Hristov, Nikolay

Conference Paper
Customer Markets, Non-Separable Utility and the Real Effects of Monetary Policy Shocks

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2010: Ökonomie der Familie - Session: Monetary Policy Under Uncertainty, No. C16-V1

Provided in Cooperation with:
Verein für Socialpolitik / German Economic Association


This Version is available at:
http://hdl.handle.net/10419/37251

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

www.econstor.eu
Customer Markets, Non-Separable Utility and the Real Effects of Monetary Policy Shocks

by

Nikolay Hristov

January 2010

Abstract

As is well known, one of the major shortcomings of the New Keynesian model (NKM) with Calvo-type price setting is the lack of a microeconomic foundation of its most important building block - price stickiness. In this paper I investigate the ability of a monetary Customer Markets model to provide the desired microeconomic foundation and to serve as an alternative to the New Keynesian approach for analyzing positive as well as normative issues. In particular, I extend a standard monetary business cycles model with fully flexible prices along two dimensions: market share competition as proposed by Phelps and Winter (1970) and non-separability of the utility function with respect to money and consumption. For a broad range of empirically plausible parameter values the monetary nonneutrality generated by the Customer Markets model is of similar magnitude and persistence as that implied by the NKM. Furthermore, as revealed by a detailed comparison between the two frameworks, the theory developed in this paper explains a standard set of business cycles facts at least as well as the NKM does.

JEL classification: E3, E4, E5

Keywords: monetary shocks, market share competition, markups, non-additively separable utility, business cycles, persistence

ifo Institute for Economic Research at the University of Munich, Poschingrstr. 5, D-81679 Munich, Germany, e-mail: hristov@ifo.de, Phone: ++49(0)89-9224-1225, Fax: ++49(0)89-9224-1462
1 Introduction

Christiano et al. (1999, 2005) provide empirical evidence indicating that positive monetary shocks are expansionary and induce highly persistent, hump-shaped dynamic responses of inflation, output, consumption and investment.\(^1\) Many economists try to explain this pattern by New Keynesian monetary business cycles models in which a very high degree of exogenously given price stickiness (e.g. of the Calvo-type) is combined with a whole battery of real rigidities and additional structural assumptions\(^2\) as well as various exotic shocks.\(^3\) Examples are Christiano et al. (2005), Walsh (2005), Trigari (2004), Altig et al. (2005), Smets and Wouters (2003, 2007) and many others. Most of these extensions are subject to debate.

Such New Keynesian models are extensively used to evaluate monetary and fiscal policy as well as to derive normative conclusions and suggestions on how monetary policy should be conducted. Unfortunately, most of these models perform rather poorly with respect to phenomena other than the impulse responses to monetary innovations such as the sample moments of many macroeconomic variables or the reactions to real supply side shocks. Another major disadvantage of the New Keynesian models is the lack of microeconomic foundation and thus, the lack of endogenous explanation of their most crucial component - the high degree of price rigidity (see Caplin and Spulber (1993), Golosov and Lucas (2007), Kehoe and Midrigan (2008)). The problem


\(^2\)For example backward indexation of the prices of the non-adjusting firms, Calvo-type nominal wage setting, adjustment costs of capital and labor, habit persistence in consumption, matching frictions and job destruction in the labor market and even bounded rationality of part of the firms.

\(^3\)For example different kinds of preference shocks, wage-markup and price-markup shocks, investment-specific shocks and even risk-premium shocks.
becomes even more severe in the light of the most recent micro evidence provided by Bils and Klenow (2004) and Klenow and Krysvost (2005) indicating that nominal prices are quite flexible, remaining unchanged for slightly more than one quarter on average. Calibrating the New Keynesian model so that it is consistent with these empirical findings leads to a dramatic worsening of its predictions: the degree of monetary nonneutrality falls sharply, the persistence and the hump-shape of the impulse responses to monetary innovations (almost completely) disappear and the markups of prices over marginal costs (*price markups*) become procyclical. These and other\(^4\) shortcomings cast doubt on the appropriateness of the sticky price models for analyzing normative issues.

The goal of the current paper is to propose an alternative theoretical framework in which price rigidity emerges endogenously and to assess the extent to which such a theory can serve as a microeconomic foundation of, and an alternative to, a typical New Keynesian model (NKM) with a high degree of price stickiness. In particular, I investigate the ability of the model developed here to generate real effects of monetary policy of similar magnitude and persistence as the NKM does and then compare the performance of both frameworks with respect to a set of standard business cycles facts. The current paper incorporates two *"novel"* features into an otherwise standard monetary business cycles model with fully flexible prices. First, monetary *nonneutrality* is introduced through the assumption that the utility function of the representative household is non-separable in money and consumption. Second, the monopolistically competitive firms do not only engage in static price competition but also in a kind of dynamic market share competition. The latter is modeled in the way proposed by Phelps and Winter (1970) in their *Customer Markets model* and makes the price-markups endogenous.

The main findings can be summarized as follows. When the degree of

\(^4\)See for example Chari *et al.* (2008).
price stickiness in the NKM is relatively high, as typically assumed in the literature, there is a broad range of empirically plausible parameter values in the monetary Customer Markets model for which both frameworks perform about equally well. However, by reducing the level of price rigidity in the NKM towards a value which is more in line with the micro evidence cited above, the performance of the NKM substantially worsens relative to that of the model developed in this paper.

What are the main mechanisms at work in the monetary Customer Markets model? At the heart of this set-up are two opposing effects of monetary policy on labor supply. The first one is negative and operates through the (reduction) of the current marginal utility of consumption. The effect can be briefly described as follows: With a non-separable utility function a positive monetary impulse induces a sharp increase in current inflation which reduces the marginal utility of consumption "today" relative to its value in the future. The result is a relatively large drop in labor supply. The second effect on this variable is positive and operates through the induced variations in the stochastic discount factor and the resulting reactions of the average price-markup: The changes in the marginal utility of consumption just mentioned imply an increase in the stochastic discount factor. Consequently future profits become more important which in turn strengthens each firm’s incentive to "invest" in its future market share. To do this, the firm has to lower its current markup. The resulting economy-wide fall in markups implies (for a given level of total factor productivity) an increase in real wages, and thus a positive effect on labor supply. As shown below, for low enough values of the short run price elasticity of demand the second (positive) effect on labor supply dominates and so, labor, output and consumption react positively to monetary expansions. Furthermore, lowering the short run elasticity of demand and, thus, making its value more realistic, makes the real effects of monetary policy stronger and the
model dynamics become more in line with the VAR-evidence. The persistence generated by the model is due to the interaction between capital accumulation and markup fluctuations and is described in section ??.

