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# Modeling the link between US inflation, output and their variabilities\*

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## Abstract

This paper employs the unrestricted extended constant conditional correlation GARCH specification proposed in Conrad and Karanasos (2010) to examine the intertemporal relationship between the uncertainties of inflation and output growth in the US. We find that inflation uncertainty effects output variability positively, while output variability has a negative effect on inflation uncertainty. In addition, we find a negative/positive relation between nominal uncertainty and output growth/inflation. Finally, both lagged inflation as well as lagged output growth have a positive/negative effect on nominal/real uncertainty.

**Keywords:** Bivariate GARCH process, negative volatility feedback, inflation uncertainty, output variability.

**JEL Classification:** C32, C51, E31.

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# 1 Introduction

During the last decade researchers have employed various bivariate GARCH-in-mean models to investigate the relationship between the uncertainties of inflation and output growth and/or to examine the impact of the uncertainties on the levels of inflation and growth (see, for example, Grier et al., 2004).<sup>1</sup>

The two most commonly used specifications are the diagonal constant conditional correlation (CCC) model (see, for example Grier and Perry, 2000, Fountas et al., 2006, and Fountas and Karanasos, 2007) and the BEKK representation (see, for example, Grier et al., 2004, and Grier and Grier, 2006). However, these two specifications are characterized by rather restrictive assumptions regarding potential volatility spillovers. At the one extreme, the former assumes that there is no link between the two uncertainties, whereas, near the other extreme, the latter only allows for a positive variance relationship.

In sharp contrast, several economic theories predict either a *positive* or a *negative* association between the variabilities of inflation and growth (for more details and a review of the literature, see, Karanasos and Kim, 2005). Obviously, the extent to which there is an interaction of either sign between the two variances is an issue that cannot be resolved on merely theoretical grounds. These considerations reinforce a widespread awareness of the need for more empirical evidence, but also make clear that a good empirical framework is lacking.

In this paper we employ the unrestricted extended CCC (UECCC) GARCH model to examine how the US nominal and real uncertainties are interrelated. This specification, defined in Conrad and Karanasos (2010), allows for feedback effects between the two volatilities that can be of either sign, i.e. positive or negative.<sup>2</sup> More specifically, Conrad and Karanasos (2010) derive necessary and sufficient conditions which ensure the positive definiteness of the conditional covariance matrix even in the case of negative volatility feedback. While negative values of the GARCH coefficients have commonly been thought of as resulting either from sampling error or model misspecification, they show that this is not necessarily the case. Interestingly, negative volatility spillovers may be in line with economic theory.

This is the first paper to apply this flexible bivariate formulation to investigating the relation between the variabilities of US inflation and output.<sup>3</sup> We find strong evidence supporting the Logue and Sweeney (1981) theory that inflation uncertainty has a positive impact on the volatility of growth. In sharp contrast, real variability affects nominal uncertainty negatively as predicted by, among others, Fuhrer (1997). Clearly, this negative effect could not have been detected by applying the restrictive DCCC or BEKK GARCH specifications.

Finally, our new flexible specification does not only allow us to investigate the relation between the variabilities of US inflation and output growth, but at the same time we can

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<sup>1</sup>We will use the terms variance, variability, uncertainty and volatility interchangeably in the remainder of the text.

<sup>2</sup>The specification is termed ‘unrestricted extended’ because it can be viewed as an unrestricted version of the extended CCC (ECCC) specification of Jeantheau (1998) which allows for positive volatility feedback only.

<sup>3</sup>Conrad et al. (2010) apply the UECCC-GARCH model to UK inflation and output growth.

control for the effects of two variabilities on their respective level variables and vice versa, the so-called ‘in-mean’ and ‘level effects’. Our findings suggest that increasing nominal uncertainty leads to higher rates of inflation, while it reduces output growth. On the other hand, real uncertainty affects output growth positively. Regarding the level effects, we find that both higher lagged inflation as well as higher lagged output tend to increase nominal uncertainty but to reduce real uncertainty.

The remainder of this article is organized as follows. Section 2 introduces the bivariate model and Section 3 discusses the economic theories. The main empirical results are presented in Section 4. A sensitivity analysis is provided in Section 5. Finally, Section 6 concludes.

