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Non-linear pricing in oligopoly

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Abstract

Price discrimination is an extensively studied subject in monopoly behavior. Increasing profits, covering fixed cost and reducing distortions are reasons to sell a homogenous good at different prices. Price discrimination is however present also in oligopolistic markets. This paper is going to analyze similarities and differences with respect to the literature on monopoly pricing on second degree price discrimination. It is shown that also oligopolies allow even for second degree price discrimination and in this case tend to increase supply even beyond the socially efficient quantity. Facing heterogeneous customers competition leads to substantial differences with respect to the determination of the optimal sales plan.

Keywords: Non-linear pricing, oligopoly.

JEL-number: D21, D43, D82.

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List of Variables

$y(p)$	individual demand
$\phi(y)$	inverse demand
N	aggregate number of individuals
$Y(p) = N \cdot y(p)$	aggregate demand
f_i	fixed fee firm i
p_i	unit price firm i
$P_i(y)$	pricing scheme firm i
c_L	(long term) marginal cost of capacity
c_S	(short term) marginal cost of delivery
x_i	capacity firm i
t_L, t_H	customers type differing in their willingness to pay
$v_t(y)$	type t 's aggregate willingness to pay

1 Introduction

There is an extensive literature on price discrimination in monopolistic markets. Price discrimination serves mainly two different aspects: It increases profits, eventually covering thereby substantial fixed cost, and allows to reduce monopolistic distortion. Perfect price discrimination even allows efficiency also in monopolistic markets. Perfect price discrimination is however in general not feasible.

If the seller can identify specific groups exhibiting different willingness to pay or differing price elasticities of demand, it can try to ask different prices. In oligopolistic markets, this third degree price discrimination may even decrease firms profits due to an increase in competition, as shown e.g. in Corts (1998) and in Armstrong and Vickers (2001).

Even if individual preferences can not be identified, e.g. by certain characteristics like sex, age, residence, or if discrimination is not allowed, pricing schemes like quantity rebate, basic fee or general non linear pricing can help to implicitly discriminate by self selection of the customers. The so called second degree price discrimination allows a monopoly in the presence of a representative consumer, to increase its supply up to the efficient quantity without negative effect on the revenue.

In case of heterogeneous consumers a non-linear tariff allows to discriminate against consumers with different willingness to pay and thus to extract more consumer rent than by offering a linear price. All these opportunities to discriminate against consumers is obviously restricted to markets where arbitrage transactions between customers are not feasible. But this holds true for most services. Second degree price discrimination might therefore occur in the telecommunication sector and grid-bound industries but also e.g. in gym, cinema, dancing school or sports club.

This paper focuses on second degree price discrimination in oligopolistic markets. In oligopolies Cournot competition gives no scope for second degree pricing behavior and Bertrand competition inhibits discrimination. The latter is straightforward since any deviation from marginal cost pricing either yields losses or opportunities to expand supply.

The literature thus focuses on deviations of the Bertrand assumptions, heterogeneity of the products provided or imperfect reaction on price differences, which yields some pricing range and thus comes close to the monopolistic results. Stole (2006) gives a comprehensive outline of the topic.

In the following paper perfect rationality of the consumers and homogeneity of the goods is maintained but another crucial assumption of Bertrand, unlimited capacities is abolished. The analysis follows the basic approach by Kreps and Scheinkman (1983), who model an endogenous capacity choice in the long run and price competition under restricted capacities à la Bertrand Edgworth in the short run. Incentives to provide capacities and opportunities for non-linear pricing (second degree price discrimination) given these capacities in oligopolies are analyzed and compared to the corresponding monopolistic markets.

In this paper neither network externalities nor behavior based pricing are taken into account in order to concentrate on the aspect of oligopolistic competition in non-linear prices.

