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# Platform Collusion in Two-Sided Markets

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## Preliminary and Incomplete

### Abstract

This paper analyses price collusion between platforms in a two-sided market model. I investigate the hypothesis that collusion is harder to sustain in presence of indirect network externalities because of the resulting feedback effects. I show that higher indirect network externalities have two opposing effects on the sustainability of a cartel. First, collusive profits increase while stage game Nash profits fall - this makes collusion more desirable. Second, the incentive to deviate increases as demand reacts more sensitively. The latter effect dominates and collusion becomes harder to sustain as indirect network externalities become stronger.

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# 1 Introduction

Cartels feature prominently in two-sided industries. For instance, the two major arts auction houses Christie's and Sotheby's fixed seller's commission fees and trading conditions for buyers for almost seven years until their cartel was uncovered by competition authorities.<sup>1</sup> Joint price-fixing has also been observed in various media markets. For instance, the German competition authority started an investigation of the two dominating players in the German private TV market, ProSiebenSat1 and RTL-Group, after a striking convergence of advertisement prices and the simultaneous announcement of a price increase which was justified by almost identical wording (see Budzinski & Wacker (2007) for details). Another case involved three German nationwide newspaper publishers that planned to build a common agency for employment advertisements.<sup>2</sup> Their contract included fixed prices for employment ad space and profit sharing rules.<sup>3</sup> Another example concerns 23 U.S. universities which colluded on financial aid awards for students for over 40 years. Their practices eventually ended after the Department of Justice charged the participating Ivy League universities and the MIT of illegal price fixing in 1991. An out-of-court settlement which forbid discussion of and coordination on prospective awards was signed by all members. An interesting detail which makes this case a suspect for two-sided collusion is that press reports and evidence presented at trial indicated that collusion on faculty salaries may have occurred as well. Hence, universities were suspected to be forming a cartel on both sides of the academic market - students and faculty members.<sup>4</sup>

As these cases indicate, there is scope for collusive behavior in two-sided markets. Since these markets are characterized by two distinct consumer groups interrelated via indirect network externalities, the aim of this paper is to analyse the effects of those indirect network effects on collusive stability. Using Armstrong's (2006) two-sided single-homing model as a stage game and assuming grim trigger punishments, I find that increasing network externalities have two opposing effects. Firstly, stronger network effects raise platforms' incentive to collude, namely the difference between collusive and Nash profits. If one market side values members on the opposing side more highly, Nash prices fall because competition for this side gets harsher. At the same time, consumers' utility from platform participation increases if they benefit more from the presence of platform users on the opposing market side. Consequently, platforms can earn larger collusive profits by setting higher cartel prices when utility grows with rising network externalities. Secondly and countervailing, platforms also gain larger profits from deviating when network effects become stronger - a result which is due to more sensitive demand reactions when both market sides are interlinked by indirect externalities. Comparing those two opposing effects and solving for the critical discount factor,

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<sup>1</sup>For details on the case, see European Commission (2002).

<sup>2</sup>Those publishers were Süddeutsche Zeitung GmbH, Druck- und Verlagshaus Frankfurt am Main GmbH and Axel Springer Verlag AG.

<sup>3</sup>For more details on the case, consider the report of the German competition authority (Bundeskartellamt 1999) or a Handelsblatt article from July 22, 2002 (<http://www.handelsblatt.com/archiv/sz-und-fr-duerfen-bei-anzeigen-kooperieren;545185>) concerning the case and the final court decision.

<sup>4</sup>For more details see e.g. Masten (1995) or Salop & White (1991). I would like to thank Patrick Rey for mentioning this case to me.

I show that the latter effect always dominates in a two-sided single-homing Hotelling model. In other words, collusion becomes harder to sustain as network externalities between the market sides grow. Furthermore, I find that an increasing asymmetry in the network benefits between both sides has a negative impact on collusive sustainability.

My results confirm Evans & Schmalensee's (2008) hypothesis that collusion is harder to sustain in two-sided markets. Their argument is that successful cartels need to coordinate prices on both sides of the market which asks for more agreements and monitoring and makes it more difficult to form an effective cartel. This paper shows that two-sided collusion becomes harder to sustain even without increased monitoring or coordination costs. A second point made by Evans & Schmalensee concerns collusion only on one market side. They argue that such one-sided agreements do not constitute an alternative since all supra-competitive profits earned on the colluding side would be competed away on the other one because of feedback effects. Given my framework, however, I cannot fully bear out their statement. Although platforms will compete away some of the collusive profits on the remaining competitive market side, they might still benefit from a one-sided cartel. Such a one-sided agreement, however, might be even harder to sustain than collusion on both markets sides.

This paper adds to the body of literature on two-sided markets which started from seminal contributions by Rochet & Tirole (2003, 2006), Caillaud & Jullien (2003) and Armstrong (2006). In particular, it enriches a strand which focuses on the impact of indirect network externalities on well established competition policy results,<sup>5</sup> providing the first formal investigation of collusive sustainability in two-sided markets. In parallel work, Dewenter, Haucap & Wenzel (2009) analyse the welfare implications of semi- and full collusion in media markets. They show that collusion only on advertisement prices might actually increase total welfare, whereas welfare implications of collusion on both market sides (readers and advertisers) are ambiguous. In contrast to the model used in this paper, they assume that firms first choose advertising levels and then compete in newspaper copy prices. Argentesi & Filistrucchi (2007) develop a structural model to empirically test for the presence of collusion in media markets. In particular, they analyse the Italian newspaper market and address the question whether observed price patterns are consistent with profit-maximizing behavior by competing firms or instead driven by some form of (tacit or explicit) coordinated practice. Their model encompasses a demand estimation for differentiated products on both sides of the market and allows for profit maximization by the publishing firms taking into account the interaction between those sides. In order to simplify the analysis, however, they assume that readers do not care about the amount of advertisement to be found in newspapers, i.e. indirect network effects are present in only one direction. They derive hypothetical markups under the two alternative conjectures of competition and joint profit maximization between newspapers and compare them with actually observed ones. Using this method, they find an indication of joint profit maximization for newspaper cover

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<sup>5</sup>Areas of competition policy that have already been covered include mergers, tying and bundling, exclusive contracts and the impact of price discrimination. See, for example, Chandra & Collard-Wexler (2009), Armstrong & Wright (2007), Rochet & Tirole (2008) or - for an overview - Evans (2003), Evans & Schmalensee (2008) as well as Rysman (2009).

prices, whereas the advertising market is closer to competition, a result which they claim to be consistent with anecdotal evidence.

The rest of the paper is organized as follows: In the next section, I will outline the framework and describe the collusive game. Section 3 presents the main result, namely that collusion is harder to sustain as indirect network effects become stronger. Section 4 is devoted to special cases of the general framework and a discussion of the main result’s robustness to optimal punishment schemes. Section 5 concludes.

## 2 The Model

The stage-game setting in which collusive behavior will be analysed is based on Armstrong’s (2006) two-sided single-homing model, a framework which fits well to media markets and - to some extent - even to academic markets.<sup>6</sup> Therefore, it addresses most of the cases mentioned in the introduction. The empirical relevance of single-homing has been investigated by Kaiser & Wright (2006). They tested different versions of Armstrong’s model assuming single- or multi-homing of advertisers in the German magazine market and concluded that competition between platforms is prevalent on both market sides. Put differently, the assumption of multi-homing on the advertisers’ side does not provide a good fit of the German magazine market. In a broader sense, real-world examples of single-homing environments can be motivated by indivisibilities, limited resources or contractual restrictions.

In Armstrong’s model, pricing for the service offered by a platform is based on membership fees rather than on a transaction-based payment. Samewise, the imposed network externalities which interconnect both market sides, are only membership-based, i.e. each consumer on one market side values the per-se presence of members on the other side. Applying this to the example of media markets indicates that advertisers care about the circulation of a certain newspaper, i.e. the size of its readership, while readers derive a certain (dis-)utility from the number of advertisements. Newspapers earn revenues from readers on a per-copy basis while advertisers pay a per-ad-space price. Hence, both sides pay once to access the opposing market side.<sup>7</sup>

Suppose there are two platforms in a two-sided market, denoted by  $A$  and  $B$ , which both serve two types of customers, group 1 and group 2. If a customer of group 1 or group 2 decides to join platform  $i$ , she receives the following respective utility:<sup>8</sup>

$$u_1^i = k + a_1 n_2^i - p_1^i \quad \text{or} \quad u_2^i = k + a_2 n_1^i - p_2^i \quad \text{with } i \in \{A, B\}$$

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<sup>6</sup>Although Armstrong’s model fits the Ivy League cartel in most aspects, I have to admit that the network benefits students or faculty members gain by joining a university are not purely membership based. Instead, the academic excellence of university members on either side of the academic market matters. Such a quality dimension, however, is not included in Armstrong’s framework.

<sup>7</sup>There exist other forms of network externalities which might be present in two-sided markets. Furthermore, a platform might apply alternative payment schemes. Therefore, I see this setting as a first step to shed light on the sustainability of collusive practices in two-sided markets.

<sup>8</sup>Note that this utility is gross of any opportunity cost of visiting platform  $i$ . Its specification is based on a updated version of the model introduced in Armstrong & Wright (2007).