The central role played by countercyclical markup variations in the model is consistent with the empirical findings by Rotemberg and Woodford (1999) and the literature cited there. According to their results, markups are countercyclical, and the output fluctuations attributable to variations of markups, which are orthogonal to fluctuations induced by shifts in the marginal cost curve, account for about 90% of the variance of output growth in the short run. Boldrin and Horvath (1996), Gomme and Greenwood (1995), Ambler and Cardia (1996) and Gali et al. (2002) also obtain negative estimates of the correlation between output and markups.

The paper is organized as follows. In subsection 1.1 I provide a short review of the empirical evidence and the related literature respectively. Section 2 describes the baseline monetary model with market share competition in the goods market. Section 3 provides details on the calibration. In subsection 3.3 I evaluate the performance of the model with adjustment costs of capital with respect to a subset of stylized business cycles facts and compare it with the performance of the New Keynesian Model with Calvo pricing. Section 4 concludes.

1.1 Related Literature

The purpose of this subsection is to provide a brief review of the literature attempting to develop an alternative to the widely used New Keynesian model with Calvo pricing as well as the most important studies focusing on markup variations.

Endogenous Price Rigidity:
Haubrich and King (1991) develop a model in which firms are able to insure
against idiosyncratic monetary shocks by signing nominal contracts. However, as the authors point out, the price-rigidity equilibrium is only one of the possible outcomes under the specific assumptions on the parameters made. Nakamura and Stensson (2007) construct a model with good-specific habit persistence in which price stickiness arises as an equilibrium outcome. However, there are again many further equilibria characterized by fully flexible prices. The sticky-information literature proposes an approach in which the Calvo-type price setting is replaced by a Calvo-type updating of information. Examples are Mankiw and Reis (2001, 2006a, 2006b), Ball et al. (2003) and others. A major shortcoming of this framework is the fact that the process of updating information is exogenous and lacks a microeconomic foundation. The menu-cost models\(^5\) generate monetary nonneutrality by assuming that there are small fixed costs of adjusting prices. However, as Bursten and Hellwig (2007) argue, to generate strong and persistent effects of monetary policy, these models need parameter values which are inconsistent with the micro evidence on the level of menu costs and the typical magnitude of price adjustments.

**Real Business Cycles Models of Endogenous Markups:**
In the already cited study by Phelps and Winter (1970) markups are endogenized by the assumption of a particular form of dynamic market share competition in continuous time. The discrete time version of that structure is used in the model presented below. In a series of real business cycles models based on the partial equilibrium model proposed by Rotemberg and Saloner (1986), Rotemberg and Woodford\(^6\) show that endogenous markup variations may arise if firms are able to collude implicitly. Ravn et al. (2006, 2007) are able to generate countercyclical markups by introducing good-specific habit formation, the so called deep habits, into a standard RBC-model with a monopolistically

---


competitive goods market. Froot and Klemperer (1989), Klemperer (1987, 1995) and Kleshchelski and Vincent (2007) develop static models of the goods market in which customers face fixed costs of switching suppliers. All these models have in common the implication that firm’s current pricing behavior has an influence on its future profits. Since the studies just mentioned neglect the money market, they do not provide any implications about the reactions of markups to monetary shocks.

2 The Model

I refer to this model as the Customer Markets Model with fixed capital.

2.1 Households

Let agents in this economy have preferences over consumption, real balances and working hours given by

\[
U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \left( aC_t^{1-b} + (1-a) \left( \frac{M_t}{P_t} \right)^{1-b} \right) \right)^{\frac{1}{1-b}} - \frac{\phi}{2} N_t^2 \right\}, \quad \phi, b > 0, \quad \beta, a \in (0, 1),
\]

where \( M_t / P_t \) and \( N_t \) denote real balances and working hours. In the above expression \( C_t \) is a composite good to be defined and explained below. For \( b \rightarrow 1 \) the current utility function which I denote by \( u_t \) reduces to

\[
u_t = C_t^a \left( \frac{M_t}{P_t} \right)^{1-a} - \frac{\phi}{2} N_t^2.
\]

The budget restriction of the representative household is given by:

\[
C_t + m_{t+1} - \frac{m_t}{\pi_t} + b_{t+1} - \frac{b_t}{\pi_t} = \frac{W_t}{P_t} N_t + \Pi_t + (1 + \theta_t) \frac{b_t}{\pi_t} + T_t \frac{P_t}{P_t},
\]

where \( W_t, \Pi_t, T_t, b_t = \frac{B_t}{P_{t-1}} \) and \( m_t = \frac{M_t}{P_{t-1}} \) denote the nominal wage, real profits, nominal net transfers form the government, the real value of nominal bonds and real balances respectively. \( i_t \) is the one-period risk free nominal interest rate.
2.1.1 First Order Conditions

The first order conditions of the representative household read:

\[
a C_t^{-b} \left( a C_t^{1-b} + (1 - a) \left( \frac{m_t}{\pi_t} \right)^{1-b} \right)^{\frac{b}{1-\pi}} = \Lambda_t, \quad (1)
\]

\[
\phi N_t = \Lambda_t \frac{W_t}{P_t}, \quad (2)
\]

\[
\frac{1}{1 + \iota_t} = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{1}{\pi_t} \right\}, \quad (3)
\]

\[
\Lambda_t = \beta E_t \left\{ (1 - a) \frac{m_{t+1}}{\pi_{t+1}} \right( a C_{t+1}^{1-b} + (1 - a) \left( \frac{m_{t+1}}{\pi_{t+1}} \right)^{1-b} \right)^{\frac{b}{1-\pi}} + \frac{\Lambda_{t+1}}{\pi_{t+1}} \right\}, \quad (4)
\]

\[
C_t + m_{t+1} - m_t + \frac{b_t}{\pi_t} = W_t \frac{N_t}{P_t} + \Pi_t + (1 + \iota_t) \frac{b_t}{\pi_t} + \frac{T_t}{P_t}, \quad (5)
\]

where \( \Lambda_t \) denotes the Lagrangean multiplier attached to the budget restriction. (3) is the bond euler equation and (4) is the euler equation with respect to money balances.

2.1.2 Key Assumption I: Non-Separable Utility

The monetary general equilibrium models developed in the last ten years usually assume that the utility function of the representative agent is separable with respect to money and consumption, e.g.

\[
\frac{C_t^{1-\eta}}{1-\eta} + \frac{\phi}{1-\chi} \left( \frac{M_t}{P_t} \right)^{1-\chi}, \quad \text{for} \ 0 < \eta, \chi \neq 1, \quad \phi > 0,
\]

\[
\ln(C_t) + \tilde{\phi} \ln \left( \frac{M_t}{P_t} \right), \quad \text{for} \ \eta = \chi = 1, \quad \tilde{\phi} > 0.
\]
Nonetheless, it is quite well known that almost all separable specifications are just special cases of more general non-separable, (nested\textsuperscript{7}) Cobb-Douglas or CES\textsuperscript{8} aggregators combining consumption and real balances. Furthermore, economic theory does not provide any convincing reason for preferring the separable to the non-separable formulation \textit{et vice versa}. The only comparative advantage of the former is perhaps its analytical simplicity. Indeed, in his seminal paper Sidrauski (1967) assumes that money and consumption enter the utility function non-separably, through a Cobb-Douglas aggregator. The early literature inspired by Sidrauski (1967), e.g. Brock (1974, 1975), Fisher (1979), Asako (1983) and others, dealing with the stability and the steady state properties of monetary general equilibrium models, also consider the non-separable utility function to be more important while the separable specification is only treated as a special case.

