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Endogenous Borrowing Constraints, Human Capital Investment and Optimal Income Taxation

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Abstract

This paper employs a two-period life-cycle model to derive the optimal tax policy when educational investments are subject to endogenous credit constraints. Credit constraints arise from the limited commitment of creditors to repay loans and from the moral hazard problem due to private information on learning effort and work effort. We show that optimal tax policy differ from that in a similar model with exogenous credit constraints. In particular, if the welfare gain of subsidizing labor income by relaxing credit constraints dominates the welfare loss of increasing borrowing demand, regressive taxation is optimal. The reason is that subsidizing labor income increases the incentive to invest in education and to work, thus mitigating the moral hazard problem adherent to credit for educational investment. Furthermore, we find that no intervention could be optimal even if private capital market is imperfect.

JEL classification: H21, I2, J2

Keywords: labor taxation, human capital investment, credit constraints
1 Introduction

Redistributive policy and public schools are often justified by the existence of credit constraints facing poor individuals when they invest in education. Using educational data of adoptees, Plug and Vijverberg (2005) find a causal effect of family income on the children’s completed schooling years. Stinebricker and Stinebricker (2008) suggest that credit constraints influence how much the student is enjoying the school and play an important causal role in the drop-out decision of students from low-income families. Although no evidence has been found on the effects of credit constraints on college entrance decision (see also Cameron and Heckmann, 2001 and Caneiro and Heckmann, 2002), Kean and Wolpin (1999) find that borrowing constraints have significant effects on students’ consumption and work choices.

Facing binding credit constraints, children from poor family cannot invest sufficiently in education, which leads to lower growth rate, lower intergenerational mobility and larger inequality (see Loury, 1981; Galor and Zeira, 1993; Mookherjee and Ray, 2003 and Galor and Moav, 2004 etc.). To mitigate the negative effects of credit constraints optimal policy should feature public provision of education and redistribution towards the poorest communities (see e.g. Glomm and Ravikumar, 1992; Fernandez and Rogerson, 1996, 1998 and Benabou, 1996). Jacobs and Yang (2009) show that progressive taxation is optimal since it mimics governmental loans by redistributing income from working and high-income period to schooling and low-income period.

However, these studies assume exogenous credit constraints and ignore the fact that credit constraints can be affected by governmental policy. Credit constraints arise from limited commitment of debitors to repay loans. In addition, the moral hazard problem due to the non-observability of effort in education as well as in labor supply makes human capital a bad collateral and thus makes the credit for educational investment more difficult to access. Governmental policy may change the endogenous credit constraints by affecting the incentive of creditors to repay. Therefore, optimal policy derived under the assumption of exogenous credit constraints could be misleading. Krueger and Perri (1999) show that redistributive taxation can exaggerate endogenous credit constraints and lead to a lower welfare. Andolfatto and Gervais (2006) argue that education subsides and a pension programm financed by tax on working population may actually lead to lower levels of human capital when credit constraints is endogenized. By simulating an educational investment model with endogenous credit constraints, Lochner and Monge-Naranjo (2002) find that education subsidies have substantially greater effect than implicated in model with exogenous credit constraints.

This paper aims to analyze optimal tax policy when individuals face endogenous credit constraints in educational investment. To that end, we employ a two-period life-cycle model with identical agents, who invest in first period in education and works in second
period. Credit constraints are determined by the condition that individuals should not be better off by defaulting. Following Lochner and Monge-Naranjo (2002, 2008), we assume that debtors can garnish a part of creditor’s earnings in case of default. Furthermore, we assume that educational investment is not verifiable. The costs of education include tuition fees, expenditure for computer and books etc, additional costs for accommodation and forgone earnings. All these costs are hardly verifiable except for tuition fees, which is only a small share of the total costs. Educational costs can also be thought as learning effort, which is not observable. Labor supply in second period is not observable either. These two assumptions imply a moral hazard problem that the collateral, i.e. the creditors’ garnishable income, is unknown when credit is extended.

We show that, when credit constraints are binding, individuals invest too low in education and consume too low in first period. When government can use age-specific transfers, the optimal policy is to transfer income from second to first period, which is in fact governmental loans. Thus, endogenizing credit constraints does not change the optimal policy qualitatively (see Jacobs and Yang, 2009), only the effects of age-specific transfers are bigger since they relax credit constraints. However, when age-specific transfers are not available, the optimality of progressive taxation as in case of exogenous credit constraints cannot be shown any more. We derive the optimal linear labor tax rate, which depends on the welfare gain of redistributing income to the constrained period, the welfare effect of the tax policy’s impact on credit constraints and the efficiency costs of tax distortion in education and labor supply. Thus, the endogeneity of credit constraints can have completely different implications for optimal policy than models with exogenous credit constraints. In particular, regressive taxation, i.e. lump-sum taxes and labor income subsidy, could be optimal. The intuition is as follows. Since individuals planning to default invest less in education and work less in second period in order to reduce default costs, subsidy to labor income, which is equivalent to education subsidy and labor supply subsidy, benefits non-defaulting individuals more than defaulting ones and can therefore relax credit constraints. As long as the welfare gain of regressive taxation by relaxing credit constraints outweighs the welfare costs of increasing credit demand, redistributing income from constrained period to high-income period is welfare-improving.

The closest to our analysis is Andolfatto and Gervais (2005), which shows that with endogenous credit constraints it is optimal to redistribute from young and old to working individuals. However, they use only age-specific transfers for redistribution, which in our model would imply an optimal policy of redistributing from working individuals to young. The completely opposite implications arise from the different assumptions about default punishment. They assume that debtors are punished by being excluded from capital market if they default. Consequently, redistributing income from old to working individuals relaxes credit constraints by increasing default costs, namely the costs of no-saving for old age. Differently, we assume that creditors can garnish part of
defaulting agents’ earnings. Redistributing from working period to young period in ways of lump-sum transfers increases the costs of losing part of the earnings and relaxes credit constraints thereafter.

