Wagner, Thomas; Jahn, Elke J.

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Do Eligibility Criteria Influence Unemployment?

Elke J. Jahn[a, b], Thomas Wagner[c *]

[a]Aarhus School of Business, Aarhus University
[b]Institute for the Study of Labor (IZA), Germany
[c]University of Applied Sciences, Nuremberg, Germany

[a] *Corresponding Author*: University of Applied Sciences, Thomas.Wagner@ohm-hochschule.de

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ABSTRACT: Economic research suggests that the amount and duration of unemployment benefits can increase equilibrium rate of unemployment, but the literature has overlooked the effects of other restrictions on benefits, namely the length of the qualifying period and the base period. We develop a Mortensen-Pissarides type matching model in which we integrate the following policy parameters: the base period, the qualifying period, unemployment benefits and a finite benefit duration. A worker is only entitled to unemployment benefits if he has completed the statutory qualifying period within the base period. We show that there is a trade-off between the eligibility parameters and the benefit parameters. A country that combines a *high* level of unemployment benefits with a *long* benefit duration can neutralize the effect on the unemployment rate with a *long* qualifying period and/or a *short* base period.

Key-words: Matching model, unemployment insurance, eligibility criteria, benefit parameters, labor market policy

JEL-Code: J41, J64, J68

1 Introduction

Base and qualifying periods are constituent parameters of the unemployment insurance system in most OECD countries. A worker must complete the qualifying period within a statutory base period to be eligible for unemployment benefits (UB). Table 1 indicates qualifying and base periods in the US, UK, Japan, and six continental European OECD countries. For example, in Italy and Germany a worker must be employed for at least 12 months during the last two years to be eligible for UB. Countries use UB eligibility criteria to balance their public unemployment insurance system budgets. For instance, the shortening of the base period from three to two years in the wake of the German labor market reform Hartz III, which came into effect in 2006, aims to reduce the budget deficit of the German public employment service (Deutscher Bundestag 2003). The intuition behind this reform is that shortening of the base period will reduce the number of eligible unemployed without affecting the number of employed workers obliged to contribute to reducing the German PES deficit.

Table 1: Characteristics of the unemployment insurance system in selected OECD countries 2005[1]

	Qualifying Period (months)	Base Period (months)	Max. Benefit Duration (months)	Replacement Rate (%)
Denmark	12	36	48	90
France	6	22	23	57 – 75
Germany (2006)	12	24	12	60
Italy	12	24	7	40
Japan	6	12	10	50 – 80
Netherlands	6	9	18	70
Spain	12	72	24	60 – 70
UK	24	n.a.	10	flat rate
USA (2006)	12	16	6	53

[1]for a 40-year-old single worker without children, with 22-year employment career, Source: OECD 2007 and US Department of Labor

The literature on the effects of the benefit and the eligibility parameters has two strands. First, microeconometric studies have investigated the behavioral effects of the level and duration of unemployment benefits on unemployment duration and the level of long-term unemployment.[1] Most studies confirm the intuition, that prolonging the level or the duration of UB in-

[1] Recent examples for Germany are Hunt (1995), Schmitz and Steiner (2007), Steiner (2001) and Dluglosz et al. (2009) and Lalive and Ours (2006) for Austria.

creases unemployment duration. Green and Riddell (1997) and Baker and Rea (1998) investigate the effect of an increase in the qualifying period for several regions in Canada during the 1990s. They find strong evidence that the prolongation of the qualifying period sets an incentive to extend the employment duration until the worker is eligible.

The second strand of literature focuses on the effects of UB on the aggregate rate of unemployment. Economic theory (Mortensen 1977, Mortensen and Pissarides 1999, Pissarides 2000, Fredriksson and Holmlund 2006, Rogerson et. al 2005, Coles and Masters 2007) as well as empirical research (Atkinson and Micklewright 1991, Layard et al. 1991, Nickell and Layard 1999, Nickell et al. 2005) show that the level and the duration of UB can increase the equilibrium rate of unemployment. Particularly the later literature has focused on the impact of just these two of the four policy parameters on the unemployment rate, while ignoring the effects and the optimal design of the two eligibility parameters, the base period and the qualifying period.

Mortensen (1977) is one of the first search models that allows for a finite duration of UB and for eligibility criteria. Possible reasons for being ineligible are: a worker is labor market entrant, may have already exhausted benefits, or may have quit. Employed and unemployed job seekers adjust to labor market conditions either through the reservation wage or the search intensity. The exit rate from unemployment to employment of a newly laid-off worker decreases with increasing maximum benefit duration, while the exit probability of an unemployed worker not currently eligible or with benefits due to expire increases. The effect of a varying duration is therefore theoretically ambiguous. Other and more recent contributions to this strand of the literature are Burdett (1978), Mortensen (1990), van den Berg (1990, 1994), Cahuc and Lehmann (2000) and Coles and Masters (2007).

Cahuc and Zylberberg (2004, 118 pp) use a Mortensen-Pissarides type (MP) matching model with exogenous separation rate, fixed search intensity and a labor force growing at a strictly positive rate. There are two risk-neutral worker types, new entrants to the labor market who are ineligible for benefits and workers who have already been employed and who are eligible for UB. Duration is infinite. Entrants, qualified workers and firms adjust to UB through the negotiated wage. The effect of higher UB for eligible workers on the aggregate unemployment rate is ambiguous. While the 'eligibility effect' of an increasing UB reduces unemployment among ineligible entrants, the unemployment rate of eligible workers increases.

This paper utilizes a positive methodology and excludes questions of optimal design to demonstrate that UB eligibility parameters have non-negligible macroeconomic effects. To analyze the impact of these parameters, we employ the MP model (Mortensen and Pissarides 1994) and show with a calibrated version that a trade-off exists between the eligibility and the benefit parameters: It is possible but not necessarily optimal for a country to offer job seekers a *high* level of UB with a *long* benefit duration, while neutralizing the effect on the equilibrium rate of unemployment and consequently on the PES budget with a *longer* qualifying period and/or a *shorter* base period.

An important feature of the model is the endogeneity of the match specific separation rates. Workers and firms adjust to changing eligibility requirements through the choice of a match specific separation rate, by which they determine the expected waiting time that elapses until the worker becomes re-entitled. A filled job can force the continuation of production until the UB entitlement is reached. However, by taking this decision the worker must accept a low wage to induce the firm to set a low reservation productivity. This pays only if the worker can expect either a high future UB, a long benefit duration, or both. A worker who is not yet qualified must therefore weigh the cost of restraining wage claims against the advantage gained from reduction in the waiting time until reaching eligibility.

The paper is structured as follows. Section 2 introduces our assumptions about the base period, the qualifying period and the duration-dependent UI. Section 3 introduces the job creation and the job destruction rules. Section 4 characterizes the model's solution. Section 5 presents numerical simulations. Section 6 concludes.[2]

2 Qualifying Period, Base Period and Benefit Duration

This paper uses a discrete-time model of job creation and job destruction. At the beginning of a period a continuum of infinitely lived job seekers look for vacancies. When a worker and a vacancy meet, they negotiate a contract and begin production. At the end of the period the output is sold, the wage is paid and firms, whose jobs are hit by idiosyncratic productivity shocks, decide whether to continue or to terminate the job.

2.1 Qualifying Period

Workers without a job register with the unemployment insurance (UI) $[E, F, T, b]$, with E the qualifying period, F the base period, T the benefit duration, and b the unemployment benefit, which is paid to eligible job seekers as a flat rate at the end of a period. E, F and T are natural numbers, with $E \geq 1$ and $F \geq E$. In order to model the UI, we use the following seven assumptions, whereby (A1) deals with the relation of T and E, (A2) – (A5) address the qualifying period E, (A6) explains the role of the base period F and (A7) is concerned with the qualifying path.

(A1) [Benefit Duration]. For brevity we assume that the qualifying period is at most as long as the benefit duration, such that $E \leq T$. The simulations in Section 5 also take into account the case $E \geq T+1$.

(A2) [Qualifying Period]. Workers are qualified for UB if previously employed for at least E periods during the base period F. Qualified workers are entitled to up to T payments of the UB b. The entitlement schedule is single-step. Workers who lose their job before completing E are not eligible and enjoy only the benefit z from leisure.

[2] Appendix OI which presents the proofs of the propositions, and Appendix OII which develops the equation system used for the numerical experiments are available online as supplementary material.

(A3) [Transferability]. Residual claims for UB from earlier employment spells are lost, while accumulated qualifying points are intertemporally transferable.[3]

(A4) [Employed worker]. Each employed worker is characterized by a tuple $[i, D]$, where the counter i represents the number of accumulated qualifying points. Workers, who have just completed qualification, have accumulated $i = E$ qualifying points, while workers, who are eligible since two or more periods, are indexed with $i = E + 1$, such that $i = 1, \ldots, E + 1$. D is either equal to T or to zero depending on whether the qualifying period is completed and the worker is entitled to UB.

(A5) [Job seeker]. Each job seeker is characterized by a tuple $[i, j]$: The counter $i = 0, \ldots, E$ shows the number of residual qualifying points and $j = 0, \ldots, T$ denotes the remaining benefit periods. An additional period of unemployment of a job seeker who still owns benefit claims reduces the counter j to $j - 1$ and raises the current spell length from $T - j$ to $T - (j - 1)$ periods.

2.2 Base Period and Waiting Time

The insurance-relevant part of the employment career of a worker covers the last F periods. In each period the worker was either employed or unemployed. Hence, there are in total 2^F employment careers of which $\binom{F}{i}$ have i qualifying points.

The waiting time is the time that passes until the onset of the next benefit entitlement. It depends on the number of qualifying points as well as on the distribution of the points over the base period.

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[3] If the claims for UB would be transferable contrary to the assumption, then the entitlement to benefits of the infinitely lived workers could grow indefinitely. To avoid this consequence we could introduce like the German social law an expiration date for benefit claims (SGB III, §127 (4)). But with the expiration date, we would impose an additional policy variable and make the model more unwieldy. If on the other hand, the qualifying points were not transferable, then there would exist only one employment career with E consecutive employment periods entitling the worker to receive UB and the base period F would be redundant.

If $F = E$, there is exactly one employment career which meets the qualification requirement: Only those workers who were continuously employed for at least E periods are eligible for UB. If $F > E$ as is the case in most OECD-countries, see Table 1, the $\binom{F}{i}$ employment careers with i qualifying points are associated with entirely different distribution-dependent waiting times, as is indicated by the following example.

The labor force is, as we assume, a unit mass with e employed and u unemployed workers, $1 = e + u$. The number of employed workers and job seekers with i qualifying points are designated by e_i and u_i, such that $e = \sum_{i=1}^{E} e_i + e_{E+1}$ and $u = \sum_{i=0}^{E} u_i$. Let A and B be two employed workers with $i < E$ qualifying points who belong to the pool e_i. Assume that A and B remain employed during the current period. Moreover assume that B was employed F periods ago while A was not. Then A makes a transition into the pool e_{i+1}, whereas B still owns i qualifying points at the end of the current period and remains in the pool e_i. Even though both workers have accumulated an additional employment period a "bifurcation" arises: Worker A replaces a period of unemployment at the beginning of the current base period with the current employment period, so that his qualifying counter increases by one. In contrast, B replaces an employment period at the beginning of F with the current employment period, so that his qualifying counter remains constant. Consequently, A's waiting time is *ceteris paribus* at least one period shorter than that of B.

The distribution of the employment periods over the base period has similar effects with the job seekers. In order to see this let A and B belong to the pool of unemployed u_{ij} with i qualifying points and j remaining benefit periods. Assume that both workers remain unemployed during the current period and that B was employed F periods ago while A was not. Then A still owns i points at the end of the current period and moves into the pool u_{ij-1}, whereas B loses one qualifying point and makes a transition to u_{i-1j-1}.

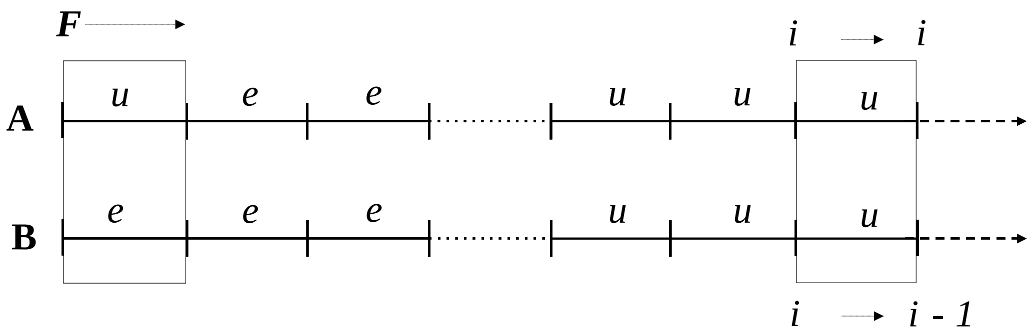

Figure 1: Base Period F

In order to model the biographic differences between workers like A and B we would have to characterize each worker by a qualification index with F components, whereby each component represents one sub-period of the base period showing whether the worker was employed in that period or not. To present a simplified version of the index and the "bifurcation effect" we introduce assumption (A6) which consists of two parts.[4]

(A6) [Employment career]. (1) Workers from e_i, who continue their job in the subsequent period, make a transition into the pool e_{i+1}. Workers from e_{E+1} will remain in e_{E+1} as long as they are employed. Whether a transition takes place, depends only on the productivity of the job, as is explained below. (2) The unemployed from u_{ij}, who have not found a job, make either a transition into the pool u_{ij-1} like worker A in the above example, or into the pool u_{i-1j-1} like worker B, where the first transition occurs with the probability $\gamma \in [0,1)$ and the second with the probability $1-\gamma \in (0,1]$.

On the macro level the transition probability has similar effects as the length of the base period. First, an increase of F or γ increases *ceteris paribus* in both cases the proportion of eligible workers. Second, if $F \to \infty$ or $\gamma \to 1$ the proportion of eligible workers approaches unity. The reason for this is the following. If $\gamma = 1$, being unemployed does not diminish the number

[4] The steady state equations for e_i and u_{ij} are developed in Appendix OII A.

of accumulated qualifying points, so that taking into account that the workers are infinitely lived, all workers in a steady state are qualified as is the case for a given $E < \infty$ and $F \to \infty$.

2.3 Qualifying Path and the Equilibrium Rate of Unemployment

We denote the output of a filled job with *yx*, with *y* a general productivity parameter and *x* an idiosyncratic one (Pissarides 2000). Idiosyncratic productivity shocks which are drawings from the distribution $G(x)$ with support $0 \le \alpha \le x \le 1$, arrive with probability λ at the end of a period. Worker and firm observe *x* and decide whether to continue the job.

In view of the assumptions (A1) and (A6) the UI establishes distributions of $E+1$ types among the employed as well as the unemployed workers. A job seeker with $i = 0,\dots,E$ residual qualifying points, was at least $E - i \ge 0$ periods without work and has therefore at most $j = T - (E - i)$ residual benefit claims, so that the UI induces in addition a distribution of $T - (E - i) + 1$ types among the job seekers with *i* qualifying points, so that $u_i = \sum_{j=0}^{T-(E-i)} u_{ij}$. Finally, the UI creates also a sequence of reservation productivities R_i, $i = 1,\dots,E+1$, which we explain next.

Successful job seekers from u_0, who do not possess qualifying points, begin their employment career in the pool e_1. Since the inflow into e_1 consists only of the job seekers from u_0 and new jobs are created with idiosyncratic productivity $x = 1$, all jobs from e_1 produce the output *y*. λe_1 of these jobs are hit by a productivity shock *x* and make a transition to e_2, if $x \ge R_2$. R_2 is the reservation productivity for the jobs which make a transition to e_2, while $\lambda G(R_2)$ is the separation rate for the jobs from e_1. The jobs from $(1-\lambda)e_1$ do not experience a productivity shock and would produce with constant productivity $x = 1$, if the worker and the firm would continue their match. Indeed, as $x > R_2$, all workers from $(1-\lambda)e_1$ make a transition to e_2.