Finally, the empirical evidence supports the assumption that utility is non-separable in consumption and real balances: in a more recent study Holman (1998) performs a GMM estimation of the Euler equation for optimal money holdings under different specifications of the utility function - Cobb-Douglas, CES and nested Cobb-Douglas or CES.\textsuperscript{9} Based on a series of tests the author rejects the separable form while the Cobb-Douglas, the CES (used here)

\textsuperscript{7}The nested Cobb-Douglas specification of the utility function is given by:

\[
\left(\frac{C_t}{\frac{M_t}{P_t}}\right)^{1-\alpha} \left(\frac{M_t}{P_t}\right)^{1-\rho}, \quad \alpha \in (0,1), \quad \rho > 0,
\]

while the non-nested case is obtained by setting \(\rho = 0\).

\textsuperscript{8}CES - Constant Elasticity of Substitution.

\textsuperscript{9}The nested CES specification of the utility function is given by:

\[
\left(\frac{aC_t^{1-b} + (1-a)\left(\frac{M_t}{P_t}\right)^{1-c}}{1-\rho}\right)^{\frac{1}{1-c}}, \quad a \in (0,1), \quad b, \rho > 0,
\]

while the non-nested case is obtained by setting \(\rho = 0\).
and the nested CES formulation can not be rejected. The quasi-likelihood-ratio tests employed indicate that the non-nested Cobb-Douglas specification performs slightly better than the other ones.

All in all, the assumption that the utility function is non-separable in money and consumption seems to be at least as plausible as the opposite one. At the same time, the non-nested CES specification chosen in the current paper, although arbitrary, is not rejected by the data.

2.2 Firms and Market Shares

There are \( n \) product varieties, each produced by a profit maximizing monopolistic firm according to the linear production function

\[
Y_{i,t} = Z_t N_{i,t},
\]

where \( N_{i,t} \) denotes labor input of firm \( i \). \( Z_t \) denotes the total factor productivity which follows a stochastic process given by:

\[
\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \epsilon_t,
\]

where \( \epsilon_t \) follows a White Noise Process with variance \( \sigma^2_t \). Real marginal costs \( \mu_t \) are easily found to be given by

\[
\mu_t = \frac{W_t}{P_t} Z_t.
\]

2.2.1 Key Assumption II: Market Share Competition

Let us assume that the consumption index is given by

\[
C_t = \left\{ \frac{1}{n} \sum_{i=1}^{n} x_{i,t}^{\frac{1}{\theta}} C_{i,t}^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}},
\]

where \( x_{i,t} \) evolves according to

\[
x_{i,t+1} = g\left( \frac{P_{i,t}}{P_t} \right) \cdot x_{i,t}
\]
The corresponding demand function faced by an arbitrary firm $i$ is given by:

$$ C_{i,t} = x_{i,t} \cdot \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} \cdot \frac{C_t}{n}. $$

This is basically the assumption underlying the "Customer Markets Model" developed by Phelps and Winter (1970).

Phelps and Winter (1970) depart from the frictionless specification of the goods market by assuming that customers can not respond instantaneously to differences in firm specific prices. As the authors note, there are various rationales for this assumption - information imperfections, habits as well as costs of decision-making. An immediate consequence of such frictions is that in the (very) short run each firm has some monopoly power over a fraction of all consumers. This fraction equals the firm’s market share. In particular, Phelps and Winter (1970) assume that the transmission of information about prices evolves (proceeds) through random encounters among customers in which they compare recent demand experience. Under this assumption the probability with which a comparison between any two firms $i$ and $j$ is made will be approximately proportional to the product of their respective market shares $x_{i,t}$ and $x_{j,t}$. Therefore, one would expect that the time rate of net customer flow from all other firms to firm $i$ will also be proportional to the product $x_{i,t}(1 - x_{i,t})$. Under the assumption $1 - x_{i,t} \approx 1$ Phelps and Winter formalize this as follows:

$$ z_{t,i,*} = g(p_{i,t}, p_t)x_{i,t}(1 - x_{i,t}) \approx g(p_i, p)x_i, $$

where $z_{t,i,*}$ is the net flow of customers to firm $i$ from all its competitors. The properties of the function $g()$ are specified below. Appendix A provides more formal details regarding the last equation and the underlying assumptions. $x_{i,t}$ can be also interpreted as an indicator of customers' satisfaction with the pricing behavior of firm $i$, or as an indicator for the subjective weight assigned to good $i$ within the consumption bundle. In the current paper $x_{i,t}$ is called
market share. I assume that the function \( g(.) \) governing its law of motion has the properties:

\[
g(1) = 1, \quad g'(\frac{P_{i,t}}{P_t}) < 0,
\]

and assume the following functional form for it

\[
g\left(\frac{P_{i,t}}{P_t}\right) = \exp\left(\gamma \left(1 - \frac{P_{i,t}}{P_t}\right)\right),
\]

where \( \gamma > 0 \) is to be calibrated via the steady state of the economy. Because \( x_{i,t} \) depends on the past pricing behavior of the firm, its profit maximization problem becomes dynamic: In this economy each firm faces a trade off between maximizing its current profits and maximizing its future market share.

