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Off-the-Record Target Zones: Theory with an Application to Hong Kong's Currency Board

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Abstract

This paper provides a modelling framework for evaluating the exchange rate dynamics in target zone regime with undisclosed bands. We generalize the literature to allow for asymmetric one-sided regimes. Market participants' beliefs about the undisclosed band change as they learn about central bank intervention policy. We apply the model to Hong Kong's one-sided currency board mechanism. In autumn 2003, the Hong Kong dollar appreciated from close to 7.80 per US dollar to 7.70, as investors feared that the currency board would be abandoned. In the wake of this appreciation, the monetary authorities finally revamped the regime into a symmetric two-sided system with a narrow exchange rate band.

Keywords: Currency Board Arrangement, Target Zone Model, Hong Kong

JEL-Classification: C61, E42, F31, F32

1 Introduction

Over the last years there has been a resurgence of interest in exchange rate regimes for emerging economies. In the aftermath of the Asian crisis in 1997-98, crisis prevention was viewed as a key criterion for choosing exchange rate regimes. Much attention was focused on the "hardness degree" of various peg systems and the standard textbook dichotomy of floats versus pegs was replaced by a more continuous grading of exchange rate regimes like free floats, various intermediate regimes, and hard pegs.¹ After the partial collapse of Europe's exchange rate mechanism (ERM) in September 1992, a standard proposition attracting widespread support was that corner solutions like free floats or super-strict pegs (currency boards, dollarisation) are preferable to intermediate regimes (soft pegs, band, crawling pegs) on the grounds that they are less crisis-prone given that today's financial markets are too powerful and volatile. According to this view, investors will overwhelm intermediate regimes like band systems sooner or later. Thus the options for exchange rate regimes have been hollowed out. Governments should either let them float, or fix them permanently with a currency board or in a monetary union.²

In fairness one should say that academics do not agree upon the best solution. Until the mid-1990s the mainstream literature has basically argued the opposite. Several objectives and trade-offs (insulation from external nominal and real shocks, flexibility vs. commitment, inflation stabilisation, exposure to international capital flows) pointed to intermediate regimes with limited flexibility, i.e. a compromise between hard pegs and free floats. Balancing risks and benefits has led to the conclusion that intermediate regimes were appropriate for a large number of countries, especially those without large exposure to international capital flows. Another fundamental reason why economists have considered the extremes - pure float or absolute fix - at that time as overdone is that economics generally prefers interiors to corner solutions.

A prime example for a hard peg is Hong Kong's currency board system.³ Hong Kong's currency board stands out among such arrangements around the world as the one with the longest history.⁴ Hong Kong's currency board was established in 1983 and the Hong Kong dollar (HKD) was pegged to the US dollar (USD) at 7.80 to 1 as a firm anchor for the external value of the HKD. The USD was an obvious choice, as the US was a major trading partner of Hong Kong. In September 1998, the convertibility rate was changed to 7.75 to 1. Subsequently the convertibility rate moved gradually from 7.75 back to the linked rate of 7.80 between April 1999 and August 2000, where it has remained. A distinctive feature of the system was that up to May 2005 no strong-side boundary existed. Thus, the currency board system was asymmetric. Finally, in May 2005 the Hong Kong Monetary Authority (HKMA) wanted to bite the bullet on appreciation and replaced the previous policy of an implicit strong band with something less muddled. More precisely, the HKMA introduced a symmetric target zone with bandwidth [7.75, 7.85].

¹More attention was also given to the distinction between *de facto* and *de jure* exchange rate regimes, as revealed by the ex-post behaviour of exchange rates.

²But free floats have a big drawback: they can overshoot and become highly unstable, especially in open emerging economies with large capital flows.

³An in-depth discussion of Hong Kong's currency board, including documentations on the technical details is available at http://www.info.gov.hk/hkma/eng/currency/link_ex/index.htm. For a perceptive and thorough discussion, see Latter (2007).

⁴In the empirical exchange rate literature, the performance of currency board systems has been discussed *ad nauseam*. For example, Ghosh et al. (2000) have found that currency boards exhibit better inflation performance than soft pegs, mostly due to a credibility effect. Regarding growth, currency boards also do better than soft pegs.

In this paper we provide new theoretical insights in the working of Hong Kong's currency board system since January 2001. Whereas there exists a huge literature on symmetric target zones, research on one-sided regimes with implicit bands is lacking. Based upon Klein (1992), we expand the scope of the existing literature and offer a new analytical framework for one-sided target zones with implicit (unknown) bands and derive the exchange rate dynamics in such regimes. The paper is structured as follows: Section 2 provides a brief self-contained overview of Hong Kong's currency board system. Section 3 then presents an analytical framework modelling Hong Kong's currency board system since January 2001. The theoretical modelling approach provides the input into the calibration analysis in Section 4. The final section offers concluding remarks.

2 Main Features of Hong Kong's Currency Board System Since 2001

We first begin by discussing the main elements of Hong Kong's currency board system which provides an anchor for our modelling work below. Contrary to actively managed fixed exchange rate systems, a currency board system is a passive "hard-fixed" peg system. The predictability and rule-based nature of currency boards are key advantages. Like a traditional peg, currency boards offer the prospect of stable exchange rates which can promote both investment and trade. Hong Kong's currency board has survived a number of booms and busts, including a massive speculative attack during Asia's financial crisis of 1997-98. Given the speculative outflow of USD, the HKMA sold large amounts of USD in October 1998 to defend the peg. Furthermore, the HKMA pursued a defensive interest rate strategy which was partly responsible for bringing about a severe recession.⁵ Despite the presumed rigidity of the currency board system, the convergence between the exchange rate in the interbank market and the fixed rate for currency did not happen in practice. A first major set of reforms in the operational framework after the Asian crisis therefore comprised the introduction of a weak-side Convertibility Undertaking (CU). This was an asymmetric weak-side commitment in that the HKMA was ready to sell USD at 7.80, but was not obliged to purchase them at a pre-announced rate.⁶ This weak-side commitment is shown by the red line in Figure 1. Since late 2003, speculative pressure for a revaluation of the Chinese Renminbi has grown, with the result of large speculative inflows.⁷ The HKD appreciated from 7.80 to about 7.70 in autumn 2003, fuelling speculation that the currency board link

⁵The result of this surprise move was that interbank interest rates jumped and the overnight rate touched 280 percent. This successfully stemmed the speculative outflow of USD. Overnight rates dropped back to about 5 percent within a few days.

⁶As a historical note, while no formal strong-side intervention point was introduced, the Subcommittee on Currency Board Operations already considered the options in this area in meetings in October 1999 and July 2000 and "agreed that there would be scope to review the arrangement again, should the need arise" [HKMA (2000)].

⁷One reason for the strengthening of the HKD was the signing of the Mainland China and Hong Kong Closer Economic Partnership Agreement (CEPA) in June 2003 which was expected to give stimulus to Hong Kong's economy. This has led some market participants to believe that any appreciation of the renminbi against the USD could also lead to an appreciation of the HKD. Another reason why Hong Kong's "iron peg" has come under attack despite strong economic fundamentals might be that the HKMA has begun to look a bit too much like a de facto central bank intervening in money markets to smooth interest rates. This kind of intervention undermines the advantages of the currency board since the system isn't regarded as automatic any more.



Figure 1: Daily HKD spot exchange rate against USD and exchange rate band;

Source: Bloomberg

to the USD might be abandoned.⁸ In these years, the exact exchange rate triggering HKMA strong-side interventions was unknown for the public. Only the last interventions provided some hints about the HKMA's intentions. Lastly, the link was defended with a combination of market interventions, including direct foreign exchange operations and manipulation of liquidity and interest rates. An assessment of the 2003-2004 episode is that the HKMA have had a tried-and-trust method for bringing to halt intermittent appreciation pressure: beyond some undisclosed threshold they have intervened. In other words, a flexible strong-side regime that was managed as if it is fixed but without explicit strong-side precommitment was in operation.

Over and above these actions, on 18 May 2005 the HKMA finally revamped the one-sided currency board mechanism into a symmetric two-sided system with a narrow exchange rate band of ± 0.6 percent. The strong-side Convertibility Undertaking (the green line in Figure 1) was fixed at HKD 7.75/USD.⁹ At the same time, the weak-side CU was changed from HKD 7.80/USD to HKD 7.85/USD. These "refinements" were intended to anchor market expectations and to stop speculative attacks. Figure 1 shows that the HKD spot rate stayed close to the strong-side CU (the green line) most of the time after May 2005. Finally, it is noticeable that the validity of the current arrangement has not been called into question by the current financial crisis.

3 Modelling Discretion on the Strong Side

Until May 2005 Hong Kong's exchange rate system comprised a credible weak-side CU and an undisclosed strong-side CU. Ultimately, the system was thus an undisclosed one-sided target zone

⁸For an analysis of the strong-side pressure on the HKD and, in particular, the HKMA's response, see the IMF (2005). The paper also offers a simple second-generation currency crisis framework modelling the trade-offs faced by the HKMA.