Our analysis is also related to Lochner and Monge-Naranjo (2002) and Krueger and Perri (1999). Monge-Naranjo (2002) show that education subsidies relax credit constraints and have greater impact on human capital compared to the case of exogenous credit constraints. Particularly, education subsidies increase educational investment and future earnings. Default becomes less attractive since default costs, i.e., losing part of the earnings, increase. Their analysis assumes exogenous labor supply and the verifiability of total educational investment. As a result, there is no moral hazard problem since the amount that banks can garnish in case of default is known when credit is extended. In this paper, however, the non-observability of education and labor supply leads to moral hazard problem that exaggerates credit constraints for educational investment. Krueger and Perri (1999) analyze quantitatively the effects of tax system on welfare with endogenous credit constraints. They show that, under plausible conditions, the increase of tax progressivity leads to lower welfare. Thus, our results seem to confirm their findings. Nevertheless, they do not consider educational investment and labor supply decision.

The rest of the paper is organized as follows. In section 2 we lay out the model environment and derive the endogenous credit constraints under non-defaulting condition. Section 3 analyzes the optimal tax policy for both the case of age-specific lump-sum transfers and the case of no age-specific transfers. The last section concludes.

2 The Model

The economy is populated by a continuum of identical individuals whose mass is normalized to one. The individuals live for two periods. In the first period individual does not work, but invests in education and consumes. Since we are interested in credit constraints, we assume that agent does not have sufficient initial wealth to finance its optimal educational investment and consumption in first period. Without loss of generality, we set the initial wealth to be zero. After acquiring human capital, agent supplies labor in second period and consumes all its wealth. There is a perfect competitive capital market providing loans for individuals. The economy is a small open economy, which implies that banks can raise funds at a constant interest rate $1 + r$.

2.1 Preference and human capital technology

The utility of the individual is separable in consumption and labor and between periods

$$U = u(c^1) + \beta (u(c^2) - v(l))$$  \hspace{1cm} (1)
with $c^1$ and $c^2$ denoting consumptions in first and second period respectively and $l$ the labor supply in second period. The separability is assumed for reason of simplicity and does not affect the main results of the paper. \( \beta \) is the time preference and is equal to \( \frac{1}{1+r} \). The sub-utility function $u(.)$ is assumed to be increasing and concave, whilst the disutility function $v(.)$ is increasing and convex. Furthermore, the Inada-conditions are fulfilled to avoid corner solutions.

When agents are young, they decide on consumption $c^1$, investment in human capital $e$ and the required borrowing $a$. The costs of human capital investment $e$ are assumed to be only monetary and may include tuition fees, forgone earnings and additional costs for computer, accommodation and books etc. than what would otherwise occur. Educational costs can also be seen as effort costs, which reduces first period utility. We assume that these costs are unobservable, since additional costs for computer, accommodation and books, forgone earnings and effort are badly verifiable. Although tuition fees are observable, the share of tuition fees in the total costs of education is small for higher education. In second period agents become adults and supply labor according to the wage rate $w(e)$ with $w'(e) > 0$ and $w''(e) < 0$. Inada-conditions for $w(e)$ are fulfilled as well.

The tax system consists of a linear labor tax with the rate $t$ and lump-sum transfers in both periods, $g^1$ and $g^2$, where the superscript denotes the periods. The agents’ budget constraints for both periods are consequently

$$c^1 = -e + a + g^1,$$

and

$$c^2 = (1-t)w(e)l - (1+r)a + g^2. \tag{3}$$

Following standard literature in optimal taxation, only gross income are observable. Since educational investment is private information, neither wage rate nor labor supply can be verified. Without loss of generality we assume that the production function of wage rate $w$ is Cobb-Douglas

$$z \equiv w(e)l = e^\epsilon l \tag{4}$$

where $\epsilon$ is the constant elasticity of wage rate w.r.t educational investment.

### 2.2 First-best allocation

For comparison we first describe the optimal allocation with perfect capital market. The representative agent maximizes utility (1) subject to (2) and (3). As a result, consumption is smoothed according to the Euler’s equation $\frac{u_{c_2}}{u_{c_1}} = 1 + r$. The assumption $\beta = \frac{1}{1+r}$ implies immediately that in optimum $c^1 = c^2$. Moreover, agent equals the marginal costs of education with its marginal return $(1-t)w'(e)l = 1+r$ and the optimal labor supply
is given by \((1 - t) w(e) = v'(l)\).

Labor tax distorts educational investment as well as labor supply. Since we have identical agents and therefore no distributional concern, a first-best allocation could be achieved by setting labor tax rate to zero and using lump-sum taxes to raise exogenous government expenditure.

### 2.3 Endogenous credit constraints

Credit constraints arise from agents’ limited commitment to repay loans. Following Lochner and Monge-Naranjo (2002, 2008) we assume that banks can garnish a fraction \(\gamma\) of the agents’ earnings if they default. However, we assume that banks cannot garnish lump-sum transfer the agents receive in second period \(g^2\). Lump-sum transfer can be thought as public goods and social insurance that cannot be seized by banks. Punishment of exclusion from credit markets after default as considered in Kehoe and Levine (1993, 2000) and Andolfatto and Gervais (2006) is not considered in our two-period model.

An agent would default if the utility of defaulting is higher than that of repaying. Unlike in Lochner and Monge-Naranjo (2002, 2008), educational investment and labor supply in our model are private information that cannot be observed by banks. As a result, the amount of earnings that banks can garnish in case of default is unknown when credit is extended to agents in first period. Agents who plan to default can reduce the costs of default by reducing educational investment and labor supply. Therefore, the non-default condition is that the maximal utility agents can achieve if they repay should not be lower than that if they default.

