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Local and Global Externalities, Environmental Policies and Growth

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Abstract

The paper analyzes the implications of local and global pollution when two types of abatement activities can be undertaken. One type (e.g., use of particulate matter filters) reduces solely local pollution while the other (e.g., application of fuel saving technologies) mitigates global pollution as well. In the framework of a 2-country endogenous growth model, the implications of different assumptions about the degree to which global externalities are internalized are analyzed. Subsequently, we derive policy rules adapted to the different scenarios as well as to implement the first-best solution. Special attention is paid to pollution, growth and optimal policy in the case of asymmetric internalization.

Keywords: economic growth, global and local externalities, government policies

JEL Classification: O4, Q53, Q54, Q58
1 Introduction

When analyzing abatement activities that aim at combating climate change, it is mostly assumed that this abatement solely reduces the emission of CO\textsubscript{2} or other greenhouse gases. In reality however, many activities that reduce emissions of greenhouse gases also reduce local pollution. Consequently, when discussing climate policies, not only the returns from abating global pollution but also from reducing local pollution should be considered. Equivalently, when deciding about policies to reduce local pollution, the analysis of available abatement options should take potential side effects on global pollution into account. Thus, local and global environmental policies should not be treated separately but rather in a unified framework. In this paper, we set up a model that accounts for local as well as global pollution and explicitly considers two types of abatement activities that differ with respect to their implications regarding local and global pollution mitigation. Using this approach, we can not only derive consequences of different degrees of internalization for both pollution types, but we can also characterize policies that result in an optimal abatement mix.\footnote{In this paper, we only differentiate between global and local pollution where the latter includes all types of non-global pollution (e.g. regional).}

Take the reduction of CO\textsubscript{2}-emissions from transport as one example that reduces global as well as local pollutants. Decreasing fossil fuel combustion by increasing technological efficiency not only decreases the emissions of greenhouse gases like CO\textsubscript{2}, CH\textsubscript{4} and NO\textsubscript{2} but also reduce emissions of, e.g., particulate matter, SO\textsubscript{2} and NO\textsubscript{x}. Consequently, negative effects of these pollutants like health problems, acid rain, and surface corrosion are mitigated as well (see, e.g., Rübbelke 2002). Similarly, aorestation and deforestation not only enhance carbon sequestration but can also reduce soil erosion and foster bio-diversity.\footnote{For specific types of additional benefits from climate policies see, e.g., Burtraw et al. (2003) regarding the mitigation of local and regional air pollution, Barker, Johnstone and O’Shea (1993) with respect to the reduction of noise, road surface damage and traffic congestion and Elbakidze and McCarl (2007) on the prevention of soil erosion and of biodiversity loss. Potential rises in employment levels, competitiveness and energy security are treated by Bovenberg (1999) and Porter (1991) and IEA 2007 respectively.}

In the literature on climate change, these additional benefits which are mostly of a local nature (see, e.g., IPCC 1996, Pearce 1992) are often referred to as ancillary benefits, implying that the main benefit lies in the reduction of greenhouse gases. A more neutral term is ‘co-benefits’ (see IPCC 2001) which leaves
undecided whether the primary target is the mitigation of global or local pollution. These co-benefits are often neglected although they are estimated to be of considerable size (see, e.g., Pearce 2000). Consequently, a comprehensive analysis of the costs and benefits of, e.g., the global warming problem should incorporate co-benefits from preventing greenhouse gas emissions (see also Morgenstern 1991, Plambeck, Hope and Anderson 1997).

In our paper we therefore include two types of pollution - local flow pollution and global stock pollution - as well as two types of abatement. Abatement either affects local pollution only, or local and global pollution simultaneously. The former could, for example, be filters that reduce the emission of particulate matter; the latter could be the aforementioned reduction of fuel combustion. The main target of the latter could be global or local, depending on the aim of the policy maker. By considering these different pollution and abatement types and their interrelations, we can analyze their effects on different internalization strategies and environmental policies.

In order to include not only the intertemporal spillovers from CO$_2$-accumulation but also their transnational nature, we consider two countries that each produce and pollute. To keep the focus on the internalization of the pollution induced externalities, we employ an AK-type endogenous growth model in which no other market failures arise. We also abstract from any flows of goods or capital between the countries, such that the economies only affect each other through transnational pollution spillovers.

A look at the related literature shows that, so far, most papers that consider both - local as well as global - benefits from pollution abatement have been case studies assessing the level of ancillary benefits for individual regions or countries (e.g., Gielen and Changhong/Chen 2001, Li 2006) or have been analytical models which employed static approaches neglecting dynamic implications (e.g., Pittel and Rübbelke 2008; Finus and Rübbelke 2008). On the other hand, the strand of analytical literature that

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3 Regarding early climate-damage estimates, for example, Pearce (1992) argued that the consideration of co-benefits would increase Nordhaus’ highest marginal-damage estimate of US$ 66 per ton of carbon to more than US$ 150.

4 For a review of the literature on ancillary benefits of climate policies, see Pittel/Rübbelke (2008).

5 Furthermore, a number of studies have previously assessed specific secondary benefits of climate policies, in particular in terms of public health; see for instance surveys of the literature by Ekins (1996), Burtraw et al. (1999), Kverndokk and Rosendahl (2000). Most existing studies explore the short-term, regional secondary benefits of climate change mitigation. Conversely, only a few studies including Eyckmans and Bertrand (2000) and Tol (2002) have examined the related long term secondary benefits at a global scale.
deals with the dynamics of economic development, and the growth-pollution nexus specifically, usually considers either flow or stock pollution but does not take potential interrelations into account (e.g., Withagen 1995, Smulders/Gradus 1996, Schou 2000, 2002). Furthermore, it is rarely distinguished between local and global pollution as most approaches assume closed economies. One exception is the paper by Bahn and Leach (2008) who consider secondary effects of climate policy due to the reduction of SO$_2$ emissions in an overlapping generation model. Their model is, however, not analytical solvable, such that transmission channels of secondary benefits and costs are not clearly identifiable. An analytical solvable paper that considers transnational spillovers in an endogenous growth setting is Ligthart and van der Ploeg (1994). Yet, their paper focusses on renewable resources and takes neither global versus local pollution nor environmental spillovers into account.\textsuperscript{6}

The remainder of the paper is organized as follows: After the introduction of the model in Section 2, we consider four different internalization scenarios that are implemented by a central planner in Section 3. These scenarios differ a) in the degree of internalization of the global externality and b) with respect to the symmetry of internalization in the two countries. In Section 4 we then consider different policy options to decentralize the planner solutions and evaluate them regarding their potential to achieve the intertemporal global welfare optimum. Section 5 concludes.

