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Equilibrium Selection and Monetary Policy

A Natural Rate Perspective

Alexander Meyer-Gohde^{*†}

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Abstract

This paper reiterates that the monetary authority can reasonably be held responsible for inflation. The bounds on monetary policy to ensure determinacy in a class of models that satisfy Lucas's (1972) natural rate hypothesis (NRH) are shown to be identical for all supply specifications, save isolated singularities. This follows, as is argued, from determinacy being a criterion of the long run when all NRH supply specifications coincide. Thus, no specific knowledge of the supply side beyond its fulfillment of the NRH is necessary to assess whether a particular monetary policy will ensure determinacy and, under the standard dynamic IS-equation, determinacy is solely a function of the parameters in the interest rate rule. Cochrane's (2007) criticism of determinacy for selecting equilibrium is verified and shown to be associated with reckless money growth accommodating the associated explosive inflation. Monetary policy's inability to control the nominal interest rate in the long run is to blame and appending policy with a credible commitment to stable long-run money growth suffices to rule out these otherwise accommodated nominal explosions.

JEL classification: C62; E31; E43; E52

Keywords: Indeterminacy; Taylor rule; Natural rate hypothesis

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1 Introduction

This paper has two main results. First, in extending the determinacy analysis to Lucas’s (1972) natural rate hypothesis (NRH) —the proposition that monetary policy cannot permanently induce a non-zero output gap— following Carlstrom and Fuerst (2002), I establish that all supply equations that satisfy the NRH for a given demand function yield the same bounds on determinacy, saving for isolated singularities. Second, I provide a monetarist interpretation for the admissibility of Cochrane’s (2007) explosive nominal equilibria; namely that the monetary authority is accommodating these equilibria with exploding money growth rates.

The first result implies that one can reasonably expect the monetary authority to know when its interest rate policy will admit many stable equilibria (indeterminacy) or a single stable equilibrium (determinacy), even if it has no specific knowledge regarding the supply side beyond that it satisfies the NRH. The analysis attempts to provide the proofs missing from the general claim of Carlstrom and Fuerst (2002) that there is a one-to-one correspondence between determinacy in models that satisfy the NRH and their corresponding frictionless counterparts. Specifically, I prove the necessity of determinacy in the latter for determinacy in the former, but disprove the sufficiency. Fortunately, the cases of insufficiency can be characterized as singular parameterizations that do not pose a general problem. Applying the result to the standard dynamic IS curve with monetary policy defined as any finite linear relationship between the nominal interest rate, inflation, and the output gap, I prove that indeterminacy is solely a function of the parameters in monetary policy.

I provide further insight into Cochrane’s (2007) criticism of determinacy as being an arbitrary elimination of explosive nominal equilibria by demonstrating that a tenet of the quantity theory provides support to his critique. Adding a standard money-demand specification, I find that the explosive paths for inflation are being accommodated by the money supply. I.e., the hyperinflationary paths are consistent with the monetarist view that, “sizable changes in the rate of change in the money stock are a necessary and sufficient condition for sizable changes in the rate of change in

money income,” (Friedman and Schwartz 1963, p. 63) and thus that monetary restraint is a necessary and sufficient condition for controlling inflation. (See Nelson and Schwartz (2008, p. 838)) Note that this does not rule out hyperinflation *per se*: it rules out hyperinflations that are speculative—i.e., non-fundamental to the money supply. The alternate, explosive equilibria of Cochrane (2007) are indeed fully valid monetary equilibria, with the monetary authority increasing the money *growth rate* exponentially commensurate with the explosive path for inflation.

The standard sticky-price New Keynesian model with Calvo (1983) contracts is known to violate the NRH. This violation is “an awkward situation in monetary economics” (Wolman 2007, p. 1366) and contradicts the consensus widely accepted by the late ’70s (Friedman 1977, p. 459). My first result implies that the standard New Keynesian model’s determinacy results and violation of the NRH are inextricably linked. This has immediate consequences as determinacy is concerned with the admissibility of multiple short-run equilibria. Determinacy analyses in standard New Keynesian models¹ must either disown the relevance of their bounds on monetary policy or defend their models’ violation of the NRH. Additionally, the sticky-price model’s violation of the NRH actually frees it from Cochrane’s (2007) critique: nominal explosions go hand in hand with real explosions that Cochrane (2007) admits economics can rule out. Yet this result, as it too rests on the violation of the NRH, is dubious.

The main focus for Cochrane’s (2007) analysis, however, is a frictionless model—i.e. a model that satisfies the NRH in the most extreme sense and that forms the basis for my determinacy analysis. In this model, pinning down the inflation rate when monetary policy controls the nominal interest rate requires a particular constellation for the interest rate rule and the elimination of explosive paths. This constellation is one that ensures determinacy and, from my first result, these are the same across a class of NRH models with a common demand specification. Remaining is then the elimination of explosive paths and thusly, for models that satisfy the NRH, Cochrane’s (2007)

¹See Bernanke and Woodford (1997), Clarida, Galí, and Gertler (1999), Bullard and Mitra (2002) and Woodford (2003), among many others.

critique does apply. The analysis of the frictionless model is directly applicable as it behaves identically in the long run to the rest of the class of NRH models, where the uniqueness of a path for real variables (should supply pose a short-run tradeoff between inflation and output) depends on the uniqueness of a path for inflation. This coincidence of determinacy follows from the frictionless model being a model where this is no liquidity effect, only the Fisher effect; and for a model to satisfy the NRH, the liquidity effect must dry up,² leaving only the Fisher effect in the long run. Thus, monetary policy has the same effects in all NRH models in the long run, impacting the economy only through the Fisher effect, where “high interest rates are a sign that monetary policy has been easy.” (Friedman 1968, p. 7)

Aside from Cochrane’s (2007) non-Ricardian fiscal solutions along the lines of Benhabib, Schmitt-Grohé, and Uribe (2001a) and Sims (1994), McCallum (2009a) and Minford and Srinivasan (2009) have attempted to answer the Cochrane’s (2007) critique. The analysis here provides an answer similar in vein to Minford and Srinivasan’s (2009) by examining money. Minford and Srinivasan (2009, p. 15) examine the question illustratively within an unrelated Cagan model and ultimately “appeal to an optimizing government [...] that sets the inflation tax” to rule out explosions in inflation. I show this to be an unnecessary and misleading detour: the underlying NRH model *reduces* to a specific Cagan model, viz. that of Sargent and Wallace (1973), and the speculative hyperinflation literature—e.g., Obstfeld and Rogoff (1983) and Gray (1984)—links Cochrane’s (2007) explosive equilibria unequivocally to reckless money growth. McCallum (2009a) rules these equilibria out by appealing to LS learning. The interpretation of LS learning in the context my analysis is not very satisfying: McCallum (2009a) rules out hyperinflation caused by an ever-increasing growth rate of money supply as the associated inflation is increasing too quickly for it to be learnable in a least-squares sense. Additionally, I argue that McCallum (2009a) misinterprets his model with money within the speculative hyperinflation literature. Upon closer inspection, his model confirms

²See Nelson (2008, p. 1804)

my claim: the explosive paths of inflation that his model cannot rule out are necessarily associated with explosive paths of money growth.

The quantity theory also provides the way out of these hyperinflationary equilibria for the monetary authority: do not accommodate such equilibria and commit credibly to not do so beforehand. But this assumption is already implicit in the New Keynesian analysis as defended by Nelson (2008). If one defines monetary policy is solely over the nominal interest rate, it should come as no surprise that this opens up the potential for problems in the long run, as “the monetary authority cannot treat the nominal interest rate as an instrument in the long run [— a position] widely shared.” (Nelson 2008, p. 1805) Yet, monetary policy can be completed through the specification of a steady-state money growth rate, and the steady-state inflation rate specified in most New Keynesian models can be interpreted as such an average money growth rate. The off-equilibrium “threat” of the monetary authority to rule out the explosive equilibria of Cochrane (2007), therefore, is nothing more than to keep money growth constant.

The importance of monetary aggregates for monetary policy has found support recently in Nelson (2003), Svensson (2003), McCallum and Nelson (2005), Nelson (2008), McCallum (2008), McCallum and Nelson (2009b). Woodford (2008) presents the case for interest rate feedback rules, as opposed to the pegging of interest rates criticized by Sargent and Wallace (1975), in the spirit of Taylor (1993). Taking nonlinearities seriously, Benhabib, Schmitt-Grohé, and Uribe (2001b) show the dangers of assessing Taylor rules in a linear framework. Additional support of interest rate rules using Evans and Honkapohja’s (2001) E-Stability can be found in Bullard and Mitra (2002) and McCallum (2003). McCallum and Nelson (2009a) provide a recent overview of the money in current analyses of inflation without dismissing the lessons of the quantity theory.

Both Woodford (2002) and Woodford (2003) acknowledge the nonverticality of the standard New Keynesian Phillips curve in the long run, which McCallum (2004) formulates into a critique of the model’s violation of the NRH. Andrés, López-Salido, and Nelson (2005) examine the NRH and

New Keynesian models both theoretically and empirically. Levin and Yun (2007) bring the standard model closer to the NRH by endogenizing the contract length.

This paper is organized as follows, section 2 sets the stage intuitively, section 3 assesses determinacy in a class of models that satisfy the NRH, section 4 links Cochrane's (2007) critique to money, section 5 examines several nonlinear money-demand specifications, section 6 provides a monetarist context for interpretation, and section 7 concludes.

2 Linking the NRH, the Long-Run, and Determinacy

To establish the necessary intuition for the mechanisms at work in the analysis and specific results, I shall build a conceptual link between the NRH, the long run, and determinacy in this section. First, I shall review the two different forms of the NRH emphasized by McCallum (2004, pp. 21–22) and argue that the stricter, or Lucas version, ought to be used in analyzing determinacy. Requiring the NRH to hold imposes restrictions only in the long-run—a stable short-run Phillips curve tradeoff does not contradict the hypothesis. Finally, I shall argue that ascertaining whether many (indeterminacy) or only one (determinacy) equilibrium paths are non-explosive is an inherently long-run exercise, though with short-run consequences (i.e., which equilibrium path prevails). Thus, intuitively, the long-run restrictions imposed by the NRH should be relevant for analyzing determinacy and, therefore, the NRH is pertinent for the short-run despite its long-run nature.

In bringing attention to the standard New Keynesian Phillips curve's violation of the NRH, McCallum (2004, pp. 21–22) draws a distinction between “Friedman's weaker version” and the “stronger Lucas version” of the NRH. The former stating that a higher, but constant, rate of inflation cannot permanently affect output and the latter that no path for prices, inflation, inflation growth, etc. can permanent keep output above its natural level. “[S]ome substitute a stable relation between the acceleration of inflation and unemployment for a stable relationship between inflation and unemployment - aware of but not concerned about the possibility that the same logic that drove them

to a second derivative will drive them to even higher derivatives.” (Friedman 1977, p. 274) It would take an infinite number of steps to get a weaker-version-NRH model to satisfy the stronger version, incorporating all possible higher derivatives. Adapting the New Keynesian Phillips curve with indexation, to either steady state or lagged inflation, is subject to Friedman’s criticism above: neither adaptation brings the model in line with the Lucas version.

