightarrow \infty$  is due to the fact that all set-up cost are financed by the bank at the competitive rate. With credit search frictions, the bank is better off financing some less productive firms than continuing to search. Firms with productivity  $\varphi^*$  leave banks indifferent.

The search costs play the same role as the sunk entry costs in Melitz (2003). Here, however, they are endogenous.<sup>27</sup>

Figure 2 depicts a numerical example of the productivity density of active firms assuming that the underlying productivity distribution is Pareto. The red line illustrates a benchmark case. The blue line depicts the steady state distribution after a shock to the exogenous productivity of the matching function that reduces a firms' probability

<sup>27</sup>As mark-ups are fixed, consumption expenditures leave the cutoff unaffected. An increase in the substitutability parameter  $\varepsilon$  raises  $\varphi^*$  (if  $p < \infty$ ). I explore the interaction of product market competition and credit frictions in Section 5.

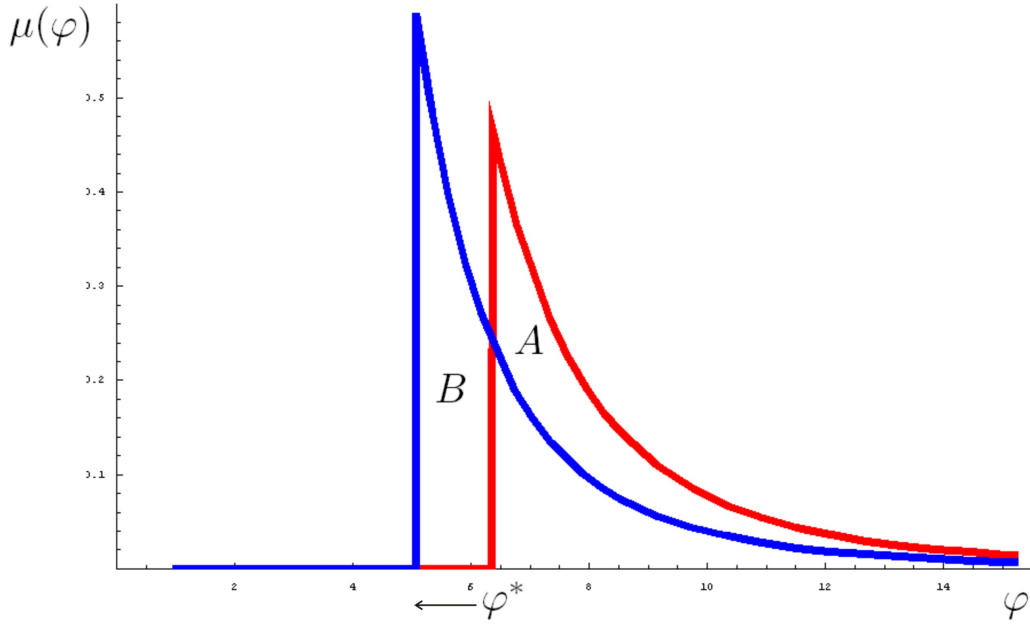


Figure 2: Red: Illustrative example for the stationary productivity density of active firms. Blue: Stationary density if matching probability is reduced to 80% of the benchmark value.

to meet with a bank persistently to 80% of the benchmark value. The shock lowers the cutoff, reduces the density at high productivity levels (region *A* in the figure), and increases the density at low productivity levels (region *B*).

The mass of firms is readily derived from the equilibrium profits of active firms,

$$\pi(\varphi, M) = \frac{E}{\epsilon M} \left[ \frac{\varphi}{\tilde{\varphi}(\varphi^*)} \right]^{\epsilon-1}. \quad (24)$$

Substituting the average productivity and noting (CC), we obtain a relationship between the cutoff and the mass of active firms

$$M^* = \left[ \frac{\varphi^*}{\tilde{\varphi}(\varphi^*)} \right]^{\epsilon-1} \frac{E}{\epsilon(r+s)\ell}. \quad (25)$$

In general, the sign of  $\nu = \partial \frac{\tilde{\varphi}(\varphi^*)}{\varphi^*} / \partial \varphi^*$  depends on the underlying distribution (cf. Melitz 2003, 1704). Intuitively,  $\nu < 0$  if  $\tilde{\varphi}$  is concave and  $\nu > 0$  if it is convex.<sup>28</sup>

**Result 2.** Suppose  $\nu < (=, >) 0$ . Credit search frictions increase (do not change, decrease) the

<sup>28</sup>Substantial empirical research strongly supports the assumption that  $G$  is Pareto (Axtell 2001, Cabral and Mata 2003, Helpman et al. 2004, Del Gatto et al. 2006, Corcos et al. 2007), in which case  $\nu = 0$ . Cf. Luttmer (2007) and the references therein.



mass of active firms.