2 The Model

Two countries $i$, $i = h, f$, produce a homogeneous output from capital. The input of capital generates two types of pollution whose impacts differ with respect to their geographical scale. For simplicity we assume the two countries to be identical with respect to their production technologies as well as preferences. It is assumed that neither capital nor goods or labor are exchanged between two countries, such that we can fully concentrate on the local and global environmental externalities.

The externality created by the first type of pollution, $P_G$, is of a global nature, i.e. it affects production in both countries. The obvious example that comes to mind is the case of CO$_2$. Due to the long-period of time it takes for CO$_2$ to be absorbed in the atmosphere, we assume that these emissions build up a renewable pollution stock, $S$,

\textsuperscript{6}For a review of the literature on economic development and pollution with a focus on more recent endogenous growth based approaches, see Pittel (2002).
that degenerates at rate \( d \). As both countries generate pollution, the pollution stock dynamics are given by

\[
\dot{S} = P_G^h + P_G^f - dS. \tag{1}
\]

with \( \dot{S} = \frac{dS}{dt} \). For simplicity we assume that capital, \( K \), generates pollution in a constant emission ratio \( p_G \). The emission of pollution can be reduced by abatement, \( A_{LG} \), such that \( P_G \) reads

\[
P_G^i = \frac{p_G K^i}{A_{LG}^i}. \tag{2}
\]

The second type of pollution we consider, \( P_L \), induces a negative local externality that only affects production negatively in the country in which it is generated. One example for this type of pollution might be emissions of \( \text{SO}_2 \) or \( \text{NO}_x \) that lead to, e.g., acid rain in a limited regional range around the point of emission. Having these types of pollution in mind, \( P_L \) is assumed to give rise to a flow externality.\(^8\) Again we assume pollution to be generated in fixed proportions to the input of capital. As the case for \( \text{CO}_2 \), abatement reduces the environmental impact of pollution. On the one hand, abatement that decreases global pollution, \( A_{LG} \), also reduces the emission of the local pollution. On the other hand, specific abatement, \( A_L \), can be conducted to reduce \( P_L \). Local pollution as a function of capital and the two types of abatement then reads

\[
P_L^i = \frac{p_L K^i}{(A_L^i)^\alpha (A_{LG}^i)^{1-\alpha}}. \tag{3}
\]

with \( p_L \) denoting the emission intensity of capital. Both, the global pollution stock and local pollution flow, exert negative effects on production.

Output, \( Y \), is produced using a linear AK-technology in the tradition of Rebelo (1991) and is subject to negative repercussions from the input of capital:

\[
Y^i = K^i (P_L^i)^{-\gamma} S^{-\delta} \quad \text{with} \quad \gamma + \delta < 1 \tag{4}
\]

It is assumed that the negative productivity effects from capital do not overcompensate the positive (i.e. \( 1 - \gamma - \delta > 0 \)). Output can be used for consumptive, investive and abatement purposes, such that the equilibrium condition for the capital market reads

\[
\dot{K}^i = Y^i - C^i - A_L^i - A_{LG}^i. \tag{5}
\]

\(^7\)As \( A_{LG} \) reduces local as well as global pollution (see below), it is indexed \( LG \) thus referring to Local and Global.

\(^8\)This is of course an approximation which seems, however, justifiable when comparing the degradation rates of, e.g., \( \text{SO}_2 \) induced pollution to \( \text{CO}_2 \).
Finally, households in each country $i$ derive utility from consumption $C$. The representative household maximizes discounted lifetime utility with respect to its intertemporal budget constraint:

$$
\max_c \int_0^\infty \frac{C_i(t)^{1-\sigma}}{1-\sigma} e^{-\rho t} dt \quad \sigma \neq 1 \quad (6)
$$

subject to

$$
\dot{W} = rW + w - C
$$

where $W$ denotes total household asset holdings. Households supply labor inelastically.

3 The Planner Solutions

In the following the will distinguish between three different types of planner solutions: First, it is assumed that the planner is myopic and only internalizes the local pollution externality, but completely neglects the global externality. Second, he integrates the local and the global externality but does not take into account that the emissions of CO$_2$ in his country also cause damages abroad. Third, we consider the social planner solution in which both externalities are perfectly internalized, independently of where the damages arise. In a final subsection we then assume the internalization regimes to differ identical across countries, i.e. we consider asymmetric internalization.

3.1 Scenario 1: Internalization of the Local Externality Only

The planner in each country maximizes intertemporal utility of the representative household, (6), subject to capital accumulation, (5). After inserting (3) and (4) the corresponding Hamiltonian in Scenario 1 for each country reads

$$
H_{S1}^i(C_i, K^i, A^i_L, A^i_{LG}, \lambda^i) = \frac{(C^i)^{1-\sigma}}{1-\sigma} e^{-\rho t} \\
+ \lambda^i (p^i L - (1-\gamma) A^i_L G A^i_{LG}) (1-\alpha) A^i_{LG} - A^i_L)
$$

where $\lambda$ is the shadow price of capital. Optimization gives rise to the following first-order conditions

$$
(C^i)^{-\sigma} e^{-\rho t} = \lambda^i \quad (8)
$$

$$
\alpha\gamma \frac{Y^i}{A^i_L} = \frac{\gamma(1-\alpha)}{A^i_{LG}} \quad (9)
$$

$$
\dot{\lambda}^i = -\lambda^i (1-\gamma) \frac{Y^i}{K^i} \quad (10)
$$
and transversality condition for the capital stock \( \lim_{t \to \infty} \lambda^i K^i = 0 \).

From (8) and (10) we get the familiar Keynes-Ramsey rule

\[ g^i_C = \sigma^{-1} \left( (1 - \gamma) Y^i_K - \rho \right) \]  

(11)

where \( g^i_C = \frac{\dot{C}}{C} \) is the growth rate of \( C \) and \( Y^i_K \) denotes the output-capital-ratio.

It will prove useful for the further analysis to determine the capital-abatement ratio \( K_A = \frac{K}{A_{LG}} \) as an indicator of the environmental friendliness of a growth path with respect to the emission of the global pollutant. The higher \( K_A \), the higher the input of polluting capital in relation to pollution reducing abatement \( A_{LG} \). Alternatively, we could have chosen the capital-abatement ratio with respect to \( A_L \), yet in this scenario a decline in \( K_A \) always implies a simultaneous increase in \( A_L \) relative to \( K \) as (9) shows that the two abatement activities will be used in a constant ratio that is determined by their respective productivity in reducing raw pollution:

\[ \frac{A^i_{LG}}{A^i_L} = \frac{1 - \alpha}{\alpha} \]  

(12)

This simple relation is of course due to the neglect of stock pollution which implies that the mitigating effects of \( A_{LG} \) on the global externality do not show up in (12).