After inserting budget constraints (2) and (3) in utility function (1), the lagrangian function for agents’ maximization problem when the credit limit is \(\bar{a}\) and when the agents repay is

\[
\max_{e,a,l} \mathcal{L} = u(-e + g^1 + a) + \beta \left( u \left( (1 - t)w(e)l - (1 + r)a + g^2 \right) - v(l) \right) + \mu (\bar{a} - a), \tag{5}
\]

where \(\mu\) is the lagrangian multiplier for credit constraint \(a \leq \bar{a}\). \(\mu\) gives the shadow price of relaxing credit limit \(\bar{a}\) by one unit. We assume that agents are credit constrained, since the case of slack credit constraints is not interesting. This assumption implies that

\[
\begin{align*}
a &= \bar{a}; & \mu &= u_1 - \beta(1 + r)u_2 > 0, \\
\frac{u_1}{\beta u_2} &= (1 - t)u'(e)l > 1 + r, \\
\frac{v'(l)}{u_2} &= (1 - t)w(e).
\end{align*}
\tag{6}
\]

Credit constrained agents cannot borrow the unconstrained optimal amount of credit to finance his consumption and educational investment. As a result, both first period
consumption and educational investment are distorted downwards compared to first-best allocation: $\frac{\mu}{\beta u_2} > 1+r$ and $(1-t)w'(e)l > 1+r$. Agents would like to consume more and invest more in education if they could borrow more than $\bar{a}$.

Binding credit constraints act like an implicit tax on borrowing and educational investment. We define this implicit tax as

$$\pi = 1 - (1+r) \frac{\beta u_2}{u_1},$$

(7)

Accordingly, we can rewrite the first order condition for educational investment as

$$(1-\pi) (1-t) w'(e) l = 1 + r.$$  

(8)

Therefore, $\pi$ measures the extent to which the intertemporal consumption and educational investment are distorted by credit constraints. The lower the credit limit $\bar{a}$, the more agents are credit constrained and the higher is the implicit tax $\pi$. Because agents save too much (or borrow too few) and invest too low in education, labor supply in second period is lower than that without credit constraints. Substituting the optimal decisions given by the first order conditions in utility function (1), the indirect utility for repaying agents can be denoted as a function of tax policy, interest rate and credit limit $V(t, g^1, g^2, r, \bar{a})$.

The lagrangian function for agents’ maximization problem if they plan to default is

$$\max_{e,a,l} L = u (-e + g^1 + a) + \beta \left( u \left( (1-\gamma) (1-t) w (e) l + g^2 \right) - v (l) \right) + \mu (\bar{a} - a).$$

(9)

Since agents do not repay loans in second period, they would borrow as much as they can, i.e. $a_d = \bar{a}$. We use the subscript $d$ to denote the variables in case of default. The first order conditions for defaulting agents are

$$a_d = \bar{a}; \mu = u_{1d} > 0,$$

$$\frac{u_{1d}}{\beta u_{2d}} = (1-t) (1-\gamma) w'(e_d) l_d,$$

$$\frac{v'(l_d)}{u_{2d}} = (1-t) (1-\gamma) w(e_d).$$

(10)

We can see that defaulting agents would choose education $e$ and labor supply $l$ differently than agents who repay. Define $\frac{u_{1d}}{\beta u_{2d}} = \frac{u_1}{\beta u_2} (1-\pi_d)$ such that $\pi_d$ measures the difference between the rate of intertemporal substitution for defaulting agents and that for non-defaulting ones. When there is no credit limit, defaulting agents would consume an indefinitely large amount in first period and $\pi_d = 1$. Using $\pi_d$, the first-order condition for education can be rewritten as

$$\frac{1-\pi}{1-\pi_d} (1-t) (1-\gamma) w'(e_d) l_d = 1 + r.$$  

(11)
Thereafter, the implicit tax on educational investment due to credit constraints is 
\[
1 - \pi_1 - \pi_d
\]
for defaulting agents. Again, the indirect utility for defaulting agents can be denoted as a function of tax policy, interest rate, credit limit and the punishment parameter, \(V_d(t, \gamma, g^1, g^2, r, \bar{a})\).

For both maximization problems (5) and (9) the second-order conditions require that the marginal utility of second period consumption should decrease sufficiently fast, the share of education in wage rate \(\epsilon\) should not be too high and the disutility of labor should increase fast enough (see Appendix A.1). These conditions ensure that the positive feedback between education and labor supply is not too strong such that interior solutions are obtained. We assume that the second-order conditions are always fulfilled.

For given tax policy, interest rate and borrowing limit, the non-default condition is
\[
V(t, g^1, g^2, r, a^*) \geq V_d(t, \gamma, g^1, g^2, r, \bar{a}) \quad (12)
\]
This condition means that agents cannot be better off by defaulting and optimizing their choices correspondingly. Notice that for \(\bar{a} = 0\) the utility of repaying is higher than that of defaulting, since defaulting brings no gain but only causes the costs of losing part of the earnings. We assume that the optimal borrowing without credit constraints and when agents repay is \(a^*\), for certain tax policy and interest rate. We make the assumption that
\[
V(t, g^1, g^2, r, a^*) < V_d(t, \gamma, g^1, g^2, r, a^*) \quad (13)
\]
which implies that agents would default if they can borrow \(a^*\). Consequently, no banks would lend the amount \(a^*\) to agents, since they know for sure that agents would default. This assumption therefore ensures the existence of endogenous credit constraints.

The indirect utility \(V_d\) is increasing and concave in \(\bar{a}\), since \(\frac{\partial V_d}{\partial \bar{a}} = \mu_d = u_{1d}\) and first period consumption always increases with borrowing limit. The indirect utility \(V\) is increasing and concave in \(\bar{a}\) as well, as long as \(\bar{a} < a^*\). This is because the marginal utility of borrowing limit is equal to \(\frac{\partial V}{\partial \bar{a}} = \mu = u_1 - (1 + r) \beta u_2\), which is positive and decreasing in \(\bar{a}\) for \(\bar{a} < a^*\) and equal to zero for \(\bar{a} \geq a^*\). From the concavity of \(V\) and \(V_d\) for \(\bar{a} < a^*\), the fact that \(V(t, g^1, g^2, r, 0) > V_d(t, \gamma, g^1, g^2, r, 0)\) and the assumption (13), we can conclude that in a \((a, V)\) diagram \(V_d\) would cut \(V\) only once from below in the interval \([0, a^*]\).

