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# Capital Market Frictions and Economic Geography

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## Abstract

The field of New Economic Geography (NEG) aims at explaining agglomeration based on increasing returns, monopolistic competition and international factor mobility. Deviating from existing approaches, this paper constructs a theoretical model based on capital market frictions. Firms compete monopolistically, but are lead by managers subject to moral hazard. Incentivizing managers is cheaper in regions with higher purchasing power, i. e., in industrialized regions where goods do not need to be imported and the price index is thus low. This simple mechanism leads to a host of predictions; some in line with traditional NEG literature, some contradictory.

**Keywords:** New Economic Geography, Capital market, Moral hazard

**JEL classifications:** F12, G15, F21.

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# 1 Introduction

The literature on New Economic Geography (NEG), based on the seminal works of Krugman (1991) and many others, explains why industrial activity clusters in some regions, but moves away from others. For example, if firms find it easier to attract workers to regions where the purchasing power of the wages is high, agglomeration can emerge endogenously. In regions with many firms, less goods need to be imported. Hence, the price index is lower and real wages are relative high, which attracts more workers and firms due to an increasing local market size. Some comparable mechanisms, i.e. intermediate trade, have been proposed. Astonishingly, none of these reasons is based on the functioning of the capital market. This is even more surprising as the literature on the finance-and-growth nexus has flourished in recent years. Our paper shows that financial market frictions alone are sufficient to explain economic agglomeration.

Our model has the following properties. An economy has two regions with a fixed number of workers who can choose to work in agriculture or manufacturing, but cannot move between regions. Manufacturing firms employ workers to produce goods they can sell on monopolistically competitive markets, partly within their own region, partly via exporting. Firms need capital, which they receive from home investors, in form of foreign direct investments (FDI). Furthermore, firms are run by managers who are subject to a standard moral-hazard problem. Managers need to be incentivized not to shirk, in which case they would receive a non-monetary private benefit. Now the interest rate that firms can pay to investors depends on a couple of factors, some real (market size) and some financial (the managers' incentive-compatible compensation). A region, in which firms can pay higher interest on capital will attract more capital, such that the industrial sector in the other region will shrink. In this type of setting, a NEG model with financial market friction, an endogenous agglomeration mechanism becomes possible.

Two main forces determine agglomeration. First, if a region already has more firms, it will be easier for new firms to sell their products in the less industrialized region. This is centrifugal force, also referred to as market-crowding effect, tends to lead to evenly distributed economic activities. But second, managers are incentivized by some compensation package. If a region already has more firms, the price index is relatively low. Hence, the same nominal compensation as in the other region has a higher value to the manager. If a firm needs to spend less on manager compensation, it can pledge higher interest on capital to investors. This leads to an attraction of more capital to a region, where already many firms are invested. This type of agglomeration mechanism implies external scale economies, which are well known in the NEG literature – but not in terms of a financial market.

The relative size of these two competing forces depends on trade costs for goods between regions. The centripetal force tends to be stronger for high trade costs. Hence, in this case there exists only a symmetric equilibrium where industrial activity is evenly spread across both regions. But for sufficiently low trade costs, centripetal forces dominate. In consequence, agglomeration becomes a stable equilibrium. Industrial activity concentrates within an industrial core, whereas the other region becomes a periphery.

The rest of the paper is organized as follows. In the remainder of this section, we give a brief overview over the relevant NEG literature and the literature on finance and growth,

both theoretical and empirical. In Section 2, we introduce a simple theoretical model, which differs from standard NEG models only in terms of a financial market with a friction. In Section 3, we characterize equilibria and their stability. Section 4 discusses the role of financial institutions. Here, we show that the quality of financial institutions is crucial especially near the point where centrifugal forces start to dominate. In Section 5, we consider a moral-hazard problem that grows with the firm size. This modification has some important implications for the structure of equilibria. Finally, Section 6 draws conclusions for economic policy and future research directions. Proofs are in the Appendix.

**Literature Review.** There are two important strands of literature to which our paper connects.

*First*, our paper is an application of approaches of the New Economic Geography (NEG). Starting from the seminal model of Krugman (1991), the NEG literature has identified a number of mechanisms that imply endogenous asymmetry among countries and regions: (i) factor mobility, (ii) inter-industry trade, and (iii) spillover effects. As one remarkable result, self-reinforcing mechanisms (cumulative causation) can lead to a core-periphery formation of initially symmetric locations. Depending on the model structure, the bifurcation structure follows two types: either a tomahawk or a pitchfork.<sup>1</sup> The tomahawk diagram features a symmetric equilibrium for high trade costs and a core-periphery equilibrium for low trade costs. At an intermediate level, both equilibrium formation are possible. Nonetheless, after passing a critical trade cost level (the break point), the symmetry is broken such that the symmetric equilibrium suddenly becomes unstable and agglomeration is the only remaining state. In contrast, the pitchfork bifurcation exhibits a smooth path rather than a catastrophic break, from the symmetric equilibrium to the core-periphery formation. The present paper directly refers to the standard model of Krugman (1991) and implements a capital input incorporating a moral-hazard problem between entrepreneurs (managers) and investors. Although an integration of capital in NEG models is not a new topic, a more extensive analysis and micro-foundation is still a neglected field. A broad survey on recent NEG models give Baldwin, Forslid, Martin, Ottaviano, and Robert-Nicoud (2003). The earliest contribution including (public) capital into a economic geography context is the *footloose-capital* model by Martin and Rogers (1995). The two-country economy consists of firms and households, whereas households supply labor and capital to firms. Here, and also in our model, the capital endowment directly determines the firm number since each firm requires one unit of capital for production. Capital is internationally mobile responding to differences in the nominal profits of firms. As a result, the spatial formation of the manufacturing industry is simply symmetric for all trade costs. This outcome allows the conclusion that in such an economy without capital-market frictions endogenous asymmetry is not possible – in contrast to our results. Later on, Robert-Nicoud (2006) extends the *footloose-capital* model by vertical industrial linkages. Not surprisingly, this version features a tomahawk bifurcation as in Krugman and Venables (1995). In a growth environment, Baldwin (1999) set up the *constructed-capital* model. The model differs from the *footloose-capital* model since also capital is immobile, it depreciates but it can also be constructed using labor as an input. As a characteristic outcome, the *constructed-capital* model exhibits a quite simple bifurcation. At a critical

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<sup>1</sup> See Fujita, Krugman, and Venables (1999), Chapter 3, or Pflueger and Suedekum (2010) for a discussion.

trade cost level, the initially stable symmetric equilibrium becomes instantaneously unstable, while the core-periphery formation becomes the only stable outcome at this point.

*Second*, the paper quite naturally attaches to the literature on finance and growth. Lucas (1990) was the first to observe that capital can flow “uphill,” from poorer to richer regions. Boyd and Smith (1997) and Hakenes and Schnabel (2010) argue theoretically that two heterogeneous regions may diverge even more due to capital market frictions. The problem of capital flowing uphill has also attracted political attention, see Economist (2006) on the Jackson Hole Symposium in 2006, or the according speech by Rajan (2006), based on the paper by Prasad, Rajan, and Subramanian (2007). The major part of the literature, however, consists of empirical work, starting with the seminal paper by King and Levine (1993). Rajan and Zingales (1998) carry out an empirical study on a firm level, finding that firms that depend more on external finance grow faster in countries with better financial institutions. Also Levine, Loayza, and Beck (2000) and Levine (2001) discuss the nexus between the quality of financial institutions and growth. Alfaro, Kalemli-Ozcan, and Volosovych (2008) explicitly address the question why capital does not always flow downhill. Guiso, Sapienza, and Zingales (2004) examine the role of financial development on growth. Klein and Olivei (2008) and Klein (2005) ask the question how financial deepness and the quality of financial institutions affect economic growth.

