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Multidimensional Measurement of Richness and the Role of Income and Wealth

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Abstract

Closely following recent innovations in the literature on the multidimensional measurement of poverty, this paper provides similar measures for the top of the distribution using a dual cutoff method to identify persons, who can be considered as rich in a multidimensional setting. Thereby, the derived measures do not only take into account the number of individuals' affluent dimensions, but are also sensitive to changes in achievements of the rich. We make use of this framework to analyze the role of wealth, in addition to income, as one dimension of multidimensional well-being. In addition we analyze the roles of health and education. We provide empirical evidence for Germany.

JEL Classification: D31, D63, I0, I31

Keywords: measurement of richness, multidimensional richness, income, wealth, health, education, Germany

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1 Introduction

In the literature on the distribution of income and well-being, unlike poverty and inequality, richness has been a field of research that was rarely regarded. Possibly, this can be explained by larger policy demand for the analysis of deprivation and/or unequal distribution of incomes and hence of well-being. While it is indisputable that society should ensure a certain minimum subsistence level, the top of the income distribution has just recently become a particular focus of attention (see, e.g. Piketty and Saez (2006); Atkinson and Piketty (2007); Atkinson (2007)).

Atkinson (2007) identifies three main reasons why one should care particularly about the rich: their command over resources (taxable capacity), their command over people (income as a source of power)¹, and their global significance. Especially in the context of income taxation and its reforms, the top of the income distribution is of special interest as, for instance, the top 10% (1%) of the taxpayers pay 50.6% (19.7%) of all income taxes in Germany (see, e.g. Merz, Hirschel, and Zwick (2005)).

The scarce research on affluence has concentrated nearly without exception on proportions of rich individuals within a given population. The associated summary index for richness, the so-called headcount ratio, only takes into account the status of individuals with incomes above some threshold value, typically a multiple of the mean or the median of the distribution under consideration. If one is, however, interested in the intensity of richness, the headcount ratio cannot be a satisfying summary index.²

The literature on the analysis of the distribution of well-being – be it inequality, poverty, or richness – is generally concerned with only one dimension, namely (monetary) income. Income is hence widely considered as an aggregate indicator for well-being. This is not unreasonable, since income satisfies two key properties

¹ Barry (2002) argues that social exclusion exists not only at the bottom of the distribution but also at the top in the form of elite separation. The rich have the ability to use 'exit' as a strategy and buy their way out of common institutions, e.g., by means of private provision of education or health care.

² Of course, the same holds for poverty measurement, for which more sophisticated measures have already been available for a long time (Foster, Greer, and Thorbecke, 1984). Similar indices for the measurement of richness have been introduced quite recently (Peichl, Schaefer, and Scheicher, 2008).

that make it popular in this context: First, it is easily *measurable* and, second, it is *comparable* among a large group of individuals (Cowell, 2008, p. 5 f.).³

Nevertheless, income does not capture every single component that arguably might influence the well-being of individuals. That is why multidimensional measurement of well-being – particularly with regard to poverty – has become of interest (see e.g. Atkinson (2003); Bourguignon and Chakravarty (2003); Alkire and Foster (2008) among others).⁴ Monetary affluence (income) fails to be an appropriate indicator for economic well-being, when markets for certain goods are imperfect or even do not exist, e.g. for public goods (Bourguignon and Chakravarty, 2003, p. 26).

Qualitative studies, like surveys on attitudes towards as well as perception and evaluation of richness, reveal that people have a multi-dimensional concept of richness: It is perceived as luxurious consumption, financial security, and as immaterial option for action. Hence, the perception of richness is not only restricted to material wealth, but it is emphasized that there is a high importance of, for example, health and education (Glatzer, Becker, Bieräugel, Hallein-Benze, Nüchter, and Schmid, 2008, p. 77).

The paper’s contribution to the literature is twofold: First, we extend the univariate richness measures developed by Peichl, Schaefer, and Scheicher (2008) to the multidimensional case by closely following the work of Alkire and Foster (2008), who proposed a class of multidimensional poverty measures based on the one-dimensional FGT poverty measures (Foster, Greer, and Thorbecke, 1984). Central to this approach is a *dual cutoff method* (Alkire and Foster, 2008, p. 2) that identifies those individuals in a population that are considered to be multidimensional rich. Second, we incorporate individual wealth as a dimension of multidimensional richness. In addition to income, health, and education, wealth is another (economic) characteristic that potentially contributes to individual well-being. Despite problems that arise with the measurement of individual wealth, especially with respect to comparability

³ In addition, income data are available for a large set of countries and over long periods of time and it is measured in a uniform way (in monetary units). Equivalence-weighting regularly improves comparability among individuals living in different-size households (Canberra Group, 2001).