### 2.2.2 Markups

The first order condition of an arbitrary firm with respect to its relative price reads:

\[
\left(\frac{P_{i,t}}{P_t}\right)^{-\theta} x_{i,t} D_t - \theta \left(\frac{P_{i,t}}{P_t} - \mu_t\right) \left(\frac{P_{i,t}}{P_t}\right)^{-\theta-1} x_{i,t} D_t + \frac{g_1\left(\frac{P_{i,t}}{P_t}\right)}{g\left(\frac{P_{i,t}}{P_t}\right)} \Omega_t = 0,
\]

where \( \mu_t \) denotes marginal costs and

\[
\Omega_t = E_t \left\{ \sum_{j=1}^{\infty} \beta^j \frac{\Lambda_{i+1}}{\Lambda_t} x_{i,j,t} \left(\frac{P_{i,t+j}}{P_{t+j}} - \mu_{t+j}\right) \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{-\theta} D_{t+j} \right\} =
\]

\[
= E_t \left\{ \beta \frac{\Lambda_{i+1}}{\Lambda_t} x_{i,t+1} \left(\frac{P_{i,t+1}}{P_{t+1}} - \mu_{t+1}\right) \left(\frac{P_{i,t+1}}{P_{t+1}}\right)^{-\theta} D_{t+1} \right\} + E_t \left\{ \beta \frac{\Lambda_{i+1}}{\Lambda_t} \Omega_{t+1} \right\}
\]

(9)

is the expected present value of future profits. Defining the markup over marginal costs as

\[
m_{u_{i,t}} = \frac{P_{i,t}}{P_t \mu_t}, \quad m_{u_t} = \frac{1}{\mu_t},
\]
one can write the FOC, evaluated at the symmetric equilibrium, as

$$m_{ut} = \frac{-\theta}{1 - \theta - \gamma_{Ct}}$$

In a symmetric intertemporal equilibrium in each period each firm sets the same price as all other firms. The most important implication regarding market shares is that $x_{i,t}$ equals one for all $t$ and all $i$. According to equation (10) the equilibrium markup depends positively on current demand and negatively on the present value of future profits. In the static monopolistic competition model markups are given by

$$m_{ut} = \frac{\theta}{\theta - 1}$$

implying that at any point in time and in any given state of the economy pass-through of marginal cost changes to prices is complete. Unlike that model, in an environment characterized by market share competition markups will be generally time varying. Whether pass-through of marginal costs to prices will turn to be greater, lower or equal to one depends on the relative strength of the reactions of $C_t$ and $\Omega_t$ to exogenous shocks. In the present model the discount factor is endogenous and strongly linked to current and next-period consumption, real balances and inflation - as shown above for $b = 1$ the discount factor is given by:

$$DF_t = \beta E_t \left\{ \frac{C_t^{a-1} \left( \frac{m_{t+1}}{\pi_{t+1}} \right)^{1-a}}{C_t^{a-1} \left( \frac{m_t}{\pi_t} \right)^{1-a}} \right\}.$$

For example, consider a positive monetary shock which at given prices increases current consumption via the positive income effect but also puts an upward pressure on current inflation as explained in the previous section. Obviously, the temporary (or even an one time) increase in current consumption will have a positive direct effect on markups but if at the same time the increase
in current inflation \( \pi_t \) and/or next period cash balances \( m_{t+1} \) is sufficiently\(^{10} \) large relative to the increase in \( C_t \) then the increase in the discount factor will be larger than that of current consumption, probably causing the term \( \frac{\Omega_t}{\pi_t} \) to rise and thus markups to fall.

### 2.3 Government

The central bank finances its lump-sum transfers to the public by changes in the nominal quantity of money:

\[
M_{t+1} - M_t = T_t.
\]

It is further assumed that in each period transfers constitute a fraction of current money supply:

\[
T_t = (\tau_t - 1)M_t,
\]

where the percentage deviation of \( \tau_t \) from its steady state \( \hat{\tau}_t \) follows a first order autoregressive process

\[
\hat{\tau}_t = \rho_{1,\tau} \hat{\tau}_{t-1} + \rho_{2,\tau} \hat{\tau}_{t-2} + u_t, \quad \rho_{\tau} \in [0, 1).
\]

\( u_t \) is assumed to be a White Noise Process with variance \( \sigma_u^2 \).

### 2.4 Equilibrium

I assume a symmetric equilibrium for simplicity. Little is lost by doing this since the main focus of the paper is on aggregate dynamics. In equilibrium, real wages and profits are given by

\[
\frac{W_t}{\bar{P}_t} = \frac{Z_t}{m_{\bar{u}_t}} \quad \text{and} \quad \Pi_t = \left( \frac{m_{\bar{u}_t} - 1}{m_{\bar{u}_t}} \right) Z_t N_t
\]

\(^{10}\)Actually one must compare the responses of \( C_t^a \) and \( m_{t+1}^{1-a} \pi_{t+1}^{-a} \).
respectively. These two results, together with the household’s optimality conditions (1) through (5), the lows of motion of markups, the present value of future profits (9) and (10) respectively and the equations specifying monetary policy describe the evolution of the economy.

3 Calibration and Steady State

In models featuring static monopolistic competition the short run price elasticity of demand for an individual good $\theta$ is restricted to be greater than unity in order to ensure that the markup of prices over marginal costs is greater than one and thus profits are positive. Usually $\theta$ is set to a value between 6 and 8 since empirically observable average markups are relatively low - according to most estimations they are smaller than 1.6. In contrast to the static monopolistic competition model in the economies featuring market share competition described above one don’t need to impose the restriction $\theta > 1$ since $\theta$ is not the sole determinant of the steady state markup $mu^*$. In fact, as I show below, any value of $\theta$ smaller than $\frac{mu^*}{mu^*-1}$ is consistent with $mu^* > 1$ and a negative first derivative of the function $g \left( \frac{P_i}{P_t} \right)$. A large part of the empirical evidence suggests that the short run price elasticity of demand for nondurables is well below one. Carrasco et al. (2005) provide panel estimates of the price elasticities of the demand for food, transport and services in Spain which take the values -0.85, -0.78 and -0.82 respectively. According to the results in Bryant and Wang (1990) based on aggregate US time series the price elasticity of total demand for nondurables is equal to -0.2078. Blanciforti et al. (1986) estimate an Almost Ideal Demand System (AIDS) based on aggregate US time series. Their results with respect to the own-price elasticities of nondurables can be summarized as follows: food - between -0.21 and -0.51; alcohol and tobacco - between -0.8 and -0.25; utilities - between -0.20 and -0.67; transportation - be-
tween -0.38 and -0.66; medical care - between -0.57 and -0.70; other nondurable goods - between -0.29 and -1.26; other services - between -0.20 and -0.36. There is also evidence supporting a short run price elasticity of demand greater than one. For example, using Finish time series Mellin and Viren (1982) come to the conclusion that the own-price elasticity of nondurables takes a value slightly below -5. However, their estimates should be interpreted with caution, since they are most likely subject to a simultaneity bias. In a more recent paper Tellis (1988) surveys the estimates of the price elasticity of demand in the marketing literature. He provides a skewed distribution of the results found in that literature with mean, mode and standard deviation equal to -1.76, -1.5 and 1.74 respectively. The bulk of the estimated elasticities take values in the range [-2,0]. In light of the empirical evidence it appears more reasonable to set $\theta$ at a value lower than one. However, for the sake of completeness and better comparability with models featuring static monopolistic competition, I decide to carry out a sensitivity analysis with respect to $\theta$ by simulating the model for several values of $\theta$ below and several values above one.

Most authors set the steady state markup at a value in the range suggested by Rotemberg and Woodford (1993) - between 1.2 and 1.4. The same is done in the current paper - $mu^* = 1.2$ is chosen as a baseline value.