If  $v < 0$ , a trade-off between the mass of available goods and the productivity of active firms exists. I show below that the productivity effect dominates the variety effect in terms of consumption utility.

Consider next the credit market for new born firms. The stationarity condition (14) delivers the mass of firms that are searching for credit as a function of the cutoff,

$$f^* = \frac{s}{r+s} \left[ \frac{\varphi^*}{\tilde{\varphi}(\varphi^*)} \right]^{\varepsilon-1} \frac{E}{\varepsilon p(\theta^*) [1 - G(\varphi^*)] \ell}. \quad (26)$$

The tightness of the credit market has a direct effect through  $p$  and an indirect effect through  $\varphi^*$ . Using (23) we find that, in equilibrium, the effect of credit search on  $f^*$  is determined by the sign of  $v$ :

$$f^* = \frac{1-\beta}{\varepsilon c} \frac{s}{r+s} \left\{ 1 - \left[ \frac{\varphi^*}{\tilde{\varphi}(\varphi^*)} \right]^{\varepsilon-1} \right\} E, \quad (27)$$

where  $\varphi^*$  is the solution to (23). Using the definition of the credit market tightness, its equilibrium value, and (27), the mass of banks searching for a firm is

$$b^* = \frac{\beta}{\varepsilon k} \frac{s}{r+s} \left\{ 1 - \left[ \frac{\varphi^*}{\tilde{\varphi}(\varphi^*)} \right]^{\varepsilon-1} \right\} E. \quad (28)$$

Since  $\frac{\partial f^*}{\partial \varphi^*} < 0$ , the following holds.

**Result 3.** *Suppose  $v < (=, >) 0$ . Credit search frictions increase (do not change, decrease) the equilibrium mass of firms (banks) searching for credit (investment projects).*

Finally, consider the equilibrium contract. As a prerequisite, substitute with  $M^*$  in (24) to obtain equilibrium profits

$$\pi^*(\varphi) = \left( \frac{\varphi}{\varphi^*} \right)^{\varepsilon-1} (r+s) \ell \quad \forall \varphi \geq \varphi^*. \quad (29)$$

More productive firms earn higher profits. The profit level is a positive function of the gap between a firm's productivity and the cutoff productivity. Inserting the equilibrium

repayment  $\rho^*(\varphi)$  in  $\pi^*(\varphi)$ , the value of an active firm is given by

$$F_1^*(\varphi) = (1 - \beta) \left[ \left( \frac{\varphi}{\varphi^*} \right)^{\varepsilon-1} - 1 \right] \ell \quad \forall \varphi \geq \varphi^*. \quad (30)$$

(29) and (30) prove the following result on equilibrium profits/firm values.

**Result 4.** *In the absence of credit search frictions, the unique profit flow is  $\pi(\bar{\varphi}) = (r + s)\ell$ . With credit search frictions, equilibrium profits are larger for all active firms,  $\pi(\varphi) \geq (r + s)\ell \quad \forall \varphi \geq \varphi^*$ .*

In equilibrium, the cutoff productivity firms earn profits equal to the periodized loan size. Credit search costs allow less productive firms to be active, and thus raise profits for all active firms.

Using (29), we find that the equilibrium repayment flow solves

$$\rho^*(\varphi) = \left\{ \beta \left[ \left( \frac{\varphi}{\varphi^*} \right)^{\varepsilon-1} - 1 \right] + 1 \right\} (r + s)\ell. \quad (31)$$

Due to the match-specificity, firms whose productivity is high relative to the cutoff productivity are charged higher repayment rates.

**Result 5.** *In the absence of credit search frictions, the unique repayment rate is  $\rho^*(\bar{\varphi}) = (r + s)\ell$ . With credit search frictions, all active firms are charged higher repayment rates,  $\rho^*(\varphi) \geq (r + s)\ell \quad \forall \varphi \geq \varphi^*$  (with equality for  $\varphi > \varphi^*$ ).*

Absent credit frictions, competition rules out the possibility that banks can bargain for a repayment rate in excess of  $r + s$ . We immediately conclude that credit search frictions raise the value of banks at the repayment stage.

**Result 6.** *In the absence of credit search frictions, the unique value of banks at the repayment stage is  $B_1^*(\bar{\varphi}) = \ell$ . With credit search frictions, the value of banks at the repayment stage varies according to the productivity of the financed firm and  $B_1^*(\varphi) \geq \ell, \varphi \geq \varphi^*$ .*

Result 6 is a direct implication of the fact that credit frictions raise the value of active firms. Banks participate in this rent due to the match-specificity.

This completes the solution of the model. I next consider the impact of credit search frictions on banks' internal rate of return.