## 2 The Bivariate GARCH Model

We use a bivariate model to simultaneously estimate the conditional means, variances, and covariances of inflation and output growth. Let  $\mathbf{y}_t = (\pi_t \ y_t)'$  represent the  $2 \times 1$  vector with the inflation rate and real output growth. The symbols  $\odot$  and  $\wedge$  denote the Hadamard product and the elementwise exponentiation respectively. Further,  $\mathcal{F}_{t-1} = \sigma(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \dots)$  is the filtration generated by the information available up through time  $t - 1$ .

Define the residual vector as  $\boldsymbol{\varepsilon}_t = (\varepsilon_{\pi,t} \ \varepsilon_{y,t})' = \mathbf{z}_t \odot \mathbf{h}_t^{\wedge 1/2}$ , where the stochastic vector  $\mathbf{z}_t = (z_{\pi,t} \ z_{y,t})'$  is independent and identically distributed (*i.i.d.*) with mean zero, finite second moments, and  $2 \times 2$  correlation matrix  $\mathbf{R} = [\rho_{ij}]_{i,j=\pi,y}$  with diagonal elements equal to one and off-diagonal elements absolutely less than one.  $\mathbf{h}_t = (h_{\pi,t} \ h_{y,t})'$  denotes a vector of  $\mathcal{F}_{t-1}$  measurable conditional variances.

We estimate the following bivariate AR( $p$ )-GARCH(1, 1)-in-mean model

$$\mathbf{y}_t = \boldsymbol{\Gamma}_0 + \sum_{l=1}^p \boldsymbol{\Gamma}_l \mathbf{y}_{t-l} + \sum_{r=0}^s \boldsymbol{\Delta}_r \mathbf{h}_{t-r} + \boldsymbol{\varepsilon}_t, \quad (1)$$

where  $\mathbf{I}$  is the  $2 \times 2$  identity matrix,  $\boldsymbol{\Gamma}_0 = [\gamma_i]_{i=\pi,y}$ ,  $\boldsymbol{\Gamma}_l = [\gamma_{ij}^{(l)}]_{i,j=\pi,y}$  and  $\boldsymbol{\Delta}_r = [\delta_{ij}^{(r)}]_{i,j=\pi,y}$ . We assume that the roots of  $|\mathbf{I} - \sum_{l=1}^p \boldsymbol{\Gamma}_l L^l|$  lie outside the unit circle. Note that our specification allows the conditional variances to effect the level variables with some time delay  $r$ .

Following Conrad and Karanasos (2010), we impose the UECCC-GARCH(1,1) structure on the conditional variances:

$$\mathbf{h}_t = \boldsymbol{\omega} + \mathbf{A} \boldsymbol{\varepsilon}_{t-1}^{\wedge 2} + \mathbf{B} \mathbf{h}_{t-1}, \quad (2)$$

where  $\boldsymbol{\omega} = [\omega_i]_{i=\pi,y}$ ,  $\mathbf{A} = [a_{ij}]_{i,j=\pi,y}$  and  $\mathbf{B} = [b_{ij}]_{i,j=\pi,y}$ .<sup>4</sup>

Finally, we assume that the above model is minimal in the sense of Jeantheau (1998, Definition 3.3) and invertible (see Assumption 2 in Conrad and Karanasos, 2010). The invertibility condition implies that the inverse roots of  $|\mathbf{I} - \mathbf{B}L|$ , denoted by  $\phi_1$  and  $\phi_2$ , lie inside the unit circle.

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<sup>4</sup>This specification nests the diagonal CCC model when  $\mathbf{A}$  and  $\mathbf{B}$  are diagonal matrices and Jeantheau’s (1998) ECCC model when  $a_{ij} \geq 0$  and  $b_{ij} \geq 0$ .