## 2 The Model

**Consumers.** Consider an economy with two regions. In each region, there are three types of agents, *workers* of mass  $L = 1$ , *managers*, and *investors* endowed with a total of  $n$  units of capital per region. All agents have the same Cobb-Douglas utility function,  $U = M^\mu A^{1-\mu}$ , where  $A$  stands for the consumption of an agricultural product, and  $M$  is the (CES) subutility derived from the consumption of manufactured products according to Dixit and Stiglitz (1977).  $A$  will also be used as an index for all quantities that belong to the agricultural sector; for the manufacturing sector, we omit the index. For a variety of  $n$  available varieties of manufactured goods, define

$$M = \left[ \int_0^n q(i)^\rho di \right]^{1/\rho}. \quad (1)$$

We set  $\sigma \equiv 1/(1-\rho)$  as the elasticity of substitution between any two varieties. Following the procedure as suggested by Fujita, Krugman, and Venables (1999), the demand for the agricultural good can be derived as

$$A = (1 - \mu) Y / p^A. \quad (2)$$

The demand for a particular variety  $j$  of continuum of manufacturing goods is

$$q(j) = \mu Y p(j)^{-\sigma} G^{\sigma-1}, \quad (3)$$

where  $G$  denotes the price index defined as

$$G = \left[ \int_0^n p(i)^{1-\sigma} di \right]^{1/(1-\sigma)}. \quad (4)$$

The price index represents the minimum cost of purchasing one unit of the composite index  $M$  of manufacturing goods. Finally, the household utility can be derived as

$$\begin{aligned} U &= \mu^\mu (1 - \mu)^{1-\mu} Y G^{-\mu} p^A^{-(1-\mu)} \\ &= \mu^\mu (1 - \mu)^{1-\mu} Y G^{-\mu} \end{aligned} \quad (5)$$

if use the price of agricultural goods as the numeraire and normalize  $p^A \equiv 1$ .

**Multiple Locations and Trade Costs.** In order to move manufactured goods from one region to another, one needs to incur trade costs. We assume an iceberg form of trade costs. Hence, if one unit of manufactures is transported, only a fraction  $1/T$  arrives at the destination. As a consequence,

$$p_{12} = T p_1 \quad \text{and} \quad p_{21} = T p_2 \quad (6)$$

where  $p_1$  is the price in region 1 of a good manufactured in region 1,  $p_2$  is the price in region 2 of a good manufactured in region 2,  $p_{12}$  is the price in region 2 of a good manufactured in region 1, and  $p_{21}$  is the price in region 1 of a good manufactured in region 2. Imported goods are more expensive in the importing country. The price index in the regions then adjusts to

$$\begin{aligned} G_1 &= \left[ n_1 (p_1)^{1-\sigma} + n_2 (T p_2)^{1-\sigma} \right]^{1/(1-\sigma)}, \\ G_2 &= \left[ n_1 (T p_1)^{1-\sigma} + n_2 (p_2)^{1-\sigma} \right]^{1/(1-\sigma)}. \end{aligned} \quad (7)$$

Aggregate consumption demand for a variety produced in region 1 and region 2, respectively is:

$$\begin{aligned} q_1 &= \mu (Y_1 p_1^{-\sigma} G_1^{\sigma-1} + Y_2 p_1^{-\sigma} G_2^{\sigma-1} T^{1-\sigma}), \\ q_2 &= \mu (Y_1 p_2^{-\sigma} G_1^{\sigma-1} T^{1-\sigma} + Y_2 p_2^{-\sigma} G_2^{\sigma-1}). \end{aligned} \quad (8)$$

Finally, as a main assumption, labor is perfectly mobile between sectors but not between locations. Thus, we do not allow for agglomeration based on migration as in the seminal model of Krugman (1991). Furthermore, also investors do not move between locations; nevertheless, they are in position to invest abroad implying an international capital market.

**Producer Behavior.** Let us now turn to the production side of the economy. Each region is endowed with two sectors: a traditional and a manufacturing sector. The agricultural sector is perfectly competitive and produces under a linear constant-return technology a homogenous good which serves as a numeraire, hence  $p^A = 1$ . Agricultural production does not use capital. Since the sector is perfectly competitive, the wage rate  $w^A$  in this sector is also equal to 1.

The monopolistic competitive manufacturing sector produces under increasing returns a continuum of differentiated goods. Because of consumer's love of variety and increasing returns, each manufacturing firm produces only one variety such that the number of varieties is equal to the number of firms. The technology is the same for all locations

and varieties and involves a fixed unit of *capital* and a marginal input  $c$  of labor per unit produced, hence:<sup>2</sup>

$$l_i = c q_i. \quad (9)$$

Thus, the profit function of a manufacturing firm is:

$$\pi_i = p_i q_i - w c q_i - \kappa_i, \quad (10)$$

where  $q_i$  is given by the demand function (8). Furthermore,  $\kappa$  is the cost of capital, payable from the firm to those investors who provide capital to the firm.

Each firm takes the price index as given. The perceived elasticity of demand is  $\sigma$ , hence each firm will optimally set

$$p_i = c w / \rho = \sigma c w / (\sigma - 1), \quad (11)$$

which implies monopolistic mark-up pricing on top marginal costs. Let us normalize  $c := (\sigma - 1) / \sigma$ . Further on, since wages in the agricultural sector are  $w^A = 1$ , and labor is mobile between agriculture and manufacturing, wages equalize in both sectors, hence  $w = 1$ . Thus, we obtain

$$p_i = p = 1. \quad (12)$$

Substituting into the profit function (10) yields

$$\pi_i = q_i (1 - c) - \kappa_i = \frac{q_i}{\sigma} - \kappa_i. \quad (13)$$

**Moral Hazard.** Each firm is run by a manager who is subject to a moral hazard problem. The manager can either behave, in which case the success probability of the firm is 1. Otherwise, the manager can shirk and receive a private, non-monetary benefit  $B$ . Shirking reduces the probability that the firm makes a profit to  $\theta$ . Hence, if the manager shirks, the labor input  $l = c q$  of the firm is simply wasted.<sup>3</sup> The cost of the effort can be interpreted as the opportunity cost of having less time to spend at home with the family. If the manager behaves, she/he generates a profit  $\pi$  as in (10). From (5), we know that the utility of a unit of income is  $\mu^\mu (1 - \mu)^{1-\mu} G^{-\mu}$ . Hence, the utility of a manager who behaves and works is:

$$U = \mu^\mu (1 - \mu)^{1-\mu} G^{-\mu} \pi. \quad (14)$$

If the manager shirks and does not spend any effort, he generates the profit  $\pi$  only with probability  $\theta$ , but he does not have any private costs. His expected utility is then  $U' =$

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<sup>2</sup>In order to obtain a finite number of firms in equilibrium, Fujita, Krugman, and Venables (1999) have fixed production costs, and  $l = F + c q$ . In our model, capital plays the role of fixed costs, limiting the number of firms.

