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Conference Paper
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Beiträge zur Jahrestagung des Vereins für Socialpolitik 2010: Ökonomie der Familie - Session: Financial Integration and Investment Decisions, No. D9-V1

Provided in Cooperation with:
Verein für Socialpolitik / German Economic Association

Suggested Citation: Heinemann, Maik; Clemens, Christiane (2010) : The Effects of International Financial Integration in a Model with Heterogeneous Firms and Credit Frictions, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2010: Ökonomie der Familie - Session: Financial Integration and Investment Decisions, No. D9-V1, Verein für Socialpolitik, Frankfurt a. M.

This Version is available at:
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The Effects of International Financial Integration in a Model with Heterogeneous Firms and Credit Frictions†

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February 20, 2010

Abstract

This paper examines the pattern of international capital flows in a two–sector dynamic general equilibrium heterogeneous agent model with financial frictions and idiosyncratic entrepreneurial risk. Countries differ only with respect to the tightness of constraints on the domestic credit market. The results provide an explanation for the ’Lucas paradox’, i.e. the empirical observation of capital flowing from poor to rich countries, where lending countries are characterized by tighter domestic constraints. International integration only indirectly mitigates negative output and welfare effects from financial constraints on domestic credit markets. The effects are triggered by adjustments in the real interest rate to global real return. We observe an accumulation–driven rise in the entrepreneurship rate and positive output effects for countries with relatively tight constraints who generally benefit from financial integration. The macroeconomic effects can be adverse for the capital–importing country which may suffer from a decrease in GNP in the integrated economy. The model is calibrated to match standard macro data, entrepreneurship rates, and Gini coefficients from OECD countries.

Keywords: DSGE, financial constraints, financial market integration, international capital flows, heterogeneous agents, occupational choice

JEL classification: C68, D3, D8, D9, F4, G0, J24

†The authors would like to thank Michael Woodford, Vincenzo Quadrini, Michael Burda, and Michael Funke for helpful comments on earlier versions of this paper. We also thank seminar participants at ETH Zurich, ZEW Mannheim, the 2009/10 meetings of the the Econometric Society in Barcelona and Atlanta and the 2009 meeting of the Canadian Economics Association for their discussion.

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1 Introduction

For the past twenty years the United States, the United Kingdom, and Australia have experienced a sustained rise in their current account deficits, while many emerging economies from East Asia, and in particular China throughout recent years, have either drastically reduced their deficit or even built up substantial surpluses (see Figure 1). Prasad et al. (2006) find perverse patterns of capital flows in the 2000s from poor to rich countries. The empirically observed pattern of international capital flows contradicts standard neoclassical theory, which predicts that capital should flow from more to less developed countries. A growing body of literature explains this empirical evidence, conveniently dubbed as ‘Lucas paradox’ (referring to Lucas, 1990), with the presence of borrowing constraints in poorer countries; see e.g. Gertler and Rogoff (1990), Boyd and Smith (1997), and Matsuyama (2004), or more recently Caballero et al. (2008), Mendoza et al. (2009), and Sandri (2009).

Our paper contributes to this strand of research. We present a two–sector neoclassical heterogeneous–agent dynamic general equilibrium model of occupational choice which allows us to relate the pattern of international capital movements to cross–country differences in financial market imperfections. We find that countries may as well as not benefit from financial market integration, the outcome depending on the tightness of domestic constraints and the resulting capital return of a small economy under autarky relative to the world real interest rate. As well–known from the literature, countries facing tighter credit constraints have lower domestic interest rates and generally become net–exporters of capital if capital markets open for international borrowing and lending. In our model economy, international capital flows only indirectly mitigate negative output effects from financial constraints on domestic credit markets. Lending countries benefit from financial integration in terms of higher per capita incomes. Countries with less restrictive credit supply may experience a long–run decrease in per capita GNP in our model which is due to foreign interest payments.

The model economy consists of two sectors of production: an unconstrained corporate sector and a non–corporate ‘small business’ sector, where firm owners are subject to financial constraints. This is consistent with previous work by Bernanke et al. (1998) suggesting that smaller firms face tighter constraints than large firms. We assume that financial constraints in our model arise from limited commitment of business owners and not from lack of financial intermediaries or under–developed financial markets. It is important to stress that financial integration in our model neither helps to alleviate agency problems nor directly facilitates small firms’ access to external funding. The international capital flows of our model explicitly originate from cross–country differences in the tightness of financial constraints. The latter are persistent, because the primary source of credit market imperfection is limited commitment of borrowers. In this context, we do not think it convincing to assume that foreign lenders and financial intermediaries are less risk averse or more effective in monitoring and law enforcement than their domestic counterparts.
Credit market frictions are a threefold impediment to economic activity in our model economy: Firstly, agents are restrained from smoothing their intertemporal consumption path by borrowing and lending. Common to Huggett (1993)–Aiyagari (1994)–type economies, the agents then undertake buffer–stock savings to self–insure on intertemporal markets against the non–diversifiable income risk.\footnote{Our analysis is also embedded in literature on dynamic general equilibrium models with idiosyncratic risks and borrowing constraints which stress the importance of buffer–stock saving and entrepreneurial activity for the explanation of the empirically observed accumulation patterns; see e.g. Quadrini (2000), Meh (2005), Boháček (2006, 2007), Cagetti and De Nardi (2006a,b), Clemens and Heinemann (2010).} Because a business owner’s wealth also serves as collateral for external financing, individual accumulation is important for occupational choice. Secondly, credit–constrained business owners are not able to operate at their profit–maximizing firm size, which reduces industry output and subsequently aggregate income. That credit frictions restrain entrepreneurship is broadly acknowledged throughout the theoretical and empirical literature; see e.g. Evans and Leighton (1989), Evans and Jovanovic (1989), Holtz–Eakin \textit{et al.} (1994b), and Gentry and Hubbard (2004). Thirdly, industry–specific financial constraints lead to an inefficient allocation of capital across sectors. The share of capital employed in the unconstrained sector is too large compared to a frictionless economy. Consequently, with diminishing returns to capital, the equilibrium real interest rate of a closed economy facing tighter constraints is more likely
to be lower than the world rate of return (cf. Caballero et al., 2008). Going from autarky to financial openness, the real interest rates rises to the global equilibrium rate and the economy becomes a net-lender. Capital is reallocated towards those foreign countries who face less tight constraints and originally higher domestic rates of return.