Conrad and Karanasos (2010) show that the following four conditions are necessary and sufficient for  $\mathbf{h}_t > 0$  for all  $t$ : (i)  $(1 - b_{yy})\omega_\pi + b_{\pi y}\omega_y > 0$  and  $(1 - b_{\pi\pi})\omega_y + b_{y\pi}\omega_\pi > 0$ , (ii)  $\phi_1$  is real and  $\phi_1 > |\phi_2|$ , (iii)  $\mathbf{A} \geq 0$  and (iv)  $[\mathbf{B} - \max(\phi_2, 0)\mathbf{I}]\mathbf{A} > 0$ . These constraints do not place any *a priori* restrictions on the signs of the coefficients in the  $\mathbf{B}$  matrix. In particular, this implies that potential negative volatility spillovers are allowed.<sup>5</sup>

Finally, we augment the variance specification in order to include the level effects:

$$\mathbf{h}_t = \boldsymbol{\omega} + \mathbf{A}\boldsymbol{\varepsilon}_{t-1}^2 + \mathbf{B}\mathbf{h}_{t-1} + \mathbf{e}^{\Lambda\mathbf{y}_{t-1}}, \quad (3)$$

where  $\Lambda = [\lambda_i]_{i=\pi, y}$ .

## 3 Economic Theories

### 3.1 The inflation-growth link

Mean inflation and output growth are interrelated. Temple (2000) presents a critical review of the emerging literature which tends to discuss how inflation affects growth. Gillman and Kejak (2005) bring together for comparison several main approaches to modelling the effect of inflation on growth by nesting them within a general monetary endogenous growth model with both human and physical capital. Their summary of the findings across the different formulations clearly establishes a robust significant negative effect. Briault (1995) argues that there is a positive relationship between growth and inflation, at least over the short run, with the direction of causation running from higher growth (at least in relation to productive potential) to higher inflation. For simplicity, in what follows we will refer to this positive influence as the Briault conjecture.

### 3.2 The relation between the variabilities of inflation and output

There are some reasons to suspect a relationship between nominal uncertainty and the volatility of real growth. In particular, Logue and Sweeney (1981) argue that producers operating in a highly inflationary economy might be unable to distinguish real shifts in demand from nominal shifts. Real growth in investment, and all other economic activity will be more variable than it would be in an environment where less guessing as to the source of an increase in nominal demand was necessary. For that matter relative price variation creates additional producer uncertainty. Moreover, models with stable inflation-unemployment trade-off imply a positive relationship between the variability of inflation and the variability of real activity. Using data from countries that are members of the OECD their cross-sectional tests show that both inflation and its uncertainty affect real variability positively. In Devereux's (1989) model higher output variability is associated with more inflation uncertainty. Further, in Fuhrer's (1997) models the short run trade-off between inflation and output implies a long-run trade-off in variability. For example, he argues that if the Fed wishes to make the variance in output small, it must allow shocks that affect inflation to persist, thus increasing the variance in inflation. Karanasos and

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<sup>5</sup>Nakatani and Teräsvirta (2008) derive a similar set of conditions for the more restricted case that the two diagonal elements of the GARCH matrix are positive.

Kim (2005) discuss a number of arguments, advanced over the last 30 years, that predict a positive association between the two variables.

### **3.3 The impact of macroeconomic uncertainty on performance**

Macroeconomists have placed considerable emphasis on the impact of economic uncertainty on the state of the macroeconomy. The profession seems to agree that the objectives of monetary policy are inflation and output stabilisation around some target levels. A detailed survey of the theories is provided, e.g., in Fountas et al. (2007).

#### **3.3.1 The effects of inflation variability**

Variability about future inflation affects the average rate of inflation. However, the direction of the effect is ambiguous from a theoretical point of view. Cukierman and Meltzer's (1986) model explains the positive association between the two variables. In the words of Holland (1995): 'The policy maker chooses monetary control procedures that are less precise, so that uncertainty about inflation is higher. The reason is that greater ambiguity about the content of monetary policy makes it easier for the government to create the monetary surprises that increase output. This causes the rate of inflation to be higher on average'. Holland (1995), on the other hand, discusses a number of theories on the association between the rate of inflation and its uncertainty. One possible reason for greater nominal variability to precede lower inflation is that an increase in uncertainty is viewed by policymakers as costly, inducing them to reduce inflation in the future (Holland, 1995). We will refer to this negative effect as the Holland conjecture.

The impact of nominal uncertainty on output growth, has received considerable attention in the literature. However, there is no consensus among macroeconomists on the direction of this effect. Theoretically speaking, the influence is ambiguous. According to Pindyck (1991) the effect might work through its impact on investment. Inflation variability increases the uncertainty regarding the potential returns of investment projects and therefore provides an incentive to delay these projects, thus contributing to lower investment and output growth. In sharp contrast, Dotsey and Sarte (2000) employ a model where money is introduced via a cash-in-advance constraint and find that nominal variability increases average growth through a precautionary savings motive.