This workhorse of the literature, the standard New Keynesian sticky-price model with Calvo (1983)-style overlapping contracts in general equilibrium, is given (in log-deviations and abstracting from exogenous driving processes) by ³

$$(1) \quad y_t = E_t [y_{t+1}] - a_1 R_t + a_1 E_t [\pi_{t+1}]$$

$$(2) \quad \pi_t = \beta E_t [\pi_{t+1}] + \kappa y_t$$

and an as of yet unspecified rule for monetary policy, where y_t is the output gap, π_t inflation, and R_t the nominal interest rate. Equation (1) is a dynamic IS-curve resulting from the Euler equation of households’ intertemporal maximization and equation (2) is the New Keynesian Phillips curve derived from Dixit-Stiglitz aggregators of individual firms’ intertemporal profit maximization reflecting the probability that prices set today remain in effect into the future.

First, one can confirm that (2) does not satisfy Lucas’s (1972) NRH by taking expectations

$$(3) \quad E[y_t] = \frac{1}{\kappa} (E[\pi_t] - \beta E[\pi_{t+1}]) \neq 0$$

Note that even in the extreme parameterization $\beta = 1$, $E[y_t] \neq 0$ should inflation be nonstationary. Requiring inflation to be stationary *a priori* precludes the possibility of an entire class of potential monetary policies, including pernicious hyperinflationary policies. As made explicit by McCallum (1998), the NRH requires that “[o]n average, output should be equal to potential output, for any monetary policy.” Nothing in this statement excludes nonstationary policies. The only way for this Phillips curve to satisfy the NRH, is if $\kappa \rightarrow \infty$, making the Phillips curve always⁴ vertical.

³Cf. McCallum (2001b, p. 152), equations (2.7) and (2.14), Woodford (2003, p. 246), or Galí (2008, p. 49).

⁴I.e., at every expectational horizon.

The sticky-price Phillips curve has been indexed, either to steady-state inflation,⁵

$$(4) \quad \pi_t - \bar{\pi} = \beta E_t [\pi_{t+1} - \bar{\pi}] + \kappa y_t$$

or past inflation⁶

$$(5) \quad \pi_t = \frac{\gamma}{1 + \gamma\beta} \pi_{t-1} + \frac{\beta}{1 + \gamma\beta} E_t [\pi_{t+1}] + \frac{\kappa}{1 + \gamma\beta} y_t$$

but both of these modifications still fail to satisfy the strict version of the NRH,⁷ for the same reason above. Only those monetary policies that lead to a stationary path for inflation allow the the output gap to be equal, on average, to zero. Certainly, indexation to steady-state inflation is meaningless, should inflation be nonstationary. As pointed out by Nelson (2008), it is monetary policy that determines steady-state inflation, or indeed whether it should exist, and without having specified monetary policy, it is almost vacuous to speak of such a value. As above, these Phillips curves can be made to satisfy the NRH, but this requires $\kappa \rightarrow \infty$, making them always vertical.

Consider a definition of the NRH, due to Carlstrom and Fuerst (2002), that holds in finite time⁸

$$(6) \quad E_{t-k} [y_t] = 0 \quad \forall t$$

This allows us to trivially express any supply function that fulfills this hypothesis as

$$(7) \quad y_t = \sum_{j=0}^{k-1} (E_{t-j} [y_t] - E_{t-j-1} [y_t])$$

Non-zero output gaps can be represented wholly as innovations or forecast errors without making any conjecture as to admissible solutions, in the words of Friedman (1977, p. 456), “[o]nly surprises matter.” Note that the effect of a surprise need not disappear immediately after impacting the output

⁵See Yun (1996)

⁶See Christiano, Eichenbaum, and Evans (2005) for $\gamma = 1$ and Smets and Wouters (2003) for $0 < \gamma \leq 1$.

⁷See McCallum (2004, pp. 21–22) and McCallum and Nelson (2009a, pp. 6–7)

⁸The list of models that satisfy this version of the NRH include: Andrés, López-Salido, and Nelson’s (2005, p. 1034) “Sticky information, staggered á la Taylor”; the Mussa-McCallum-Barro-Grossmann “P-bar model” — see McCallum (1994) and McCallum and Nelson (2001); models of staggered predetermined prices such as Fischer (1977) and Blanchard and Fischer (1989, pp. 390–394); Carlstrom and Fuerst’s (2002, p 81-82) model in the spirit of Fischer (1977); as well as the expectational Phillips curve of Lucas (1973)—see also Sargent and Wallace (1975)— that formalized the rational expectations revolution. Though one might argue that a NRH in finite time is overly restrictive, this subset covers every model to my knowledge that purports to satisfy Lucas’s (1972) NRH with one exception: Mankiw and Reis’s (2002) sticky-information model staggered á la Calvo, whose determinacy properties are examined separately here in Meyer-Gohde (2009) and coincide with those of this analysis for the demand and monetary policy specifications examined there. In any case, k is completely arbitrary here, it makes no difference for the conclusions whether the long run sets in after four quarters, four years, or four millennia.

gap, it can have a lasting—but not permanent—effect. That is, there can be a stable short-run tradeoff between the output gap and inflation, but this tradeoff must not be permanent if the model is to satisfy the NRH.

In the frictionless counterpart model, there is no impediment to firms' setting the optimal, full-information price every period. It follows by definition that the output gap is always zero, which can be viewed as an extreme version of the NRH

$$(8) \quad y_t = 0 \quad \forall t$$

the special case of $k = 0$ in (6). In this case, (1) reduces to

$$(9) \quad R_t = E_t [\pi_{t+1}]$$

this is identical to the Fisher-type equation in Woodford's (2003, Ch. 2) analysis of nominal (price-level) determinacy in a frictionless economy, as well as the simple model found in the discussion of Cochrane (2007) and McCallum (2009a) regarding the appropriateness of determinacy as an equilibrium criterion in monetary models. After k periods have passed since some disturbance from equilibrium, the supply side described by (6) behaves identically to that of (8), i.e., applying the conditional expectations operator to the LHS of both supply sides yields zero— $E_{t-k}[(8)] = E_{t-k}[(6)] = 0$. Hence, given a common specification for the remainder of the model, any two models that satisfy (6) for some k are identical in the long-run (or indeed, after k).

Determinacy is most frequently ascertained by the eigenvalue counting method of Blanchard and Kahn (1980). Roughly speaking, a model is brought into first-order form $E_t [G_{t+1}] = HG_t$, where some variables in G_t might be predetermined, and is said to be determinate if the number of stable eigenvalues in H is exactly equal to the number of these predetermined variables. Thus, the instantaneous reaction of G_t to some disturbance is sufficient to ascertain whether some equilibrium path will lead to explosive or stable behavior. While this remains technically true of the models that satisfy (6),⁹ it is easy to forget that the explosiveness being ruled out need not occur instantaneously

⁹By defining sufficient dummy variables to capture the information structure. See, e.g., Sims (2001).

in the variables of interest (i.e., a subset of G_t) and, in general, any finite value at any finite horizon is permissible.¹⁰ Determinacy rules out paths that would lead to explosive, i.e., infinite values of, variables of interest.

To illustrate, assume that inflation is required to be stable. That is, inflation must converge back to equilibrium following any disturbance. Examining Figure 1a, all the paths pictured (here with initial log-deviations of inflation to some unit exogenous disturbance in demand between 0.54 and 0.56) certainly appear to be uniformly explosive (within five periods, inflation on all paths exceeds the initial deviations), violating the required stability. Yet this is deceiving: there is nothing that violates the requirement of stability for inflation in the figure but for one's own imagined extrapolation of the behavior depicted into the infinite future. To see this, examine Figure 1b, the same picture as before, but now extended out to thirty periods after the initial disturbance. The initial common explosiveness dissipates rather swiftly as some variables are below their initial values and some are above. One could imagine now that some path, here highlighted as a more heavily weighted line, is uniquely convergent, with all paths that started above diverging to positive infinity and all that started below diverging to negative infinity. Again, this is the result of one's extrapolation of the first thirty periods on out into the infinite future, the same "shift" that occurred between Figures 1a and 1b could certainly occur again at a more distant horizon. It is the behavior in the long run that establishes whether a particular path is diverging, yet the particular path chosen by the long-run is associated with specific short run reactions of variables. That is, the long run is decisive for the short run through the selection of valid equilibrium paths.

The eigenvalue counting method, by bringing the model into a first order form, ensures that the system described by $E_t[G_{t+1}] = HG_t$ behaves instantaneously exactly as it would asymptotically. This is convenient, but upon reflection highlights an important shortcoming of the standard New Keynesian model: it behaves in the short run as it does in the long run with the same stable tradeoff

¹⁰Exceptions would be, e.g., finite but negative values for prices, but assuming that variables are transformed, as they usually are, appropriately to allow the range of the transformed variable to encompass the reals, e.g., for prices, the log of the price would be included in the system.

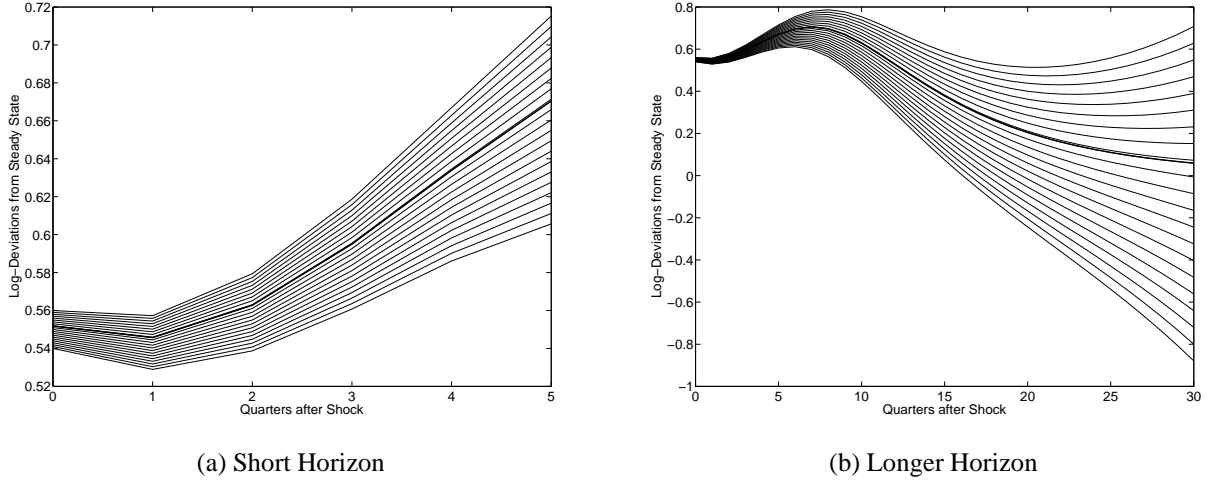


Figure 1: Impulse Responses of Inflation to a Demand Shock for Different Initial Conditions

between output and inflation.¹¹ Allowing a short-run tradeoff in this model implies a long-run tradeoff that can impact determinacy, admitting a single path (determinacy) for some parameter regions where a NRH model might admit many (indeterminacy). Of course and as was seen in Figure 1, different paths are usually associated with different instantaneous reactions of variables to disturbances as well. NRH models of the class satisfying (6) can also be brought into first-order form by defining dummy variables. The variables of interest, like inflation in the illustration, are a subset of G_t and may differ in their behavior before and after k . Thus, all models that differ only in their supply side and that satisfy the NRH will display the same behavior after k and thusly ought to have identical determinacy regions in parameters, regardless of their behavior in the short run.