⁹In other words, the HKMA moved from a single-edge shield towards a double-edged shield and thus the idea of target zones made a comeback. One underlying motivation was that if the appreciation pressure was to happen repeatedly, this could undermine the credibility of the currency board arrangement. For a theoretical framework analysing this lightning-bolt solution, see Chen et al. (2010).

model.¹⁰ Keeping in mind this specific one-sided nature of the currency board system, we study the associated credibility issues and the exchange rate dynamics in a formal model. Roughly speaking, we attempt to model the machinery inside the clock. The new aspects and insights of this paper are to make an extension of previous work by Klein (1992). Klein (1992) has analysed the dynamics of the exchange rate in a target zone with unknown band width. In our model past exchange rate interventions then convey information about the undisclosed bands and affect the exchange rate dynamics via the changed fundamentals and the triggered revision of exchange rate expectations.

3.1 The Basic Model

It is useful to preface a detailed discussion of the one-sided target zone model with some reference to Krugman's (1991) seminal paper. Although the review pretends to be brief, it will be adequate to demonstrate the conceptual and research framework. His model starts from the observation that due to the forward-looking nature of rational expectations, the presence of a credible symmetric band exerts an influence on the dynamics of the exchange rate. The model starts from the log-linear asset pricing equation that expresses the log exchange rate, $s(t)$, as the sum of the logarithm of the fundamental, f , and its own expected change:

$$(1) \quad s(t) = f(t) + \tau \frac{E(ds(t))}{dt},$$

where $E[\cdot]$ denotes the rational expectations operator and $\tau > 0$. The factors affecting the exchange rate are the fundamentals and financial markets' expectations about the future movement of the exchange rate. The fundamental f consists of the logarithm of money supply, m , and velocity, v .

$$(2) \quad f(t) = m(t) + v(t).$$

Except for intervention periods, $m(t)$ is constant. In the case of an intervention at $t = T$, the money supply takes a new value in T . Beyond intervention periods, the driving force of f is v . Therefore, the values of the function f will be denoted by V for possible strong/weak band thresholds. It is assumed that the log of the velocity follows an arithmetic Brownian motion without drift:

$$(3) \quad dv(t) = \sigma dz,$$

where σ is the risk parameter and z denotes a standard Brownian motion. To handle this process, we introduce a function g with

$$(4) \quad g(f) = s(t).$$

Applying Itô's lemma to the expectations term yields

$$(5) \quad \frac{E(ds)}{dt} = \frac{\sigma^2}{2} g'',$$

¹⁰An implicit strong-side band is likely because history has shown that Hong Kong's policymakers are putting a large weight on exchange rate stability.

which means that the logarithm of the exchange rate is subject to the second-order differential equation

$$(6) \quad s(t) = f(t) + \tau \frac{\sigma^2}{2} g''.$$

The innovation in the paper is that we shall solve the second-order ordinary differential equation (6) describing the dynamics of the exchange rate in the target zone for the special case of a one-sided target zone with an undisclosed strong-side band. This provides a sound modelling framework with considerable rigour allowing to understand the dynamics of the HKD since January 2001.¹¹

3.2 The One-Sided Target Zone Framework

In Krugman's (1991) model the central bank credibly commits to maintaining the symmetric target zone regime. Given Hong Kong's asymmetric exchange rate regime, the question becomes: How do we introduce such asymmetric dynamics into the target zone model? In our one-sided target zone model, the weak-side band \bar{S} is credibly fixed. On the contrary, the central bank's strong-side band \underline{S} is undisclosed. In other words, we assume a "high" confidence weak-side band and a "low" confidence strong-side band. Market participants form expectations of the undisclosed strong-side band \underline{S} . Their expectations depend on the unknown threshold which triggers a central bank intervention.

Let's assume that the intervention triggering exchange rate against appreciation pressure is expected to be located anywhere in the interval $[S_1, S_2]$, $S_1, S_2 \in \mathbb{R}$ with $S_1 < S_2$, whereas the corresponding fundamental values form the interval $[V_1, V_2]$ with $V_1 < V_2$.

In order to obtain a solution for equation (6) in the case of a one-side target zone, we implement the value matching and smooth pasting conditions and substitute the fundamental value V^l , $V^l \in [V_1, V_2]$, triggering the next intervention at the strong side:¹²

$$(7) \quad s(t) = f(t) + A_1(V^l)e^{rf(t)} + A_2(V^l)e^{-rf(t)},$$

where

$$\begin{aligned} r &= \sqrt{\frac{2}{\tau\sigma^2}} \\ A_1(V^l) &= -\frac{\tau\sigma^2}{2\omega(V^l)} \left(e^{-rV^l} - e^{-r\bar{V}} \right) r \\ A_2(V^l) &= \frac{\tau\sigma^2}{2\omega(V^l)} \left(e^{r\bar{V}} - e^{rV^l} \right) r \\ \omega(V^l) &= e^{r\bar{V}-rV^l} - e^{rV^l-r\bar{V}}. \end{aligned}$$

The three terms $A_1(V^l)$, $A_2(V^l)$ and $\omega(V^l)$ typify the uncertainty inherent in the model since they depend on the uncertain fundamental value V^l .

¹¹Unfortunately, Klein's (1992) modelling approach does not lend itself naturally to the asymmetric one-sided band exchange rate regime case. That may explain why the current paper is the first to model an asymmetric one-sided target zone. It also explains why we depart from Klein's (1992) approach on technical grounds.

¹²A thorough description of the approach is provided by Sarno and Taylor (2003, pp. 177-184).

Rearranging $A_1(V^l)$ and $A_2(V^l)$ yields

$$(8) \quad \begin{aligned} A_1(V^l) &= -\frac{r\sigma^2\tau}{2(e^{rV^l} + e^{r\bar{V}})} \\ A_2(V^l) &= \frac{e^{r(V^l+\bar{V})}r\sigma^2\tau}{2(e^{rV^l} + e^{r\bar{V}})}. \end{aligned}$$

Next, we shall describe the sequence of events and the strategic interactions between the central bank and the market participants.

3.3 The Situation at the Outset

This section provides an outline of the initial situation. The market participants' expectations depend upon the perception of the central bank's behaviour. We postulate that market participants act on the assumption of a uniform distribution of possible (unknown) trigger values of the fundamentals in the range $[V_1, V_2]$. The uniform distribution makes sound sense because no a priori information about the intervention trigger points is available. This implies that the probability for an intervention at time t is given by

$$(9) \quad P(V_t = V_T) = \frac{1}{V_2 - V_1} \mathbf{1}_{\{V_t \in [V_1, V_2]\}},$$

where V_T is the intervention triggering fundamental value. At the outset, the exchange rate has not yet moved outside the range $(S_2, \bar{S}]$. The value-matching condition implies

$$(10) \quad s(t_0) = E(s(t_0)).$$

Taking the expectation value of equation (7) yields

$$(11) \quad E(s(t_0)) = f(t_0) + E(A_1(V^l(t_0)))e^{rf(t_0)} + E(A_2(V^l(t_0)))e^{-rf(t_0)},$$

where

$$(12) \quad E(A_1(V^l)) = -\int_{-\infty}^{\infty} \frac{r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} dP = -\int_{V_1}^{V_2} \frac{r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \frac{1}{V_2 - V_1} dv$$

and

$$(13) \quad E(A_2(V^l)) = \int_{-\infty}^{\infty} \frac{e^{r(v+\bar{V})}r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} dP = \int_{V_1}^{V_2} \frac{e^{r(v+\bar{V})}r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \frac{1}{V_2 - V_1} dv.$$

Inserting equations (12) and (13) into equation (11), we get

$$(14) \quad \begin{aligned} s(t_0) &= f(t_0) - \left(\int_{V_1}^{V_2} \frac{r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \frac{1}{V_2 - V_1} dv \right) e^{rf(t_0)} \\ &\quad + \left(\int_{V_1}^{V_2} \frac{e^{r(v+\bar{V})}r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \frac{1}{V_2 - V_1} dv \right) e^{-rf(t_0)}. \end{aligned}$$

The general closed-form solution of equation (14) is given by

$$(15) \quad s(t_0) = f(t_0) - e^{-r(f(t_0)+\bar{V})} \sigma^2 \tau \frac{\left(e^{2f(t_0)r} r (V_1 - V_2) - \left(e^{2f(t_0)r} + e^{2r\bar{V}} \right) \left(\ln \left(e^{rV_1} + e^{r\bar{V}} \right) - \ln \left(e^{rV_2} + e^{r\bar{V}} \right) \right) \right)}{2(V_1 - V_2)}.$$

Equation (15) describes the dynamics of the exchange rate when the central bank has discretion on the strong side.¹³

To explain intuitively, consider the graphical illustration of the exchange rate against the fundamentals in Figure 2. The segment ab in Figure 2 displays the relationship between the exchange rate and the fundamentals in case that the exchange hasn't moved outside the range $(S_2, \bar{S}]$. The segment ac represents the exchange rate dynamics for the unattained velocity interval $[V_1, V_2)$. In a nutshell, all possible exchange rate curves are located in the area abc .