⁴ Additional dimensions, beyond income, that have been proposed and used in empirical applications are e.g. health and education. While the health status of an individual can be considered quite obviously as an indicator for well-being, education can be seen as a proxy for potential lifetime income, that is not necessarily captured by conventional income measures.

(see (OECD, 2008, p. 254 ff.)), we believe it to be worthwhile to integrate wealth as an additional dimension in the multidimensional measurement of well-being, since it performs several tasks that are related to it. The developed framework for the measurement of multidimensional richness is applied to Germany with a special focus on income, wealth, health, and education. The paper is further organized as follows: In Section 2 we provide an overview of the literature on wealth. Section 3 introduces our concept for the measurement of multidimensional richness. After a description of the data in Section 4 we present our results in Section 5. The paper is concluded in Section 6.

2 Overview

In this section we provide an overview of the literature on wealth, that characterizes this dimension of economic well-being that will be incorporated in our analysis of multi-dimensional richness.

Wealth is judged to be a "key dimension of economic resources" (OECD, 2008, p. 254) and can be defined as "*a person's total immediate command over resources*", while income can be defined as "*the increase in a person's command over resources during a given time period*" (Cowell, 2008, p. 4 f.). Hence, it deals with the stock dimension of monetary well-being rather than the flow dimension of income. Wealth fulfills several functions: It serves as a source of income (from capital), it provides utility from occupation of property or tangible assets, provides economic and/or political power as well as social status and shapes behavior through socialization and bequest motives (Frick and Grabka, 2009). In addition wealth holdings help risk-averse individuals to stabilize consumption over time and hence can serve as a measure of "sustainable consumption" (Wolff and Zacharias, 2009, p. 83) and of the vulnerability of households in times of crisis (Azpitarte, 2010). Moreover, it can be seen as an indicator for "permanent income" (Michelangeli, Peluso, and Trannoy, 2009).

The distributions of wealth in many countries are characterized by some stylized facts: First, wealth is highly unequally distributed. In most countries the Gini coefficient is very large, especially compared with coefficients for income or consumption, and in some cases even above 0.9⁵. Second, wealth is positively correlated with income, but not perfectly (OECD, 2008; Davies, Sandström, Shorrocks, and Wolff, 2009; Wolff and Zacharias, 2009). Hence, income seems not to be the only factor that determines the resources available to households (OECD, 2008, p. 270).

⁵ The Gini coefficient of the global distribution of wealth is estimated to be 0.802 (i.e. inequality for wealth is larger than for consumption or income). The average wealth holding worldwide is 44,024 US\$ per adult in PPP terms and the median of 8,635 US\$ (Davies, Sandström, Shorrocks, and Wolff, 2009).

3 Measuring Multidimensional Richness

In this section, we describe in detail the measures of multi-dimensional richness that we will employ in the following. Thereby, we make use of the multi-dimensional poverty measures introduced by Alkire and Foster (2008) and the uni-dimensional richness measures that were derived by Peichl, Schaefer, and Scheicher (2008). The dual cutoff method in connection with the measurement of multi-dimensional richness works as follows: In a first step, by comparing their achievements to dimension-specific cutoff values, we distinguish whether an individual is affluent with respect to a specific dimension of well-being or not. An individual is considered as *dimension-specific affluent* when its achievement does not fall behind the cutoff value associated with the dimension under consideration. In a second step, with the help of a *counting methodology* (Atkinson, 2003; Alkire and Foster, 2008) we define which individuals (among those who are affluent with respect to at least one dimension) are considered as rich in a multi-dimensional sense. An affluent individual is defined to be *multi-dimensional rich*, when the number of its affluence counts across all dimensions under consideration does not fall behind a certain threshold (second cutoff). After having identified the multi-dimensional rich persons, their individual achievements are aggregated to a single-value multi-dimensional measure of richness. Here, similar to the one-dimensional measurement of richness, we propose two different ways of aggregation, namely a concave and a convex way respectively (Peichl, Schaefer, and Scheicher, 2008). These aggregation procedures are not only sensitive to the number of individuals' affluent dimensions but also take into account changes in the achievements of the rich.

3.1 Definition of dimension-specific affluence

The number of individuals in the population is denoted with n , while $d \geq 2$ denotes the number of dimensions of affluence under consideration. Define the matrix of achievements with

$$\mathbf{Y} = [y_{ij}]_{n \times d}, \quad (1)$$

where y_{ij} denotes the achievement of individual $i \in \{1, \dots, n\}$ in dimension $j \in \{1, \dots, d\}$. For each dimension j , there is some cutoff value γ_j . Hence, γ denotes a $1 \times d$ vector of dimension-specific cutoffs. With the help of this vector of dimension-specific cutoffs, it is possible to identify, whether individual i is affluent with respect to dimension j or not. Therefore, define an indicator function θ_{ij} :

$$\theta_{ij}(y_{ij}; \gamma) = \begin{cases} 1 & \text{if } y_{ij} > \gamma_j, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

and with its help construct a 0 – 1 matrix of dimension-specific affluence:

$$\Theta^0 = [\theta_{ij}]_{n \times d}, \quad (3)$$

where each row vector of Θ^0 , denoted with θ_i , is equivalent to individual i 's affluence vector. Hence, this yields us a vector of affluence counts, denoted $\mathbf{c} = (c_1, \dots, c_n)'$. Its elements $c_i = |\theta_i|$ are equal to the number of dimensions, in which an individual i is defined to be affluent.

