

Pethig, Rüdiger; Eichner, Thomas

Conference Paper

International taxing and trading carbon emissions: Distributional impacts of double regulation

Beiträge zur Jahrestagung des Vereins für Socialpolitik 2010: Ökonomie der Familie - Session:
Regulatory Overlap in Climate Policy, No. C2-V1

Provided in Cooperation with:

Verein für Socialpolitik / German Economic Association

Suggested Citation: Pethig, Rüdiger; Eichner, Thomas (2010) : International taxing and trading carbon emissions: Distributional impacts of double regulation, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2010: Ökonomie der Familie - Session: Regulatory Overlap in Climate Policy, No. C2-V1, Verein für Socialpolitik, Frankfurt a. M.

This Version is available at:

<https://hdl.handle.net/10419/37152>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

International taxing and trading carbon emissions: Distributional impacts of double regulation

Thomas Eichner^a and Rüdiger Pethig^b

a) Department of Economics, University of Hagen, Universitätsstr. 41, 58097 Hagen, Germany. E-mail: thomas.eichner@fernuni-hagen.de

b) Department of Economics, University of Siegen, Hoelderlinstr. 3, 57068 Siegen, Germany. E-mail: pethig@vwl.wiwi.uni-siegen.de

Abstract:

We focus on carbon emissions control in a group of countries to explore the distributional incidence of mixed policies consisting of a joint emissions trading scheme (ETS) and of national emissions taxes overlapping with the ETS. Such policies impact on national welfares through both the overlapping taxes and the distribution of national emissions caps. Our main result is that for every mixed policy, there exists an ETS policy without overlapping taxes yielding the same levels of national welfare as the mixed policy. We also suggest two measures of the net distributional incidence of mixed policies.

JEL-classification: H23, Q52

Keywords: emissions cap, emissions tax, emissions trading

1 Introduction

We envisage an international agreement on mitigating climate change like the Kyoto Protocol, in which a group of countries commits to reduce carbon emissions relative to their baseline emissions. Each country in that group is assumed to observe an emissions limit, called national emissions cap. Varying the distribution of national emissions caps under the constraint of keeping constant the total emissions limit for the entire group is known to have consequences for the countries' distribution of welfares. Since the national emissions cap is a valuable asset for the individual country, the larger is that asset the better off the country tends to be at the expense of the other countries. To put it differently, each country's share in the burden of implementing the group emissions cap tends to be the smaller the larger is its national emissions cap. Although that observation is well understood, in general, the precise impact of any given distribution of caps on the distribution of burdens depends on the institutional design of emissions control. In particular, the distribution of burdens depends on whether or not:

- (a) The group of countries operates a joint emissions trading scheme (ETS);
- (b) The ETS covers only part of each country's economy (which we will call ETS sector);
- (c) The individual country regulates carbon emissions not only in its non-ETS sector, which is a necessary condition for cost-effective regulation, but also in its ETS sector, which constitutes potentially harmful *double regulation*.

In an analytical framework with the features (a), (b) and (c), the present paper aims to analyze the impact of double regulation via ETS and overlapping national emissions taxes on the distribution of national welfares. That investigation can claim empirical relevance, because the arrangements (a), (b) and (c) of emissions control characterize the European Union's current approach to fulfilling its Kyoto emissions reduction obligations. In fact, the EU has complemented its member states' national emissions control by a joint ETS in 2005 (EU 2003a) which covers only part of each member state's economy. In their non-ETS sectors, national governments are responsible for curbing emissions by means of domestic policies. There also (pre-)exist various national regulatory policies, notably energy taxes, in the ETS sector overlapping with the ETS

(Johnstone 2003, Sorell and Sijm 2003).¹ The focus of the paper on the distributional impact of double regulation may become even more relevant, if in the near future - which is conceivable, though not yet clear at present - some post-Kyoto agreement should enter into force which extends the EU-type approach to carbon reduction beyond the EU to some (or even many?) non-EU countries.

For any given distribution of national emissions caps, emissions taxes in the countries' ETS sectors impact on national welfares in that they affect the equilibrium permit price and thus each country's export or import of permits. It is therefore the overlapping tax as well as the distribution of national emissions caps that determine the distributional incidence of mixed policies. As an implication, the calculations of national net burdens of mixed policies are not correct unless the interaction of both instruments is accounted for. To address this issue we will consider variations in double regulation and investigate their *net* effect on national welfares resulting from an integrated account of the *partial* welfare effects of both instruments that may point either in the same or in opposite directions. Policies using a tax without ETS or an ETS without a tax are included as limiting cases as illustrated in Table 1.