For the general case consider a filled job with $i \le E$ qualifying points. All jobs from e_i have an idiosyncratic productivity component *x* for which $x \in [R_i, 1]$, where R_i is the endogenous reservation productivity for the jobs which make a transition to e_i. We can decompose e_i into two parts: $e_i = e_i^+ + e_i^-$. e_i^+ is the strictly positive measure of jobs with *i* qualifying points and the idiosyncratic productivity $x = 1$. The distribution of the idiosyncratic productivity component among the jobs from e_i^- is determined by the distribution function $G(x)$ truncated at the reservation productivity R_i. Thus, the probability that a job from e_i^- has an idiosyncratic productivity component which is larger than or equal to $X \in [R_i, 1]$ is $\eta_X = \frac{1-G(X)}{1-G(R_i)}$.

λe_i of the jobs from e_i are hit by a productivity shock, and $\lambda[1-G(R_i)]e_i$ of these jobs make a transition to e_{i+1}. On the other hand $(1-\lambda)e_i$ of the jobs from e_i experience no shock. Jobs, which receive no shock, do not necessarily make a transition. However, the jobs from $(1-\lambda)e_i^+$ move to e_{i+1}. The fraction of jobs which move from $(1-\lambda)e_i^-$ to e_{i+1} is denoted by η_{i+1}.[5]

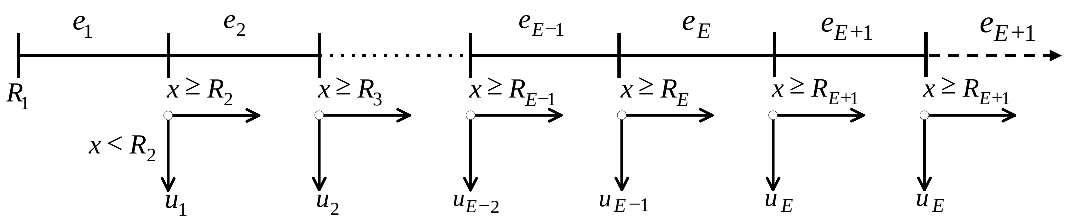

Figure 2: Qualifying path

Matches from e_{E-1} close to the completion of the qualifying period continue to e_E, if for the match specific productivity parameter $x \ge R_E$, see Figure 2. R_E is the reservation productivity for the jobs which make a transition to e_E. Finally, jobs from e_E make a transition to

[5] The definition of η_{i+1} is in the Appendix OII.

e_{E+1}, if for the job specific productivity parameter $x \geq R_{E+1}$. Otherwise, the job is destroyed, the worker becomes unemployed with claim to UB and makes a transition to u_{ET}.

We call the path of the reservation productivities $\Psi_E = [R_2, \ldots, R_E]$ the qualifying path: Every worker must, possibly interrupted by unemployment spells, pass through the qualifying path before the qualifying period is completed and the worker becomes entitled to UB. With respect to the qualifying path we introduce a final hypothesis which mainly helps to simplify the presentation and will be confirmed by the simulation results.

(A7) [Qualifying path]. The qualifying path is monotonically decreasing.

Workers face the following trade-off from which results the intuition for (A7). The shock parameter $x \in [\alpha, 1]$ is bounded from below. Consequently, a match can force the continuation of production until the qualifying period is completed and the UI entitlement is reached. Thus, for example, a worker with *i* qualifying points can reduce his waiting time to exactly $E - i$ periods, if the firm fixes the reservation productivity along the remaining qualifying path at the level of the lower support of the shock distribution. However, by taking this decision the worker must accept low, possibly negative wages until the qualifying period is completed.

Instead, workers will prefer a decreasing wage path in light of the fact that the farther away the end of the qualifying period is, the lower is the present value of the future entitlement to UB. From this it follows that the reservation productivity of a worker's job will be the higher, the longer the residual qualifying period is.

Out of e_i employed workers with *i* qualifying points, $(1-\lambda)(1-\eta_{i+1})e_i^- + \lambda G(R_{i+1})e_i$ lose their job at the end of the period. In the ensuing matching $(1-p)\left[(1-\lambda)(1-\eta_{i+1})e_i^- + \lambda G(R_{i+1})e_i\right]$ do not meet a vacancy and make a transition into the pool of unemployed u_i, where *p* is the transition probability into employment. Taking into account that from (A7) it follows that $\eta_i = 1$ for $i = 2, \ldots, E$, we get the steady state condition

$\left(1-p\right)\left[\sum_{i=1}^{E}\lambda G\left(R_{i+1}\right)e_i+\left(1-\lambda\right)\left(1-\eta_{E+1}\right)e_{\bar{E}}+\lambda G\left(R_{E+1}\right)e_{E+1}\right]=pu$, where we allow for the possibility which is confirmed by the simulation results that $R_{E+1}>R_E$ and hence $\eta_{E+1}\in\left(0,1\right)$. If we divide both sides of the steady state condition by e and observe that $e=1-u$ we obtain the steady state unemployment rate

$$(1)\qquad u\left(\theta,\Psi_E,R_{E+1}\right)=\frac{\sum_{i=1}^{E}\lambda G\left(R_{i+1}\right)\varepsilon_i+\left(1-\lambda\right)\left(1-\eta_{E+1}\right)\varepsilon_{\bar{E}}+\lambda G\left(R_{E+1}\right)\varepsilon_{E+1}}{\sum_{i=1}^{E}\lambda G\left(R_{i+1}\right)\varepsilon_i+\left(1-\lambda\right)\left(1-\eta_{E+1}\right)\varepsilon_{\bar{E}}+\lambda G\left(R_{E+1}\right)\varepsilon_{E+1}+\pi\left(\theta\right)},$$

where ε_i, with $\varepsilon_i=e_i/e$, is the fraction of the employed workers with the qualifying counter i, hence $\sum_{i=1}^{E+1}\varepsilon_i=1$; while $\pi\left(\theta\right)=p\left(\theta\right)/\left[1-p\left(\theta\right)\right]$ is the odds in favor of the transition into employment.[6]

The transition probability into employment p results from the matching function $m\left(u,v\right)$, where m is the number of jobs filled with an input of u job seekers and v vacancies. $m\left(u,v\right)$ is linear homogenous, concave and strictly increasing in both arguments. For a given vacancy, $q\left(\theta\right)\equiv m\left(1/\theta,1\right)=m\left(u,v\right)/v$ is the probability of an application, where the ratio of vacancies to job seekers, $\theta=v/u$, is the tightness of the labor market, so that $p\left(\theta\right)=\theta q\left(\theta\right)$. For convenience, we will write $q=q\left(\theta\right)$ and $p=p\left(\theta\right)$.

3 Job Creation and Job Destruction

3.1 Filled Jobs and Employed Workers

Let $\Pi_{E+1}\left(x\right)$ be the present value of a filled job with a completed qualifying period. As we will use the generalized Nash bargaining solution to determine the wage it follows that worker and firm both want to continue the match as long as $\Pi_{E+1}\left(x\right)\geq 0$ and agree on job destruc-

[6] Lemma A3 in Appendix OII B. characterizes the shares ε_i.

tion as soon as $\Pi_{E+1}(x) < 0$. Since $\Pi_{E+1}(x)$ is an increasing function, a reservation productivity R_{E+1} exists, for which

(2) $$\Pi_{E+1}(R_{E+1}) = 0 .$$

Taking account of the time structure of the firm's cash flow, the steady state equation for $\Pi_{E+1}(x)$ is

(3) $$\Pi_{E+1}(x) = \rho\left\{ yx - w_{E+1}(x) + \lambda \int_{R_{E+1}}^{1} \Pi_{E+1}(h)dG(h) + (1-\lambda)\Pi_{E+1}(x) \right\},$$

where $w_{E+1} : [R_{E+1}, 1] \to \Re$ is the function of the bargained inside wage. Flow and stock variables are discounted at factor ρ, where $0 < \rho = 1/(1+r) < 1$ with the interest rate $r > 0$. If the productivity of the job is hit by a shock it changes into state h. If $h \geq R_{E+1}$, the match is continued and the continuation value of the job becomes $\Pi_{E+1}(h)$. If the match specific productivity does not change, an event that occurs with probability $1-\lambda$, the continuation value of the job is $\Pi_{E+1}(x)$.

The present value of a worker with a completed qualifying period who is entitled to UB is

(4) $$W_{E+1}(x) = \rho\left\{ w_{E+1}(x) + \lambda\left[\int_{R_{E+1}}^{1} W_{E+1}(h)dG(h) + G(R_{E+1})U_{ET} \right] + (1-\lambda)W_{E+1}(x) \right\},$$

where U_{ET} is the value of a job seeker whose qualifying period and benefit entitlement are both complete.

The continuation value of a job with i qualifying points and the idiosyncratic productivity component $x \geq R_i$ is

(5) $$\Pi_i(x) = \rho\left\{ yx - w_i(x) + \lambda \int_{R_{i+1}}^{1} \Pi_{i+1}(h)dG(h) + (1-\lambda)\max\{0, \Pi_{i+1}(x)\} \right\},$$

where $w_i : [R_i, 1] \to \Re$ is the function of the bargained inside wage. The firm sets the reservation productivity R_i, which is determined by the reservation rule

$$\Pi_i(R_i) = 0. \tag{6}$$

If the match is hit by a shock and draws the productivity $h \geq R_{i+1}$, the continuation value of the job becomes $\Pi_{i+1}(h)$. If no shock arrives, firm and worker must still decide whether to proceed. If $x \geq R_{i+1}$ the match continues, the worker will transit to e_{i+1} and the value of the job in the continuation period will become $\Pi_{i+1}(x)$. Since the profit maximizing firm is free to destroy the job at no charge, it opts for the alternative $\max\{0, \Pi_{i+1}(x)\}$.

The steady state present value of an employed worker with i qualifying points is

$$W_i(x) = \rho\left\{ w_i(x) + \lambda\left[\int_{R_{i+1}}^{1} W_{i+1}(h)\,dG(h) + G(R_{i+1})U \right] + (1-\lambda)\max\{U, W_{i+1}(x)\} \right\}. \tag{7}$$

If the job is hit by a shock $h < R_{i+1}$, it is destroyed and the worker becomes unemployed, where the value of the unemployed equals U. If $i = E$, the worker has just completed the qualifying period and is entitled to UB, such that $U = U_{ET}$. If $i = 1, \ldots, E-1$, the worker is not yet entitled and the value of the worker's fallback position is $U = U_{i0}$, where U_{i0} denotes the present value of a job seeker with i qualifying points and no benefit claims. If no shock occurs and $W_{i+1}(x) \geq U$ the worker decides to continue the match; otherwise he quits the job and transits to unemployment.

The initial value of a newly filled job, Π_{ij}, the initial value of an outsider, who accepts a new job, W_{ij}, the value of an unemployed worker, U_{ij}, and the sharing rule that job seekers and vacancies use in their initial contract negotiations are developed in Appendix OI.

Since all job seekers look for jobs with the same search intensity, for a given vacancy $\mu_{ij} = u_{ij}/u$ is the conditional probability of an application from a job seeker with type $[i,\ j]$.[7] Access to the labor market is free, so that the following job creation condition applies, given the recruiting costs *k* and the probability *q* of an application:

$$0 = -k + q\sum_{i=0}^{E}\sum_{j=0}^{T-(E-i)}\mu_{ij}\Pi_{ij}\ . \tag{8}$$

3.2 Wage Determination

As a consequence of search frictions, each match generates a monopoly rent, which is distributed between the worker and the firm through the wage. To account for the different outside options of applicants and employed workers, we assume a two-tier wage structure (*Pissarides* 2000). w_{ij} is the bargained outside wage which is paid to entrants of type $[i, j]$ at the end of the initial employment period, and $w_i(x)$ is the bargained inside wage which is paid to employed workers of type $[i, D]$ at the end of a continuation period.

The outside options of applicants and employed workers differ for the following reasons. First, an applicant who is of type $[i, j]$ at the time of wage negotiation, with $j > 0$ and $i < E$, and an employed worker of type $[i, 0]$ own the same number of qualifying points. But while the applicant has a right to *j* residual benefit payments, the employed worker is by the assumptions (A2) - (A3) and $i < E$ neither entitled to UB nor in possession of residual claims. Second, while applicants negotiate after the event of the job matching, insiders negotiate before. If the bargaining fails, the applicant must wait until the next period to get another opportunity to participate in the job matching. The insider in contrast if dismissed can search for a new job without delay.

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[7] The probabilities μ_{ij} are developed in Lemma A4, Appendix OII B.

The sharing rules for outside and inside wages are obtained according to the generalized Nash solution to a bargaining problem, with $\beta \in (0,1)$ denoting the bargaining strength of the worker.[8] Given the free entry condition and the fact that the bargaining takes place before the job matching, the sharing rules implemented by the negotiation with an insider who has *i* qualifying points are

$$(9)\quad \begin{aligned} W_{i+1}(x) - U_{i0} &= \frac{\beta}{1-\beta} \Pi_{i+1}(x), \quad i = 1, \ldots, E-1 \\ W_{E+1}(x) - U_{ET} &= \frac{\beta}{1-\beta} \Pi_{E+1}(x), \quad i = E, E+1 \end{aligned}.$$

$W_{i+1}(x) - U_{i0}$ ($W_{E+1}(x) - U_{ET}$) denotes the worker's and $\Pi_{i+1}(x)$ ($\Pi_{E+1}(x)$) the firm's contribution to the joint surplus of the job, where the value of the worker's outside option is either equal to U_{ET} or to U_{i0} depending on whether the negotiating worker's qualifying period is complete. Equation (A4), Appendix OI, specifies the asset values U_{ij}, while the inside and outside wages are developed in Lemma 1 below, which is proved in the Appendix OI.

Lemma 1 [Bargained Wages]. (1) *Let* $x \geq R_{E+1}$. *The bargained inside wage of a worker who is at the time of negotiation of the type* $i = E, E+1$ *is*

$$(10)\quad w_{E+1}(x) = rU_{ET} + \beta(yx - rU_{ET}).$$

Let $x \geq R_{i+1}$. *The inside wage of a worker who is at the time of negotiation of the type* $i = 0, \ldots, E-1$ *is*

$$(11)\quad w_{i+1}(x) = rU_{i0} + \beta[yx - rU_{i0}] - (1-\beta)[U - U_{i0}] \quad ,$$

where $U = U_{i+10}$ *if* $i = 0, \ldots, E-2$ *and* $U = U_{ET}$ *if* $i = E-1$.

(2) *Since newly filled jobs produce with the productivity* $x = 1$, *an entrant of type* $[i, j]$, $j = 0, \ldots, T-(E-i)$, *obtains the initial wage*

$$(12)\quad w_{ij} = w_{i+1}(1) - (1-\beta)(U - O_{ij})\rho^{-1},$$

where $w_{i+1}(1)$ *is respectively the inside-wage* (10) *or* (11), *and* O_{ij}, *the fallback position of the applicant, is specified in equation* (A1), *Appendix* OI.

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[8] The sharing rule for outside wages is to be found in Appendix OI.

The bargained inside wage $w_{i+1}(x)$ is, as Equation (11) shows, equal to the worker's reservation income, rU_{i0}, plus the worker's share of the current match rent, $\beta[yx - rU_{i0}]$, minus the firm's share of the qualifying rent, $(1-\beta)[U - U_{i0}]$. To understand the reason for the deduction of the firm's share of the qualifying rent, note that at the end of the preceding period, the worker owned *i* qualifying points, so that the value of the worker's outside option is equal to U_{i0}. If the match continues, the qualifying counter increases by one to $i+1$ and the reservation value of the worker changes by $U_{i+10} - U_{i0}$ ($U_{ET} - U_{E-10}$). The current employer extracts the share $1-\beta$ of this differential rent.

The initial wage of a jobseeker of type *i*, who is entitled to *j* payments of UB, see Equation (12), is equal to the inside wage $w_{i+1}(1)$ minus the employer's share of the differential rent between the two outside options, $U - O_{ij}$, where *U* is equal to either U_{i0} or U_{ET}. Consider, for example, an applicant with no qualifying points and no claims to UB, so that $i = j = 0$. Then we have $U - O_{00} = U_{00} - \rho[z + U_{00}]$, which is equal to $\rho[rU_{00} - z] > 0$, see Lemma A6, Appendix OII, such that $w_{00} < w_1(1)$. Now let $i = 0$ and $j = 1$. Then the differential rent between the two outside options is $U - O_{01} = \rho[(rU_{00} - z) - b]$, which may, depending on *b*, be negative.