In case of cardinal variables in the achievement matrix \mathbf{Y} , it is possible to construct matrices that, in addition, do not only provide the information whether an individual i is affluent with respect to dimension j or not, but also inform about the intensity of affluence associated with the dimension under consideration. Thereby, one can distinguish between two ways of evaluating the intensity of affluence, namely a concave or a convex way. If we are interested in the *convex* case, we replace the matrix of dimension-specific affluence Θ^0 and instead look at the following matrix:

$$\Theta^\alpha = \left[\left(\frac{y_{ij} - \gamma_j}{\gamma_j} \right)_+^\alpha \right]_{n \times d} \quad \text{for } \alpha \geq 0. \quad (4)$$

In the *concave* case we have

$$\Theta^\beta = \left[\left(1 - \left(\frac{\gamma_j}{y_{ij}} \right)^\beta \right)_+ \right]_{n \times d} \quad \text{for } \beta > 0. \quad (5)$$

The subscript ”+” indicates that the entries of matrices Θ^α and Θ^β respectively

must be positive. If the expressions in brackets should happen to be negative for single individuals, they are replaced with a zero entry. This is equivalent to multiplying the expressions with the indicator function $\theta_{ij}(y_{ij}; \gamma)$. The parameters α and β are sensitivity parameters for the intensity of richness. For larger (smaller) values of α (β) more weight is put on more intense affluence.⁶ The main difference between the convex and the concave measure of richness is the reaction to a rank-preserving progressive transfer between two affluent individuals. While such a transfer *decreases* richness when it is measured in the convex way, it *increases* richness in case of the concave measure.⁷

3.2 Definition of multi-dimensional richness

After having described in the previous section how affluence is defined with respect to single dimensions, we now describe how we define multi-dimensional richness with the help of the dual cutoff method of identification.

For an integer number $k \in \{1, \dots, d\}$ define the identification method as

$$\phi_i^k(y_i, \gamma) = \begin{cases} 1 & \text{if } c_i \geq k, \\ 0 & \text{if } c_i < k. \end{cases} \quad (6)$$

This yields a 0 – 1 vector $\phi^{\mathbf{k}}$ with entries ϕ_i^k equal to one if the number of affluent dimensions of individual i is not less than k , and is zero otherwise. In other words, individual i is considered to be *multi-dimensional* or *multivariate rich*, if the number of dimensions in which its achievement is considered as affluent attains a certain threshold.⁸ So, we can define the subset of multivariate rich individuals among the whole population as $\Phi_k \subseteq \{1, \dots, n\}$ with $\Phi_k = \{i : \phi_i^k(y_i, \gamma) = 1\}$. The number of

⁶ Note that Θ^0 is simply a special case of Θ^α for $\alpha = 0$ and of Θ^β for $\beta \rightarrow \infty$ respectively.

⁷ See Peichl, Schaefer, and Scheicher (2008) for a detailed discussion of concave and convex measures of richness in the one-dimensional case.

⁸ Note that, throughout the paper, we speak of *affluence*, when we refer to affluence *with respect to a specific dimension* (or a set of dimensions). In contrast, we consider an individual to be (*multi-dimensional*) *rich*, if and only if its number of affluent dimensions (c_i) is not smaller than the multi-dimensional threshold (k). So, an individual i can be affluent in one or more dimensions and, at the same time, not be rich (when it holds that $c_i < k$), while a rich person by definition is always affluent in at least k dimensions.

multivariate rich individuals is denoted with $s = |\Phi_k|$.⁹

In order to obtain matrices that provide information on rich individuals only, we replace the row i of Θ^α and Θ^β respectively with vectors of zeros, whenever it holds that $\phi_i^k(y_i, \gamma) = 0$. Formally, define

$$\Theta^\alpha(\mathbf{k}) = \left[\left(\frac{y_{ij} - \gamma_j}{\gamma_j} \right)^\alpha \cdot \phi_i^k(y_i, \gamma) \right]_{n \times d} \quad \text{and} \quad (7a)$$

$$\Theta^\beta(\mathbf{k}) = \left[\left(1 - \left(\frac{\gamma_j}{y_{ij}} \right)^\beta \right) \cdot \phi_i^k(y_i, \gamma) \right]_{n \times d} \quad (7b)$$

respectively. Since, according to the focus axiom, a measure of (multivariate) richness must only take into account information on the individuals considered to be rich, we replace the elements of the vector of affluence counts \mathbf{c} with zero, when the number of affluence counts of the according individual i does not attain the threshold k . Formally:

$$c_i^k = \begin{cases} c_i & \text{if } c_i \geq k, \\ 0 & \text{if } c_i < k. \end{cases} \quad (8)$$