Comparing steady states of the closed vis-à-vis the financially integrated (albeit constrained) economy, we find that rising interest rates trigger wealth accumulation. Household wealth serves as collateral for external funding of non-corporate enterprises. Wealthier households are more likely to be members of the entrepreneurial class than poorer ones, which is consistent with recent empirical findings (Quadrini, 1999; Holtz-Eakin et al., 1994a). The increase in the interest rate partially relieves business owners from financing constraints and stimulates economic activity, even though the fundamental market conditions remain unchanged. We observe an accumulation-driven rise in the entrepreneurship rate, an associated increase in sectoral production which in turn positively affects aggregate output and national income. From this, we conclude that financial integration already mitigates adverse effects from imperfect capital markets without necessarily having to improve the functioning of credit markets.

Our analysis extends the results of Mendoza et al. (2009), who explain the ‘up-stream’ pattern of international capital flows primarily with excess saving due to the presence of borrowing constraints and idiosyncratic risk in the household sector in a Huggett/Aiyagari-type economy. Their work does not take account of financing constraints in production, which is an empirically important issue. Our model is devised to fill this gap in the literature.

We develop a model economy which is closely related to modern growth theory and draw on previous work (Clemens, 2006a,b, 2008; Clemens and Heinemann, 2006, 2010). We combine occupational choice under risk à la Kihlstrom and Laffont (1979) and Kanbur (1979) with the two-sector approach of Romer (1990), but without endogenous growth. In each period of time, the risk-averse agents choose between two alternative occupations. They either set up an enterprise in the intermediate goods industry or supply their labor endowment to the production of a final good in a perfectly competitive market. Producers of the final good use capital and labor inputs, and differentiated varieties of the intermediate good. All households are subject to an idiosyncratic income risk which cannot be pooled. Managerial ability and productivity as a worker follow independent random processes. Entrepreneurial activity is rewarded with a higher expected income. There is no aggregate risk.

An entrepreneur employs capital from own and borrowed resources. Due to the two-sector general equilibrium nature of our model, the optimal business size and the demand for credit are endogenously determined. The firm owner seeks credit if his individual wealth is too small to operate his business at the profit-maximizing firm size and faces financial constraints on the market for credit.

The model is broadly consistent with macro data from industrialized countries. Naturally, the model cannot draw a realistic picture of the economy over the entire domain of financial constraints under consideration. For this reason, we define a
benchmark economy with an empirically plausible debt equity ratio, and calibrate the model to match macroeconomic key variables, for instance, for the U.S. and other OECD members.

We find substantial gains of financial market integration for our benchmark (capital–exporting) economy, on average amounting to a 2.5–3% gain in national income per p.p. increase in the real interest rate, and observe likewise improvements in average wealth holdings and welfare. Interestingly, if there are little constraints on entrepreneurial activity, the economy actually might be negatively affected by financial integration. A country with only little or no constraints becomes a net–debtor and might suffer from an associated decrease in GNP, although GDP increases. Here, the final goods sector of production—which relies more heavily on external financing—experiences a bigger increase in capital input than the entrepreneurial sector, which is in accordance with empirical evidence by Rajan and Zingales (1998).

Our paper does not address the role of intermediaries for the growth process, total factor productivity, and the relationship between financial market integration and development; cf. Greenwood and Jovanovic (1990), Bencivenga and Smith (1993), King and Levine (1993a,b), Boyd and Smith (1997), Gourinchas and Jeanne (2006, 2007), Aoki et al. (2009), Buera and Shi (2009). The general insight from this body of literature is that capital account liberalization speeds up the process of convergence and promotes growth.

The paper is organized as follows: Section 2 develops the two–sector model. We describe the closed–economy equilibrium associated with a stationary earnings and wealth distribution. Since the formal structure of the model does not allow for analytical solutions, we perform numerical simulations of a calibrated model in order to examine the general equilibrium effects of financial market integration. Section 3 gives information on the calibration procedure and related empirical evidence. Section 4 discusses the simulation results. Regarding credit availability, we compare three settings: the baseline model with a debt equity ratio of unity, a frictionless economy and the case of no–credit for entrepreneurial firms. Section 5 concludes. Technical details are relegated to the Appendix.

2 A Small–Country Model with Financial Constraints

2.1 Overview

We consider a neoclassical growth model with two sectors of production. Drawing from Quadrini (2000) and Romer (1990), we consider a corporate sector with perfectly competitive large firms who hire capital and labor services and use an intermediate good in order to produce a homogeneous output which can be consumed or invested respectively. The intermediate goods industry (non–corporate sector) consists of a large number of small firms operating under the regime of monopolistic competition. Each firm in this sector is owned and managed by an entrepreneur. Both sectors of production are essential. Firms of the two sectors differ with respect to credit–availability, which is assumed to be unconstrained for members of the final goods industry, whereas the monopolistic entrepreneurs face financial con-
Entrepreneurs of the intermediate goods industry seek external financing on the credit–market if they lack sufficient own funds to run their business at the profit–maximizing firm size.

Because our focus lies on the effects of financial market integration, we assume labor to be immobile and output goods to be nontradeable. Hence, all trade between economies is financial.\(^2\) The capital market is perfectly competitive.

The economy is populated by a continuum \([0, 1]\) of infinitely–lived households, each endowed with one unit of labor. In each period of time, individuals follow their occupation predetermined from the previous period and make a decision regarding their future profession, which is either to become producers of the intermediate good or to supply their labor services to the production of the final good. Labor efficiency as well as entrepreneurial productivity are idiosyncratic random variables. Regarding the associated income risk, we assume that wage incomes are less risky than profit incomes. There is no aggregate risk.

With respect to the timing of events, we assume that individual occupational choice takes place before the resolution of uncertainty. Once the draw of nature has occurred, entrepreneurs as well as workers in the final goods sector know their individual productivity. Those monopolists, who now discover their own wealth being too low to operate at the optimal firms size, will express their capital demand on the credit market, probably become subject to credit–constraints, and then start production. After labor and profit income is realized, the households decide on how much to consume and to invest. There is no capital income risk and no risk of production in the corporate sector.

### 2.2 Final Goods Sector

The representative firm of the final goods sector produces a homogeneous good \(Y\) using capital \(K_F\), labor \(L\), and varieties of an intermediate good \(x(i), i \in [0, \lambda]\) as inputs. Production in this sector takes place under perfect competition and the price of \(Y\) is normalized to unity. The production function is of the generalized CES–form\(^3\)

\[
Y = (K_F^\gamma L^{1-\gamma})^{1-\alpha} \int_0^\lambda x(i)^\alpha \, di, \quad 0 < \alpha < 1, \quad 0 < \gamma < 1.
\]

Each type of intermediate good employed in the production of the final good is identified with one monopolistic producer in the intermediate goods sector. Consequently, the number of different types is identical with the population share \(\lambda\) of entrepreneurs in the population. The number of entrepreneurs is determined endogenously through occupational choices of the agents, which will be described below. Additive–separability of (1) in intermediate goods ensures that the marginal product of input \(i\) is independent of the quantity employed of \(i' \neq i\). Intermediate goods are close but not perfect substitutes in production.