3.3 Job Destruction

The asset values of the filled jobs and the job destruction rules, which determine the waiting time of a worker, are developed in the following proposition, which is proved in Appendix OI.

Proposition. (1) [Filled Jobs]. *The value of a filled job with a completed qualifying period is*

(13) $$\Pi_{E+1}(x) = (1-\beta)y\frac{x - R_{E+1}}{\lambda + r}.$$

The value of a job filled with a worker who owns $i = 1, \ldots, E$ *qualifying points, is*

(14) $$\varPi_i(x) = \rho\left\{(1-\beta)y(x-R_i) + (1-\lambda)\left[\max\left\{0, \varPi_{i+1}(x)\right\} - \max\left\{0, \varPi_{i+1}(R_i)\right\}\right]\right\}.$$

(2) [Job Destruction]. *For a job with a completed qualifying period, the job destruction rule can be derived by evaluating the asset equation* (3) *at the reservation productivity* R_{E+1}. *Taking into account the wage equation* (10) *we obtain*

(15) $$R_{E+1} = \frac{rU_{ET}}{y} - \frac{\lambda}{(1-\beta)y}\int_{R_{E+1}}^{1} \varPi_{E+1}(h)dG(h).$$

For a job with the qualifying counter i, $i = 1, \ldots, E$, *the job destruction rule can be derived from the asset equation* (5), *the reservation condition* (6) *and the wage equation* (11) *with*

(16) $$R_i = \frac{rU_{i-10}}{y} - \frac{U - U_{i-10}}{y} - \frac{1}{(1-\beta)y}\left[\lambda\int_{R_{i+1}}^{1} \varPi_{i+1}(h)dG(h) + (1-\lambda)\max\left\{0, \varPi_{i+1}(R_i)\right\}\right],$$

where the value of the worker's outside option *U is either equal to* U_{ET} *or to* U_{i0}, *depending on whether* $i = E$ *or* $i = 1, \ldots, E-1$, *respectively.*

As the Equations (15) and (16) illustrate, the current reservation output of a match is lower than the match's reservation income both during the waiting time of the worker, see Equations (16), and after the completion of the qualifying period, see Equation (15).

With the termination rule (15), the firm chooses the reservation productivity such that $yR_{E+1} < rU_{ET}$. The firm is willing to hoard labor and to supply the market, even if hit by severe shocks. The reasons are the recruiting costs and the resulting option value of a filled job. The option value is the expected market value of a job weighted with the shock probability λ. If the productivity changes in favor of the job, the hoarded workers are immediately available to restart production, since neither search nor recruiting costs arise on the ”internal labor market”. If the match would separate as soon as the output falls below the reservation income, they would sacrifice this option.

A non-eligible worker will weigh the disadvantages of restraining his wage demand against the benefit of a reduction of the expected waiting time. The worker's willingness to restrain his wage claims, as the job destruction rule (16) indicates, is bounded by three factors: the path of the reservation incomes, the qualifying rents the worker can expect to capture, and the option value of the filled job.

The option value of the filled job is measured by the integral expression in Equation (16). Since the worker makes a transition independent of the prevailing market conditions from e_i to e_{i+1} when the job is continued, the lower bound of the integral is the reservation productivity R_{i+1} that is the threshold productivity for the transition to e_{i+1}.

If the firm currently produces at the "break-even point" with the reservation productivity R_i and is not hit by a shock, the employer opts for the alternative $\max\{0, \Pi_{i+1}(R_i)\}$ as a consequence of the free disposal.

Finally, the worker's willingness to accept a sequence of low wages on the qualifying path is bounded by the qualifying rents. If the firm chooses the reservation productivity R_i, and the match is continued, the reservation value of the worker changes from U_{i-10} to U_{i0}. The worker is willing to accept a reduction of the reservation output by an amount just equal to the qualifying rent $U_{i0} - U_{i-10}$.

4 Solution

To solve the model, we must determine the equilibrium path of the reservation productivities R_i, $i = 1, \ldots, E+1$, and the steady state tightness θ of the labor market, in total $E+2$ unknowns. The $E+1$ reservation productivities are determined by the $E+1$ job destruction conditions (15) and (16), the market tightness follows from the job creation condition (8). The reservation productivities depend on the reservation income of the workers, the qualifying rents, and the market values of the filled jobs. The market values of the filled jobs are in turn functions of the reservation productivities, as the Equations (13) and (14) show. The qualifying rents and the reservation income of the workers depend on the labor market tightness and the value of the filled jobs as Lemma A6 in the Appendix OII makes clear. The job creation condition depends on the conditional probabilities μ_{ij} and the initial values of the jobs occupied by an entrant. The job values follow from the asset equations (A2) and the qualifying rents, which are determined in Lemma A6. Finally, the probabilities μ_{ij} are derived in Lemma A4 in Appendix OII.

5 Simulation

The benchmark case of our simulations is the German labor market before the Hartz III reform came into effect in 2006. Due to the Hartz reforms the base period was lowered from 12 to 8 quarters and the benefit duration to 12 months. In the years 2004 and 2005, the standardized German unemployment rate amounted to 9.5 % (OECD 2006). We use this rate as a yardstick for the calibration of the model. The baseline parameters for the simulations are reported in Table 2.

Table 2: Baseline parameters

Qualifying period E	4	Output y	100
Base period γ	0.30	Bargaining power β	0.50
Benefit duration T	8	Real interest rate r	0.012
Unemployment benefit b	35	Probability of a shock λ	0.10
Leisure z	30	Elasticity ϕ	$1-\beta$
Recruiting costs k	30	Total factor productivity κ	0.5141071

We set the length of one model period equal to a quarter of a year. Then, see Table 1, the length of the German qualifying period is $E=4$. Before 2006, the benefit duration lasted from 12 to 36 months depending on age and employment history. We assume an average value equal to two years, which means that we have to set $T=8$. UB are $b=35$, the flow value of leisure is $z=30$. z can also be interpreted as unemployment assistance, so that $z+b$ corresponds to the German “Arbeitslosengeld I” (ALG I), while z reflects the “Arbeitslosengeld II” (ALG II). The replacement income $z+b$ matches the German replacement rate which amounts to 60 % - 67 % of the last net-income of the unemployed. The recruiting costs of a vacancy amount to $k=30$. The bargaining power of the workers is $\beta=0.50$. The interest rate r is 1.2 % per quarter; the probability of a productivity shock λ is 10 %. The distribution function $G(x)$ of the shocks is assumed to be uniform on $[\alpha,1]$, with the lower support $\alpha=0$. The matching function is of the Cobb-Douglas type (Petrongolo/ Pissarides 2001). For a given vacancy, the probability of a contact with a job seeker is $q(\theta)=\kappa\theta^{-(1-\phi)}$. For the vacancy elasticity we use the Hosios condition $\phi=1-\beta$ (Hosios 1990). The length of the base period $\gamma=0.30$ and our target of a German unemployment rate of 9.5 % require setting $\kappa=0.5141071$.

With this calibration the model generates an equilibrium unemployment rate of 9.5 % and an average duration of unemployment of $1/p(\theta)=2.14$ quarters. This is in line with the unemployment duration in Germany which lasted for about 3 quarters or 39 weeks on average in 2004 and 2005 (Bundesagentur für Arbeit 2006). Moreover, in equilibrium 1.6 % of the employed workers have not yet completed their qualifying period and are not entitled to UB, while 1.2 % of the job seekers are not eligible for UB. In comparison, in 2004 1.35 % of the German unemployed were ineligible for benefit payments (Statistisches Bundesamt 2005).

The Figures 3a - d depict the impact of the eligibility parameters (E,γ) and the benefit parameters (T,b) on the equilibrium rate of unemployment *u*. The unemployment rate is measured on the vertical axes, and the policy parameters are depicted on the horizontal axes. Figures 4a - b present the underlying qualifying and wage paths to illustrate the channels through which the eligibility parameter impact the unemployment rate.

The labor market policy has four parameters available. The “iso-unemployment curves” in the policy space are therefore four-dimensional hyperplanes. The Figures 5a - d graph two-dimensional sectors of these iso-unemployment curves and show the trade-offs between the benefit parameters on the one hand, and the eligibility parameters on the other hand, for the given unemployment rates of 9.25 %, 9.50 % and 9.75 %.

Raising the qualifying period *E*, decreasing the base period *γ*, the benefit duration *T*, or the benefit payments *b* would reduce the German unemployment rate below the pre-Hartz level of 9.5 %, marked by a circle. All four relations are strictly monotone and confirm the intuition with the exception of the special case $E=1$ in Figure 3b, which will be explained below.

Figure 3a. [*Impact of the qualifying period E***].** There are two channels through which E influences u. First, an increase of E raises the number of ineligible workers and extends the time, in which these workers are willing to trade lower wages for a lower separation probability and a shorter waiting time. For example, if $E=1$ as in the standard MP model, all employed workers are entitled and are confronted with the same probability of separation of approximately 9.2 %. If $E=4$, then in contrast to the MP model only 98.4 % of the employed workers are entitled, while the probability of a separation is 7.2 % for the workers with one qualifying point and decreases to 1.6 % for the workers with four qualifying points. Second, the longer the qualifying period E, the more vacancies are created. The result is a rising labor market tightness and increasing odds in favor of a transition into employment.

[Figures 3a and 3b about here]

Figure 3b. [*Impact of the base period γ***].** If $E=1$ as in the MP model, all employed workers are qualified because finding a job is sufficient for the re-entitlement. In the MP case, the length of the base period γ does not matter. The unemployment rate remains constant and equal to 9.86 % at all values of γ. Figure 3b illustrates furthermore that the equilibrium unemployment rates for $E \geq 2$, converge from below to the unemployment rate of the MP case. If γ approaches unity, the length of the qualifying period E no longer matters and the unemployment rate behaves as if E would be equal to one. The reason is that in the steady state with finite E and infinitely lived workers the total labor force is qualified and entitled to UB.

Why does a rising base period increase the equilibrium rate of unemployment? If the policymaker extends the base period γ, the waiting time of the workers decreases. The effect on the unemployment rate is ambiguous. Workers on the qualifying path who are not yet eligible reduce their wage claims. Hence, firms lower their reservation productivities and consequently the separation probabilities fall. Despite this effect, which lowers the unemployment rate, the aggregate rate of unemployment increases, as Figure 3b shows. The reason is the following entitlement effect. Workers who are currently eligible demand higher wages because their prospective waiting time is now shorter. Consequently employers increase the reservation productivity R_{E+1}, such that the separation probability $\lambda G(R_{E+1})$ rises. Even a small increase in R_{E+1} suffices to boost the unemployment rate because the large majority of the workers is eligible.

Even if the curves depicted in Figure 3b would decrease initially because the negative impact of an increase of γ on u outweighs the positive effect, they must eventually converge to the steady state value of the MP model where all employed workers are eligible.

Figure 3c and 3d. [*Impact of T and b***].** Figure 3c and 3d confirm the well known results of the search theory and empirical literature (see Section I). We find increasing relationships between the benefit duration and the level of UB on the one hand and the equilibrium unemployment rate on the other. While the elasticity of the unemployment rate with respect to duration is rather high when duration is low, the elasticity decreases with increasing duration. For example, if the benefit duration increases from 8 to 9 periods in the benchmark case of $E=4$, the unemployment rate increases by 0.22 percentage points, while the elasticity approaches zero for high values of the duration.

In the case of the level of UB we find a weakly convex relationship between the benefit amount and the unemployment ratio, where the elasticity of u with respect to b is rather high, (see Figure 3d). If, for instance, the unemployment benefits increase from 35 to 40 percent the unemployment rate climbs by 0.8 percentage points from 9.5 to 10.3 percent.

[Figures 3c and 3d about here]

Figure 4a. [*Decreasing Qualifying Path***].** Three qualifying paths for the qualifying periods $E=1$, $E=4$ and $E=8$ are shown in Figure 4a, while Figure 4b depicts the corresponding wage paths. The accumulated qualifying points $i=1,\ldots,E+1$ are plotted on the horizontal axis and the corresponding reservation productivities R_i are shown on the vertical axis of Fig.

4a. Fig. 4b displays on the vertical axis the marginal inside wages $w_i(R_i)$, see equations (10) and (11). Note, that the marginal inside wages and the reservation productivities are simultaneously computed by the job destruction equations (15) and (16).

The qualifying paths and the wage paths follow the same pattern for all $E \geq 2$: First, the workers moderate their wage claims the closer they come to the completion of their qualifying period. Wages and reservation productivities therefore are monotonically decreasing and reach a minimum in the last period before re-entitlement. In that period the workers cut their wage demand by more than 20 %. The firms correspondingly reduce their reservation productivities such that the qualifying paths and the corresponding wage paths are strictly decreasing and strictly concave.

[Figures 4a and 4b about here]

Second, as soon as the firms and the workers have extracted all qualifying rents and the workers are entitled to UB, the marginal wage, the reservation productivity, and the quit rate jump to the levels of the jobs with a completed qualifying period. Consequently, we find that the separation rate on the qualifying path strictly decreases until in the last period before completion it reaches $\lambda G(R_E)$ which is roughly equal to 1.6 % for all shown qualifying paths. As soon as the workers are eligible the separation rate jumps to $\lambda G(R_{E+1})$ which is approximately 9.2 %.

For a fixed number of qualifying points *i* the reservation productivity R_i and the marginal wage $w_i(R_i)$ are the higher the longer the qualifying period *E* is. The reason for these differences in the wages of otherwise identical workers with the same number of accumulated qualifying points is the following. If the policymaker extends the qualifying period *E*, the expected waiting time for the employed workers with $i = 1, \ldots, E-1$ qualifying points grows. As the waiting time increases the present value of the next entitlement decreases. The workers, who are on the qualifying path, demand higher wages, while the employers react to the wage claims with higher reservation productivities. Consequently the qualifying paths as well as the wage paths for higher E run above those with a shorter qualifying period, see Figure 4a and 4b.

[Figures 5a and 5b about here]

Figures 5a - d [*Iso-Unemployment Curves***]**. The insurance system has four policy parameters. The "iso-unemployment curves" in the policy space are therefore four-dimensional hyperplanes. Figures 5a and 5b depict two-dimensional sectors of the iso-unemployment curves for $u = 9.25$ %, 9.50 % and 9.75 % with trade-offs between the eligibility parameters and the benefit parameters.

Figures 5a and 5b illustrate the trade-offs between the qualifying period, the benefit duration, and the unemployment benefit, respectively. Figure 5a demonstrates that the policy maker can neutralize a longer benefit duration with an extended qualifying period, so that the unemployment rate remains constant. For example the policymaker can compensate for a given $u = 9.50$ % a rise of the benefit duration from 8 to 9 quarters with an increase of the length of the qualifying period from 4 to 6 quarters. Moreover, as Figure 5a makes clear, the benefit duration elasticity with regard to the qualifying period increases with rising unemployment, so that the incentive for the policy maker rises to exploit the trade-off.

[Figures 5c and 5d about here]

Finally, Figures 5c and 5d illustrate that the policy maker can also neutralize the employment effects of high unemployment benefits or a long benefit duration with a shorter base period.

6 Conclusion

Base and qualifying periods are labor market policy instruments that so far have received little attention in labor market theory, macroeconomic theory, and empirical research. We develop a Mortensen-Pissarides type search model in which we integrate the following policy instruments: The base period, the qualifying period, the unemployment benefit and a finite benefit duration. A worker is entitled to UB if he has completed the statutory qualifying period within the base period.

The qualifying period lowers both the incidence and the duration of unemployment and therefore reduces the aggregate unemployment rate. On the other hand, an increasing base period weakens the effect of the qualifying period by providing workers with a time margin to meet the criterion of the qualifying rule. The longer the base period, the higher therefore the equilibrium rate of unemployment.