Because the credit market is perfectly competitive, the equilibrium borrowing limit would be such that agents are indifferent between repaying and defaulting. Denote the equilibrium borrowing limit as \(A\), then \(A\) is determined by the condition
\[
V(t, g^1, g^2, r, A) = V_d(t, \gamma, g^1, g^2, r, A) \quad (14)
\]
Agents would have a higher utility by repaying if the credit limit is lower than \(A\) and
they would have a higher utility by defaulting if the credit limit is higher than $A$. Solving the equation (14), we can denote the equilibrium credit limit as a function $A = A(t, g^1, g^2, r, \gamma)$. By construction we have $A < a^*$.

The comparative statics of endogenous credit constraints depends on the effects of the parameters on the indirect utility $V$ and $V_d$. Applying Roy’s lemma yields:

$$\frac{\partial V}{\partial t} = -\beta u_2 w(e) l, \quad \frac{\partial V_d}{\partial t} = -\beta u_{2d} (1 - \gamma) w(e_d) l_d; \quad (15)$$

$$\frac{\partial V}{\partial g^1} = u_1, \quad \frac{\partial V_d}{\partial g^1} = u_{1d}, \quad (16)$$

$$\frac{\partial V}{\partial g^2} = \beta u_2, \quad \frac{\partial V_d}{\partial g^2} = \beta u_{2d}, \quad (17)$$

$$\frac{\partial V}{\partial r} = - (1 + r) \beta u_2, \quad \frac{\partial V_d}{\partial r} = 0. \quad (18)$$

Since defaulting agents have a lower marginal return to educational investment, agents would invest less in education and work less if they plan to default.\(^1\) It follows that $c^1 < c^1_d$, $e > e_d$ and $l > l_d$. Furthermore, since $V = V_d$ is always fulfilled in equilibrium, we have $c^2 > c^2_d$. Consequently, the rate of intertemporal substitution is lower for defaulting agents than for repaying ones, $\frac{u_{1d}}{\pi_{1d}} < \frac{u_1}{\pi_2}$ and $\pi_d > 0$.

Increasing first period transfer $g^1$ increases the utility of repaying more than increasing the utility of defaulting, since $u_1 > u_{1d}$. Thereafter, endogenous borrowing limit increases. Intuitively, since defaulting agents consume more in first period, increasing first period consumption benefits the non-defaulting agents more than defaulting ones. On the other hand, increasing second period transfer $g^2$ tightens the incentive constraints and lowers the borrowing limit. This is because lump-sum transfer in second period is not seizable by banks and makes the punishment of losing part of the earnings less severe. A higher interest rate tightens the incentive constraints as well, which is very intuitive. The higher the interest rate, the higher the costs of repaying and the more attractive is defaulting.

However, the effect of a higher labor tax $t$ is ambiguous. On the one hand, a higher labor tax harms defaulting agents more by reducing second period income, since defaulting agents consume less in second period and have a higher marginal utility of consumption. On the other hand, a higher tax rate harms defaulting agents less since they have lower earnings due to lower educational investment and labor supply and pay less taxes for a marginal increase in tax rate. Therefore, the total effect of a higher labor tax depends on which effect dominates.

\(^1\)The optimal choices in case of repaying is not optimal any more if agents plan to default, as can be seen by $\frac{u_{1d}}{\pi_{1d}} > (1 - t) (1 - \gamma) w^* (e^*) t^*$ and $\frac{u_{1d}}{\pi_{1d}} > (1 - t) (1 - \gamma) w(e^*)$, where $e^*$ and $t^*$ are the optimal choices of agents in case of repaying. Hence, agent would invest less in education and work less if it plans to default than if it plans to repay loans.
3 Optimal Tax Policy

In this section we first formulate the governmental problem and then derive the optimal tax policy. The government is benevolent and the tax system consists of a flat labor tax and lump-sum transfers in both periods. We assume without loss of generality that there is no exogenous government expenditure requirement.\(^2\) The government budget constraint is given by

\[
 tw(e)l = (1 + r) g^1 + g^2. \tag{19}
\]

The government chooses \(g^1, g^2\) and \(t\) to maximize the indirect utility of a representative agent. The lagrangian function for governmental problem is

\[
 W = V + \eta \left( tw(e)l - (1 + r) g^1 - g^2 \right), \tag{20}
\]

where the lagrangian multiplier \(\eta\) measures the shadow price of governmental revenue. When age-specific transfer is available, government can choose \(g^1\) differently from \(g^2\); when age-specific transfer is not available, we have \(g^1 = g^2\).

3.1 Age-specific transfer

We first derive the optimal tax policy when government can use age-specific transfers. With exogenous credit constraints, Jacobs and Yang (2009) argue that the availability of age-specific transfers enables the government to remove credit constraints perfectly. The government only has to provide the amount of credit that individuals could not borrow on the private loan market and requires them to pay it back plus interest income in form of lump-sum taxes. In fact, government acts through age-specific lump-sum transfers as a lender to replace missing or imperfect private credit market. As a result, agents are not credit constrained any more and the optimal labor tax rate is zero.

If credit constraints are endogenous, such age-specific transfers would change the individuals’ incentive to repay and thus affect the endogenous borrowing limit. As shown before, a higher first period lump-sum transfer relaxes the incentive constraints of repaying whilst a higher second period transfer tightens the incentive constraints. Consequently, age-specific transfers that redistribute income from second to first period \((g^1 = -(1 + r) g^2 > 0)\) would increase agents’ incentive to repay and increase borrowing limit \(A\). The lowest level of lump-sum transfers needed to remove binding credit constraints is characterized by

\[
 A(g^{1*}) + g^{1*} = a^{LP}, \tag{21}
\]

\(^2\)Any exogenous government expenditure requirement can be financed by lump-sum taxes in such a way that credit constraints are not affected.
where \(a^{LP}\) is the optimal borrowing in a laissez-fair economy with perfect credit market. The government should provide the amount of credit to young agents such that the endogenous credit constraints are not binding any more. If government transfers more resources than \(g^1\), banks would like to lend more than agents want to borrow. The public lending crowds out private lending. For \(g^1 = a^{LP}\) there would be no private lending any more.