#### **3.3.2 The effects of growth variability**

Next, real variability may affect the rate of inflation. First, it would be expected to have a negative impact on inflation via a combination of the Fuhrer and the Cukierman-Meltzer effects. In sharp contrast, the approach in Cukierman and Gerlach (2003) implies a positive relation between inflation and the variance of growth where causality runs from the latter to the former.

Finally, of particular interest has been the relationship between growth and its variance with different analyses reaching different conclusions depending on what type of model is employed, what values for parameters are assumed and what types of disturbance are considered (see Blackburn and Pelloni, 2005, and the references therein). In another class of models the relation between short-term variance and long-term growth is positive (see

Blackburn, 1999, and the references therein). Blackburn(1999) presents a model of imperfect competition with nominal rigidities and ‘learning by doing’ technology. He argues that it is possible that the additional learning during expansions more than compensates for the loss of learning during recessions so that, on average, the rate of technological progress increases when there is an increase in real volatility. Under such circumstances, there is a positive relationship between growth and its uncertainty. A positive correlation between the two variables does not imply a causal link. However, in our analysis a positive effect from real variability to growth implies a positive correlation between the two variables.

### **3.4 The impact of macroeconomic performance on uncertainty**

#### **3.4.1 The impact of inflation**

The positive effect of inflation on its uncertainty has often been noted. According to Holland (1993) if regime changes cause unpredictable changes in the persistence of inflation, then lagged inflation squared is positively related to volatility. In addition, Ungar and Zilberfarb (1993) provide a theoretical framework in order to specify the necessary conditions for the existence of a positive or negative impact.

A number of theories have been put forward to examine the impact of inflation on real uncertainty. In a nutshell, the sign of such an effect is ambiguous. The models developed by Ball et al. (1988) assume menu costs and imply that the slope of the short-run Phillips curve should be steeper when average inflation is higher. In their New Keynesian model, nominal shocks have real effects because nominal prices change infrequently. Higher average inflation reduces the real effects of nominal disturbances and hence also lowers the variance of output. Dotsey and Sarte (2000) present a model which suggests that as average money growth rises nominal variability increases and real growth rates become more volatile.

#### **3.4.2 The impact of output growth**

The sign of the impact of output growth on macroeconomic volatility is also ambiguous. Consider first the influence on nominal uncertainty. First, a higher growth rate will raise inflation according to the Briault conjecture, and therefore this raises/lowers its variability, as predicted by the Ungar-Zilberfarb theory. In addition, as Brunner (1993) puts it: ‘While Friedman’s hypothesis is plausible, one could also imagine that when economic activity falls off, there is some uncertainty generated about the future path of monetary policy, and consequently, about the future path of inflation’. We will use the term ‘Brunner conjecture’ as a shorthand for this negative effect.

Finally, consider now the effect of growth on its variability. An increase in growth, given that the Briault conjecture and the Ball-Mankiw-Romer theory hold, reduces its variance. However, if the impact of inflation on real uncertainty is positive (the Dotsey-Sarte conjecture), the opposite conclusion applies.

In Table 1 we present a summary of the signs implied by the respective theories.

Table 1: Economic Theories

Columns	(1) $\pi_t$	(2) $y_t$	(3) $h_{\pi,t}$	(4) $h_{y,t}$
$\pi_t$	x	Briault: + (conjecture)	Cukierman-Meltzer: + Holland: - (conjecture)	Fuhrer(-) and Cukierman-Meltzer (+): - Cukierman-Gerlach: +
$y_t$	Gillman-Kejak: -	x	Pindyck: - Dotsey-Sarte: +	Blackburn-Pelloni: $\pm$
$h_{\pi,t}$	Ungar-Zilberfard: $\pm$	Briault(+) and Ungar-Zilberfard ( $\pm$ ): $\pm$ Brunner: - (conjecture)	x	Fuhrer: - Devereux: + (conjecture)
$h_{y,t}$	Ball et al. : - Dotsey-Sarte (conjecture): +	Briault(+) and Ball et al. (-): - Briault(+) and Dotsey-Sarte(+): +	Logue-Sweeney: + Fuhrer: -	x

**Notes:** The effects of inflation(uncertainty) on the other three variables are presented in the first(third) column.

The effects of growth(uncertainty) on the other three variables are presented in the second(fourth) column.