In accordance with most empirical estimates, distribution parameter appearing in the utility function $a$ is set at 0.995 assigning a very high weight to consumption $C_t$ and a very low, but significant, weight to money balances $\frac{M_t}{P_t}$. The benchmark value of the inverse of the elasticity of substitution between $C_t$ and $\frac{M_t}{P_t}$, $b$, is set at $b = 8$ which implies that the velocity of money with respect to consumption is approximately 1.2. To investigate the sensitivity of the results with respect to the choice of $b$, I vary this parameter in the range [0.8; 20].

The second part of the calibration involves finding the parameter values
of $\gamma$ as well as the steady state values $C^*$ and $\pi^*$ satisfying the economy’s non-stochastic stationary equilibrium.

To be able to determine the value of $\gamma$ one needs to compute $\frac{\Omega^*}{D^*}$ first. To find the value of $\Omega^*$ just observe that the steady state is characterized by the following relationships $\Lambda_{t+1} = \Lambda_t$, $(\frac{D}{P})^* = 1$, $x_i^* = 1$ and $\frac{P}{D} - \mu^* = \frac{\mu^* - 1}{\mu^*}$, and then insert them into the definition of $\Omega_t$. After some algebraic manipulations one arrives at

$$\frac{\Omega^*}{D^*} = \beta \frac{\mu^* - 1}{1 - \beta \mu^*}.$$  

$\gamma$ can then be derived from (10) evaluated at the steady state. This equation is reproduced here for convenience:

$$\mu^* = \frac{-\theta}{1 - \theta - \gamma \frac{D^*}{D}}.$$  

For $\gamma$ to be positive $\theta$ should be smaller than $\frac{\mu^*}{\mu^* - 1}$ which in the case $\mu^* = 1.2$ is equivalent to the restriction $\theta < 6$. Next, in the models without capital, for a given value of $N^*$, $C^*$ can be derived from the goods market equilibrium condition

$$Y^* = N^* = C^*.$$  

I do not make an attempt to estimate the properties of the Solow-residual based on the current model since they would be strongly affected by the choice of the steady state markup $\mu^*$, but borrow the estimates provided by Gomme and Rupert (2006) obtained with US-data. The process estimated by them takes the form

$$\ln(Z_t) = 0.9641 \ln(Z_{t-1}) + \epsilon_t,$$  

where $\epsilon_t$ follows a White Noise process with standard deviation $\sigma_\epsilon$ equal to 0.0082. The implied unconditional standard deviation of the Solow-residual,
\( \sigma_z \), is given by

\[
\sigma_z = \frac{\sigma_{\epsilon}}{\sqrt{1 - 0.9641^2}} = 0.03088.
\]

The properties of the money supply process were estimated by fitting an AR\((p)\) process to the growth rate of the monetary aggregate M1. The process chosen by minimizing the Akaike information criterion is given by:\(^{11}\)

\[ g_{M1,t} = 0.0037^{**} + 0.5097^{**} g_{M1,t-1} + 0.2251^{**} g_{M1,t-2} + \tilde{u}_t, \]

where \( g_{M1,t} \) denotes the growth rate of M1,\(^{12}\) \( \tilde{u}_t \) the residual term and ** indicates significance at the 5% level. The estimated standard deviation of the unsystematic component of money supply \( \sigma_u \) equals 0.0092. The unconditional mean and standard deviation of \( g_{M1,t} \) take the values 0.0138 and 0.0125 respectively. Therefore, I choose \( \tau^* = 1.0138 \) which implies that the steady state value of the gross rate of inflation is also equal to 1.0138.

The subjective discount factor is set at 0.991 which is a standard value often found in the literature. \( \phi \) is chosen to be consistent with the observable average fraction of time spent working \( N^* \).\(^{13}\) Table 1 summarizes the calibration of the model.

### 3.1 Understanding Key Features of the Model

Figure (1) in Appendix B depict the impulse responses to a one time monetary expansion (without autocorrelation), equal to one standard deviation, for the

\(^{11}\)I used quarterly data from 1970:Q1 through 2003:Q3. According to the Ljung-Box-Q statistic and White’s heteroscedasticity test the estimated residuals display neither serial correlation nor heteroscedasticity.

\(^{12}\)Note that the stochastic process generating \( \tau_t = M_{t+1}/M_t \) introduced in section ?? can be identified as the AR(2) process in (13) since

\[ g_{M1,t} = \ln(M_t) - \ln(M_{t-1}) = \ln(\tau_{t-1}). \]

\(^{13}\)See Hristov (2008b) for details about the calibration of \( N^* \).
Parameter combinations $[\theta = 0.3; b = 8]$, $[\theta = 1.2; b = 8]$, $[\theta = 0.3; b = 20]$ and $[\theta = 0.3; b = 0.8]$. As can be seen, the lower value of $\theta$ (case $[\theta = 0.3; b = 8]$) leads to a positive reaction of output to the monetary shock while $\theta = 1.2$ (case $[\theta = 1.2; b = 8]$) implies that monetary expansions induce economic contractions.

What’s the intuition behind these results? Since households expect next period inflation to exactly offset any positive wealth effects stemming from the increase in real balances $m_{t+1}$ and at the same time all future inflation rates, markups and productivity levels to remain constant they will have no incentive to set consumption, labor supply and savings at values different from their respective steady state values. As a consequence, the expected discounted present value of firm’s profits $\Omega_t$ changes only because the discount factor $DF_t$ changes. The latter, in turn, deviates from its steady state level only because the product $C_t^{1-a} \pi_t^{1-a}$ does. Hence, the log-deviation of the markup from its steady state level can be represented as:

$$\hat{\mu}_t = -\xi((1-a)\hat{C}_t + (1-a)\hat{\pi}_t - \hat{C}_t) = \xi a\hat{C}_t - \xi(1-a)\hat{\pi}_t,$$

where $\xi = \frac{\gamma \Omega^*}{\gamma \Omega^* + \theta - 1}$. With $a = 0.9$ the difference between the log-deviation of the discount factor and that of current consumption

$$DF_t - \hat{C}_t = -a\hat{C}_t + (1-a)\hat{\pi}_t$$
will be positive as long as the increase in inflation is sufficiently large relative to the reaction of consumption. The latter is the case in all simulations performed. The optimal reaction of firms to an increase in $\Omega_t$ relative to $C_t$ is to lower markups. As a result real wages rise forcing households to increase labor supply. This is the positive effect of the monetary shock on labor stemming from the implied reactions of the discount factor and the markup. However, as mentioned in the introduction, the positive nominal impulse also induces a negative effect on labor supply which can be described as follows: Everything else given, the above average inflation reduce the marginal utility of consumption, generating an incentive for households to reduce labor supply. Whether working hours will rise or fall depends on the relative strength of the positive effect of the markup and the negative effect of the fall in the marginal utility of consumption. Which of this two effects dominates depends on the short run elasticity of demand $\theta$. Why? Optimal labor supply is given by

$$N_t = a C_t^{a-1} \left( \frac{m_t}{\pi_t} \right)^{1-a} \frac{W_t}{P_t}.$$  

Its relative deviation from the steady state can be written as

$$\hat{N}_t = -\hat{C}_t + (\xi - 1)((1 - a)\hat{\pi}_t - a\hat{C}_t),$$  

and by imposing the equilibrium condition $N_t = C_t$ we get:

$$\hat{N}_t = \frac{(\xi - 1)(1-a)}{2 + a(\xi - 1)} \hat{\pi}_t.$$  

(14)