3 Determinacy in Natural Rate Models

Here, I shall establish an equivalence between nominal determinacy in the frictionless model with (8) and real determinacy in the general model with any supply side satisfying (6). This equivalence was asserted, but without proof, by Carlstrom and Fuerst (2002) to be one-to-one. I shall prove that nominal determinacy in the frictionless model is a necessary condition but not sufficient, one

¹¹To see this, simply note that (2) is already in first-order form. The relation between the output gap and current and future inflation is the same no matter what horizon is examined, a very stable tradeoff indeed.

must guard against singular cases. Saving for such cases, the equivalence alluded to intuitively in the previous section is established and this intuition is extended. Additionally, the questionability of existing determinacy analyses using the standard New Keynesian model is highlighted.

In what follows, I will analyze linear rational-expectations models of the following class:

$$(10) \quad 0 = \sum_{i=0}^p \sum_{j=-m}^n Q(i, j) E_{t-i} X_{t+j}, \quad X_t = \begin{bmatrix} R_t \\ \pi_t \\ y_t \end{bmatrix}, \quad 0 \leq p, m, n < \infty$$

where the $Q(i, j)$'s are matrices of dimensions 3×3 . That is, the model is composed of three structural equations determining the supply side, demand side, and monetary policy. The class encompasses all linear rational-expectations models in the three variables of interest that (i) have a finite number of leads (given by n), (ii) have a finite number of lags (given by m), and (iii) have expectations formed at differing horizons from t into the finite past $t - p$.¹² This, of course, encompasses the models discussed in Section 2.

To close out any of models of the foregoing section, monetary policy needs to be specified. The only restriction I shall impose on monetary policy is that it fits into the class defined in (10). Accordingly, let monetary policy be the third equation of (10),¹³ given by

$$(11) \quad 0 = \sum_{i=0}^p \sum_{j=-m}^n Q_{3,\cdot}(i, j) E_{t-i} [X_{t+j}]$$

This captures a wide range of interest rate rules found in the literature, including the current and forward-looking inflation targeting, interest rate smoothing, and output-gap targeting as examined in Woodford (2003) and all the rules of Bullard and Mitra (2002).

Lemma 3.1. *For the system (10) to be determinate, i.e., to have a unique stationary solution,*

1. *The model*

$$(12) \quad 0 = \sum_{j=-m}^n \tilde{Q}_j X_{t+j}$$

where $\tilde{Q}_j = \sum_{i=0}^p Q(i, j)$, must have a unique saddle-point stable solution.

¹²Note that the absence of exogenous driving forces in (10) is not restrictive. The conditions for determinacy remain the same if (10) is appended with stationary driving forces —i.e., I am investigating the properties of the homogeneous component of the system of difference equations, but one has the additional task of associating the exogenous driving forces with the expectation errors —see Sims (2001).

¹³Where $Q_{3,\cdot}(i, j)$ is the row vector given by the the third row of $Q(i, j)$.

2. The matrix

$$(13) \quad \begin{bmatrix} \mathbf{Q} \\ \mathbf{B} \end{bmatrix}$$

must be non-singular. \mathbf{Q} and \mathbf{B} are block matrices of dimensions $3p \times 3(p+n)$ and $3n \times 3(p+n)$ respectively with blocks of dimension 3×3 . The s 'th block row of \mathbf{Q} is given by

$$(14) \quad [0_{\max(0, s-1-m)} \quad \tilde{Q}(s-1, -\min(s-1, m), n) \quad 0_{p-s}]$$

where 0_i is a $3 \times 3i$ block vector of zeros and $\tilde{Q}(a, b, c) = [\tilde{Q}(a, b) \quad \tilde{Q}(a, b+1) \quad \dots \tilde{Q}(a, c)]$

with $\tilde{Q}(a, b) = \sum_{i=0}^{\min p, a} Q(i, b)$. The s 'th block row of \mathbf{B} is given by

$$(15) \quad [0_{\max(0, s+p-m-1)} \quad -\tilde{B}(\min(p+s-1, m)) \quad I \quad 0_{n-s}]$$

where I is a 3×3 identity matrix and $\tilde{B}(a)$ being the last $3 \times 3a$ elements of the $3 \times 3m$ matrix B that forms Anderson's (2010, p. 7) convergent autoregressive solution to (12).

Proof. See Appendix □

The first condition requires that the model be determinate if all lagged expectations are replaced with time t expectations and the second condition requires additionally that one can uniquely resolve the lagged expectations. Whiteman (1983, pp. 29–36) shows that resolving lagged expectations, “withholding” constraints in his language, is not generally a trivial task.

Carlstrom and Fuerst (2002, p. 82) make a quite general claim, without proof, regarding the conditions under which a model that satisfies the NRH is determinate: “[I]n a model that satisfies the NRH, there is real determinacy if and only if there is nominal determinacy in the corresponding flexible-price economy.” In the two propositions that follow, I will substantiate the necessity component of their claim but refute the sufficiency component.

Proposition 3.2. *Consider a model in (10) that satisfies the NRH defined in (6). The model is determinate only if the corresponding frictionless model, i.e. that satisfies (8), is determinate.*

Proof. See Appendix □

Thus, a necessary condition for determinacy in any model that satisfies the NRH is that the corresponding frictionless model is determinate. In the latter, real variables are determinate by definition, so the question of determinacy pertains only to nominal variables. In the former, the output gap is jointly determined with nominal variables and thus determinacy relates to real as well as nominal variables. So the foregoing proposition corroborates the “only if” component of Carlstrom and Fuerst’s (2002) claim, showing essentially that the eigenvalue counting method of Blanchard and Kahn (1980) is the same regardless of actual value of k .

Proposition 3.3. *Consider a determinate frictionless model, i.e. that satisfies (8), in (10). There exist corresponding NRH models, i.e. that satisfy (6) for $k > 0$, that are not determinate.*

Proof. See Appendix □

Therefore, it does not necessarily follow that a model that satisfies the NRH is determinate when its frictionless counterpart is, refuting the sufficiency component of Carlstrom and Fuerst (2002). Lemma (3.1) shows that, while necessary, the saddle-point property of the underlying matrix polynomial is insufficient to conclusively establish determinacy. As Whiteman (1983, p. 33) points out, “the conditions for existence and uniqueness of solutions to withholding equations are quite different from those for the general expectational difference equation.” The class of models in (10) combines the latter —i.e., forward looking difference equations—with withholding equations—i.e., lagged expectations—and, thus, it is not surprising that one has to take both the standard—i.e. saddle-point—and these quite different conditions into account.

Fortunately, it should be more the exception than the rule that the “if” is not fulfilled. This pertains to the non-singularity of the matrix $[\mathbf{Q}' \quad \mathbf{B}']'$, which cannot be guaranteed due to the generality of the class of models specified in (10). Yet, there is nothing in the class of models to induce this matrix to be singular in general. Even should one encounter a particular model parameterization leading to singularity, it should be expected that a minor perturbation of the model or its parameterization will lead to non-singularity. This is reminiscent of King and Watson’s (1998, p. 1017) “mundane

source” of indeterminacy, requiring here the structure of the model to be such that it leaves no linear combination of forecast errors unrestricted.

Moving past this additional source of mundanity, the close relationship between determinacy under the NRH and determinacy in the corresponding frictionless model has some strong implications. Indeed, if one restricts attention to models that satisfy the condition of non-singularity in (3.1), the following proposition can be made

Proposition 3.4. *Consider a model in (10) restricted to rule out the singularity of (13) and fix the demand equation and monetary policy.*

1. *If the model is determinate under one supply equation that satisfies (6), it is determinate under all supply equations that satisfy (6).*
2. *If the model is not determinate under one supply equation that satisfies (6), it is not determinate under all supply equations that satisfy (6).*

In other words, for any given demand specification, the bounds on monetary policy to ensure determinacy are same for all supply equations that satisfy (6).

Proof. See Appendix □

With demand given by (1), restricting supply equations to satisfy the NRH (6), but leaving monetary policy still generically specified as in (11), a more specific statement can be made

Corollary 3.5. *Consider a model in (10) with demand given by (1) and any supply equation satisfying (6) and restricted to rule out the singularity of (13). Determinacy is a function solely of the parameters in the interest rate rule (11) pertaining to inflation and the interest rate.*

Proof. See Appendix. □

If the model satisfies the NRH, then the output gap must on average be equal to zero independent of monetary policy (see McCallum (1998, p. 359)). From (1):

$$(16) \quad E[y_t - y_{t+1}] = a_1 E[\pi_{t+1} - R_t]$$

which posits a relationship between the average output gap and monetary policy (defined over the nominal interest rate R_t). One could certainly specify a process for the nominal interest rate such that the average output gap would be equal to zero, but the NRH requires that this hold *regardless* of monetary policy. Thus, that the output gap on average is equal to zero must follow from the supply side equation and must hold independently of (1).

The NRH delivers, then, the existence but not necessarily the uniqueness of a bounded path for the output gap irrespective of the existence and uniqueness of bounded paths for inflation and the nominal interest rate. However, from (16) it must then be the case that the real interest rate $R_t - E_t[\pi_{t+1}]$ also converges. Furthermore, if the bounded path for the real interest rate is uniquely determined, then so is the bounded path for the output gap and vice-versa.

The uniqueness of a bounded path for inflation and the nominal interest rate is, thus, given by the rule for monetary policy and (9).¹⁴ Determinacy, therefore, corresponds to nominal determinacy in the frictionless counterpart.