Hence, an agent of group 1 enjoys a benefit  $a_1$  from the presence of each agent on the other market side and therefore a total indirect benefit of  $a_1$  times the number of customers  $n_2^i$  which join platform  $i$  on side 2.<sup>9</sup> The consumer's utility is reduced by the lump-sum price  $p_1^i$  charged for using platform  $i$ . Finally,  $k$  describes the intrinsic benefit from joining a platform, e.g. the utility from content provided by a newspaper, and is assumed to be high enough to guarantee that all agents on either market side wish to subscribe to a platform in the competitive equilibrium.<sup>10</sup>

Using these utility functions, demand for  $i$  is derived by assuming that platforms compete in a Hotelling world: A unit mass of agents on side  $i$  is uniformly distributed along a unit interval and each agent is assumed to purchase one single unit. Platforms  $A$  and  $B$  are located at positions 0 and 1 of both lines, respectively. The product differentiation or transport cost parameters are labelled  $t_1, t_2 > 0$  for side 1 and side 2, respectively. The differentiation or transport cost function is assumed to be linear,<sup>11</sup> i.e. a type- $j$  customer located at  $x \in [0, 1]$  incurs differentiation costs of  $t_j |0 - x|$  if joining platform  $A$  and  $t_j |1 - x|$  if joining platform  $B$ . According to this Hotelling specification, market shares for platform  $i$  are given as follows:<sup>12</sup>

$$n_1^i = \frac{1}{2} + \frac{a_1(p_2^j - p_2^i) + t_2(p_1^j - p_1^i)}{2(t_1 t_2 - a_1 a_2)} ; n_2^i = \frac{1}{2} + \frac{a_2(p_1^j - p_1^i) + t_1(p_2^j - p_2^i)}{2(t_1 t_2 - a_1 a_2)} \quad (1)$$

$i, j \in \{A, B\} ; i \neq j$

In line with Armstrong (2006), I will assume that network externality parameters  $a_1$  and  $a_2$  are small enough in comparison to differentiation parameters  $t_1$  and  $t_2$  for a market-sharing equilibria to exist. In other words, the following sufficient and necessary condition must be fulfilled to guarantee that both platforms will be active in the competitive equilibrium:

**Assumption 1.**  $4t_1 t_2 > (a_1 + a_2)^2$

Given this demand specification, I will analyse a standard infinitely repeated price game where both platforms choose prices simultaneously in each period and discount their profits with a common discount factor  $\delta$ . To evaluate the sustainability of collusion, the critical discount factor above which a collusive agreement on monopoly prices on both sides can be supported by a grim trigger strategy is derived.<sup>13</sup> This critical discount factor equates the sum of discounted future losses and the one-time gain of an optimal defection from the collusive agreement. For a one-sided market, i.e. hor-

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<sup>9</sup>For a possible micro foundation of this reduced-form network benefit structure, see for example Belleflamme & Peitz (2010).

<sup>10</sup>Explicit thresholds for  $k$  will be defined later on.

<sup>11</sup>The results presented in section 3 are robust against a switch to quadratic transport costs.

<sup>12</sup>For a detailed derivation of these market shares see Armstrong (2006).

<sup>13</sup>As introduced by Friedman (1971), this strategy states that firms set prices at the monopoly level in period 1 and stick to this price in all following periods as long as both firms adhered to the agreement in the past. If either platform deviates from this collusive price, both immediately revert to the static Nash equilibrium in prices and keep this price forever. In case of two-sided markets, such a grim-trigger strategy will imply that platforms set monopoly prices on both market sides and stick to them as long as no deviation is observed on either side. In case of defection, Nash reversion on both market sides is assumed.

izontally differentiated firms which sell only one product to one group of customers, this collusive game has been analyzed by Chang (1991).

Finally, to simplify the analysis and to allow for clear-cut comparative statics results on the critical discount factor, I assume from now on that the transport cost parameter is identical on both sides, i.e.  $t_1 = t_2 = t$ , and that platforms' costs of production on either side are equal to zero.<sup>14</sup>

### 3 Collusion in Two-Sided Markets

To analyse whether the presence of indirect network externalities makes collusion easier or harder to sustain, its impact on the critical discount factor  $\hat{\delta}$  above which monopoly prices can be sustained as a subgame-perfect Nash equilibrium in the repeated game is investigated. The critical discount factor results from solving both platforms' incentive constraints as shown in equation (2) with equality:

$$\underbrace{\frac{\delta}{1-\delta}(\pi_i^C - \pi_i^N)}_{\text{sum of discounted future losses}} \geq \underbrace{(\pi_i^D - \pi_i^C)}_{\text{one-time gain from optimal defection}} \Rightarrow \hat{\delta} = \frac{(\pi_i^D - \pi_i^C)}{(\pi_i^D - \pi_i^N)} \quad (2)$$

with  $i \in \{A, B\}$

For all discount factors above  $\hat{\delta}$ , platforms find it more profitable to collude on monopoly prices than to deviate. In other words, the one-time gain of optimal defection from the collusive agreement is smaller than the sum of forgone profits which would result from Nash reversion in all future. If  $\hat{\delta}$  increases in response to an increase in one side's network parameter, then collusion becomes harder when this side values its opposing market participants more highly. On the contrary, if  $\hat{\delta}$  decreases in a network benefit parameter, then collusion is more likely as network benefits become greater. It is important to note that such an overall effect needs to be disentangled to fully understand its mechanism. Therefore, I will firstly analyse the effect of growing indirect network effects on a platform's gain from colluding and secondly on its incentive to deviate before summing up both effects in  $\hat{\delta}$ .

For the above described game, each platform's punishment profit is given by its stage game Nash profit. Rearranging first order conditions, I obtain:

$$p_1^N = t - \frac{a_2}{t} (a_1 + p_2^N) ; \quad p_2^N = t - \frac{a_1}{t} (a_2 + p_1^N) \quad (3)$$

Note that the classical Hotelling price on each side, which would be equal to  $t$ , is reduced by the external benefit a platform enjoys from attracting one additional consumer on this side. Solving those two simultaneous equations yields the symmetric Nash equilibrium prices  $p_1^N = t - a_2$  and  $p_2^N = t - a_1$ . Thus, one side of the market will be targeted more aggressively than the other if that side's consumers impose larger external benefits on the other side's consumers than vice versa.

In order to guarantee that both market sides are fully covered given the Nash equilibrium prices of the stage game, I will assume that the intrinsic utility level  $k$  is

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<sup>14</sup>Numerical analysis indicates that the obtained results are robust to relaxing those assumptions.

large enough throughout the remaining analysis.

**Assumption 2.**  $\infty > k \geq \max\{\frac{3}{2}t - \frac{a_1}{2} - a_2, \frac{3}{2}t - \frac{a_2}{2} - a_1\}$

Under this assumption, each platform gains a fifty percent market share on either side and Nash profits are given by:

$$\pi^N = \frac{1}{2}p_1^N + \frac{1}{2}p_2^N = t - \frac{a_1 + a_2}{2} \quad (4)$$

Hence, punishment profits fall as network externalities increase while they increase when product differentiation gets stronger.

The maximum collusive prices platforms can set in a cartel correspond to those prices which set the indifferent consumer's utility equal to zero on each side. Since platforms are located symmetrically, monopoly prices differ only with respect to network externality parameters, namely  $p_i^C = k - \frac{t}{2} + \frac{a_i}{2}$  with  $i = 1, 2$ . Collusive profits follow directly if firms split both market sides equally:

$$\pi^C = k - \frac{t}{2} + \frac{a_1 + a_2}{4} \quad (5)$$

Summing up, it is easy to see that a platform's gain from colluding ( $\pi^C - \pi^N$ ) is increasing in both network externality parameters and the following lemma can be stated.

**Lemma 1.** *A two-sided platform's incentive to collude on prices increases as indirect network externalities become stronger. Firstly, increasing network benefits on one side reduce Nash prices and thereby profits earned on the opposing market side. Secondly, increasing network benefits allow for a higher maximum collusive price and thereby larger profits on the respective market side.*

In the newspaper industry example, this result would imply that competing newspapers would earn higher collusive profits if network effects grow as market cartelization enables them to skim away all utility from readers and advertisers. The more an advertiser cares about the number of readers seeing his advertisement, the higher his reservation price. The less readers dislike ads (or the more they like them) the higher is the cover price they are willing to pay. In contrast, competitive Nash profits are decreasing in the indirect network effects because platforms have to price in any external benefit they earn from attracting additional consumers. If, for example, readers become more valuable to advertisers over time, newspapers demand a lower cover price in order to attract more of those readers. Overall, newspapers' incentives to collude are the higher the more advertisers and readers value each other.

Counteracting the above described effect, firms always have an incentive to deviate from the collusive agreement in order to earn larger stage-game profits. The optimal defection strategy of platform  $i$  is given by its price reaction functions  $R_1^i$  and  $R_2^i$ :

$$R_1^i(p_1^j, p_2^i, p_2^j) = \begin{cases} \frac{p_1^j + t}{2} + a_1 \left( \frac{p_2^j - p_2^i}{2t} \right) - \frac{a_2}{t} \left( \frac{a_1 + p_2^i}{2} \right) & \text{if } p_1^j < 3t - \frac{(a_1 + a_2)^2}{2t} - \frac{(a_1 + a_2)(t + a_1 + p_2^j)}{2t} \\ p_1^j - t + \frac{a_1 a_2}{t} + \frac{a_1}{t} (p_2^j - p_2^i) & \text{if } p_1^j \geq 3t - \frac{(a_1 + a_2)^2}{2t} - \frac{(a_1 + a_2)(t + a_1 + p_2^j)}{2t} \end{cases} \quad (6)$$



$$R_2^i(p_1^i, p_1^j, p_2^j) = \begin{cases} \frac{p_2^j+t}{2} + a_2 \left( \frac{p_1^j-p_1^i}{2t} \right) - \frac{a_1}{t} \left( \frac{a_2+p_1^i}{2} \right) & \text{if } p_2^j < 3t - \frac{(a_1+a_2)^2}{2t} - \frac{(a_1+a_2)(t+a_2+p_1^j)}{2t} \\ p_2^j - t + \frac{a_1 a_2}{t} + \frac{a_2}{t} (p_1^j - p_1^i) & \text{if } p_2^j \geq 3t - \frac{(a_1+a_2)^2}{2t} - \frac{(a_1+a_2)(t+a_2+p_1^j)}{2t} \end{cases} \quad (7)$$