Since for $\theta \in (0, 1)$ $\xi > 1$, while $\theta \geq 1$ implies $\xi \in (0, 1]$, working hours respond positively (for $\theta < 1$) and negatively (for $\theta > 1$) to fluctuations of the inflation rate. In the case of $\theta \in (0, 1)$ and thus $\xi > 1$ the slope of the first derivative of the current profit function with respect to the individual relative price is relatively small in absolute value. As a result, when changes of current inflation and/or current consumption occur firms need a relatively large adjustment of
the markup in order to ensure that their respective Euler equations are still satisfied. Put differently, if current demand is relatively inelastic (the case of a low \( \theta \)) the economy needs a larger adjustment of the markup to restore equilibrium after a monetary shock. In this case, the fall of the markup is more pronounced than the decrease of the marginal utility of consumption, both caused by the increase in inflation. As a consequence, working hours increase. In contrast, for a given level of \( C_t \theta > 1 \) and thus \( \xi \in (0, 1) \) implies that the fall in the marginal utility of consumption is stronger than the increase in the real wage, both caused by the jump of the inflation rate. The result is a drop in hours shifting income and consumption down. The reaction of consumption, implies a slight weakening of the effects induced by the rise in \( \pi_t \).

Another way to gain intuition about the key mechanism in this model is as follows: Suppose, initially firms miss the occurrence of the monetary shock and do not adjust the markup. Then consumption and inflation will react in exactly the way as if there were constant markups - there will be a drop in current consumption and a large jump in current inflation. But can this situation be an equilibrium? The negative (positive) reaction of consumption (inflation) will induce an unambiguous\(^\footnote{\( \frac{b}{\pi_{t+1}} \) as well as all other future variables are expected to remain unchanged.} \) increase in

\[
\hat{\Omega}_t - \hat{C}_t = DF_t - \hat{C}_t = -a\hat{C}_t + (1 - a)\hat{\pi}_t.
\]

Hence, each firm will find it optimal to lower its markup. As a result, the real wage will rise generating an incentive for households to increase labor supply. Thus, in this model for any level of consumption, labor supply will be higher than in an economy without market share competition and thus, with constant markups.

Higher values of \( b \) imply that consumption and real balances are less close
substitutes.\textsuperscript{15} The better the substitutability between both variables the weaker the reaction of the marginal utility of consumption for any given change in real balances. This is the reason why \( b = 20 \) implies much stronger responses to the monetary disturbance than \( b = 0.8 \) does (see figure 1). While the quantitative results are sensitive to the choice of \( b \), the qualitative implications of the model are almost unaffected by this parameter.\textsuperscript{16}

According to figure (1) the major shortcoming of the model is that the one-time monetary disturbance \((\rho_{1,\tau} = \rho_{2,\tau} = 0)\) induces purely temporary, one-time reactions of the main economic aggregates. This absence of any persistence is at odds with the empirical evidence provided by a vast number of studies employing structural VARs.\textsuperscript{17} As expected, setting the autocorrelation parameters \( \rho_{1,\tau} \) and \( \rho_{2,\tau} \) at their estimates given in table 1 makes the effects of the monetary expansion more persistent (see figure 2).

### 3.2 A Comparison with the New Keynesian Model

For the sake of better comparability I use a version of the New Keynesian model characterized by the same utility function, the same production technology and the same money supply rule as in the Customer Markets model. From a technical point of view the only difference between the two models concerns the firm’s condition for optimal price setting evaluated at the symmetric equilibrium. In the NKM its loglinear version is the well known \textit{New Keynesian Phillipps Curve}. It reads:

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \frac{(1 - \varphi)(1 - \varphi\beta)}{\varphi} \hat{\mu}_t, \tag{15}
\]

\textsuperscript{15}The elasticity of substitution between consumption and real balances equals \( 1/b \).

\textsuperscript{16}The impulse responses in the case \([(\theta = 0.3, b = 0.8)] \) are of very limited magnitude but have the same sign as that implied by \([(\theta = 0.3, b = 8)] \) and \([(\theta = 0.3, b = 20)] \).

\textsuperscript{17}See for example Christiano et al. (1999, 2005).
where $\varphi$ denotes the so called Calvo parameter representing the fraction of firms which are not allowed to adjust their prices within a period. (15) replaces the loglinearized versions of (9) and (10). To bias the results towards a better performance of the NKM, I first assume a relative high degree of price stickiness, $\varphi = 0.75$. This value implies that firms adjust their prices once per year on average which is much more frequently than what is consistent with the evidence provided by Bils and Klenov (2004) and Klenov and Kryvstov (2005): $\varphi \approx 0.33$. Figure 3 depicts the impulse responses to an autocorrelated monetary shock for $\varphi = 0.75$ and $\varphi = 0.33$. $\theta$ was set equal to 6 in order to ensure that the steady state markup equals 1.2. $a$ and $b$ again take their benchmark values 0.995 and 8. The economic mechanisms present in the NKM are similar to that in the model developed in this paper. The response of labor is again largely driven by the variations in the marginal utility of consumption and the average price markup (or equivalently the real wage). Again, a sufficiently strong negative reaction of the price-markup is needed to offset the negative effect of inflation on labor supply via the marginal utility of consumption. In the case of high price rigidity ($\varphi = 0.75$) output does not react on impact since both opposing effects on labor supply (almost) exactly offset each other. In the period after the shock consumption increases due to the positive wealth effect of the additionally accumulated real balances. Since nominal prices are sticky, this increase in demand drives output and thus the demand for labor and the real wage up. As can be seen, in the $\varphi = 0.75$-case the reactions to the monetary innovation are of similar magnitude and persistence as that implied by the benchmark calibration of the Customer Markets model (see figure 2). However, there is an important qualitative difference since, unlike the model developed above, in the NKM the responses of output, markups and real wages reach their maximum in the period after the shock. Consequently, it remains an open question whether the form of market share competition used in this
paper is able to serve as a microeconomic foundation of the New Keynesian model. Reducing the degree of price rigidity from $\varphi = 0.75$ to the empirically more plausible $\varphi = 0.33$ dramatically worsens the qualitative predictions of the NKM (see figure 3): Since the higher flexibility of nominal prices implies a much weaker markup reaction, output drops sharply and remains below average for about two years. Such a prediction is completely at odds with the empirical evidence regarding the real reactions to monetary innovations.