	Emissions control in the ETS sector via		
	ETS	ETS and sectoral tax	Sectoral tax
Emissions control in the non-ETS sector via sectoral tax	1	2	3

Table 1: Emissions control in a two-sector economy

To keep the focus on distribution as clear as possible, we assume the overall emissions cap \bar{c} constant throughout the paper. First, we analyze cost-ineffective policies, i.e. double regulation with overlapping taxes whose rates differ across countries. Proposition 1 shows that for every cost-ineffective mixed policy there exists a (cost-effective) ETS-only policy providing all countries with a level of welfare higher than that in the mixed policy by some uniform percentage rate.

¹For example, coal in industry is taxed in Austria and in Finland; heavy fuel oil for electricity generation is taxed in Austria, Germany, Ireland and Poland (International Energy Agency 2007). Moreover, the EU Energy Tax Directive (EU 2003b) widens the scope of the EU's minimum energy tax system, previously limited to mineral oils, to all energy products including coal, natural gas and electricity.

In the rest of the paper we restrict attention to cost-effective policies. For the study of those policies we introduce the simplifying assumptions^{2,3}

- that in their non-ETS sectors national governments effectively control emissions through a domestic sectoral emissions tax;
- that the rate of the emissions tax overlapping with the ETS sector is uniform across countries (which renders the double regulation cost effective);⁴
- and that all countries choose their permit cap efficiently, as to equalize marginal abatement cost across the ETS sector and the non-ETS sector.

Having characterized the associated class of cost-effective hybrid policies we consider the limiting case of a tax-only policy (box 3 in Table 1) showing that the associated equilibrium allocation is unaffected by the introduction of an ETS and its partition into national caps (Proposition 2). This finding turns out to be a useful benchmark. Next we demonstrate the *distributional* consequences of mixed policies (box 2 in Table 1) by showing how a country's welfare varies in response either to changes in the emissions tax rate in the ETS sectors or to changes in its national emissions cap (with compensating changes in the caps of all other countries). Due to the interdependence of markets, the distributional effects of policy changes turn out to be not monotone, in general, and hence not easy to characterize (Proposition 3). Next, we establish an equivalence result (Proposition 4) stating that for every mixed policy (box 2 in Table 1) there exists an ETS-only policy (box 1 in Table 1) which provides all countries with the same level of welfare and the same allocation as the mixed policy. It is also possible to specify how the national caps in the equivalent ETS-only policy deviate from the caps assigned to the countries in the actually prevailing mixed policy (Proposition 5).

²Here we follow Eichner and Pethig (2009) who established conditions under which the policy mix is cost effective for the group of countries. They show, in particular, that the emissions tax can be fixed at different levels without compromising cost effectiveness if the overlapping tax is uniform across countries (and if some other qualifications are met). For more details see also Section 2.

³The relation between cost effectiveness and pareto efficiency has been clarified by Chichilnisky and Heal (1994) and Shiell (2003).

⁴More recently Nordhaus (2006) brought forward arguments in favor of an internationally harmonized emissions tax.

Making use of our analytical findings we then propose two measures for the distributional incidence of emissions control. The first measure is non-monetary taking advantage of both the equivalence result and the benchmark property of the tax-only policy. This measure allows to identify winners and losers of mixed policies relative to the tax-only policy. In the spirit of the welfare measure of equivalent variation the second measure consists of a monetary transfer payment a country needs to pay or receive in order to be indifferent between some given mixed policy and the tax-only benchmark policy.