2 The basic model

There are two firms offering a homogeneous good in a market with N identical consumers¹ with decreasing demand $y(p)$, and inverse demand $\phi(y)$ respectively. The analysis follows the basic approach by Kreps and Scheinkman (1983). It is assumed that in a two-step procedure firms simultaneously decide on their capacity in the first stage while simultaneously choosing their pricing scheme under the restriction of capacity in a second. Firms are assumed to have identical cost, constant marginal cost c_L providing capacity x_i and constant marginal cost c_S delivering the demanded quantity $Y_i \leq x_i$. On the second stage firms thus perceive fixed cost ($c_L x_i$) and short term marginal cost c_S up to the capacity level x_i . Individual demand $y(p)$ is strictly decreasing and preferences are quasi-linear. Individual utility from consuming y paying P is thus determined by $\int_0^y \phi(\zeta) d\zeta - P = v(y) - P$. Aggregate demand is $Y(p) = N \cdot y(p)$.

Two different settings are under consideration: In the first, consumers can only satisfy their demand with one single firm. This is the case e.g. for electricity demand. The consumer has to choose a firm and subsequently her demand, maximizing her utility $\max_i (\max_y v(y) - P_i(y))$. It is assumed, that individuals know the pricing schemes of all firms and are not stuck to a firm. Firms can only sell contracts up to their capacity, taking the optimal choice of individuals into account.² It is assumed that on the second stage when pricing decisions are made capacities are common knowledge to the firms³.

The second approach assumes that individuals are free to satisfy their demand with different firms. In this case efficient rationing is assumed, that is the first units are purchased at the constrained (cheaper) firm. Again consumers rationally choose their optimal contract, taking firms pricing schemes into account.

In the first approach, because all customers are identical, we can restrict our analysis at the second stage to just one contract (y_i, P_i) for each firm.

Two cases have to be treated separately:

- Overall capacity is more than sufficient to serve all consumer's demand at short term marginal cost ($\sum_i x_i > Ny(c_S)$).
- Overall capacity is at most sufficient to serve this demand ($\sum_i x_i \leq Ny(c_S)$).

In the second case competition between firms is restricted. There is a unique equilibrium (in pure strategies):

Lemma 1 *Let $y^s = \phi(c_s)$ and aggregate capacities $\sum_i x_i \leq N \cdot y^s$. Then price equilibrium on the second stage is determined by $y = \frac{\sum_i x_i}{N}$ and $P_i = v(y)$.*

¹ N is assumed to be large s.t. in the following analysis firm specific demand can be seen as a continuous variable in order to use calculus to determine equilibrium capacities.

²Assuming rational individuals, this can also be interpreted as rational expectations about demand and capacities of firms. We assume that switching between firms even in case of long-ranging contracts is quicker than long run investments in capacity.

³It is also assumed that each consumer can meet its demand at the conditions of the contract, i.e. firms can not sell more contracts than they are able to satisfy. This seems to be plausible in the long run respectively under the assumption of perfectly informed costumers

Equilibrium profits are

$$\Pi_i^* = \frac{x_i}{y}v(y) - (c_S + c_L)x_i. \quad (1)$$

Firms determine the market clearing price and then collect the complete consumer rent. There is no incentive to deviate from this strategy in the short run since capacity is completely exhausted and there is no opportunity to achieve higher revenues. Any other strategy combination will lead to

1. excess capacity at marginal revenue higher than short term marginal cost,
2. excess demand or
3. different marginal revenues to the firms respectively,

which in any case cannot be part of a subgame perfect equilibrium.

In the first case, i.e. if aggregate capacity exceeds aggregate demand at short term marginal cost, there is no equilibrium in pure strategies except in the case where each firm by oneself can serve the whole market at short term marginal cost. The reasoning is similar to the one in Bertrand-Edgworth competition. No firm has an incentive to provide any quantity below short run marginal cost. Therefore firms hold excess capacities in equilibrium. This gives reason to either undercut the opponents offer, or to exhaust the monopoly power with respect to the residual demand.

Residual demand will be zero if each firm by oneself can serve the whole market at short term marginal cost. Similarly to Bertrand competition, firms will then perfectly compete and provide their services at short term zero profits.

In all other cases equilibria of the subgames are in mixed strategies, that is firms choose one of multiple strategies with certain probabilities. Equilibrium payoffs are therefore expected values. In the following it is assumed that firms are risk neutral.