The internal rate of return  $\zeta$  for a loan of size  $\ell$  solves

$$\ell = \frac{\rho(\varphi)}{\zeta(\varphi) + s}. \quad (32)$$

Combining this definition with the equilibrium repayment (31) yields

$$\zeta(\varphi) = r + \beta(r + s) \left[ \left( \frac{\varphi}{\varphi^*} \right)^{\varepsilon-1} - 1 \right]. \quad (33)$$

**Result 7.** *In the absence of credit market imperfections, the internal rate of return is  $r$  for all banks. With credit search frictions,  $\zeta(\varphi) \geq r$  with strict inequality for all  $\varphi > \varphi^*$ .*

I conclude this section by showing that raising the cutoff productivity raises utility from consumption.<sup>29</sup>

**Result 8.** *Consumption utility is strictly decreasing in the degree of credit market imperfections.*

*Proof.* Substituting for firms with productivities, utility in the stationary equilibrium can be written as (see Appendix A.3)

$$U = \left[ \int_0^{M^*} x^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} = \left( 1 - \frac{1}{\varepsilon} \right) (M^*)^{\frac{1}{\varepsilon-1}} \tilde{\varphi}(\varphi^*) E. \quad (34)$$

Inserting the equilibrium expression for  $M^*$ ,

$$U = \left[ \frac{E^\varepsilon}{\varepsilon(r+s)\ell} \right]^{\frac{1}{\varepsilon-1}} \varphi^*. \quad (35)$$

Using the previous comparative static results on  $\varphi^*$  completes the proof.  $\square$

If increasing the cutoff productivity reduces the mass of available products (i.e. if  $\nu < 0$ ), the productivity gains outweigh the loss of variety.

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<sup>29</sup>This neglects the impact of search costs. It is, however, instructive to access the tension between firms' productivity and the mass of available products in cases where  $\nu < 0$ .

## 4 Comparative Statics Results

In this section, I show how key parameters of the model interact with the credit friction in the determination of equilibrium outcomes.

To this end let  $\varepsilon_p = -\frac{\partial p(\theta^*)}{\partial \theta^*} \frac{\theta^*}{p(\theta^*)} \in (0, 1)$  denote the elasticity of the credit matching function evaluated at equilibrium. I begin by looking at the determinants of the productivity distribution (for which  $\varphi^*$  is a sufficient statistic). Consider first the effects of search costs.

**Result 9.** *The cutoff productivity is decreasing in banks' search cost  $k$  and firms' search costs  $c$ . The effect of  $c$  on  $\varphi^*$  is more pronounced if  $\varepsilon_p$  is small, whereas the effect of  $k$  on  $\varphi^*$  is more pronounced if  $\varepsilon_p$  is large.*

*Proof.* Rearrange (23) to define

$$\Gamma \equiv \frac{c}{(1-\beta)p(\theta^*)\ell} - \left\{ \left[ \frac{\tilde{\varphi}(\varphi^*)}{\varphi^*} \right]^{\varepsilon-1} - 1 \right\} [1 - G(\varphi^*)] = 0. \quad (36)$$

Appendix A.4 shows that, for  $\tilde{\varphi} > \varphi^*$ ,

$$\frac{\partial \Gamma}{\partial \varphi^*} = \left\{ \left[ \frac{\tilde{\varphi}(\varphi^*)}{\varphi^*} \right]^{\varepsilon-1} - 1 \right\} G'(\varphi^*) - (\varepsilon-1) [1 - G(\varphi^*)] \left[ \frac{\tilde{\varphi}(\varphi^*)}{\varphi^*} \right]^{\varepsilon-2} \nu > 0. \quad (37)$$

Using  $\frac{\partial \theta^*}{\partial c} = -\frac{\theta^*}{c}$  and  $\frac{\partial \theta^*}{\partial k} = \frac{\theta^*}{k}$ ,

$$\frac{\partial \Gamma}{\partial c} = \frac{1 - \varepsilon_p}{(1-\beta)p(\theta^*)\ell} > 0, \quad (38)$$

$$\frac{\partial \Gamma}{\partial k} = \frac{\varepsilon_p}{\beta \tilde{p}(\theta^*)\ell} > 0. \quad (39)$$

Hence, by the implicit function theorem,

$$\frac{\partial \varphi^*}{\partial k} < 0, \quad \frac{\partial \varphi^*}{\partial c} < 0, \quad \text{and} \quad \frac{\partial \left( \frac{\partial \varphi^*}{\partial k} \right)}{\partial \varepsilon_p} > 0, \quad \frac{\partial \left( \frac{\partial \varphi^*}{\partial c} \right)}{\partial \varepsilon_p} < 0. \quad (40)$$

□

Figure 3 depicts the equilibrium cutoff and average profits before and after increases in the search costs  $c$  and  $k$ . Intuitively, banks are willing to accept less profitable projects