$i, j \in \{A, B\} ; i \neq j$

First, note that the reaction function of platform  $i$  on side 1 depends on both prices of her competitor as well as on her own price on the opposite market side. Furthermore, both reaction functions consist of two parts, which is due to the fact that it might be an optimal reaction for platform  $i$  to monopolize one or even both market sides when prices set by the competitor are high enough. In case that monopolization is optimal, platform  $i$  will choose the lowest price that allows her to gain a market share equal to one on the corresponding market side. In the following, I interpret those reaction functions in more detail to shed some light on the influence of network effects on the optimal defection strategy.<sup>15</sup>

**Case 1 - No market monopolization** If the competitor's prices  $p_1^j$  and  $p_2^j$  are low enough, platform  $i$ 's optimal deviation corresponds to the first line of equation (6). The first part of this term,  $\frac{p_1^j+t}{2}$ , corresponds to the classical Hotelling reaction function, i.e. the optimal price  $i$  imposes on side-1 customers depends positively on the differentiation parameter  $t$  and on the price of her competitor on this side. In addition, however, the optimal side-1 price is influenced by the second market side through existing indirect network externalities. *Assume for a moment that network externalities  $a_1$  and  $a_2$  are positive.* Then, the second term of equation (6),  $a_1 \left( \frac{p_2^j-p_2^i}{2t} \right)$ , indicates that an increase in the price differential between platform  $i$  and her competitor on the opposite market side increases  $i$ 's price on side 1. First, a price advantage of  $(p_2^j - p_2^i) > 0$  increases demand for  $i$  on side 2 by  $1/2t$ . This, in return, raises a consumer's utility on side 1 by  $a_1$  times the demand increase. Hence, platform  $i$  can charge a higher price on side 1 when it is less expensive than her competitor on side 2. The third term in equation (6) measures the external benefit of a decrease in  $p_1^i$ . Suppose that  $p_1^i$  is decreased by the amount which causes an additional type-1 consumer to join  $i$ . In return, this will attract  $a_2/t$  additional type-2 consumers. Those additional type-2 consumers will generate an extra revenue of  $p_2^i$  times  $a_2/t$ . Furthermore, they increase a type-1 consumer's utility by  $a_1$  and thus the revenue which can be extracted on side 1. Summing up, the larger the external benefit becomes, the smaller will be the optimal price reaction on side 1. *If network externalities are both negative,* then the second term of equation (6) will be negative if  $(p_2^j - p_2^i) > 0$ , i.e. a price advantage on market side 2 will decrease the optimal price on side 1. The overall influence of the third term - the external benefit - is ambiguous and depends on the size of  $a_1$  and  $p_2^i$ . In general, equations (6) and (7) show that platforms' best responses on one side might depend positively or negatively on its own price set on the opposite market side. For given rival's prices, platform  $i$ 's side-1 reaction function will be decreasing in  $i$ 's side-2 price if the total external benefit  $(a_1 + a_2)$  that consumers enjoy is positive. In contrast, when consumers' total network benefit is negative, i.e. when  $(a_1 + a_2) < 0$ , then the price reaction function for side 1 is increasing in  $p_2^i$ . Hence, a positive network externality  $a_1$  of consumers on market

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<sup>15</sup>Since both reaction functions are symmetric, interpretation will be limited to the side-1 reaction function.

side 1 implies that  $R_1^i$  is an increasing function of prices set by the competitor, but it will only increase in  $p_2^i$  if consumers on side 2 have a negative externality which is larger than  $a_1$  in absolute terms. If consumers on side 2 also have a positive indirect network valuation, then platform  $i$  will always reduce its side-1 price in reaction to an increase in  $p_2^i$ .

**Case 2 - market monopolization** If the competitor's prices are so high that monopolization of market side 1 is the best response, then the second line of equation (6) indicates the optimal price choice. Its first part,  $p_1^j - t$ , is once again equal to the classical Hotelling reaction function. The second part, however, indicates that the maximum price which guarantees market monopolization on side 1 depends on the other market side as well. If  $a_1$  and  $a_2$  have the same sign, then  $p_1^i$  will be increased by  $a_1 a_2 / t$  in comparison to a market without network externalities. In addition, a positive network effect on side 1 allows to further increase the optimal price if platform  $i$ 's price on side 2 is lower than the one of her competitor. As it has been the case for the optimal reaction without market monopolization, a better price on the market side 2 increases demand on this side, which in return allows to charge higher prices on side-1 consumers. Finally, if  $a_1 > 0$ , then the side-1 reaction function of platform  $i$  is decreasing in  $p_2^i$ , i.e. if consumers have a positive network benefit, then a price increase on one market side which causes the number of customers on this side to fall, has to be compensated by a price decrease on the opposite side to avoid that those consumers also switch to the competitor because of their loss in utility.

From the above remarks on optimal deviation strategies, it is easy to see that the impact of network externalities on deviation prices and the resulting deviation profits is not as clear cut as it is on collusive or Nash profits. Instead, equations (6) and (7) imply that deviation prices might be increasing or decreasing in  $a_1$  and  $a_2$ . To figure out if deviation profits in total will effectively rise or fall in response to a change in network effects, the following two-step procedure is chosen. First, focus will be on the case when network externalities are symmetric, namely when  $a_1 = a_2 = a$ . Taking the results for symmetric externalities as a benchmark, the second step will be to investigate the impact of an increasing asymmetry between network externalities.

### 3.1 Symmetric Externalities

When network externalities are symmetric, solving both reaction functions simultaneously under the assumption that the rival platform sticks to collusive prices yields the following simple expressions:

$$R_1^i(p_1^C) = \begin{cases} \frac{p_1^C + t}{2} - \frac{a}{2} & \text{if } k < \frac{7}{2}t - \frac{7}{2}a \\ p_1^C - t + a & \text{if } k \geq \frac{7}{2}t - \frac{7}{2}a \end{cases}$$

$$R_2^i(p_2^C) = \begin{cases} \frac{p_2^C + t}{2} - \frac{a}{2} & \text{if } k < \frac{7}{2}t - \frac{7}{2}a \\ p_2^C - t + a & \text{if } k \geq \frac{7}{2}t - \frac{7}{2}a \end{cases} \quad \text{with } i, j \in \{A, B\}$$

In essence, the standard Hotelling price reaction is decreased by the external ben-

efit from attracting an extra agent on the opposite side. The influence of prices on the other market side as shown in equations (6) and (7), however, fully cancels out in case of symmetric externalities. Note that if collusive prices are high enough or, as stated in equations (6) and (7), if the intrinsic utility  $k$  is low enough compared to the difference between transport costs and network benefits, a deviating platform will find it profitable to monopolize both market sides.

Plugging those prices as well as monopoly prices for the rival platform into the demand equations, deviation profits are derived as follows:<sup>16</sup>

$$\pi_{SE}^D = \begin{cases} \frac{(k + \frac{t}{2} - \frac{a}{2})^2}{4(t-a)} & \text{if } k < \frac{7}{2}t - \frac{7}{2}a \\ 2k - 3t + 3a & \text{if } k \geq \frac{7}{2}t - \frac{7}{2}a \end{cases} \quad (8)$$

Equation (8) is an increasing function of the network externality parameter  $a$ . In other words, if consumers on one side value consumers' presence on the other side more highly, then platforms earn higher profits from optimal defection. While prices react less strongly as network effects increase, the demand reactions are higher. Overall, deviation becomes more profitable. Taking the derivative of  $(\pi^D - \pi^C)$  with respect to  $a$ , it is easy to show that platforms' one-time gain from defection is increasing in the indirect network externality parameter  $a$  if assumption 1 is fulfilled - a result which is summarized in the next lemma.

**Lemma 2.** *A two-sided platform's gain from defection increases as indirect network externalities become stronger. While deviation prices fall, demand reacts more sensitively leading to an increase in deviation profits that outweigh the increase in collusive profits due to stronger network effects.*

Recalling lemma 1, one can conclude that increasing network benefits have two opposing effects. In consequence, the overall impact of an increase in  $a$  on collusive sustainability depends on whether the increased deviation incentive is dominated by larger gains from colluding or vice versa. Solving equation 2 with equality for  $\delta$  yields the critical discount factor  $\hat{\delta}_{SE}$ :

$$\hat{\delta}_{SE} = \frac{(\pi_{SE}^D - \pi_{SE}^C)}{(\pi_{SE}^D - \pi_{SE}^N)} = \begin{cases} \frac{2k - 3t + 3a}{2k + 5t - 5a} & \text{if } k < \frac{7}{2}t - \frac{7}{2}a \\ \frac{2k - 5t + 5a}{4k - 8t + 8a} & \text{if } k \geq \frac{7}{2}t - \frac{7}{2}a \end{cases} \quad (9)$$

As comparative statics show,  $\hat{\delta}_{SE}$  is increasing in  $a$  if  $k > 0$ .<sup>17</sup> I can therefore sum up the above analysis in the following proposition.

**Proposition 1.** *For a given increase in indirect network externalities the rising incentive to collude is always dominated by larger gains from optimal defection. Thus, the critical discount factor above which monopoly prices can be sustained in a two-sided market increases as symmetric network externalities become stronger.*

<sup>16</sup>Note that for symmetric network effects, Nash and collusive prices are identical on both market sides, namely  $p^N = t - a$  and  $p^C = k - \frac{t}{2} + \frac{a}{2}$ . Hence, Nash and collusive profits are just given by the respective prices, i.e.  $\pi_{SE}^N = t - a$  and  $\pi_{SE}^C = k - \frac{t}{2} + \frac{a}{2}$ .