Lemma 2 *If aggregate capacity of the firms is higher than aggregate demand at short run marginal cost pricing, but lower than Bertrand capacities ($Ny(c_S) < \sum_i x_i$, $x_i < Ny(c_S)$) then expected gains equal*

$$E[\Pi_i] = \frac{Ny(c_S) - x_j}{y(c_S)} f^{max} - c_L x_i \quad (2)$$

where $f^{max} = \int_0^{y(c_S)} (\phi(\eta) - c_S) d\eta$.

Proof: Consider a two-part tariff with basic charge f_i and price per unit p_i . Because of excess capacity, any contract offering a unit price not equal to short run marginal cost is dominated by a contract with $p_i = c_S$ keeping the consumer indifferent. Hence firms will supply at a unit price $p_i = c_S$ equal to short run marginal cost, which allows for the highest consumer rent per unit and compete via different fixed fees. The first $x_i/y(c_S)$ customers are served by the low price firm, the rest by the high price firm.

In equilibrium the fixed fee f_i is chosen from a closed interval $[f^{min}, f^{max}]$. The equilibrium distribution has to be continuous over the whole interval except at most at the lower end. f^{min} may be adopted with positive probability by at most one of the two firms.

f^{max} must not exceed $\int_0^{y(c_S)} (\phi(\eta) - c_S) d\eta$ which equals the maximal fixed fee that the representative consumer is willing to pay. On the other hand this maximal fee must be reached at the top of the interval in order to have no incentive to increase the fee further. The bottom of the interval f^{min} can be determined by the following reasoning: At least one firm has to be indifferent between f^{min} and f^{max}

$$\frac{N \cdot y(c_S) - x_j}{y(c_S)} f^{max} = x_i f^{min}$$

the competing firm will be no better of choosing f^{min}

$$\frac{N \cdot y(c_S) - x_i}{y(c_S)} f^{max} \geq x_j f^{min}.$$

In equilibrium this leads to

$$f^{min} = \min \left\{ \frac{N y(c_S) - x_1}{x_2} f^{max}, \frac{N y(c_S) - x_2}{x_1} f^{max} \right\}.$$

□

Turning to the first stage, equation (2) implies that capacities chosen will not exceed realised demand in equilibrium. Therefore expected payoffs at the second stage are given by equation (1).

$$\Pi_i = \int_0^{x_i+x_j} \phi(\eta) d\eta \cdot \frac{x_i}{x_i+x_j} - (c_L + c_S)x_i.$$

$$\frac{\partial \Pi_i}{\partial x_i} = \phi(x_i+x_j) \frac{x_i}{x_i+x_j} + \frac{\int_0^{x_i+x_j} \phi(\eta) d\eta}{x_i+x_j} \frac{x_j}{x_i+x_j} - (c_L + c_S) = 0$$

This condition must hold true for both firms in equilibrium. Adding up both equalities gives

$$p^* + \frac{\int_0^{y(p^*)} \phi(\eta) d\eta}{y(p^*)} = 2(c_L + c_S), \quad (3)$$

which together with the fact $p^* < \frac{\int_0^{y(p^*)} p(\eta) d\eta}{y(p^*)}$ (decreasing demand) implies that $p^* < c_L + c_S$, i.e. firms will supply more than the (socially) efficient quantity.⁴

Corollary 1 *In an oligopoly, firms that charge a non linear pricing schedule will supply more than the efficient quantity.*

This result can be illustrated on the basis of a linear demand function $\phi(y) = 1 - y$ (normalize N to $N = 1$). Equilibrium payoffs at the second stage (given aggregate capacity does not exceed $1 - c_S$) are

$$\Pi_i(x_i, x_j) = \int_0^{x_i+x_j} (1 - \eta) d\eta \cdot \frac{x_i}{x_i+x_j} - (c_L + c_S)x_i$$

⁴The above argument neglects the fact, that in equilibrium unit prices at the second stage must exceed short run marginal cost c_S . The result however still holds true as long as there are positive marginal cost of capacity $c_L > 0$. The consequences are illustrated in the example below.