\(^2\)Wynne (2005), Ju and Wei (2006), Manova (2008a,b), Antrás and Caballero (2009) discuss the relevance of credit constraints for international trade patterns, an issue which is beyond the scope of the present paper.

\(^3\)All macroeconomic variables are time–dependent. For notational convenience, we will drop the explicit time–notation unless necessary. If needed, the \(i\) symbol denotes next period variables.
The profit of the representative firm in the final goods sector, \( \pi_F \), is given in each period by

\[
\pi_F = Y - wL - (r + \delta)K_F - \int_0^\lambda p(i)x(i) \, di ,
\]

where \( p(i) \) denotes the price of intermediate good \( i \). We further assume physical capital to depreciate over time at the constant rate \( \delta \), such that the interest factor is given by \( R = 1 + r - \delta \). Optimization yields the profit maximizing factor demands consistent with marginal productivity theory

\[
K_F = (1 - \alpha) \gamma \frac{Y}{r + \delta} ,
\]

\[
L = (1 - \alpha)(1 - \gamma) \frac{Y}{w} ,
\]

\[
x(i) = K_i^\gamma L^{1 - \gamma} \left( \frac{\alpha}{p(i)} \right)^{\frac{1}{1 - \alpha}} .
\]

The monopolistic producer of intermediate good \( x(i) \) faces the isoelastic demand function \((5)\), where the direct price elasticity of demand is given by \(-1/(1 - \alpha)\). Condition \((4)\) describes aggregate labor demand in efficiency units. Equation \((3)\) is the final good sector demand for capital services.

### 2.3 Intermediate Goods Sector

The intermediate goods sector consists of the population fraction \( \lambda \) of entrepreneurs who self-employ their labor endowment by operating a monopolistic firm. Each monopolist produces a single variety \( i \) of the differentiated intermediate good by employing capital from own wealth and borrowed resources according to the identical constant returns to scale technology of the form

\[
x(i) = \theta(i)^e k(i) .
\]

Firm owners are heterogeneous in terms of their talent as entrepreneurs. They differ with respect to the realization of an idiosyncratic productivity shock \( \theta(i)^e \) which is assumed to be non-diversifiable and uncorrelated across firms. The producer maximizes his profit

\[
\pi(k(i), \theta(i)^e) = p(i)x(i) - (r + \delta)k(i) .
\]

Utilizing the demand function for intermediate good type–\( i \), \((5)\), and the production technology \((6)\), the optimal firm decision can be expressed in terms of the optimal firm size \( k(i)^* \), given by:

\[
k(i)^* = L \theta(i)^e \gamma \left( \frac{\gamma w}{1 - \gamma (r + \delta)} \right)^\gamma \left( \frac{\alpha^2}{r + \delta} \right)^{\frac{1}{1 - \alpha}} .
\]

Because capital demand takes place after the draw of nature has occurred, there is no individual capital risk and no under-employment of input factors. The optimal firm size increases with random individual productivity \( \theta(i)^e \), such that more productive business owners express a larger demand for capital.
2.4 Capital Market and Financial Constraints

Firms of the two sectors of production differ with respect to access to external financing. Those monopolistic firm owners, who are wealth-constrained in operating their business at the optimal size \((8)\), would want to borrow from financial intermediaries. The credit market is imperfect with respect to lenders not being able to enforce loan–repayment due to limited commitment of borrowers (cf. Banerjee and Newman, 1993). In order not to default on loan contracts, borrowing amounts are limited, and individual wealth acts as collateral. We do not explicitly model financial intermediaries and assume that there is no difference between borrowing and lending rates.

In case of default, the financial intermediary is able to seize a fraction of the borrowers gross capital income \((1 + r) a(i)\). It will lend only the amount consistent with the borrower's incentive–compatibility constraint, such that it is in the borrower's interest to repay the loan, and there is no credit default in equilibrium. Let \(k(i) = a(i) + b(i)\) be the firm size an entrepreneur is able to operate at from own wealth \(a(i)\) and borrowed resources \(b(i)\). An entrepreneur with individual wealth \(a(i)\) lower than \(k(i)^*\) seeks external financing \(k(i)^* - a(i)\). In case of \(k(i) < k(i)^*\) the firm faces a borrowing constraint. Incentive–compatibility requires a self–enforcing contract. It is never optimal for the borrower to default, if

\[
\pi(i) + (1 + r) a(i) \geq \pi(i) + b(i) (1 + r) + (1 - \phi) (1 + r) a(i)
\]

which boils down to

\[
b(i) \leq \phi a(i).
\]

The borrowing amount is limited such that the maximum possible loan is proportional to the borrowers individual wealth \(a(i)\). The parameter \(\phi\) can be viewed as a credit multiplier and is a measure for the extent to which a lender can use the borrower’s wealth income as collateral. Credit constraints become less tight with rising \(\phi\) and vanish for large \(\phi\). The limiting cases consequently reflect the two cases of either complete enforceability \(\phi \rightarrow \infty\) or no enforceability \(\phi = 0\).

Using the collateral constraint in the entrepreneurial budget constraint yields \(k(i) \leq (1 + \phi) a(i)\). The operating firm size \(k(i)\) of entrepreneur \(i\) with productivity \(\theta(i)\), and wealth \(a(i)\) can then be written as:

\[
k(\theta(i), a(i)) = \min \left[k(i)^*, (1 + \phi) a(i)\right].
\]

Workers’ savings together with the fraction of entrepreneurial wealth, which is not employed in the production of the intermediate good, are supplied to the (domestic or international) capital market at the equilibrium interest rate. Capital demand of the closed economy consists of credit–constrained entrepreneurs and firms from the final good industry.

2.5 Idiosyncratic Risks

In each period of time, workers are endowed with one unit of raw labor and are subject to an idiosyncratic shock \(\theta_w\) affecting labor supply in efficiency units, and
exposing each of them to an uninsurable income risk. We assume that labor productivity $\theta_w$ evolves according to a first–order Markov process with $h = 1, \ldots, H$ states, and $\theta_{w,h} > 0$. The transition matrix associated with the Markov process is $P_w$.