<sup>17</sup>Since assumption 1 simplifies to  $t^2 > a^2$  in case of symmetric externalities, it follows directly from assumption 2 that  $k$  must be larger than zero.

For given intrinsic utility levels and transport cost parameters,  $\hat{\delta}_{SE}$  is plotted as a function of  $a$  in figure 1.

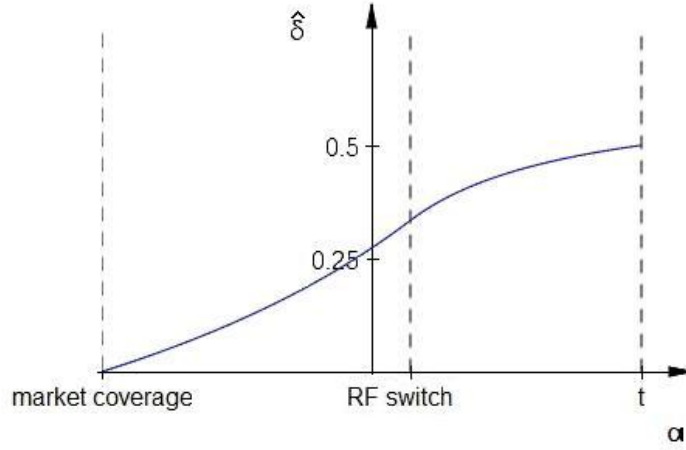


Figure 1: critical discount factor for symmetric externalities

As this graph illustrates,  $\hat{\delta}_{SE}$  increases monotonically in  $a$ . At the lowest possible value for  $a$  which still satisfies assumption 2 (full market coverage), maximum collusive profits are equal to Nash profits and, as a natural consequence, sustainable for all discount factors between 0 and 1. Hence,  $\hat{\delta}_{SE}$  is equal to zero. The maximum feasible value for  $a$  is given by assumption 1 which guarantees existence of a market sharing equilibrium. In case of symmetric externalities, this assumption simplifies to  $t > |a|$ . For  $a \rightarrow t$ ,  $\hat{\delta}_{SE}$  converges to the critical discount factor for homogenous goods Bertrand competition,  $\hat{\delta}_{SE} \rightarrow \frac{1}{2}$ . The vertical line named "RF switch" refers to the value of  $a$  at which optimal defection switches from market sharing to market monopolization.

Finally, the impact of symmetric network externalities on the sustainability of collusion is decreasing in the other model parameters  $k$  and  $t$ . In other words, if the intrinsic utility of platform participation grows or if both platforms become more differentiated,  $\hat{\delta}_{SE}$  responds less strongly to an increase in network effects.

### 3.2 Asymmetric Externalities

Having in mind the result of the previous subsection, the question is whether collusion will also be harder to sustain as network externalities grow if those externalities are asymmetric. To this end, it is assumed from now on that  $a_1 = a + \Delta$  and  $a_2 = a - \Delta$  with  $a, \Delta > 0$ . In the newspaper example, this would imply that advertisers (on side 1) care more about readers than readers do about ads. If  $\Delta$  is large enough, it might even be true that readers dislike all advertisements.

Given this network effects specification, equations (6) and (7) imply the following optimal price defection from a collusive agreement with prices equal to  $p_1^C = k - \frac{t}{2} + \frac{a_1}{2}$

and  $p_2^C = k - \frac{t}{2} + \frac{a_2}{2}$ :

$$p_1^i = \begin{cases} \frac{p_1^C + t}{2} + \frac{(a+\Delta)(p_2^C - p_2^i)}{2t} - \frac{a-\Delta}{t} \left( \frac{a+\Delta+p_2^i}{2} \right) & \text{if } k < \frac{7}{2}(t-a) + \frac{\Delta(t-a)}{2(t+a)} \\ p_1^C - t + (a+\Delta) & \text{if } k \geq \frac{7}{2}(t-a) + \frac{\Delta(t-a)}{2(t+a)} \end{cases} \quad (10)$$

$$p_2^i = \begin{cases} \frac{p_2^C + t}{2} + \frac{(a-\Delta)(p_1^C - p_1^i)}{2t} - \frac{a+\Delta}{t} \left( \frac{a-\Delta+p_1^i}{2} \right) & \text{if } k < \frac{7}{2}(t-a) - \frac{\Delta(t-a)}{2(t+a)} \\ p_2^C - \frac{t^2 - (a+\Delta)(a-\Delta)}{t} + \frac{(a-\Delta)(p_1^C - p_1^i)}{t} & \text{if } \frac{7(t-a)}{2} + \frac{\Delta(t-a)}{2(t+a)} > k \geq \frac{7(t-a)}{2} - \frac{\Delta(t-a)}{2(t+a)} \\ p_2^C - t + (a-\Delta) & \text{if } k \geq \frac{7}{2}(t-a) + \frac{\Delta(t-a)}{2(t+a)} \end{cases} \quad (11)$$

$i, j \in \{A, B\}; i \neq j$

Since  $\Delta > 0$ , it might be the case that the intrinsic utility parameter  $k$  is of such size that it is optimal to conquer all of market side 2 while still sharing market side 1 with the competitor. The opposing case, however, is never optimal. If  $k$  is above a certain threshold, namely if  $k \geq \frac{7}{2}(t-a) + \frac{\Delta(t-a)}{2(t+a)}$ , the deviating platform will monopolize both market sides. Using (10) and (11) and plugging in collusive prices, deviation profits are calculated as follows:

$$\pi_{ASE}^D = \begin{cases} \frac{(k + \frac{t}{2} - \frac{a}{2})^2}{4(t-a)} + \frac{\Delta^2}{16(t+a)} & \text{if } k < \frac{7}{2}(t-a) - \frac{\Delta(t-a)}{2(t+a)} \\ p_1^D \left( \frac{1}{2} + \frac{(a+\Delta)(p_2^C - p_2^D) + t(p_1^C - p_1^D)}{2(t^2 - (a+\Delta)(a-\Delta))} \right) + p_2^D & \text{if } \frac{7(t-a)}{2} + \frac{\Delta(t-a)}{2(t+a)} > k \geq \frac{7(t-a)}{2} - \frac{\Delta(t-a)}{2(t+a)} \\ 2k - 3t + 3a & \text{if } k \geq \frac{7}{2}(t-a) + \frac{\Delta(t-a)}{2(t+a)} \end{cases}$$

It is easy to show that deviation profits are non-decreasing in  $a$  as well as in  $\Delta$  for  $k < \frac{7}{2}(t-a) - \frac{\Delta(t-a)}{2(t+a)}$ , i.e. when neither market side is monopolized. Moreover, in the extreme case of deviation to monopolization on both sides, deviation profits do not depend on the asymmetry of network externalities at all, but they increase in  $a$ . In the intermediate case when it is optimal to monopolize market side 2 but to still share side 1, deviation profits are very complex. This is due to the asymmetry between both sides' reaction functions. The exact term is not exposed here, but numerical analysis indicates that deviation profits are still increasing in  $a$  while its dependence on  $\Delta$  is non-monotonic.<sup>18</sup> Summing up, platforms' incentive to deviate from the collusive agreement grows as network externalities between both market sides become more asymmetric given that collusive prices are not too large, i.e. if  $k < \frac{7}{2}(t-a) + \frac{\Delta(t-a)}{2(t+a)}$ . Hence, if readers dislike ads while advertisers value a large readership, deviation from a cartel between newspapers is more attractive than when both readers and advertisers value each other by the same amount - given that the total external benefit  $a$  is fixed.

In order to draw a conclusion concerning the total effect on collusive sustainability, it is important to note that collusive and Nash profits did not change compared to the symmetric case. Nash prices, however, are now given by  $p_1^N = t - a + \Delta$  and  $p_2^N = t - a - \Delta$  while maximum collusive prices changed to be equal to  $p_1^C = k + \frac{a+\Delta}{2} - \frac{t}{2}$  and  $p_2^C = k + \frac{a-\Delta}{2} - \frac{t}{2}$ . Since a platform's gain from collusion ( $\pi^C - \pi^N$ ) has not changed, the left-hand side of the incentive constraint described in equation (2) stays constant

<sup>18</sup>A detailed analysis is available from the author upon request. Please note that I am still working on the analysis of  $\Delta$ 's influence on  $\pi^D$  in this intermediate case.

while the right-hand side increases. The critical discount factor  $\hat{\delta}_{ASE}$  which solves this constraint with equality must therefore be larger than  $\hat{\delta}_{SE}$ . This leads to the following proposition.

**Proposition 2.** *Assume that  $k < \frac{7}{2}(t - a) - \frac{\Delta(t-a)}{2(t+a)}$ , which ensures that collusive prices are not too high. Then the following result holds true:*

*If the total network benefit which a platform  $i$  can extract from its customers on both sides does not change but the difference between the externality parameters  $a_1$  and  $a_2$  increases, then the incentive to deviate from the collusive agreement increases while the gains from collusion stay constant. Thus, the more asymmetric network effects become, the higher the critical discount factor gets and the less likely it is for platforms to form a cartel.*

Note that the effect of  $\Delta$  on deviation profits and thus on the critical discount factor is stronger if both the transport cost parameter and the total network benefit  $a$  are small.