$$\begin{aligned}
&= \left((x_i + x_j) - \frac{1}{2}(x_i + x_j)^2 \right) \frac{x_i}{x_i + x_j} - (c_L + c_S)x_i = \left(1 - \frac{1}{2}(x_i + x_j) \right) x_i - (c_L + c_S)x_i \\
&\frac{\partial \Pi_i}{\partial x_i} = 1 - \frac{1}{2}(2x_i + x_j) - (c_L + c_S) = 0 \tag{4} \\
&x_i = x_j = \frac{2}{3}(1 - c_L - c_S) \quad p^* = \frac{4}{3}(c_L + c_S) - \frac{1}{3} \quad \text{if } 4c_L + c_S \geq 1.
\end{aligned}$$

Aggregate quantity supplied ($x_i + x_j$) is larger than the efficient quantity ($x^* = 1 - c_L - c_S$) that is supplied by a monopolist.

If marginal cost are relatively low ($4c_L + c_S < 1$) the above incentives would lead to aggregate capacities equal to $y(c_S)$, which is an upper limit as shown above (equation (2)). In this case there are multiple equilibria: Any combination of capacities (x_i, x_j) satisfying $x_i + x_j = 1 - c_S$ and $2c_L \leq x_i \leq 1 - 2c_L - c_S$ is part of an equilibrium that leads to unit prices $p^* = c_S$. Equation (2) shows that there is no incentive to increase capacity, the left hand side of equation (4) on the other hand shows that there is no incentive to decrease capacity. Again oligopoly firms will supply more than the efficient quantity as long as $c_L > 0$, that is there are capacity cost.

3 Heterogenous consumers

In the basic model it was assumed that all customers are identical. In fact non-linear pricing schemes are also an appropriate screening device to detect different willingness to pay. The following analysis is restricted to a simple model of two different types of customers, differing in their willingness to pay, types t_H and t_L respectively. Both types are assumed to be equally likely ($N_L = N_H$). It is assumed that the firm can not observe the type, but it can offer a non-linear pricing schedule in order to extract higher revenues. In case of two types of customers it is sufficient to offer a sales plan consisting of two price-quantity combinations $\{(P_1, y_1), (P_2, y_2)\}$, that is the monopolist offers a fixed quantity y_i upon payment of P_i . Customers then choose the price-quantity combination (respectively $(0, 0)$) that offers the higher consumer surplus:

$$\int_0^{y_i} \phi_t(\eta) d\eta - P_i \geq 0$$

This implies that the following incentive constraints must hold

$$\begin{aligned}
&\int_0^{y_1} \phi_{t_L}(\eta) d\eta - P_1 \geq 0 \quad \text{and} \\
&\int_{y_1}^{y_2} \phi_{t_H}(\eta) d\eta - P_2 + P_1 \geq 0.
\end{aligned}$$

The optimal sales plan by a monopolist exhibits no distortion at the top, i.e. $\phi_{t_H}(y_2) = c_L + c_S$, distortion at the bottom ($\phi_{t_L}(y_1) > c_L + c_S$), no surpluses at bottom and surplus at the top (unless $y_1 = 0$). Distortion at the bottom is due to the fact, that the monopolist can thus extract a higher rent

from type t_H customer, whereas the marginal loss from type t_L customer at no-distortion quantity is at zero. The optimal sales plan satisfies

$$2(\phi_{t_L}(y_1) - (c_L + c_S)) = \phi_{t_H}(y_1) - (c_H + c_S). \quad (5)$$

In oligopoly the approach is completely different. Let us assume, capacities are small in order to allow for an equilibrium in pure strategies ($x_1 + x_2 \leq N_L y_L(c_S) + N_H y_H(c_S)$). Again the two price-quantity combinations⁵ have to satisfy the incentive constraints:

$$\int_0^{y_1} (\phi_{t_L}(\eta) d\eta - P_1) \geq 0 \quad \text{and} \quad (6)$$