Entrepreneurial productivity $\theta_e$ also evolves according to a first–order Markov process with $h = 1, \ldots, H$ different states $\theta_{e,1}, \ldots, \theta_{e,H}$; $\theta_{e,h} > 0$, and transition probability $P_e$. Since agents can be either workers or entrepreneurs, it is possible to identify the occupational status of an agent with his productivity in the respective occupation. We assume worker productivities to be more evenly distributed than managerial skills, such that profit incomes in general are more risky than wage incomes. We assume the processes $\theta_w$ and $\theta_e$ to be uncorrelated, such that for an individual the conditional expectation of entrepreneurial productivity is independent of the labor efficiency, if employed as a worker. A presently high productivity as a worker does not necessarily indicate an equivalently high future productivity as an entrepreneur. The associated probabilities are summarized in a $H \times H$ transition matrices $P_j, j'$ describing the transition from productivity state $\theta_{j,h}$ to state $\theta_{j',h'}$ for $h, h' = 1, \ldots, H, j = e, w$ and $j \neq j'$.

2.6 Intertemporal Decision and Occupational Choice

Each household $i$ has preferences over consumption and maximizes discounted expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U[c_t(i)] \quad 0 < \beta < 1.$$  

$E_0$ is the expectation operator conditional on information at date 0 and $\beta$ is the discount factor. Individuals are identical with respect to their preferences regarding momentary consumption $c(i)$ which are described by constant relative risk aversion

$$U[c(i)] = \frac{c(i)^{1-\rho}}{1-\rho} \quad \text{for } \rho > 0, \rho \neq 1$$

and $\ln c(i)$ for $\rho = 1$, where $\rho$ denotes the Arrow/Pratt index of relative risk aversion.

The single household also makes a decision on his future occupation in each period, which either is to be business owner or to inelastically supply his labor services in efficiency units to the production of the final good. Occupational choice, once made, is irreversible in the same period.

Let $V^w(a(i), \theta(i)_w)$ denote the optimal value function of an agent currently being a worker with wealth $a$, who is in a given productivity state $\theta_w$. If he decides to remain a worker, his productivity evolves according to the transition matrix $P_w$ of the underlying Markov process. If he becomes an entrepreneur in the following period, he gets a new draw $\theta'_e$ from the invariant distribution of entrepreneurial productivities. The next period productivity is determined by the transition matrix $P_{w,e}$.

There are no markets for pooling idiosyncratic risks. There is limited scope to which agents are able to smooth their intertemporal consumption flow by borrowing and lending. The standard approach of the literature is to assume that individual asset holdings are bounded from below. In what follows, we assume a lowest possible wealth level of $a = 0$. 

9
The maximized value function for a typical individual currently being a worker is given by

\[
V^w(i, a(i)_w) = \max_{c(i) \geq 0, a(i)_w'} \left\{ U[c(i)] + \beta \max_{q(i') \in \{0, 1\}} (1 - q(i')) E \left[ V^w(a(i')', \theta(i')_w) | \theta(i)_w \right] + q(i') E \left[ V^c(a(i')', \theta(i')_c) \right] \right\}
\]

s.t. \( a(i)' = (1 + r) a(i) + \theta(i)_w - c(i) \).

(11)

\( q \) is a boolean variable which takes on the values 0 or 1, depending on whether or not the agent decides to switch between occupations. \( r \) and \( w \) denote the equilibrium returns to capital and labor in efficiency units, which are constant over time for a stationary distribution of wealth and occupational statuses over agents. The optimal decision associated with the problem (11) is described by the two decision rules for individual asset holdings \( a(i)'_w = A_w(a(i), \theta(i)_w) \) and the future professional state \( q(i)'_w = Q_w(a(i), \theta(i)_w) \).

Let \( V^e(a(i), \theta(i)_e) \) denote the maximized value function of an entrepreneur with wealth \( a \) in productivity state \( \theta_e \), who faces a decision problem similar to those of a worker. If he decides to remain an entrepreneur, his productivity evolves according to the transition matrix \( P_e \) of the underlying Markov process. If, instead, he decides to switch between occupations by becoming a worker in the next period, his future productivity \( \theta'_w \) is determined by the transition matrix \( P_{e,w} \). With \( k(i)^* \) denoting the optimal firm size, the intertemporal problem of an entrepreneur can be written as

\[
V^e(a(i), \theta(i)_e) = \max_{c(i) \geq 0, a(i)'_e = a} \left\{ U[c(i)] + \beta \max_{q(i') \in \{0, 1\}} (1 - q(i')) E \left[ V^e(a(i')', \theta(i')_e | \theta(i)_e \right] + q(i') E \left[ V^w(a(i')', \theta(i')_w) \right] \right\}
\]

s.t. \( a(i)' = (1 + r) a(i) + \pi(k(i), \theta(i)_e) - c(i) \)

\( k(i) = \min[k(i)^*, (1 + \delta) a(i)] \)

\( \pi(\theta(i)_e, k(i)) = p(x(i)) x(\theta(i)_e, k(i)) - (r + \delta) k(i) \)

(12)

Again, \( q \) is a boolean variable, indicating the agent’s decision on leaving or remaining in his present occupation. The optimal decision is described by the decision rules for individual asset holdings \( a(i)'_e = A_e(a(i), \theta(i)_e) \) and the future professional state \( q(i)'_e = Q_e(a(i), \theta(i)_e) \).

In general, our model generates the same implications for individual savings and wealth accumulation under risk, as discussed in Huggett (1993) or Aiyagari (1994). Similar to Quadrini (2000), we additionally consider occupational choice. The higher entrepreneurial wealth, the less likely a business owner is credit-constrained for a given realization of the productivity shock.
2.7 Closed–Economy Stationary Recursive Equilibrium

A stationary recursive competitive general equilibrium is an allocation, where equilibrium prices generate a distribution of wealth and occupations over agents which is consistent with these prices given the exogenous process for the idiosyncratic shocks and the agents’ optimal decision rules.

We obtain aggregate labor supply by summing up individual labor supplies in efficiency units over the population fraction \(1 - \lambda\) of workers. The stationary recursive equilibrium is a set of value functions \(V_w(a, \theta_w), V_e(a, \theta_e)\), decision rules \(A_w(a, \theta_w), Q_w(a, \theta_w)\) and \(A_e(a, \theta_e), Q_e(a, \theta_e)\), prices \(w, r, p(i)\) and a distribution \(\lambda, 1 - \lambda\) of households over occupations such that:

(i) the decision rules \(A_w(a, \theta_w), Q_w(a, \theta_w)\) and \(A_e(a, \theta_e), Q_e(a, \theta_e)\) solve the workers’ and entrepreneurs’ problems (11) and (12) at prices \(w, r, p(i)\),

(ii) the aggregate demands of consumption, labor, capital and intermediate goods are the aggregation of individual demands. Factor and commodity markets clear at constant prices \(w, r, p(i)\), where factor inputs are paid according to their marginal product.