In an unemployment insurance system without a qualifying rule, as for example in the standard MP model, becoming employed means the worker also becomes entitled to UB. The qualifying period endogenizes the waiting time and confronts the workers with the following trade-off. The lower the separation rate of a match, the longer the durability of the job, the shorter the waiting time of the worker, but also the lower the worker's wage. The decision to reduce the waiting time by lowering it's wage claims during the wage negotiations is more attractive the higher the UB and the longer the benefit duration.

For a match on the qualifying path, the separation rate falls together with the marginal wage from period to period, until it reaches a minimum in the last period before the re-qualification. At this point, the qualifying rents created by the unemployment insurance are extracted, the reservation productivity as well as the separation rate, and the wage of the workers, who are now re-entitled to UB, increase sharply.

The model demonstrates that a trade-off exists between the eligibility parameters and the benefit parameters: It is in principle possible for a policymaker to offer job seekers a high level of UB or a long benefit duration, while neutralizing the effect on the unemployment rate and the PES budget with a longer qualifying period or a shorter base period. An adjustment of the eligibility criteria to reduce the equilibrium unemployment rate may be easier to implement for polit-economic reasons than a reduction of the benefit parameters - namely, because the qualified median voter may not be directly affected by the latter reform.

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Figures

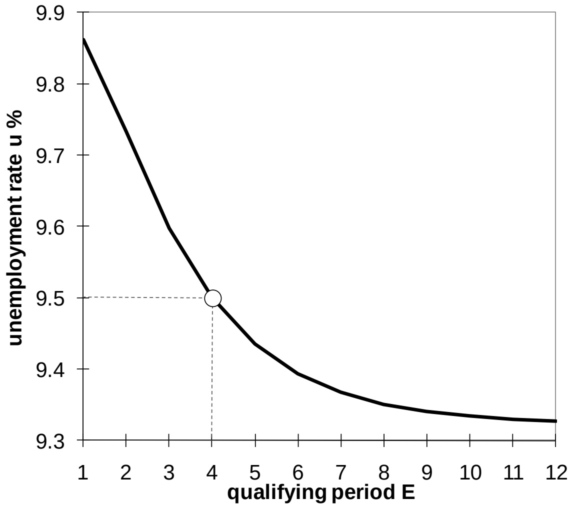


Figure 3a

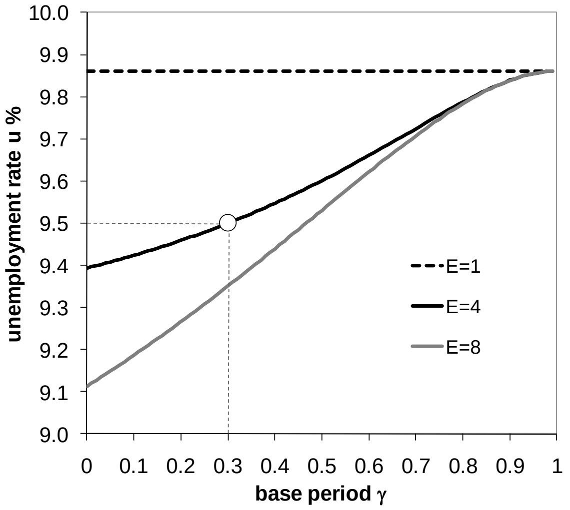


Figure 3b

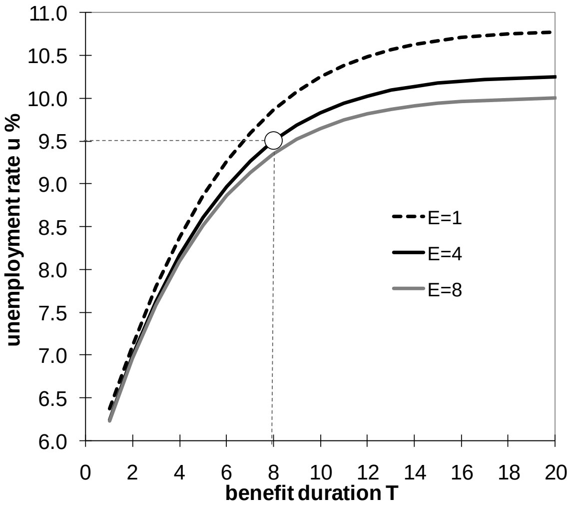


Figure 3c

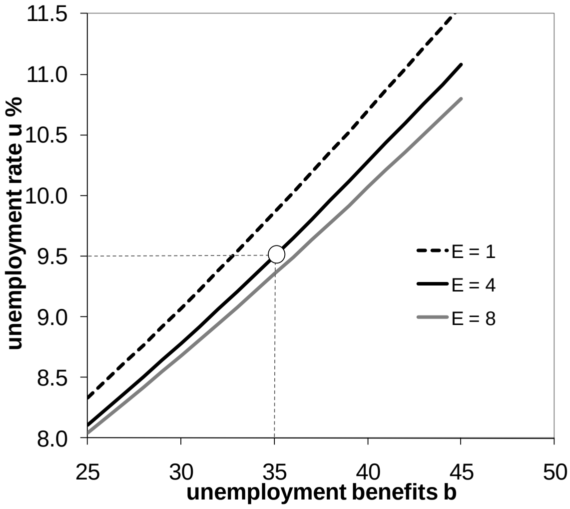


Figure 3d

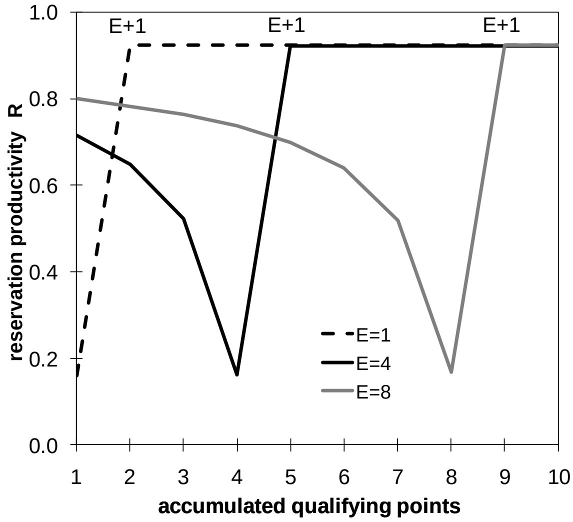
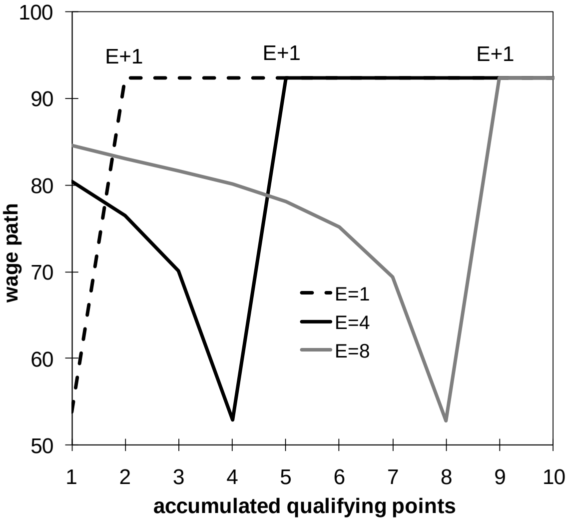


Figure 4a



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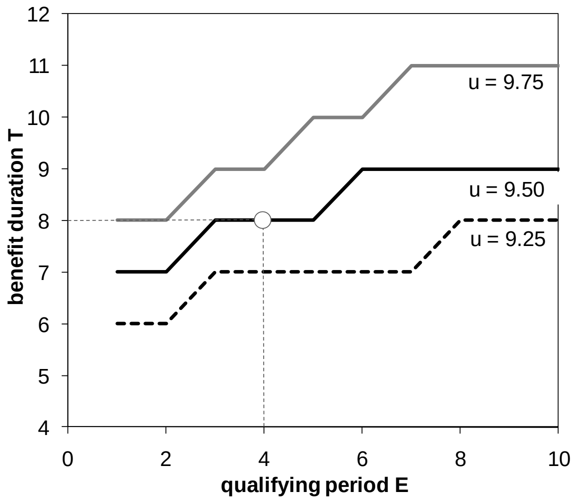

Figure 5a

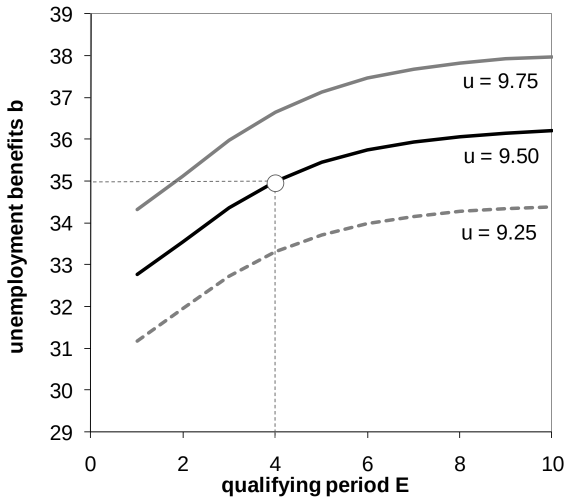

Figure 5b

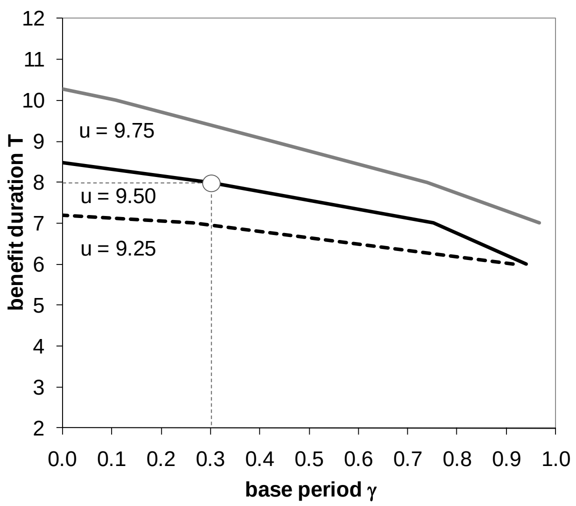

Figure 5c

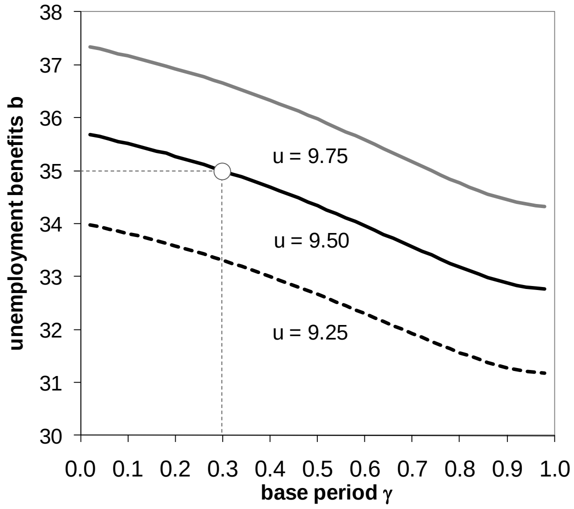

Figure 5d

Appendix OI Proofs (available as electronic supplementary material)

While the index $i = 1, \ldots, E$ indicates the number of accumulated qualifying points, the index $j = 0, \ldots, T$ denotes the remaining benefit periods of a job seeker of type $[i, j]$. From the assumptions (A1); (A5) and (A6) it follows that the length of the unemployment spell of the job seeker must be at least as large as the number of used up qualifying points: $T - j \geq E - i$, such that $j \leq T - (E - i)$. Without (A1) we can only conclude that $j \leq j_i \equiv \max\{0, T - (E - i)\}$. If $i \geq i_T + 1$, where $i_T = \max\{0, E - T\}$, then $j_i \geq 1$ and *vice versa*. We denote the odds in favor of the transition into employment with $\pi = p/(1-p)$, and write for the relationship between the bargaining strength of the worker and the firm $\delta = \beta/(1-\beta)$.

The sharing rule for wage negotiations between a vacancy and a job seeker of type $[i, j]$ is given by

(A1) $$W_{ij} - O_{ij} = \delta \Pi_{ij}, \; i = 0, \ldots, E, \; j = 0, \ldots, j_i,$$

where the value of the fallback position of the job seeker, O_{ij}, is

$$O_{ij} = \begin{cases} \rho[z + U_{00}], & i = 0, \; j = 0 \\ \rho[z + b + U_{0j-1}], & i = 0, \; j = 1, \ldots, j_i \\ \rho[z + \gamma U_{i0} + (1-\gamma) U_{i-10}], & i = 1, \ldots, E, \; j = 0 \\ \rho[z + b + \gamma U_{ij-1} + (1-\gamma) U_{i-1j-1}], & i = i_T + 1, \ldots, E, \; j = 1, \ldots, j_i \end{cases}.$$

The outside options of the employed workers, U_{ij}, are introduced below. The second line of the specification of O_{ij} is valid only, if $T \geq E + 1$. W_{ij} is the prospective value of an applicant with *i* qualifying points and a remaining benefit duration of *j* periods, and Π_{ij} is the initial value of a job occupied by an applicant with the characteristics $[i, j]$. Π_{ij} depends on the job seeker's residual claims and the current number of qualifying points, where in view of the initial productivity $x = 1$, the outside wage w_{ij} and the asset Equations (3) and (5):

(A2) $$\Pi_{ij} = \Pi_{i+1}(1) + \rho[w_{i+1}(1) - w_{ij}], \; i = 0, \ldots, E, \; j = 0, \ldots, j_i \; .$$

From Equation (12), Lemma 1, follows: $\rho[w_{i+1}(1) - w_{ij}] = (1-\beta)(U - O_{ij})$, where *U* is either equal to U_{ET} or to U_{i0} depending on whether the applicant's qualifying period is complete, $i = E$, or not, $i = 0, \ldots, E-1$. For the distribution of the initial value of the job seekers, W_{ij}, analogously we have:

(A3) $$W_{ij} = W_{i+1}(1) + \rho[w_{ij} - w_{i+1}(1)], \; i = 0, \ldots, E, \; j = 0, \ldots, j_i \; .$$

Finally, the distribution of the outside options of the employed workers is given by:

(A4) $$U_{ij}=\begin{cases} pW_{00}+(1-p)O_{00}\ , \ i=0,\ j=0 \\ pW_{i0}+(1-p)O_{i0}\ , \ i=1,\ldots,E,\ j=0 \\ pW_{ij}+(1-p)O_{ij}\ , \ i=i_T+1,\ldots,E,\ j=1,\ldots,\ j_i \\ pW_{0j}+(1-p)O_{0j}\ , \ i=0,\ j=1,\ldots,\ j_0 \end{cases}$$

If $i_T=E-T\geq 1$, then $j=j_i=0$ holds for all types with $i=1,\ldots,i_T$ qualifying points, such that $U_{ij}=U_{i0}$. The fourth equation of (A4) presupposes that $T\geq E+1$, otherwise, if $T\leq E$, types $[i,j]$ with $i=0$ and $j\geq 1$ do not exist.

Proof of Lemma 1. (1) From the sharing rule (9) it follows that: $(1-\beta)W_{E+1}(x)-\beta\Pi_{E+1}(x)=(1-\beta)U_{ET}$. Using the asset Equations (3) – (4) and rearranging terms provides the wage Equation (10). Similar from (5), (7) and (9) we get the wage equation (11).

(2) Rewrite the sharing rule (A1): $(1-\beta)W_{ij}-\beta\Pi_{ij}=(1-\beta)O_{ij}$, and insert the asset Equations (A2) and (A3) to obtain the wage Equation (12).

Proof of the Proposition. (1) If we solve the asset Equation (3) for $\Pi_{E+1}(x)$ and take the wage Equation (10) into account, we obtain:

(A5) $$\Pi_{E+1}(x)=\frac{1}{\lambda+r}\left\{(1-\beta)yx-(1-\beta)rU_{ET}+\lambda\int_{R_{E+1}}^{1}\Pi_{E+1}(h)dG(h)\right\}.$$

Let $x=R_{E+1}$ in (A5) then by virtue of $\Pi_{E+1}(R_{E+1})=0$, we obtain the asset Equation (13). If we use the wage Equation (11) in (5), we obtain, for $i=1,\ldots,E$:

(A6) $$\Pi_i(x)=\rho\left\{ (1-\beta)yx-(1-\beta)rU_{i-10}+(1-\beta)[U-U_{i-10}]+\right.$$
$$\left.\lambda\int_{R_{i+1}}^{1}\Pi_{i+1}(h)dG(h)+(1-\lambda)\max\{0,\Pi_{i+1}(x)\}\right\}.$$

If we use $x=R_i$ in (A6) and consider the reservation condition (6), we obtain the continuation value (14).