This yields the vector $\mathbf{c}^k = (c_1^k, \dots, c_n^k)'$, that contains zeros for individuals not considered to be multivariate rich and the number of dimensions, in which the multivariate rich individuals are considered as affluent. I.e., even when an individual is affluent in several dimensions, its entry in \mathbf{c}^k nevertheless might be zero. This is the case, when its number of affluent dimensions is smaller than the threshold k .

3.3 Measures of multi-dimensional richness

Now we are able to define measures of multi-dimensional richness based on the definitions that were introduced in the previous two subsections. In order to derive

⁹ Hereby, one can think of two extreme cases. First, in case of $k = 1$, individual i is multivariate rich when it is considered as affluent in only one single dimensions (union approach). Second, in case of $k = d$, an individual is only considered as rich, if it is considered as affluent in all dimensions (intersection approach). In case of $1 < k < d$ we have an intermediate approach (Alkire and Foster, 2008, p. 7 f.).

a first multivariate measure of richness, define the *headcount ratio* (HR) as

$$HR = \frac{s}{n}, \quad (9)$$

which is simply the proportion of rich individuals among total population and the *average affluence share* (AAS) as

$$AAS = \frac{|\mathbf{c}^k|}{s \cdot d}, \quad (10)$$

where $|\mathbf{c}^k|$ denotes the number of affluence counts among the multivariate rich population. The average affluence share is hence equal to the relation of this number to the maximum number of affluence counts that would be observed when *all rich* individuals were rich among *all* dimensions. It holds that $1/d \leq AAS \leq 1$. For a given number of dimensions under consideration d the value of AAS is close to one, when there is a very strong correlation of affluence across dimensions, i.e. those who are rich tend to be affluent in nearly all dimensions. The value becomes smaller when the number of dimensions, according to which the rich are affluent, decreases. This means, the rich population is not necessarily affluent in every single dimension. It reaches its minimum value of $1/d$, when all multivariate rich individuals are in effect only affluent with respect to one single dimension.

Now, we can define a first measure of multivariate richness by simply multiplying the headcount ratio and the average affluence share. I.e., the **dimension adjusted headcount ratio** is defined as

$$R_{HR}^M = HR \cdot AAS = \frac{|\mathbf{c}^k|}{n \cdot d}, \quad (11)$$

which is equal to the proportion of the total number of affluence counts to the maximum number of affluence counts that one would observe when every single individual in the population under consideration would be affluent with respect to every single dimension. Contrary to the simple headcount ratio HR the measure R_{HR}^M satisfies the property of *dimensional monotonicity*, which requires that a measure of multivariate richness increases (decreases) when a rich individual ($c_i \geq k$) becomes (is no

more) affluent in some dimension. That is why the *AAS* is incorporated in R_{HR}^M .¹⁰ However, the dimension adjusted headcount ratio does not satisfy the property of *monotonicity*, i.e. the measure R_{HR}^M does not necessarily increase (decrease) when the achievement y_{ij} of a rich individual i in dimension j increases (decreases).¹¹ Hence, it only reveals information about the width and not the depth of affluence among the population under consideration.

The following additional measures of multivariate richness by contrast do satisfy the monotonicity property. Again, one can distinguish between a convex and a concave measure respectively. The **dimension adjusted multivariate richness measures** R_α^M and R_β^M are defined as

$$R_\alpha^M = HR \cdot AAS \cdot \frac{|\Theta^\alpha(\mathbf{k})|}{|\mathbf{c}^{\mathbf{k}}|} = \frac{|\Theta^\alpha(\mathbf{k})|}{n \cdot d} \text{ and} \quad (12a)$$

$$R_\beta^M = HR \cdot AAS \cdot \frac{|\Theta^\beta(\mathbf{k})|}{|\mathbf{c}^{\mathbf{k}}|} = \frac{|\Theta^\beta(\mathbf{k})|}{n \cdot d} \quad (12b)$$

and hence are equal to the sum of the elements of the matrices $\Theta^\alpha(\mathbf{k})$ and $\Theta^\beta(\mathbf{k})$ divided by their maximum value $n \cdot d$ respectively.¹²

Since we are interested in analyzing the role of certain dimensions (especially income and wealth) with respect to the measurement of multi-dimensional richness, it seems helpful to formally disentangle the dimensions-specific contributions. Therefore, we write equations (12a) and (12b) respectively as

$$R_c^M = \frac{|\Theta^c(\mathbf{k})|}{n \cdot d} = \frac{\sum_{j=1}^d |\theta_j^c(\mathbf{k})|}{n \cdot d} = \frac{1}{d} \cdot \sum_{j=1}^d \frac{|\theta_j^c(\mathbf{k})|}{n} = \frac{1}{d} \cdot \sum_{j=1}^d \Pi_j^c(\mathbf{k}) \quad (13)$$

for $c \in \{\alpha, \beta\}$. Hence, $\Pi_j^c(k)$ denotes the contribution of each dimension j multiplied by the total number of dimensions d . More intuitively, it is equal to the

¹⁰ The richness measure R_{HR}^M is perfectly equivalent to the multi-dimensional poverty measure M_0 (Alkire and Foster, 2008, p. 10 f.).