Using (4) and (12), \( K_A \) can be expressed as a function of \( S \) only:

\[ K^i_A(S) = \left[ \gamma(1 - \alpha) \right]^{-\frac{1-\alpha\gamma}{1-\gamma}} \left[ \frac{p^*_L S^{\delta}}{b(\alpha \gamma)^{\alpha \gamma}} \right]^{\frac{1}{1-\gamma}} \]  

(13)

As \( K^i_A \) only depends on the pollution stock and no country specific variables, the capital-abatement ratio will be identical across countries, i.e. \( K^h_A = K^f_A \).\(^9\) (13) also shows the positive relation between \( K_A \) and the pollution stock, i.e. a higher \( S \) is due to less abatement relative to capital accumulation.

As from (9) the capital-abatement ratio can be expressed as \( Y^i_K = [\gamma(1 - \alpha)K^i_A]^{-1} \) where \( K_A \) is determined by (13), the dynamics of consumption at any point in time depend on the pollution stock only:

\[ g^i_C = \sigma^{-1} \left( \frac{1 - \gamma}{\gamma} \frac{1}{1 - \alpha} K^i_A(S)^{-1} - \rho \right) \]  

(14)

(14) also shows that growth depends negatively on the negative productivity effect of pollution. As the relation between the capital-abatement stock and the pollution

\(^9\)In case of differences in the countries technologies and/or preferences, \( K_A \) would of course differ across countries, yet would still only be influenced by the other country through the pollution stock.
stock is positive, a higher $S$ implies lower growth. As the negative externality from $S$ is not internalized in this scenario, this effect is solely due to the negative productivity effect of the pollution stock and does not result from lower capital accumulation due to increased abatement activities.

Along any balanced growth path (BGP),\textsuperscript{10} \[ g_{Y}^{i} = g_{C}^{i} = g_{K}^{i} = g_{A_{L}}^{i} = g_{A_{LG}}^{i} = 0 \quad \text{and} \quad g_{S} = 0 \] (15) have to hold. The constant pollution stock along the BGP is given by

\[ S = \frac{p_{G}(K_{h}^{i} + K_{A}^{i})}{a} = 2\frac{p_{G}K_{A}^{i}}{a} \] (16)
as the capital-abatement ratio in Scenario 1 is equal across countries. Consequently, $K_{A}^{i}$ in the long-run equilibrium is given by

\[ K_{A}^{i} = \left[ \frac{p_{L}^{\gamma}(2p_{G}a)^{\delta}}{\gamma(1 - \alpha)} \left( \frac{1 - \alpha}{\alpha} \right)^{\alpha\gamma} \right]^{\frac{1}{1 - \gamma - \delta}}. \] (17)

As $K_{A}^{i}$ in (17) is constant along the BGP, this implies that - even if the global externality is not internalized - the long-run stock of pollution is constant, albeit higher than under internalization of the stock externality as the next sections will show.

The RHS of (17) depends positively on the pollution reagibility of output with respect to stock pollution, i.e. $\delta$, as well as the pollution intensities of capital, $p_{L}$ and $p_{G}$. With respect to $\gamma$, two effects – the reagibility effect as well as a productivity effect – arise and at least partially offset each other. On the one hand, a higher $\gamma$ implies a higher reagibility of output with respect to flow pollution which raises $K_{A}$. On the other hand, the higher $\gamma$, the higher the marginal productivity of abatement which lowers the capital-abatement ratio. Depending on which of the two effects dominates, $K_{A}$ rises or falls with $\gamma$. As can be seen from \[ \frac{\partial K_{A}^{i}}{\partial \gamma} = \frac{K_{A}^{i}[\log(p_{L}^{i})]^{-1/\gamma}}{1 - \gamma - \delta}, \] the higher the equilibrium pollution flow, the more likely the reagibility effect is to dominate the productivity effect.

It can be shown that the BGP is locally saddle-path stable by rewriting the dynamic system in terms of $S$ and the consumption-capital-ratio, $C_{K}^{i}$, which is constant.

\textsuperscript{10}A BGP or long-run equilibrium is defined as a growth path along which all variables grow at constant rates.
along the BGP. From (1), (5) and (14) we get

\[ \dot{S} = 2 p_G K_A^j(S) - a S \]

\[ \dot{C}_K^i = \left( \frac{1 - \sigma}{\sigma} \frac{1 - \gamma}{\gamma} (1 - \alpha) K_A^j(S)^{-1} - \frac{\rho}{\sigma} - C_K^i \right) C_K^i \]

The eigenvalues in the proximity of the steady state are \( EV_1 = -a \frac{1 - \gamma - \delta}{1 - \gamma} \) which is < 0 for \( 1 - \gamma > \delta \) and \( EV_2 = C_K^i > 0 \). As one eigenvalue is negative and our system contains one jump variable and one predetermined variable, the economy is saddle-path stable if, as assumed, the negative externalities from capital do not outweigh its positive internal effects.

### 3.2 Scenario 2: Decentral Internalization of Global Externality

In contrast to the previous section, we now assume that the planner in each country takes the effects of the global externality on its own economy into account yet fails to internalize the transnational effects it exerts on others. The planner maximizes intertemporal welfare subject to the equations of motion of the capital stock as well as of the pollution stock. The corresponding Hamiltonian is given by

\[
H_{S2}(C^i, K^i, A_L^i, A_{LG}^i, S, \lambda^i) = H_{S1}(\cdot) + \mu^i \left( p_G (K_A^j + K_A^i) - a S \right).
\]

where \( \mu \) denotes the shadow price of the pollution stock which is negative as welfare is negatively related to \( S \). The new set of FOC’s includes adapted FOCs for \( A_{LG} \) and \( K \), and an additional FOC for the pollution stock that – after some rearranging – read

\[
\gamma(1 - \alpha) Y_K^i K_A^j = 1 + p_G \left( \frac{\mu^i}{\lambda^i} \frac{1}{A_{LG}} \right) K_A^j
\]

\[
g_{\lambda^i} = -(1 - \gamma) Y_K^i - p_G \left( \frac{\mu^i}{\lambda^i} \frac{1}{A_{LG}} \right)
\]

\[
g_{\mu^i} = \frac{\delta}{S} \left( \frac{\mu^i}{\lambda^i} \frac{1}{A_{LG}} \right)^{-1} Y_K^j K_A^i + a
\]

plus a second transversality condition \( \lim_{t \to \infty} \mu S = 0 \).