<sup>3</sup>This modeling choice resembles the fixed input version of Holmström and Tirole (1997), as presented in (Tirole, 2006, chapter 3.4).

$\mu^\mu (1 - \mu)^{1-\mu} G^{-\mu} \theta \pi + B$ . Hence, the manager will behave only if:

$$\begin{aligned} \mu^\mu (1 - \mu)^{1-\mu} G^{-\mu} \pi &\geq \mu^\mu (1 - \mu)^{1-\mu} G^{-\mu} \theta \pi + B, \\ \frac{q}{\sigma} - \kappa &\geq \frac{G^\mu B}{(1 - \theta) \mu^\mu (1 - \mu)^{1-\mu}}, \\ \kappa &\leq \frac{q}{\sigma} - G^\mu b, \end{aligned} \quad (15)$$

where we have defined  $b := B/((1 - \theta) \mu^\mu (1 - \mu)^{1-\mu})$  as an auxiliary variable, measuring the degree of the moral hazard problem. Based on equation (15), we can set up the location specific capital rates:

$$\kappa_1 = \frac{q_1}{\sigma} - G_1^\mu b \quad \text{and} \quad \kappa_2 = \frac{q_2}{\sigma} - G_2^\mu b. \quad (16)$$

Substituting these capital rates given into firm profits (10), we get:

$$\pi_1 = b G_1^\mu \quad \text{and} \quad \pi_2 = b G_2^\mu. \quad (17)$$

Note that the utility of managers according to equation (14) is constant at:  $\mu^\mu (1 - \mu)^{1-\mu} b$  and, thus, independent from the degree of trade integration.

**The Capital Market.** Investors invest in the region featuring a higher capital rate. Without loss of generality, consider the case that capital flows from region 2 into region 1, hence that  $n_1 \geq n_2$ . As a consequence, firm in region 1 pay part of their cost of capital to investors in region 2. Thus, aggregate regional income amounts to:

$$\begin{aligned} Y_1 &= L_1^A w_1^A + L_1^M w_1^M + n_1 \pi_1 + n \kappa_1 \\ &= (1 - L_1^M) 1 + L_1^M 1 + n_1 \pi_1 + n \kappa_1 \\ &= 1 + n_1 \pi_1 + n \kappa_1, \quad \text{and} \\ Y_2 &= 1 + n_2 \pi_2 + n_2 \kappa_2 + (n - n_2) \kappa_1. \end{aligned} \quad (18)$$

For  $n_2 > n_1$ , the equations would have to be mirrored.

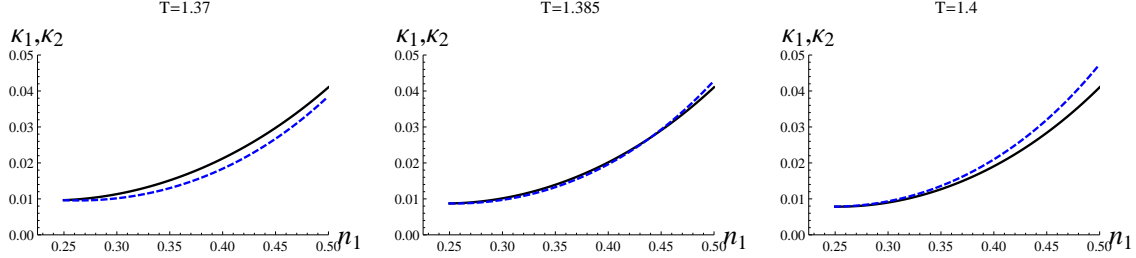
### 3 Equilibrium

The variables  $\kappa_1$  and  $\kappa_2$  measure the capital rates that can be paid in both regions. If  $\kappa_1 > \kappa_2$ , investors can benefit from moving their capital from region 2 into region 1, and vice versa for  $\kappa_2 > \kappa_1$ . Consequently, there are several kinds of equilibrium. If  $\kappa_1 = \kappa_2$  for an  $\delta_n < n$ , capital rates are equal in both regions, and capital stops flowing into region 1. The economy is in an (inner) equilibrium. If, at this point, the (solid)  $\kappa_1$  function is steeper than the (dashed)  $\kappa_2$  function, the equilibrium is instable. If  $n_1$  increases slightly from the equilibrium point, the capital rate in region 1 will exceed that in region 2, hence region 1 will attract even more capital, and the number of firms will grow, away from the initial equilibrium point. If, at an inner equilibrium point, the (solid)  $\kappa_1$  function is *shallower* than the (dashed)  $\kappa_2$  function, the equilibrium is stable. If  $n_1$  increases slightly from the equilibrium point, the capital rate in region 1 will fall short of



that in region 2, hence capital will move *from* region 1, and the number of firms will drop, back to the initial equilibrium point. Note that the point  $\delta_n = 0$ , hence  $n_1 = n_2 = n$ , is always an inner equilibrium point, either stable or unstable. Finally, there can be a corner equilibrium. If  $\delta_n = n$ , hence  $n_1 = 2n$  and  $n_2 = 0$ , the economy is in equilibrium if  $\kappa_1 > \kappa_2$ . The same is true if  $n_1 = 0$  and  $\kappa_2 > \kappa_1$ .

Figure 1: Cost of Capital



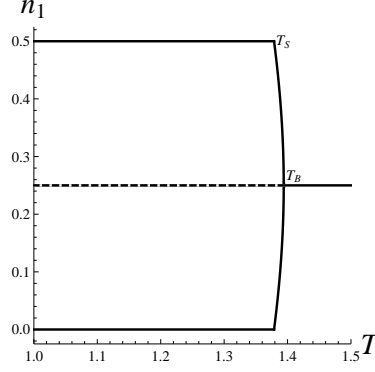
Parameters are  $n = 0.25$ ,  $b = 0.5$ ,  $\sigma = 3$ , and  $\mu = 0.4$ . The capital rate  $\kappa_1$  in region 1 is solid, that in region 2  $\kappa_2$  is dashed. The argument is  $n_1$ , and consequently  $n_2 = 2n - n_1$ . Hence for  $n_1 = n = 0.25$ , the number of firms in the regions is identical. For  $n_1 = 2n = 0.5$ , region 1 has attracted all capital, and region 2 is void of capital (and firms). We consider the case  $n_2 \leq n$  and thus  $n_1 \geq n_2$  without loss of generality. For the opposite case, the pictures would have to be mirrored.

The pictures in Figure 1 show capital rates in the two regions (solid for  $\kappa_1$ , dashed for  $\kappa_2$ ), depending on the number of firms in the regions. In the left picture, trade costs are relatively low. There is an unstable equilibrium at  $n_1 = n$ . At  $n_1 = n$ , capital rates are equal. But increasing  $n_1$  only slightly implies higher capital rates  $\kappa_1$ . Region 1 will start to attract even more capital, moving the economies away from the equilibrium. Another (stable) corner equilibrium is at  $n_1 = 2n$ . At this point, and in the neighborhood,  $\kappa_1 > \kappa_2$ . There is no reason for capital to move back into region 2. On the contrary, region 1 would be able to attract more capital, if there were any capital left in region 2 to be attracted. This equilibrium has an industrial core (region 1) and a periphery (region 2).