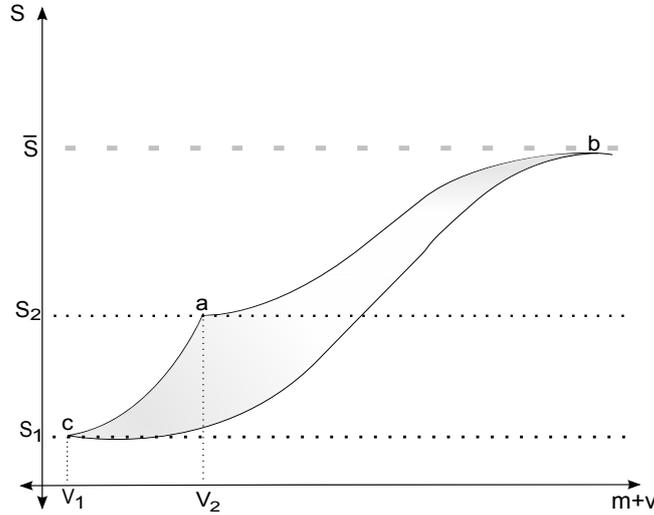


Figure 2: The relationship between the fundamental and the exchange rate with a continuum of possible intervention triggering exchange rates on the strong side

Once the exchange rate has moved outside the range $(S_2, \bar{S}]$ by taking a value \tilde{S} , $S_1 < \tilde{S} < S_2$, without an intervention response, the system is changed on two counts. First, the range of expected intervention triggering exchange rates degenerates to $[S_1, \tilde{S})$. Second, the relation between the fundamentals and the exchange rate undergoes a change. Denoting \mathbf{f} , $V_1 < \mathbf{f} < V_2$, as the smallest value of the fundamental which has occurred so far, the updated relationship is

$$(16) \quad s(t_0) = f(t_0) - e^{-r(f(t_0)+\bar{V})} \sigma^2 \tau \frac{\left(e^{2f(t_0)r} r (V_1 - \mathbf{f}) - \left(e^{2f(t_0)r} + e^{2r\bar{V}} \right) \left(\ln \left(e^{rV_1} + e^{r\bar{V}} \right) - \ln \left(e^{r\mathbf{f}} + e^{r\bar{V}} \right) \right) \right)}{2(V_1 - \mathbf{f})}.$$

Substituting \mathbf{f} for V_2 in equation (16) implies that fundamental values larger than \mathbf{f} are irrelevant. Analogously, the expected probability for an intervention at time t given that the fundamental

¹³The interested reader is referred to Appendix A.1 for the derivation. It may be a matter worthy of comment that the closed-form solution in Klein (1992) has a more appealing kind because it is derived for the symmetric case whereby the integrals are characteristic.

value \mathbf{f} has been observed is modified to

$$(17) \quad P(V_t = V_T | \mathbf{f}) = \frac{1}{\mathbf{f} - V_1} \mathbb{1}_{\{V_t \in [V_1, \mathbf{f}]\}} \quad \forall \mathbf{f} \in (V_1, V_2],$$

having the feature

$$(18) \quad \lim_{\mathbf{f} \searrow V_1} P(V_t = V_T | \mathbf{f}) = 1.$$

The expectation terms in (12) and (13) are altered accordingly. The updated relationship between the exchange rate and the fundamentals also changes the segments in Figure 2. More precisely, the segment ab becomes steeper and Krugman's (1991) honeymoon effect weakens.

The framework described above acts on the assumption that the non-appearance of an intervention is a mechanism to signal the unknown edge of the strong-side band. It reveals the central bank's true preferences and alters market participants' expectations. By observing the exchange rate \tilde{S} market participants from now on expect that the intervention triggering exchange rate is located in the smaller range $[S_1, \tilde{S}]$ and no intervention will happen as long as the exchange rate fluctuates within the range $[\tilde{S}, \bar{S}]$. In addition, the more the exchange rate appreciates, the higher the expected intervention probability is.

In the next subsection, we offer a proper assessment of the post-intervention exchange rate dynamics.

3.4 Post-Intervention Exchange Rate Dynamics

As time evolves, an intervention takes place at $t = T_1$. This brings the undisclosed strong-side band to life. Yet at the same time the problem gets more complicated. One obvious implication is that market participants expect a higher probability that the central bank will intervene close to S_{T_1} again.¹⁴ This demands the replacement of the uniform distribution in equation (9) and (17) by a density function which puts more weight on V_{T_1} and thus on S_{T_1} . On the other hand, the first intervention may not be a landmark decision for the entire future, i.e. the intervention triggering point may still be a moving target.¹⁵ Clearly expectations about this target are dependent on the success of the last intervention. Therefore we define the market participants' expectations about the intervention triggering exchange rate in $t = T_2$ conditional upon the actual exchange rate being located in the upper, $(S_{T_1}, S_2]$, or lower interval, $[S_1, S_{T_1}]$.¹⁶

Starting with the exchange rate dynamics in the lower interval, we consider the conditional probability function $P(V_t = V_{T_2} | S_1 \leq s(t) \leq S_{T_1})$. As aforementioned, the distribution function should put more weight on S_{T_1} . To simplify the problem somewhat, we assume that the density

¹⁴In dynamic economic models backward-looking expectations with systematic forecasting errors are inconsistent with rational behaviour. In nonlinear dynamic models, exhibiting seemingly unpredictable breaks due to the sporadic nature of the interventions, however, simple "rule of thumb" backward-looking expectation rules may yield non-systematic forecasting errors. Furthermore, a large number of survey studies such as Cheung and Chinn (2001) and Menkhoff (1998) uniformly confirm that speculators in foreign exchange markets generally do not rely on mathematically well-defined econometric or economic models, but instead follow simple backward-looking trading rules.

¹⁵In Klein's (1992) model, the first intervention is such a landmark decision for the future. This implies that after the first intervention the model with undisclosed band width collapsed to the standard Krugman (1991) model with full faith in the target zone.

¹⁶Whether S_{T_1} is determined to belong to the upper or the lower interval, which influences the conditions in the conditional probability functions, is negligible for the exchange rate movements, as only a null set is integrated.

is convex and defined by

$$(19) \quad \varphi(v) = \lambda e^{2(v-V_{T_1})} \mathbb{1}_{\{v \in [V_1, V_{T_1}]\}},$$

whereas $0 < \lambda \leq 1$ controls how much weight is put on V_{T_1} and thus on S_{T_1} .¹⁷

When the exchange rate appreciates beyond S_T without triggering another intervention, then this newly acquired information serves as feedback to the market participants and provides the basis for updating prior expectations. To be precise, we alter the probability function by using the smallest observed fundamental \mathbf{f} to update the prior beliefs. In our notation, we will therefore write $P(V_t = V_{T_2} \mid \mathbf{f}, S_1 \leq s(t) \leq S_{T_1})$, in order to show the relation to the updating \mathbf{f} , so that equation (19) becomes

$$(20) \quad \varphi(v, \mathbf{f}) = \lambda e^{2(v-\mathbf{f})} \mathbb{1}_{\{v \in [V_1, \mathbf{f}]\}} \quad \forall \mathbf{f} \in (V_1, V_{T_1}).$$

The quintessence is that the lower \mathbf{f} is, the higher the intervention probability is. Ultimately, it is reasonable to assign measure 1 to $P(V_t = V_{T_2} \mid \mathbf{f}, S_1 \leq s(t) \leq S_{T_1})$ on the set $\{V_1, \dots, V_{T_1}\}$. In other words, the central bank will definitely intervene in the lower range $[S_1, S_{T_1}]$. This is tantamount to

$$(21) \quad \int_{V_1}^{\mathbf{f}} \lambda e^{2(v-\mathbf{f})} dv = 1.$$

Equation (21) signifies that λ is a function of \mathbf{f} .¹⁸

An illustration of the updating-mechanism is provided in Figure 3. The convex curves show the density in equation (20) for different fundamentals \mathbf{f} . The horizontal curves show the density of the uniform distribution over the same interval $(V_1, \mathbf{f}]$. Apparently, the convex density functions puts more weight on \mathbf{f} and penalize more distant values in the interval $(V_1, \mathbf{f}]$.¹⁹ Next, we derive the closed-form expression for the exchange rate in the lower range along the lines in subsection 3.3. For $t \in (T_1, T_2]$ we obtain

$$(22) \quad s(t) = f(t_0) + \underline{E}(A_1(V^l(t_0)))e^{rf(t_0)} + \underline{E}(A_2(V^l(t_0)))e^{-rf(t_0)} \quad \forall s(t) \in [S_1, S_{T_1}],$$

where

$$(23) \quad \underline{E}(A_1(V^l)) = - \int_{V_1}^{\mathbf{f}} \frac{r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \lambda e^{2(v-\mathbf{f})} dv$$

¹⁷Convex functions are typically used in macroeconomic models with adjustment costs to penalise swift changes in variables and thereby to induce gradual movements over time. Across the many models with convex adjustment costs, quadratic functions have been by far the most common specification, essentially for sake of tractability. Without loss of generality and for mathematical convenience, we also assume a quadratic specification.

¹⁸In our framework, we approximate the relevant considerations with the simplest functional forms to keep the model tractable and the conclusions less susceptible to certain twists in the functions. The derivation of λ is shown in Appendix A.2. However, another functional shape for λ is also technically feasible, if it obeys condition (21).

¹⁹In other words, Figure 3 highlights the role of choosing V_1 . The larger the interval $[V_1, \mathbf{f}]$ is set, the flatter the density in (19) and the uniform distribution get. Therefore a different choice of V_1 implies a different exchange rate curve progression. If V_1 takes a value close to \mathbf{f} , the curve pastes smoothly near to the exchange rate \mathbf{S} corresponding to \mathbf{f} . Otherwise, the more distant V_1 is from \mathbf{f} , the more is the smooth pasting effect departing from \mathbf{S} . This provides a helpful instrument, as the size of the interval $[V_1, \mathbf{f}]$ is the mirror-image of the publicly perceived need for an intervention. The following formulas show how this mirror-image is transferred first to the expectations about a monetary operation, which is then affecting the exchange rate curve progression and the smooth pasting behavior.