In comparison to (9), the new term on the RHS of (18) represents the internalized return to \( A_{LG} \) from lowering the pollution \( S \). The return is the higher, the higher the negative impact of pollution on welfare relative to the positive welfare effect from capital accumulation, i.e., \( \left| \frac{\mu^i}{\lambda^i} \right| \), and the higher the marginal impact of abatement on pollution, i.e. the higher \( \left| \frac{\partial p_G}{\partial A_{LG}} \right| = p_G \frac{K_A^i}{A_{LG}} \). Due to the additional return, the social
costs of $A_{LG}$ are lower than unity and, ceteris paribus, higher investment in $A_{LG}$ is optimal. Setting the FOCs for the two types of abatement into relation shows $\frac{A_{LG}}{A_L} > \frac{1-\alpha}{\alpha}$, i.e. in comparison to Scenario 1 the optimal ratio of $A_{LG}$ to $A_L$ rises due to the internalization as the planner now considers the additional returns from $A_{LG}$. With respect to $K$ in (19), the additional term in its FOC reflects the internalized costs of capital in terms of pollution.

From the FOCs we get the new capital-abatement ratio along the BGP (for the derivation see Appendix 6.1)

$$K_A(S,C_K) = \left( \frac{\delta}{C_K + \alpha a} \right) - \frac{1}{1-\gamma} \left( \frac{\gamma (1-\alpha)}{b(\alpha \gamma)} \right)^{\frac{1}{1-\gamma}}. \quad (21)$$

Comparison with (13) shows that the capital-abatement ratio is lower for any value of the pollution stock if the global externality is internalized by the social planner. The additional term in (21) reflects the reduced incentives to invest in capital due to the negative effects on productivity from pollution. The higher the reagibility of production to stock pollution, i.e. the higher $\delta$, the lower the optimal capital-abatement ratio. In contrast to $K_A$ in (13), the capital-abatement ratio not only depends on the pollution stock but also on the consumption-capital ratio. In the long-run equilibrium, however, $K_A$ will be equal across countries due to our simplifying assumption of identical economies.

From (16) it follows immediately that the pollution stock along the BGP is the lower, the lower the capital-abatement ratio. So, as to be expected, internalization of the global externality includes the long-run pollution stock to fall.\footnote{The stability of the transition path under imperfect internalization can be analyzed using a 5x5 dynamic system that considers the development of $C_h$, $C_f$, $C_A$, $K_A$ and $S$. (As the countries can now follow different transitional paths depending on their initial endowment with capital, it does not suffice to consider the transition path of each economy separately as we did in Section 3.1.) The resulting eigenvalues’ functional form is quite involved, yet it can be shown numerically that the system has one negative and four positive eigenvalues for a wide range of parameter values. Consequently, the system is saddle-path stable as it contains one predetermined and four jump variables.}

(8), (18) and (19) give the new growth rate of consumption

$$g^i_C = \frac{1}{\sigma} \left( (1-\alpha)Y^i_K - K_A^i(S)^{-1} - \rho \right). \quad (22)$$

For $Y_K = [\gamma(1-\alpha)K_A]^{-1}$, as holds from (9) in Scenario 1, (22) replicates (14). Inserting $Y^i_K = [\gamma(1-\alpha)K_A^i]^{-1} \left[ 1 + p_G \left( \frac{\mu}{A_{LG}} \right) K_A^i \right]$, as holds in Scenario 2 (see
shows that internalization of the stock externality gives rise to two opposing effects on the growth rate. On the one hand, less investment into capital accumulation reduces growth, but on the other hand, the pollution mitigating effect of internalization increases productivity and thereby growth. Which effects dominates, depends on the parametrization of the model.

From (4) and (9), we get an expression for the output-capital ratio that solely depends on the capital-abatement ratio

\[ Y^i_K = \left( b p^\gamma L^\gamma \left( \frac{2 p_L}{a} \right)^{-\delta} \right)^{\frac{1}{1-\alpha\gamma}} (K^A)^{\frac{1-\alpha\gamma+\delta}{\alpha(1-\alpha)}}. \] (23)

As (23) only reflects the production technology and the optimal input of \( A_L \), the same functional relationship between \( Y_K \) and \( K_A \) can be derived for Scenario 1. Yet, as the optimal capital-abatement ratio is lower in Scenario 2 than in Scenario 1, the output-capital ratio is higher. Multiplying (23) by \( K^A \) gives \( Y^A_i \), the output-abatement ratio, which negatively depends on \( K_A \) implying that the share of output that is used for abatement purposes rises due to the internalization of \( P_G \). Lower investment also raises the consumption-capital ratio, yet whether the consumed share of output, \( C^i_Y \), rises depends crucially on whether capital or abatement react stronger to the internalization. (5), (22) and (23) give – when considering that \( g_{cK} = 0 \) in the steady state – the long-run consumption-output ratio

\[ C^i_Y = \frac{C^i}{Y^i} = \frac{1 - \sigma}{\sigma} \left( (1 - \alpha\gamma) - (K^A Y^i_K)^{-1} \right) - \frac{\rho}{\sigma} (Y^i_K)^{-1}. \] (24)

that for (9) is equal to the consumption share in Scenario 1. The increased output-capital ratio in Scenario 2 leaves a higher share of output for non-investive purposes which, assuming that the intertemporal elasticity of utility, \( \sigma \), is below unity, exerts a positive effect on \( C_Y \). This effect is represented by the last term on the RHS of (24). But internalization also induces the planner to devote a larger share of output to abatement, such that \( \frac{Y^i_A}{A^i_{LG}} = Y^i_K K^A_i \) rises. Which effect dominates depends on the model’s parametrization. Higher impatience, e.g., works in favor of an increase of \( C^i_Y \) as the planner favors consumption compared to investment in abatement that not only rises productivity today but also constitutes an investment in future productivity as it mitigates stock pollution.\(^{12}\)

\(^{12}\)Please note that by inserting (23) and (24) into (21) it can be shown that (21) determines a unique equilibrium value of \( K^A_i \).
Although we have seen that the capital-abatement is higher and the pollution stock is lower than in Subsection 3.1, they are still suboptimal. As neither country so far considers the negative spill-overs of its emissions on the other country, the capital-abatement ratio as well as the pollution stock are still suboptimally high as will be shown in the next subsection.