+/-: the effect is positive/negative.



## 4 Empirical Results

Monthly data, obtained from Datastream, are used to provide a reasonable number of observations. The inflation and output growth series are calculated as the monthly difference in the natural log of the Consumer Price Index and Industrial Production Index, respectively. The data range from 1960:01 to 2007:12 and, hence, comprise 576 usable observations. Applying various unit root tests to both series, we came to the conclusion that inflation as well as output growth can be treated as stationary variables.

Within the bivariate UECCC-GARCH-in-mean framework we will analyze the dynamic adjustments of the conditional means and variances of US inflation and output growth, as well as the implications of these dynamics for the direction of causality between the two variables and their respective uncertainties. Parameter estimates were obtained by quasi maximum likelihood estimation (QMLE). To check for the robustness of our estimates we used a range of starting values and, hence, ensured that the estimation procedure converged to a global maximum. The best model was chosen on the basis of Likelihood Ratio (LR) tests and three alternative information criteria. For reasons of brevity, we refrain from presenting the estimation results for the autoregressive parameters; instead, in Table 2 we concentrate on the main parameters of interest.

In what follows we analyse the results from the various specifications and examine the sign and the significance of the estimated coefficients to provide some statistical evidence on the nature of the relationship between the four variables. First, there is strong evidence supporting the Gillman-Kejak theory and the Briault conjecture. That is, there is strong bidirectional feedback between inflation and output growth. In particular, inflation affects growth negatively (or results not reported).

Second, we analyze the potential spillover effects between the two volatilities by discussing the implications of equation (6) in Table 2. First, since  $b_{y\pi}$  is positive and significant there is strong evidence that nominal uncertainty has a positive impact on real volatility, as predicted by Logue and Sweeney (1981). Second, the negative and significant value of  $b_{\pi y}$  indicates that output variability affects the uncertainty of inflation negatively and provides support for the Fuhrer (1997) theory. Most importantly, although one off-diagonal GARCH coefficient is negative the necessary and sufficient conditions for the positive definiteness of the conditional covariance matrix are satisfied (see Section 2).

Next, the parameter estimates in equation (4) in Table 2 show that higher nominal uncertainty leads to higher inflation rates as suggested by Cukierman and Meltzer (1986), while increasing output volatility appears to lower the average inflation rate. Recall that, theoretically speaking, this effect is based on the interaction of the Fuhrer and the Cukierman-Meltzer theories. The evidence for both theories confirms the negative impact of real uncertainty on inflation.

The results in equation (5) support the Pindyck (1991) hypothesis that increasing inflation uncertainty affects output growth negatively,<sup>6</sup> whereas higher real variability appears to increase output growth as predicted by Blackburn and Pelloni (2005). Note that in three out of the four cases the effects from the uncertainties to the levels arise with some time delay (insignificant contemporaneous parameters are not presented), which is

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<sup>6</sup>Both cross effects, i.e., the coefficients  $\delta_{\pi y}^{(0)}$  and  $\delta_{y\pi}^{(1)}$  are significant at the 10% level.

Table 2: Parameter estimates for the AR-UECCC-GARCH(1,1)-in-mean model.

$$\pi_t = \dots + \underset{(0.043)}{0.130}h_{\pi,t-1} - \underset{(0.004)}{0.006}h_{y,t} + \varepsilon_{\pi,t} \quad (4)$$

$$y_t = \dots - \underset{(0.081)}{0.137}h_{\pi,t-1} + \underset{(0.006)}{0.017}h_{y,t-3} + \varepsilon_{y,t} \quad (5)$$

$$\mathbf{h}_t = \begin{pmatrix} 0.3816 \\ (0.119) \\ 5.3901 \\ (3.098) \end{pmatrix} + \begin{pmatrix} 0.0684 & 0.0064 \\ (0.013) & (0.003) \\ - & 0.2945 \\ & (0.051) \end{pmatrix} \varepsilon_{t-1}^{\wedge 2} + \begin{pmatrix} 0.9145 & -0.0077 \\ (0.028) & (0.004) \\ 1.0250 & 0.5259 \\ (0.345) & (0.058) \end{pmatrix} \mathbf{h}_{t-1}, \quad (6)$$

with  $\rho = -0.0396$  (0.0449).