(2) If we use $x=R_{E+1}$ in (A5) and solve the equation for R_{E+1}, considering the reservation condition (6), we get the job-destruction rule (15). Correspondingly, if we use $x=R_i$ in (A6) and solve for the reservation productivity R_i, we get the job-destruction-rule (16).

Appendix OII Simulation Equations (available as electronic supplementary material))

In Section A we deal with the steady state equations for the number of employed workers, e_i, $i=1,\ldots,E+1$, and the job seekers, u_{ij}, $i=0,\ldots,E$, $j=0,\ldots,\ j_i$. In Section B we derive the conditional probabilities μ_{ij} to encounter a job seeker with characteristics $[i,j]$. Finally, in Section C we convert the guarantee income of the workers and the qualifying rents into expressions which depend on the reservation productivities R_i and the tightness of the labor market θ.

A. Pool equations

1. *Jobs which get no productivity shock*

If $R_i \geq R_{i+1}$ all the jobs from $(1-\lambda)e_i^-$ will move to e_{i+1}, where R_{i+1} is the reservation productivity for the jobs which make a transition to e_{i+1}. Otherwise, the jobs with idiosyncratic productivity $x \in [R_i, R_{i+1})$ will be terminated, and only the jobs with $x \in [R_{i+1}, 1]$ will make a transition. The fraction of jobs which move from $(1-\lambda)e_i^-$ to e_{i+1} is denoted by η_{i+1}, where $\eta_{i+1} = \min\left\{1, \frac{1-G(R_{i+1})}{1-G(R_i)}\right\}$. Thus, the number of jobs which make a transition from e_i to e_{i+1} is $(1-\lambda)\left[e_i^+ + \eta_{i+1}e_i^-\right] + \lambda[1-G(R_{i+1})]e_i$ what is equal to $[1-\lambda G(R_{i+1})]e_i$, if $\eta_{i+1} = 1$.

2. *Employed Workers:* e_i, $i = 1, \ldots, E+1$

In the steady state, the following equations hold for the number of the employed workers.

$$
\text{(S1)} \qquad e_i = \begin{cases} [1-\lambda G(R_{E+1})]e_{E+1} + p\lambda G(R_{E+1})e_{E+1} + \lambda[1-G(R_{E+1})]e_E + p\lambda G(R_{E+1})e_E + \\ \qquad (1-\lambda)\left\{[\eta_{E+1} + p(1-\eta_{E+1})]e_E^- + e_E^+\right\} + pu_E, \; i = E+1 \\ [1-\lambda G(R_i)]e_{i-1} + p\lambda G(R_i)e_{i-1} + pu_{i-1}, \; i = 2, \ldots, E \\ pu_0, \; i = 1 \end{cases}
$$

Ad $i = 1$: The inflow into the pool e_1 consists solely of the successful job seekers whose qualifying counter is equal to zero, pu_0.

Ad $i = 2, \ldots, E$: The inflow into the pool e_i has four parts. First, in the light of the monotonicity assumption (A7) the $(1-\lambda)e_{i-1}$ workers from e_{i-1} who experience no productivity shock belong to the inflow. Second, the workers who experience a productivity shock $x \geq R_i$ as well as the workers who make a job-to-job transition belong to the inflow, together their measure is equal to $\lambda[1-G(R_i)]e_{i-1} + p\lambda G(R_i)e_{i-1}$. Finally, the pu_{i-1} successful job seekers from u_{i-1} belong to the inflow into e_i.

Ad $i = E+1$: e_{E+1} is the measure of employed workers who have been eligible for at least one period. The inflow into e_{E+1} consists first of $[1-\lambda G(R_{E+1})]e_{E+1}$ workers, who already belonged to e_{E+1}; second, $p\lambda G(R_{E+1})e_{E+1}$ workers who make a job-to-job transition; third, $\lambda[1-G(R_{E+1})]e_E$ workers of the pool e_E who experience an idiosyncratic productivity shock $x \geq R_{E+1}$, and $p\lambda G(R_{E+1})e_E$ workers of e_E who suffer a negative shock and make at the beginning of the next period a transition to e_{E+1}; fourth, in the inflow are $(1-\lambda)\left\{[\eta_{E+1} + p(1-\eta_{E+1})]e_E^- + e_E^+\right\}$ workers of the pool e_E who experience no productivity shock, where $\eta_{E+1} = \min\left\{1, \frac{1-G(R_{E+1})}{1-G(R_E)}\right\}$ is the fraction of workers from e_E^- whose job has an idiosyncratic productivity component $x \geq R_{E+1}$. $(1-\lambda)e_E^+$ of the workers without shock make a transition because they produce with $x = 1$; $(1-\lambda)\eta_{E+1}e_E^-$ of the workers without shock produce with a productivity $x \geq R_{E+1}$, and $p(1-\lambda)(1-\eta_{E+1})e_E^-$ make a job-to-job transition. Fifth, the pu_E successful job seekers with a qualifying counter equal to E belong also to the inflow into e_{E+1}. Next we calculate the number of workers from e_E who produce with $x = 1$, e_E^+, and the number of workers e_E^- whose idiosyncratic productivity component $x \geq R_E$ is determined by the distribution $G(x)$.

In the steady state, the following equations hold for e_i^+ and e_i^-, where $e_i = e_i^+ + e_i^-$:

$$\text{(S2)}\qquad \begin{pmatrix} e_i^+ \\ e_i^- \end{pmatrix} = \begin{cases} \begin{pmatrix} pu_0 \\ 0 \end{pmatrix}, i = 1 \\[2ex] \begin{pmatrix} (1-\lambda) + p\lambda G(R_2) \\ \lambda[1 - G(R_2)] \end{pmatrix} e_1 + \begin{pmatrix} pu_1 \\ 0 \end{pmatrix}, i = 2 \\[2ex] A_i \begin{pmatrix} e_{i-1}^+ \\ e_{i-1}^- \end{pmatrix} + \begin{pmatrix} pu_{i-1} \\ 0 \end{pmatrix}, i = 3, \ldots, E \end{cases}$$

where for the 2×2 matrix A_i

$$A_i = \begin{pmatrix} (1-\lambda) + p\lambda G(R_i) & p[(1-\lambda)(1-\eta_i) + \lambda G(R_i)] \\ \lambda[1 - G(R_i)] & (1-\lambda)\eta_i + \lambda[1 - G(R_i)] \end{pmatrix}.$$

Note that $\eta_i = 1$ for $i = 2, \ldots, E$, because of assumption (A7). The solution of the above difference equation for $i = 4, \ldots, E$ is given by

$$\begin{pmatrix} e_i^+ \\ e_i^- \end{pmatrix} = \prod_{t=3}^{i} A_t \begin{pmatrix} e_2^+ \\ e_2^- \end{pmatrix} + \sum_{k=2}^{i-2} \prod_{t=2+k}^{i} A_t \begin{pmatrix} pu_k \\ 0 \end{pmatrix} + \begin{pmatrix} pu_{i-1} \\ 0 \end{pmatrix}.$$

Inserting (S2) into the last equation gives the result

$$\text{(S3)}\qquad \begin{pmatrix} e_i^+ \\ e_i^- \end{pmatrix} = \prod_{t=3}^{i} A_t \begin{pmatrix} (1-\lambda) + p\lambda G(R_2) \\ \lambda[1 - G(R_2)] \end{pmatrix} pu_0 + \sum_{k=1}^{i-2} \prod_{t=2+k}^{i} A_t \begin{pmatrix} pu_k \\ 0 \end{pmatrix} + \begin{pmatrix} pu_{i-1} \\ 0 \end{pmatrix}.$$

From (S1) the following system of equations for e_i, $i = 1, \ldots, E$, results

$$\text{(S4)}\qquad \begin{aligned} e_1 &= pu_0 \\ e_2 &= [1 - (1-p)\lambda G(R_2)]pu_0 + pu_1 \\ e_3 &= [1 - (1-p)\lambda G(R_3)][1 - (1-p)\lambda G(R_2)]pu_0 + [1 - (1-p)\lambda G(R_3)]pu_1 + pu_2 \\ &\vdots \\ e_i &= \sum_{t=0}^{i-2} \prod_{k=2+t}^{i} [1 - (1-p)\lambda G(R_k)]pu_t + pu_{i-1},\ i = 2, \ldots, E \end{aligned}$$

We can write (S4) as matrix equation:

$$\text{(S5)}\qquad \boldsymbol{e} = \boldsymbol{B}p\boldsymbol{u},$$

where $\boldsymbol{e}^T = (e_1, e_2, \cdots, e_E)$ and $\boldsymbol{u}^T = (u_0, u_1, \cdots, u_{E-1})$ are the transpose of the E×1-column vectors ***e*** and ***u***, and ***B*** is a E×E-triangular matrix:

$$
\boldsymbol{B}(\theta,\Psi_{\boldsymbol{E}})=\begin{pmatrix}
1 & 0 & 0 & 0 & \cdots & 0\\
[1-(1-p)\lambda G(R_2)] & 1 & 0 & 0 & \cdots & 0\\
\prod_{k=2}^{3}[1-(1-p)\lambda G(R_k)] & [1-(1-p)\lambda G(R_3)] & 1 & 0 & \cdots & 0\\
\prod_{k=2}^{4}[1-(1-p)\lambda G(R_k)] & \prod_{k=3}^{4}[1-(1-p)\lambda G(R_k)] & [1-(1-p)\lambda G(R_4)] & 1 & \cdots & 0\\
\vdots & \vdots & \vdots & \vdots & \cdots & \vdots\\
\prod_{k=2}^{E}[1-(1-p)\lambda G(R_k)] & \prod_{k=3}^{E}[1-(1-p)\lambda G(R_k)] & \prod_{k=4}^{E}[1-(1-p)\lambda G(R_k)] & \cdots & \cdots & 1
\end{pmatrix}
$$

3. *Job Seekers:* u_{ij}, $i=0,\ldots,E$, $j=0,\ldots,j_i$

2.1 For the steady state measure of job seekers of type $[E,j]$, who have a completed qualifying period and *j* residual benefit payments, the following equations hold

(S6) $$u_{Ej}=\begin{cases}(1-p)\left[\lambda G(R_{E+1})(e_{E+1}+e_E)+(1-\lambda)(1-\eta_{E+1})e_E^-\right], & j=T\\ \gamma(1-p)u_{Ej+1}, & j=1,\ldots,T-1\\ \gamma(1-p)(u_{E0}+u_{E1}), & j=0\end{cases}.$$

Ad $j=T$**:** u_{ET} is the pool of the job seekers with a completed qualifying period and full entitlement to UB. The inflow to u_{ET} consists of workers with a completed qualifying period who lost their job in the previous period and did not meet a vacancy during the last matching.

Ad $j=1,\ldots,T-1$**:** A type $[E,j]$ with $j=1,\ldots,T-1$ remaining benefit periods, whose steady state number is determined by the second equation of (S6), exists if, and only if, $T-1\geq 1$.

Ad $j=0$**:** The third line of (S6) shows the inflow to the pool of job seekers with a completed qualifying period, but no residual claims to unemployment insurance, u_{E0}. The inflow consists of job seekers from the pool $u_{E0}+u_{E1}$ who, although without a match, retain their qualifying points, an event, which has the probability $\gamma(1-p)$.

2.2 For the number of job seekers with a qualifying counter equal to $i=1,\ldots,E-1$, the following steady state conditions hold

(S7) $$u_{ij}=\begin{cases}(1-\gamma)(1-p)u_{i+1j+1}, & i=i_T+1,\ldots,E-1,\ j=j_i\\ (1-p)\left[\gamma u_{ij+1}+(1-\gamma)u_{i+1j+1}\right], & i=i_T+2,\ldots,E-1,\ j=1,\ldots,j_{i-1}\\ (1-p)[\lambda G(R_{i+1})e_i+\gamma(u_{i0}+u_{i1})+(1-\gamma)(u_{i+10}+u_{i+11})], & i=i_T+1,\ldots,E-1,\ j=0\\ (1-p)[\lambda G(R_{i+1})e_i+\gamma u_{i0}+(1-\gamma)(u_{i+10}+u_{i+11})], & i=i_T,\ j=0\\ (1-p)[\lambda G(R_{i+1})e_i+\gamma u_{i0}+(1-\gamma)u_{i+10}], & i=1,\ldots,i_T-1,\ j=0\end{cases}$$

If $E=1$, the equations of (S7) do not apply. In the following remarks we therefore assume that $E\geq 2$.

Ad $i=i_T+1,\ldots,E-1$, $j=j_i$**:** u_{ij_i} is the pool of job seekers with qualifying counter $i=i_T+1,\ldots,E-1$ who for given *i* have the largest possible number of residual claims for unemployment benefits j_i. For $j_i\geq 1$ to hold, the inequality $i\geq i_T+1$ must be fulfilled. There are two cases to distinguish. First, the case $i_T=0$, in which the equation one of (S7) applies for all $i=1,\ldots,E-1$. Second, the case $i_T\geq 1$. In this case all types $[i,j]$ with $i=1,\ldots,i_T$ have no residual claims for benefits, so that $j=0$. The steady state measure of these types is determined by the third, fourth or fifth equation of (S7).

A type whose steady state measure is determined by the first equation of (S7) exists if, and only if, $E-1 \geq i_T+1$, or equivalently if $T \geq 2$. In the case $T=1$, we have $E=i_T+1$ and a type whose steady state measure is determined by the first equation of (S7) does not exist.

Ad $i=i_T+2,\ldots,E-1$, $j=1,\ldots,j_{i-1}$: For $j_{i-1} \geq 1$ to be the case, $i-1 \geq (E-T)+1$ or equivalently $i \geq i_T+2$ must be true. The steady state measure of types with $i=1,\ldots,i_T+1$ qualifying points is determined by the third, the fourth or the fifth equation of (S7). A type to which the second equation of (S7) applies, exists only if $E-1 \geq i_T+2$ or equivalently if $T \geq 3$.

Ad $i=i_T+1,\ldots,E-1,\ j=0$**:** The inflow to the pool u_{i0} is first composed of workers who lost their job because of an adverse shock and did not meet a vacancy during the subsequent matching, $(1-p)\lambda G(R_{i+1})e_i$. Secondly, the fraction of the unsuccessful job seekers from $u_{i0}+u_{i1}$ makes a transition to u_{i0} who retain their qualifying points *i*. Finally the fraction of unsuccessful job seekers from $u_{i+10}+u_{i+11}$ who lose a qualifying point belong also to the inflow to u_{i0}.

Ad $i=i_T$, $j=0$: The length of the unemployment spell of a type with $i=i_T \geq 1$ qualifying periods is at least equal to $E-i$ periods. As in this case $E-i=T$, it follows that $j=0$. Therefore, in this case, the type $[i,1]$ does not exist and we have to omit the job seeker pool u_{i1} from the steady state condition.

Ad $i=1,\ldots,i_T-1$, $j=0$: A type, whose steady state measure is determined by the fifth equation of (S7), exists only if $i_T-1 \geq 1$. The length of the unemployment spell of such a type is at least equal to $E-i \geq E-(i_T-1)=T+1$ periods, so that the job seeker has no residual benefit claims and $j=0$. Consequently the fifth equation does not apply for types with the characteristics $[i,1]$ and $[i+1,1]$ and therefore we must omit the job seeker pools u_{i1} and u_{i+11} from the equation.

2.3 A job seeker type $[0,j]$, with $j \geq 1$, whose qualifying counter is equal to zero and who holds residual claims for unemployment benefits, exists if, and only if, $T-E \geq 1$.