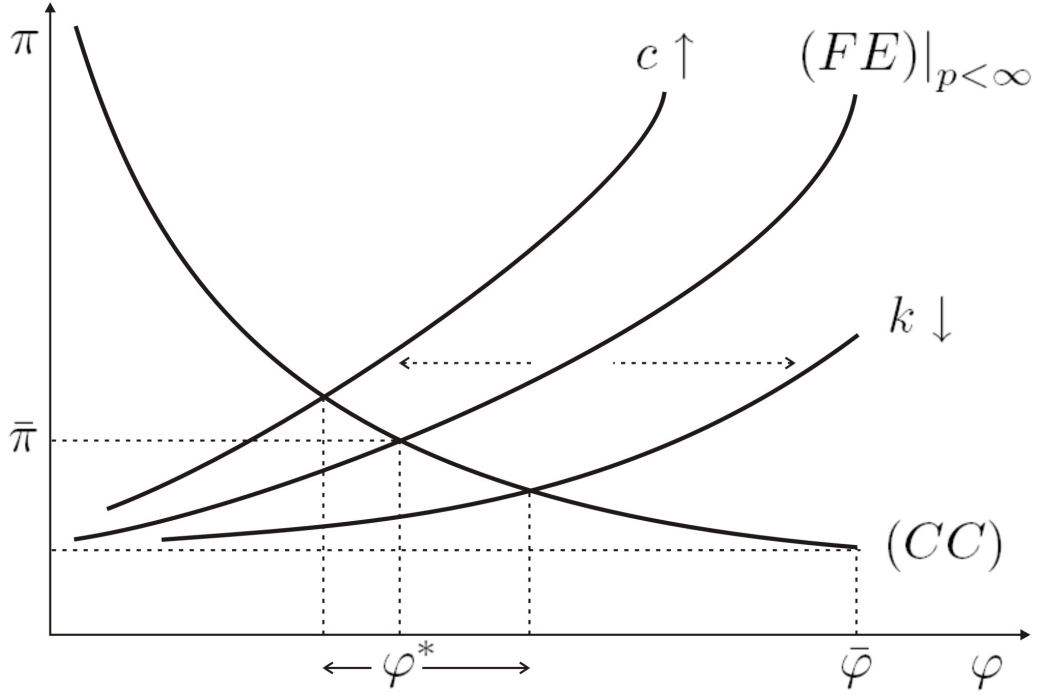


Figure 3: Effects of changing search costs  $c$  and  $k$  on the cutoff.

if total expected search costs go up.<sup>30</sup> From the point of view of a bank, the increase in search costs dominates the increase in the probability of matching with a firm (i.e.,  $\partial \frac{k}{\bar{p}(\theta^*)} / \partial c > 0$ ,  $\partial \frac{k}{\bar{p}(\theta^*)} / \partial k > 0$ ). From the point of view of a firm, an increase in  $k$  tightens the credit market and raises expected search costs. This reduces entry of firms. Similarly, if  $c$  increases, less firms enter, and the matching probability for banks decreases. This reduces the necessary minimum productivity level for firms to obtain a credit.

We now turn to the effects of credit search costs on the mass of active firms and the “number” of declined credit applicants. In the benchmark case where matching occurs instantaneously,  $M^* = \frac{E}{\epsilon(r+s)\ell}$ . With credit search frictions, the mass of firms depends, through the cutoff, also on the costs of search.

**Result 10.** *Let  $v \leq 0$ . The stationary mass of active firms is decreasing in  $c$  and  $k$ . If  $v < 0$ , the mass of searching firms  $f^*$  is increasing in  $k$ . The mass of searching banks  $b^*$  is increasing in  $c$ . If  $v = 0$ ,  $f^*$  is decreasing in  $c$ ,  $b^*$  is decreasing in  $k$ , and  $f^*$  and  $b^*$  do not depend on  $k$  and  $c$ , respectively.*

<sup>30</sup>The “direct” effect of the credit tightness is negative,  $\frac{\partial \varphi^*}{\partial \theta^*} < 0$ .

*Proof.* Result 9 established  $\frac{\partial \varphi^*}{\partial k} < 0$  and  $\frac{\partial \varphi^*}{\partial c} < 0$ .  $\frac{\partial M^*}{\partial \varphi^*} > 0$  completes the first part of the proof. The second part follows, if  $\varphi^*$  is linear, from  $\partial \left\{ 1 - \left[ \frac{\varphi^*}{\bar{\varphi}(\varphi^*)} \right]^{\varepsilon-1} \right\} / \partial \varphi^* < 0$  and (40).  $\square$

Intuitively, if the ratio of average and cutoff productivity is fixed (as is the case if  $G$  is Pareto), an increase in  $c$  ( $k$ ) leads less firms (banks) to enter because matching with a partner gets less likely and search becomes more costly.

Since an increase in search costs lowers the cutoff, it is intuitive that the mass of firms that are denied credit in each instant,  $f_f^* = f^* p(\theta^*) G(\varphi^*)$ , is decreasing in the severeness of the credit friction.