Higher values of $b$ reduce the elasticity of substitution between consumption and real balances, and so magnify the variations of the marginal utility of consumption. As a result, the impact reaction of output becomes significantly negative.\footnote{The results are not reported here but are available upon request.} Lower values $b$ imply the opposite, making the real reactions to nominal disturbances in the NKM more empirically plausible. However, relatively low degrees of price stickiness, e.g. $\varphi = 0.33$, are associated with less persistence than present in the Customer Markets model for the same value of $b$.

All in all, it is not possible to give a clear cut answer to the question whether the model developed here generates more plausible reactions to monetary shocks than the NKM does, or whether the latter can be seen as a microeconomic foundation of the former. The main tendency can be summarized as follows: For high degrees of price stickiness and values of $b$ lower than the benchmark, $b = 8$, the NKM performs better than the Customer markets model. For lower levels of price rigidity and/or higher choices of $b$ the opposite is true.

### 3.3 Business Cycles Moments

Table 2 summarizes the results obtained for the Customer Markets model with $\theta = 0.3$ and the New Keynesian model with $\varphi = 0.75$ and $\varphi = 0.33$. In all sim-
ulation \( b \) is set to its benchmark value, \( b = 8 \). The empirical second moments are taken from Hristov (2008b). It is readily seen that the model developed in this paper has to important shortcomings. First, it implies a large positive correlation between output and the markup. While as a reaction to a monetary innovation production and output tend to move in opposite directions, technological disturbances induce strong and persistent comovements of these variables. Obviously, the effects triggered off by technology shocks dominate. Second, the model performs poorly with regard to the cross correlation of inflation with output. Nevertheless, the overall performance of the model should be seen as average with a slight tendency to understate the autocorrelations of most variables.

The New Keynesian model with \( \varphi = 0.75 \) performs better than the one developed here with respect to the cross correlations with output of inflation and the markup. The former equals 0.15 and is not far from its empirical counterpart, 0.317. The correlation between output and the markup is closer to empirical value but it has still the wrong sign. In contrast the Customer Markets model implies a negative correlation between output and inflation, -0.01, and a large positive one between output and the markup. In sum, the Customer Markets model is better able to match the standard deviations of the aggregates under consideration as well as the correlation between the real wage and output. The New Keynesian model performs better with respect to the cyclical properties of the inflation rate and the markup.

Unfortunately, the careful inspection of the second moments of the two models does only reveal that each of them has as many important advantages as significant shortcomings. Therefore, neither model can be considered better than the other one.
<table>
<thead>
<tr>
<th>Variable</th>
<th>sd(x)</th>
<th>acorr(x)</th>
<th>corr(x,y)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CMM $\theta = 0.3$</td>
<td>3.73</td>
<td>0.96</td>
<td>1.00</td>
</tr>
<tr>
<td>NKM $\varphi = 0.75$</td>
<td>5.27</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>NKM $\varphi = 0.33$</td>
<td>5.86</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>US Data</td>
<td>1.547</td>
<td>0.863</td>
<td>1.000</td>
</tr>
<tr>
<td><strong>Real Wage</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CMM $\theta = 0.3$</td>
<td>1.60</td>
<td>0.74</td>
<td>0.56</td>
</tr>
<tr>
<td>NKM $\varphi = 0.75$</td>
<td>3.22</td>
<td>0.91</td>
<td>0.80</td>
</tr>
<tr>
<td>NKM $\varphi = 0.33$</td>
<td>2.96</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>US Data</td>
<td>0.815</td>
<td>0.637</td>
<td>0.472</td>
</tr>
<tr>
<td><strong>Real Balances</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CMM $\theta = 0.3$</td>
<td>3.61</td>
<td>0.96</td>
<td>0.99</td>
</tr>
<tr>
<td>NKM $\varphi = 0.75$</td>
<td>5.25</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>NKM $\varphi = 0.33$</td>
<td>5.69</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>US Data</td>
<td>3.222</td>
<td>0.941</td>
<td>0.280</td>
</tr>
<tr>
<td><strong>Inflation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CMM $\theta = 0.3$</td>
<td>1.50</td>
<td>0.49</td>
<td>-0.01</td>
</tr>
<tr>
<td>NKM $\varphi = 0.75$</td>
<td>1.20</td>
<td>0.82</td>
<td>0.15</td>
</tr>
<tr>
<td>NKM $\varphi = 0.33$</td>
<td>1.84</td>
<td>0.42</td>
<td>-0.11</td>
</tr>
<tr>
<td>US Data</td>
<td>0.387</td>
<td>0.497</td>
<td>0.317</td>
</tr>
<tr>
<td><strong>Markups</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CMM $\theta = 0.3$</td>
<td>2.74</td>
<td>0.88</td>
<td>0.79</td>
</tr>
<tr>
<td>NKM $\varphi = 0.75$</td>
<td>3.04</td>
<td>0.77</td>
<td>0.69</td>
</tr>
<tr>
<td>NKM $\varphi = 0.33$</td>
<td>0.87</td>
<td>0.22</td>
<td>0.23</td>
</tr>
<tr>
<td>US Data</td>
<td>0.538</td>
<td>0.727</td>
<td>-0.058</td>
</tr>
</tbody>
</table>

CMM - Customer Markets Model, NKM - New Keynesian Model; $\varphi$ denotes the Calvo parameter; $\mu^* = 1.2; \theta = 0.3; b = 8$; serially correlated monetary shock with $\rho_{1, \tau} = 0.5097, \rho_{2, \tau} = 0.2251$; serially correlated technology shock; sd(x) - standard deviation of x; acorr(x) - first order autocorrelation of x; corr(x,y) - contemporaneous correlation between x and output.

The second moments refer to HP-filtered empirical and simulated data. The second moments implied by the model refer to averages over 300 simulations. Each simulated series consists of 135 observations.
4 Conclusion

The model presented in this paper extends the standard monetary business cycles model with non-additively separable utility function and fully flexible prices by introducing market share competition and thus endogenizing markups. This new feature substantially approves the quantitative and qualitative properties of the model. In particular, positive monetary shocks become expansionary while the reactions of output, employment and real wages become delayed by one period, much as indicated by many VAR studies.