Compared to the case of exogenous credit constraints, a lower level of lump-sum transfers are needed to remove binding credit constraints perfectly. Thus, the endogeneity of credit constraints does not change the optimal policy qualitatively when age-specific transfers are available.

Andolfatto and Gervais (2006) also analyze the optimal age-specific lump-sum transfers with endogenous credit constraints. However, they argue that the optimal policy would be transferring income from young and old to working agents. The difference of our result to theirs arises from the different assumptions about the punishment of defaulting. In their model default is punished by being excluded from capital market, which means that agents cannot save any more for old age if they default. Transferring income from old and young to middle-aged agents makes the costs of no-saving higher and relaxes the incentive constraints thereafter. In our two-period model, however, the punishment of exclusion from capital market is not considered. Defaulting agents are punished by losing part of their labor income. Transferring income from adults to young agents not only reduces the credit demand but also relaxes incentive constraints, since defaulting agents benefit less from the (mandatory) public lending on which they cannot default.

The first-best result with age-specific transfers arises from the assumption that government has higher enforcement than private banks and agents cannot default on governmental loans. This assumption is not very harmful since government does have higher enforcement through tax system. In addition, government faces lower costs of collecting repayment and a less severe tracking problem than private banks.

### 3.2 Optimal tax without age-specific lump-sum transfer

Age-specific lump-sum transfers (or taxes) are normally difficult to implement in an overlapping-generation world due to no age-discrimination. In some countries age-discrimination is legally forbidden. Therefore, in this section, we analyze the optimal tax policy when age-specific lump-sum transfers are not available, i.e. \(g^1 = g^2 = g\). The lagrangian function for governmental optimization becomes

\[
W = V + \eta (tw(e)l - (2 + r)g) .
\]

Are credit constraints exogenous, optimal tax policy should be progressive, i.e. a positive tax rate on labor income and positive lump-sum transfers. This is because only
progressive taxation can redistribute income from high-income to low-income (and con-
strained) period and thus alleviate the exogenous credit constraints. In fact, government
still acts like a lender through progressive taxation to supplement the imperfect capital
market (see Jacobs and Yang (2009)). With endogenous credit constraints, there is an
additional effect of tax policy, namely the effect on endogenous borrowing constraints,
that has to be considered when designing the optimal policy.

Analogous to the definition of net social marginal valuation of income by Diamond
(1975) we define the net social marginal valuation of increasing borrowing limit as

$$\psi \equiv \frac{u_1 - (1 + r) \beta u_2}{\eta} + tw' t \frac{\partial e}{\partial A} + tw \frac{\partial l}{\partial A}. \quad (23)$$

The increase of borrowing limit by one unit increases the utility of a representative
agent by $u_1 - (1 + r) \beta u_2$, which is positive for constrained agents. $\psi$ is therefore the
welfare effect of an marginal increase in $A$ including income effect, measured in terms of
governmental tax revenue.

Using definition (23) we can derive the optimal lump-sum transfers as (see Appendix
A.3)

$$\frac{u_1 + \beta u_2}{\eta} + tw' t \frac{\partial e}{\partial g} + tw \frac{\partial l}{\partial g} + \psi \frac{\partial A}{\partial g} = 2 + r. \quad (24)$$

The left-hand-side of equation (24) gives the net social marginal valuation of income
including income effect on tax revenue, where $\psi \frac{\partial A}{\partial g}$ measures the welfare effect of income
through affecting endogenous credit constraints. If higher income increases borrowing
limit, the net social marginal valuation of income is higher than that in case of exogenous
credit constraints due to the welfare-improving effect of a higher borrowing limit; and
vice versa. In optimum, lump-sum transfers equal the net social marginal valuation of
income to its resource costs, $2 + r$, both measured in second period income.

The first-order-condition for optimal labor tax $t$ can be reformulated as (see Appendix
A.3)

$$(1 - \rho) \pi + \left( \rho \frac{\partial A}{\partial g} + \frac{\partial A}{\partial l} \frac{1}{\zeta} \right) \psi = \frac{t}{1 - \tau} (\varepsilon_e + \varepsilon_l), \quad (25)$$

where $\varepsilon_e \equiv -\frac{\partial e}{\partial t} \frac{1}{\epsilon}$ and $\varepsilon_l \equiv -\frac{\partial l}{\partial t} \frac{1}{\epsilon}$ are the compensated tax elasticities of education
and labor supply and $\rho \equiv \frac{1 - \pi}{2\pi - \pi}$. The optimal labor tax balances the welfare gain from
alleviating credit constraints with the efficiency costs of doing so. The latter, as given by
the right-hand-side of equation (25), arise from tax distortions in educational investment
and labor supply, measured by the compensated tax elasticities. $\epsilon = \frac{w'(e)e}{w(e)}$ is the elasticity
of gross wage rate w.r.t education. The more important educational investment is, the
higher is $\epsilon$ and the higher are the efficiency costs of tax distortion.

The welfare gain of taxing labor income is given by the left-hand side of equation (25).
The first summand is the welfare effect of labor taxation for given credit constraints. Tax-
ing labor income and reimbursing tax revenue in forms of lump-sum transfers redistribute income from second to first period. Since credit constrained agents value first period income more than second period income \((u_1 - \beta(1+r)u_2 > 0)\), such redistribution increases welfare. The more agents are credit constrained, as shown by a higher \(\pi\), the higher is the welfare gain of transferring one unit income from second to first period.

However, since the same amount of transfers has to be given in second period, only part of the tax revenue can be transferred to first period. The parameter \(\rho \equiv \frac{1 - \pi}{2 + r - \pi} = \frac{1 + r}{2 + r - \pi} < 1\) gives the increase in lump-sum transfers if tax revenue is increased by one unit, while taking into account that the shadow price for first period income compared to second period income is \(\frac{1 + r}{2 + r - \pi}\). Note that for one unit tax revenue we have \(\frac{1 + r}{2 + r - \pi} \rho + \rho = 1\), i.e., the values of first and second period transfer should sum up to 1. Consequently, \(1 - \rho = \frac{1 + r}{2 + r - \pi} \rho\) gives the value of first period transfer. The higher the value of first period transfer, the more of one unit tax revenue can be transferred to first period and the higher is the welfare gain of taxation. A higher interest rate and more severe credit constraints increase the value of first period transfer and thus increase the welfare gain of taxation.