In the middle picture, trade costs are intermediate. There is still an unstable equilibrium at  $n_1 = n$ . Instead of the corner equilibrium, there is an inner equilibrium at  $n_1 \approx 0.44$ . At this point,  $\kappa_1 = \kappa_2$ . Increasing  $n_1$  from this point,  $\kappa_2 > \kappa_1$ , hence capital will move back into region 2. Reducing  $n_1$  from this point, the opposite will happen: the equilibrium is stable. In the right picture, trade costs are relatively high. There is now only a stable (symmetric) equilibrium at  $n_1 = n_2 = n$ . Capital always moves into the less industrialized region.

Plotting the stable (solid) and unstable (dashed) equilibria as a function of  $n_1$ , and now allowing  $n_1 \in [0; 2n]$ , we get a pitchfork diagram (Figure 2). Again, we see that the only equilibrium is symmetric if trade costs are high. For low trade costs, the only stable equilibria exhibit a core-periphery structure. Because the regions are initially symmetric, both can end up as the industrial core, or as the agricultural periphery. Supposing that trade costs  $T$  fall over time due to globalization, we get the following pseudo-dynamics. For very high trade costs, there is only the stable symmetric equilibrium. The manufacturing sectors of both regions have the same size, and both regions export some of their manufactures. As trade costs fall, the bifurcation point (break point, here at  $T_B \approx 1.390$ ) is

Figure 2: Core-Periphery Bifurcation



reached. The symmetric equilibrium becomes unstable, hence minor distortions entail an asymmetric equilibrium with a larger manufacturing sector in one region (say, region 1), and a larger agricultural sector in the other (say, region 2).<sup>4</sup> As a prerequisite, investors from region 2 put some of their capital into firms in region 1. In equilibrium, the rate on capital is the same in both regions. On the one hand, region 1 has more firms, hence the price index  $G_1$  is *smaller*, and managers can easier be incentivized not to shirk. Thus, the moral hazard problem is smaller. On the other hand, more firms imply a smaller firm size  $q_1$ , which leads to a smaller capacity to pay interest on capital. In the equilibrium both effects cancel out each other.

For  $T$  just below the break point,  $n_1$  will only slightly outnumber  $n_2$ . But as trade costs drop further, the stable equilibria become more and more asymmetric. At a certain level of trade costs, the sustain point, one of the regions (say, region 2) will be void of capital (here at  $T_S \approx 1.378$ ). Then, all manufacturing goods are produces in region 1. Region 2 imports all manufactures, and exports agricultural goods.

**Forces at Work.** In order to identify the prevalent centripetal and centrifugal forces, we derive the total differential of the equation system (7), (8), (16), and (18) in the symmetric equilibrium at  $n_1 = n$ . As a result, we obtain

$$\frac{dG}{G} = \left( \frac{Z}{1 - \sigma} \right) \frac{dn}{n}, \quad (19)$$

$$dq = \mu G^{\sigma-1} (1 - T^{1-\sigma}) [dY + (\sigma - 1) Y] \frac{dG}{G}, \quad (20)$$

$$dY = b G^{-\mu} \left( dn - \mu n \frac{dG}{G} \right) + n d\kappa, \quad (21)$$

$$d\kappa = \frac{dq}{\sigma} - \mu b G^{\mu} \frac{dG}{G}. \quad (22)$$

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<sup>4</sup>Note that, at the break point  $T_B$ , it is undetermined which region becomes the industrial core, and which region will produce the food.

where the price index in the symmetric equilibrium is  $G = [n(1 + T^{1-\sigma})]^{1/(1-\sigma)}$ , the corresponding income is  $Y = \sigma/(\sigma - \mu)$ , and

$$Z = \frac{1 - T^{1-\sigma}}{1 + T^{1-\sigma}} \quad (23)$$

is a renormalization of  $T \in [1; \infty)$  on the interval  $[0; 1]$ , thus  $T = 1 \Leftrightarrow Z = 0$  and  $T = \infty \Leftrightarrow Z = 1$ .

Equation (19) reflects the price-index effect: an increasing number of firms reduces the price index. The second equation (20) illustrates the market-crowding or business-stealing effect. The higher is the firm number (and the lower the price index) the smaller is the firm size as well as firm profits. The third equation (21) provides a information about the home-market effect: the higher is the number of firms, which corresponds to the size of the manufacturing industry, the higher is the local market size in terms of income. Finally, equation (22) demonstrates the moral-hazard effect: the more firms are in the market the lower is the real benefit of moral hazard due to a lower price index, and thus, the lower is the capital rate.

**The Break Point.** Figure 2 captures the consequences of product market globalization on the regional economies, and the figure is most prominently characterized by two statistics, the break point  $T_B$  at which the symmetric equilibrium becomes unstable, and the sustain point  $T_S$  at which the corner equilibrium starts to exist. As a consequence, we are extremely interested in the determinants and properties of the break and sustain point. Luckily, we can give implicit equations for both points.

We determine the break point following the same approach as Fujita, Krugman, and Venables (1999). As illustrated in the wiggle diagrams in Figure 1, the break point occurs for a value of  $T$  where the slopes of the capital-rate curves in the symmetric equilibrium are equalized. Formally, at the break point holds:  $d\kappa_1/dn_1 = d\kappa_2/dn_1$  in the symmetric equilibrium:  $n_1 = n$ . Substitution of equations (19), (20), and (21) into (22) and solving for  $d\kappa/dn$  yields the first derivative of the capital-rate differential in the symmetric equilibrium with respect to the firm number in region 1. This derivative becomes zero at the break-point level of trade costs, given by an implicit equation,

$$bG^\mu \left( \frac{1 - 2\sigma + \mu Z}{1 - \sigma} \right) - \frac{YZ}{n} = 0. \quad (24)$$

Remember that both  $G$  and  $Z$  implicitly depend on  $T_B$ .

Now that we have derived an implicit equation for the break point, we can ask how the point depends on exogenous parameters, most notably the moral hazard parameter  $b$ . From equation (24), one would guess that, as  $b$  tends to zero,  $YZ/n$  must also go to zero, hence  $Z \rightarrow 0$ , hence  $T_B \rightarrow 1$ . This would also match the intuition that, as the capital market friction becomes less distinctive, the point where it starts to for firm location choice (the break point) moves towards extreme degrees of economic integration ( $T \approx 1$ ). This is made precise in the following proposition.

**Proposition 1 (Impact of the Capital Market Friction)**

*Assume that  $\sigma \geq (9 - \sqrt{17})/4 \approx 1.22$ . Then*

- $dT_B/db > 0$ , a larger moral hazard includes a break point at lower levels of economic integration (higher  $T$ ), and
- $\lim_{b \rightarrow 0} T_B = 1$ , in the limit for vanishing moral hazard, the break point moves toward extreme levels of integration (no trade costs).

Note that the two propositions imply that, even for very small moral hazard problems, the pitchfork structure of equilibrium capital allocations never disappears. The bifurcation only moves to the left (smaller  $T$ ).

Another interesting question is how the equilibrium allocation reacts after a meltdown in capital, for example after an economic crisis. Remember that  $n$  is the aggregate amount of capital per region, and as a consequence the average number of firms per region. Hence, after a contraction in capital, does the phenomenon of agglomeration becomes more pronounced (higher  $T_B$ ), or less so?