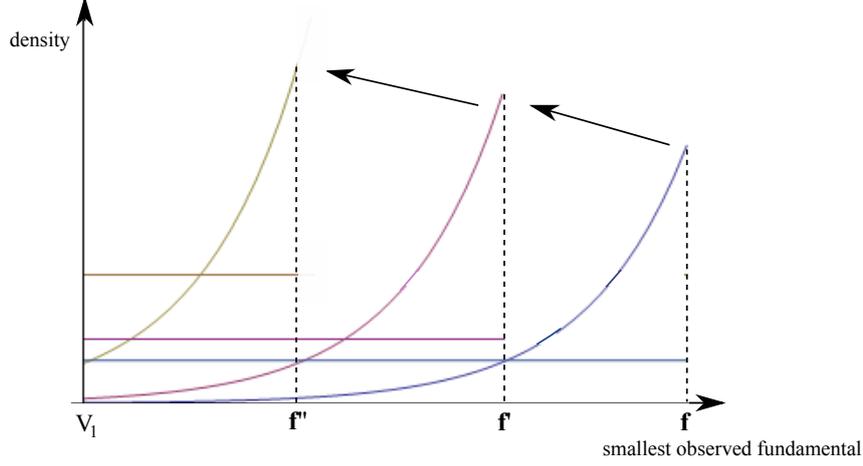


Figure 3: The density function in equation (20) and the density function of the uniform distribution on the interval $(V_1, \mathbf{f}]$ for different fundamentals \mathbf{f} .

and

$$(24) \quad \underline{E}(A_2(V^l)) = \int_{V_1}^{\mathbf{f}} \frac{e^{r(v+\bar{V})} r \sigma^2 \tau}{2(e^{rv} + e^{r\bar{V}})} \lambda e^{2(v-\mathbf{f})} dv.$$

Rearranging (23) and (24) using a Hypergeometric function ${}_2F_1$, we finally obtain the closed form solution for the exchange rate dynamics in the lower range $s(t) \in [S_1, S_{T_1}]$:²⁰

$$(25) \quad s(t) = f(t) + \frac{1}{2(2+r)} e^{-2\mathbf{f}-f(t)r} r \lambda \sigma^2 \tau \left(e^{\mathbf{f}(2+r)} {}_2F_1 \left[1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(\mathbf{f}-\bar{V})} \right] - e^{V_1(2+r)} {}_2F_1 \left[1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(V_1-\bar{V})} \right] \right) - \frac{1}{4} e^{-2\mathbf{f}-r\bar{V}+f(t)r} r \lambda \sigma^2 \tau \left(e^{2\mathbf{f}} {}_2F_1 \left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(\mathbf{f}-\bar{V})} \right] - e^{2V_1} {}_2F_1 \left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(V_1-\bar{V})} \right] \right).$$

In the upper range of exchange rates $(S_{T_1}, S_2]$, however, we face a different situation concerning the public's expectations. As long as the exchange rate fluctuates above S_{T_1} , the market participants might remember the last intervention and keep it in mind as the off-the-record strong-side band. This means implementing Krugman's model (1991) with the target zone $[S_{T_1}, \bar{S}]$. However, including an expectation updating process is reasonable, when the exchange rate does not approach to S_{T_1} for a longer time. This might be reasoned by changed economic developments. In this situation the market participants update by taking into account their observations after the first intervention.

For implementation, we fix a period of time in which the exchange rate behavior is assumed to fluctuate like in Krugman's basic target zone model with $V^l = V_{T_1}$ in equation (7). After this period the public updates its expectations if the exchange rate has been departed from S_{T_1} . If

²⁰The interested reader may take a look at the derivation in Appendix A.3. A Hypergeometric function can be defined in the form of a convergent Hypergeometric series. Many functions can be expressed as special cases of a Hypergeometric function (for example, the Exponential, the Gamma, the Trigometrical and the Bessel function).

the exchange rate has come close to S_{T_1} , no updating occurs and the basic model by Krugman is valid.²¹

For an expectations update after a fixed period of time \hat{t} , the smallest observed fundamental \mathbf{f} between T_1 and $T_1 + \hat{t}$ is used to recondition the relationship between the exchange rate and the fundamental. The exchange rate \mathbf{S} corresponding to \mathbf{f} is thus used to divide the upper range $(S_{T_1}, S_2]$ into two subsets (S_{T_1}, \mathbf{S}) and $[\mathbf{S}, S_2]$, whereas the exchange rate dynamics in the lower subset (S_{T_1}, \mathbf{S}) is the same as explained above for the lower range $[S_1, S_{T_1}]$. For consistency the exchange rate fluctuations in the subset $[\mathbf{S}, S_2]$ are modelled like in Krugman's basic target model.

In our mathematical framework, the exchange rate dynamics in the upper interval $(S_{T_1}, S_2]$ is expressed as follows. As long as no expectation update is implemented, the exchange rate behaves according to

$$(26) \quad s(t) = f(t) + A_1(V_{T_1})e^{rf(t)} + A_2(V_{T_1})e^{-rf(t)} \quad \forall s(t) > S_{T_1}, \quad \forall t: T_1 < t < T_1 + \hat{t} < T_2,$$

where A_1 and A_2 arise from (8).

The smallest observed fundamental between the i 'th and $i + 1$ 'th, $i \in \mathbb{N}$, expectation update is denoted by \mathbf{f}_i . After the first expectation update the exchange rate, in case $s(t) \geq \mathbf{S}_i$, moves like

$$(27) \quad s(t) = f(t) + A_1(\mathbf{f}_i)e^{rf(t)} + A_2(\mathbf{f}_i)e^{-rf(t)} \quad \forall t: i(T_1 + \hat{t}) \leq t < (i + 1)(T_1 + \hat{t}) < T_2.$$

In contrast, the dynamics of the exchange rate $s(t) < \mathbf{S}_i$ is determined according to the mechanism in equation (25).

3.5 The Information Content of Further Interventions

In the last subsection, we have analysed the exchange rate dynamics assuming that an intervention only occurs once. This setup may be unrealistic for economies in which (a) the structure of the economy is constantly evolving in ways that are imperfectly understood by both the public and policymakers and (b) the policymakers' objective function may change over time and is not fully known by private agents.²² What happens once further interventions are carried out? Where does that leave us? For sake of simplicity we assume that market participants use a weighted average of past intervention triggering exchange rates as a predictor for future interventions. Hence, in terms of our notation we define mathematical

$$(28) \quad \mathcal{S}_{\mathcal{T}_N} = \sum_{i=1}^N a_i S_{T_i} \quad \forall N \in \mathbb{N},$$

where $a_i \in [0, 1]$ and $\sum_{i=1}^N a_i = 1$.

The weighted average exchange rate $\mathcal{S}_{\mathcal{T}_N}$ yields the two intervals $[S_1, \mathcal{S}_{\mathcal{T}_N}]$ and $(\mathcal{S}_{\mathcal{T}_N}, S_2]$ for the mechanism in section 3.4. Corresponding to $\mathcal{S}_{\mathcal{T}_N}$ is the fundamental $\mathcal{V}_{\mathcal{T}_N}$. Equation (28)

²¹The HKMA's foreign currency market interventions are carried out in an open and transparent manner and are therefore public knowledge. In all cases, the interventions are announced the same day they occurred. Agents can therefore distinguish between movements in f arising from interventions and fluctuations due to equation (3)

²²Market participants realize, however, that a currency board is no free lunch. For example, countries with currency boards imported the loose monetary conditions of the advanced economies. This policy put a floor under asset prices and eventually created the conditions for the credit crunch of 2008.

implies that the extent to which intervention expectations are anchored can change, depending on economic developments and (most important) the current and past conduct of monetary policies.²³

3.6 The Introduction of a Symmetric Band

On 18 May 2005 the currency board arrangement was altered when finally a narrow symmetric target zone of 0.6 percent was introduced with a strong-side CU at HKD 7.75/USD. For the first time, this added a ceiling to the floor by which it had traditionally managed the currency, in a move to discourage investors from using the HKD to speculate on a RMB appreciation. At the same time, the weak-side CU was shifted from HKD 7.80/USD to HKD 7.85/USD. These "refinements" intended to anchor market expectations and to promote a smooth functioning of the currency board arrangement.²⁴

Viewed in retrospect, it is reasonable to say that different considerations and assessments may have prevailed after May 2005. Judged by the HKD exchange rate since January 2004, market participants may have assessed the newly introduced symmetric band as generously dimensioned. Alternatively, one can well imagine that financial markets may not based their expectations on a blind faith in the working of the currency board mechanism and the will and commitment of the monetary authorities to defend the edges of the band.²⁵ If markets can figure out the potential fragility of the edges and perform the requisite backward induction, then a target zone may lose its reputation and stabilising power.²⁶

We implement the credibility issues arising in the new regime via a new shape for equation (28). Credibility is defined as the capacity of the policymakers to announce a policy which is trusted by market participants. For analytical convenience we modify the model such that $\mathcal{S}_{\mathcal{T}_N}$ is given by

$$(29) \quad \mathcal{S}_{\mathcal{T}_N} = \sum_{i=1}^N a_i S_{T_i} + a_A S_A, \quad \forall N \in \mathbb{N},$$

where S_A is the announced strong-side band, the coefficients a_N , $a_i \in [0, 1]$ and $\sum_{i=1}^N a_i = 1 - a_A$. In other words, $\mathcal{S}_{\mathcal{T}_N}$ is the weighted average of the past N interventions and the announced strong-side band S_A , and the coefficient a_A gauges the extent to which the announcement is seen as credible. A larger a_A coefficient ties $\mathcal{S}_{\mathcal{T}_N}$ closer to S_A . Subsequently, the full credibility

²³Technically expressed, the coefficients a_i in equation (28) show the importance of the last interventions. However, expectations are also influenced by V_1 (compare the effects of different V_1 by means of Figure 3). Therefore, there is also room for expectations, which are not primarily anchored by past interventions.