3.3 Scenario 3: Perfect Internalization

It is now assumed that both countries internalize the negative effects of their own emissions not only on their own economy but also on the other country. As both countries perfectly internalize all market failures, the resulting growth path is identical to the integrated growth path a global social planner would choose. The corresponding Hamiltonian considers the development in both countries and therefore reads

\[ H_{S3}(C^i, K^i, A^i_L, A^i_{LG}, S, \lambda^i) = \sum_i H_{S1}(\cdot) + \mu^f \left( p_G(K^h_A + K^f_A) - aS \right). \]

The resulting set of FOCs for each country is identical to the previous section with exception of the FOC for the pollution stock which now reads

\[ g_{\mu^h} = \frac{\delta}{S} \left( \frac{\mu}{\lambda^h} \frac{1}{A_{LG}^h} \right)^{-1} Y^h_K K^h_A + \frac{\delta}{S} \left( \frac{\mu^f}{\lambda^f} \frac{1}{A_{LG}^f} \right)^{-1} Y^f_K K^f_A + a. \]  

(25) reflects that an increase of the pollution stock induces negative externalities in both countries which both reduce the value of the objective functions.

Following the same line of reasoning as in the previous section, it can be shown that the respective country’s growth rates in the integrated economy are still given by (22) while the capital-abatement ratio in the symmetric equilibrium reads

\[ K_A(S, C_K)^i = \left( \gamma(1 - \alpha) + \frac{a\delta}{C^i_K + a} \right)^{-\frac{1 - \alpha\gamma}{1 - \gamma - \delta}} \left( \frac{p_L}{b(\alpha\gamma)^\alpha} \right)^{\frac{1}{1 - \gamma - \delta}}. \]  

(26)

The effect of the integration of the foreign country’s damages from domestic pollution show up in the first term on the RHS. Compared to the previous section, perfect internalization doubles the feedback effect of stock pollution damages on the capital-abatement ratio. This induces a further decline of \( K^i_A \) and – as follows straightforwardly – the pollution stock.
3.4 Scenario 4: Asymmetric Internalization

So far it has been assumed that all countries internalize the global and local externalities symmetrically, yet the current debate on climate policies shows that this is hardly the case. In reality, a number of countries largely ignores global externalities and focuses on the internalization of local externalities. This holds especially for developing countries – although not exclusively – while industrialized, higher income countries tend to take the feedback effects of the global externality on their economy into account. This is, for example, reflected by the fact that all industrialized countries (except for the US) committed to GHG emission reductions by ratifying the Kyoto Protocol. Although our simplified set-up assumes that both countries are identical with respect to preferences as well as production technologies such that our analysis does not account for the systemic differences between developing and industrialized economies, we can derive basic implications on growth, abatement and pollution that result from asymmetric internalization strategies.

In this section we assume that country $h$ internalizes the global externality while country $f$ solely takes the local externality into account. Under this assumption, the optimization problem of country $f$ is again given by the Hamiltonian of Subsection 3.1. If country $h$ only considers the domestic consequences of its pollution, its optimization problem is represented by the Hamiltonian in Subsection 3.2. If it, however, also considers international spill-overs, it would maximize the Hamiltonian of Subsection 3.3.

In both of the above scenarios, country $f$’s capital-abatement ratio is given by (13) while it is equal to (54) in country $h$ if only the domestic externalities are internalized and

$$K_A(S, C_K)^i = \left( \gamma (1 - \alpha) + 2 \frac{a \delta}{C_K + a K_A^i + K_A^j} \right)^{\frac{1-\alpha}{1-\gamma}} \left( \frac{p^2 L S^d}{b(\alpha \gamma)^{\alpha \gamma}} \right)^{\frac{1}{1-\gamma}}$$  (27)

if additionally the international spill-overs are considered. As to be expected, the capital-abatement ratio will be higher in country $f$ than in country $h$. The long-run pollution stock will be lower than in Scenario 1 but higher than in Scenario 2. Considering this in (13) and (54), see appendix, we observe that – compared to Scenario 1 – the capital-abatement ratio in country $f$ is lower while – compared to Scenario 2 – the capital-abatement ratio in country $h$ is higher. So, internalization in country $h$ induces the non-internalizing country $f$ to raise its abatement effort relative
to capital accumulation. As the two abatement activities are conducted at the fixed ratio $\frac{1-\alpha}{\alpha}$ in this country, investment in both types of abatement increase and not only global pollution is lower due to the internalization of country $h$, but also local pollution in country $f$ decreases. Country $h$ experiences the opposite: Due to higher global pollution comparison to a situation with internalization by both countries, it conducts less stock reducing abatement relative to capital accumulation. And parallel to the development in country $h$, the reduction in $A_{h}^{L_{G}}$ will induce a reduction in $A_{h}^{L}$. So, the asymmetric behavior not only affects the level of abatement regarding global pollution in both countries but spills over to the abatement of local pollution as well.

From the preceding analysis of the planner solutions, the straightforward question arises, how the different scenarios could be implemented in a market economy. Depending on the degree of internalization of the externality, the instruments to be employed as well as their optimal level will vary. Of special interest is the question of optimal policies in the asymmetric case. How should the policy maker in country $h$ optimally react if country $f$ ignores the global externality?

4 Market Solution and Environmental Policies

4.1 Symmetric Scenarios 1 to 3

Let us first consider the optimal policies a policy maker would adopt in the symmetric Scenarios 1 to 3. Due to the specific production technology adopted in this paper, the only market failures are due to environmental externalities which makes the policy choice more straightforward than in an economy in which market failures additionally arise from, e.g., knowledge spill-overs or monopolistic competition as in Pittel/Bretschger (2009) or Grimaud/Rougé (20xx).

In Scenario 1, both countries ignore the global externality such that only the local market failure remains to be internalized. The policy maker can in reality obviously choose between different instruments to implement the optimum, in this paper we stick for simplicity to environmental taxation by assuming that a tax $\tau_{L}$ is levied on local pollution $P_{L}$. (In the following we drop country indices for convenience as optimal policies in both countries are identical along the BGP in the symmetric scenarios.)\(^{13}\)

\(^{13}\)Please note, that the stability properties of the system during the transition to the BGP are of course identical to those of the planner scenarios when the planner conducts an optimal policy.

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In the market economy, households maximize their intertemporal utility subject to their budget constraint as given in (6) which gives the standard Keynes-Ramsey rule
\[ g_C = \sigma^{-1} (r - \rho) \tag{28} \]

Firms maximize profits leading to two FOCs for abatement and capital. As individual firms do not internalize the externalities arising from pollution, their return to abatement solely consists in the taxes saved due to abatement. Consequently, the FOCs for the two abatement activities and capital read
\[ 1 = (1 - \alpha) \tau_L \frac{P_L}{A_{LG}} = \alpha \tau_L \frac{P_L}{A_L} \tag{29} \]
\[ r = Y_K - \tau_L \frac{P_L}{K} \tag{30} \]

The optimal policy in this case is given by
\[ \tau_L = \gamma \frac{Y}{P_L} = \gamma \left( \frac{\alpha}{1 - \alpha} \right) p_L Y_K A_{LG}, \tag{31} \]
i.e. the optimal tax rate has to equal the marginal externality. Inserting (31) into the above FOCs and the Keynes-Ramsey condition replicates the growth path of Scenario 1. From (31) follows that the tax rate has to rise over time in order to mirror the increasing scarcity of pollution in a growing economy.