Were $k = 0$, there would be complete separation between the real and nominal sides of the economy and monetary policy through the nominal interest rate would serve only to establish nominal determinacy. Otherwise if $k > 0$, the lack of a complete separation but fulfillment of the NRH by assumption links nominal and real determinacy: without a unique path for the nominal side, the link between the output gap and the nominals at horizons less than k implies that although every path for the output gap be bounded, a unique path for the output gap cannot be pinned down. If a unique path for the nominal side can be determined by (9) and monetary policy, this path selects, through the link at horizons less than k , a single path for the output gap.

¹⁴I.e., the Fisher-type equation with the real interest rate normalized to zero or as derived from the dynamic IS equation (16) with the output gap always closed.

Therefore, there is a unique convergent path for the output gap if and only if there is a unique convergent path for inflation and the nominal interest rate in the counterpart model (9).¹⁵

The situation is exemplified graphically in Figure 2. All the different paths of the output gap in Figure 2a converge even though all but one of the paths for inflation, depicted in Figure 2b, diverge. If one has reason, say by requiring inflation to be stable, to select among the different paths for inflation, the selected path for inflation corresponds to a particular path for the output gap, thus determining both through consideration solely over inflation.

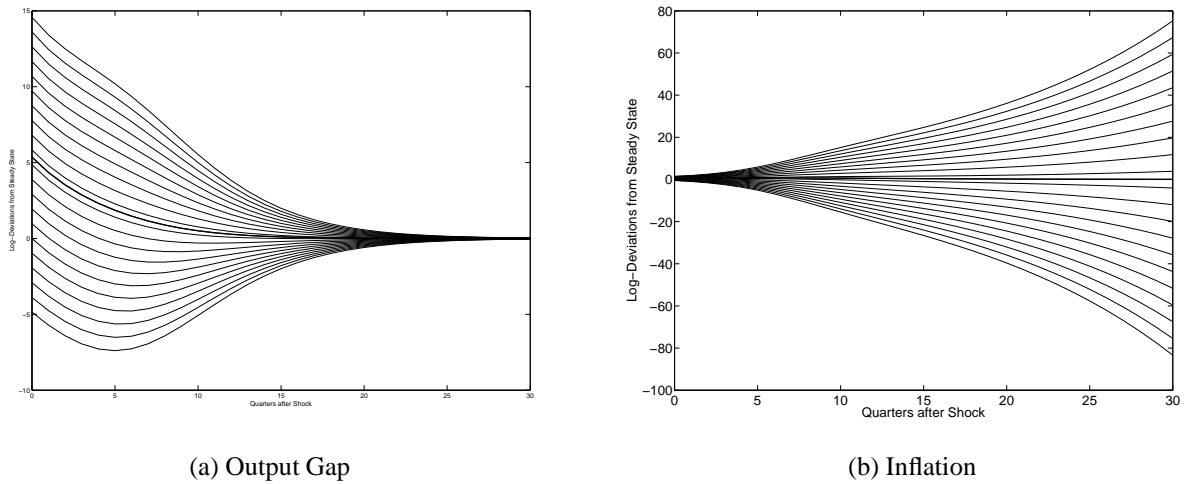


Figure 2: Impulse Responses to a Demand Shock for Different Initial Conditions

A few comments are in order here. Real business cycle models are generally of the type that the NRH holds but does so already at $k = 0$, as complete flexibility in prices is assumed.¹⁶ In the sticky-price New Keynesian model, the NRH does not hold at any horizon. As a consequence, the sticky-price model is not even asymptotically isomorphic to its frictionless equivalent, and there is no reason to expect a general equivalence between determinacy conditions in the two models. With there being a permanent link between the nominal and real side of the economy, nominal and real determinacy must be simultaneously ascertained. As discussed previously, many modifications of the standard sticky-price model do satisfy the NRH assuming inflation be stationary. Since we are

¹⁵Saving, of course, for the caveat of the singularity of (13).

¹⁶cf. Woodford (2003, p. 6)

only interested in stationary equilibria, there would not seem to be a contradiction. This is unfortunately mistaken as establishing determinacy requires one to look at all possible equilibria, including explosive equilibria, in the hope that only one is non-explosive. Thus, in assessing determinacy in the standard New Keynesian model, one is forced to look at paths along which both the NRH is violated and its violation is consequential for the ensuing path.

When the NRH does not hold at every horizon (i.e., $k > 0$), nominal and real determinacy are linked as in standard sticky-price models. That the NRH holds at all, however, ensures that this link dissolves such that conditions necessary to determine this determinacy are identical to the conditions for nominal determinacy that would prevail were the NRH to hold at all horizons. This conceptual link between nominal determinacy in RBC models and both real and nominal determinacy in NRH models provides for a simple means to establish nominal and real determinacy: one need only to examine the conditions for nominal determinacy in the corresponding frictionless equivalent. This is generally a much simpler task.

Meyer-Gohde's (2009, p. 17) Table 1 juxtaposes the bounds on several standard interest rate rules both with the standard sticky-price Phillips curve and Mankiw and Reis's (2002) sticky-information Phillips curve.¹⁷ As noted by its authors, the latter satisfies the NRH—but only asymptotically as opposed to the $k < \infty$ assumed here following Carlstrom and Fuerst (2002). The bounds derived by Meyer-Gohde (2009) for determinacy coincide with those required for nominal determinacy in the frictionless model for the set of standard interest rate rules examined.

Thus a broad class of models, those satisfying Lucas's (1972) NRH, achieve determinacy under the same conditions and do so independently of parameters outside of the monetary policy rule. Wherefore, the bounds derived under the NRH pass the additional criticism of Cochrane (2007, p. 27) that the bounds for determinacy ought to not be complex functions of the entire parameter space of the model. This follows from the reduction of the system to the interest rate rule and

¹⁷It is astounding that Cochrane (2007, p. 24) claims, "Mankiw and Reis (2002) argue for a return to mechanical or adaptive expectations, [...] though this means throwing out economic microfoundations." Mankiw and Reis's (2002, p. 1297) model has fully rational expectations and is microfounded (see Reis (2006)).

the Fisher-type equation, which makes no reference to any parameters in either the demand or the supply side. The common trait is a long-run vertical Phillips curve,¹⁸ that “by 1980 even self-styled Keynesian economists were agreeing to.” (McCallum 2004, p. 21)

The analysis here should make one wary of the conclusions from determinacy analysis in the New Keynesian literature: its policy recommendations or restrictions in terms of bounds on monetary policy are a consequence of the New Keynesian Phillips curve’s violation of the NRH. This does not mean that the literature standard sticky-price model ought to be rejected, merely that we should not ask it to perform tasks for which it was not intended. Among these is the assessment of determinacy, a long-run question¹⁹ that requires the examination of explosive paths, and when addressing it, we should use models whose long-run properties are defensible.

4 Determinacy and the Cochrane (2007) Critique

With the general results for determinacy of models that satisfy the NRH, I shall confront the issue, raised by Cochrane (2007), of whether determinacy is an appropriate means to justify a unique equilibrium. The equilibria ruled out by determinacy are in fact legitimate monetarist equilibria resulting from the deficiency of defining monetary policy solely over the nominal interest rate. Interpreting steady-state inflation as a long run monetary target provides the missing mechanism to select establish the determinate equilibrium as the only permissible one.

Cochrane (2007) has challenged the determinacy analysis in the New Keynesian literature. It notes that explosive paths are ruled out for both nominal and for real variables. One can generally rule out explosive paths for real variables by appealing to a transversality condition, but such a condition is lacking for nominal variables. In the foregoing section, I imposed saddle-point stability on a real variable, the output gap, and two nominal variables, the nominal interest rate and inflation.

¹⁸The NRH and vertical Phillips curves are central to the rational expectations revolution, see Lucas (1972) and Sargent (1973), with Sargent (1987b, p. 7) calling Friedman’s (1968) address its “opening shot”.

¹⁹Emphasized also in Meyer-Gohde (2009).

Cochrane's (2007) critique is directly relevant for the analysis of the foregoing section: if the model is determinate, there is one stable path and a continuum of explosive paths; if the model is indeterminate there is a continuum of stable paths. Conveniently, the model of dialogue between Cochrane (2007), Cochrane (2009) and McCallum (2009a), McCallum (2009b) is the frictionless model of the foregoing section, upon whose stability the determinacy analysis of all NRH models with the standard dynamic IS equation depends.

Interestingly, Cochrane's (2007) critique, however, does not actually apply to the standard three equation New Keynesian model. As was laid in previous section,²⁰ the lack of a long-run vertical Phillips curve implies quite generally that an explosive path for inflation implies an explosive path for the output gap. Thus, if "[e]conomics can rule out real explosions", then a supply schedule that violates the NRH, by associating explosive paths for inflation with explosive paths for the output gap, will give one the means to rule out the nominal explosions as well. This situation, depicted by Cochrane (2007, p. 28), is reproduced in figure 3: the explosiveness for the nominals is associated with explosiveness for the real variables. Cochrane (2007, p. 25) admits that the output gap, a real variable, explodes in all equilibria except for the equilibrium chosen in standard New Keynesian analysis, but softens his distinction between real and nominal variables with the statement, "[n]o economic consideration rules out the explosive solutions." I believe he is mistaken with the claim that the situation here is exactly the same as in the frictionless case. In the frictionless case, the problem was the legitimate one of a nominal explosion without a real explosion, whereas here the two go hand-in-hand. This permanent tradeoff makes the New Keynesian Phillips curve ill-suited to examine or even exclude hyperinflationary paths, reiterating the analysis of the foregoing sections. Long-run questions—like determinacy as well—require a model whose long-run properties are defensible.²¹

²⁰And, of course, within the linear(ized) framework of the previous section.

²¹McCallum (2003, p. 1157) actually anticipates this discussion: "the [Calvo] form of sticky prices [...] is such that the model continues to include nominal variables even when monetary policy supplies no nominal anchor, because private behavior involves a type of dynamic money illusion [as the model violates the NRH.]"