²⁴For a graphical display, see Figure 1.

²⁵The 1-year forward rate of the HKD was consistently outside the convertibility zone between October 2005 and the beginning of 2007. This is known as Svensson's (1992, 1994) 100% credibility test and indicates that financial market participants have initially revealed scepticism about the ability of the new strong side CU to limit exchange rate fluctuations. Another intermittent upward pressure on the HKD occurred in autumn 2007 when HKMA interventions took place once again in another move to anchor market expectations. Related empirical analysis of the credibility of the two-sided system is scarce, exceptions being Fung and Yu (2007) and Hui and Fong (2007).

²⁶The HKMA would not be the first central bank doing this. For example, in the European Exchange Rate Mechanism (ERM) currencies were initially allowed to fluctuate no more than 2.25 percent above or below fixed bilateral rates. The UK joined in 1990 but was forced to leave in 1992 when sterling came under speculative pressure. Fluctuation bands were then widened to ± 15 percent in 1993 to avoid defending the indefensible.

scenario is given by $\mathcal{S}_{T_N} = S_A$ and $\mathcal{V}_{T_N} = V_1$ (see Figure 2). For $S_{T_N} > S_A$ the credibility constraint is not binding. Lastly, the imperfect credibility case, where the public doubt the monetary authority's ability to defend the announced band, is given by $\mathcal{S}_{T_N} < S_A$.

In the next section we conduct an analysis of the model by resorting to numerical methods.

4 Putting it all Together

The idea is to make our model match exchange rate data of interest.²⁷ Figure 4 illustrates the interventions of the HKMA over the period 2001 to 2007.²⁸ Contrary to common views, Hong Kong's currency board isn't a simple rule-based monetary policy but involves discretion. Figure 4 also unveils that the currency board was one-sided until May 2005, i.e. there was a commitment to sell, but not to buy, USD at 7.80 HKD/USD. The spot exchange rate of the HKD against the USD remained on the strong side from autumn 2003 throughout January 2004, with repeated foreign exchange market interventions by the HKMA to stem the upward pressure amidst continued capital inflows.²⁹ This stepping in and "leaning against the wind" in late 2003 to defend the HKD's peg to the USD highlights the fear that an appreciation of the HKD may hurt the reputation of the currency board system. Ultimately, there was a slight ease-back in the strength of the HKD in late January 2004, amidst growing concern about the spread of avian flu in East Asia.

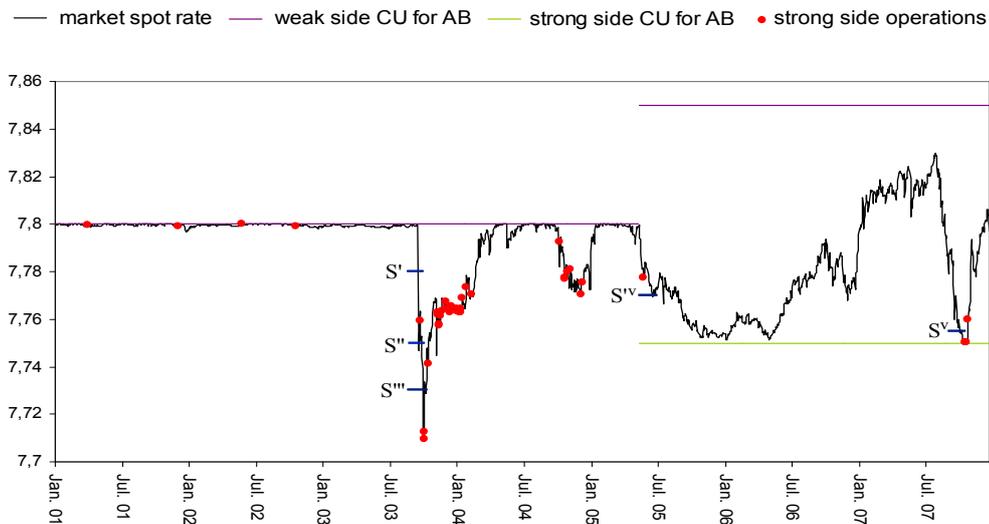


Figure 4: Strong side operations

²⁷At the end of the day, it is always useful to acknowledge that the theoretical framework and calibrations primarily serve as a communication device. The modelling approach introduced in this paper goes some way towards achieving the purpose of understanding the dynamics of the HKD. Yet we do not claim empirical accuracy for the model but use it rather for qualitative features and predictions.

²⁸Interventions refer to the net injection or withdrawal of funds by the HKMA in the interbank money market. For the daily market operations data, see <http://www.info.gov.hk/hkma/eng/statistics/msb/index.htm>.

²⁹Interventions were necessary because markets believed that the HKD would appreciate alongside the RMB made the automatic adjustment mechanisms of the currency board system ineffective. For an econometric logit analysis of monetary operations conducted by the HKMA, using daily data for the one-sided regime between September 1998 and December 2001, see Gerlach (2005).

In the graph, we have also highlighted five points indicated by S' - S'' . Below we shall calibrate the dynamics of the exchange rate at these given instants.

First we derive the exchange rate dynamics prior to the strong-side interventions at S_{T_1} . The benchmark set of parameters for the simulation is $\sigma = 0.01$, $\tau = 0.9$, $V_1 = 2.035 \approx \ln(7.65)$.

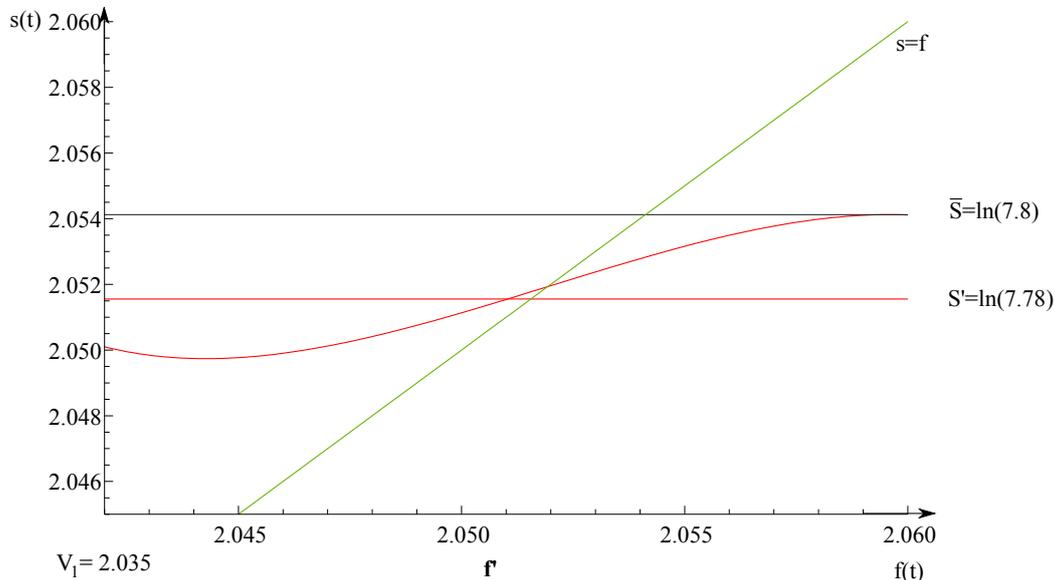


Figure 5: S -shaped curve for $S' = \ln(7.78)$

The first point to note is that the relationship between the fundamentals and the exchange rate is an S -shaped curve, i.e. the exchange rate is a function of the fundamentals and the expected exchange rate leading to a disconnection of fundamental and nominal volatility. Two properties of the solution are apparent from Figure 5. First, the upper weak-side band is fully credible. Second, as long as no intervention occurs, market participants expect a further appreciation of the HKD beyond S' . Where does that leave us? For $S' = \ln(7.78)$, for example, the "lens" below the horizontal line indicates that the perceived exchange rate band ranges up to approximately $2.05 \approx \ln(7.768)$.

In Figure 5, we have illustrated the dynamics of the exchange rate for a given level of V_1 determining the lower bound of the perceived exchange rate interval prior to the first intervention.³⁰ Next we explore the sensitivity of the conclusions presented above to assumptions about V_1 . The effect of alternative V_1 parameters is indicated in Figure 6. The S -shaped curves illustrate that the exchange rate is perceived to be more credible for larger V_1 parameters. This leads to a narrower perceived target zone range. The intuition for the result is straightforward and can be sketched as follows. The moderating honeymoon effect is the stronger, the better the reputation of the policymaker leading to a narrower interval for the fundamentals, which may trigger an intervention. Formally stated, given the uniform distribution in equations (9) and (17)

³⁰In the theoretical modelling framework, V_1 is assumed to be exogenous neglecting the central bank's incentives to influence expectations with announcements. Rational central banks choose "verbal interventions" as a toolkit since it has the ability to enhance the predictability of monetary policy decisions, and potentially to help achieve central banks' macroeconomic objectives. On the other hand, when optimal policies are dynamically inconsistent, then announcements will only be considered to be cheap talk. For a survey of this partially credible commitment device, see Blinder et al. (2008).

the probability of an intervention becomes progressively more likely the larger V_1 is chosen.