### 3.3 Discussion

An increase in indirect network externalities has two opposing effects on collusion. First, if one side values members on the other market side more highly, Nash prices fall because competition for this side gets harsher. As a consequence, punishment profits are a falling function of network effects. In addition, consumers' utility from platform participation increases if they enjoy a larger benefit from the presence of platform members on the opposing market side. Hence, two-sided platforms can earn larger collusive profits as network externalities grow. Second and countervailing, however, platforms also earn larger profits from deviation as network effects become stronger - a result which is mainly due to more sensitive demand reactions. Comparing those opposing effects and solving for the critical discount factor, it can be shown that the latter effect always dominates for the specific framework of a two-sided single-homing Hotelling model. Collusion becomes harder as network externalities grow. Furthermore, increasing asymmetry between both sides for a given total network benefit has a negative impact on collusive sustainability. This is due to the fact that the external benefits of a price decrease are smaller when network externalities are asymmetric rather than symmetric. In consequence, the deviation price on side 1 increases by a larger amount than the deviation price on side 2 falls after an increase of the network benefit parameter  $a_1$  by  $\Delta$  and an equally sized reduction in  $a_2$ . In contrast, demand reactions to these changes of size  $\Delta$  are perfectly symmetric: deviation demand on side 1 falls while side-2 demand increases by the same amount. In sum, deviation profits are higher than under symmetric network externalities.

The above results are in line with Evans & Schmalensee's hypothesis that collusion is harder to sustain in two-sided markets. In contrast to their argumentation, however, my results do not hinge on increased monitoring or coordination costs. The result that collusion becomes harder as network effects grow rather follows from balancing the two opposing effects of increased network externalities. While it might be due to the chosen framework that the deviation effect always dominates, I would like to argue that the existence of those two countervailing effects caused by network externalities

in a two-sided market - namely higher gains from colluding as well as larger benefits from defection - is of a more general nature.

Another way to interpret my results would be to compare the impact of indirect network effects in my framework to a situation where platforms are compatible.<sup>19</sup> In the latter case, users of one market side enjoy a network benefit from all consumers on the other market independent of their platform choice. In consequence, a platform's market share on one side of the market does not influence consumers decision making on the other - the only things that matter are a consumer's location on the respective Hotelling line and the price he faces on his market side. In consequence, a platform's incentive to deviate from the collusive agreement is much smaller in case of compatibility because of the lacking feedback effects. Thus, collusion will always be harder to sustain in case of no compatibility whereas the scope for collusion is larger when platforms do not have to compete for indirect network externalities.

## 4 Special Cases and Robustness Checks

In the following subsections, the robustness of my main result that the critical discount factor is increasing both in the size and in the asymmetry of indirect network externalities is tested against special types of two-sided market structures, the possibility to collude only on one market side and the introduction of optimal symmetric punishment schemes.

### 4.1 Flexible prices only on one market side

This case is a common phenomenon in several media markets, such as free-to-air TV channels or free newspapers. In these industries, advertisers gain utility from a large viewer- or readership. Thus, their network parameter is positive and they are willing to pay a positive price for accessing readers. Readers or viewers, on the other hand, very often dislike ads.<sup>20</sup> As a consequence, newspapers or TV channels would sometimes even like to subsidize them in order to attract enough readers or viewers to maximize profits gained from advertisers. Negative prices, however, are seldomly feasible and this subsection serves to investigate what happens to collusive sustainability if prices on readers' market side are fixed to be zero. More precisely, suppose w.l.o.g. that  $a_1 > 0$ ,  $a_2 < 0$  and  $p_2^A = p_2^B = 0$ .<sup>21</sup>

Given these assumptions, demand simplifies to:

$$n_1^i = \frac{1}{2} + \frac{t(p_1^j - p_1^i)}{2(t^2 - a_1 a_2)} ; n_2^i = \frac{1}{2} + \frac{a_2(p_1^j - p_1^i)}{2(t^2 - a_1 a_2)} ; i, j \in \{A, B\} ; i \neq j \quad (12)$$

<sup>19</sup>See Katz & Shapiro (1985) for a seminal contribution on network externalities and compatibility.

<sup>20</sup>See for example Wilbur (2008) who finds that consumers are strongly advertising averse in the television industry.

<sup>21</sup>Armstrong & Wright (2007) analyse the case when non-negative prices are not allowed and find Nash prices identical to the ones presented below. They assume, however, that network externalities are both positive.

Note that demand on both sides is influenced by the price difference on side 1. While platforms can earn profits only on side 1, they have to take into account the positive external benefit an additional consumer on side 2 imposes on side-1 consumers when making their pricing decision. In consequence, the side-1 demand a platform faces depends more strongly on this side's price difference than the classical Hotelling demand.

Nash prices in this special case of the framework are equal to  $p_1^N = t - \frac{a_1 a_2}{t}$  and profits amount to  $\pi^N = \frac{t}{2} - \frac{a_1 a_2}{2t}$ . Hence, the Hotelling price  $t$  is increased by the negative external benefit from attracting an extra side-1 agent.<sup>22</sup> Collusive profits follow from setting  $p_1^C$  equal to  $k - \frac{t}{2} + \frac{a_1}{2}$  and thereby extracting all utility from the indifferent advertiser on side 1 and is given by  $\pi_C = \frac{k}{2} - \frac{t}{4} + \frac{a_1}{4}$ . Thus, collusive profits increase if advertisers value readers more highly. The effect of a higher readership valuation on Nash profits, however, is different from section 3. For a given negative externality that ads impose on readers, the price a newspaper demands from advertisers rises in  $a_1$ . As a consequence, the gain from colluding only increases in advertisers' benefit from readers if readers do not hate ads too much. On the other hand, the gain from colluding always decreases if readers dislike ads more strongly. In this case, newspapers can demand a higher competitive ad price while collusive prices stay constant.

Turning to a newspaper's incentive to deviate from the collusive agreement, deviation prices can be inferred from a platform's reaction function which is given as follows

$$p_1^i = \begin{cases} \frac{p_1^C + t}{2} - \frac{a_1 a_2}{2t} & \text{if } p_1^C < \frac{3(t^2 - a_1 a_2)}{t} \\ p_1^C - t + \frac{a_1 a_2}{t} & \text{if } p_1^C \geq \frac{3(t^2 - a_1 a_2)}{t} \end{cases} \quad (13)$$

Plugging in collusive prices and taking first derivatives, it is easy to show that deviation prices are not monotonic in either network effect. As a consequence, the critical discount factor  $\hat{\delta}_{PF}$  might not be monotonically increasing in the network externality parameters, too. In other words, if one price is fixed to be equal to zero, it might be the case that collusion becomes easier to sustain as network externalities become stronger.

Noticing that  $\hat{\delta}_{PF}$  is given by the following equation:

$$\hat{\delta}_{PF} = \begin{cases} \frac{2k - 3t + a_1 + 2\frac{a_1 a_2}{t}}{2k + 5t + a_1 - 6\frac{a_1 a_2}{t}} & \text{if } p_1^C < \frac{3(t^2 - a_1 a_2)}{t} \\ \frac{2k - 5t + a_1 + 4\frac{a_1 a_2}{t}}{4k - 8t + 2a_1 + 6\frac{a_1 a_2}{t}} & \text{if } p_1^C \geq \frac{3(t^2 - a_1 a_2)}{t} \end{cases} \quad (14)$$

it is straightforward to show that  $\hat{\delta}_{PF}$  increases monotonically in  $a_2$ , but is non-monotonic in  $a_1$ . If  $a_2 < \frac{t^2}{t-2k}$ , then  $\hat{\delta}_{PF}$  falls in  $a_1$ , whereas it increases in  $a_1$  for  $a_2 > \frac{t^2}{t-2k}$ . Put differently, the less readers dislike ads, the harder it is for newspapers to collude. Further, if readers' distaste of ads is not too high, then collusion is less likely

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<sup>22</sup>In case of no price on side 2, the external benefit simply amounts to the additional benefit  $a_1$  each side-1 agent enjoys from one more agent on the other side, i.e. the extra revenue a newspaper can extract from its advertisers. This amount is multiplied by the fraction  $a_2/t$  of readers which are lost by one more ad displayed in the newspaper.



when advertisers gain a larger utility from their readership. If, however, readers hate to read ads very much, collusion actually becomes easier when advertisers' valuation of readers increases. The following remark summarizes these findings:

**Remark 1.** *If prices on one market side are fixed to zero and network externalities on this side are negative whereas they are positive on the opposite market side, then collusion might become easier to sustain as the positive network effect grows as long as the the negative network externality is large enough in absolute value.*

The intuition for this result, which differs from the one presented in section 3, hinges on the fact that readers cannot be priced. Thus, newspapers or free-to-air TV channels have to earn all their money from advertisers. When readers dislike ads, they will prefer to join a platform which has few advertisers. Hence, as  $a_1$  increases, newspapers might prefer to collude and share the number of advertisers, rather than to deviate and loose readers. The result, however, depends on the fact that full market coverage is assumed for both market sides, i.e. readers always buy one of the newspapers no matter how much they dislike the advertisements they will have to face.

## 4.2 One-way network externalities

There might be two-sided markets where only one side values platform participants on the opposing side. Argentesi & Filistrucchi (2007) argue that this is actually the case for Italian newspapers where readers do not care about the amount of advertising. This finding is also confirmed for the Belgian newspaper market by Van Cayseele & Vanormelingen (2009). Furthermore, this special case of asymmetric externalities allows me to analyse what happens to collusive sustainability when monopoly prices are so high that the defector finds it optimal to fully conquer one of the market sides - a case which is not analysed in the more general framework of section 3.2.