**Proposition 2 (Impact of the Amount of Capital)**

Again, assume that  $\sigma \geq (9 - \sqrt{17})/4 \approx 1.22$ . Then  $dT_B/dn > 0$ .

This proposition implies that an increase in the capital stock leads to agglomeration already at a higher level of trade costs. For an economy that is already in a core-periphery equilibrium, a larger capital stock is irrelevant. The situation changes, if the economy faces a decrease in the capital stock, e.g., as a result of an economic crisis, war etc.. In this case, the concentration may be reversed.

**The Sustain Point.** In a pitchfork bifurcation the sustain point indicates the trade cost level, where the interior asymmetric equilibria meet the corner solutions, in which the whole manufacturing sector is located in the core region. In terms of stability, at this point the stable core-periphery formation becomes unstable with increasing trade costs. In order to determine this threshold it is necessary to evaluate the capital rates with the core-periphery values. Note that the capital of the periphery (say of region 2) has to be considered as a hypothetical capital rate since no capital is invested in that location.

At the sustain point  $T_S$ ,

$$\frac{\mu}{\sigma - \mu} \left[ 1 - \left( \frac{T^{\sigma-1} + T^{1-\sigma}}{2} \right) \right] + n b G_1^\mu \left[ \frac{\mu}{\sigma} \left( \frac{T^{\sigma-1} - T^{1-\sigma}}{2} \right) + T^\mu - 1 \right] = 0. \quad (25)$$

**Some Parameter Restrictions.** In principle, the algebraic solution of the equilibrium conditions can yield negative costs of capital,  $\kappa < 0$ . This, of course, cannot be an equilibrium outcome; investors would then prefer not to lend their capital at all. This becomes apparent at equations (13) and (15). The moral hazard problem escalates such that no positive interest rate can ensure that the incentive compatibility condition is fulfilled. The

according condition can be derived from (16) evaluating with infinite trade costs at the symmetric equilibrium:

$$b \leq \left( \frac{\mu}{\sigma - \mu} \right) n^{\frac{1-\sigma+\mu}{\sigma-1}}. \quad (26)$$

Furthermore, in order to ensure that we do not obtain a corner solution as the being the only stable equilibrium for the whole domain of trade costs, we derive the corresponding equivalent of the no-black-hole condition of Fujita, Krugman, and Venables (1999). This condition can be directly derived from equation (24) evaluated for infinite trade costs ( $Z = 1$ ):

$$b < \left( \frac{\mu}{\sigma - \mu} \right) \left( \frac{1 - \sigma}{1 - 2\sigma + \mu} \right) n^{\frac{1-\sigma+\mu}{\sigma-1}}. \quad (27)$$

Since the upper bound of equation (27) is higher than of equation (26), only the non-negativity condition for the capital rates binds.

**Welfare.** Welfare in region 1 is defined as

$$W_1 = \mu^\mu (1 - \mu)^{1-\mu} G_1^{-\mu} Y_1, \quad (28)$$

where  $Y_1$  again consists of wages, firm profits and investor's rents.  $W_2$  is defined analogously. Consider first the symmetric equilibrium. With falling trade costs  $T$ , aggregate nominal wages are constant. Domestic prices are constant, but the prices of imports fall, hence the price index  $G_1$  decreases. This implies that it becomes cheaper to incentivize managers. As a consequence, firm profits drop with proceeding globalization, and gains from capital increase.<sup>5</sup> However, because payments to investors are a mere redistribution of goods, the sum of firm profits and capital gains is constant. Consequently,  $Y_1$  does not depend on trade costs. But because  $G_1$  falls as  $T$  decreases, globalization increases welfare in the symmetric equilibrium. This is visible in the middle curve of Figure 3 (with  $n_1 = n$ ).

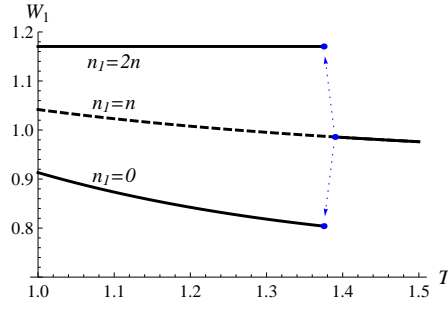
Now, what are the welfare effects of moving to an asymmetric equilibrium? The region that attracts capital (say, region 1) profits in several ways. First, the number of manufactures in region 1 increases, leading to a decreasing price index  $G_1$  (and an increasing  $G_2$ ). Consequently, managers in region 1 can pay a higher capital rate  $\kappa_1$ , implying decreasing firm profits  $\Pi_1$ , but this cancels out with higher gains from capital in region 1. A third effect is important. Because there are more firms in region 1, the aggregate amount of firm profits increases. As a result, welfare in region 1 increases. As soon as the corner equilibrium is reached, welfare does not longer depend on  $T$ . The reason is straightforward: since all manufactures are already produced in region 1 the price index independent from  $T$ . This can be seen in the upper curve in Figure 3 (with  $n_1 = 2n$ ).

Region 2 suffers from capital moving away into region 1. At least, as trade costs decrease further, welfare in region 2 increases. All manufactures are imported from region 1, so the

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<sup>5</sup>Note that, in our model, the reason for falling profits is not increased international competition.

Figure 3: Welfare Levels



price in region 2 falls as trade costs decrease. This is visible in the lower curve in Figure 3 (with  $n_1 = 0$ ).

Summing up, this model features agglomeration winners and losers. Comparing the utility of workers, managers and entrepreneurs in both equilibrium states, symmetry and agglomeration, we can derive the following results:

**Remark 1 (Utilities of factor groups)** *For all trade costs,  $T > 1$ ,*

- *the utility of workers in the core region is higher than of workers in the symmetric equilibrium. The utility of workers in the symmetric equilibrium is higher than of workers in the periphery.*
- *the utility of managers is independent from trade costs and, thus, the same in agglomeration and symmetric equilibrium.*
- *the utility of investors in the core is higher than of investors in the periphery and in the symmetric equilibrium.*

The utility ratio of investors in the periphery compared to investors in the symmetry is ambiguous. The reason is straightforward. On one hand, investors in the periphery receive the capital rate of the core region, which is higher than of the symmetric equilibrium. On the other hand, the price index, and thus, the real value of capital returns are lower. A critical trade cost value, at which one effect compensates the other solves:

$$(1 + T^{1-\sigma})^{\frac{\mu}{\sigma-1}} + 2^{\frac{\mu}{\sigma-1}} T^{-\mu} - b(1 - T^{-\mu}) n^{\frac{1-\sigma+\mu}{1-\sigma}} \left( \frac{\sigma - \mu}{\mu} \right) = 0. \quad (29)$$

## 4 Financial Institutions

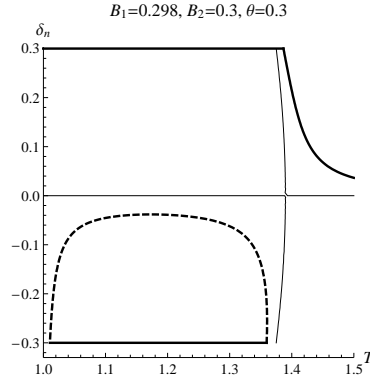
In Section 3, regions were assumed to have identical characteristics. Especially, the moral hazard problem (as characterized by  $B$  and  $\theta$ ) was the same in both regions. However, financial institutions can be seen as a means to overcome informational frictions. For

example, banks might monitor borrowers, such that the scope for shirking shrinks. The same applies for venture capital funds or general investment funds who have some control over the firms under management. Third, also public institutions, like development banks, will try and allocate capital in an efficient way and monitor recipients.