$$\int_{y_1}^{y_2} (\phi_{t_H}(\eta) d\eta - P_2 + P_1) \geq 0. \quad (7)$$

But now there is also competition for the attractive consumers. Assume the two different customers differ in average revenue per unit. Then it would pay to supply the preferred customer until expected average revenue equals. So in equilibrium average expected return is equal for both customers:

$$\frac{P_1}{y_1} = \frac{P_2}{y_2}. \quad (8)$$

Similar to the monopoly case there is no surplus at the bottom and surplus at the top (unless $y_1 = 0$). But in contrast to monopoly quantity y_1 and in consequence quantity y_2 are not determined by marginal willingness to pay (equation (5)) but rather average willingness to pay (equation (8)).

This aspect can be illustrated on the basis of the following demand ($N_L = N_H = 1$):

$$\phi_L(y) = 4 - 4y, \quad \phi_H(y) = 5 - 2y.$$

In the monopoly case supply y_i depends on marginal willingness to pay:

$$2(4 - 4y_1 - (c_S + c_L)) = 5 - 2y_1 - (c_S + c_L) \quad c_S + c_L = 5 - 2y_2$$

$$y_1 = \frac{3 - (c_S + c_L)}{6} \quad y_2 = \frac{5 - (c_S + c_L)}{2},$$

prices are determined in order to satisfy the incentive constraints

$$P_1 = \frac{(3 - (c_S + c_L))(6 + (c_S + c_L))}{9}, \quad P_2 = \frac{(6 - (c_S + c_L))(3 + (c_S + c_L))}{9}.$$

In the duopoly case in contrast incentive constraints give

$$P_1 = (4 - 2y_1)y_1 \quad \text{and} \quad P_2 = 5(y_2 - y_1) - (y_2^2 - y_1^2) + P_1.$$

Equation (8) leads to

$$4 - 2y_1 = 5 - (y_1 + y_2),$$

⁵Firms will offer the same price-quantity combination in any pure strategy equilibrium.

together with $x_1 + x_2 = y_1 + y_2$ this gives

$$y_1 = \frac{x_1 + x_2 - 1}{2} \quad \text{and} \quad y_2 = \frac{x_1 + x_2 + 1}{2},$$

in the second stage equilibrium.

Profits are then determined by

$$\Pi_i(x_i, x_j) = \frac{P_1}{y_1} \cdot x_i - (c_S + c_L) \cdot x_i = (5 - x_i - x_j)x_i - (c_S + c_L)x_i \Rightarrow \max$$

Equilibrium capacities are $x_1^* = x_2^* = \frac{5-(c_S+c_L)}{3}$.⁶

Equilibrium quantity is again higher in duopoly than in monopoly. If aggregate marginal costs ($c = c_S + c_L$) are high ($c > 2$) equilibrium quantity even exceeds efficient quantity. Consumer with low willingness to pay are more likely to be served in duopoly, which in consequence leads to higher rents for consumer with a high willingness to pay.

4 Incompletely informed customers

The analysis above applies usually to a long run relationship between customer and supplier. So it is quite reasonable, that costumers are not fully informed about their (future) demand. In the following it is assumed that there is a representative consumer who's willingness to pay depends on some random variable t , distributed according to the density $f(t)$. In order to simplify the analysis, aggregate demand is assumed to even out by the law of large numbers.

A monopolist facing a risk neutral consumer will again supply at long run marginal cost and fully extract the consumer surplus:

$$p = c_S + c_L$$

$$f = \int_0^1 \int_0^{y(p,t)} \phi(\eta, t) d\eta f(t) dt.$$

Under the assumption that capacities are long range compared to consumer contracts, in a duopoly on the second stage there is again competition restricted by capacities. Expected consumer surplus is maximized under the condition of equal marginal willingness to pay. Therefore also in a duopoly unit prices are constant (p^*).