¹¹ It does so only marginally around dimension-specific thresholds γ_j .

¹² In the convex case (R_α^M) the multivariate measure again is perfectly equivalent to the multivariate poverty measure M_α . Then the fraction $|\Theta^\alpha(\mathbf{k})|/|\mathbf{c}^{\mathbf{k}}|$ is equal to the *average affluence gap* for $\alpha = 1$ and the *average severity of affluence* for $\alpha = 2$ (Alkire and Foster, 2008, p. 11 f.). Note that the concave measure R_β^M is normalized between zero and one, while the convex measure R_α^M is not. Although one would prefer to have normalized measures only, this is not possible in the convex case in general without violating the monotonicity axiom. Hence, the choice of R_α^M implies a certain normative view, since it emphasizes intense rather than moderate richness.

proportion of individuals that are multi-dimensional rich *and* affluent with respect to dimension j at the same time (Alkire and Foster, 2008, p. 27 f.). The simple mean of all these contributions over the d dimensions yields the overall multi-dimensional richness measure R_c^M . One can show that the proportional contribution of dimension j to the overall measure R_c^M , denoted with $\pi_j^c(k)$, can be written as

$$\pi_j^c(k) = \frac{|\theta_j^c(\mathbf{k})|}{|\Theta^c(\mathbf{k})|}. \quad (14)$$

Obviously, it holds that $\sum_{j=1}^d \pi_j^c(k) = 1$. Hence, it is possible to decompose the measures proportionally into the contributions of the single dimensions. In Section 5 we provide evidence for this.

4 Data

The German Socio-Economic Panel Study (GSOEP) is a panel survey of households and individuals in the Federal Republic of Germany that has been conducted annually since 1984. A weighting procedure allows to make respondents' data to be representative for the German population.¹³

The **income** variable that we use is the individual equivalent post-government income, which defined as follows (Grabka, 2007, p. 41 f.): A household's post-government income encompasses pre-government income¹⁴, public transfers, and social security pensions from all household members minus total tax-payments of all household members.¹⁵ We use the modified OECD equivalence scale for equivalence weighting in order to make incomes of individuals living in different-size households comparable to each other.¹⁶

The 2002 and 2007 waves of the GSOEP contain information on **wealth** that was surveyed in additional questionnaires. Different from most other surveys that provide information on wealth the GSOEP data were collected at the *individual* level rather than on the *household* level (Frick, Grabka, and Marcus, 2007; Frick and Grabka, 2009). The variable that provides information on net worth of individuals aged 17 and older aggregates the following single components: owner-occupied housing and other property (net of mortgage debt), financial assets, business assets, tangible assets, private pensions and consumer credits. Frick and Grabka (2009) provide a detailed overview and description of the distribution of overall wealth as

¹³ A detailed overview of the GSOEP is provided by its Desktop-Companion (Haisken-DeNew and Frick, 2005) or by Wagner, Frick, and Schupp (2007).

¹⁴ Pre-government income consists of labor earnings, asset flows, private retirement income and private transfers from all household members. Labor earnings include wages and salary from all employment including training, self-employment income, and bonuses, overtime, and profit-sharing. Asset flows include income from interest, dividends, and rent. Private transfers include payments from individuals outside of the household including alimony and child support payments (Grabka, 2007, p. 41).

¹⁵ Public transfers include housing allowances, child benefits, subsistence assistance, government student assistance, maternity benefits, unemployment benefits, unemployment assistance, and unemployment subsistence allowance. Social security pensions include payments from old age, disability, and widowhood pension schemes. The tax burden includes income taxes and payroll taxes (health, unemployment, retirement insurance and nursing home insurance taxes)(Grabka, 2007, p. 42).

¹⁶ The modified OECD scale assigns a weight of 1.0 to the first (adult) household member. Every additional adult is assigned a weight of 0.5 and every child a weight of 0.3 (OECD, 2005).

well of the single components based on the 2007 wave of the GSOEP wealth data.¹⁷ In order to handle the problem of measurement error arising from item or unit non-response, the GSOEP provides editing and multiple imputation procedures that are described in detail in Frick, Grabka, and Marcus (2007).