The second summand of the right-hand-side of (25) is the welfare effect of taxation through affecting endogenous credit constraints. As defined by (23), \(\psi\) is the welfare effect of relaxing credit constraints by one unit. The term \(\rho \frac{\partial A}{\partial g} + \frac{\partial A}{\partial t} z\) gives the total change in credit constraints for one unit increase in tax revenue. \(\rho\) is by definition the increase in \(g\), while taking the relative price of first period income into account. Therefore, \(\rho \frac{\partial A}{\partial g}\) gives the change in credit constraints due to higher lump-sum transfers if tax revenue is increased by one unit. Similarly, \(\frac{\partial A}{\partial t} z\) is the change in credit constraints due to a higher labor tax rate, since \(\frac{1}{t}\) is the required increase in labor tax rate for one unit increase in tax revenue, ceteris paribus.

To summarize, the welfare effect of taxing labor income is the sum of the welfare gain from alleviating credit constraints while keeping credit constraints as given and the welfare effect from changing endogenous credit constraints. The latter effect can be either positive or negative, depending on how lump-sum transfers and labor taxation affect credit constraints.

Rewriting condition (25), we can characterize the optimal tax rate as

\[
\frac{t}{1 - t} = \frac{(1 - \rho) \pi + \left(\rho \frac{\partial A}{\partial g} + \frac{\partial A}{\partial t} \frac{1}{z}\right) \psi}{\epsilon e + \epsilon_l}
\]

(26)

The optimal tax rate depends on the welfare gain from alleviating credit constraints and its efficiency costs. The higher the total welfare gain and the lower the tax distortions, the higher is the optimal tax rate.
If credit constraints are exogenous, the optimal tax rate reduces to

\[
\frac{t}{1-t} = \frac{(1-\rho)}{\epsilon\varepsilon_e + \varepsilon_l},
\]

(27)
since \(\frac{\partial A}{\partial g} = \frac{\partial A}{\partial t} = 0\) for exogenous credit constraints. This result is the same as that in Jacobs and Yang (2009) for identical agents, where exogenous credit constraints are assumed. In this case, the optimal tax rate is unambiguously positive. The reason is that for exogenous credit constraints only redistribution of income from second to first period can alleviate credit constraints, which means taxing labor income and giving lump-sum transfers.

When credit constraints are endogenous, labor tax and lump-sum transfers change credit constraints by changing the agents’ incentive to repay. The aforementioned comparative statics (equations (15) to (17)) imply that both labor tax \(t\) and lump-sum transfers \(g\) have ambiguous effects on endogenous credit constraints. Therefore, contrary to the case of exogenous credit constraints, the sign of optimal tax rate with endogenous credit constraints is ambiguous.

If increasing labor tax raises borrowing limit by relaxing the incentive constraints of repaying, the optimal tax rate is unambiguously positive. In this case, taxing labor income increases welfare not only by reducing credit demand but also by relaxing endogenous credit constraints.

If increasing labor tax decrease borrowing limit by tightening the incentive constraints, taxing labor income and giving lump-sum transfers have adverse effects on welfare. Dominate the negative effects of reducing credit limit the positive effects of transferring income to first period, the optimal tax rate turns negative. In this case, government should raise lump-sum taxes and subsidize labor income to relax credit constraints. Subsidizing labor income is another way to subsidize educational investment and is a subsidy to labor supply as well. Since defaulting agents invest less in education and work less, subsidies to education and labor supply benefit repaying agents more than defaulting ones and can relax the incentive constraints of repaying thereafter. Lochner and Monge-Naranjo (2002) show that subsidizing educational investment has a bigger welfare effect than in a model with exogenous credit constraints due to the fact that subsidizing education reduces the incentive to default. Because of the non-observability of education in our model it is not possible to subsidize education. Since education subsidies mimic age-specific transfers, the availability of education subsidies could reduce the desirability of labor taxation. However, as long as the share of observable costs in total costs are not very high, the effects of education subsidies are limited. Furthermore, subsidizing observable costs would distort the efficient composition of observable and unobservable investment (see Bovenberg and Jacobs, 2005) and exaggerate the moral hazard problem.

Interestingly, the optimal tax rate can also be zero, if the positive effects of transferring
income to first period exactly compensate for the negative effects of tightening borrowing constraints. Thus, notwithstanding credit constraints, government should not intervene. Andolfatto and Gervais (2006) find as well that no intervention could be optimal when government can use age-specific transfers. We summarize our results in the following proposition:

**Proposition 1** The optimal labor tax equals the welfare gain from alleviating credit constraints to the efficiency costs of distorting educational investment and labor supply. The welfare gain from a higher tax rate is the sum of the welfare gain of redistributing income from second to first period and the welfare effect of changing the endogenous credit constraints. In case that a higher tax rate and higher lump-sum transfers tighten borrowing constraints, the optimal tax policy can be regressive, i.e. there should be a labor income subsidy and lump-sum taxes.

4 Conclusion

This paper derives the optimal tax policy when educational investments are subject to endogenous credit constraints. We show that when government cannot discriminate according to age, the optimal tax policy differ from that in a similar model with exogenous credit constraints. In particular, if the welfare gain of labor income subsidization by relaxing credit constraints dominates the welfare loss of increasing borrowing demand, regressive taxation is optimal. The reason is that subsidizing labor income increases educational investment and labor supply and mitigates therefore the moral hazard problem adherent to credit for educational investment. Furthermore, we find that no intervention could be optimal even if private capital market is imperfect.

In future research it could be interesting to analyze optimal tax policy with endogenous credit constraints when individuals differ in ability and initial wealth. In this case, the level of borrowing limit at which the individual is indifferent between defaulting and repaying would differ individually and is private information due to the unobservable ability. Consequently, conditions for equilibrium credit constraints could change, which would also have different political implications.