Similarly to the analysis above profits on the second stage then are

$$\Pi_i = \int_0^1 \int_0^{z(p^*,t)} \phi(\eta, t) d\eta f(t) dt \frac{x_i}{\int_0^1 z(p^*,t) f(t) dt} - (c_S + c_L)x_i.$$

$$\frac{\partial \Pi_i}{\partial x_i} = p^* \frac{x_i}{x_i + x_j} + \frac{\int_0^1 \int_0^{z(p^*,t)} \phi(\eta, t) d\eta f(t) dt}{x_i + x_j} \frac{x_j}{x_i + x_j} - (c_L + c_S) = 0$$

Again equilibrium prices are below long run marginal cost and demand exceeds the efficient quantity.

⁶The analysis above holds true if $c_S + 4c_L > 2$ and $2c_L + 2 > c_S$.

Facing a risk averse consumer complicates the analysis. On the one hand the firm as a risk neutral agent would like to bear the risk instead of passing on the risk premium. On the other hand this would imply that there cannot be a constant marginal price. The costumer facing lower demand will get a reduction, whereas the costumer facing a high demand has to pay a relatively higher total. This would imply higher marginal prices decreasing equilibrium quantity at the top. There is no specific difference between oligopoly and monopoly in this respect. In both cases, in equilibrium a positive basic charge will emerge.

5 Conclusion

Non-linear pricing offers the opportunity to extract a larger part of the willingness to pay, compared to a linear pricing scheme. This on the other hand raises the incentive to increase quantity supplied. In a monopolistic market the firm facing a representative consumer is able to extract all consumer rent and therefore has an incentive to supply the efficient quantity. In duopoly firms have similar opportunities. If capacities are not too large they can likewise extract consumer rent. If however capacities are large (i.e. firms provide excess capacities in the second stage equilibrium), competition will aggravate this possibility. Firms will in this case offer their capacity at short run marginal cost and cover their desired rent by some fixed fee.

In equilibrium however capacities will be chosen to avoid excess capacities. Nevertheless the supply will stay higher than in the corresponding monopoly case but surprisingly will be even higher than the socially efficient quantity.

If customers are heterogenous, duopoly competition will put firms to supply the different customers at identical average unit cost. This fact however should not be seen as a restriction to linear pricing rules. Quite the contrary, firms force the consumer to demand quantities that might even be larger than the efficient quantity. But due to incentive constraints firms are in general no longer in a position to completely extract consumer rent. Even to a larger extend than a monopoly, firms in duopoly are under compulsion to serve customers and therefore admit higher consumer rent in equilibrium.

References

- [1] Armstrong, M. and Vickers, J. (2001), Competitive price discrimination. *The RAND Journal of Economics*, 32(4):579–605.
- [2] Armstrong, M., (1998), “Network interconnection in telecommunications”, *Economic Journal*, **108**, 545-564.
- [3] Armstrong, M., (1996), “Multiproduct nonlinear pricing”, *Econometrica*, **64**, 51-75.

- [4] Corts, K. (1998), "Third-degree price discrimination in oligopoly: all-out competition and strategic commitment", *RAND Journal of Economics*, **29**: 306-23.
- [5] Dessein, W., (2003), "Network competition in nonlinear pricing", *RAND Journal of Economics*, forthcoming.
- [6] Gans, J. S. and King, S. P., (1999), "Termination charges for mobile phone networks", *mimeo*, University of Melbourne.
- [7] Hahn, J.-H., (2003), "Nonlinear pricing of telecommunications with call and network externalities", *International Journal of Industrial Organization*, **21**, 949-967.
- [8] Kreps, D. M. and Scheinkman, J. A., (1983), "Quantity Precommitment and Bertrand competition yield Cournot outcomes", *Bell Journal of Economics*, **14**, 326-337.
- [9] Snyder, Ch. M., (1998), "Why do large buyers pay lower prices? Intense supplier competition", *Economics Letters*, **58**, 205-209.
- [10] Stole, L.A., (2007) "Price Discrimination and Competition", in Mark Armstrong and Robert H. Porter, editors, *Handbook of Industrial Organization*, Volume **3**, North Holland.
- [11] Thomas, L., (1998), "Optimal non-linear pricing with indirect prior beliefs", *Economics Letters*, **61**, 321-325.
- [12] Wilson, R. W., (1993), *Nonlinear Pricing* New York: Oxford University Press.