Holmström and Tirole (1997) model monitors as institutions who can reduce the benefit of shirking from some level  $B$  to a smaller  $b < B$ , at some cost. Within the framework of our model, let us assume that investors themselves have the ability to reduce the benefit.<sup>6</sup> In order to see the effects of different degree of financial development, assume that in region 1,  $B_1$  can be reduced, whereas in region 2, it cannot.

The following Figure 4 shows the stable and unstable equilibria for  $B_1 = 0.298$  and  $B_2 = 0.300$ . Monitoring is thus slightly more efficient in region 1, but only by 0.67%. Remember that in Figure 2, we had  $B_1 = B_2 = B = 0.300$ . In that numerical example, starting with high trade costs  $T$ , regions were exactly symmetric, and at the break point at  $T_B \approx 1.390$ , one of the regions started to attract more capital. However, it was undetermined which of the regions would be the industrial core. Now in Figure 4, although the difference in  $B$  is only small, as  $T$  decreases and approaches the critical value  $T_B \approx 1.390$ , region 1 already attracts more and more capital. Region 2, at the same time, is emptied of firms. For smaller  $T$ , there is another stable equilibrium where region 2 is the industrial core. But this equilibrium is never reached along the equilibrium path.

Figure 4: Core-Periphery Bifurcation



This numerical exercise illustrates one important implication. If the economy is far from the break point  $T_B$ , the magnitude of the friction does not matter much. For  $T < T_B$ , a slightly lower market friction  $B$  just implies that wealth is shifted from firms to investors, but the regions remain in the same branch of the set of equilibria. For  $T \gg T_B$ , a region with a marginally smaller friction can attract marginally more capital and have marginally more firms. But for  $T \approx T_B$ , there are two reasons why the size of the friction becomes important. First, as  $T$  approaches  $T_B$ , the number of firms in each region reacts extremely sensitively to small differences in the friction  $B$ . Second, and more importantly, at  $T \approx T_B$  the regions end up on the upper or lower branch of the equilibrium set. The

<sup>6</sup>Otherwise, monitoring would have to be delegated. But the delegation of monitoring comes at zero delegation costs if the number of monitored firms is large, see Diamond (1984). In our model, there is a continuum of firms.

discussion on welfare (see Figure 3) shows that the welfare consequences for each region will be substantial.

**Remark 2** *The quality of financial institutions is decisive especially around the bifurcation point  $\bar{T}$ .*

To be concrete, consider a process of increasing globalization, and assume that  $B_2 < B_1$  (but with  $B_1 \approx B_2$ ); region 2 has the better financial system. But assume that just around  $T \approx T_B$ , region 1 gets a slight advantage,  $B_1 < B_2$ . Then the advantage of region 1 disappears again. Nevertheless, region 1 will end up as the industrial core in this thought experiment.

In the above discussion, regions were asymmetric only in one aspect; region 1 had a smaller moral hazard problem. However, regions can differ in many more parameters. The size may differ ( $L_1 \neq L_2$ ), consumer preferences may differ ( $\mu_1 \neq \mu_2$  and/or  $\sigma_1 \neq \sigma_2$ ), trade costs may be asymmetric, and most prominently, production technologies may differ. For example, firms in one region may be able to produce with less capital, or at lower marginal costs  $c$ . For concreteness, assume that  $c_1 < c_2$ . Such an asymmetry entails equilibrium paths that look similar to Figure 4. One will want to know that the relative importance in the different dimensions of asymmetries is. For example, if region 1 has a cost advantage over region 2 by  $x\%$ , but region 2 has lower financial frictions (lower benefit  $B$  by  $y\%$ ), which region will then be able to become the industrial core when trade costs approach the critical break point? The following remark states that, for small  $B$ , the technical dimension ( $c$ ) and the financial dimension ( $B$ ) have exactly the same importance. And  $x$  and  $y$  of the same size lead to identical core-periphery diagrams with identical equilibrium paths. Capital market frictions are just as decisive for the question of agglomeration as productivity advantages.

**Remark 3** *For small capital market frictions  $B$ , the reduction of production costs  $c$  in one region by a factor has the same effect on the equilibrium allocation of capital as a reduction of  $B$  in that region by the same factor.*

## 5 Larger Moral Hazard for Larger Firm Size

Up to now, we have assumed that the private benefit a manager receives from shirking was fixed at  $B$ . Just as well, one could argue that the benefit increases as the firm size (proportional to the output  $q$ ) increases, hence that the benefit is  $qB$ .<sup>7</sup> For example, the manager of a bigger firm might have to work harder/more to make the firm successful; the opportunity costs of hard work are hence larger for big firms. Summing up, instead of (15), we have

$$\begin{aligned} \mu^\mu (1 - \mu)^{1-\mu} G^{-\mu} \pi &\geq \mu^\mu (1 - \mu)^{1-\mu} G^{-\mu} \theta \pi + qB \\ \kappa &\leq \frac{q}{\sigma} - q G^\mu b \end{aligned} \tag{30}$$

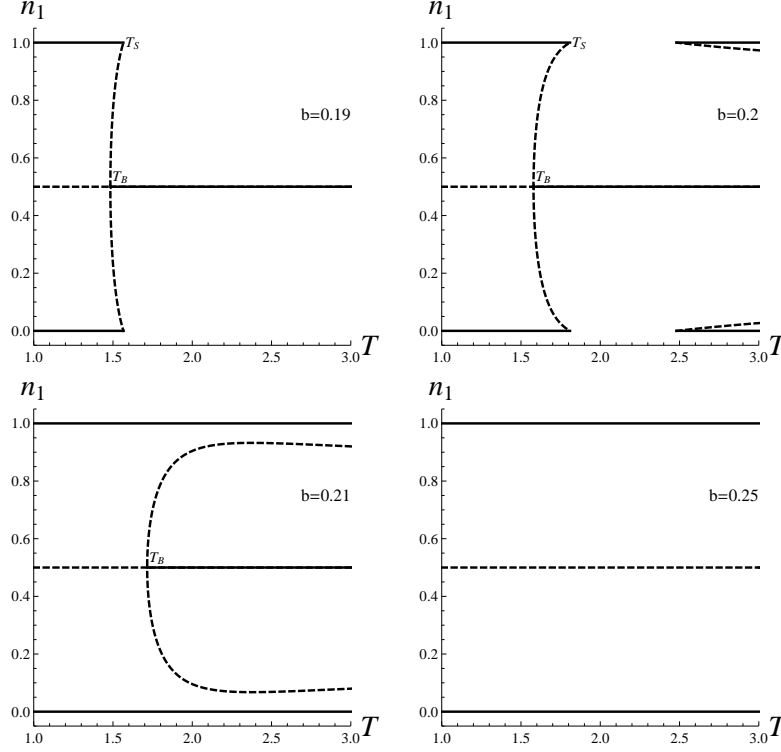
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<sup>7</sup>This modeling choice resembles the variable input version of Holmström and Tirole (1997), as presented in (Tirole, 2006, chapter 3.2).



with  $b$  defined as above. The size of the moral hazard problem depends on  $b$  and  $q$ . The larger  $b$ , the more beneficial is shirking for the manager, and the more costly it is to incentivize the manager to behave. The same applies for  $q$ . In the following figure, we plot the bifurcation diagrams for different values of  $b$ .