**Result 11.** *Let  $\nu \leq 0$ . An increase in search costs  $c$  and  $k$  reduces the instantaneous mass of firms that are denied credit.*

*Proof.* From the stationarity condition:

$$f_f^* = \frac{G(\varphi^*)}{1 - G(\varphi^*)} s M^*. \quad (41)$$

Since  $\partial \frac{G(\varphi^*)}{1 - G(\varphi^*)} / \partial \varphi^* > 0$  and  $\frac{\partial M^*}{\partial \varphi^*} > 0$ ,  $\frac{\partial f_f^*}{\partial \varphi^*} > 0$ .  $\frac{\partial \varphi^*}{\partial k} < 0$  and  $\frac{\partial \varphi^*}{\partial c} < 0$  complete the proof.  $\square$

This results substantiates the claim that we cannot conclude from a large observed numbers of declined credit application that credit markets are not functioning. In this model, if a trade-off between productivity and product variety exists, the opposite is true: if the credit market works poorly, less firms are denied credit.

There are two types of fixed costs in the model: the endogenous search costs and the loan, which can be interpreted as present-value of overhead costs. In Hopenhayn-Melitz models, sunk entry costs allow less productive firms to survive and therefore lower the average productivity. "Overhead" or setup costs on the other hand increase the average productivity level because they force the least productive firms to exit (cf. Felbermayr and Prat 2009).<sup>31</sup> Result 9 characterized the productivity effects of entry

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<sup>31</sup>In the Felbermayr and Prat (2009) model, sunk entry costs and overhead costs affect equilibrium employment through their impact on the productivity distribution. Here, I provide an environment where the (positive) selection effect from overhead costs disappears if the costs are fully financed at a competitive in-

costs, which here are determined by (exogenous) search costs and (endogenous) matching probabilities. The next result shows how search costs and set-up costs interact in the determination of the cutoff productivity.

**Result 12.** *In the absence of credit frictions, the size of  $\ell$  has no effect on the cutoff. With credit search frictions, the cutoff falls in  $\ell$ . This effect is more pronounced if search costs  $c$  and  $k$  are large and the banks' bargaining weight  $\beta$  is small.*

*Proof.* As  $p \rightarrow \infty$ , the size of  $\ell$  does not matter for the determination of the cutoff. If  $p < \infty$ , (36) and (37) imply  $\frac{\partial \varphi^*}{\partial \ell} > 0$  since  $\frac{\partial \Gamma}{\partial \ell} < 0$ . The derivative is

$$\frac{\partial \varphi^*}{\partial \ell} = \frac{c}{(1 - \beta) p (\theta^*)^2 \ell^2 \frac{\partial \Gamma}{\partial \varphi^*}}. \quad (42)$$

Since  $\frac{\partial \Gamma}{\partial \varphi^*}$  only depends on the exogenously given distribution and  $\varepsilon$ , we have

$$\begin{aligned} \frac{\partial^2 \varphi^*}{\partial \ell \partial c} &= \frac{1 - \varepsilon_p}{(1 - \beta) p (\theta^*)^2 \ell^2 \frac{\partial \Gamma}{\partial \varphi^*}} > 0, \\ \frac{\partial^2 \varphi^*}{\partial \ell \partial k} &= \frac{\varepsilon_p c}{(1 - \beta) p (\theta^*)^2 \ell^2 \frac{\partial \Gamma}{\partial \varphi^*}} > 0, \\ \frac{\partial^2 \varphi^*}{\partial \ell \partial \beta} &= -\frac{\varepsilon_p c}{\beta (1 - \beta)^2 p (\theta^*)^2 \ell^2 \frac{\partial \Gamma}{\partial \varphi^*}} < 0. \end{aligned}$$

□

To interpret this result, it is instructive to contrast the limiting case of vanishing search frictions with an economy where new firms obtain credit directly at a perfect credit market. In the latter, firms decide whether to enter the market based on the expected present value of profits. As setup costs fall, less productive firms enter. In this model, by screening and selecting investment projects, banks participate in the entry decision. Anticipating the outcome of the repayment rate bargain, banks in a sense make the market entry decision on behalf of the firm. Importantly, however, the size of the set-up costs (which coincides with the size of the loan) affects this decision only to the extent that the repayment rate differs from the banks' discount rate. Result 5

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terest rate. See Pflüger and Südekum (2009) for an analysis of the welfare costs of barriers to entry and the benefits of entry subsidies.

showed that both coincide in the absence of credit market imperfections. The relevant set-up costs for the bank's decision then have zero net present value (independently of the size of  $\ell$ ). With credit search frictions, the net present value of the loan is positive since a bank's discount rate falls short the repayment rate. Lower set-up costs then have the same selection effect as in models without banks and perfect credit markets: a decline in effective fixed costs allows less productive firms to operate (i.e. obtain a credit). Formally, the downward shift of (FE) due to additional entry is dominated by the downward shift of (CC) due to the lower net present value of the loan. In the absence of credit frictions, lower set-up costs do not reduce the cutoff productivity as long as they are fully financed by banks at their discount rate.