Suppose in the following that network externalities on side 2 are equal to zero while they amount to  $2a > 0$  on side 1.<sup>23</sup> This implies that the maximum external benefit all consumers can enjoy is the same as in section 3, although competition on side 2 is now reduced to a standard Hotelling model. Demand functions change accordingly to:

$$n_1^i = \frac{1}{2} + \frac{2a(p_2^j - p_2^i) + t(p_1^j - p_1^i)}{2t^2} ; n_2^i = \frac{1}{2} + \frac{(p_2^j - p_2^i)}{2t} \quad i, j \in \{A, B\} ; i \neq j$$

As a consequence, Nash prices are given by  $p_1^N = t$  and  $p_2^N = t - 2a$  and punishment profits amount to  $\pi^N = t - a$ . Maximum collusive prices follow from skimming away all utility of the indifferent consumer on either side, i.e.  $p_1^C = k + a - \frac{t}{2}$  and  $p_2^C = k - \frac{t}{2}$ , and yield a profit of  $\pi^C = k + \frac{a}{2} - \frac{t}{2}$ . It is thus straightforward to see that platforms' incentive to collude increases if side 1 values the other side more strongly.

Turning to the incentive to deviate, reaction functions change accordingly to:

$$p_1^i = \begin{cases} \frac{p_1^j + t}{2} + \frac{a(p_2^j - p_2^i)}{t} & \text{if } k < \frac{7}{2}(t - a) + \frac{a(t-a)}{2(t+a)} \\ p_1^j - t + 2a & \text{if } k \geq \frac{7}{2}(t - a) + \frac{a(t-a)}{2(t+a)} \end{cases} \quad (15)$$

<sup>23</sup>This case results from assuming  $\Delta = a$  in the framework of section 3.2.

$$p_2^i = \begin{cases} \frac{p_2^j + t}{2} - \frac{ap_1^i}{t} & \text{if } k < \frac{7}{2}(t-a) - \frac{a(t-a)}{2(t+a)} \\ p_2^j - t & \text{if } k \geq \frac{7}{2}(t-a) - \frac{a(t-a)}{2(t+a)} \end{cases} \quad (16)$$

Since  $a > 0$ , it might be profitable to conquer all of market side 2 while still sharing side 1 for intermediate levels of  $k$ . In consequence, deviation profits consist of three parts, which are all increasing in the network effect  $a$ :

$$\pi_{OWE}^D = \begin{cases} \frac{(k + \frac{t}{2} - \frac{a}{2})^2}{4(t-a)} + \frac{a^2}{16(t+a)} & \text{if } k < \frac{7}{2}(t-a) - \frac{a(t-a)}{2(t+a)} \\ k - \frac{3}{2}t + \frac{(k + 3a + \frac{t}{2})^2}{8t} & \text{if } \frac{7}{2}t - 3a > k \geq \frac{7}{2}(t-a) - \frac{a(t-a)}{2(t+a)} \\ 2k + 3a - 3t & \text{if } k \geq \frac{7}{2}t - 3a \end{cases} \quad (17)$$

Summing up, a platform's gain from defection rises if side-1 customers value their market opponents more strongly. Furthermore, tedious calculations which are relegated to the appendix show that deviation profits as given in equation (17) are always larger than those for symmetric externalities as expressed in equation (8) as long as the optimal deviation strategy does not ask for monopolization of both market sides. Thus, the following remark can be stated.

**Remark 2.** *A one-sided positive network effect which amounts to the same maximum external benefit as two-sided symmetric network externalities makes collusion harder to sustain than those symmetric network effects. Furthermore, it still holds true that the critical discount factor is an increasing function of the indirect network effect, i.e.  $\frac{\partial \hat{\delta}_{OE}}{\partial a} > 0$ .*

### 4.3 One-sided price collusion

Some of the cartel cases described in the introduction imply that platforms might find it profitable to collude only on one market side. One might thus ask whether this form of collusion might actually be easier to implement than full collusion on both sides. Furthermore, analyzing one-sided collusion allows me to evaluate whether Evans & Schmalensee (2008) have been correct in arguing that all supra-competitive profits earned on one side will be competed away on the other and therefore one-sided collusion should not be profitable at all.

Let me start by analysing the symmetric case of  $a_1 = a_2 = a > 0$  and assuming w.l.o.g. that platforms collude on side 1. If firms collude on the highest possible price given that markets are split equally when making the agreement, they will set  $p_1^{OC} = k + a/2 - t/2$ . On side 2, they will now maximize profits separately taking into account  $p_1^{OC}$ , which yields prices equal to  $p_2^{OC} = t - \frac{a}{t}(a + p_1^{OC})$ . Recalling assumption 2, it is easy to show that  $p_2^{OC}$  is always smaller than  $p_2^N = t - a$ . Thus, platforms do compete away some of the supra-competitive profits earned on side 1 as Evans & Schmalensee (2008) expected it to be the case. Actually, the collusive profit amounts to  $\pi^{OC} = \frac{(t-a)}{2t}(t+a+p_1^{OC}) = \frac{(t-a)}{4t}(2k+3a+t)$ , which is always smaller than the collusive profit when platforms fully collude on both sides. Nevertheless, it is still beneficial to collude, i.e.  $(\pi^{OC} - \pi^N) \geq 0$ , for all feasible parameter values. In contrast to section 3.1, however, the gain from colluding is no longer monotonically increasing in  $a$ , but

instead  $\frac{\partial(\pi^{OC}-\pi^N)}{\partial a} \geq 0$  if  $k \leq 3t - 3a$  while  $\frac{\partial(\pi^{OC}-\pi^N)}{\partial a} < 0$  if  $k > 3t - 3a$ .

Turning to a platform's incentive to deviate, it is important to note that it is never an optimal reaction for the defecting platform to fully conquer market side 2. While it is optimal to decrease prices on market side 1, which in return raises market shares on this side, optimal defection prices on side 2 are actually higher than  $p_2^{OC}$ . In fact, the deviation price on side 2 is increased by such an amount that the corresponding market share of the defecting platform stays constant at 1/2. Hence, reaction functions and deviation profits are given as follows:

$$R_1^D(p_1^{OC}) = \begin{cases} \frac{p_1^{OC} + t}{2} - \frac{a}{2} & \text{if } k < \frac{7}{2}t - \frac{3}{2}a \\ \frac{(t^2 - a^2)}{t^2}(p_1^{OC} - t + a) & \text{if } k \geq \frac{7}{2}t - \frac{3}{2}a \end{cases} \quad (18)$$

$$R_2^D(p_2^{OC}) = \frac{p_2^{OC} + t}{2} - \frac{a}{2} \quad (19)$$

$$\pi_{OC}^D = \begin{cases} \frac{15(t^2 - a^2) + 2t(t - a) + 4k(k + t - a)}{4kt - 4t^2 - 2ka + 5at - 3a^2} & \text{if } k < \frac{7}{2}t - \frac{3}{2}a \\ \frac{32t}{4t} & \text{if } k \geq \frac{7}{2}t - \frac{3}{2}a \end{cases} \quad (20)$$

One can easily show that the gain from deviation ( $\pi_{OC}^D - \pi^{OC}$ ) is an increasing function of  $a$ . Hence, the larger network externalities become, the more profitable it is for a platform to deviate from the one-sided collusive agreement. Balancing this gain from deviation and the incentive to collude yields the critical discount factor  $\hat{\delta}_{OC}$  above which monopoly prices on market side 1 can be sustained:

$$\hat{\delta}_{OC} = \begin{cases} \frac{2k + 3a - 3t}{2k - 5a + 5t} & \text{if } k < \frac{7}{2}t - \frac{3}{2}a \\ \frac{t(2k + 3a - 5t)}{t(2k + 3a - 5t) - a(2k + 3a - t)} & \text{if } k \geq \frac{7}{2}t - \frac{3}{2}a \end{cases} \quad (21)$$

Comparing this discount factor to the one for two-sided collusion (see equation (9)) indicates that sustainability conditions for one- and two-sided collusion are identical if  $k < \frac{7}{2}t - \frac{7}{2}a$ . In other words, when network externalities are small compared to transport costs and intrinsic utility levels, it is not harder to sustain collusion on both sides rather than only on side 1. If network externalities become large enough such that  $k \geq \frac{7}{2}t - \frac{7}{2}a$ , however, it is always harder to sustain collusion only on one side. For given values of  $t$  and  $k$ , both critical discount factors are plotted as a function of  $a$  in figure 2.<sup>24</sup>

Summing up the above findings, it is not true that platforms will compete away *all* supra-competitive profits earned on side 1 when they still compete on the other side. One-sided collusion, however, always yields smaller profits than colluding on both mar-

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<sup>24</sup>The label "RF switch" indicates the value of  $a$  for given parameter values of  $k$  and  $t$  at which  $k = \frac{7}{2}t - \frac{7}{2}a$ . Below this value, the critical discount factors for one- and two-sided collusion are identical, whereas they differ for larger values of  $a$ . Note that in case of  $k < \frac{7}{2}t - \frac{7}{2}a$ , it is optimal to share both market sides during deviation in the two-sided collusive game while it is optimal to fully capture both sides for  $k \geq \frac{7}{2}t - \frac{7}{2}a$ .

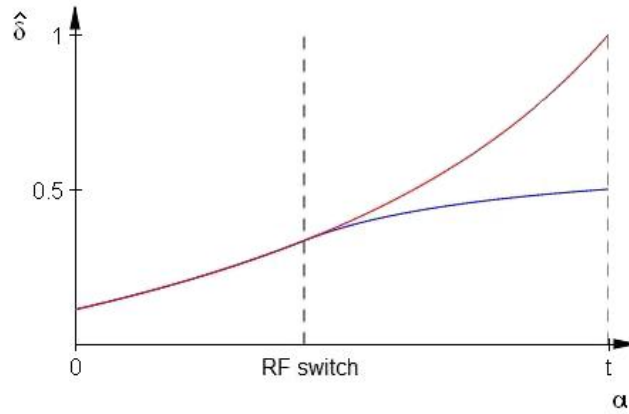


Figure 2: critical discount factors for one-sided (red) and two-sided (blue) collusion

ket sides. In addition, it is not as easy to sustain as two-sided collusion when network effects are large.