Figure 5: Core-Periphery Bifurcation

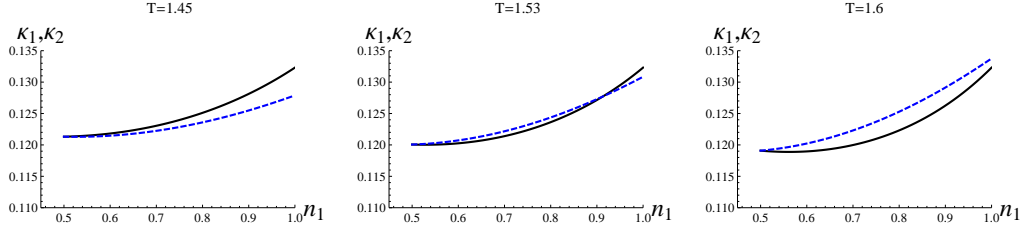


In Section 3, the set of equilibrium allocations always had the form of a pitchfork. Here, the forms differ depending on the size of the capital market friction. For small frictions, the diagram exhibits the shape of a tomahawk (upper left picture with  $b = 0.19$ ). This slightly different shape has consequences for the adjustment dynamics for falling trade costs. Here, as trade costs fall short of  $T_B$ , there would be a sudden flow of capital out of one region (which then becomes the periphery) into the industrial core. Then, if trade costs were to increase again, regions would remain in these corner equilibria until trade costs reach the sustain point  $T_S$ . Above this point, there would be another abrupt flow of capital. In other words, the dynamics of capital adjustments here is hysteretic, where in Section 3 it was not.

For this first numerical example, let us look at a couple of wiggle diagrams to see which of the equilibria are stable. For *low*  $T$  (*left* picture), the diagram looks similar to that in Figure 1;  $\kappa_1 = \kappa_2$  for  $n_1 = n = n_2$ , and  $\kappa_1 > \kappa_2$  for  $n_1 > n > n_2$ . Therefore, similar to Figure 2, there is a symmetric unstable equilibrium and two stable corner equilibria for small  $T$ . For *high*  $T$  (*right* picture), the diagram looks similar to that in Figure 1;  $\kappa_1 = \kappa_2$  for  $n_1 = n = n_2$ , and  $\kappa_1 < \kappa_2$  for  $n_1 > n > n_2$ . Therefore, similar to Figure 2, there is only a symmetric stable equilibrium high  $T$ . Now for *medium*  $T$  (*middle* picture), there is a major difference. For  $n_1 \approx n$ ,  $\kappa_2 > \kappa_1$ . Therefore, the symmetric equilibrium is now

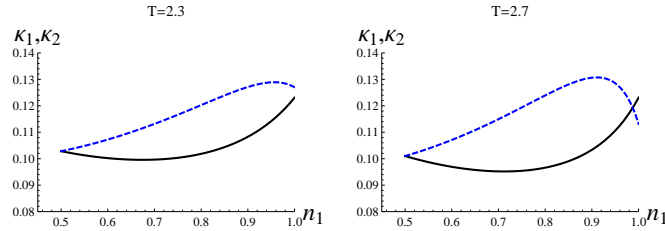
stable. There is another stable corner equilibrium, and in between an unstable equilibrium with  $\kappa_1 = \kappa_2$ .

Figure 6: Cost of Capital



Now in the next bifurcation diagram (with  $b = 0.2$ , upper right picture in Figure 5), there is a little surprise. For large  $T$ , new stable and unstable equilibria appear. These equilibria, however, can only be reached by extreme shocks to the symmetric equilibrium. The following wiggle diagrams again show the costs of capital in the two regions for  $b = 0.2$  and different values for  $T$ .

Figure 7: Cost of Capital



For larger capital market frictions (with  $b = 0.21$ , lower left picture in Figure 5), the tomahawk diagram again changes its character. Now, there is always a stable corner equilibrium. Finally, for even higher capital market frictions (with  $b = 0.25$ , lower right picture in Figure 5), there are only three equilibria for each single  $T$ ; an unstable symmetric equilibrium and two stable corner equilibria.

The different shape of the equilibrium diagrams in Figure 5 has important implications for the industrial development of the regions. For concreteness, assume we start with infinite trade costs  $T$ , then product markets integrate and  $T$  falls (below the break point), and then states erect trade barriers and  $T$  rises again. What will happen? The answer depends on the size of the capital market friction  $b$ . If  $b$  is low (e. g.,  $b = 0.19$ ), the regions will start symmetrically, but when  $T$  drops below the break point, the tiniest perturbation will move the economy to a core-periphery equilibrium. When  $T$  increases again and moves above the sustain point, regions will again become symmetric.

If  $b$  is higher (e. g.,  $b = 0.20$ ), the dynamics are similar. If regions start off symmetrical, once  $T$  falls below the break point, a core-periphery equilibrium will emerge. When  $T$  moves back above the sustain point, the symmetrical equilibrium obtains again. The asymmetric equilibrium for large  $T$  is never reached. This is different for even higher

capital market friction  $b$  (e. g.,  $b = 0.21$ ). Starting with a symmetric equilibrium, if  $T$  falls below the break point, a core-periphery equilibrium will obtain. However, if  $T$  increases again, the equilibrium will always maintain the core-periphery structure. A symmetric equilibrium cannot be reached any more. For yet higher  $b$  (e. g.,  $b = 0.25$ ), the dynamics become extremely simple. The symmetric equilibrium is unstable right away. A tiny perturbation will immediately move the economy to a core-periphery structure.

## 6 Conclusion

Although the type of bifurcation crucially depends on the specification of the moral-hazard problem, we obtain a robust result that including a corresponding capital-market friction leads to a self-reinforcing agglomeration process. Nonetheless the path from the symmetric to the core-periphery formation differs (smooth in the case of the pitchfork bifurcation vs. catastrophic in case of the tomahawk), the characteristic core-periphery constellation for sufficiently low trade costs is the same: the core region focuses on manufacturing, while the periphery produces the agricultural numeraire. The question, which modeling version is the better choice to describe real industries may depend on specific characteristics and institutional arrangements.

In regard to social welfare, the model results are also clear cut. The aggregate utility in the core is higher than in the periphery – a fact, which influences political programmes on national or regional levels. As demonstrated in Section 4, small institutional differences may lead to different realization of the moral hazard problem and corresponding agglomeration advantages or even disadvantages. Interestingly, the moral hazard parameter exhibits the same impact as comparative advantages in (marginal) production costs. This allows the conclusion, that also the organization of financial institutions is decisive for agglomeration dynamics. Considering this outcome from a supra-national perspective, like in case of the European Union, spatial divergence of countries and regions inevitably leads to agglomeration winners and losers (also in respect to different factor groups).