(S8)
$$u_{0j}=\begin{cases}(1-\gamma)(1-p)u_{1j+1},\ j=T-E\\ (1-p)\left[u_{0j+1}+(1-\gamma)u_{1j+1}\right],\ j=1,\ldots,(T-E)-1\\ (1-p)\left[u_{00}+u_{01}+(1-\gamma)(u_{10}+u_{11})\right],\ (T-E)-1 \geq 0,\ \ j=0\\ (1-p)\left[u_{00}+(1-\gamma)(u_{10}+u_{11})\right],\ T-E=0,\ \ j=0\\ (1-p)\left[u_{00}+(1-\gamma)u_{10}\right],\ T-(E-1) \leq 0,\ \ j=0\end{cases}$$

Ad $j=1,\ldots,(T-E)-1$: A type with $j=1,\ldots,(T-E)-1$ remaining benefit periods only exists if $(T-E)-1 \geq 1$.

Ad $T-(E-1) \leq 0$, $T-E=0$, or $(T-E)-1 \geq 0$, and $j=0$: The pool u_{00} consists of job seekers who have neither qualifying points nor residual claims for unemployment insurance. The inflow to u_{00} depends on the proportion between *T* and *E*. If $(T-E)-1 \geq 0$ as in the third equation of (S8), the inflow is composed of unsuccessful job seekers first from $u_{00}+u_{01}$ and second from $u_{10}+u_{11}$ who lose the last qualifying point at the transition. If $T-E=0$, the type $[0,1]$ and therefore the pool u_{01} do not exist. Finally, if $T-(E-1) \leq 0$, neither the type $[0,1]$ nor the type $[1,1]$ do exist.

B. Conditional Probabilities $\mu_{ij}=u_{ij}/u$ (Lemmas A1 – A5)

Next we present the Lemmas A1 – A3, to be used to solve the above difference equations and to determine the fractions $\varepsilon_i(\theta,\Psi_E,R_{E+1})$. Lemma A4 derives the conditional probabilities μ_{ij} to meet a job seeker with characteristics $[i,j]$.

1. *Lemmas A1 – A4*

Lemma A1 presents solutions of the difference equations (S6) – (S8), where we use the conditional probability $a(\theta) \equiv \dfrac{(1-\gamma)[1-p(\theta)]}{p(\theta)+(1-\gamma)[1-p(\theta)]}$. A job seeker with i qualifying points makes a transition from u_i either because he met a vacancy or because he did not meet a vacancy and loses a qualifying point. The first event occurs with the probability p, the second with the probability $(1-\gamma)(1-p)$. a is the conditional probability that a job seeker who makes a transition will not find a job and loses a qualifying point. $1-a$ is the probability that a job seeker who makes a transition will find a new job.

Lemma A1 (1) [Job Seekers]. *1. For the job seeker pool u_{ij}, with $i = i_T + 1, \ldots, E-1$ qualifying points and $j = 1, \ldots, j_i$ remaining benefit claims, the following is true:*

(S9)
$$u_{ij} = \binom{T-j}{E-i}(1-p)^{T-j}(1-\gamma)^{E-i}\gamma^{(T-j)-(E-i)}u_{ET}.$$

2. For the job seeker pool u_{i0}, with $i = 1, \ldots, E-1$ and $j = 0$, we have:

(S10)
$$u_{i0} = \begin{cases} \dfrac{1-p}{1-\gamma(1-p)}\left[(1-p)^{T-1}u_{ET}\sum_{k=0}^{E-i}\binom{T}{k}a^{E-(i+k)}(1-\gamma)^k\gamma^{T-k} + \sum_{k=i}^{E-1}a^{k-i}\lambda G(R_{k+1})e_k\right], & i = i_T + 1, \ldots, E-1 \\ \dfrac{1-p}{1-\gamma(1-p)}\left[(1-p)^{T-1}u_{ET}\sum_{k=0}^{E-i_T}\binom{T}{k}a^{E-(i+k)}(1-\gamma)^k\gamma^{T-k} + \sum_{k=i}^{E-1}a^{k-i}\lambda G(R_{k+1})e_k\right], & i = 1, \ldots, i_T \end{cases}$$

The second equation is valid only if $i_T \geq 1$.

3. For the pool u_{E0}, with $i = E$ and $j = 0$, we can prove:

(S11)
$$u_{E0} = \frac{1-p}{1-\gamma(1-p)}(1-p)^{T-1}\gamma^T u_{ET}.$$

4. If job seeker types $[0, j]$ with $j = 1, \ldots, T-E$ exist, such that $T-E \geq 1$, the following is the case:

(S12)
$$u_{0j} = (1-p)^{T-j}(1-\gamma)^E u_{ET}\sum_{k=0}^{T-(E+j)}\binom{E-1+k}{E-1}\gamma^k.$$

5. For the pool u_{00} with $i = j = 0$, the following is true:

(S13)
$$u_{00} = \begin{cases} \dfrac{1-p}{p}\left[\sum_{k=1}^{E-1}a^k\lambda G(R_{k+1})e_k + (1-p)^{-1}u_{ET}\left[a^E - (1-p)^E(1-\gamma)^E\sum_{k=0}^{T-E}\binom{E-1+k}{E-1}(1-p)^k\gamma^k\left[1-(1-p)^{T-(E+k)}\right]\right]\right], & T \geq E+1 \\ \dfrac{1-p}{p}\left[\sum_{k=1}^{E-1}a^k\lambda G(R_{k+1})e_k + (1-p)^{T-1}u_{ET}\sum_{k=0}^{E}\binom{T}{k}a^{E-k}(1-\gamma)^k\gamma^{T-k}\right], & T = E \\ \dfrac{1-p}{p}\left[\sum_{k=1}^{E-1}a^k\lambda G(R_{k+1})e_k + (1-p)^{T-1}u_{ET}\sum_{k=0}^{E-i_T}\binom{T}{k}a^{E-k}(1-\gamma)^k\gamma^{T-k}\right], & T \leq E-1 \end{cases}$$

(2) [Aggregate Pools]. *1. In the steady state, the aggregate pool* u_i, $i=1,\dots,E$, *is determined by*

(S14) $$u_i = \begin{cases} \dfrac{1}{1-\gamma(1-p)}\left[(1-p)\sum_{k=i}^{E-1} a^{k-i}\lambda G(R_{k+1})e_k + a^{E-i}u_{ET}\right], \; i=1,\dots,E-1 \\ \\ \dfrac{1}{1-\gamma(1-p)}u_{ET}, \; i=E \end{cases},$$

2. Finally for u_0 *the following steady state equation holds:*

(S15) $$u_0 = \frac{1}{p}\left[(1-p)\sum_{k=1}^{E-1} a^k \lambda G(R_{k+1})e_k + a^E u_{ET}\right].$$

From (S14) and (S15) we get the following matrix equation for the E×1 column vector of unemployed workers, $\boldsymbol{u}$, where $\boldsymbol{u}^T = (u_0, u_1, \cdots, u_{E-1})$:

(S16) $$\boldsymbol{u} = \frac{1}{1-\gamma(1-p)}\left[(1-p)\lambda \boldsymbol{C}\,\boldsymbol{G}\,\tilde{\boldsymbol{e}} + \boldsymbol{a}\,u_{ET}\right],$$

with $\tilde{\boldsymbol{e}}^T = (e_1, \cdots, e_{E-1})$, $\boldsymbol{a}^T = \left(\dfrac{a^E}{1-a}, a^{E-1}, \cdots, a\right)$ and where for the E×(E-1)-matrix $\boldsymbol{C}$ and the (E-1)×(E-1) diagonal matrix $\boldsymbol{G}$:

$$\boldsymbol{C}(\theta) = \begin{pmatrix} \dfrac{a}{1-a} & \dfrac{a^2}{1-a} & \cdots & \dfrac{a^{E-1}}{1-a} \\ 1 & a & \cdots & a^{E-2} \\ 0 & 1 & \cdots & a^{E-3} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \boldsymbol{G}(\Psi_{\boldsymbol{E}}) = \begin{pmatrix} G(R_2) & 0 & \cdots & 0 \\ 0 & G(R_3) & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & G(R_E) \end{pmatrix}.$$

Inserting (S16) in (S5) yields

(S17) $$\boldsymbol{e} = (1-a)\left[(1-p)\lambda \boldsymbol{B}\,\boldsymbol{C}\,\boldsymbol{G}\,\tilde{\boldsymbol{e}} + \boldsymbol{B}\,\boldsymbol{a}\,u_{ET}\right],$$

where $\boldsymbol{BCG}$ is a E×(E-1)-matrix, while $\boldsymbol{BCG}\,\tilde{\boldsymbol{e}}$ and $\boldsymbol{Ba}$ are E×1-column vectors. $\boldsymbol{D}$ is the (E-1)×(E-1)-matrix, which results from $\boldsymbol{BCG}$ through deleting the last row, while $\boldsymbol{d}$ is a (E-1)×1-column vector which results from $\boldsymbol{Ba}$ through deleting the last entry. With $\boldsymbol{D}$ and $\boldsymbol{d}$ we get from (S17) the following equation for the vector $\tilde{\boldsymbol{e}}$:

(S18) $$\tilde{\boldsymbol{e}} = (1-a)\left[(1-p)\lambda \boldsymbol{D}\,\tilde{\boldsymbol{e}} + \boldsymbol{d}\,u_{ET}\right].$$

From (S18) it follows that $\left[\boldsymbol{I}_{E-1} - (1-a)(1-p)\lambda \boldsymbol{D}\right]\tilde{\boldsymbol{e}} = (1-a)\,\boldsymbol{d}\,u_{ET}$, where $\boldsymbol{I}_{E-1}$ is the (E-1)×(E-1) identity matrix. If the (E-1)×(E-1)-matrix $\left[\boldsymbol{I}_{E-1} - (1-a)(1-p)\lambda \boldsymbol{D}\right]$ is invertible, then we get from (S18) the following solution for $\tilde{\boldsymbol{e}}$:

(S19) $$\tilde{\boldsymbol{e}} = (1-a)\left[\boldsymbol{I}_{E-1} - (1-a)(1-p)\lambda \boldsymbol{D}\right]^{-1}\boldsymbol{d}\,u_{ET}.$$

If we insert (S19) in (S16) it follows that

(S20) $$\boldsymbol{u} = \boldsymbol{u}(\theta, \Psi_E) u_{ET},$$

where for the E×1 column vector $\boldsymbol{u}(\theta, \Psi_E)$:

$$\boldsymbol{u}(\theta, \Psi_E) = \frac{1}{1-\gamma(1-p)} \Big[(1-p)(1-a)\lambda \boldsymbol{C}(\theta) \boldsymbol{G}(\Psi_{\boldsymbol{E}}) [\boldsymbol{I}_{E-1} - (1-a)(1-p)\lambda \boldsymbol{D}(\theta, \Psi_{\boldsymbol{E}})]^{-1} \boldsymbol{d}(\theta, \Psi_{\boldsymbol{E}}) + \boldsymbol{a}(\theta)\Big],$$

so that for the pool of unemployed with i, $i = 0, \ldots, E-1$, qualifying points: $u_i = \boldsymbol{u_i}(\theta, \Psi_E) u_{ET}$, with $\boldsymbol{u_i}(\theta, \Psi_E)$ the ith entry of the column vector $\boldsymbol{u}(\theta, \Psi_E)$. From (S14) it follows that $u_E = \boldsymbol{u}_E(\theta, \Psi_E) u_{ET}$, where $\boldsymbol{u}_E(\theta, \Psi_E) = [1-\gamma(1-p)]^{-1}$.
Next, we insert (S20) in (S5) and get

(S21) $$\boldsymbol{e} = \boldsymbol{e}(\theta, \Psi_E) u_{ET},$$

where for the E×1 column vector $\boldsymbol{e}(\theta, \Psi_E)$:

$$\boldsymbol{e}(\theta, \Psi_{\boldsymbol{E}}) = (1-a)\Big[(1-p)(1-a)\lambda \boldsymbol{B}(\theta, \Psi_{\boldsymbol{E}}) \boldsymbol{C}(\theta) \boldsymbol{G}(\Psi_{\boldsymbol{E}}) [\boldsymbol{I}_{E-1} - (1-a)(1-p)\lambda \boldsymbol{D}(\theta, \Psi_{\boldsymbol{E}})]^{-1} \boldsymbol{d}(\theta, \Psi_{\boldsymbol{E}}) + \boldsymbol{B}(\theta, \Psi_{\boldsymbol{E}}) \boldsymbol{a}(\theta)\Big]$$

For the pool of employed workers with i, $i = 1, \ldots, E$, qualifying points we thus can write: $e_i = \boldsymbol{e_i}(\theta, \Psi_E) u_{ET}$, where $\boldsymbol{e_i}(\theta, \Psi_E)$ is the ith entry of the E×1 column vector $\boldsymbol{e}(\theta, \Psi_E)$.
Inserting (S20) into the equation (S3) gives

(S22) $$\begin{pmatrix} e_i^+ \\ e_i^- \end{pmatrix} = \begin{pmatrix} e_i^+(\theta, \Psi_E) \\ e_i^-(\theta, \Psi_E) \end{pmatrix} u_{ET},$$

where for $i = 4, \ldots, E$:

$$\begin{pmatrix} e_i^+(\theta, \Psi_E) \\ e_i^-(\theta, \Psi_E) \end{pmatrix} = \prod_{t=3}^{i} A_t \begin{pmatrix} (1-\lambda) + p\lambda G(R_2) \\ \lambda[1-G(R_2)] \end{pmatrix} p\boldsymbol{u}_0(\theta, \Psi_E) + \sum_{k=1}^{i-2} \prod_{t=2+k}^{i} A_t \begin{pmatrix} p\boldsymbol{u}_k(\theta, \Psi_E) \\ 0 \end{pmatrix} + \begin{pmatrix} p\boldsymbol{u}_{i-1}(\theta, \Psi_E) \\ 0 \end{pmatrix}.$$

Lemma A2 [Eligible Workers]. (1) *For the pool of employed workers, who are eligible since one or more period, the following equation holds:*

(S23) $$e_{E+1} = e_{E+1}(\theta, \Psi_E, R_{E+1}) u_{ET},$$

where

$$e_{E+1}(\theta, \Psi_E, R_{E+1}) = \frac{1}{(1-p)\lambda G(R_{E+1})} \Big[\lambda [1-(1-p)G(R_{E+1})] \boldsymbol{e}_E(\theta, \Psi_E) + (1-\lambda)\big[[\eta_{E+1} + p(1-\eta_{E+1})] e_E^-(\theta, \Psi_E) + e_E^+(\theta, \Psi_E)\big] + p u_E(\theta, \Psi_E)\Big]$$

(2) *For the pool of jobseekers with E qualifying points and an entitlement for T periods,* u_{ET}*, the following equation holds:*

(S24) $$u_{ET} = \frac{1}{\boldsymbol{u}_0(\theta, \Psi_E) + \sum_{i=1}^{E} [\boldsymbol{e}_i(\theta, \Psi_E) + \boldsymbol{u}_i(\theta, \Psi_E)] + e_{E+1}(\theta, \Psi_E, R_{E+1})}.$$

Next we introduce the fractions $\varepsilon_i(\theta, \Psi_E, R_{E+1})$ and the conditional probabilities $\mu_{ij} = u_{ij}/u$.