A Appendix

A.1 Second-order conditions

We first derive the second-order conditions for the individuals’ maximization problem when they repay loans. With binding credit constraints, savings are equal to the borrowing limit $\bar{a}$. Hence, we can obtain an unconstrained maximization problem upon
Define the labor supply function $u(-e + g + \bar{a}) + \beta (u((1 - t)w(e)l - (1 + r)\bar{a} + g^2) - v(l))$ (28)

The first-order conditions are given by

\[-u_1(.) + \beta u_2(.) (1 - t)w'(e)l = 0,\]
\[\beta u_2(.) (1 - t)w(e) - \beta v'(l) = 0.\] (29, 30)

The Hessian matrix $H$ with second-order partial derivatives is given by

\[
H \equiv \begin{pmatrix}
(u_{11} + u_{22} \beta (1 - t)^2 w^2(e)l^2) + \beta u_2(1 - t)w'(e)l & \beta u_{22}(1 - t)^2 w w'(e)l \\
(\beta u_{22}(1 - t)^2 w(e)w'(e)l) & (\beta u_{22}(1 - t)^2 w^2(e)) \\
+ \beta u_2(1 - t)w'(e)l & (\beta u_{22}(1 - t)^2 w^2(e))
\end{pmatrix}
\] (31)

For the Hessian matrix to be negative semi-definite, the principal minors should switch signs. The first principal minor is negative if the utility functions and the wage rate function are concave, which we assume to be fulfilled. The second principal minor should be positive:

\[
(u_{11} + u_{22} \beta (1 - t)^2 w^2(e)l^2 + \beta u_2(1 - t)w'(e)l) \times (\beta u_{22}(1 - t)^2 w^2(e) - \beta v''(l)) - (\beta u_{22}(1 - t)^2 w(e)w'(e)l + \beta u_2(1 - t)w'(e))^2 > 0.
\] (32)

Next, define $\varepsilon_l \equiv \left(\frac{wu}{wu}\right)^{-1}$ and $(1 - t)w(e)l = Z$. Use the first-order conditions and $w(e) = e^\varepsilon$ to formulate the above inequality as

\[
\left(u_{11} + u_{22} \beta \left(\frac{\varepsilon}{e}\right)^2 Z^2 + \beta u_2 \frac{\varepsilon(\varepsilon - 1)}{e^2} \right) \times \left(\beta u_{22} \frac{Z^2}{l^2} - \beta \varepsilon_l^{-1} \frac{u_l}{l}\right) > \left(\beta u_{22} \left(\frac{\varepsilon}{e}\right)^2 Z^2 + \beta u_2 \frac{\varepsilon}{e} Z \right)^2.
\] (33)

Using first-order condition for labor supply to get

\[
\left(\frac{u_{11}}{\beta} \left(\frac{\varepsilon}{e}\right)^2 + u_{22} Z^2 + u_2 \frac{\varepsilon - 1}{\varepsilon} Z\right) \times \left(u_{22} - \frac{u_2}{Z} \varepsilon_l^{-1}\right) > (u_{22}Z + u_2)^2.
\] (34)

Define $-\frac{wu}{u_2} c_2 = \sigma$ as the elasticity of marginal utility of second period consumption. $\sigma$ measures how fast the marginal utility of second period consumption decline. We have therefore

\[
\varepsilon_l^{-1} > \frac{1 - \sigma Zc_2}{\frac{wu}{u_2} \left(\frac{\varepsilon}{e}\right)^2 \frac{1}{Z} + 1 + \frac{1}{\varepsilon}}
\] (35)
If $1 - \sigma \frac{Z}{c_2} \left( -\frac{w_1}{\beta w_2} \left( \frac{\epsilon}{\epsilon} \right)^2 \frac{1}{Z} + \frac{1+\epsilon}{\epsilon} \right) < 0$, equation (35) is always fulfilled. Therefore, the second order condition is fulfilled if $1 - \sigma \frac{Z}{c_2} \left( -\frac{w_1}{\beta w_2} \left( \frac{\epsilon}{\epsilon} \right)^2 \frac{1}{Z} + \frac{1+\epsilon}{\epsilon} \right) > 0$, which implies that $\sigma$ should be sufficiently small and $\epsilon$ should be sufficiently high. If not, then $\epsilon_1$ should not be too high. In other words, the second order condition requires that the marginal utility of second period consumption declines fast enough, the share of education in wage rate is not too high and the marginal disutility of labor increases fast enough. The unconstrained maximization problem for defaulting agent is

$$\max_{\{\epsilon, l\}} u(-e + g^1 + \bar{w}) + \beta \left( u \left( (1 - \gamma) (1 - t) w(e) l + g^2 \right) - v(l) \right)$$

(36)

Thus, the second order condition is the same as for repaying agents except that the after-tax income becomes

$$Z = (1 - \gamma) (1 - t) w(e) l,$$

which does not change the results qualitatively.

### A.2 Slutsky equations

To derive the Slutsky equations we totally differentiate utility and the budget constraints of the households:

$$dU = u_1 dc^1 + \beta u_2 dc^2 - \beta v_1 dl,$$  

(37)

$$dc^1 = -de + dg^1 - da,$$  

(38)

$$dc^2 = (1 - t)w^lde + (1 - t)wdl - wldt + (1 + r)da + dg^2.$$  

(39)

Substitute $dc^1$ and $dc^2$ in $dU$ to get

$$dU = (\beta u_2 (1 - t)w^l u_1) de + u_1 dg^1 + (\beta u_2 (1 + r) - u_1) da$$

$$+ \beta (u_2 (1 - t)w - v_1) dl - u_2 wdt + u_2 dg^2$$

$$= 0.$$  

(40)