In the presence of credit search frictions, reducing the loan size has two counteracting effects. First, as  $\ell$  falls, more firms will enter. The (FE) curve shifts downwards. The (CC) curve, however, shifts downwards as well since the reduced loan implies smaller repayments (which act like the overhead costs in Melitz 2003). It turns out that the latter effect dominates.

Consider next the effects of search costs on repayment rates and the internal rate of return for banks. To this end, recall that high search costs increase profits earned by active firms, see (29) and (40), by reducing entry of high productivity type firms.

**Result 13.** *An increase in  $c$  or  $k$  increases repayment rates and the internal rate of return of banks.*

*Proof.* (31) and (33) imply  $\frac{\partial \rho(\varphi)}{\partial \varphi^*} < 0$  and  $\frac{\partial \xi(\varphi)}{\partial \varphi^*} < 0$ .  $\frac{\partial \varphi^*}{\partial k} < 0$  and  $\frac{\partial \varphi^*}{\partial c} < 0$  complete the proof.  $\square$

## 5 Interaction with Changes in the Environment

This section explores the implications of changes in the environment for productivity and selection. I consider more efficient credit matching, more intense product market competition (PMC), and changes in the relative bargaining power of firms and banks in turn.

*More efficient credit matching.* Denote the exogenously given productivity of the



matching function by  $\gamma$  ( $\frac{\partial p}{\partial \gamma}, \frac{\partial \bar{p}}{\partial \gamma} > 0$ ). The previous sections immediately imply that credit market innovations that lead to more efficient credit matching entail selection effects that are qualitatively similar to the selection effects resulting from a reduction of credit search frictions. That is, if  $\gamma$  increases (and  $\nu \leq 0$ ), the cutoff increases, the mass of active firms decreases or stays constant, the mass of searching banks and firms decreases or stays constant, profits of active firms decrease, repayment rates decrease, and utility from consumption rises.

*Changing bargaining weights.* On the one hand, an increase of  $\beta$  allows banks to obtain a higher effective share from the rent realized by a match, and thus reduces their incentive to search for more profitable investments. On the other hand, as the return for banks increases, more banks enter and drive down the equilibrium credit tightness. As search costs increase, banks will only accept sufficiently profitable projects that at last compensate for the increased search costs. The net impact of these counteracting effects depends on the elasticity of the matching function.

**Result 14.** *Suppose  $\varepsilon_p < (=, >) \beta$ . An increase in the banks' bargaining weight  $\beta$  reduces (has no effect, increases) the cutoff.*

*Proof.* From (36),

$$\frac{\partial \Gamma}{\partial \beta} = \frac{c \left(1 - \frac{\varepsilon_p}{\beta}\right)}{(1 - \beta)^2 p(\theta^*) \ell}. \quad (43)$$

Hence,  $\frac{\partial \varphi^*}{\partial \beta} >, =, < 0$  if  $\beta <, =, > \varepsilon_p$ . □

If  $\varepsilon_p$  is sufficiently large ( $\varepsilon_p > \beta$ ) the increase in search costs dominates the increase in the share of the rent (and vice versa).

Combing this result with (41), I conclude that an increase in banks' bargaining weight reduces (raises) the instantaneous mass of firms that are denied credit if  $\beta > \varepsilon_p$  ( $\beta < \varepsilon_p$ ).

Suppose policy makers can affect the distribution of bargaining weights. This may e.g. be possible through tighter monetary policy, which may induce banks to become less patient in repayment rate negotiations. Applying result 8, the model then suggests that raising (reducing) firms' bargaining power is desirable in environments where banks have a sufficiently high (low) bargaining weight.

In the absence of credit search frictions, the distribution of bargaining weights naturally has no effect on the stationary mass of firms. With credit search frictions,  $\beta$  affects the incentives for entry for both firms and banks.

**Result 15.** *Let  $\nu \leq 0$ . The stationary mass of active firms is decreasing (increasing) in  $\beta$  if  $\beta > \varepsilon_p$  ( $\beta < \varepsilon_p$ ).*

*Proof.* Follows directly from result 14 and  $\frac{\partial M^*}{\partial \varphi^*} > 0$ . □

In view of the counteracting effects, it is not surprising that changes in  $\beta$  in general have ambiguous effects on credit supply.