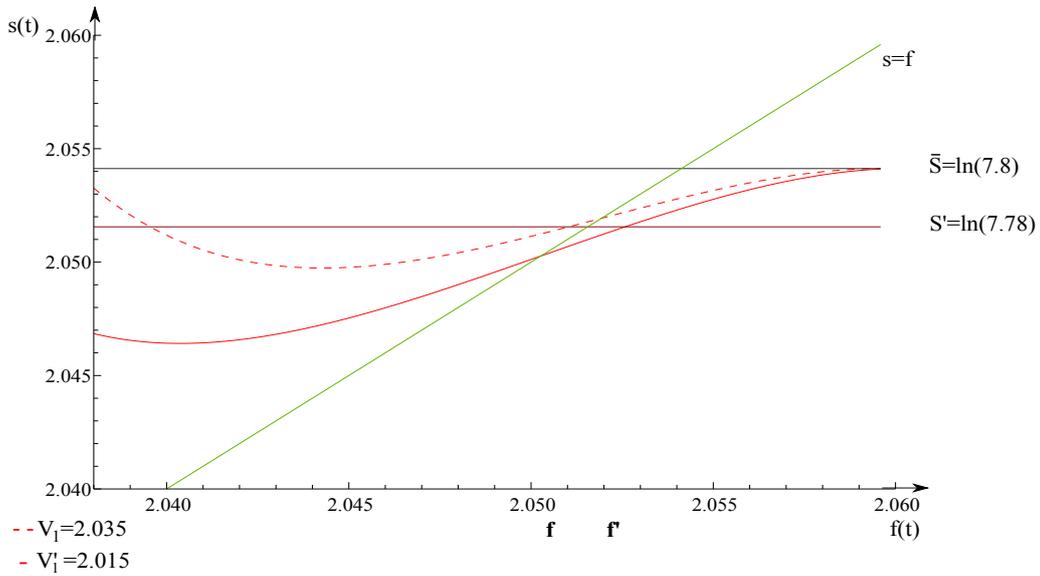


Figure 6: Set of S -shaped curves for $S' = \ln(7.78)$ and $V_1 \approx \ln(7.65)$ vs. $V_1' \approx \ln(7.50)$

We now return to our main theme and consider the exchange rate dynamics after the first intervention, i.e. S'' and S''' in Figure 4. Interventions are used by market participants to make inferences about the HKMA preferences, i.e. they are used to predict the implicit strong-side band. How does the intervention affect the belief of the public? As demonstrated in subsection 3.5 of the theoretical framework, credibility rises discretely with successive interventions. Furthermore, interventions influence future behaviour until the learning process brings beliefs closely in line with reality. The resulting exchange rate dynamics in S'' and S''' after the first intervention can be studied with the help of Figure 7.

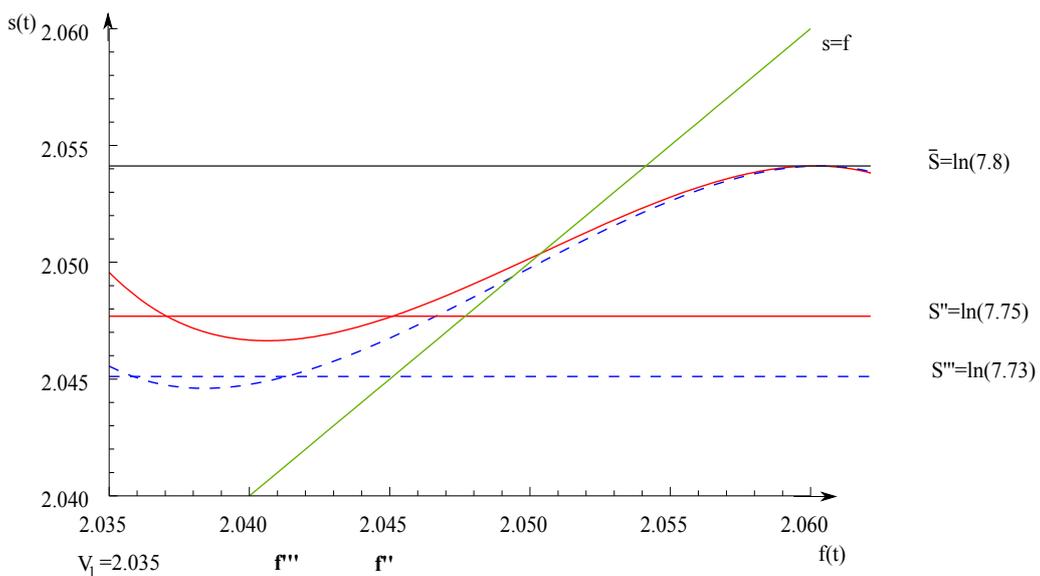


Figure 7: S -shaped curves with learning about the first intervention

Again we obtain a family of S -shaped curves. When comparing the exchange rate dynamics for $S'' = \ln(7.76)$ and $S''' = \ln(7.74)$, three differences are apparent. First, the relationship between the fundamentals and the exchange rate becomes steeper, and the non-linear effect is reduced for $S''' = \ln(7.74)$. Second, since the exchange rate has appreciated beyond $S'' = \ln(7.76)$, the perceived bandwidth has increased. Third, and not less importantly, central bank interventions are expected to be more likely and more intensive at $S''' = \ln(7.74)$. This means that expectations of further appreciations will be lower and the stabilizing effect of the undisclosed target zone is increasing.³¹

Over the last decade, central banks have implemented new breeds of target zone exchange rate regimes. In Hong Kong, a symmetric band (\underline{S}, \bar{S}) forming upper and lower limits for HKD fluctuations around the central parity was adopted in May 2005, as an integral part of the currency board regime (see Figure 1). By way of example, we finally calibrate the dynamics of the exchange rate according to equation (25) and (29) for S'^v (5 July 2005) and S^v (25 October 2007).

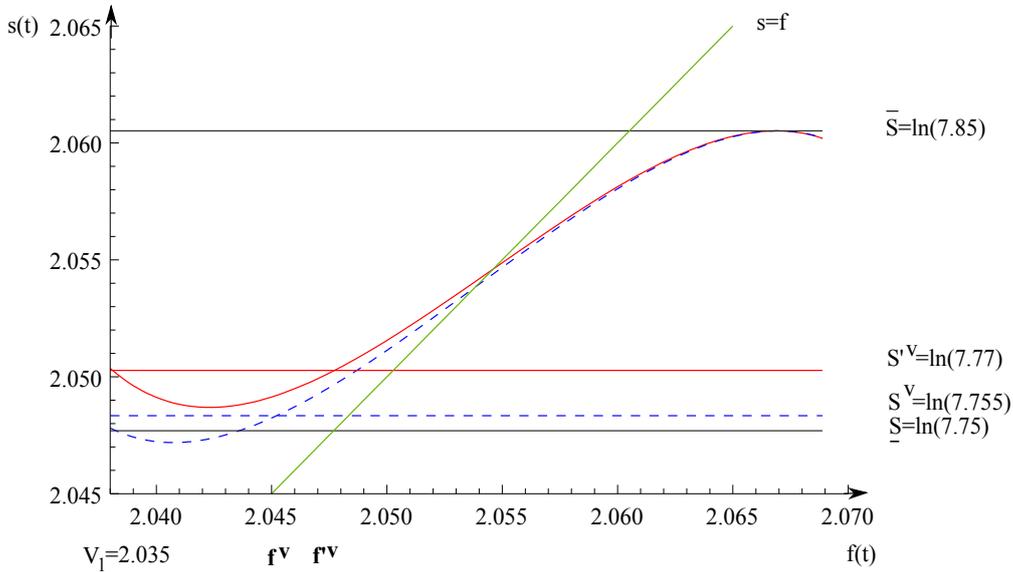


Figure 8: S -shaped exchange rate dynamics in the symmetric target zone

Viewed in retrospect, Figure 8 displays the exchange rate dynamics for $S'^v = \ln(7.77) \approx 2.0502$ and $S^v = \ln(7.755) \approx 2.0483$. We have assumed $N = 8$ in equation (29). The resultant a_A -coefficients for S'^v and S^v are $a_A \approx 0.309$ and $a_A \approx 0.827$, respectively (see Appendix A.4).

The calibration exercise suggests several conclusions. As a start, Figure 8 tells essentially the same qualitative story from the exchange rate dynamics point of view. Again we obtain an S -shaped behaviour of the exchange rate. The difference is in the detail. First, the exchange rate is now stabilised at the credible upper edge $\bar{S} = \ln(7.85) \approx 2.0605$.³² Second, at S'^v the lower (strong-side) edge of the band was perceived to be credible according to the calibrations. Market participants formed beliefs that the automaticity of the currency board system and/or market

³¹Further points after the next interventions yielded qualitatively similar, although quantitatively different, results. For brevity of exposition, only the exchange rate dynamics for S'' and S''' is presented here. Interested readers may obtain further calibrations of the model dynamics from the authors upon request.