In **Scenario 2** both planners internalize the local and, at least partly, the global externality. As two different externalities are now considered, each policy maker requires two instruments to internalize both. Although both pollution types arise as a constant share of the capital input, two instruments are necessary as the induced market failures are not perfectly correlated – in which case one policy instrument would be sufficient.\(^{14}\) In addition to the tax on local pollution we now consider a second tax on pollution that adds to the global pollution stock, \(\tau_G\).

Due to the additional tax the FOC for \(K\) and \(A_{LG}\) now read
\[ 1 = (1 - \alpha) \tau_L \frac{P_L}{A_{LG}} + \tau_G \frac{P_G}{A_{LG}} \tag{32} \]
\[ r = Y_K - \tau_L \frac{P_L}{K} - \tau_G \frac{P_G}{K} \tag{33} \]

\(^{14}\)Employing a unified tax rate on capital in this scenario could generate the optimal growth rate, but would not lead to an optimal abatement mix as the price ratio between the two abatement types would remain distorted.
while the FOC for $A_L$ remains unchanged. As the local externality was already perfectly internalized in Scenario 1, the optimal tax rate on local pollution is again set according to (31). The optimal tax on $P_G$ can then be shown to equal the marginal externality arising from global pollution (see Appendix 6.2)

$$\tau_G = \delta \frac{Y}{S} \frac{1}{C_K + a}.$$ (34)

In comparison to $\tau_L$ which is set equal to the currently arising negative productivity effect of $P_L$, a second term appears in (34). $\frac{1}{C_K + a}$ accounts for future externalities from today’s addition to the stock of pollution. The tax rate depends therefore negatively on the regeneration rate $a$, as faster regeneration implies that pollution is absorbed faster. Also, a higher consumption-capital ratio, i.e. lower investment in (polluting) capital compared to consumption, means that less pollution is generated from today’s production and therefore less is added to the pollution stock.

Finally, let us consider Scenario 3 in which both externalities are perfectly internalized and the same tax instruments can be employed. Regarding the local externality, (31) still represents the optimal tax rate while (34) has to be adjusted in order to capture the international spill-overs of domestic CO$_2$ emissions. Recalling the results of Subsection 3.3 where perfect internalization doubled the $K_A$ reducing effect, it follows straightforwardly that $\tau_G$ has to be doubled as well.

### 4.2 Asymmetric Scenario 4

Given that the two countries do not follow the same internalization strategy, the question arises which policy mix would be optimal in this case. As country $f$ only taxes local pollution, the resulting stock of pollution is higher than in Scenario 2, resp. 3, which are seen as optimal by country $h$. So, setting the tax rates according to the policy rules of the symmetric scenarios will not result in optimal growth in this case.

One option for country $h$ could be to tax domestic pollution, local and/or global, at different rates than in the previous sections. First, it could choose to tax global pollution at a higher rate, thus inducing a further reduction which would yield as ancillary benefit an accompanying reduction of local pollution. This policy can, however, not be optimal. Under policy rules we derived in the previous sections, country $h$ sets the tax rate, $\tau_G$, equal to the marginal damage from pollution. For a $\tau_G$ above this level, the marginal damages would be smaller than the tax rate which would lower welfare. If the country would – as compensation for the $\tau_G$ – lower the tax on local
pollution, this could result in the optimal local and global pollution level, yet the costs of attaining these optimal levels would be suboptimally high as the price ratio between the two types of pollution would be distorted.

A second option would be to subsidize either pollution reduction or abatement in country \(f\). As we will show in the following, the two policies, although closely related, have very different implications for global welfare and especially the receiving country.

**Subsidy on Pollution Reduction**

First we consider a subsidy to pollution reduction, \(s_P\), i.e. country \(h\) pays for a reduction of \(P^f_G\) below its status quo level.\(^{15}\) In a world with perfect information – as assumed in our model – this is feasible since status quo pollution in country \(f\) is known to the policy maker in country \(h\). In reality, in which such perfect information is hardly given, this policy would, however, set incentives for the country \(f\) to exaggerate its pre-subsidization pollution. If we stick for the moment to the assumption that no information related market failure arises, optimal policy in case is straightforward.

Profits of firms in country \(f\) are given by

\[
\Pi^f = Y^f - r^f K^f - A_L^f - A_{LG}^f - \tau_L^f P_L^f + s_G (P^r_G - P^f_G) \tag{35}
\]

where \(P^r_G\) denotes pollution in the reference scenario. Optimization leads to the following FOCs:

\[
\gamma (1 - \alpha) Y^f K^f A_L^f = 1 - s_G \frac{P^G A_L^f}{A_{LG}^f} \tag{36}
\]

\[
1 = \alpha \gamma \frac{Y^f}{A_L^f} \tag{37}
\]

\[
r = (1 - \gamma) Y^f K^f - s_G \frac{P^G}{A_{LG}^f} \tag{38}
\]

where we have already considered that country \(f\) sets the tax rate on local pollution at its optimal level, i.e. \(\tau_L = \gamma \frac{Y^f}{A_L^f}\). Comparison of (37) to the planner scenario 2, (9), shows that the FOC for \(A_L\) is already corrected for by \(\tau_L\). Further comparison of (36) and (38) to (18) and (19) reveals that the subsidy that induces optimality equals the negative ratio of the shadow prices for capital and stock pollution

\[
s_G = -\frac{\mu^f}{X^f}. \tag{39}
\]

\(^{15}\)For simplicity we assume that subsidies are financed by lump-sum taxation in country \(h\).
This implies that the optimal subsidy rate paid to country \( f \) has to follow the same rule as the tax rate \( \tau_G \) in country \( h \). To induce country \( f \) to reduce its pollution to the optimal level, country \( h \) has to pay a marginal compensation which equals the marginal damages from pollution. As the marginal damages that arise from \( P_G \) are independent from their geographical origin, the marginal avoided damage in country \( f \) due to the subsidy exactly equals the marginal damage avoided in country \( h \). Consequently, the subsidy equals both, the marginal damages at home and abroad, i.e.

\[
s_G = \tau_G = \delta \frac{Y^h}{S^h} \frac{1}{C^h_K + a} = \delta \frac{Y^f}{S^f} \frac{1}{C^f_K + a}.
\]

Please note, that if we assumed country 1 to internalize the transnational spillovers from \( P_G \) (Scenario 3) this would simply imply an adjustment of the tax and subsidy rates by a factor of two.