**Result 16.** *Let  $\nu \leq 0$ . An increase in  $\beta$  reduces the mass of credit searching firms. The mass of searching banks may either increase or decrease.*

*Proof.* If  $\nu \leq 0$ , differentiating (27) yields  $\frac{\partial f^*}{\partial \beta} < 0$ . Differentiating (28) implies

$$\frac{\partial b^*}{\partial \beta} >, =, < 0 \Leftrightarrow \left[ \frac{\tilde{\varphi}(\varphi^*)}{\varphi^*} \right]^\varepsilon - 1 + \beta(\varepsilon - 1)\nu >, =, < 0$$

□

An increase in bank's bargaining weight will lead to less searching banks if, *c.p.*,  $\beta$  is close to 0,  $\varepsilon$  is close to 1, or  $\varphi^*$  is close to  $\bar{\varphi}$ . If  $G$  is Pareto, the mass of searching banks is increasing in  $\beta$ .

*More intense PMC.* We conclude this section by looking at the effects of product market competition (PMC). Following Aghion et al. (2001, 471), we explore the substitutability parameter  $\varepsilon$ , which measures both the price elasticity of demand for each good and the elasticity of substitution between any two goods, to assess the impact of PMC on the productivity distribution.

**Result 17.** *Let  $\nu \leq 0$ . An increase in PMC increases the productivity cutoff. More intense PMC also raises repayment rates, the internal rate of return for banks, and mitigates the effects of higher search costs and changes in  $\beta$  on the cutoff in absolute terms.*

*Proof.* The first part follows from inspecting the right-hand side of (23), which is decreasing in  $\varphi^*$ . The second part follows from (31) and (33) and the fact that  $\frac{\partial \Gamma}{\partial c}$ ,  $\frac{\partial \Gamma}{\partial k}$ , and  $\frac{\partial \Gamma}{\partial \beta}$  are independent of  $\varepsilon$ , while  $\partial \left( \frac{\partial \Gamma}{\partial \varphi^*} \right) / \partial \varepsilon > 0$  if  $\nu \leq 0$ . □

Increasing PMC is effective in reducing the adverse selection effects of credit search frictions. Intuitively, raising  $\varepsilon$  reduces profits at any productivity level. Banks thus require firms to be more productive (the (CC) curve shifts upwards).

## 6 Conclusion

This paper sheds new light on how distortions on firm entry, credit search costs in particular, affect the productivity distribution within a given industry. It shows that specificity in creditor-borrower relationships will cause average productivity to fall and productivity dispersion to rise. More severe credit market imperfections will allow less productive firms to enter and will simultaneously prevent more productive firms from entry. I conclude that credit market imperfections may contribute to the observed systematic variation in within-industry productivity distributions across different sectors and countries.

To assess these effects, I constructed an analytically tractable dynamic model of credit market imperfections and heterogeneous firms. Crucially, the model predicts that tighter credit markets reduce the average productivity among active firms. Models without specificity in borrower-creditor relationships predict that financial frictions lead to a positive selection of active firms. I thus find that, to the extent that credit frictions play an adverse role in the determination of measured productivity, match specificity is important. This conclusion is in line with Levine's (1997, 715) assertion, emphasized also by Wasmer and Weil (2004), that "the durability of the bank-borrower relationship is valuable".

Policy conclusions must be stated with ample qualifications. The analysis suggests that policies aiming at reducing the red tape involved in financing young firms, deregulating product markets, or improving credit conditions are unambiguously beneficial for productivity, while policies aiming at increasing lending must be carefully evaluated as to how they affect selection incentives. Large numbers of denied credit applications may well indicate efficient credit markets. This possibility is easily dismissed in the public debate.

An important task for future research is to test the model's predictions and quantify

the magnitude of the selection effect caused by credit market imperfections. In terms of modeling, future research should shed light on the potential limitations of the standard monopolistic competition model. In particular, it may be instructive to relax the assumption of constant elasticities of substitution. Also, future research could dispense with constant productivities across time and assess the dynamic effects of misallocation distortions. More research along these lines is needed to derive robust policy implications.

# A Appendix

## A.1 Relative Profits

Consider a representative consumer with preferences given by a C.E.S. utility function over a continuum of goods indexed by  $j$  ( $J$  is the set of available goods):

$$U = \left[ \int_{j \in J} x(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}. \quad (\text{A.1})$$

$x(j)$  denotes the quantity consumed of good  $j$ . Call  $E$  the consumption expenditures of the representative consumer and let  $p(j)$  denote the price of good  $j$ . By standard arguments, utility maximization subject to the budget constraint yields  $x(j) = \frac{EP^{\varepsilon-1}}{p(j)^\varepsilon}$ , where  $P$  is the Dixit-Stiglitz price index defined in the main text. Firms charge the usual markup  $\frac{\varepsilon}{\varepsilon-1}$  over their respective marginal costs. Normalizing wages to unity, equilibrium profits are  $\pi(\varphi) = \left(\frac{\varepsilon-1}{\varepsilon}\varphi P\right)^{\varepsilon-1} \frac{E}{\varepsilon}$ . Combining  $\pi(\varphi)$  and  $\pi(\varphi')$  for any  $\varphi, \varphi' \geq \varphi^*$  gives (8).