However, the question of spatial inequality is also strongly connected to the question of spatial efficiency. As we shown in the previous section, the moral hazard problem and, thus, the corresponding capital costs are lower in highly industrialized areas.

A second remarkable result is the sensitivity of the spatial distribution of industries with respect to changes in the capital stock. Capital accumulation implies according to Proposition 2 a faster agglomeration process in the course of globalization. In contrast, a financial crisis as recently observed may foster industrial dispersion. In terms of the recent economic and financial crisis, the model predicts an increasing convergence of core and peripheral regions – if they are at a critical trade cost value.

Another question, which occurs considering the results of our model, concerns sectoral specialization. Having a look on real-world agglomeration clearly reveals that we do not observe only industrial concentration but also the emergence and growth of financial centers, i.e. London, Frankfurt or Zurich. In our model, the financial sector works costless and under perfect competition. Potential extensions may focus on a more explicit modeling of this sector with its prevalent characteristics. Finally, with respect to further research, the modeling results derived in this paper also suggest a stronger micro-foundation of economic geography models.

## A Appendix

**Symmetric Equilibrium Values.** For  $n_1 = n_2 = n$ , we obtain:

$$G^{1-\sigma} = n (1 + T^{1-\sigma}) \quad (31)$$

$$Y = \frac{\sigma}{\sigma - \mu} \quad (32)$$

$$q = \frac{\mu Y}{n} = \left( \frac{\sigma}{\sigma - \mu} \right) \frac{\mu}{n} \quad (33)$$

$$\kappa = \frac{1}{n} \left( \frac{\mu}{\sigma - \mu} \right) - bG^\mu \quad (34)$$

**Core-Periphery Values.** For  $n_1 = 2n$  and  $n_2 = 0$ , we obtain:

$$G_1^{1-\sigma} = 2n \quad \text{and} \quad G_2 = G_1 T \quad (35)$$

$$Y_1 = \frac{\sigma}{\sigma - \mu} \quad \text{and} \quad Y_2 = Y_1 - nbG_1^\mu \quad (36)$$

$$q_1 = \frac{\mu}{n} \left( \frac{\sigma}{\sigma - \mu} \right) - \frac{\mu}{2} bG_1^\mu \quad (37)$$

$$q_2 = \frac{\mu}{2} \left[ \frac{1}{n} \left( \frac{\sigma}{\sigma - \mu} \right) (T^{\sigma-1} + T^{1-\sigma}) - bG_1^\mu T^{\sigma-1} \right] \quad (38)$$

$$\kappa_1 = \left( \frac{\mu}{\sigma - \mu} \right) \frac{1}{n} - bG_1^\mu \quad (39)$$

$$\kappa_2 = bG_1^\mu \left[ \frac{\mu}{2\sigma} (T^{1-\sigma} - T^{\sigma-1}) - T^\mu \right] + \frac{\mu}{2n\sigma} \left[ 1 + (T^{1-\sigma} + T^{\sigma-1}) \left( \frac{\mu}{\sigma - \mu} \right) \right] \quad (40)$$

**Proof of proposition 1.** The break point  $T_B$  is implicitly defined by

$$\begin{aligned} 0 &= bG^\mu \frac{2\sigma - 1 - \mu Z}{\sigma - 1} - \frac{Y Z}{n}, \\ 0 &= \Psi := b \left( \frac{1 + Z}{2n} \right)^{\frac{\mu}{\sigma-1}} \frac{2\sigma - 1 - \mu Z}{\sigma - 1} - \frac{\sigma Z}{(\sigma - \mu)n}, \end{aligned} \quad (41)$$

where we have substituted  $G = (n + nT^{1-\sigma})^{1/(1-\sigma)}$  and  $Y = \sigma/(\sigma - \mu)$  and  $1 + T_B^{1-\sigma} = 2/(1 + Z)$  because  $Z = (1 - T_B^{1-\sigma})/(1 + T_B^{1-\sigma})$ . We are originally interested in the sign

of the derivative  $\partial\Psi/\partial T_B$ , but because  $Z$  depends strictly monotonically on  $T$ , we may consider the derivative  $\partial\Psi/\partial Z$  instead. We have

$$\frac{\partial\Psi}{\partial Z} = b \frac{1}{2n} \frac{\mu}{\sigma-1} \left(\frac{1+Z}{2n}\right)^{\frac{\mu}{\sigma-1}-1} \frac{2\sigma-1-\mu Z}{\sigma-1} + b \left(\frac{1+Z}{2n}\right)^{\frac{\mu}{\sigma-1}} \frac{\mu}{\sigma-1} - \frac{\sigma}{(\sigma-\mu)n}.$$

However, because  $\Psi = 0$ , we know that

$$\left(\frac{1+Z}{2n}\right)^{\frac{\mu}{\sigma-1}} = \frac{\sigma Z}{b(\sigma-\mu)n} \frac{\sigma-1}{2\sigma-1-\mu Z}.$$

Substituting into the above  $\partial\Psi/\partial Z$ , we receive

$$\begin{aligned} \frac{\partial\Psi}{\partial Z} &= -\frac{\sigma}{n(\sigma-\mu)} \left(1 + \mu Z \left(\frac{1}{2\sigma-1-\mu Z} - \frac{1}{(1+Z)(\sigma-1)}\right)\right) \\ &= -\frac{\sigma}{n(\sigma-\mu)} \frac{(2\sigma^2 - 3\sigma + 1) + \mu^2 Z^2 + Z(2\sigma-1)(\sigma-\mu-1)}{(Z+1)(\sigma-1)(2\sigma-1-\mu Z)}. \end{aligned}$$

Numerator and denominator of the first fraction are positive, and so is the denominator of the second fraction. Hence consider the numerator of the second fraction. The first bracket is positive, the second part is positive, and the third part can be negative only for  $\sigma < 2$ . Hence for  $\sigma \geq 2$ , the complete derivative  $\partial\Psi/\partial T$  is negative. Some more algebra shows that the complete second fraction can be negative only for  $\sigma < (9 - \sqrt{17})/4 \approx 1.2192$ . Hence for  $\sigma > 1.2192$ ,  $\Psi$  decreases in  $Z$ , hence it also decreases in  $T_B$ . Because  $\partial\Psi/\partial b > 0$ , the implicit function theorem yields that  $dT_B/db > 0$ .

It remains to show that  $\lim_{b \rightarrow 0} T_B = 1$ . Clearly, for  $b = 0$ ,  $Z = 0$  and hence  $T_B = 1$ . All functions are continuous, hence the limit property must also hold. ■

**Proof of proposition 2.** Multiply (41) with  $n$  to get

$$0 = n\Psi = b n^{1-\frac{\mu}{\sigma-1}} \left(\frac{1+Z}{2}\right)^{\frac{\mu}{\sigma-1}} \frac{2\sigma-1-\mu Z}{\sigma-1} - \frac{\sigma Z}{(\sigma-\mu)}.$$

The exponent of  $n$  is  $1 - \mu/(\sigma-1)$ , which is positive if  $\sigma-1 > \mu$ , hence especially if  $\sigma \geq 2$ . As a consequence, the effect of an increase in  $n$  on  $Z$  has the same sign as an increase of  $b$  on  $Z$ . Now the fact that  $T_B$  strictly increases with  $Z$  in combination with proposition 1 completes to proof. ■

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