Thus, for country \( h \) to induce optimal pollution, it has to forego its gains from reducing the global pollution stock by paying the subsidy to country 2. Thus, if it receives no additional benefits from reducing pollution, country \( h \) will be indifferent between subsidization and the business usual scenario. Hence, if raising and transferring the public funds needed for subsidization would be accompanied by any social costs, country \( h \) would rationally decide not to subsidize country \( f \). Country \( f \), on the other hand, profits from the subsidy, as the externalities from global pollution decline and it receives additionally the subsidy from country \( h \).

In comparison to other possible subsidy schemes – as, e.g., subsidizing abatement (see next paragraph) – the advantage of subsidizing pollution reduction is that it corrects not only for the returns to \( A_{LG} \)-abatement but also for the returns to capitals. The price ratios between the two abatement types as well as between abatement and capital are thus optimal. From a global perspective, subsidizing pollution reduction is optimal as it can achieve the first-best allocation, yet due to the uneven distribution of gains, this policy might not be implemented. As we will see in the next paragraph, abatement subsidization – the second option for country \( h \) – might have a better chance of being implemented. Yet, while it equally corrects for the distortion in the relative price of the two abatement types, the price ratio between abatement and capital remains distorted and the first-best solution is consequently not implemented.
Subsidy to Abatement

In the real world, the advantage of subsidizing abatement in comparison to pollution reduction would be its better observability and thus the lower potential to exaggerate abatement activities in order to receive higher payments. In our world, however, information is perfect, such that observability plays no role regarding a comparison of the two instruments.

If country $h$ subsidizes abatement in country $f$, profits of firms in country $f$ are given by

$$\Pi^f = Y^f - r^f K^f - A^f_L - (1 - s_A)A^f_{LG} - \tau L^f P^f_L$$

which yields the following FOCs (assuming optimal internalization of $P_L$)

$$\gamma (1 - \alpha) Y^f K^f_A = 1 - s_A$$
$$\alpha \gamma \frac{Y^f}{A^f_L} = 1$$
$$r^f = (1 - \gamma) Y^f K^f.$$  (44)

Again, the price of $A_L$ equals its social return. Comparison of the FOC for $A_{LG}$, (42), to (18) shows that the optimal subsidy rate has to equal

$$s_A = -\frac{\mu^f P^f_G}{\lambda A^f_{LG}}.$$  (45)

In contrast to the subsidy on pollution, the subsidy on abatement has to mirror not only the relative shadow price of pollution but also has to take the productivity of abatement in pollution reduction, $\frac{dP^f_G}{dA^f_{LG}} = P^f_G A^f_{LG}$, into account. Consequently, the higher the productivity, the more abatement is optimal and the higher the subsidy rate. The question arises whether this optimal subsidy rate is higher or lower than the induced marginal damage reduction in country $h$. Recall that the marginal damage is equal to the negative ratio of the shadow prices, $-\frac{\mu^f}{\lambda}$. So, for $s_G < -\frac{\mu^f}{\lambda}$, the ratio of pollution to abatement has to be smaller than unity. As in the optimum, pollution is constant while abatement increases over time, the ratio falls along the BGP and approaches zero, such that the condition holds in the long run. The economic intuition is that the larger the economy, the more difficult is gets to keep pollution at bay.

Along the BGP, the falling productivity of abatement, $\frac{P^f_G}{A^f_{LG}}$, and the increase of the shadow price of pollution, $\frac{\mu^f}{\lambda}$, exactly offset each other, such that the social return
to abatement – and thus the subsidy rate – is constant over time. The pollution tax, on the other hand, increases over time as pollution gets scarcer and thus more valuable. On first sight, it seems that country \( h \) should favor subsidizing abatement in contrast to pollution reduction as it is less costly and even gets relatively cheaper over time. Yet, while the subsidy rate on \( A_{LG}^f \) remains constant over time, the quantity of abatement activities rises in the optimum, such that the aggregate subsidies on pollution and abatement not only grow at the same rate, but are identical at each point in time – as the following simple calculation shows:

\[
s_p P_G^f = \frac{\mu_f}{\lambda_f} P_G^f = \left( \frac{\mu_f}{\lambda_f} \right) A_{LG}^f = s_A A_{LG}^f.
\]

(46)

We have thus confirmed that no difference exists between the two alternatives regarding the amount of funds required for subsidization. Yet, regarding the accumulation of capital, (44) shows that a subsidy on abatement leaves the FOC for capital unchanged. As the subsidy does not capture the negative externality from capital, the return to capital as well as growth are higher than in the optimum. In order to show this, substitute (44) into the Keynes-Ramsey rule, (28), which gives the growth rate of country \( f \) in terms of the capital-abatement ratio

\[
g_c^f = \sigma^{-1} \left( \frac{1 - \gamma}{\gamma} \frac{1}{1 - \alpha K_A^f} \right).
\]

(47)

The growth rate under internalization of the domestic consequences of global pollution would, however, be given by (22). Using \( Y_K = \frac{1}{K_A^h + P_G^h} \left( \frac{\gamma}{\gamma(1 - \alpha)} \right) \) from (18), it can be shown that the country’s optimal growth rate in this scenario equals

\[
g_c^h = \frac{1}{\sigma} \left[ \frac{1 - \gamma}{\gamma} \frac{1}{1 - \alpha K_A^h} + \frac{1 - \alpha \gamma}{\gamma(1 - \alpha)} \left( \frac{\mu_h}{\lambda_h} \frac{P_G^h}{A_{LG}^h} \right) - \rho \right].
\]

(48)

As the capital-abatement ratios in (47) and (48) are identical under optimal subsidization and \( \mu^h \) is negative, it follows that country \( f \) grows at a suboptimally high rate due to the uninternalized externalities from capital.\(^{16}\)

If country \( h \) would not only internalize the domestic damages from \( P_{LG}^h \) but also the transnational externalities, the gap between the growth rate under abatement

\[^{16}\]Under subsidization, the capital-abatement ratio in country \( f \)

\[
K_A^f = \left( \frac{\gamma(1 - \alpha)}{1 - s_A} \right)^{\frac{1 - \alpha}{\alpha}} \left( \frac{p_s^f S_s^f}{b(\alpha \gamma)^{\alpha \gamma}} \right)^{\frac{1 - \gamma}{\gamma}}.
\]

depends, as to be expected, negatively on the subsidy rate. Equalization of the equation above to
subsidization and the optimal (in this case also first-best) growth rate would even be wider.