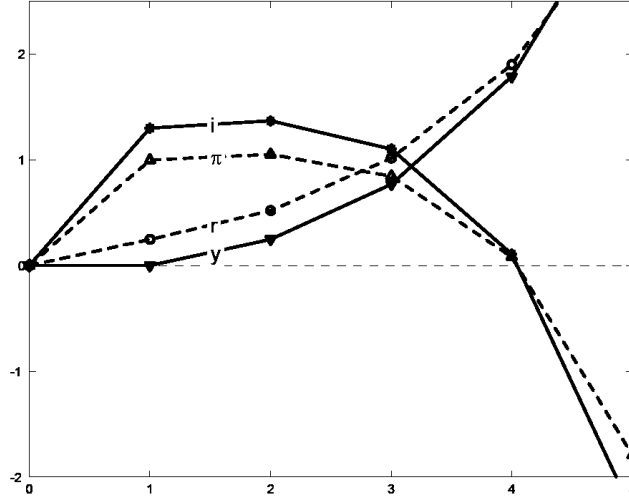


Figure 3: Response of the Three-Equation New-Keynesian Model to a One-Percent Off-Equilibrium Inflation Innovation, with No Change in Output. From Cochrane (2007, p. 28)

Returning to the NRH model of the previous sections to address Cochrane’s (2007) critique within the NRH, temporarily replace the assumption of a Taylor rule with monetary policy defined as control over the money supply. Append the model with a standard money demand function²² in first difference form

$$(17) \quad \mu_t - \pi_t = \eta_y \gamma_t - \eta_R \Delta R_t + \Delta \varepsilon_t^m$$

where μ_t is the money growth rate, γ_t the growth rate of output, and ε_t^m a money demand shock. The output gap is necessarily stationary due with the NRH being fulfilled so we can neglect both γ_t and ε_t^m for the purposes of asymptotic behavior if it can be assumed that the natural rate of output and the money demand shock are at least difference stationary²³

$$(18) \quad \mu_t - \pi_t = -\eta_R \Delta R_t$$

Thus, (18), the Fisher equation $R_t = E_t[\pi_{t+1}]$, and a process for the money supply constitute a

²²I adopt the notation of Woodford (2008) for ease. Note that as discussed in, e.g., Woodford (2008), McCallum (2008), and Nelson (2008), adding a money demand relation does not alter the previous analysis. It adds one variable and one equation and is ‘superfluous’ according to McCallum (2008, p. 1785) with monetary policy defined over the interest rate or the previous analysis was ‘self-contained’ in its absence according to Nelson (2008, p. 1799). The nonlinear origin of this standard equation is of importance only insofar as it provides transversality conditions to rule out particular paths of variables or insofar as its linearization leads to spurious artifacts. In the next section, some specific origins will be examined and an artifact of linearization will be addressed.

²³As emphasized by McCallum and Nelson (2009a, pp. 13–15), the key element for the quantity theory is the unitary relation between money and prices—a stability of the money demand function with respect to other parameters and variables is not necessary for the theory’s relations.

specification for inflation, money growth, and the nominal interest rate. This is identical to the Cagan model under rational expectations of Sargent and Wallace (1973),²⁴ but the focus here—due to Cochrane (2007)—is on potentially explosive inflation and not just the price level.

Consider the case of a constant money supply constant ($\mu_t = 0$), reducing the system to

$$(19) \quad R_t = \eta_R E_t [\Delta R_{t+1}] = \frac{\eta_R}{1 + \eta_R} E_t [R_{t+1}]$$

One solution is $R_t = \pi_t = 0$. McCallum (2001a, p. 26) labels this the “monetarist solution”. But a whole continuum of solutions exists with R_t and π_t diverging to positive or negative infinity. These hyperinflations and -deflations are speculative in nature, as they are not accompanied by equivalent movements in the money supply. Although Sargent and Wallace (1973) rule them out with an arbitrary terminal condition, this continuum of additional solutions *can* be ruled out by economic theory. I shall address this in the next section by, e.g., postulating that money is essential.²⁵

But this, of course, does not mean that the model is incompatible with hyperinflation. Assume that the monetary authority follows an extraordinarily money creation scheme, whereby *the growth rate* of the money supply is increasing exponentially ($\mu_t = \lambda \mu_{t-1}$, $1 < \lambda < \frac{1+\eta_R}{\eta_R}$).²⁶ Thus,

$$(20) \quad R_t = \eta_R E_t [\Delta R_{t+1}] + E_t [\mu_{t+1}] = \frac{\eta_R}{1 + \eta_R} E_t [R_{t+1}] + \frac{1}{1 + \eta_R} E_t [\mu_{t+1}]$$

One equilibrium has R_t and π_t increasing at the same rate as μ_t —the monetarist solution

$$(21) \quad \pi_t = \frac{1}{1 - \eta_R(\lambda - 1)} \mu_t, \quad R_t = \frac{\lambda}{1 - \eta_R(\lambda - 1)} \mu_t$$

Defining $\tilde{\pi}_t$ and \tilde{R}_t as the difference of inflation and the nominal interest rate from their values in

²⁴See their Equation (4), where the only difference is the first difference of a “stochastic term with central tendency equal to zero” that I have omitted here.

²⁵Note that the essentiality of money rules out speculative hyperinflation. Speculative hyperdeflation can typically be ruled out under weaker restrictions and I, like McCallum (2009a, pp. 1106–1107), will not dwell on them in the following. Gray (1984) shows that such paths can always be ruled out in the class of money-in-the-utility models she examines as they provide households with an open-ended arbitrage opportunity. Obstfeld and Rogoff (1986, pp. 355–358) demonstrate that such paths can be ruled out in a transactions-technology model and provide some intuition for off-equilibrium threats that can rule out speculative hyperdeflation even in some extreme cases.

²⁶The restriction on the growth rate of the money growth rate is required for “process consistency” reasons, see Flood and Garber (1980a) and McCallum (1983). Essentially, the rate of money growth would be growing too quickly to be commensurate with the (linearized) money demand function. Taking, e.g., Ball’s (2001) estimate for the interest semi-elasticity of money demand, $\eta_R = 0.05$, the process consistency limit is equal to 21—limiting the period-over-period change in the *growth rate* to a fantastical 2000%. The next section will show this to be an artifact of linearization.

the monetarist solution, the system can be reduced to the case of a constant money supply in the redefined variables and, thus, there is a whole continuum of solutions with R_t and π_t diverging from μ_t . All the paths off of the monetarist hyperinflation path can be ruled out under the same conditions as before—e.g. the essentiality of money.

Cochrane's (2007) critique need not, therefore, be referring to speculative abberations. What, then, goes awry with interest rate rules? Define monetary policy over the nominal interest rate,

$$(22) \quad R_t = \phi_\pi \pi_t$$

Given this rule, and the Fisher equation ($\pi_t = \frac{1}{\phi_\pi} E_t [\pi_{t+1}]$) one solution is $\pi_t = R_t = 0$, which implies through (18) that $\mu_t = 0$. But a whole continuum of solutions satisfying

$$(23) \quad \pi_t = \phi_\pi \pi_{t-1}$$

are also potential equilibria. In the context of determinacy, one would require $\phi_\pi > 1$.²⁷ A $\phi_\pi > 1$ means the potential equilibria are characterized by explosive paths for inflation and the nominal interest rate. Combining (18) with (22) and (23)

$$(24) \quad \mu_t = [1 - \eta_R (\phi_\pi - 1)] \pi_t$$

But this implies that the money supply growth rate is increasing proportionally with the inflation rate. Monetary policy is accommodating this hyperinflationary equilibrium, making this explosive path of inflation a consistent “moneterist solution” through extraordinary money supply growth.²⁸

This highlights where the New Keynesian sticky-price model breaks down: monetary policy cannot pursue the aggressively inflationary money-supply growth associated with these explosive equilibria, as this policy, through the violation of the NRH, would be associated with an explosion in the output gap, which can be ruled out by appealing to a transversality argument. Cochrane (2007, p. 25) states, “sensible economic models work in hyperinflation or deflation. If they don't, it usually reveals something wrong with the model.” This statement needs to be tempered, I believe, with the

²⁷ Assuming the interest rate react positively to inflation.

²⁸ As explained in footnote 26, a process consistency constraint is present here as well: $\phi_\pi < \frac{1+\eta_R}{\eta_R}$.

assessment that the New Keynesian model was never intended as an explanation of hyperinflation. However, this certainly does mean that one must be wary of drawing any conclusions that implicitly rest on the analysis of hyperinflation, such as determinacy, in such models.

The sticky-price model was conceived as a model for short-term fluctuations. In the background and in the back of the modelers' minds is an RBC model with full neutrality in the long run.²⁹ Woodford (2008) shows that the standard sticky-price model fulfills a list of neutrality properties. What has not garnered attention is that these properties may only be fulfilled by the determinate solution itself. Indeed, the examination of determinacy—though short-run in its consequences through potential sunspot equilibrium—is an examination of the long-run: does a particular equilibrium path converge asymptotically to the steady state or does it diverge? The New Keynesian model through its violation of the NRH and inability to give an accurate picture of equilibria on divergent (i.e., hyperinflationary) paths is not suitable for such long-run analyses as determinacy.

Thus, Cochrane's (2007) critique is wholly valid in the set of models examined in the foregoing section. Should the model be associated with determinacy, all of the explosive paths constitute fully valid equilibria. But the reasoning of Cochrane (2007)—the absence of transversality conditions for nominal variables—obfuscates the real reason for the validity of these equilibria. An equilibrium with inflation diverging towards infinity is valid precisely because the monetary authority keeps increasing the growth rate of the money supply, accommodating the ever increasing inflation rates.

McCallum (2009a) offers LS learning as a means to "select" the determinate solution. If Cochrane's (2007) explosive equilibria are legitimate, McCallum's (2009a) argument must have some defect. Reinterpreting the explosive equilibria in terms of an exogenous process for the money growth as I have done, Cochrane's (2007) explosive equilibria are associated with explosive processes for money growth. But McCallum (2009a, p. 1103), following Evans and Honkapohja (2001, pp. 198& 229), requires the exogenous processes to be stationary. That is, McCallum's (2009a) LS learning rules

²⁹See Woodford (2003, Ch. 3, esp. p. 142)

out Cochrane's (2007) explosive equilibria by assumption. With *least-squares (LS)* learning, agents' expectation formation progresses to slowly for explosive money growth: this is not a reassuring mechanism to prevent hyperinflation.

With monetary policy defined solely over control of the short-run nominal interest rate, there is, therefore, an entire continuum of valid equilibrium paths in the absence of any fundamental shock ranging from hyperinflation to hyperdeflation when the determinacy conditions of the previous section are satisfied.³⁰ That is, there must be some defect in defining monetary policy solely in terms of the short-run nominal interest rate. This is precisely the point made by Nelson (2008, p. 1805): "the monetary authority cannot treat the nominal interest rate as an instrument in the long run." What is his proposed solution? "Long-run money growth determines long-run inflation,"

Though they no longer affect real interest rates, and no longer can affect nominal rates via a *liquidity effect*, the central bank's open market operations continue in the long run to affect nominal money growth. So nominal money growth is unambiguously and undeniably susceptible to central bank influence even in the long run... Reaching [an] inflation target means a specified quantity of open market operations in the steady state; specifically, open market operations that deliver a steady-state money growth [consistent with the inflation target and the secular growth]. There it is: the sense in which steady-state inflation can be regarded as pinned down by steady-state money growth. Nelson (2008, p. 1805)[emphasis in the original]

Let monetary policy be fully specified by adding a steady-state inflation rate, which can be sensibly interpreted as an average growth rate for the money supply. Thus the main result:

Proposition 4.1. *Consider the NRH model of the foregoing section appended with (18). Monetary policy is specified by an interest rate rule and an average money growth rate. If the interest-rate rule is associated with a determinate equilibrium, this equilibrium is the unique equilibrium.*

³⁰When they are not, there is an additional dimension of indeterminacy

Proof. See Appendix

□

Cochrane's (2007) "threat" of monetary policy, is not hyperinflation, hyperdeflation, or "to blow up the world", but rather to simply keep money growth constant. All that is needed here is the commitment on behalf of the central bank to ensure the unconditional expectation of the money growth rate be equal to the steady-state value it selects. Note that this still allows for the multiple equilibria in case of indeterminacy, not curing all the ills of interest rate policy. All of the multiple equilibria in case of indeterminacy converge back to the steady state allowing the average money growth rate to be satisfied and thusly cannot be ruled out.