(iii) the stationary distribution \(\Gamma\) of agents over individual wealth holdings, occupations and associated productivities is the fixed point of the law of motion which is consistent with the individual decision rules and equilibrium prices. The distribution \(\lambda, 1 - \lambda\) of agents over occupations is time–invariant.

The decision rules for workers, \(A_w(a, \theta_w), Q_w(a, \theta_w)\), and entrepreneurs, \(A_e(a, \theta_e), Q_e(a, \theta_e)\), together with the stochastic processes for individual labor productivity and entrepreneurial productivity, determine the stationary distribution \(\Gamma\) at equilibrium prices \(w, r\). The stationary distribution \(\Gamma\) governs the entrepreneurship rate, the efficiency units of labor supplied by workers, capital demand of the intermediate goods sector, and the aggregate capital supply. Once the entrepreneurship rate \(\lambda\) is derived, this—together with the stationary distribution of entrepreneurial productivities—determines the supply of intermediate goods (for details, see Appendix A).

3 Calibration

In order to evaluate the macroeconomic effects of financial integration, our first step is to define a benchmark economy which matches standard macro data from OECD countries. We calibrate the model to replicate empirical observations regarding the functional and personal distribution of income and wealth, capital return, entrepreneurship rates, and social mobility. The benchmark value for the debt equity ratio is set to \(\phi = 1\), i.e., the maximum loan equals half the amount of operating capital (cf. Evans and Jovanovic, 1989; Gentry and Hubbard, 2004). Table 1 summarizes the parameterization of the model and our calibration targets. We find that it is sufficient to mimic unlimited access to external financing \((\phi \to \infty)\) in our simulations by choosing \(\phi = 1\).
Table 1: Calibration Values for the Baseline Model

<table>
<thead>
<tr>
<th>Calibrated parameter</th>
<th>Calibration target (approx.)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology</td>
<td>Interest rate $r$</td>
<td>2–4%</td>
</tr>
<tr>
<td></td>
<td>Source</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Obstfeld, 1994; King and Rebelo, 1999)</td>
<td></td>
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<tr>
<td>α γ δ</td>
<td>Factor income shares</td>
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<tr>
<td></td>
<td>Source</td>
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<tr>
<td></td>
<td>(King and Rebelo, 1999, PSID)</td>
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<td>Preferences</td>
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<td>ρ β</td>
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<td></td>
<td>Gini index of wealth</td>
<td>0.75–0.78</td>
</tr>
<tr>
<td></td>
<td>Source</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(PSID, SCE)</td>
<td></td>
</tr>
<tr>
<td>Shocks</td>
<td>Income persistence $p_w$</td>
<td>0.5–0.9</td>
</tr>
<tr>
<td>σw p_w σ_e p_e</td>
<td>Source</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Aiyagari, 1994; Guvenen, 2009, &amp; references therein)</td>
<td></td>
</tr>
<tr>
<td>Financial frictions</td>
<td>Entrepreneurship rate</td>
<td>15–25%</td>
</tr>
<tr>
<td>φ</td>
<td>Exit / entry rates</td>
<td>20–35%</td>
</tr>
<tr>
<td></td>
<td>Source</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(GEM, 2005)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Quadrini, 2000; Vale, 2006; Aghion et al., 2007)</td>
<td></td>
</tr>
</tbody>
</table>

$\phi = 1000$ as largest value, where virtually no entrepreneur is restrained. The resulting allocation is confronted with the baseline model and the no–credit case $\phi = 0$.\(^5\)

We adopt a broad notion of entrepreneurship. Following the notions of Schumpeter and Knight, we consider an entrepreneur as someone, who owns and operates a small business, and who is willing to take risks, to be innovative, and to exploit profit opportunities. Definitions of self–employment and entrepreneurial activity differ widely across countries. According to the OECD, self–employment encompasses “... those jobs, where the remuneration is directly dependent upon the profits derived from the goods and services produced. The incumbents make the operational decisions affecting the enterprise, or delegate such decisions while retaining responsibility for the welfare of the enterprise.” (OECD, 2000, Ch. 5, p. 191). Our model generates self–employment business ownership rates around 20%, which is somewhat more at the upper range of values for OECD countries (including owner–managers), matching countries like New Zealand (20.8%), Italy (24.8%), or Spain (18.3%); see also the annual Global Entrepreneurship Monitor (GEM 2005, Minniti et al.).

We set the discount factor $\beta$ and the coefficient of relative risk aversion $\rho$ according to estimates from the literature (cf. Obstfeld, 1994; King and Rebelo, 1999). The parameters of production technology, $\alpha$ and $\gamma$, are chosen such as to generate an equilibrium labor income share of 0.63 which matches empirical observations e.g. for the U.S. economy (King and Rebelo, 1999) or the average of EU 15. The corresponding capital and profit income shares of the unconstrained economy ($\phi \rightarrow \infty$) are 0.16

\(^5\)Recall that $\phi$ is a measure for the credit multiplier in the intermediate goods sector. Restrictions on household consumption, borrowing and lending do not vanish, if there is unlimited access to credit in this industry.
and 0.21. PSID data report a income share for entrepreneurs of around 22%. The depreciation rate is fixed at 6%, which also is a standard choice in the literature.

The steady state of the simulated benchmark economy replicates the Gini coefficient of wealth inequality for the U.S. (PSID, 1989) but also matches OECD countries like Sweden, France, and Switzerland.

To take account of empirically observed income persistence, we assume that the processes for labor efficiency $\theta_w$ and entrepreneurial productivity $\theta_e$ are lognormal with normalized mean $\ln \theta_w \sim \mathcal{N} \left( -\sigma_w^2 / 2, \sigma_w^2 \right)$, $\ln \theta_e \sim \mathcal{N} \left( -\sigma_e^2 / 2, \sigma_e^2 \right)$ and $AR(1)$ of the general form:

$$\ln \theta_j' = (p_j - 1) \frac{\sigma_j^2}{2} + p_j \ln \theta_j + \sigma_j \sqrt{1 - p_j^2} \varepsilon, \quad j = e, w \quad \text{and} \quad \varepsilon \sim \mathcal{N}(0, 1) \quad (13)$$

The labor income process is parameterized following Aiyagari (1994) and Guvenen (2009) with $p_w = 0.6$ and $\sigma_w = 0.2$. We assume higher serial correlation and dispersion for the entrepreneurial income process and set $p_e = 0.9$ and $\sigma_e = 1.8$ in order to generate empirically plausible exit/entry rates and wealth inequality.