In order to shed some light on what happens when network effects are asymmetric, assume from now on that  $a_1 = 2a > 0$  while  $a_2 = 0$ .<sup>25</sup> Furthermore, I suppose that platforms can choose to collude on side 1, i.e. the market side with higher maximum profits. In consequence, collusive prices are equal to  $p_1^{OC} = k + a - \frac{t}{2}$  while platforms' prices on the competitive side amount to  $p_2^{OC} = t - \frac{2a}{t}p_1^{OC}$ . Hence, as in the case of symmetric externalities, prices on side 2 are below the competitive price level  $p_2^N = t - 2a$ . The gain from colluding, i.e. the difference between collusive and Nash profits, is given by  $(\pi^{OC} - \pi^N) = \frac{(t-2a)}{4t}(2k - 3t + 2a)$  and it is easy to check that platforms will only gain from collusion if  $t > 2a$ . In other words, if side-1 customers value the opposing market side too highly, platforms will prefer not to collude at all. Thus, in case of an asymmetric network structure, Evans and Schmalensee's hypothesis is true for large network effects  $a$ .

Suppose from now on, that  $t > 2a$ . The gain from colluding is increasing in  $a$  if  $k \leq 2(t - a)$  while it is decreasing in  $a$  for  $k > 2(t - a)$ . Hence, the non-monotonicity property found above still holds for asymmetric externalities. Turning to platforms' incentive to deviate, it is still never optimal for a defecting platform to fully conquer market side 2. Although the side-2 defection market share is no longer equal to 1/2, assumption 2 now guarantees that it does not become larger than 1. It might, however, be optimal to conquer market side 1 when the collusive price is high enough, i.e. when  $p_1^{OC} \geq 2t + \frac{t^3}{t^2 - 2a^2}$ . When optimal defection implies market sharing, it is straightforward to show that the gains from defection are an increasing function of  $a$ . Furthermore, the critical discount factor is equal to:

$$\hat{\delta}_{OC} = \frac{t^2(2k - 3t + 2a)}{t^2(2k - 3t + 2a) + 8(t^2 - a^2)(t - 2a)}$$

This critical discount factor is larger than the one for symmetric network effects. Thus, for a given level of total external benefits, one-sided collusion is harder to sustain

<sup>25</sup>This corresponds to the network externality structure discussed in section 4.2.

when network effects are asymmetric in comparison to when they are symmetric - a result which is in line with the findings for two-sided collusion presented in section 4.2. When collusive prices on side 1 are large enough for optimal defection to imply full market conquest on this side, expressions become more complicated. Numerical analysis indicates that the critical discount factor is larger under asymmetric rather than under symmetric externalities even in this case.

The following remark sums up the above analysis:

**Remark 3.** *Price collusion on one side of a two-sided market has the following properties:*

- *One-sided price collusion is always profitable in case of symmetric network effects and might be profitable in case of asymmetric network effects given that the total network benefit is not too large.*
- *Some of the supra-competitive profits earned on the colluding market side will always be competed away by setting prices below the competitive level on the opposite side.*
- *In case of symmetric network effects, one-sided collusion is harder to sustain than collusion on both sides if those network effects are large compared to transport costs and the intrinsic utility enjoyed from platform participation. Otherwise, both forms of collusion have the same critical discount factor.*

This analysis partially refutes Evans & Schmalensee's hypothesis that one-sided collusion is never optimal. Although they have been right in claiming that supra-competitive profits earned on the colluding market side will be competed away on the other, this effect is not strong enough to make cartelization overall unprofitable. Instead, platforms will always gain from one-sided collusion in case of symmetric externalities. Further, platforms also have an incentive to fix prices on the market side which enjoys higher network benefits if externalities are asymmetric - as long as those positive benefits are not too large compared to product differentiation. Finally, one-sided collusion has different welfare implications than collusion on both sides of the market. First, platforms extract all utility from the indifferent consumer on the market side with higher network benefits. Hence, consumer surplus on the collusive side decreases. Market participants on the non-colluding side, however, benefit from the collusive agreement via a reduction in their membership fees. Since they impose high external effects on the opposing side which faces monopoly prices, platforms have an increased incentive to attract them and subsidize their participation by setting a price below the competitive level. In consequence, the distributional effects of collusion differ from those of two-sided collusion: consumers on the colluding side suffer whereas those on the non-colluding side profit from cartelization.

#### 4.4 Optimal Symmetric Punishments

An optimal punishment can be defined as the most severe subgame-perfect Nash equilibrium punishment allowing firms to charge the best collusive price for a given discount

factor (Abreu 1986). In other words, the maximal collusive price that firms can sustain, which is an increasing function of the discount factor, will be higher if firms are using optimal punishment schemes rather than the grim-trigger strategy profile used in earlier sections. In case of one-sided horizontally differentiated firms competing in a Hotelling framework, Häckner (1996) shows that optimal symmetric punishments have a two-phase structure as introduced in Abreu (1986). These so-called "stick-and-carrot" strategy profiles are characterized by just one period of an intense price war followed by an immediate return to the collusive outcome whenever a deviation is observed. In particular, the strategy profile demands that in any period  $\tau$ , firms set the highest sustainable collusive price  $p^*$  if everyone set  $p^*$  or the punishment price  $p^P$  in period  $\tau - 1$  and that they set  $p^P$  otherwise. Häckner (1996) shows that the relationship between cartel stability and product differentiation obtained in Chang (1991) is fairly robust to a change in punishment schemes from grim trigger to stick-and-carrot profiles.

In flavor of Häckner's analysis, I will derive the best collusive prices that platforms can set on both market sides given a "stick-and-carrot" strategy profile and analyse whether the result that collusion becomes harder to sustain as network effects become stronger is robust to this change. As in section 3, I will proceed in a two-step procedure starting with the case of symmetric externalities and subsequently turning to the impact of increasing asymmetries between both sides' network benefits.

**Symmetric externalities.** Note first that we can derive the optimal collusive prices given grim-trigger punishments by choosing the collusive price levels such that the incentive constraint (equation (2)) just binds for a given  $\delta$  with  $\delta < \hat{\delta}_{SE}$ . By symmetry, *platforms will set identical prices on both market sides*. Hence, the following condition is solved with equality for the optimal collusive price  $p_{grim}^*$ :<sup>26</sup>

$$\begin{aligned} \frac{\delta}{1-\delta}(p_{grim}^* - (t-a)) &\geq (\pi^D(p_{grim}^*) - p_{grim}^*) \Leftrightarrow \\ \frac{\delta}{1-\delta}(p_{grim}^* - (t-a)) &\geq \frac{(p_{grim}^* + t - a)^2}{4(t-a)} - p_{grim}^* \end{aligned}$$

which yields:

$$p_{grim}^* = \begin{cases} \frac{(t-a)(1+3\delta)}{(1-2\delta)} & \text{if } p_{grim}^* < 3t - 3a \Leftrightarrow \delta < \frac{1}{3} \\ \frac{(t-a)(2-3\delta)}{(1-2\delta)} & \text{if } p_{grim}^* \geq 3t - 3a \Leftrightarrow \delta \geq \frac{1}{3} \end{cases} \quad (22)$$

It is easily shown that this collusive price is decreasing in the network externality  $a$  as long as  $\delta < \frac{1}{2}$  which is guaranteed by assumption 1.<sup>27</sup> In addition, it equals  $p^C = k - \frac{t}{2} + \frac{a}{2}$  if  $\delta = \hat{\delta}_{SE}$ .

Turning to the case of optimal punishments, the following characteristics have to be fulfilled for the existence of an optimal symmetric punishment (Häckner 1996):<sup>28</sup>

<sup>26</sup>Platforms share both market sides equally in the competitive equilibrium as well as under collusion. Therefore, profits in those cases are identical to prices.

<sup>27</sup>Recall figure 1 which clarified that  $\hat{\delta}_{SE}$  converges to  $\frac{1}{2}$  as  $a \rightarrow t$ .

<sup>28</sup>Those properties allow to apply the existence proof from Abreu (1988).

- (I) there exists a symmetric pure strategy equilibrium in the one-shot stage game
- (II) the strategy set is compact
- (III) the platforms' profit functions are continuous
- (IV) the static best reply payoff is increasing in the competitor's prices

In case of symmetric network externalities, it is easy to see that conditions (I), (II) and (IV) are fulfilled.<sup>29</sup> Condition (II) asks for an upper and a lower bound on prices. An upper bound is given by the monopoly levels whereas a lower bound is somewhat more difficult to define if prices are not restricted to be non-negative.<sup>30</sup> Given the existence of an optimal symmetric punishment, it is only needed to show that a simple stick-and-carrot path sustains the same collusive price as this optimal symmetric punishment in a game characterized by conditions (I) to (IV). Häckner proves this by assuming that there exists some optimal, symmetric and credible punishment path  $Y$  and showing that the punishment price in a stick-and-carrot profile  $Y'$  can be chosen low enough to guarantee that both punishment schemes yield the same discounted sum of profits in period 1. Therefore, it is not profitable to break the cartel under  $Y'$  if it had not been profitable under  $Y$ . Furthermore, firms will find it profitable to participate in the price war. Noting that they do not find it profitable to deviate from  $Y$  and that the punishment price in  $Y'$  must be lower than the price of  $Y$  to yield the same discounted profit stream, we can infer that the deviation profit during the price war in the stick-and-carrot regime must be smaller than the deviation profit under  $Y$  (by property (IV)). Since the punishment following the deviation is equally severe under both  $Y$  and  $Y'$  by construction, it must be less tempting to deviate under  $Y'$ .