Lemma A3 [Fractions ε_i]. *The fraction of employed workers with the qualifying counter i,* $\varepsilon_i = e_i / e$, *is*

(S25) $$\varepsilon_i(\theta, \Psi_E, R_{E+1}) = \begin{cases} \dfrac{\boldsymbol{e}_i(\theta, \Psi_E)}{\sum_{k=1}^{E} \boldsymbol{e}_k(\theta, \Psi_E) + e_{E+1}(\theta, \Psi_E, R_{E+1})}, & i = 1, \ldots, E \\[2ex] \dfrac{e_{E+1}(\theta, \Psi_E, R_{E+1})}{\sum_{k=1}^{E} \boldsymbol{e}_k(\theta, \Psi_E) + e_{E+1}(\theta, \Psi_E, R_{E+1})}, & i = E+1 \end{cases}$$

Lemma A4. *For the conditional probabilities* $\mu_{ij} = u_{ij} / u$, *we obtain with*

$$F(\theta, \Psi_E) \equiv \left[\sum_{i=0}^{E} \boldsymbol{u}_i(\theta, \Psi_E) \right]^{-1}$$

in view of $u(\theta, \Psi_E, R_{E+1}) = F(\theta, \Psi_E)^{-1} u_{ET}$ *the following equations:*

(S26) $$\mu_{ij} = \binom{T-j}{E-i} (1-p)^{T-j} (1-\gamma)^{E-i} \gamma^{(T-j)-(E-i)} F(\theta, \Psi_E), i = i_T + 1, \ldots, E, \; j = 1, \ldots, j_i$$

(S27) $$\mu_{E0} = \frac{1-p}{1-\gamma(1-p)} (1-p)^{T-1} \gamma^T F(\theta, \Psi_E).$$

(S28)

$$\mu_{i0} = \begin{cases} \dfrac{1-p}{1-\gamma(1-p)} F(\theta, \Psi_E) \left[(1-p)^{T-1} \sum_{k=0}^{E-i} \binom{T}{k} a^{E-(i+k)} (1-\gamma)^k \gamma^{T-k} + \right. \\ \qquad \left. \sum_{k=i}^{E-1} a^{k-i} \lambda G(R_{k+1}) \boldsymbol{e}_k(\theta, \Psi_E) \right], \; i = i_T + 1, \ldots, E-1 \\[2ex] \dfrac{1-p}{1-\gamma(1-p)} F(\theta, \Psi_E) \left[(1-p)^{T-1} \sum_{k=0}^{E-i_T} \binom{T}{k} a^{E-(i+k)} (1-\gamma)^k \gamma^{T-k} + \right. \\ \qquad \left. \sum_{k=i}^{E-1} a^{k-i} \lambda G(R_{k+1}) \boldsymbol{e}_k(\theta, \Psi_E) \right], \; i = 1, \ldots, i_T \end{cases},$$

where the second equation of (S28) *is valid only if* $i_T \geq 1$.

(S29) $$\mu_{0j} = (1-p)^{T-j} (1-\gamma)^E F(\theta, \Psi_E) \sum_{k=0}^{T-(E+j)} \binom{E-1+k}{E-1} \gamma^k, \; j = 1, \ldots, T-E.$$

(S30)

$$\mu_{00}=\begin{cases}\dfrac{1-p}{p}F(\theta,\Psi_E)\left[\sum_{k=1}^{E-1}a^k\lambda G(R_{k+1})e_k(\theta,\Psi_E)+(1-p)^{-1}\left[a^E-(1-p)^E(1-\gamma)^E\sum_{k=0}^{T-E}\binom{E-1+k}{E-1}(1-p)^k\gamma^k\left[1-(1-p)^{T-(E+k)}\right]\right]\right], & T\geq E+1\\[2ex] \dfrac{1-p}{p}F(\theta,\Psi_E)\left[\sum_{k=1}^{E-1}a^k\lambda G(R_{k+1})e_k(\theta,\Psi_E)+(1-p)^{T-1}\sum_{k=0}^{E-i_T}\binom{T}{k}a^{E-k}(1-\gamma)^k\gamma^{T-k}\right], & T\leq E\end{cases}$$

2. *Proofs of the Lemmas A1 – A4*

Proof of Lemma A1. **(1) [JOB SEEKERS] Ad** 1. Let $i=i_T+1,\ldots,E-1$ and $j=j_i$, then in view of (S7) the statement follows from $u_{ij}=(1-p)(1-\gamma)u_{i+1j+1}=(1-p)^{T-j}(1-\gamma)^{E-i}u_{ET}$.

Now, let $j<j_i$, then by virtue of (S7), we get the following results by solving the underlying difference equations:

$$\begin{aligned}u_{ij}&=(1-p)[\gamma u_{ij+1}+(1-\gamma)u_{i+1j+1}]\\&=(1-p)\left[\gamma\binom{T-(j+1)}{E-i}(1-\gamma)^{E-i}(1-p)^{T-(j+1)}\gamma^{T-(j+1)-(E-i)}+\right.\\&\qquad\left.(1-\gamma)\binom{T-(j+1)}{E-(i+1)}(1-\gamma)^{E-(i+1)}(1-p)^{T-(j+1)}\gamma^{T-(j+1)-(E-(i+1))}\right]u_{ET}\\&=\left[\binom{T-(j+1)}{E-i}+\binom{T-(j+1)}{E-(i+1)}\right](1-p)(1-\gamma)^{E-i}(1-p)^{T-(j+1)}\gamma^{[T-(j+1)]-[E-(i+1)]}u_{ET}\\&=\binom{T-j}{E-i}(1-p)^{T-j}(1-\gamma)^{E-i}\gamma^{(T-j)-(E-i)}u_{ET}.\end{aligned}$$

Ad 2. Let $i=i_T+1,\ldots,E-1$. With (S7),

$$\begin{aligned}u_{i0}&=(1-p)[\lambda G(R_{i+1})e_i+\gamma(u_{i0}+u_{i1})+(1-\gamma)(u_{i+10}+u_{i+11})]\\&=\frac{(1-p)}{1-\gamma(1-p)}[\lambda G(R_{i+1})e_i+\gamma u_{i1}+(1-\gamma)(u_{i+10}+u_{i+11})].\end{aligned}$$

We eliminate u_{i1} and u_{i+11} using (S9):

$$\begin{aligned}u_{i0}&=\frac{(1-p)}{1-\gamma(1-p)}\left[\lambda G(R_{i+1})e_i+(1-\gamma)u_{i+10}+(1-p)^{T-1}(1-\gamma)^{E-i}\gamma^{T-(E-i)}u_{ET}\left[\binom{T-1}{E-i}+\binom{T-1}{E-(i+1)}\right]\right]\\&=\frac{(1-p)}{1-\gamma(1-p)}\left[\lambda G(R_{i+1})e_i+(1-\gamma)u_{i+10}+\binom{T}{E-i}(1-p)^{T-1}(1-\gamma)^{E-i}\gamma^{T-(E-i)}u_{ET}\right]\end{aligned}$$

Solving the difference equation we get:

$$\begin{aligned}u_{i0}&=\frac{(1-p)}{1-\gamma(1-p)}\left[\lambda G(R_{i+1})e_i+a\left[(1-p)^{T-1}u_{ET}\sum_{k=0}^{E-(i+1)}\binom{T}{k}a^{E-(i+1+k)}(1-\gamma)^k\gamma^{T-k}+\right.\right.\\&\qquad\left.\left.\sum_{k=i+1}^{E-1}a^{k-(i+1)}\lambda G(R_{k+1})e_k\right]+\binom{T}{E-i}(1-p)^{T-1}(1-\gamma)^{E-i}\gamma^{T-(E-i)}u_{ET}\right]\\&=\frac{(1-p)}{1-\gamma(1-p)}\left[\lambda G(R_{i+1})e_i+(1-p)^{T-1}u_{ET}\sum_{k=0}^{E-(i+1)}\binom{T}{k}a^{E-(i+k)}(1-\gamma)^k\gamma^{T-k}+\right.\\&\qquad\left.\sum_{k=i+1}^{E-1}a^{k-i}\lambda G(R_{k+1})e_k+\binom{T}{E-i}(1-p)^{T-1}(1-\gamma)^{E-i}\gamma^{T-(E-i)}u_{ET}\right]\end{aligned}$$

$$=\frac{(1-p)}{1-\gamma(1-p)}\left[(1-p)^{T-1}u_{ET}\sum_{k=0}^{E-i}\binom{T}{k}a^{E-(i+k)}(1-\gamma)^k\gamma^{T-k}+\sum_{k=i}^{E-1}a^{k-i}\lambda G(R_{k+1})e_k\right].$$

Let $i=i_T$, so that $i+1=i_T+1$. With (S7),

$$u_{i0}=(1-p)[\lambda G(R_{i+1})e_i+\gamma u_{i0}+(1-\gamma)(u_{i+10}+u_{i+11})]$$
$$=\frac{(1-p)}{1-\gamma(1-p)}[\lambda G(R_{i+1})e_i+(1-\gamma)(u_{i+10}+u_{i+11})].$$

We eliminate u_{i+11} using (S9) and u_{i+10} using (S10):

$$u_{i0}=\frac{(1-p)}{1-\gamma(1-p)}\left[(1-p)^{T-1}u_{ET}\left[\sum_{k=0}^{E-(i+1)}\binom{T}{k}a^{E-(i+k)}(1-\gamma)^k\gamma^{T-k}+\right.\right.$$
$$\left.\left.\binom{T-1}{E-(i+1)}(1-\gamma)^{E-i}\gamma^{T-(E-i)}\right]+\sum_{k=i}^{E-1}a^{k-i}\lambda G(R_{k+1})e_k\right]$$

As $i=i_T=E-T$ it follows that $\binom{T}{E-i}=\binom{T-1}{E-(i+1)}$. Thus

$$u_{i0}=\frac{(1-p)}{1-\gamma(1-p)}\left[(1-p)^T u_{ET}\sum_{k=0}^{E-i}\binom{T}{k}a^{E-(i+k)}(1-\gamma)^k\gamma^{T-k}+\sum_{k=i}^{E-1}a^{k-i}\lambda G(R_{k+1})e_k\right].$$

Let $i=1,\ldots i_T-1$. From (S7) it follows that $u_{i0}=(1-p)[\lambda G(R_{i+1})e_i+\gamma u_{i0}+(1-\gamma)u_{i+10}]$. With (S10) we find

$$u_{i0}=\frac{(1-p)}{1-\gamma(1-p)}\left[\lambda G(R_{k+1})e_k+a\left[(1-p)^T u_{ET}\sum_{k=0}^{E-i_T}\binom{T}{k}a^{E-(i+1+k)}(1-\gamma)^k\gamma^{T-k}+\sum_{k=i+1}^{E-1}a^{k-(i+1)}\lambda G(R_{k+1})e_k\right]\right],$$

from which equation the proposition follows.

Ad 3. With (S6) $u_{E0}=\gamma(1-p)(u_{E0}+u_{E1})$ results. If we eliminate u_{E1} with (S9) and solve for u_{E0}, the statement follows.

Ad 4. From (S8) $u_{0j}=(1-p)[u_{0j+1}+(1-\gamma)u_{1j+1}]$. If we eliminate u_{1j+1} with (S9) we get:

$$u_{0j}=(1-p)\left[u_{0j+1}+\binom{T-(j+1)}{E-1}(1-p)^{T-(j+1)}(1-\gamma)^E\gamma^{(T-j)-E}u_{ET}\right].$$

By solving the difference equation we get

$$u_{0j}=(1-p)^{T-j}(1-\gamma)^E u_{ET}\left[\sum_{k=0}^{T-(E+j+1)}\binom{E-1+k}{E-1}\gamma^k+\binom{T-(j+1)}{E-1}\gamma^{(T-j)-E}\right],$$

from which equation the result follows.

Ad 5. There are three cases to distinguish. First, let $T-(E+1)\geq 0$. From (S8): $u_{00}=\frac{(1-p)}{p}[u_{01}+(1-\gamma)(u_{10}+u_{11})]$. Replace u_{01} with (S12), u_{10} with (S10) and u_{11} with (S9), to get:

$$u_{00}=\frac{(1-p)}{p}\left[\sum_{k=1}^{E-1}a^k\lambda G(R_{k+1})e_k+\right.$$
$$\left.(1-p)^{T-1}(1-\gamma)^E u_{ET}\left[\sum_{k=0}^{T-E}\binom{E-1+k}{E-1}\gamma^k+\sum_{k=0}^{E-1}\binom{T}{k}a^{E-k}(1-\gamma)^{k-E}\gamma^{T-k}\right]\right]$$

$$=\frac{(1-p)}{p}\left[\sum_{k=1}^{E-1}a^k\lambda G(R_{k+1})e_k+(1-p)^{-1}u_{ET}\left[(1-p)^T(1-\gamma)^E\sum_{k=0}^{T-E}\binom{E-1+k}{E-1}\gamma^k+\right.\right.$$
$$\left.\left.a^E\sum_{k=0}^{E-1}\binom{T}{k}[1-\gamma(1-p)]^k(1-p)^{T-k}\gamma^{T-k}\right]\right]$$

In view of Equation (S31), Lemma A5 below and $\binom{E-1+k}{k}=\binom{E-1+k}{E-1}$, we can write:

$$u_{00}=\frac{(1-p)}{p}\left[\sum_{k=1}^{E-1}a^k\lambda G(R_{k+1})e_k+(1-p)^{-1}u_{ET}\left[(1-\gamma)^E(1-p)^T\sum_{k=0}^{T-E}\binom{E-1+k}{E-1}\gamma^k+\right.\right.$$

$$\left.\left.a^E\left[1-[1-\gamma(1-p)]^E\sum_{k=0}^{T-E}\binom{E-1+k}{E-1}(1-p)^k\gamma^k\right]\right]\right]$$

$$=\frac{1-p}{p}\left[\sum_{k=1}^{E-1}a^k\lambda G(R_{k+1})e_k+(1-p)^{-1}u_{ET}\left[a^E-\right.\right.$$

$$\left.\left.(1-p)^E(1-\gamma)^E\sum_{k=0}^{T-E}\binom{E-1+k}{E-1}(1-p)^k\gamma^k\left[1-(1-p)^{T-(E+k)}\right]\right]\right].$$

Second, let $T-E=0$. From (S8): $u_{00}=\frac{(1-\gamma)(1-p)}{p}(u_{10}+u_{11})$. Replace u_{10} with (S10) and u_{11} with (S9), to get the result. Third, let $T-(E-1)\leq 0$. From (S8): $u_{00}=\frac{(1-\gamma)(1-p)}{p}u_{10}$. Taking account of $i=1\leq i_T$ replace u_{10} with (S10) to get the result.

(2) [Aggregate Pools]. The equations for the aggregate pools (S14) – (S15) can be derived from assumption (A7) and the following steady state conditions.

Ad 1. In the steady state the inflow into the job seeker pool u_E, $(1-p)\left[\lambda G(R_{E+1})(e_E+e_{E+1})+(1-\lambda)(1-\eta_{E+1})e_{\bar{E}}\right]$, must be equal to u_{ET}, see also (S6), while the outflow is given by $pu_E+(1-\gamma)(1-p)u_E$, so that the steady state equation (S14) for $i=E$ follows.

The inflow into u_i, $(1-p)\lambda G(R_{i+1})e_i+(1-\gamma)(1-p)u_{i+1}$, is equal to the outflow, $pu_i+(1-\gamma)(1-p)u_i$, such that:

$$u_i=\frac{1-p}{1-\gamma(1-p)}\left[\lambda G(R_{i+1})e_i+(1-\gamma)u_{i+1}\right].$$

Solving this difference equation yields:

$$u_i=\frac{1-p}{1-\gamma(1-p)}\sum_{k=i}^{E-1}a^{k-i}\lambda G(R_{k+1})e_k+a^{E-i}u_E.$$

Inserting the steady state equation for u_E yields the result.

Ad 2. For the pool of unemployed without qualifying points, u_0, the following steady state condition holds: $(1-\gamma)(1-p)u_1=pu_0$. In view of (S14) it follows:

$$u_0=\frac{1-p}{p}\left[\sum_{k=1}^{E-1}a^k\lambda G(R_{k+1})e_k+a^E(1-p)^{-1}u_{ET}\right].$$

Proof of Lemma A2. (1) The proposition follows from (S1) and (S20)-(S22). (2) The proposition follows from $1=e+u$ together with (S20)-(S22).

Proof of Lemma A3. The results follow from the equations (S21) and (S23).

Proof of Lemma A4. The conditional probabilities μ_{ij} - that an applicant has *i* qualifying points and a residual claim to the UB *b* of j periods – directly follow from Lemma A1 5., where we make use of Lemma A5 below.

3. Lemma A5

Lemma A5. *Let* $T \geq m+1 \geq 1$, *then the following equation holds:*

$$\text{(S31)} \quad 1 = \sum_{k=0}^{m} \binom{T}{k} [1-\gamma(1-p)]^k (1-p)^{T-k} \gamma^{T-k} + [1-\gamma(1-p)]^{m+1} \sum_{k=0}^{T-(m+1)} \binom{m+k}{k} (1-p)^k \gamma^k .$$

Proof of Lemma A5. 1. Let $m=0$, then clearly $(1-p)^T \gamma^T + [1-\gamma(1-p)] \sum_{k=0}^{T-1} (1-p)^k \gamma^k = 1$ holds.