Entry rates into entrepreneurship equal exit rates in the stationary recursive equilibrium. Our model is calibrated to generate exit rates of around 4% of the population ($\approx 20\%$ of intermediate industry members) which consistent with the evidence reported by Quadrini (2000) but higher than the rates documented by Evans (1987) for the U.S., and also in the upper range of empirically plausible values for OECD countries (cf. Vale, 2006; Aghion et al., 2007).

The income processes are approximated with a five-state Markov chain by using the method described in Tauchen (1986). The transition matrices for individuals switching occupations are derived from the stationary distributions of the Markov processes. The probability for a worker (entrepreneur) of ending up in a specific state of entrepreneurial (worker) productivity $\theta_{e,h}$ ($\theta_{w,h}$) is given by the stationary (unconditional) probabilities of this state. The algorithm for finding the equilibrium consists of three nested loops, starting from an initial guess on factor prices $w, r$ and employment $L$, then iterating until markets clear and the conditions of a stationary recursive equilibrium are met (see Appendix A).

4 Results

Before we examine the effects of international financial integration, we briefly describe the major implications of financial constraints for the macroeconomic equilibrium of the closed economy. Wealth accumulation, occupational choice and economic performance in the two sectors of production are interdependent due to the general equilibrium nature of our approach. Although the value of $\phi$ is fixed exogenously, the credit demand as well as the magnitude of rationing is determined endogenously and depends on firm-specific factors, such as optimal business size (8), individual wealth, factor prices and the ability shock.

Our analysis proceeds as follows: We first investigate to what extent our model is able to replicate empirical evidence on wealth distributions. We then examine
how the presence of credit market imperfections and changes in the tightness of constraints affects aggregate output, business sizes, and the real interest rate.

This provides the starting point for the analysis of the open economy. The macroeconomic effects following the opening of the economy to international capital flows are primarily triggered by adjustments of the domestic capital return to the world real interest rate. This affects wealth accumulation, the allocation of capital across sectors and subsequently occupational choice and output. We describe how the steady state values of the macroeconomic key variables respond to changes in the equilibrium real interest rate and confront the baseline model \( \phi = 1 \) with the frictionless economy \( \phi \to \infty \) and the no-credit case \( \phi = 0 \).

### 4.1 Distribution of Wealth and Business Size

Table 2 reports the percentiles and Gini coefficients for household wealth computed from the PSID and SCF and the associated values of our model economy for three different degrees of financial constraints, \( \phi \in \{0, 1, 1000\} \). Similar to related work by Quadrini (2000), Cagetti and De Nardi (2006a), and Boháček (2006) we observe that workers are more concentrated at lower wealth levels. There exists a significant mass of wealthy entrepreneurs but also a comparably large share of poorer ones. This matches empirical findings by Gentry and Hubbard (2004), and Hamilton (2000).

<table>
<thead>
<tr>
<th></th>
<th>Top percentiles (in %)</th>
<th>Gini</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
<td>5%</td>
</tr>
<tr>
<td>PSID 1994</td>
<td>22.6</td>
<td>44.8</td>
</tr>
<tr>
<td>SCF 1992</td>
<td>29.5</td>
<td>53.5</td>
</tr>
<tr>
<td>( \phi = 0 )</td>
<td>21.95</td>
<td>56.59</td>
</tr>
<tr>
<td>( \phi = 1.0 )</td>
<td>20.24</td>
<td>54.57</td>
</tr>
<tr>
<td>( \phi = 1000 )</td>
<td>20.16</td>
<td>56.16</td>
</tr>
</tbody>
</table>

Source: PSID and SCF data, Quadrini (2000, p. 6)

Our benchmark model economy \( \phi = 1 \) replicates the Gini coefficient for the U.S. economy and closely matches the wealth distribution in the top percentiles. As can be seen, the presence of financial constraints tends to increase the concentration of wealth at the top of the wealth distribution. Moreover, we observe an increase in overall wealth inequality, the Gini coefficient rising by 6.5 p.p.

Figure 2 shows the cumulative distribution of firm sizes in the intermediate goods sector for \( \phi \in \{0, 1, \infty\} \). Each entrepreneur is able to operate his business at the optimal firm size \( (8) \) in case of unconstrained credit markets \( \phi \to \infty \). Consequently, we observe a stepwise CDF, each step corresponding to the optimal firm size associated with one out of the five underlying possible productivity states \( \theta_{e,h} \).

Consider next the case \( \phi = 1 \). The first observation is that the optimal firm sizes rise slightly for each possible state of entrepreneurial talent \( \theta_{e,h} \). This increase in firm sizes can be ascribed to the factor price effect. Borrowing constraints prevent the
efficient allocation of capital among sectors such that too much capital is employed in the production of the final good. This is associated with a decline in the real interest rate, which in turn raises the optimal firm size in the intermediate sector for each state of productivity.

The second, major observation in the credit–constrained economy is that there exists a positive mass of entrepreneurs between each two subsequent steps of optimal firm sizes, and the distribution is more concentrated at smaller firm sizes. Constraints affect more entrepreneurs, who have to operate their enterprise at a suboptimally low scale. For $\phi = 0$, steps in the CDF almost vanish, which means that even more business owners are now subject to constraints. The optimal levels of firm sizes for the different states of productivity rise even further, due to the factor price effect.

4.2 Macroeconomic Effects of Changes in the Tightness of Constraints

Figure 3 shows the response of GDP (equaling GNP in a closed economy) and the real interest rate to a change in the tightness of financial constraints. The general picture reflects the outcome one would expect from credit market improvements. Aggregate output $Y$ increases if we relax borrowing constraints. The overall loss in output in a perfectly constrained compared to the unconstrained economy ($\phi \rightarrow \infty$) lies at about 21%. We also see from Figure 3 that the response of output to a change in $\phi$ is monotonous and concave. The marginal gains from improving credit markets are much higher for small values of $\phi$, especially in the range of debt equity ratios between $0 < \phi < 2$, which is the empirically plausible domain. This interval accounts for more than three–quarter of the overall output loss associated with financial constraints. For a more detailed discussion of the macroeconomic effects, see Clemens and Heinemann (2010).

Given the general equilibrium nature of the underlying model, one would expect adjustments in the real interest rate as a response to the reduction in the amount of external financing associated with tighter constraints. If there is only limited or no capital demand from the intermediate goods industry, we observe a capital–relocation effect between sectors. More capital is employed in the final goods industry. With
diminishing marginal returns and starting from the frictionless economy, the equilibrium interest rate $r$ declines by almost 350 basis points over the domain of $\phi$.