## A.2 Derivation of the Equilibrium Credit Market Tightness

Applying free entry and destruction of all specificity (i.e., equations (10), (11), (12), and (13)) to (2), (3), (4) and (6) yields

$$\frac{c}{p(\theta)} = \int_{\varphi^*}^{\infty} F_1(\varphi) dG(\varphi), \quad (\text{A.2})$$

$$F_1(\varphi) = \frac{\pi(\varphi) - \rho(\varphi)}{r+s} \quad \forall \varphi \geq \varphi^*, \quad (\text{A.3})$$

$$\frac{k}{\tilde{p}(\theta)} = \int_{\varphi^*}^{\infty} B_1(\varphi) dG(\varphi) - \ell [1 - G(\varphi^*)], \quad (\text{A.4})$$

$$B_1(\varphi) = \frac{\rho(\varphi)}{r+s}. \quad (\text{A.5})$$

Combining (A.2) and (A.3) yields a free entry condition for firms:

$$\frac{c}{p(\theta)} = \int_{\varphi^*}^{\infty} \frac{\pi(\varphi) - \rho(\varphi)}{r+s} dG(\varphi). \quad (\text{A.6})$$

Combining (A.4) and (A.5) yields a free entry condition for banks:

$$\frac{k}{\tilde{p}(\theta)} + \ell [1 - G(\varphi^*)] = \int_{\varphi^*}^{\infty} \frac{\rho(\varphi)}{r+s} dG(\varphi). \quad (\text{A.7})$$

Combining (16) with (A.6) and (A.7) yields

$$\frac{c}{(1-\beta)p(\theta)} = \int_{\varphi^*}^{\infty} \frac{\pi(\varphi)}{r+s} dG(\varphi) - \ell [1 - G(\varphi^*)], \quad (\text{A.8})$$

$$\frac{k}{\beta\tilde{p}(\theta)} = \int_{\varphi^*}^{\infty} \frac{\pi(\varphi)}{r+s} dG(\varphi) - \ell [1 - G(\varphi^*)]. \quad (\text{A.9})$$

Combining (A.8) and (A.9) yields the equilibrium credit market tightness in (21).

### A.3 Utility

Substituting for the firm index  $j$  with productivities  $\varphi$  and optimal demands  $x(\varphi, P, E)$  in (A.1), utility can be written as

$$U = \left(1 - \frac{1}{\varepsilon}\right)^{\varepsilon} P^{\varepsilon-1} E \left[ M \int_{\varphi^*}^{\bar{\varphi}} \varphi^{\varepsilon-1} d\mu(\varphi) \right]^{\frac{\varepsilon}{\varepsilon-1}}. \quad (\text{A.10})$$

Combining this expression with  $P = \left[ (1 - \frac{1}{\varepsilon}) \tilde{\varphi}(\varphi^*) M^{\frac{1}{\varepsilon-1}} \right]^{-1}$  and the definition of  $\tilde{\varphi}(\varphi^*)$ , we obtain (34):  $U = (1 - \frac{1}{\varepsilon}) \tilde{\varphi}(\varphi^*) M^{\frac{1}{\varepsilon-1}} E$ .

### A.4 Existence and Uniqueness of Equilibrium

This section adapts Melitz' (2003) proof for existence and uniqueness of equilibrium.

Consider the equilibrium condition for  $\varphi^*$ , (23):

$$\frac{c}{(1-\beta)p(\theta^*)\ell} = \theta(\varphi^*), \quad \theta(\varphi) \equiv \omega(\varphi) [1 - G(\varphi)], \quad \omega(\varphi) \equiv \left[ \frac{\tilde{\varphi}(\varphi)}{\varphi} \right]^{\varepsilon-1} - 1. \quad (\text{A.11})$$

Using the definition of  $\varphi$ ,  $\omega'(\varphi)$  can be written as  $\omega'(\varphi) = \frac{g(\varphi)}{1-G(\varphi)}\omega(\varphi) - (\varepsilon-1)\frac{\omega(\varphi)+1}{\varphi}$ . Hence,

$$\theta'(\varphi) = -(\varepsilon-1)[1 - G(\varphi)] \frac{\omega(\varphi)+1}{\varphi} < 0 \quad \forall \varphi < \bar{\varphi}. \quad (\text{A.12})$$

$\lim_{\varphi \downarrow 0} \theta(\varphi) = +\infty$  and  $\lim_{\varphi \uparrow \bar{\varphi}} \theta(\varphi) = 0$  complete the proof.

Since  $\frac{\partial \Gamma}{\partial \varphi^*} = -\theta'(\varphi^*) > 0 \forall \varphi^* < \bar{\varphi}$ , (A.12) also completes the proof to result (9).

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