Our measure of **health** is the number of days per year *without* doctoral visit.¹⁸ **Education** is measured by the number of years of education (Grabka, 2007, p. 23). Table 1 provides descriptive information on the dimension that we choose to include

Table 1: Descriptive statistics, dimension-specific cutoffs, and poverty lines (Germany 2007)

dimension	mean	median	cutoff	poverty line
income	19,845 (145)	17,669 (117)	35,339 (234)	10,601 (70)
wealth	83,243 (1,930)	32,000 (1,607)	128,000 (6,429)	19,200 (964)
health	355.5 (0.14)	361 (0.00)	363 (0.00)	350 (0.00)
education	9.5 (0.05)	10.5 (0.00)	12 (0.00)	9 (0.00)

Note: Income and wealth are measured in Euros, equivalence weighted with modified OECD scale. Income and wealth data were trimmed (i.e. bottom and top 0.5% of respective distribution dropped). Health: number of days without doctoral visit per year. Education: years of education. Cutoff (poverty line) for wealth corresponds to 400% (60%) of median wealth. Cutoff (poverty line) for income corresponds to 200% (60%) of median income. Bootstrapped standard errors of empirical distribution in parentheses (1,000 replications). Source: GSOEP, own calculations.

in our analysis. Note that we trimmed the income and wealth data by dropping the bottom and top 0.5% of the respective distributions in order to rule out bias by extreme values. The table also reports the cutoff values that we employ in the

¹⁷ Due to the difficulty of collecting information on pension claims of individuals that are still in the labor force, these information are not included in the wealth measure of the GSOEP. Frick and Grabka (2010) report results from a statistical matching procedure of the GSOEP wealth data with data from the *German Statutory Pension Insurance Scheme*. It turns out that, compared to financial and tangible assets only, the inclusion of a discounted (present) value of pension claims increases mean (+76%) as well as median wealth (+430%) and decreases inequality (-20%). However, these data are not freely available and hence cannot be included into our analysis.

¹⁸ However, this might be controversial as an indicator of individual health and should be replaced by a more acknowledged measure of health. The GSOEP data contains information on the Mental and Physical Component Scale (MCS/PCS) that is widely recognized in the literature as an indicator for health (Nübling, Andersen, Mühlbacher, Schupp, and Wagner, 2007).

analysis. The main results are presented in the next section. We define the cutoff value for income to be twice the median value. Hence, an individual is considered to be affluent with respect to income when its equivalence weighted disposable income exceeds the threshold of 35,339 Euros per year. The cutoff value for wealth is also defined as a multiple of the median value. Here, we define an individual to be affluent in the wealth dimension if the sum of its wealth holdings exceeds 128,000 Euros which corresponds to 400% of median wealth. We set the cutoff for health to 363 days without doctoral visit and the cutoff for education is 12 years of education.

5 Results

In this section we present the results for our analysis with respect to the multi-dimensional measures for richness, that were derived in Section 3.

Table 2 provides information on the one-dimensional distributions of the dimensions under consideration, i.e. one-dimensional richness and poverty measures as well as the Gini coefficient as a measure of inequality. It turns out that the richness headcount ratio for income (7.4%) is relative small compared to the headcount ratios of the other dimensions: 22.4% are affluent in wealth, 24.7% in health and 20.9% in education. Concerning wealth, one can see that it is distributed very unequally, since its Gini coefficient is very large (about 0.7), compared to a Gini of about 0.27 for income, 0.29 for education, and 0.02 for health. The poverty rate for wealth (with a poverty line at 60% of the median) is very high, too. This means, only about one third of the population form the "wealth middle-class", i.e. are neither affluent nor poor with respect to wealth. Table 3 reports Spearman's rank correlation coeffi-

Table 2: One-dimensional Measures (Germany 2007)

dimension	R_{HR}	$R_{\alpha=1}$	$R_{\alpha=2}$	$R_{\beta=1}$	$R_{\beta=3}$	I_{Gini}	P_{HR}
income	0.074 (0.003)	0.024 (0.002)	0.014 (0.002)	0.016 (0.001)	0.034 (0.002)	0.267 (0.003)	0.137 (0.004)
wealth	0.224 (0.011)	0.247 (0.023)	0.674 (0.105)	0.089 (0.006)	0.152 (0.009)	0.694 (0.006)	0.431 (0.003)
health	0.247 (0.004)	0.001 (0.000)	0.000 (0.000)	0.001 (0.000)	0.004 (0.000)	0.016 (0.000)	0.195 (0.004)
education	0.209 (0.004)	0.062 (0.001)	0.024 (0.001)	0.045 (0.001)	0.103 (0.002)	0.290 (0.004)	0.217 (0.004)