4.3 Macroeconomic Effects from Financial Integration

Figure 4 summarizes the macroeconomic effects of international financial integration for the small economy. We plot the steady state values of the macroeconomic key variables as functions of real capital return, which necessarily equals the world real interest rate in equilibrium. The letter A denotes the corresponding autarky levels for the three small-country settings under consideration, which differ with respect to the tightness of domestic constraints on external financing of firms: (a) unlimited access $\phi = 1000$, (b) no credit $\phi = 0$, and (c) limited access $\phi = 1$. The major difference to a fully-fledged multi-country model is that there the world interest rate is endogenously determined, while it is exogenous in our model.

The macroeconomic effects of capital market integration are entirely driven by the initial wedge between the autarky equilibrium and the world equilibrium interest rate. Figures 4a to 4c show how going from autarky to openness affects the equilibrium allocation of capital across sectors and the direction of international capital flows. The autarky position of a country with comparably strong domestic financial constraints can be described as follows: only a small fraction of the aggregate capital stock is employed in the non-corporate sector, the remaining part is used in the production of the final good, and the domestic equilibrium return to capital is low. Allowing for international capital movements, a new steady state is characterized by a higher interest rate and capital exports. On the contrary, a country with unlimited access to external business financing becomes a net-importer of capital in the stationary equilibrium, as the domestically high marginal productivity of capital adjusts to the lower world return.
Figure 4: Macroeconomic Effects of International Financial Integration
The constrained economy  The reduction in capital inputs in the completely constrained economy for a percentage point increase in the interest rate on average amounts to 8% in the production of the final good and 4% in the intermediate goods industry. The comparably small negative response of capital inputs in the non–corporate sector can be attributed to several (partly counter–acting) general equilibrium effects. On the one hand, the user–cost of capital increase with a rise in the equilibrium interest rate, which leads to an associated decrease in average entrepreneurial profits (–3.2%) and reduces the average firm size by 4% per p.p. change in \( r \); see Figures 4d and 4e. On the other hand, a higher interest rate triggers accumulation, as displayed in Figure 4k. Financial constraints affect especially households with little wealth. An increase in average wealth holdings by almost 15% per p.p. increase in the interest rate relaxes business owners’ dependency on external financing and exerts a positive impulse on occupational choice. There is an average rise in the entrepreneurship rate by 2.5% per p.p. change in \( r \). From this we conclude that international capital market liberalization primarily helps to overcome domestic financial constraints in an indirect manner. The observations are qualitatively identical but less pronounced for the case of \( \phi = 1 \).

Summarizing, we observe a larger number of (albeit) smaller firms. Capital inputs are reduced in all sectors of production for an economy, where firms are subject to comparably tight credit–constraints or do not have access to credit at all. The constraints.

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Figure 4: Macroeconomic Effects of International Financial Integration (cont’d)
next natural step would be to ask, how this affects aggregate output and household incomes.

We find that the increase in the entrepreneurship rate more than compensates for the reduction in average firm size, such that industry output in the intermediate goods sector increases. Intermediate goods are substituted for capital in the production of the final good, such that aggregate output (GDP) increases, too. The country is a net–lender and receives returns on foreign assets, such that gross national income on average increases by 3.5% per p.p. increase in $r$ for the no–credit economy, and by 3% for the case of $\phi = 1$. Figure 4g displays the results. Average worker incomes rise for two reasons. Firstly, a rise in the entrepreneurship rate reduces aggregate labor input in efficiency units. Accordingly, the wage rate increases; see Figure 4m. Secondly, the capital income share in household income rises with an increase in wealth holdings. This is also true for average entrepreneurial incomes, which increase, despite the fact that average profits decline; see Figures 4e and 4h. Average consumption follows the general pattern of household incomes, see Figure 4j. For the two constrained economies under consideration, the average gain in consumption per p.p. increase in the interest rate following international capital market integration, amounts to 2.5% in the no–credit economy and 1.85% for the case of $\phi = 1$.

Regarding wealth inequality, international financial integration also exerts an equalizing effect on the distribution of wealth across households in a credit constrained economy. The overall shift in the Gini coefficient for a change in the equilibrium real interest rate is strongest for the completely constrained and much less pronounced in the frictionless economy.

*The unconstrained economy* Interestingly our simulation results show that the frictionless economy might actually suffer from international financial integration. Figures 4g and 4j show that the domestic equilibrium interest rate is comparatively high and bound for a decline in the world equilibrium. Capital imports are accompanied by building up foreign liabilities and an outbound flow of capital incomes. Domestic capital is also substituted with foreign capital, which becomes obvious in the reduction of average domestic wealth holdings displayed in Figure 4k. This reduces household incomes. The average income of workers declines despite an increase in the wage rate for moderate decreases in capital return. The same is true for entrepreneurial incomes which also decline, although average profits and firm sizes are rising. Here the negative wealth effect dominates, and only for a sharp drop in the interest rate the positive effects of additional employment of (foreign) capital in production and the associated increase in aggregate output prevail.

## 5 Concluding Remarks

In this paper, we examined financial constraints and idiosyncratic risks in a two–sector heterogeneous agent dynamic general equilibrium model of occupational choice. We discuss the macroeconomic and distributional effects of financial market integration for small economies which differ only with respect to the tightness of constraints on
the domestic credit market. Workers and firm owners are subject to idiosyncratic shocks. Entrepreneurship in the intermediate (non–corporate) goods industry is the riskier occupation. The stationary wealth distribution generated in the model is consistent with empirical findings.

As a general result, we find that tighter financial constraints on the domestic level cause substantial losses in aggregate output and lower the equilibrium capital return of the closed economy. The latter effect can mainly be attributed to the inefficient allocation of capital across sectors.

The general equilibrium nature of our model, where optimal firm sizes and the demand for credit are determined endogenously, gives rise to interesting implications regarding the steady state effects of international financial integration. We find that international capital flows only indirectly mitigate negative output and welfare effects from borrowing constraints on domestic capital markets. The effects are triggered by adjustments in the real interest rate, and the magnitude depends on the initial wedge between the autarky equilibrium capital return and the world interest rate.

In our model, countries which are characterized by tighter domestic constraints will ultimately export capital. This stands in contrast to the widely acknowledged view in the literature that capital account liberalization relaxes financial constraints and causes capital inflows, but can be reconciled with empirical evidence for some developing countries. To this extent our model contributes to the line of research by Gertler and Rogoff (1990), Boyd and Smith (1997), Matsuyama (2004), or more recently Caballero et al. (2008) and Mendoza et al. (2009). Countries with serious credit market frictions will benefit from a positive wealth effect of international financial integration. A higher world return to capital reinforces accumulation, thereby indirectly weakening the negative impact financial constraints have as a barrier to entrepreneurship. As a consequence the entrepreneurship rate of the economy rises. Ultimately this raises output and welfare, and reduces wealth inequality in the economy.