Summing up, the consequences for country \( h \) from subsidizing abatement or pollution are the same. In both cases its payments to country \( f \) are equal to its gains from internalization. In either case, the capital-abatement ratios in both countries and the global pollution stock are optimal. Yet, the consequences for country \( f \) and global welfare differ as under abatement subsidization, country \( f \) accumulates capital faster and therefore grows at a higher rate than is welfare optimal.

5 Conclusions

This paper analyzed local and global pollution in the framework of a 2-country endogenous growth model. It considered two different technologies to mitigate pollution, one of which reduced only local pollution while the other mitigated global pollution as well. We first derived implications for the optimal abatement mix, capital accumulation and growth under different assumptions about the degree to which the local and global externalities are internalized. Subsequently, policies were derived in order to achieve optimal growth for the different internalization scenarios. Special attention was paid to the case of asymmetric internalization.

It was shown that the capital-abatement ratio expectedly declines with an increasing degree of internalization. In case countries only internalize the local externality, their transition and growth path were shown to exhibit the same growth rate and capital-abatement ratio at every point in time. If, however, countries also internalize global pollution, their growth paths might differ during transition. In this case stock pollution gives rise to a feedback effect from capital accumulation to future production. This feedback is ignored by countries that solely internalize pollution. Internalization of the stock externality, however, requires an adjustment of the optimal accumulation path. This strength of the feedback effect depends on whether countries internalize only the domestic or also the transnational effects from the global pollutant.

(21) gives the optimal subsidy rate

\[
s_A = \left( \frac{\delta}{C^f_K + a} \frac{P^f_G}{S} \right)^{-1} \left( \gamma(1 - \alpha) + \frac{\delta}{C^f_K + a} \frac{P^f_G}{S} \right)
\]

which again can be shown to be smaller than the marginal damage from pollution, \( \frac{\delta}{C^f_K + a} \frac{y^f}{S} \), if pollution is smaller than abatement.
We showed that, although both types of pollution are caused by capital accumulation in our model, two instruments are required to achieve the first-best solution. In the case of taxes, the local pollutant is optimally taxed according to the current marginal damages it generates. The tax on the global pollutant, however, also has to take intertemporal externalities from adding to the pollution stock into account.

The paper also considered asymmetric internalization in which countries internalize the global externality to different degrees. Without international environmental policy neither country attains the development path it considers to be optimal in this case. We analyzed two policy options – a subsidy on the reduction of pollution and a subsidy on abatement. It was shown that either policy can implement the optimal capital-abatement ratio and the optimal pollution stock. Yet, the global welfare optimum can only be achieved by subsidizing pollution reduction since subsidizing abatement does not correct for the negative externalities from capital accumulation. Consequently, capital accumulation and growth are suboptimally high in the subsidy receiving country while they are at their optimal level in the subsidizing country.

In our analysis we have assumed perfect information about pollution and abatement between the countries. Although, we shortly pointed to implications of imperfect information, a more detailed analysis of its consequences as well as of strategic incentives arising in the case of international environmental policy are certainly desirable. Also, dropping the simplifying assumption of identical technologies and preferences would allow, for example, to better capture asymmetric internalization in the case of industrialized and developing countries. Integrating these aspects into the present framework might yield further interesting results.
6 Appendix

6.1 Scenario 2: Derivation of BGP Capital-abatement Ratio

From the FOCs for $A_{LG}$ and $K$, (18) and (19), we get

$$g_{\lambda} = -(1 - \alpha \gamma)Y_K^i + K_A^i - 1$$

(49)

while combining (18) and (20) gives

$$g_{\mu} = \delta \frac{1}{S} P_G Y_K^i \left( \gamma (1 - \alpha)Y_K^i - K_A^i - 1 \right) + a.$$  

(50)

From differentiating (18) with respect to time, we get a second expression for the dynamics of $g_{\mu}$

$$g_{\mu}^i = g_{\lambda}^i + g_{A_{LG}}^i - g_K^i + \frac{\gamma (1 - \alpha) Y_K^i}{\gamma (1 - \alpha) Y_K^i - 1} Y_K^i A_{LG} - g_{A_{LG}}^i$$

(51)

Along the BGP $g_{\mu}^i = g_{\lambda}^i + g_{A_{LG}}^i$ again has to hold, such that we get from (51) that along the BGP $g_{\mu}^i = g_{\lambda} + g_{K}$. Using also $g_{K}^i = (1 - \alpha \gamma) Y_K^i - C_K^i - K_A^i$ from (5) where we employed (9), we get

$$g_{\mu}^i = -C_K^i$$

(52)

and from equating (50) and (52)

$$-(C_K^i + a) \frac{\delta S}{P_G} = Y_K^i \left( \gamma (1 - \alpha) Y_K^i - K_A^i - 1 \right).$$

(53)

Using (23) gives

$$K_A(S, C_K)^i = \left( \gamma (1 - \alpha) + \frac{a \delta}{C_K^i + a K_A^i + K_A^i} \right)^{-\frac{1}{1 - \alpha \gamma}} \left( \frac{p_G \gamma S^\gamma}{b(\alpha \gamma)^{\alpha \gamma}} \right)^{\frac{1}{1 - \gamma}}.$$  

(54)

Finally, by employing (16) we can now derive the capital-abatement ratio along the BGP to equal (21).

6.2 Scenario 2: Derivation of Optimal $\tau_G$

To determine the optimal $\tau_G$, first insert $\tau_L$ from (31) into (33) which gives

$$\tau_G \frac{P_G}{A_{LG}} = 1 - (1 - \alpha \gamma) Y_K^i K_A.$$  

(55)
From (18) we know that $1 - (1 - \alpha)\gamma Y_KK_A = -\frac{\mu P_G}{\lambda A_{LG}}$ has to hold in the optimum. Equating the two expressions shows that the tax rate has to equal the negative ratio of the shadow prices of stock pollution and capital,

$$\tau_G = -\frac{\mu}{\lambda}. \quad (56)$$

Equating the two expressions for $g_\mu$ from (20) and (52) gives

$$-C_K = \frac{\delta}{S} \left( \frac{\mu}{\lambda^i A_{LG}^i} \right)^{-1} Y_K^i K_A^i + a \quad (57)$$

which yields after some rearranging

$$-\frac{\mu}{\lambda} = \delta \frac{Y}{S} \frac{1}{C_K + a}. \quad (58)$$

Combining (56) and (58) finally gives the optimal tax rate on $P_G$ in (34).
References


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