Note: One-dimensional richness (poverty) is measured by the head count ratio R_{HR} (P_{HR}), inequality is measured by the Gini coefficient I_{Gini} . Bootstrapped standard errors of empirical distribution in parentheses (1,000 replications). Source: GSOEP, own calculations.

icients of the four dimensions under consideration. It turns out that the position of an individual within the respective distribution of the the single dimensions are not very much correlated. The only sizeable rank-correlation coefficient of 0.47 is the one for income and wealth. But as it has been mentioned before, the correlation (of

ranks) between these two dimensions is positive, but far from perfect.¹⁹ In addition, the (rank) correlation between the other dimensions are very weak. Hence, it is justified to look at richness in a multi-dimensional way, since it indeed turns out that an individual's position in the income distribution is a very poor predictor of its position in the distributions of the other dimensions. In addition to the rank

Table 3: Rank correlation coefficients between dimensions (Germany 2007)

	income	wealth	health	education
income	1 (0.000)			
wealth	0.469 (0.001)	1 (0.000)		
health	0.017 (0.001)	-0.047 (0.000)	1 (0.000)	
education	0.261 (0.001)	0.156 (0.001)	-0.102 (0.000)	1 (0.000)

Note: Bootstrapped standard errors of empirical distribution in parentheses (100 replications). Source: GSOEP, own calculations.

correlations, Table 4 lists the population proportions of the combinations of affluent dimensions. According to this table, about 46.7% of the German population are not considered to be affluent in any dimension. Hence, the population is more or less split up into two halves, one has zero affluence counts, the other half has at least one. Besides the combination of no affluence counts, the most frequent ones can be found within the group of individuals with exactly one affluence count: about 14.7% are affluent only in health, 11.1% only in wealth and 9.1% only in education. These three combinations make up about one third of the population. Only 1.1% are only affluent in income. With respect to the one-dimensional headcount ratio of 7.4%, this means that the vast majority of those affluent in income are also affluent in at least one additional dimension. A very small fraction of the population, about 0.65%, is affluent in every single dimension. In Table 5 we present our results for the different multi-dimensional richness measures, that are described in detail in Section 3: the headcount ratio (see Equation (11)) as well as the multi-dimensional richness measures (see Equations (12a) and (12b)). We present results for different

¹⁹ The results for the correlation coefficients of levels are very similar.

Table 4: Combinations of dimension-specific affluence: population proportions (in per cent) (Germany 2007)

affluent in dimension							
	income	wealth	health	education	%	counts	%
1	0	0	0	0	46.89	0	46.89
2	1	0	0	0	1.09	1	35.93
3	0	1	0	0	11.11		
4	0	0	1	0	14.65		
5	0	0	0	1	9.08		
6	1	1	0	0	1.67	2	12.76
7	1	0	1	0	0.30		
8	1	0	0	1	1.12		
9	0	1	1	0	2.90		
10	0	1	0	1	2.78		
11	0	0	1	1	3.99		
12	1	1	0	1	1.57	3	3.78
13	1	1	1	0	0.50		
14	1	0	1	1	0.53		
15	0	1	1	1	1.18		
16	1	1	1	1	0.65	4	0.65

Note: Source: GSOEP, own calculations.

values of the second cutoff threshold k and for different values of α and β respectively. Corresponding to the results from Table 4, about 53% of the population are rich, when it is sufficient to be affluent in at least one dimension ($R_{HR}^M = 0.531$ for $k = 1$), since about 47% are not affluent in any of the four dimensions discussed here. For larger values of k , the proportion of the rich decreases considerably to 17.2% ($k = 2$) and 4.4% ($k = 3$). And as has become clear from Table 4, less than 1% are rich if one requires the rich to be affluent in every single dimension ($k = 4$). The resulting values for the other multi-dimensional richness measures, R_α^M and R_β^M respectively, also decrease with the the value for k . Especially from $k = 3$ to $k = 4$ there is substantial drop, which is not surprising, since there is only a very small number of people with four affluence counts (see above). However, looking at the

Table 5: Multi-dimensional Measures (Germany 2007)

k	R_{HR}^M	$R_{\alpha=1}^M$	$R_{\alpha=2}^M$	$R_{\beta=1}^M$	$R_{\beta=3}^M$
1	0.531 (0.008)	0.083 (0.006)	0.178 (0.026)	0.038 (0.002)	0.073 (0.002)
2	0.172 (0.005)	0.052 (0.004)	0.123 (0.019)	0.023 (0.001)	0.043 (0.002)
3	0.044 (0.002)	0.022 (0.002)	0.058 (0.010)	0.009 (0.001)	0.017 (0.001)
4	0.007 (0.001)	0.004 (0.001)	0.010 (0.002)	0.002 (0.000)	0.003 (0.000)

Note: k denotes the second cutoff threshold. Bootstrapped standard errors of empirical distribution in parentheses (1,000 replications). Source: GSOEP, own calculations.