³²Please note that the upper band of the target zone hasn't been tested to the limit during the sample period.

operations conducted in light of market conditions would be effective. This calibration result is consistent with the empirical fact that on 5 July 2005 the 1-year forward rate $S^{1-Year} = 7.7528$ was slightly above the strong-side band at 7.75 HKD/USD, i.e. no HKD appreciation beyond the strong-side band was expected by the market. Third, in autumn 2007 another episode with strong pressure for revaluation occurred (see Figure 4).³³ Subsequently, the HKMA started to conduct operations in the foreign exchange market with the intent to stabilise the rate. In the end, stability was maintained. The simulated *S*-shaped dynamics for 25 October 2007 indicates that this time appreciations of the HKD beyond the strong-side band $\underline{S} = \ln(7.75)$ were expected by the market, i.e. market participants had doubts about the ability and/or commitment of the HKMA to defend the narrow band in the face of exchange rate shocks.³⁴

5 Summary and Conclusion

The modelling framework in the paper provides a variation of the standard hard edge target zone concept. The theoretical literature provides little clue to understand the phenomenon of one-sided target zones. The classical model of Krugman (1991) describes the behaviour of the exchange rate in a perfectly known and credible symmetric target zone. Klein (1992) presents a model in which the width of the symmetric band is unknown to the public. The exchange rate is thus a function of the fundamentals and the expected bandwidth. Unfortunately, he only analyses symmetric bands.

We have considered a model of an asymmetric target zone with an undisclosed strong-side band and learning by market participants. Depending upon the occurrence or non-occurrence of interventions, market participants revise their estimates of the intervention probabilities and the location of the off-the-record strong-side band. To study the expectations-updating scheme and the mechanisms that give rise to the dynamics of the exchange rate, we consider Hong Kong's exchange rate regime since the turn of the millenium. We believe the model allows an elegant and parsimonious treatment of undisclosed asymmetric target zones. The model in this paper is stylised, but the results are rich and may be of relevance to other target zone economies. Nevertheless, the analysis may be pressed further, to show how uncertainty about exchange rate policy might result from uncertainty about broader government objectives like the Pearl River Delta integration, and how this uncertainty can be reduced by precommitments.

A Appendix

A.1 Derivation of equation (15)

At first we solve the integrals in (14). Manipulations of the integrand provide the primitive $\ln(e^{rv} + e^{r\bar{V}})$ of $\frac{re^{rv}}{e^{rv} + e^{r\bar{V}}}$. Therefore we obtain

³³Our analysis invites a number of questions. One of these is why the HKMA cannot take actions which would provide an unambiguous signal of its intentions. For a model with Markov switching and recurrent interventions to keep the HKD within the band, see Chen et al. (2010). The regime-dependent toolkit makes the perceived nonlinear swings in exchange rate pressure tractable.

³⁴This calibration result is consistent with the empirical fact that on 25 October 2007 the 1-year forward rate $S^{1-Year} = 7.728$ was below the strong-side band at 7.75 HKD/USD. One perceived risk was that the newly introduced symmetric bands might be widened in the future.

$$\begin{aligned}
E(A_1(V^l)) &= \int_{V_1}^{V_2} -\frac{r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \frac{1}{V_2 - V_1} dv \\
&= \frac{\sigma^2\tau}{2(V_1 - V_2)} \int_{V_1}^{V_2} \frac{re^{-r\bar{V}}e^{r\bar{V}}}{e^{rv} + e^{r\bar{V}}} dv \\
&= \frac{e^{-r\bar{V}}\sigma^2\tau}{2(V_1 - V_2)} \int_{V_1}^{V_2} \frac{re^{r\bar{V}} + re^{rv} - re^{rv}}{e^{rv} + e^{r\bar{V}}} dv \\
&= \frac{e^{-r\bar{V}}\sigma^2\tau}{2(V_1 - V_2)} \int_{V_1}^{V_2} r \frac{(e^{rv} + e^{r\bar{V}})}{e^{rv} + e^{r\bar{V}}} - \frac{re^{rv}}{e^{rv} + e^{r\bar{V}}} dv \\
&= \frac{e^{-r\bar{V}}\sigma^2\tau}{2(V_1 - V_2)} \int_{V_1}^{V_2} r - \frac{re^{rv}}{e^{rv} + e^{r\bar{V}}} dv \\
&= \frac{e^{-r\bar{V}}\sigma^2\tau}{2(V_1 - V_2)} \left(rv - \ln(e^{rv} + e^{r\bar{V}}) \right) \Big|_{V_1}^{V_2} \\
&= \frac{-e^{-r\bar{V}}\sigma^2\tau \left(r(V_1 - V_2) + \ln(e^{rV_2} + e^{r\bar{V}}) - \ln(e^{rV_1} + e^{r\bar{V}}) \right)}{2(V_1 - V_2)}
\end{aligned}$$

and

$$\begin{aligned}
E(A_2(V^l)) &= \int_{V_1}^{V_2} \frac{e^{r(v+\bar{V})}r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \frac{1}{V_2 - V_1} dv \\
&= -\frac{e^{r\bar{V}}\sigma^2\tau}{2(V_1 - V_2)} \int_{V_1}^{V_2} \frac{re^{rv}}{e^{rv} + e^{r\bar{V}}} dv \\
&= -\frac{e^{r\bar{V}}\sigma^2\tau}{2(V_1 - V_2)} \ln(e^{rv} + e^{r\bar{V}}) \Big|_{V_1}^{V_2} \\
&= \frac{e^{r\bar{V}}\sigma^2\tau \left(\ln(e^{rV_1} + e^{r\bar{V}}) - \ln(e^{rV_2} + e^{r\bar{V}}) \right)}{2(V_1 - V_2)}.
\end{aligned}$$

Now we derive the closed form expression (15) by using both expectation values in equation (14).

$$\begin{aligned}
s(t_0) &= f(t_0) - \frac{e^{f(t_0)r-r\bar{V}}\sigma^2\tau \left(r(V_1 - V_2) + \ln(e^{rV_2} + e^{r\bar{V}}) - \ln(e^{rV_1} + e^{r\bar{V}}) \right)}{2(V_1 - V_2)} \\
&\quad + \frac{e^{-f(t_0)r+r\bar{V}}\sigma^2\tau \left(\ln(e^{rV_1} + e^{r\bar{V}}) - \ln(e^{rV_2} + e^{r\bar{V}}) \right)}{2(V_1 - V_2)} \\
&= f(t_0) - e^{-r(f(t_0)+\bar{V})}\sigma^2\tau \\
&\quad \frac{\left(e^{2f(t_0)r}r(V_1 - V_2) - \left(e^{2f(t_0)r} + e^{2r\bar{V}} \right) \left(\ln(e^{rV_1} + e^{r\bar{V}}) - \ln(e^{rV_2} + e^{r\bar{V}}) \right) \right)}{2(V_1 - V_2)}
\end{aligned}$$

A.2 Derivation of λ

A easy way to choose λ properly is to derive it from condition (21), which claims

$$\begin{aligned} 1 &\stackrel{!}{=} \int_{V_1}^{\mathbf{f}} \lambda e^{2(v-\mathbf{f})} dv = \lambda \frac{1}{2} e^{2v-2\mathbf{f}} \Big|_{V_1}^{\mathbf{f}} = \lambda \frac{1}{2} - \lambda \frac{1}{2} e^{2V_1-2\mathbf{f}} \\ &\Rightarrow \lambda = \frac{1}{\frac{1}{2} - \frac{1}{2} e^{2V_1-2\mathbf{f}}}. \end{aligned}$$

A.3 Derivation of equation (25)

Before we prove equation (25), we provide a short introduction to the Hypergeometric function. The Hypergeometric function ${}_2F_1$ is the convergent Gauss Hypergeometric series

$${}_2F_1[a, b; c; z] = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{z^n}{n!},$$

where the circle of convergence is the unit circle $|z| = 1$ and $\Gamma(\cdot)$ denotes the Gamma function. The relationship between the factorial and the Gamma function is defined to be $\Gamma(n+1) = n!$ for all $n \in \mathbb{N}$. The functional equation of the Gamma function is $x\Gamma(x) = \Gamma(x+1)$ for all $x \in \mathbb{R}^+$. An important property of Hypergeometric functions is that the six functions ${}_2F_1[a \pm 1, b; c; z]$, ${}_2F_1[a, b \pm 1; c; z]$ and ${}_2F_1[a, b; c \pm 1; z]$ are contiguous to ${}_2F_1[a, b; c; z]$. They are used to express one of them as a linear combination of any two of the other contiguous functions and are derived by Gauss. The two relations, which are applied in the following, are

Property 1: $\frac{bz}{c} {}_2F_1[a+1, b+1; c+1; z] = {}_2F_1[a+1, b; c; z] - {}_2F_1[a, b; c; z]$

Property 2: $a {}_2F_1[a+1, b; c; z] = b {}_2F_1[a, b+1; c; z] - (b-a) {}_2F_1[a, b; c; z]$

Another important property is

Property 3: ${}_2F_1[a, b; b; z] = (1-z)^{-a}$

An useful overview of the linear combinations and other interesting relations is given in Abramowitz and Stegun (1972).