I also evaluate the theory developed in this paper by comparing its implications with that of the New Keynesian model with Calvo pricing. Both models perform about equally well/bad in explaining the reactions to monetary shocks as well as a set of standard stylized facts. However there are some important qualitative differences. Hence, it is not possible to give a clear cut answer to the question whether the model presented here can be seen as a microeconomic foundation of the Calvo assumption, or whether it is the ”better” theoretical framework at all.
References


[58] Tellis, Gerard J., 1988, The Price Elasticity of Selected Demand: A Meta-
Analysis of Econometric Models of Sales, *Journal of Marketing Research*, 25,
331:341


[60] Walsh, Carl E., 2003, Monetary Theory and Policy, Massachusetts Insti-
tute of Technology, 2nd ed.

Rate Policies, *Review of Economic Dynamics*, 8, 829:849


[63] Yun, T., 1996, Nominal Price Rigidity, Money Supply Endogeneity, and
A Market Share Competition

Phelps and Winter (1970) depart from the frictionless specification of the goods market by assuming that customers can not respond instantaneously to differences in firm specific prices. As the authors note, there are various rationales for this assumption - information imperfections, habits as well as costs of decision-making, none of which is explicitly modeled in their paper. An immediate consequence of such frictions is that in the (very) short run each firm has some monopoly power over a fraction of all consumers. This fraction equals the firm’s market share. In particular, Phelps and Winter (1970) assume that the transmission of information about prices evolves (proceeds) through random encounters among customers in which they compare recent demand experience. Under this assumption the probability with which a comparison between any two firms $i$ and $j$ is made will be approximately proportional to the product of their respective market shares $x_i$ and $x_j$. Therefore, one would expect that the time rate of net customer flow from firm $j$ to firm $i$ will also be proportional to the product $x_i x_j$. Phelps and Winter formalize this as follows:

$$z_{i,j} = \delta(p_i, p_j)x_i x_j,$$

where $z_{i,j}$ is the net flow of customers from $j$ to $i$. The function $\delta(p_i, p_j)$ has the properties:

$$\text{sgn}(\delta(p_i, p_j)) = \text{sgn}(p_j - p_i), \quad \delta(p_i, p_j) = -\delta(p_j, p_i), \quad \delta_1 < 0, \quad \delta_2 > 0.$$

The market share $x_i$ then evolves according to:

$$\dot{x}_i = \sum_{j=1}^{m} z_{i,j} = x_i \sum_{j=1}^{m} \delta(p_i, p_j)x_j = x_i \sum_{j=1, j\neq i}^{m} \delta(p_i, p_j)x_j,$$

where $m$ is the number of firms. Defining the customer-weighted mean of other firms’ prices $\bar{p}_i$ by

$$\bar{p}_i = \frac{\sum_{j \neq i}^{m} p_j x_j}{\sum_{j \neq i}^{m} x_j} = \frac{\sum_{j \neq i}^{m} p_j x_j}{1 - x_i}$$
and expanding $\delta(p_i, p_j)$, $\forall j \neq i$ in a first order Taylor’s series with respect to its second argument one obtains:

$$
\dot{x}_i \approx x_i (1-x_i) \delta(p_i, \bar{p}_i) + x_i \delta_2(p_i, \bar{p}_i) \left( \sum_{j \neq i} p_j x_j - \bar{p}_i (1-x_i) \right) = x_i (1-x_i) \delta(p_i, \bar{p}_i).
$$

Assuming that each supplier is small enough, so that the following relations hold:

$$
1 - x_i \approx 1 \Rightarrow \bar{p}_i \approx \sum_{j \neq i} p_j x_j = \bar{p},
$$

where $\bar{p}$ is the overall mean price in the goods market, the law of motion of $x_i$ reduces to

$$
\dot{x}_i \approx \delta(p_i, \bar{p}) x_i. \quad (16)
$$

The discrete-time version of (16) used in the following sections reads:

$$
x_{i,t+1} = g \left( \frac{p_{i,t}}{\bar{p}_t} \right) x_{i,t},
$$

where $\delta(p_{i,t}, \bar{p}_t) = g \left( \frac{p_{i,t}}{\bar{p}_t} \right) - 1$.\(^\mathbf{20}\) Now assume that the demand of each individual belonging to the customer stock of firm $i$ is given by $D \left( p_{i,t}, \bar{p}_t \right)$. Then the

\(^\mathbf{19}\) Actually, Phelps and Winter approximate $\delta(p_i, p_j)$ by a second order Taylor’s series but then assume that the second order terms are negligible and drop them. Consequently, their results are identical with that delivered in this section.

\(^\mathbf{20}\) To see that, write the discrete-time version of (16) in the more general form:

$$
x_{i,t} - x_{i,t-h} = \left( g \left( \frac{p_{i,t}}{\bar{p}_t} \right) - 1 \right) h \cdot x_{i,t-h},
$$

where $\left( g \left( \frac{p_{i,t}}{\bar{p}_t} \right) - 1 \right) h$ measures the net customer flow to firm $i$ over a time interval of length $h$. Divide both sides of the last equation by $h$, let $h$ go to zero and assume that $x_{i,t}$ is differentiable from the left (from below) with respect to $t$. The resulting equation is:

$$
\dot{x}_{i,t} = \left( g \left( \frac{p_{i,t}}{\bar{p}_t} \right) - 1 \right) x_{i,t}.
$$
demand curve faced by firm $i$ is given by:

$$x_{i,t}D\left(\frac{p_{i,t}}{\bar{p}_t}\right) = g\left(\frac{p_{i,t-1}}{\bar{p}_{t-1}}\right) x_{i,t-1}D\left(\frac{p_{i,t}}{\bar{p}_t}\right),$$

Hence, the price setting problem of the typical firm becomes dynamic.
B Impulse Responses

In each case the shock $u_t$ in $\hat{\tau}_t = \rho_{1,\tau} \hat{\tau}_{t-1} + \rho_2 \tau \hat{\tau}_{t-2} + u_t$ takes place in the first period ($t = 1$) and equals one standard deviation $\sigma_u$.

Figure 1: MIU-model with market share competition. Impulse responses to a monetary shock, $\rho_{1,\tau} = \rho_{2,\tau} = 0$, $\theta = \{0.3, 1.2\}$, $a = 0.995$, $b = \{8, 20, 0.8\}$. Percentage deviations from steady state.

\[\theta\] denotes the parameter $\theta$. 

37
Figure 2: MIU-model with market share competition. Impulse responses to a monetary shock, $\rho_{1,\tau} = 0.5097$, $\rho_{2,\tau} = 0.2251$, $\theta = \{0.3, 1.2\}$, $a = 0.995$, $b = \{8, 20, 0.8\}$. Percentage deviations from steady state.
Figure 3: New Keynesian model with Calvo pricing. Impulse responses to a monetary shock, $\rho_1,\tau = 0.5097$, $\rho_2,\tau = 0.2251$, $\varphi = \{0.75, 0.33\}$, $a = 0.995$, $b = 8$. Percentage deviations from steady state.