**Notes:** This table reports parameter estimates of the bivariate AR-UECCC-GARCH(1,1)-in-mean model for the US inflation ( $\pi_t$ ) and output ( $y_t$ ) data.  $h_{\pi,t}$  and  $h_{y,t}$  denote the conditional variances of inflation and output, respectively. The numbers in parenthesis are robust standard errors.

to be expected when working with monthly data.<sup>7</sup>

## 5 Sensitivity Analysis

### 5.1 Sub-periods

In order to check for the robustness of our results and to control for possible monetary policy changes we reestimated our favored specification by interacting the main variables of interest with dummy variables for the periods 1960-1979 and 1980-2007. While our conclusions regarding the link between the variabilities of inflation and output remained unchanged (results not reported), we found some changes in the in-mean effects:

$$\pi_t = \dots + \underset{(0.045)}{0.138} \cdot \mathbf{1}_{\{<80\}} h_{\pi,t-1} + \underset{(0.061)}{0.190} \cdot \mathbf{1}_{\{\geq 80\}} h_{\pi,t-1} \\ - \underset{(0.003)}{0.004} \cdot \mathbf{1}_{\{<80\}} h_{y,t} - \underset{(0.005)}{0.009} \cdot \mathbf{1}_{\{\geq 80\}} h_{y,t} \quad (7)$$

$$y_t = \dots - \underset{(0.077)}{0.025} \cdot \mathbf{1}_{\{<80\}} h_{\pi,t-1} - \underset{(0.142)}{0.515} \cdot \mathbf{1}_{\{\geq 80\}} h_{\pi,t-1} \\ + \underset{(0.006)}{0.003} \cdot \mathbf{1}_{\{<80\}} h_{y,t-3} + \underset{(0.017)}{0.047} \cdot \mathbf{1}_{\{\geq 80\}} h_{y,t-3} \quad (8)$$

The estimated parameters in equations (7) and (8) suggest that - apart from the effect of nominal uncertainty on inflation - all other in-mean effects are mainly a phenomenon of the second sub-period.

<sup>7</sup>In the previous studies which employed GARCH-in-mean models the uncertainties were restricted to affecting the levels contemporaneously, often resulting in insignificant parameter estimates (see Section 6).

## 5.2 Level effects

Finally, we controlled for level effects by including the lagged values of inflation and output in the variance equation, i.e. we reestimated our model with the augmented variance specification given by

$$\mathbf{h}_t = \boldsymbol{\omega} + \mathbf{A}\varepsilon_{t-1}^2 + \mathbf{B}\mathbf{h}_{t-1} + \begin{pmatrix} \exp(\lambda_{\pi\pi}\pi_{t-1}) & \exp(\lambda_{\pi y}y_{t-1}) \\ \exp(\lambda_{y\pi}\pi_{t-1}) & \exp(\lambda_{yy}y_{t-1}) \end{pmatrix}.$$

Choosing this exponential specification for the level effects ensures that our non-negativity conditions still apply without any modification.

Since the sign of the level effects is of great interest for itself, the following matrix presents the estimates for the  $\lambda_{ij}$  coefficients.

$$\begin{pmatrix} 0.1158 & 0.0184 \\ (0.013) & (0.008) \\ -0.4558 & -0.1332 \\ (0.229) & (0.016) \end{pmatrix}$$

The coefficient estimates indicate that higher lagged inflation tends to increase nominal uncertainty and reduce growth variability, thus supporting the Ungar-Zilberfarb and Ball-Mankiw-Romer theories respectively. We now turn to the effects of growth on the two volatilities. The positive impact of growth on inflation uncertainty is in line with the indirect effect that works through the inflation channel. That is, it is based on the interaction of the Briault conjecture and the Ungar-Zilberfarb theory. In sharp contrast, growth affects its variability negatively. This result is consistent with the theoretical underpinnings that predict a negative effect because of the interaction of the Briault conjecture with the Ball-Mankiw-Romer theory.