Monetary policy is not bound by any restriction to accommodate the hyperinflationary or hyperdeflationary paths. The threat that monetary policy will not keep increasing [decreasing] the rate of money growth boundlessly would seem credible and is already incorporated in the framework of several central banks. Most notably the monetary analysis pillar of the ECB, but also Section 2a of the Federal Reserve Act requiring that the Federal Reserve "shall maintain long run growth of the monetary and credit aggregates [...] so as to promote effectively [...] stable prices."³¹

Both of these central banks have committed, implicitly or explicitly, to keeping the rate of growth of the money supply at very least finite. So long as this commitment is credible, no explosive path for inflation can be an equilibrium. Following, e.g., Friedman and Schwartz (1963), monetary restraint is necessary and sufficient to controlling inflation, at least in the long-run. And, as emphasized by Nelson (2008), monetary policy defined over control of the nominal interest rate is incomplete, as the monetary authority cannot control this variable in the long run. There is thusly, no contradiction between monetary policy being defined over control of the nominal interest rate at all finite horizons and over the rate of money growth asymptotically.³²

³¹Paraphrasing the Chairman of the Board of Governors slightly: Bernanke (2008, pp. 317 & 319) emphasizes that although they have not played a central role in recent times, monetary data is and will continue to be monitored by the Federal Reserve as a sensible part of the framework of monetary policy.

³²Indeed, Friedman (1960, p. 35) states, "[t]he sufficiency of open market operations as a tool for monetary policy is not, of course, a decisive reason for relying on this tool alone." Likewise, Brunner and Meltzer's (1976, pp. 98–99) analysis differentiates between the "accumulated effects of past policies" and one-off impulses.

One immediately appealing equivalent measure to the average growth rate of the money supply in Proposition 4.1 is a direct inflation target. One could interpret Proposition 4.1 as wholly consistent with such a form of direct inflation targeting: if the inflation target is credible, any equilibrium path that diverges from the target contradicts the target's credibility.³³ However, this is the "high-level assumption" that Nelson (2008, p. 1803) argues is deceiving, as it assumes a permanent liquidity effect. It is exactly this permanent liquidity effect that imbues the nominal interest rate with an always and everywhere stabilizing effect, which Cochrane (2007) criticizes as the New Keynesian literature's intuitive reliance on "old Keynesian" thinking. Likewise Meltzer (1999, p. 268) notes that the reliance on the nominal interest rate to indicate the expansiveness of monetary policy has misled the Federal Reserve on a number of occasions. As one should not neglect the NRH and its short-run implications in assessing determinacy, one should not neglect that monetary policy has no direct control over the nominal interest rate or inflation in the long run.

However, keeping the foregoing reservations in mind, the notion of an inflation target for the long run as being a key element of a well-formulated monetary policy is germane to the "constrained discretion" interpretation of inflation targeting by Bernanke, Laubach, Mishkin, and Posen (1999, p. 22), under which "inflation targets keep the economic ship in the right area in the long term," but where the interpretation of inflation targeting as a strict rule is rejected. In sum, a particular inflation rate in the long-run is the target and the commitment to keeping the money growth rate finite, monitoring mid-term developments in the monetary aggregates, and/or a commitment to an average money growth rate consistent with the inflation target the rule.³⁴

³³Such a policy was rejected half a century ago by Friedman (1960, p. 88): "[W]e will [...] further the ultimate end of achieving a reasonably stable price level better by specifying the role of the monetary authorities in terms of magnitudes they effectively control and for whose behavior they can properly be held responsible[...] In this as in so many human activities what seems the long way round may be the short way home."

³⁴Such a rule is easily implemented here as there are no impediments to the central bank committing to set policy according to the interest rate rule along a determinate equilibrium and or by keeping money growth equal to the target on off-determinate equilibrium paths—the "threat" from above. True welfare- or loss-function-based assessments as to the credibility of such an immediate switch is beyond the analysis here. However, with all off-determinate equilibrium paths associated with infinite divergence of inflation, there would seem to be a great a priori incentive for the central bank to avoid such paths. Nelson (2008, p. 1806) also notes that "what needs to be kept in mind is that such an approach is a shortcut or an abstraction that takes for granted the underlying operations involving money on the part of the central

5 Nonlinear Money Demand and the Monetarist Equilibrium

In this section, I wish to justify the selection of the monetarist equilibrium in the foregoing selection that validated Cochrane's (2007) explosive inflation by reckless money growth. For the sake of brevity, I would only note that significant price level movements in the absence of corresponding movements in the money supply are inconsistent with the empirical evidence. Yet, as Meltzer (1999, p. 262) notes, "[e]conomists are rarely satisfied with evidence that something works in practice. They are inclined to be more interested in whether it works in theory." So despite the compelling reasons to dismiss speculative inflation and deflation *a priori*, as conceded by Obstfeld and Rogoff (1986), I shall also offer formal arguments in the context of the models presented by Cochrane (2007) and McCallum (2009a) that an explosive equilibrium for inflation is only admissible with an associated explosive money growth rate.

As to practice, Friedman (1958, p. 172) noted, "[t]here is perhaps no empirical regularity among economic phenomena that is based on so much evidence for so wide a range of circumstances as the connection between substantial changes in the stock of money and in the level of prices." Flood and Garber (1980b) reject the hypothesis of a bubble in the German hyperinflation of the '20s and, in the face of such empirical evidence, Flood and Garber (1980b, p. 760) state that "this artifact of dynamic models is unimportant; a special case of these models adequately predicts behavior, and further elaboration of the model to explain unobserved phenomena is unnecessary." More recently, McCallum and Nelson (2009a, p. 37) conclude, "[n]ominal homogeneity of money demand is not rejected irrespective of the inflation series used, the definition of money chosen, or sample period considered."

Theoretically, explosive paths of inflation *can* be associated with explosive paths of the money growth rate. The question at hand from the foregoing section is whether this *must* be the case. Cochrane (2007, p. 22) mentions and McCallum (2009a, p. 1106) discusses the literature that ad-

bank."

addresses this question, that of speculative hyperinflations, but both fail to note the decisive role of money. This literature does not purport to address whether explosions in inflation can be ruled out in general, as my reading of McCallum (2009a, p. 1106) might lead one to believe, but seeks to address whether those explosions can be ruled out that are “unrelated to monetary growth.” (Obstfeld and Rogoff 1983, p. 675) The question of whether explosive price paths can exist *without* monetary growth cannot be equated to the credibility of the “threat of the government to take the economy to a configuration (hyperinflation or deflation) in which the [sic] we all know the economy will blow up on its own.” (Cochrane 2007, pp. 22–23) In a nutshell, Scheinkman (1980), Obstfeld and Rogoff (1983), Gray (1984) and Woodford (1994) demonstrate that the speculative hyperinflations in separable money-in-the-utility-function, medium-of-exchange, and cash-in-advance setups can be ruled out by requiring money to be essential or have intrinsic value. Intuitively, if real balances are necessary or necessarily of worth, a hyperinflationary path initiated by a whim and not accompanied by money growth would rob utility maximizers of this necessity, bringing the rational origin of such a whim into question.

Turning to the specific discussion of Cochrane (2007) and McCallum (2009a), Cochrane (2007, p. 22) lays out a two-equation nonlinear model under perfect foresight to address the issue, whose necessary conditions are

$$(25) \quad 1 + i_t = \beta^{-1} \Pi_{t+1} \frac{u_c(Y, M_t/P_t)}{u_c(Y, M_{t+1}/P_{t+1})}$$

$$(26) \quad M_t/P_t = L(Y, i_t)$$

along with a specification of monetary policy. In his appendix, Cochrane (2007) solves for the latter of the foregoing using a first-order condition relating bond and money holdings,

$$(27) \quad \frac{i_t}{1 + i_t} u_c(Y, M_t/P_t) = u_m(Y, M_t/P_t)$$

The foregoing, or more generally (26), can be linearized and first-differenced to yield (17).

Let us eliminate the only possibility mentioned by Cochrane (2007, pp. 21–23) to rule out explosions with the extension to money, namely the possibility of the real interest rate going to in-

finiteness due to monetary distortions—i.e., the passive “blow-up threat by the government.” One easy way to do this is to assume separability ($u_{cm} = 0$), reducing the model to $1 + i_t = \beta^{-1}\Pi_{t+1}$ and $\frac{i_t}{1+i_t}u_c(Y) = u_m(M_t/P_t)$. From the latter, it follows immediately that $\lim_{i_t \rightarrow \infty} u_m(M_t/P_t) = u_c(Y)$. Thus, real balances must be constant if $i_t \rightarrow \infty$, necessarily requiring the growth rate of money to be equal to inflation. If the interest rate follows an active Taylor rule (i.e. $i_t = \Phi(\Pi_t)$, with $\Phi' > 0$), explosive inflation leads to an explosive nominal interest rate. In terms of the process consistency requirement of the preceding section, there is no upper bound on the elasticity of the nominal interest rate with respect to inflation here. More generally, the assumption that money is essential, following Obstfeld and Rogoff (1983, p. 681) and Gray (1984, p. 100),

$$(28) \quad \lim_{m_t \rightarrow 0} m_t u_m(Y, m_t) > 0$$

would suffice to ensure that any hyperinflation or deflation is *necessarily* associated with a corresponding path of money.

Using a standard money-in-the-utility function from Galí’s (2008, p. 27),

$$(29) \quad U(C_t, \frac{M_t}{P_t}, \dots) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{(M_t/P_t)^{1-\nu}}{1-\nu}$$

the foregoing condition holds for all $\nu > 1$, i.e. elasticities of utility with respect to real balances greater than unity—not a severe restriction.³⁵ With these preferences, optimality requires,

$$(30) \quad \frac{M_t}{P_t} = C_t^{\frac{\sigma}{\nu}} \left(1 - \frac{1}{R_t}\right)^{-\frac{1}{\nu}}$$

which can be linearized, combined with market-clearing, and first-differenced to yield (17). The process-consistency restrictions come from the interest elasticity of money demand, η_R , which is a constant after linearizing. In the nonlinear version, however, it is equal to $\frac{1}{\nu} \frac{1}{R_t - 1}$ and with an active interest rate rule, this elasticity will approach zero as inflation explodes, again confirming the process-consistency restrictions to be an artifact of linearization.