$(\beta u_2 (1 - t)w^l u_1) de$ and $\beta (u_2 (1 - t)w - v_1) dl$ are both equal to zero from the first-order conditions. The term $(u_2 (1 + r) - u_1) da$ is equal to zero as with a binding credit constraint $da = 0$. Thus, we have

$$dU = -\beta u_2 wldt + u_1 dg^1 + \beta u_2 dg^2 = 0,$$  

(41)

or

$$dU = -\beta u_2 wldt + (u_1 + u_2) dg = 0,$$  

(42)
if \( g^1 = g^2 \). For compensation given in both periods, which is the case when age-specific lump-sum transfers are not available, the Slutsky equations are therefore

\[
\begin{align*}
\frac{\partial e}{\partial t} &= \frac{\partial c}{\partial t} - \frac{\beta u_2 w l}{u_1 + \beta u_2 \frac{\partial g}{\partial t}}, \\
\frac{\partial l}{\partial t} &= \frac{\partial c}{\partial t} - \frac{\beta u_2 w l}{u_1 + \beta u_2 \frac{\partial g}{\partial t}}.
\end{align*}
\]

(A.3) Optimal tax policy without age-specific lump-sum transfers

In this appendix we derive the optimal tax policy when government has no access to age-specific lump-sum transfers, which implies \( g^1 = g^2 \equiv g \). The lagrangian function for governmental maximization problem becomes

\[
L = V + \eta (tw(e)l - (2 + r)g).
\]

The first order conditions are respectively

\[
\begin{align*}
\frac{\partial L}{\partial t} &= \frac{\partial V}{\partial t} + \eta \left( w(e)l + tw' \left( \frac{\partial e}{\partial t} \frac{\partial c}{\partial t} + \frac{\partial e}{\partial A} \frac{\partial A}{\partial t} \right) + tw \left( \frac{\partial l}{\partial t} \frac{\partial c}{\partial t} + \frac{\partial l}{\partial A} \frac{\partial A}{\partial t} + \frac{\partial l}{\partial g} \frac{\partial A}{\partial g} \right) \right) = 0 \quad (46) \\
\frac{\partial L}{\partial g} &= \frac{\partial V}{\partial g} + \eta \left( -(2 + r) + tw' \left( \frac{\partial e}{\partial g} \frac{\partial c}{\partial t} + \frac{\partial e}{\partial A} \frac{\partial A}{\partial g} \right) + tw \left( \frac{\partial l}{\partial g} \frac{\partial c}{\partial t} + \frac{\partial l}{\partial A} \frac{\partial A}{\partial g} + \frac{\partial l}{\partial g} \frac{\partial A}{\partial g} \right) \right) = 0 \quad (47)
\end{align*}
\]

since for constrained agents \( a = A \) and \( e = e(t, g, A(t, g)) \). Assuming that credit constraints remain binding in the neighborhood of policy parameters we use general envelope theorem to get

\[
\begin{align*}
\frac{\partial V}{\partial g} &= u_1 + \beta u_2 + \mu \frac{\partial A}{\partial g} = u_1 + \beta u_2 + (u_1 - (1 + r) \beta u_2) \frac{\partial A}{\partial g}, \quad (48) \\
\frac{\partial V}{\partial t} &= -\beta u_2 \phi(e)l + \mu \frac{\partial A}{\partial t} = -\beta u_2 \phi(e)l + (u_1 - (1 + r) \beta u_2) \frac{\partial A}{\partial t}, \quad (49)
\end{align*}
\]

where \( \mu \) is the marginal utility of an increase in borrowing limit by one unit and is equal to \( u_1 - (1 + r) \beta u_2 \). Using (48) we can reformulate (47) for \( g \) as

\[
\frac{u_1 + \beta u_2}{\eta} + \left( \frac{u_1 - (1 + r) \beta u_2}{\eta} + t\phi' l \frac{\partial e}{\partial A} + t\phi l \frac{\partial c}{\partial A} \right) \frac{\partial A}{\partial g} + t\phi' l \frac{\partial e}{\partial g} + t\phi l \frac{\partial l}{\partial g} = 2 + r.
\]

Define

\[
\psi \equiv \frac{u_1 - (1 + r) \beta u_2}{\eta} + t\phi' l \frac{\partial e}{\partial A} + t\phi l \frac{\partial l}{\partial A} \quad (50)
\]

as the net marginal valuation of increasing credit limit measured in terms of government tax revenue, including the income effect. We can then rewrite the first-order condition for \( g \) as

\[
\frac{u_1 + \beta u_2}{\eta} + \psi \frac{\partial A}{\partial g} + t\phi' l \frac{\partial e}{\partial g} + t\phi l \frac{\partial l}{\partial g} = 2 + r. \quad (51)
\]
The optimal lump-sum transfers require that the net marginal valuation of income should be equal to its resource costs $2 + r$, whereby the effects of income on endogenous credit constraints are taken into account. Using general envelop theorem (49), the Slutsky’s equations (43) and (44), the definition of $\psi$ (50) and the optimal lump-sum transfers (51), the first-order condition for tax rate (46) can be reformulated after as

$$z - \frac{\beta u_2 z}{u_1 + \beta u_2} (2 + r) + \frac{\beta u_2 z}{u_1 + \beta u_2} \psi \frac{\partial A}{\partial g} + \frac{\partial A}{\partial t} \psi = \frac{t}{1 - t} \epsilon \epsilon e z + \frac{t}{1 - t} \epsilon l z$$  \hspace{1cm} (52)

where we define the tax elasticities of education and labor supply as $\epsilon e = -\frac{\partial e c}{\partial t} \frac{1 - t}{1 - r}$ and $\epsilon l = -\frac{\partial l c}{\partial t} \frac{1 - t}{1 - r}$ respectively. Substituting the definitions $\pi \equiv 1 - (1 + r) \frac{\beta u_2}{u_1}$ and $\rho \equiv \frac{1 - \pi}{2(1 + r - \pi)}$ leads further to

$$(1 - \rho) \pi + \left( \frac{\rho}{\partial g} + \frac{\partial A}{\partial t} \right) \frac{1}{z} \psi = \frac{t}{1 - t} (\epsilon \epsilon e + \epsilon l) .$$  \hspace{1cm} (53)

Thereafter, the optimal tax rate balances the welfare gain from alleviating credit constraints, including the welfare effect of changing the endogenous credit constraints, and the welfare loss of distorting educational investment and labor supply.

**References**