Being equipped with this short introduction to Hypergeometric functions ${}_2F_1$, we can turn to the calculation of the expectation value $\underline{E}(A_1(V^l))$.

$$\begin{aligned} \underline{E}(A_1(V^l)) &= \int_{V_1}^{\mathbf{f}} -\frac{r\sigma^2\tau}{2(e^{rv} + e^{r\bar{V}})} \lambda e^{2(v-\mathbf{f})} dv \\ &= -\frac{1}{2} r\sigma^2\tau \lambda \int_{V_1}^{\mathbf{f}} \frac{e^{2(v-\mathbf{f})}}{e^{rv} + e^{r\bar{V}}} dv \\ &= -\frac{1}{4} r \lambda \sigma^2 \tau e^{-2\mathbf{f}+2v-r\bar{V}} {}_2F_1\left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})}\right] \Big|_{V_1}^{\mathbf{f}} \\ &= -\frac{1}{4} e^{-2\mathbf{f}-r\bar{V}} r \lambda \sigma^2 \tau \\ &\quad \left(e^{2\mathbf{f}} {}_2F_1\left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(\mathbf{f}-\bar{V})}\right] - e^{2V_1} {}_2F_1\left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(V_1-\bar{V})}\right] \right) \end{aligned}$$

In order to prove the third equals sign, we show that the derivative of $\frac{1}{2} e^{-2\mathbf{f}+2v-r\bar{V}} {}_2F_1 \left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right]$ equals the integrand in the second equation. As preliminary work we consider the derivative of the Hypergeometric function at first. The derivation of the considered Hypergeometric function is

$$\begin{aligned}
& \frac{d}{dv} \left({}_2F_1 \left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right] \right) \\
&= \frac{d}{dv} \left(\frac{\Gamma(\frac{2}{r}+1)}{\Gamma(1)\Gamma(\frac{2}{r})} \sum_{n=0}^{\infty} \frac{\Gamma(1+n)\Gamma(\frac{2}{r}+n)}{\Gamma(\frac{2}{r}+1+n)} \frac{-e^{nr(v-\bar{V})}}{n!} \right) \\
&= \frac{\Gamma(\frac{2}{r}+1)}{\Gamma(1)\Gamma(\frac{2}{r})} \sum_{n=0}^{\infty} \frac{\Gamma(1+n)\Gamma(\frac{2}{r}+n)}{\Gamma(\frac{2}{r}+1+n)} \frac{nr(-e^{nr(v-\bar{V})})}{n!} \quad TF \\
&= \frac{\Gamma(\frac{2}{r}+1)}{\Gamma(1)\Gamma(\frac{2}{r})} \sum_{n=0}^{\infty} \frac{\Gamma(2+n)\Gamma(\frac{2}{r}+n+1)}{\Gamma(\frac{2}{r}+n+2)} \frac{r(-e^{r(v-\bar{V})})^{n+1}}{n!} \quad TF \\
&= -re^{r(v-\bar{V})} \frac{2}{2+r} \frac{\Gamma(\frac{2}{r}+2)}{\Gamma(2)\Gamma(\frac{2}{r}+1)} \sum_{n=0}^{\infty} \frac{\Gamma(2+n)\Gamma(\frac{2}{r}+1+n)}{\Gamma(\frac{2}{r}+2+n)} \frac{(-e^{r(v-\bar{V})})^n}{n!} \quad TF \\
&= -re^{r(v-\bar{V})} \frac{2}{2+r} {}_2F_1 \left[2, \frac{2}{r}+1; \frac{2}{r}+2; -e^{r(v-\bar{V})} \right].
\end{aligned}$$

The third equals sign holds, because the first summand in the line above is zero. For the fourth equals sign the functional equation of the Gamma function is used, which results in a Hypergeometric function with new parameters.

According to the order of the equals signs the properties 1-3 are applied now.

$$\begin{aligned}
& r \frac{-2e^{r(v-\bar{V})}}{2+r} {}_2F_1 \left[2, \frac{2}{r}+1; \frac{2}{r}+2; -e^{r(v-\bar{V})} \right] \\
&= r \left({}_2F_1 \left[2, \frac{2}{r}; \frac{2}{r}+1; -e^{r(v-\bar{V})} \right] - {}_2F_1 \left[1, \frac{2}{r}; \frac{2}{r}+1; -e^{r(v-\bar{V})} \right] \right) \\
&= r \left(\frac{2}{r} {}_2F_1 \left[1, \frac{2}{r}+1; \frac{2}{r}+1; -e^{r(v-\bar{V})} \right] - \frac{2}{r} {}_2F_1 \left[1, \frac{2}{r}; \frac{2}{r}+1; -e^{r(v-\bar{V})} \right] \right) \\
&= 2 \left(\frac{1}{1+e^{r(v-\bar{V})}} - {}_2F_1 \left[1, \frac{2}{r}; \frac{2}{r}+1; -e^{r(v-\bar{V})} \right] \right).
\end{aligned}$$

Now we have obtained all ingredients for differentiating the above mentioned primitive.

$$\begin{aligned}
& \frac{d}{dv} \left(\frac{1}{2} e^{-2\mathbf{f}+2v-r\bar{V}} {}_2F_1 \left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right] \right) \\
&= e^{-2\mathbf{f}+2v-r\bar{V}} {}_2F_1 \left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right] \\
& \quad + e^{-2\mathbf{f}+2v-r\bar{V}} \left(\frac{1}{1+e^{r(v-\bar{V})}} - {}_2F_1 \left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right] \right) \\
&= e^{-2\mathbf{f}+2v-r\bar{V}} \left(\frac{1}{1+e^{r(v-\bar{V})}} - {}_2F_1 \left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right] + {}_2F_1 \left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(v-\bar{V})} \right] \right) \\
&= \frac{e^{2(v-f)}}{e^{r\bar{V}}(1+e^{r(v-\bar{V})})} \\
&= \frac{e^{2(v-f)}}{e^{r\bar{V}}+e^{rv}}.
\end{aligned}$$

The other expectation value is given without a prove for the primitive, as it is derived likewise.

$$\begin{aligned}
\underline{E}(A_2(V^l)) &= \int_{V_1}^{\mathbf{f}} \frac{e^{r(v+\bar{V})} r \sigma^2 \tau}{2(e^{rv} + e^{r\bar{V}})} \lambda e^{2(v-\mathbf{f})} dv \\
&= \frac{e^{-2\mathbf{f}+(2+r)v} r \lambda \sigma^2 \tau {}_2F_1 \left[1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(v-\bar{V})} \right]}{2(2+r)} \Bigg|_{V_1}^{\mathbf{f}} \\
&= \frac{1}{2(2+r)} e^{-2\mathbf{f}} r \lambda \sigma^2 \tau \\
&\quad \left(e^{\mathbf{f}(2+r)} {}_2F_1 \left[1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(\mathbf{f}-\bar{V})} \right] - e^{V_1(2+r)} {}_2F_1 \left[1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(V_1-\bar{V})} \right] \right)
\end{aligned}$$

Hence the closed form solution results in

$$\begin{aligned}
s(t) &= f(t) + \frac{1}{2(2+r)} e^{-2\mathbf{f}-f(t)r} r \lambda \sigma^2 \tau \\
&\quad \left(e^{\mathbf{f}(2+r)} {}_2F_1 \left[1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(\mathbf{f}-\bar{V})} \right] - e^{V_1(2+r)} {}_2F_1 \left[1, \frac{2+r}{r}; 2 + \frac{2}{r}; -e^{r(V_1-\bar{V})} \right] \right) \\
&\quad - \frac{1}{4} e^{-2\mathbf{f}-r\bar{V}+f(t)r} r \lambda \sigma^2 \tau \\
&\quad \left(e^{2\mathbf{f}} {}_2F_1 \left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(\mathbf{f}-\bar{V})} \right] - e^{2V_1} {}_2F_1 \left[1, \frac{2}{r}; \frac{2+r}{r}; -e^{r(V_1-\bar{V})} \right] \right).
\end{aligned}$$

A.4 Calculation of the coefficient a_A

Let's suppose that market participants assign the same weight to the last $N = 8$ interventions in their expectations formation process. In case of $S'^v = \ln(7.77)$ and $S^v = \ln(7.755)$, these market operation dates and the corresponding HKD spot exchange rates are:

12.10.04	7.7925	10.11.04	7.7812
25.10.04	7.7771	08.12.04	7.7705
27.10.04	7.7777	10.12.04	7.7753
01.11.04	7.7799	27.05.05	7.7775

As a start, this allows to calculate the logarithmized exchange rates $S_{T_1} - S_{T_8}$. The value of a_A for $S'^v = \ln(7.77)$ can then be solved from

$$(1 - a_A) \frac{1}{8} \sum_{i=1}^8 S_{T_i} + a_A S_A = S'^v.$$

This equation is derived from equation (29). However, the question arises, why the left hand side is equalized with S'^v . One has to take into consideration that whenever a smaller fundamental \mathbf{f} is observed, the interval of possible intervention triggering exchange rates is truncated. Where does that leave us? In case of S'^v , the original interval $[S_1, \mathcal{S}_{\mathcal{T}_N}]$ is cut to $[S_1, S'^v]$ and therefore we can use S'^v to calculate the unknown coefficient a_A . The resulting parameter is $a_A \approx 0.309$. In an analogous manner, $a_A \approx 0.827$ is obtained for S^v .

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