The essentiality of money required by (28) might seem too much to require of a model. Indeed,

³⁵Of course, this does not contradict Obstfeld and Rogoff’s (1983) assessment that this is an extreme restriction on preferences. The functional form itself of preferences over real balances is what here might justifiably be called “extreme.”

McCallum (2009a, p. 1106) cites Obstfeld and Rogoff's (1983, p. 675) conclusion in their money-in-the-utility framework that this constitutes an extreme restrictions on preferences and goes on to claim that, "a model specification that drives consumption to zero (as real money holdings decrease) implies that a barter economy would necessarily feature zero consumption. That should be regarded as an inadmissible assumption." (McCallum 2009a, p. 1107) Yet, McCallum (2009a, p. 1107) adopts a transaction function that does just this.

Gray (1984, p. 106) requires the limit of real balances times the marginal transaction cost to be negative infinity as real balances approach zero. McCallum (2009a) mistakenly states that Gray's (1984) analysis lacks an extension of a transaction-costs function dependant on the quantity of transactions. Gray (1984), however, does not address the case that combines this extended transaction-costs function with convex utility. Yet, her results extend straightforwardly to this case, as I show in the appendix, and with the sufficiency conditions fulfilled by McCallum's (2009a) transaction function, Gray's (1984, p. 113) requirement is necessarily fulfilled

$$(31) \quad \lim_{m \rightarrow 0} m\Phi_2(C, m) = \lim_{m \rightarrow 0} -a_2 a_1 C^{1+a_2} m^{-a_2} = -\infty < 0$$

as a_1, a_2 are both positive.³⁶ So McCallum (2009a) does rule out speculative hyperinflations and -deflations. Implicitly, McCallum (2009a, p. 1107) finds the monetarist hyperinflation: "as inflation explodes, [... real balances do] not approach zero." If inflation explodes, the price level explodes at an exploding rate. With real balances approaching a constant, money is exploding at the same exploding rate as prices. That is, inflation and the rate of money growth are exploding together.

Beyond essentiality of money, Obstfeld and Rogoff (1983) show that if money has some intrinsic value, however fleetingly small, speculative hyperinflations will be impossible. Despite having technically ruled out the possibility, McCallum (2009a, p. 1107) reasons for the existence of speculative hyperinflation, as their impossibility would require a barter economy to be associated with zero consumption. Yet the return to a barter economy along such paths is not an inexorable conclusion, as

³⁶Additionally, note that in McCallum's (2009a, p. 1107) model, money demand is given by $\Phi_2(C_t, m_t) = -\frac{i_t}{1+i_t}$ which, again, can be linearized, combined with market-clearing, and first-differenced to yield (17).

the transactions-chain approach to the medium-of-exchange explanation of money in Brunner and Meltzer (1971, p. 801) demonstrates.³⁷ Friedman and Schwartz (1970, p. 108) too, in their discussion of money as a medium of exchange, note an “irreducible minimum [real-value quantity of money] necessary for transactions purposes” that make the necessity of money absolute. One could, alternatively, assert that there is a discontinuous difference between approaching a barter economy through rampant hyperinflation and actually being in a functional barter economy.

Theoretical and empirical considerations aside, ruling out speculative hyperinflations is necessary for maintaining the proposition that “the central bank can reasonably be held accountable for controlling inflation.” (Woodford 2008, p. 1563) The central bank would certainly be relieved of this accountability if it were—at any moment of time—probable (or even if it were merely possible) that the price-level or inflation could go spiralling out of control despite a constant money supply or growth rate thereof.

Thus, in any sensible monetary description, it ought to hold that “[t]here is a one-to-one relation between monetary changes and changes in [...] prices”—at very least in the long run or for “major economic fluctuations”. (Friedman and Schwartz 1963, p. 50) This requires ruling out non-monetary divergences. At the same time, accepting this one-to-one relation and the empirical evidence that hyperinflations have occurred forces one to dismiss specifications or equilibrium-selection devices that would rule out *fundamental* divergences. The associated skepticism applies not only to the analyses where monetary policy drives the real interest rate to infinity to rule out hyperinflations as argued in Cochrane (2007, pp. 22–23) or to a transversality-based argument on real variables in a model like the standard New Keynesian model with a non-vertical long-run Phillips curve, but also to the LS-learnability analysis of McCallum (2009a, pp.3–13) that would rule out explosive money supply growth rates as inflation in the associated equilibria would accelerate more quickly than could be learned by least-squares agents.

³⁷Though this approach simultaneously appears to rule out all hyperinflations, speculative or monetary, as new mediums of exchange are sought out.

6 The Nominal Interest Rate

From the foregoing sections, it should be clear that some mention of money is advantageous in a monetary model. Monetary policy should make some reference, implicit or explicit, to the money supply as a monetary policy defined solely over the nominal interest rate is insufficient to control inflation. As Cochrane (2007, p. 42) rightfully criticizes, one cannot “use old Keynesian stabilizing logic” to describe the mechanism of inflation control at work with an interest rate rule in a New Keynesian model.³⁸ The old Keynesian stabilizing logic focuses on the liquidity effect and neglects the Fisher effect,³⁹ leading to difficulties for New Keynesian explanations of Friedman’s (1968, p. 7) observation that “low interest rates are a sign that monetary policy has been tight—in the sense that the quantity of money has grown slowly; high interest rates are a sign that monetary policy has been easy—in the sense that the quantity of money has grown rapidly.” Raising the nominal interest rate once is associated with tight monetary policy via the liquidity effect, but raising the nominal interest rate continually must certainly be associated with easy monetary policy:

Add only one wrinkle to Wicksell—the Irving Fisher distinction between the nominal and the real rate of interest. Let the monetary authority keep the nominal market rate for a time below the natural rate by inflation. That in turn will raise the nominal natural rate itself, once anticipations of inflation become widespread, thus requiring still more rapid inflation to hold down the market rate. (Friedman 1968, p. 8)

Cochrane’s (2007) explosive equilibria under an active interest rate rule (i.e., $\partial R_t / \partial \pi_t > 0$), though caused by some exogenous shift in belief, can be brought into the reasoning of the foregoing statement: (1) Let anticipations of inflation become widespread ($E_t [\pi_{t+1}] > 0$), (2) this raises the nominal natural rate itself ($R_t = E_t [\pi_{t+1}]$), (3) meaning that monetary policy kept the nominal market rate for

³⁸Perhaps, the alternative nomenclature “New Neoclassical” noted again by McCallum (2009a, p. 1102) is indeed more appropriate with the NRH supply of this paper, reserving “New Keynesian” for those models that possess the stable long-run tradeoff.

³⁹See, e.g., Nelson and Schwartz (2008, p. 844).

a time below the natural rate by inflation ($\pi_t = \frac{1}{\partial R_t / \partial \pi_t} R_t > R_{t-1} \equiv 0$), (4) requiring now still more rapid inflation ($E_t [\pi_{t+2}] = E_t [R_{t+1}] = \frac{\partial R_t}{\partial \pi_t} E_t [\pi_{t+1}] > E_t [\pi_{t+1}]$) to hold down the market rate. Thus, the multiple equilibria of Cochrane (2007) can be interpreted as the Fisher effect of monetarism rearing its ugly head in the New Keynesian model.

According to Bordo and Schwartz (1999, p. 193), “[t]he dangers of operating with an interest rate instrument became clear when rising interest rates from the mid-1960s on reflected growing fears of inflation, not restrictive monetary policy. Rising interest rates were accompanied by high money growth.” With the meliorative policies of the Great Moderation having dulled the memory of the Great Inflation, Issing (2008, p. 266) surmised, “[i]t is not surprising that in a world of low inflation, the interest in ‘money’ in central banks as well as in academia has declined, if not disappeared. I do, however, hope that the world does not have to go through the same process of pathological learning as at the end of the last century.” With the apparent end of the Great Moderation, Leijonhufvud (2009, p. 6) reiterates that “[i]t is a dangerous illusion that you can always control the price level in an economy where the money stock however measured is left to vary in purely endogenous fashion.”

Though monetary restraint is necessary for monetary policy to control inflation, the framework of interest rate rules need not be discarded. Nelson (2008) has given a very appealing justification for the use of an interest rate rule by appending the rule with steady state money growth. Combining this with the determinacy bounds of section 3 provides clear guidance to the monetary authority on the interest rate independent of the actual short-run mechanism at work on the supply side.

7 Conclusion

It should be clear that Cochrane’s (2007) critique is substantially correct: there are explosive nominal paths associated with interest rate rules that cannot be ruled out. The requirement that the economy ought to fulfill Lucas’s (1972) NRH means, through determinacy, that Cochrane’s (2007) critique applies—for a given demand specification—to all non-degenerate models at the same policy

specifications. As a consequence, the monetary authority needs no knowledge of the supply side to ascertain whether its policy will ensure determinacy. Indeed, in the case of the literature standard dynamic IS equation, no parameters of the model other than those in the interest rate rule can affect whether determinacy is achieved. Asserting additionally that monetary policy can be held reasonably accountable for inflation demands monetary restraint and thus the hyperinflations or -deflations of Cochrane (2007) can only occur if the monetary authority allows them to. These explosive equilibrium paths are admissible not for lack of LS-learnability (McCallum 2009a) or a non-Ricardian fiscal regime (Cochrane 2007), but simply because the monetary authority is increasing or decreasing the growth rate of money commensurate with accelerating inflation or deflation. Monetary policy associated with a determinate equilibrium, therefore, must additionally credibly commit to “prevent[ing] money itself from being a major source of economic disturbance,” (Friedman 1968, p. 12) and a commitment to an average money growth rate following Nelson (2008) is offered as a means to that end. Thus money still plays a decisive role for the short run even when relegated to the very long run for monetary policy.

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A Appendix

A.1 Proof of Lemma 3.1

By the Wold theorem,⁴⁰ any stationary process can be represented as

$$(A-1) \quad X_t = \sum_{l=0}^{\infty} \theta_l \varepsilon_{t-l} + \Xi_t, \text{ where } E\varepsilon_t = 0 \text{ and } E\varepsilon_t \varepsilon'_{t+j} = 0, \forall j \neq 0$$

and Ξ_t is an orthogonal linearly deterministic process, forecastable perfectly from its own history.