2. Assume the statement is true for *m*, then for $m+1$ and $T \geq m+2$ with

$$RHS(T) \equiv \sum_{k=0}^{m+1} \binom{T}{k} [1-\gamma(1-p)]^k (1-p)^{T-k} \gamma^{T-k} + [1-\gamma(1-p)]^{m+2} \sum_{k=0}^{T-(m+2)} \binom{m+1+k}{k} (1-p)^k \gamma^k$$

it follows that

$$\begin{aligned} RHS(T) &= 1 + [1-\gamma(1-p)]^{m+1} \Bigg[\binom{T}{m+1} (1-p)^{T-(m+1)} \gamma^{T-(m+1)} + \\ &\qquad [1-\gamma(1-p)] \sum_{k=0}^{T-(m+2)} \binom{m+1+k}{m+1} (1-p)^k \gamma^k - \sum_{k=0}^{T-(m+1)} \binom{m+k}{m} (1-p)^k \gamma^k \Bigg] \\ &= 1 + [1-\gamma(1-p)]^{m+1} \Bigg[(1-p)^{T-(m+1)} \gamma^{T-(m+1)} \left[\binom{T}{m+1} - \binom{T-1}{m} \right] + \\ &\qquad \sum_{k=0}^{T-(m+2)} (1-p)^k \gamma^k \left[[1-\gamma(1-p)] \binom{m+1+k}{m+1} - \binom{m+k}{m} \right] \Bigg]. \end{aligned}$$

The second summand in the above equation is equal to zero! We prove this statement by induction over the benefit duration $T \geq m+2$. Clearly, for $T = m+2$, $RHS(m+2) = 1$ holds. For the conclusion from *T* to $T+1$, in view of the induction hypothesis, it then holds that:

$$\begin{aligned} RHS(T+1) &= 1 + [1-\gamma(1-p)]^{m+1} \Bigg[(1-p)^{T-m} \gamma^{T-m} \left[\binom{T}{m+1} \frac{T+1}{T-m} - \binom{T-1}{m} \frac{T}{T-m} \right] + \\ &\sum_{k=0}^{T-(m+2)} (1-p)^k \gamma^k \left[[1-\gamma(1-p)] \binom{m+1+k}{m+1} - \binom{m+k}{m} \right] + (1-p)^{T-(m+1)} \gamma^{T-(m+1)} \left[[1-\gamma(1-p)] \binom{T}{m+1} - \binom{T-1}{m} \right] \Bigg] \\ &= 1 + [1-\gamma(1-p)]^{m+1} \Bigg[(1-p)^{T-m} \gamma^{T-m} \left[\binom{T}{m+1} \frac{T+1}{T-m} - \binom{T-1}{m} \frac{T}{T-m} \right] - (1-p)^{T-m} \gamma^{T-m} \binom{T}{m+1} \Bigg] \\ &= 1 . \end{aligned}$$

C. Reservation Income and Rents (Lemma A6)

With Lemma A6 we convert the guarantee income of the workers and the qualifying rents into expressions which depend on the model parameters, the tightness θ, the odds in favor of the transition into employment, $\pi = p/(1-p)$, and the asset values of the occupied jobs, Π_i, $i = 1, \ldots, E+1$. We denote the relative bargaining strength of the jobseekers with $\delta = \beta/(1-\beta)$.

Lemma A6 (i) [Filled Jobs]. *In view of the assumption (A7) the value of a filled job with i qualifying points,* $i = 1, \ldots, E$, *is given by*

$$\text{(S32)} \quad \Pi_i(x) = \frac{(1-\beta)}{\lambda + r} y \left\{ \left[1 - [\rho(1-\lambda)]^{E-(i-1)} \right] (x - R_i) + [\rho(1-\lambda)]^{E-(i-1)} [\max\{0, x - R_{E+1}\} - \max\{0, R_i - R_{E+1}\}] \right\}.$$

(ii) [Reservation Income]. 1. *The reservation income of a job seeker who neither owns qualifying points nor residual claims for UB is:*

(S33) $$rU_{00} = z + \delta\pi\Pi_1(1)\rho^{-1}.$$

2. *Let* $T - E \geq 1$, *then the value of a job seeker without qualifying points who still owns claims for UB for another* $j = 1,\ldots,T-E$ *periods is:*

(S34) $$U_{0j} = U_{00} + b\sum_{k=1}^{j} d^k ,$$

where $d \equiv \dfrac{\rho(1-p)}{1-p(1-\beta)} < 1$.

3. *For the reservation income of an employed worker with a qualifying counter equal to* $i = 1,\ldots,E-1$, *who owns no residual claims for UB the following is true*

(S35) $$rU_{i0} = z + \delta\pi\tau^i\Pi_1(1)\rho^{-1} + \frac{\delta\pi}{(1-\rho\gamma)}\sum_{k=1}^{i}\tau^{i-k} r\Pi_{k+1}(1),$$

where $\tau \equiv \dfrac{\rho(1-\gamma)}{1-\rho\gamma} < 1$.

4. *The value of a job seeker with* $i = i_T + 1,\ldots,E-1$ *qualifying points and* $j = 1,\ldots,j_i$ *remaining benefit periods, is:*

(S36) $$U_{ij} = U_{i0} + b\sum_{k=1}^{j} d^k .$$

5. *For the reservation income of an employed worker with a completed qualifying period we have*

(S37) $$rU_{ET} = \frac{(1-\rho)(1-d\gamma)}{1-\rho\gamma}\frac{1-d^T}{1-d}b + z + \delta\pi\tau^E\Pi_1(1)\rho^{-1} + \frac{\delta\pi}{(1-\rho\gamma)}\sum_{k=1}^{E}\tau^{E-k} r\Pi_{k+1}(1).$$

6. *A job seeker with a completed qualifying period and residual claims for UB over* $j = 0,\ldots,T-1$ *periods, has the value:*

(S38) $$U_{Ej} = U_{ET} - b\sum_{k=j+1}^{T} d^k .$$

(iii) [Rents]. 1. *The qualifying rent for a match that makes a transition from* e_{i-1} *to* e_i, *where* $i = 2,\ldots,E-1$, *is:*

(S39) $$U_{i0} - U_{i-10} = \frac{\delta\pi}{(1-\rho\gamma)}\sum_{k=1}^{i}\tau^{i-k}\left[\Pi_{k+1}(1) - \Pi_k(1)\right].$$

2. *The qualifying rent for* $i = E$ *is given by:*

(S40) $$U_{ET} - U_{E-10} = \frac{\rho(1-d\gamma)}{1-\rho\gamma}\frac{1-d^T}{1-d}b + \frac{\delta\pi}{(1-\rho\gamma)}\sum_{k=1}^{E}\tau^{E-k}\left[\Pi_{k+1}(1) - \Pi_k(1)\right].$$

3. *Lemma 1, Equation* (12), *shows that for two workers with a completed qualifying period – one is an outsider, the other an insider -, the outsider has the weaker bargaining position. The side payment he must accept,* $(1-\beta)(U_{ET} - U_{Ej})$, *is equal to*

(S41) $$(1-\beta)(U_{ET} - U_{Ej}) = (1-\beta)b\sum_{k=j+1}^{T} d^k .$$

4. *If we compare two workers with i qualifying points – one is an outsider with a residual benefit duration of j periods, the other is an insider -, then the outsider is better off, (see Lemma 1, Equation* (12)*), because he receives a wage bonus for which:*

(S42) $$U_{ij} - U_{i0} = b\sum_{k=1}^{j} d^k .$$

Proof of Lemma A6. **(i)** *From the asset equation (14) and assumption (A7) we get for* $i = 1,\ldots,E$ *:*

$$\Pi_i(x) = \rho\{(1-\beta)y(x - R_i) + (1-\lambda)[\Pi_{i+1}(x) - \Pi_{i+1}(R_i)]\}.$$

$$= \rho\left\{[1+\rho(1-\lambda)](1-\beta)y(x-R_i)+(1-\lambda)^2\rho[\Pi_{i+2}(x)-\Pi_{i+2}(R_i)]\right\}$$
$$\vdots$$
$$= \rho\left\{(1-\beta)y(x-R_i)\sum_{k=0}^{E-i}\rho^k(1-\lambda)^k + \rho^{E-i}(1-\lambda)^{E-(i-1)}[\max\{0,\Pi_{E+1}(x)-\max\{0,\Pi_{E+1}(R_i)\}\}]\right\}$$
$$= \frac{(1-\beta)}{\lambda+r}y\left\{\left[1-[\rho(1-\lambda)]^{E-(i-1)}\right](x-R_i)+[\rho(1-\lambda)]^{E-(i-1)}[\max\{0,x-R_{E+1}\}-\max\{0,R_i-R_{E+1}\}]\right\}$$

(ii) Ad 1. The statement follows with $i = j = 0$ from the asset Equations (A2), (A4) and the sharing rule (A1).

Ad 2. Let $i = 0$ and $j = 1,\ldots,T-E$. Then from the asset Equations (A2), (A4) and the sharing rule (A1) we get:

$$U_{0j} = \frac{\delta p}{1-(1-\beta)p}\Pi_1(1)+\frac{\beta p}{1-(1-\beta)p}U_{00}+d[z+b+U_{0j-1}].$$

Replace $\Pi_1(1)$ using (S33), and rearrange terms, to get

$$U_{0j} = \left[\frac{\beta p}{1-(1-\beta)p}+rd\right]U_{00}+d[b+U_{0j-1}].$$

Solve the difference equation

$$U_{0j} = \left[\left(\frac{\beta p}{1-(1-\beta)p}+rd\right)\frac{1-d^j}{1-d}+d^j\right]U_{00}+b\sum_{k=1}^{j}d^k .$$

The sum in the square brackets is equal to one, so that the statement follows.

Ad 3. From the asset Equations (A2), (A4) and the sharing rule (27) we get

$$U_{i0} = \frac{\delta\pi}{1-\rho\gamma}\Pi_{i+1}(1)+\frac{\rho}{1-\rho\gamma}[z+(1-\gamma)U_{i-10}].$$

Solve the difference equation, replace U_{00} with Equation (S33) and the statement follows.

Ad 4. With the asset Equation (A4), the sharing rule (A1) and the initial value of a filled job (A2) we obtain the following difference equation:

(S43) $$U_{ij} = \frac{\delta p}{1-p(1-\beta)}[\Pi_{i+1}(1)+(1-\beta)U_{i0}]+d[z+b+\gamma U_{ij-1}+(1-\gamma)U_{i-1j-1}]$$

First, we show that the proposition holds for $j=1$. For $j=1$, it follows from (S43) that

$$U_{i1} = \frac{\delta p}{1-p(1-\beta)}[\Pi_{i+1}(1)+(1-\beta)U_{i0}]+d[z+b+\gamma U_{i0}+(1-\gamma)U_{i-10}]$$

If we replace U_{i-10} with (S35), we get:

$$U_{i1} = d\left[b+\left(1+\frac{\rho(1-\gamma)}{1-\rho}\right)z+\left(\gamma+\beta\pi\rho^{-1}\right)U_{i0}\right]+\frac{d(1-\rho\gamma)}{(1-\rho)}\left[\delta\pi\tau^i\Pi_1(1)\rho^{-1}+\frac{\delta\pi}{(1-\rho\gamma)}\sum_{k=1}^{i}\tau^{i-k}r\Pi_{k+1}(1)\right]$$

Substitute the expression in the last square brackets with (S35) by $rU_{i0}-z$ and rearrange terms, to obtain the statement: $U_{i1} = U_{i0}+db$. In case $i=1$, we must replace $U_{i-10} = U_{00}$ with (S33), beyond that the calculations are similar. For the conclusion from j to $j+1$ we eliminate U_{ij-1} and U_{i-1j-1} in (S43) with (S36) and obtain $U_{ij+1} = U_{i0}+b\sum_{k=1}^{j+1}d^k$.

Ad 5. With (A4), (A2) and the sharing rule (9), we obtain the following equation for the guarantee value of an employed insider with a completed qualifying period:

(S44) $$U_{ET} = \delta\pi\Pi_{E+1}(1)+\rho[z+b+\gamma U_{ET-1}+(1-\gamma)U_{E-1T-1}].$$

To solve the difference equation, we need to know the guarantee value of a job seeker with a completed qualifying period and an unemployment spell of one period, U_{ET-1}. The value U_{E-1T-1} results from (S36).

With (A4), (A2), the sharing rule (A1) and the wage Equation (12) we get:

(S45) $$U_{Ej} = \frac{\delta p}{1-p(1-\beta)}\left[\Pi_{E+1}(1) + (1-\beta)U_{ET}\right] + d[z + b + \gamma U_{Ej-1} + (1-\gamma)U_{E-1j-1}].$$

Solve the difference Equation (S45) to obtain:

(S46) $$U_{Ej} = \frac{\delta p}{1-p(1-\beta)}\frac{1-(d\gamma)^j}{1-d\gamma}\left[\Pi_{E+1}(1) + (1-\beta)U_{ET}\right] + \frac{1-(d\gamma)^j}{1-d\gamma}d(z+b) + (d\gamma)^j U_{E0} + \frac{1-\gamma}{\gamma}\sum_{k=1}^{j}(d\gamma)^k U_{E-1j-k}.$$

For U_{E0}, we get from (A4), (A2), the sharing rule (A1) and the wage Equation (12):

(S47) $$U_{E0} = \frac{\delta\pi}{(1-\rho\gamma)+\beta\pi}\left[\Pi_{E+1}(1) + (1-\beta)U_{ET}\right] + \frac{\rho}{(1-\rho\gamma)+\beta\pi}\left[z + (1-\gamma)U_{E-10}\right].$$

Insert (S47) and (S44) into (S47), to obtain the following equation for $j = T-1$:

(S48) $$\gamma U_{ET-1} + (1-\gamma)U_{E-1T-1} = \frac{\gamma\delta\pi}{1-\rho\gamma}\Pi_{E+1}(1) + \frac{\gamma\rho}{1-\rho\gamma}z + \frac{\rho\gamma-(d\gamma)^T}{1-\rho\gamma}b + \frac{1-\gamma}{1-\rho\gamma}\left[(d\gamma)^T U_{E-10} + \frac{(1-p)(1-\rho\gamma)+\beta p}{1-p(1-\beta)}\sum_{k=0}^{T-1}(d\gamma)^k U_{E-1T-(1+k)}\right].$$

Insert (S48) into (S44) and the statement follows by virtue of (S35) and (S36).

Ad 6. From (S44) and (S44) it follows that

$$U_{ET} - U_{Ej} = \delta\left[\pi - \frac{p}{1-p(1-\beta)}\right]\Pi_{E+1}(1) + (\rho - d)(z+b) + \rho\left[\gamma U_{ET-1} + (1-\gamma)U_{E-1T-1}\right] - \frac{\beta p}{1-p(1-\beta)}U_{ET} - d\left[\gamma U_{Ej-1} + (1-\gamma)U_{E-1j-1}\right].$$

Inserting (S44) on the right hand side of the above equation and rearranging terms yields

(S49) $$U_{ET} - U_{Ej} = d\gamma[U_{ET-1} - U_{Ej-1}] + d(1-\gamma)[U_{E-1T-1} - U_{E-1j-1}].$$

From (S36) it follows that $U_{E-1T-1} - U_{E-1j-1} = b\sum_{k=j}^{T-1} d^k$, so that

$$U_{ET} - U_{Ej} = d\gamma[U_{ET-1} - U_{Ej-1}] + b(1-\gamma)\sum_{k=j+1}^{T} d^k.$$

Solving this difference equation gives:

(S50) $$U_{ET} - U_{Ej} = (d\gamma)^j\left[U_{ET-j} - U_{E0}\right] + b\left(1-\gamma^j\right)\sum_{k=j+1}^{T} d^k.$$

From (A4), (A3), the wage equation (12), (S36) and $U_{E1} - U_{E0} = db$ it follows that $U_{ET-j} - U_{E0} = b\sum_{k=1}^{T-j} d^k$. Inserting this expression into (S50) and rearranging terms gives (S38).