To solve for the stick-and-carrot profile, the best collusive price  $p_{stick}^*$  has to be maximized subject to the following conditions:

$$\pi^D(p_{stick}^*) - \pi(p_{stick}^*) \leq \delta (\pi(p_{stick}^*) - \pi(p_{stick}^P)) \quad (23)$$

$$\pi^D(p_{stick}^P) - \pi(p_{stick}^P) \leq \delta (\pi(p_{stick}^*) - \pi(p_{stick}^P)) \quad (24)$$

where  $\pi(p_{stick}^*)$  and  $\pi(p_{stick}^P)$  are the symmetric payoffs when every firm plays  $p_{stick}^*$  and  $p_{stick}^P$  respectively.  $\pi^D(p_{stick}^*)$  and  $\pi^D(p_{stick}^P)$  are the best response payoffs given that the other firm plays  $p_{stick}^*$  or  $p_{stick}^P$ .<sup>31</sup> These two conditions ensure that it is not optimal to deviate neither during a collusive period (equation (23)) nor during a punishment period (equation (24)). Similar to the case with grim trigger punishments, both conditions must hold with equality for  $\delta < \hat{\delta}$ , i.e. when monopoly prices cannot be sustained. Thus, the following price pairs define the optimal stick-and-carrot punishment:

$$p_{stick}^* = \begin{cases} (t-a)(1+8\delta) & \text{if } p^* < 3t-3a \Leftrightarrow \delta < \frac{1}{4} \\ \frac{2(t-a)(1-\delta)}{(1-2\delta)} & \text{if } p^* \geq 3t-3a \Leftrightarrow \delta \geq \frac{1}{4} \end{cases} \quad (25)$$

<sup>29</sup>The pure strategy equilibrium is derived in section 3.1 and it is easily checked that platforms' profit functions are continuous in prices. The static best reply payoff is stated in equation (22).

<sup>30</sup>I refer to Häckner's footnote 9 for a detailed explanation.

<sup>31</sup>Recall that platforms set identical prices on both sides in case of symmetric externalities. In consequence, those two conditions include only two different prices.

$$p_{stick}^P = \begin{cases} (t-a)(1-8\delta) & \text{if } p^* < 3t-3a \Leftrightarrow \delta < \frac{1}{4} \\ \frac{-2(t-a)\delta}{(1-2\delta)} & \text{if } p^* \geq 3t-3a \Leftrightarrow \delta \geq \frac{1}{4} \end{cases} \quad (26)$$

It is easy to check that the best collusive price under optimal symmetric punishments is always larger than the one under grim trigger strategies. More importantly,  $p_{stick}^*$  decreases in the indirect network externality parameter  $a$  for any given discount factor below  $\hat{\delta}$ . This allows for the following remark:

**Remark 4.** *In case of symmetric network externalities, a change in punishment schemes from grim trigger to stick-and-carrot strategy profiles does not alter the result that collusion becomes less successful when network effects increase.*

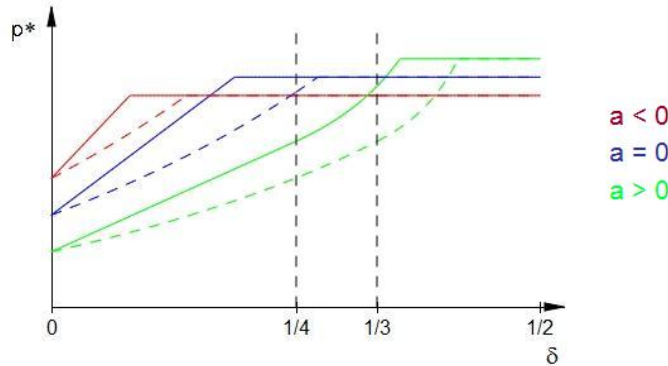


Figure 3: Best collusive prices under optimal symmetric punishments (solid lines) and grim trigger punishments (dashed lines)

Figure 3 illustrates this result for given values of the intrinsic utility level  $k$ , the transport cost  $t$  and for three different sizes of the symmetric network effects parameter  $a$ . The solid lines represent best collusive prices given stick-and-carrot punishment while the dashed lines indicate the best collusive prices for grim trigger strategies. Both types of punishment schemes yield a best collusive price which is increasing in the discount factor  $\delta$  until it reaches the monopoly pricing level at  $\delta = \hat{\delta}$ . Stick-and-carrot paths allow to sustain a higher collusive price at any given discount factor and for any given level of network effects as long as  $\delta < \hat{\delta}$ . It is easy to see from the figure that the collusive price level increases in  $a$  while the Nash price level, which corresponds to the best collusive price level when  $\delta = 0$ , is decreasing in  $a$ . Whenever monopoly prices cannot be sustained, the best collusive prices which can be self-enforced in a cartel decrease when symmetric network externalities become larger. In other words, collusion is the less successful the higher network benefits from platform participation are.

The above analysis comes with one major disadvantage: it demands to set negative punishment prices for  $\delta > 1/8$ . Häckner (1996) shows that it is possible to circumvent this problem and to find an optimal *positive* symmetric punishment which is characterized by setting punishment prices equal to zero and prolonging the punishment period accordingly before switching back to collusive prices.<sup>32</sup>

<sup>32</sup>I will not analyse the case of non-negative punishment prices in detail, but my conjecture is that



## 5 Conclusion

This paper is a first step in understanding the impact of indirect network externalities on the sustainability of collusion in two-sided markets. It shows that collusion is harder to sustain when network externalities between market sides increase. This is the result of two countervailing effects. First, if one side values members on the other market side more highly, Nash prices fall because competition for this side gets harsher. As a consequence, punishment profits are a falling function of network effects. In addition, consumers' utility from platform participation increases if they enjoy a larger benefit from the presence of platform members on the opposing market side. Hence, two-sided platforms can earn larger collusive profits as network externalities grow. Second and countervailing, however, platforms also earn larger profits from deviation as network effects become stronger - a result which is mainly due to more sensitive demand reactions. Comparing those opposing effects, it can be shown that the latter effect always dominates for the specific framework of a two-sided single-homing Hotelling model. Furthermore, collusion becomes less attractive if network effects are asymmetric rather than symmetric given that the maximum external benefit that a platform can possibly extract from consumers on both sides stays constant. The reason for this is that deviation profits rise in response to growing asymmetries in network externalities while the gain from colluding stays constant.

The robustness of these results is tested against changes in the structure of two-sided markets. If prices on one market side are fixed to be zero while network effects on this side are negative, as it might be the case for readers of free newspapers or TV-channels, the main result of a positive and monotone relationship between network externalities and the critical discount factor does no longer hold. Instead, if negative externalities imposed on the non-paying side are very large, newspapers are actually more likely to collude as the positive network effects on the paying market side (namely advertisers) grow. In contrast, the case of one-way network externalities emphasizes once again that a larger network effect as well as a larger asymmetry between market sides make collusion between platforms less likely. The analysis of collusion on one market side shows that Evans & Schmalensee (2008) have only been partially right. When firms collude only on one market side, they compete away some, but not all, of the supra-competitive profits by setting prices below the competitive level on the other side. Hence, it might actually be profitable to collude only on one market side if cartelization on both sides is not possible for some reason. Finally, the result that increasing network effects make collusion less successful is robust to a change in punishment schemes from grim trigger to optimal stick-and-carrot profiles when externalities are symmetric.

Although my analysis of collusion in Armstrong's (2006) often-cited model of competition between two-sided platforms provides interesting insights, it has some caveats.

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the analysis by Häckner (1996) can be carried over completely.

It limits the scope of analysis by not allowing for platforms to charge transaction-based prices. In addition, its focus is on homogeneous membership externalities. Although this form of indirect network effects is well suited to media markets, other two-sided industries might be better described by other forms of network effects. Finally, the chosen framework has to face the general critique addressing Hotelling specifications. In particular, it does not allow for a meaningful normative analysis that focuses on total welfare instead of consumer surplus. Future research should thus focus on possible generalizations of the presented model to overcome some of its caveats and provide further robustness checks for its main results.

## A Proof of remark 2

The second part of the remark follows directly from taking the first derivative with respect to  $a$  and applying assumptions 1 and 2.

To prove the first part, subtract equation (17) from (8) and rearrange. The difference in deviation profits between one-way and symmetric network externalities can be written as follows:

$$\pi_{OWE}^D - \pi_{SE}^D = \begin{cases} \frac{a^2}{16(t+a)} & \text{if } k < \frac{7}{2}(t-a) - \frac{a(t-a)}{2(t+a)} \\ * & \text{if } \frac{7}{2}(t-a) > k \geq \frac{7}{2}(t-a) - \frac{a(t-a)}{2(t+a)} \\ \frac{(2k-7t+6a)^2}{32t} & \text{if } \frac{7}{2}t - 3a > k \geq \frac{7}{2}(t-a) \\ 0 & \text{if } k \geq \frac{7}{2}t - 3a \end{cases} \quad (\text{A.1})$$

with

$$* = -\frac{4(t+a)\left(k - \frac{7}{2}(t-a)\right)\left(k - \frac{7}{2}(t-a) + \frac{a(t-a)}{2(t+a)}\right)}{32t(t-a)} - \frac{2a\left(k - \frac{7}{2}(t-a)\right)}{32t} + \frac{a^2}{32t}$$

It is easy to see that the first and third part of (A.1) are larger than zero given that  $a, t > 0$  by assumption. The second part, denoted by  $*$ , is also larger than zero since  $\frac{7}{2}(t-a) > k \geq \frac{7}{2}(t-a) - \frac{a(t-a)}{2(t+a)}$  and assumption 1 implies that  $(t-a) > 0$ .

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