Starting with the indeterministic part,⁴¹ and inserting into (10)

$$(A-2) \quad 0 = \sum_{j=0}^n \left[\sum_{l=0}^{\infty} \left(\sum_{i=0}^{\min(p,l)} Q(i,j) \right) \theta_{l+j} \varepsilon_{t-l} \right] + \sum_{j=1}^m \left[\sum_{l=0}^{\infty} \left(\sum_{i=0}^{\min(p,l+j)} Q(i,j) \right) \theta_l \varepsilon_{t-l-j} \right]$$

Using the definition of $\tilde{Q}(i,j)$ yields

$$(A-3) \quad 0 = \sum_{j=0}^n \left[\sum_{l=0}^{\infty} \tilde{Q}(l,j) \theta_{l+j} \varepsilon_{t-l} \right] + \sum_{j=1}^m \left[\sum_{l=0}^{\infty} \tilde{Q}(l+j,j) \theta_l \varepsilon_{t-l-j} \right]$$

This must hold for all realizations of ε_t . Comparing coefficients yields

$$(A-4) \quad 0 = \sum_{j=0}^n \tilde{Q}(l,j) \theta_{l+j} + \sum_{j=1}^m \tilde{Q}(l,j) \theta_{l-j}$$

a time-varying system of difference equations with initial conditions $\sum_{j=1}^m \theta_{-j} = 0$. But as $\tilde{Q}(p+i,j) = \tilde{Q}(p,j)$, $\forall i \geq 0$, the system of difference equations has constant coefficients, after and including p . This system can be written as (12) and coincides with Anderson's (2010) canonical form.

If the solution to this system is unique, its stable solution can be written as

$$(A-5) \quad \theta_l = B \begin{bmatrix} \theta_{l-m} \\ \vdots \\ \theta_{l-1} \end{bmatrix}, \quad \forall l \geq p$$

The first p (block) equations —remembering the initial conditions— can be gathered into

$$(A-6) \quad \mathbf{Q} \begin{bmatrix} \theta_0 \\ \vdots \\ \theta_{n+p-1} \end{bmatrix} = 0$$

giving $3p$ equations in $3(p+n)$ variables. (A-5) yields $3n$ more equations that can be gathered into

$$(A-7) \quad \mathbf{B} \begin{bmatrix} \theta_0 \\ \vdots \\ \theta_{n+p-1} \end{bmatrix} = 0$$

⁴⁰See, e.g., Sargent (1987a, pp. 286–290), as well as Priestley (1981, pp. 756–758).

⁴¹Whittle (1983, p. 31) and Sargent (1987a, p. 290) focus primarily on the purely indeterministic case. This forms the basis for the time-domain solution methods of Muth (1961) and Taylor (1986).

stacking the two yields (13).⁴²

The system (A-4) is homogenous. Thus, one stationary solution is given by $\theta_i = 0, \forall i$, the fundamental solution in the absence of exogenous driving forces. If (13) is invertible and if (12) is saddle-point stable, then this is the the only solution.

Only Ξ_t remains. Inserting it into (10), it follows that this can also be written as (12). If there is a unique solution in past values of Ξ_t , the solution can be written in the same form as (A-5), which must be zero when taken to its remote past from the stability of (A-5).

A.2 Proof of Proposition 3.2

Assume the opposite is true. Thus, the NRH model is determinate and the frictionless model is not. From the former, according to lemma 3.1, (13) is invertible and the system (12) is saddle-point stable. But the system (12) is the same for both models and (13) is lower triangular for the frictionless model. Thus, the frictionless model that satisfies (8) is determinate, a contradiction.

A.3 Proof of Proposition 3.3

As the frictionless model is determinate, the system (12) is saddle-point stable. This system is the same for the NRH (6) model. The second requirement (13) is lower triangular for the frictionless model, but is unrestricted for the NRH model. Thus, there exist NRH models with a singular (13) that are thusly indeterminate, even though the corresponding frictionless model is determinate.

A.4 Proof of Proposition 3.4

Ruling out the singularity of (13), proposition 3.3 has been ruled out by assumption. Thus, a model in this class that satisfies the NRH defined in (6) is determinate if and only if the corresponding model that satisfies (8) is determinate. This must hold for all k and thus holds for all $\tilde{k} < k$. Any supply equation that satisfies the NRH at a horizon $\tilde{k} < k$, necessarily satisfies it at the horizon k as

⁴²This extends equation 12 in Meyer-Gohde (2010) to Anderson's (2010) higher leads and lags.

well. Thus, for a given k , all supply equations that satisfy the NRH are determinate if and only if the corresponding frictionless model is determinate.

A.5 Proof of Corollary 3.5

It follows from proposition (3.4) that one may choose any supply equation to establish determinacy. Choosing (8) reduces the demand equation to (9), thus eliminating the parameters in the demand equation. Additionally, (8) removes the parameters in monetary policy pertaining to the output gap. Furthermore, from proposition (3.4), it follows that the parameters in the supply equation are irrelevant. Thus the only parameters in the model remaining that can affect determinacy are those in the interest rate rule pertaining to inflation and the interest rate.

A.6 Proof of Proposition 4.1

If the interest-rate rule induces determinacy, all nominal paths but one diverge. Thus, the money growth rate diverges for all paths but one. All divergent paths for the money growth rate contradict the assumption that monetary policy chose the average money growth rate. Therefore, the only consistent path is the non-divergent one, which is unique following from determinacy.

A.7 Extension of Gray (1984)

Gray (1984, pp. 101–116) provides criteria to rule out speculative hyperinflation and -deflation with a transactions cost model of money assuming linear utility from consumption and a transaction cost function that depends solely on real balances. Gray (1984, p. 118) relaxes the two assumptions individually, but not jointly. In the following, I will allow for diminishing marginal utility and the generalization of the transaction cost function to include the quantity of transactions—i.e., the level of consumption. This entails neither great difficulty nor significant insight and is thusly relegated to the appendix here.

Following Gray (1984, p. 102), the representative household seeks to maximize its lifetime dis-

counted utility

$$(A-8) \quad Z = \int_0^{\infty} e^{-\rho t} U(c_t) dt$$

subject to

$$(A-9) \quad P_t y = P_t c_t + P_t \phi(c_t, m_t) + \dot{M}_t$$

where $\phi(c_t, m_t)$ is McCallum's (2009a) transaction cost function with $\phi_c > 0$, $\phi_{cc} < 0$, $\phi_m < 0$ and $\phi_{mm} > 0$. Additionally, $U_c > 0$ and $U_{cc} \leq 0$. Finally, c_t is consumption, ρ the rate of time preference, P_t the price level, y real income "rain[ing] from heaven at a fixed rate of y units per period" (Gray 1984, p. 97), and M_t nominal and $m_t = M_t/P_t$ real money balances.

The resulting optimization problem produces the following Euler equation

$$(A-10) \quad \frac{U_{cc}(c_t)}{U_c(c_t)} \dot{c}_t - \rho = \phi_m(c_t, m_t) + \frac{\dot{P}_t}{P_t} + \frac{\phi_{cc}(c_t, m_t) \dot{c}_t + \phi_{cm}(c_t, m_t) \dot{m}_t}{1 + \phi_c(c_t, m_t)}$$

Holding nominal balances constant⁴³ yields

$$(A-11) \quad \dot{m}_t = -m_t \frac{\dot{P}_t}{P_t}$$

and subsequently differentiating the budget constraint with respect to time yields,

$$(A-12) \quad \dot{c}_t = -\frac{\phi_m(c_t, m_t)}{1 + \phi_c(c_t, m_t)} \dot{m}_t$$

Combining the foregoing three yields

$$(A-13) \quad \frac{\dot{P}_t}{P_t} = -\frac{\rho + \phi_m}{1 + \frac{m_t}{1 + \phi_c} \left[\frac{\phi_m \phi_{cc}}{1 + \phi_c} - \frac{\phi_m U_{cc}}{U_c} - \phi_{cm} \right]}$$

It suffices that the denominator of (A-13) not be negative for her condition under linear utility and transaction costs only dependent on real balances for ruling out speculative hyperinflation

$$(A-14) \quad \lim_{m_t \rightarrow 0} m_t \phi_m < 0$$

to carry over to this more general case.

Note, firstly, that setting $\phi_c = 0$ yields

$$(A-15) \quad \frac{\dot{P}_t}{P_t} = -\frac{\rho + \phi_m}{1 - m_t \frac{\phi_m U_{cc}}{U_c}}$$

i.e., the special case of transaction costs independent of the level of transactions, but with nonlinear

⁴³This is the assumption maintained throughout Gray (1984). The issue at hand, remember, is whether *speculative* hyperinflation and -deflation can be ruled out.

utility. This corresponds to Gray's (1984, p. 118) Equation (36). As she notes, a condition to ensure the denominator always be positive is for $-\phi_m U_{cc}$ to be positive, which is contradicted by assumption. Gray (1984, p. 118) interprets this compound term as, "the effect on the marginal utility of consumption of the change in consumption generated by a change in real balances."

Also examined by Gray (1984, p. 118) is the special case of linear utility, but with the general transaction function. Setting U_{cc} to zero yields

$$(A-16) \quad \frac{\dot{P}_t}{P_t} = - \frac{\rho + \phi_m}{1 + \frac{m_t}{1+\phi_c} \left[\frac{\phi_m \phi_{cc}}{1+\phi_c} - \phi_{cm} \right]}$$

which corresponds to the equation in Gray's (1984, p. 118) Footnote 30. As interpreted there, a sufficient condition is now $\phi_{cm} < 0$. McCallum (2001b, p. 148) argues for setting this "cross partial derivative negative, so that the marginal benefit of holding money—i.e., the reduction in transaction costs—increases with the volume of consumption spending." And indeed the transaction function in both McCallum (2001b) and McCallum (2009a) does this.⁴⁴

Using the foregoing two special cases, a sufficient condition would be that the marginal benefit of holding money increase *sufficiently* with an increase in consumption spending to outweigh the associated decrease in marginal utility from such a consumption spending increase. I.e., $-\phi_{cm} U_c > \phi_m U_{cc}$. Thus, the cross partial derivative being sufficiently negative constitutes a sufficient condition.

As this cross partial derivative is in no way constrained by the general transaction function of McCallum (2009a), assume the foregoing condition is fulfilled, and hence it suffices that

$$(A-17) \quad \lim_{m_t \rightarrow 0} m_t \phi_m < 0$$

for speculative hyperinflation to be ruled out.⁴⁵ As noted in the main text, this assumption is fulfilled by McCallum's (2009a, p. 1107) specific transaction function.

⁴⁴Nevermind that this function does not satisfy McCallum's (2009a, p. 1106) own requirement that $\phi_{cc} < 0$ as Gray (1984, p. 118) too requires.

⁴⁵Note that speculative hyperdeflation is ruled out with a transversality condition that would be violated along such a path given that the saddle-point property is ensured by the assumption $-\phi_{cm} U_c > \phi_m U_{cc}$.