Our conclusions regarding the i) inflation-growth link and the ii) volatility feedback, were not affected by controlling for the level effects (results not reported). However, we found some interesting changes concerning the impact of macroeconomic uncertainty on performance. The effect of nominal uncertainty on inflation (0.641 (0.128)) and on output growth (-0.432 (0.134)) are considerably stronger when the level effects in the conditional variance specification are taken into account.

Table 3 summarizes all twelve effects. As seen in table 3 there are two mixed bidirectional feedbacks. That is, inflation and real uncertainty have a negative impact on growth and nominal variability respectively, whereas the effects in the opposite direction are positive. Moreover, the two own in-mean effects are positive whereas the two cross in-mean effects are negative. Finally, inflation and growth affect nominal uncertainty positively and real variability negatively.

## 6 Comparison with Related Literature

The GARCH time series studies that examine the inflation-growth link in the US use various sample periods, frequency data sets and empirical methodologies. Some GARCH studies of this issue utilize the simultaneous-estimation approach. For example, Baillie

Table 3: The relationship between inflation, growth and their uncertainties.

		Columns →			
		(1)	(2)	(3)	(4)
Rows ↓		$\pi_t$	$y_t$	$h_{\pi,t}$	$h_{y,t}$
(1)	$\pi_t$	x	+	+	-
(2)	$y_t$	-	x	-	+
(3)	$h_{\pi,t}$	+	+	x	-
(4)	$h_{y,t}$	-	-	+	x

**Notes:** The first column presents the effects of inflation on the other three variables.

+/-: the effect is positive/negative.

et al. (1996) and Fountas and Karanasos (2006) employ univariate GARCH models that allow for simultaneous feedback between the conditional mean and variance of inflation and growth respectively. Other recent studies use bivariate GARCH-in-mean models—either the CCC (Grier and Perry, 2000) or the BEKK specification (Grier et al., 2004, Bordin and Fountas, 2005, Shields et al., 2005)—to examine the impact of macroeconomic uncertainty on performance. Some researchers employ the Granger-causality approach. For example, Grier and Perry (1998), Conrad and Karanasos (2005) and Fountas and Karanasos (2007) estimate univariate GARCH models, while Karanasos and Kim (2005) and Fountas et al. (2006) use bivariate BEKK and CCC GARCH formulations respectively. Table 4 presents a summary of these studies and their findings. The results are as follows:

*Inflation-growth link:* Only Grier et al. (2004) and Shields et al. (2005) find that growth affects inflation positively.

*Volatility feedback:* Only Karanasos and Kim (2005) find evidence for a mixed volatility feedback. Our result, that nominal variability has a positive impact on real uncertainty whereas the effect on the opposite direction is negative, is in line with their findings. The studies that employ bivariate GARCH-in-mean models use either a CCC or a BEKK GARCH specification, and hence, impose either no feedback or a positive one.

*In-mean effects:* All five studies that employ bivariate GARCH-in-mean models find that nominal uncertainty affects growth negatively, and three of them find that real uncertainty has a positive impact on growth. Our results are in line with these findings. In sharp contrast, in Bordin and Fountas (2005) the two uncertainties have no impact on inflation. Nevertheless, the results of this study must be considered with caution. Doubts arise as to whether in-mean effects were sufficiently accounted for by not including lags of the conditional variances in the two mean equations. Only Grier et al. (2004) and Shields et al. (2005) find that real variability has a negative effect on inflation. Interestingly, these two studies also find that nominal variability affects inflation negatively, whereas we find that this effects is positive. However, as mentioned above, they only test for a contemporaneous effect. It should also be mentioned that Fountas et al. (2006) and Fountas and Karanasos (2007), who use Granger causality tests, find significant in-mean effects, with same signs as the present study, in three out of the four cases.

Table 4: The inflation-growth link in the USA. Summary of previous studies.