over-all values of the multi-dimensional richness measures is more interesting when making comparisons over time or across countries. Hence, in additon, we provide information how the different dimensions of affluence contribute to the over-all measures of multi-dimensional richness. The graph in Figure 1 shows the proportional contributions of the four dimensions to the richness measures, again for different values of k as well as for α and β . The graph reveals that the contributions are more or less evenly distributed across dimensions for the multi-dimensional headcount ratio (denoted HR in Figure 1). Taken together, health and education make up about 60% of the headcount ratio for $k = 1$. Their joint contribution is also above 50% for

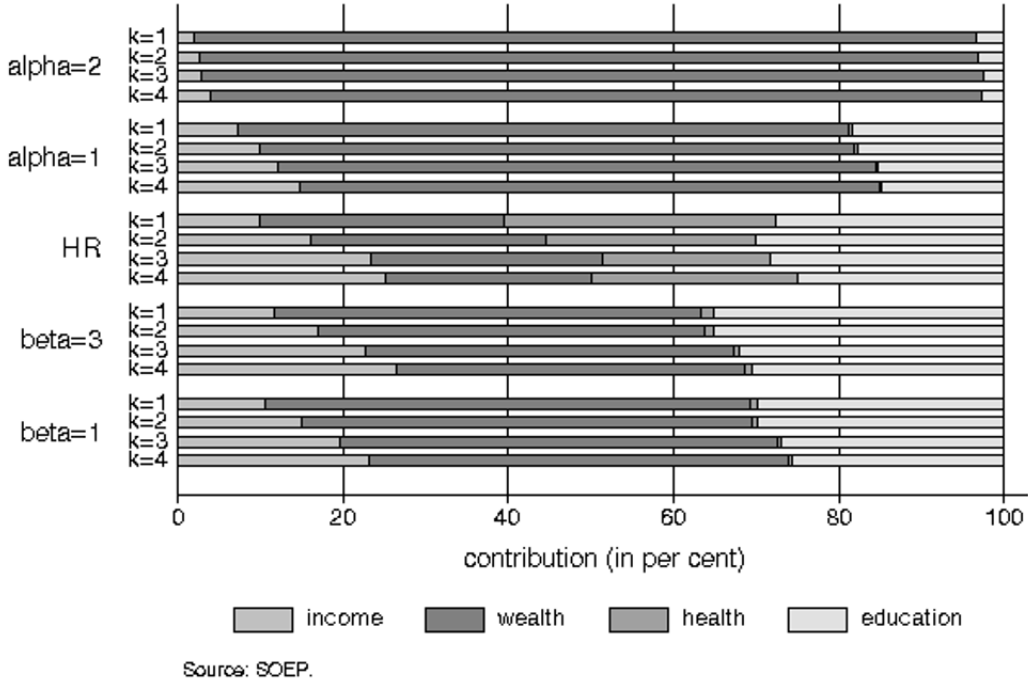


Figure 1: Contributions per dimension

larger values of k . However, besides for the headcount ratio, health plays virtually no role for multi-dimensional richness. Its contributions are only marginal for R_α^M and R_β^M respectively, irrespective of the level of k , while the contributions of education to R_β^M are well above 20% and slightly below 20% for $R_{\alpha=1}^M$. For $R_{\alpha=2}^M$, we see that wealth plays an overwhelmingly dominant role for multi-dimensional richness. Of course, this is due to the fact that the convex measure emphasizes intense richness, especially for larger values of α . However, the same is true for the concave measure for smaller values of β . The contribution to multi-dimensional richness of income is quite small: In most cases it does not exceed 25%. However, what can be recognized is a pattern of increasing relative importance of income for increasing values of k . This might be due to the fact that the over-all proportion of individuals who are affluent in income is relatively small (about 7.4%). Hence, it is not surprising that income plays a more important role for larger values of the second cutoff threshold k .

6 Conclusions

In this paper, we derive a methodology for the measurement of richness in a multi-dimensional setting. We argue that economic well-being, and especially the top of its distribution, should not only consider income as a single dimension, but in addition take into account further dimensions, since richness is not only perceived as a monetary concept. That is why we suggest a multi-dimensional approach in order to provide a more-sided picture of economic well-being.

Using income, wealth, health, and education as dimensions of multi-dimensional well-being and based on survey data from the GSOEP we provide evidence for Germany. We find that it is justified to incorporate additional dimensions of well-being beyond income, since the (rank) correlation across dimensions is relative weak. I.e., an individual's position in the income distribution does not necessarily predict its position in the distribution of other dimensions. While more than 50% of the German population are affluent in at least one the four dimensions, less than 1% is affluent in every single dimension. Moreover, we find that every dimension evenly contributes when multi-dimensional richness is measured by the multi-dimensional headcount ratio. However, when more emphasis is put on the intensity of richness, health plays virtually no role, while the contribution of wealth is predominant.

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