The macroeconomic effects can be adverse for the capital–importing country, which is characterized by less tight constraints and may suffer from a decrease in GNP in the internationally integrated economy.

There are many important issues this paper does not address. In its present form there is no trade in output goods, and all trade is financial. Taking account of international trade will be the next step of our research. We discuss steady state effects for a small open economy, where the world interest rate is exogenously given. In order to assess the welfare effects of financial integration it is important to take account of transitory adjustments towards the steady state. It is a worthwhile extension to consider a multi–country model, where the world interest rate is determined endogenously in equilibrium. This allows us discuss the magnitude of macroeconomic effects from financial integration in more detail. Last, the paper explicitly assumes that capital market liberalization does not affect the tightness of domestic constraints, an assumption which could also be relaxed in future research.
References


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A   Computational Issues

The state space of wealth is approximated by a grid of $N$ wealth levels $a_n$ for $n = 1, \ldots, N$ with $a_1 = 0$ and $a_N = \bar{k}$. The macroeconomic equilibrium is recursively computed. In the closed economy, we start with an initial guess on factor prices $\tilde{w}, \tilde{r}, \tilde{L}$, and the equilibrium level of employment in efficiency units $\tilde{L}$. Let $\mu = \{\tilde{w}, \tilde{r}, \tilde{L}\}$ denote the vector of the initial guesses. From this first solution trial we obtain factor proportions in the final goods sector according to the marginal productivity conditions. The underlying production technology implies $K_F = L (w + \gamma r) \frac{1}{\gamma}$. Moreover, $(\tilde{K}_F, L^{1-\gamma})^{1-\alpha}$ equals $L (\tilde{w} + \frac{\tilde{r}}{\gamma})^{1-\gamma}$. In the open-economy case, the interest rate is exogenously fixed.

Let $k_{n,h} (\mu)$ denote the firm size an entrepreneur with productivity $\theta_{c,h}$ and wealth $a_n$ is able to operate at for a given extent of borrowing constraints and the initial guess $\mu$. His profit is given by

$$\pi_{n,h} (\mu) = \alpha (\theta_{c,h} k_{n,h} (\mu))^{1/\sigma} L \left( \frac{\tilde{w}}{\tilde{r} + \tilde{δ} (1-\gamma)} \right)^{\gamma} - (\tilde{r} + \tilde{δ}) k_{n,h} (\mu).$$

Let $A^w_{n,h} (\mu)$ and $Q^w_{n,h} (\mu)$ as well as $A^e_{n,h} (\mu)$ and $Q^e_{n,h} (\mu)$ for $n = 1, \ldots, N$ and $h = 1, \ldots, H$ denote the optimal policies associated with the discrete formulation of the optimization problems (11) and (12) for the given initial guess $\mu$ on prices and employment. We characterize agents by their wealth holdings $a_n$, their occupational status $\zeta$, where $\zeta \in \{w, e\}$ and their current productivity state $\theta_h$, $h = 1, \ldots, H$.

Knowing the policy functions and transition matrices for the underlying productivity shocks, we are able to compute a stationary distribution and the probability for an agent to have wealth $a_n$, occupational status $\zeta$ and productivity state $\theta_h$. Let $\Psi_{n,\zeta,h} (\mu)$ denote the respective probability for $n = 1, \ldots, N$, $\zeta \in \{w, e\}$ and $h = 1, \ldots, H$ given the initial guess $\mu$. These probabilities can now be used to compute aggregate quantities. The aggregate capital stock (i.e. mean wealth holdings) can be determined as:

$$K(\mu) = \sum_{n=1}^{N} \sum_{\zeta \in \{w, e\}} \sum_{h=1}^{H} \Psi_{n,\zeta,h} (\mu) a_n$$

The population share of entrepreneurs results as

$$\lambda(\mu) = \sum_{n=1}^{N} \sum_{h=1}^{H} \Psi_{n,c,h} (\mu)$$

while labor supply in efficiency units is given by

$$L(\mu) = \sum_{n=1}^{N} \sum_{h=1}^{M} \Psi_{n,w,h} (\mu) \theta_{w,h}$$

Capital demand of the intermediate goods sector can be computed as:

$$K^I_1 (\mu) = \sum_{n=1}^{N} \sum_{h=1}^{H} \Psi_{n,c,h} (\mu) k_{n,h} (\mu)$$

The supply of capital to the final goods sector is thus given by $K_F^\tilde{L} (\mu) = K(\mu) - K^I_1 (\mu)$. Finally, the initial guess $\mu$ together with the production decision of entrepreneurs generates an
aggregate output of

\[ Y(\mu) = (\bar{K}_F^T \bar{L}^{1-\gamma})^{1-\alpha} \sum_{n=1}^{N} \sum_{j=1}^{H} \Psi_{n,\epsilon,\theta} \theta_{\epsilon,\theta} (\theta_{\epsilon,\theta})^\alpha \]

The initial solution guess represents an equilibrium only if the following conditions hold:

(i) Labor supply in efficiency units must equal the initial guess, i.e.

\[ L(\mu) = \bar{L} \]  \hspace{1cm} (A.1)

(ii) Labor demand in the final goods sector equals labor supply:

\[ L(\mu) = (1 - \alpha)(1 - \gamma) \frac{Y(\mu)}{\bar{w}} \]  \hspace{1cm} (A.2)

(iii) Capital demand in the final goods sector equals supply of capital to the final goods sector:

\[ K_S^F(\mu) = K(\mu) - K_D^I(\mu) = (1 - \alpha)\gamma \frac{Y(\mu)}{\bar{r} + \delta} \]  \hspace{1cm} (A.3)

The algorithm for finding the equilibrium values consists of three nested loops over \( \bar{L}, \bar{w} \) and \( \bar{r} \). The first loop iteratively computes the value \( \bar{L} \) which meets condition (A.1) for given factor prices \( \bar{w} \) and \( \bar{r} \). Then, factor prices \( \bar{w} \) and \( \bar{r} \) are adjusted according to the resulting excess demands for labor and capital according to conditions (A.2) and (A.3). The whole procedure is repeated until the equilibrium conditions (A.1) to (A.3) are satisfied, except for a tolerably small approximation error.

To implement the algorithm, we used the programming language C++. The underlying source code and the data are available from the authors upon request.