Papers	Sample; Data	Cross (mean and variance) effects						Level effects						Signific. effects
		In mean effects			In mean effects			Level effects			Level effects			
		$\pi_t$ $\rightarrow$ $y_t$	$y_t$ $\rightarrow$ $\pi_t$	$h_{\pi,t}$ $\rightarrow$ $h_{y,t}$	$h_{\pi,t}$ $\rightarrow$ $y_t$	$h_{y,t}$ $\rightarrow$ $\pi_t$	$h_{y,t}$ $\rightarrow$ $h_{y,t}$	$\pi_t$ $\rightarrow$ $h_{\pi,t}$	$h_{\pi,t}$ $\rightarrow$ $h_{y,t}$	$y_t$ $\rightarrow$ $h_{\pi,t}$	$h_{y,t}$ $\rightarrow$ $h_{y,t}$	$\pi_t$ $\rightarrow$ $h_{\pi,t}$	$h_{\pi,t}$ $\rightarrow$ $h_{y,t}$	
In-mean-level models														
Univariate specifications														
BCI, 96		x	x	x	x	x	x	x	x	x	x	x	x	0
FK, 06	1860-1999; IPI	x	x	x	x	x	x	x	x	0	x	x	x	1
Bivariate specifications														
GP, 00	48-96; PPI, IPI	-	x	x	x	-C	0	0	0	0	x	x	x	2;1C
E*, 04	66-00; CPI, IPI	-	x	x	x	-	x	x	x	x	x	x	x	2
GHOS, 04	47-00; PPI, IPI	-	+	+R	+R	-C	-C	+C	+C	+C	x	x	x	8;2R,4C
BF, 05	57-03; PPI,IPI	0	0	+R	+R	-C	0	+C	+C	0	x	x	x	4;2R,2C
SOHB, 05	47-00; PPI,IPI	-	+	+R	+R	-C	-C	-C	-C	-C	x	x	x	8;2R,4C
CK <sup>c</sup> , 10	60-07; CPI, IPI	-	+	+	-	-	-	+	+	+	+	+	+	12
Granger causality tests														
Univariate cases														
GP,98	48-93; CPI	+	x	x	x	-	x	x	x	x	+	+	+	3
CK, 05	62-00; CPI	x	x	x	x	0	x	x	x	x	+	+	+	1
Bivariate cases														
KK, 05	57-00; PPI,IPI	x	x	+	-	x	x	x	x	x	x	x	x	2
FKK, 06	57-00; WPI, IPI	-	0	0	0	±	-	+	+	+	+	+	+	6
FK, 07	57-00; PPI, IPI	-	+	x	x	+	-	0	0	+	+	+	+	6
Overall effect:		-	+	+	-	±	-	+	+	+	+	+	+	12

**Notes:** The  $x_t$   $\rightarrow$  column presents the effect of  $x_t$  on  $z_t$ ,  $+/-$ : the effect is positive/negative. 0: the effect is zero;

$z_t$

x: the link is not examined. \*: This is a trivariate model. °: This is the present study.

R: the effect is restricted to positive; C: this is a contemporaneous effect.

BCI: Baillie, Chung and Tieslau; BF: Bredin and Fountas; CK: Conrad and Karanasos;

E: Elder; GHOS: Grier, Henry, Olekalns and Shields; GP: Grier and Perry;

FKK: Fountas, Karanasos and Kim; FK: Fountas and Karanasos; KK: Karanasos and Kim;

SOHB: Shields, Olekalns, Henry and Brooks.

*Level effects:* Only Fountas and Karanasos (2006) and Fountas, Karanasos and Kim (2006) test and find, as the present study does, a negative effect of growth on its uncertainty. None of the aforementioned studies test for cross level effects. Only the studies that employ Granger causality tests examine the (positive) impact of inflation on its uncertainty. In fact the level effects that are shown as being important have not been accounted for in the previous studies who employed bivariate GARCH-in-mean models.

## 7 Conclusions

This is the first paper which employs the UECCC GARCH model to investigate the inflation-growth uncertainty link using US data. The main advantage of this new specification is that it allows for volatility feedback of either sign, i.e., positive or negative. Thus, we are able to test economic theories which suggest a trade-off between the variabilities of inflation and output. Our results show that real volatility affects nominal uncertainty negatively, as predicted by Fuhrer (1997). In sharp contrast, we find strong evidence supporting the Logue and Sweeney (1981) theory that inflation uncertainty has a positive impact on the volatility of growth. In addition, the empirical results show that inflation and real uncertainty have a negative impact on growth and nominal variability respectively. The effects in the opposite direction are positive. Finally, the level effects of inflation and growth are positive on nominal uncertainty but negative on real variability.

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