Most of the literature on hybrid carbon emissions control deals with allocative distortions of existing policies and/or with issues of policy design for allocative efficiency, e.g. Bovenberg and de Mooij (1994), Babiker et al. (2003), Bento and Jacobsen (2006), Böhringer et al. (2008) and Eichner and Pethig (2009). Only a few studies address the international distribution of national welfares and burdens. The issue of *equitable* burden sharing has been studied e.g. by Phylipsen et al. (1998) and Marklund and Samakovlis (2007). Yet our focus is not on equity or fairness but rather on the positive analysis of the distributional impact of mixed policies. There is an applied literature of numerical analysis in large-scale CGE models in which some distributional issues are investigated although not in a systematic *analytical* way. For example, Böhringer et al. (2008) consider a group of countries operating an ETS and they calculate how burdens change when an individual country successively raises the rate of the emissions tax in its ETS sector. Peterson and Klepper (2007) compare a harmonized international carbon tax to an ETS with different allocation rules for the emissions caps without considering the issue of overlapping instruments. Hence the distributional incidence of mixed policies appears to be under-researched.

The paper is organized as follows: Section 2 sets up the model. Section 3 characterizes cost-effective equilibria and specifies the distributional impacts of cost-ineffective policies. Section 4 establishes the tax-only policy as a benchmark and analyzes the welfare effects of changes in policy parameters. Section 5 points out that every mixed cost-effective policy can be transformed into an cost-effective ETS-only policy without changing the associated welfare distribution. Section 6 suggests two measures of the distributional impact of mixed policies and Section 7 concludes.

2 The model

We consider an economy of n countries that are open to the rest of the world and that operate a joint ETS. Each country's economy consists of two sectors: One sector that is covered by the ETS, called the ETS sector, and the rest of the economy, called the non-ETS sector. The non-ETS sector of country $i = 1, \dots, n$ uses the fossil fuel input e_{xi} to produce the output $x_{si} = X^i(e_{xi})$. Likewise, the ETS sector uses the fossil fuel input e_{yi} to produce the output $y_{si} = Y^i(e_{yi})$. All fossil fuel is assumed to be imported from the world market at the fixed price p_e . The energy costs of the firms in country i 's ETS sector are $(p_e + t_{yi})e_{yi}$ if country i levies an energy tax at rate t_{yi} . We consider that tax as a tax on carbon emissions because the release of CO₂ is approximately proportional to the amount of fossil fuel burned. The imports of fossil fuel are (mainly) paid for by exporting good Y at the world market price p_y .⁵ Good X^i is traded on a domestic market at price p_{xi} and the corresponding market clearance condition is

$$x_{si} = x_i \quad \text{for } i = 1, \dots, n, \quad (1)$$

where x_i is the domestic demand for good X^i . Given the overall emissions cap \bar{c} for the group of countries and some partition (c_1, \dots, c_n) of \bar{c} (as outlined in the Introduction) the government of each country i chooses the permit budget $c_{yi} \in [0, c_i]$. It issues and hands over to its ETS sector for free⁶ the amount c_{yi} of emissions permits which can then be traded at price π_e among all firms in the ETS sectors of all countries. The condition for equilibrium on that permit market is

$$\sum_j c_{yj} = \sum_j e_{yj}. \quad (2)$$

Each country also levies an emissions tax in its non-ETS sector whose rate t_{xi} is assumed to be chosen as to satisfy

$$c_i - c_{yi} = e_{xi} \quad \text{for } i = 1, \dots, n. \quad (3)$$

⁵Part of the import bill may also be paid for by revenues from exporting permits. However, if permits are imported, the import of both fossil fuel and permits need to be paid for by revenues from exports of good Y . See the trade balance equation (5) below.

⁶At the high level of abstraction in the present analysis, free allocation and auctioning of emissions permits are equivalent allocation procedures. For an analysis where the allocation rule matters see Rosendahl (2008).

Summing (3) over i and invoking (2) shows immediately that the overall emissions cap \bar{c} is met:

$$\sum_j c_j = \sum_j (e_{xj} + e_{yj}) = \bar{c}.$$

The representative consumer of country i derives utility $U^i(x_i, y_i)$ from consuming the amounts x_i and y_i of the goods X^i and Y , respectively. His or her income is $z_i := g_{xi} + g_{yi} + t_{xi}e_{xi} + t_{yi}e_{yi}$. That income consists of transferred profits $g_{xi} := p_{xi}x_{si} - (p_e + t_{xi})e_{xi}$ and $g_{yi} := p_y y_{si} - \pi_e(e_{yi} - c_{yi}) - (p_e + t_{yi})e_{yi}$ and of recycled tax revenues $t_{xi}e_{xi}$ and $t_{yi}e_{yi}$. The consumer spends her income on the goods X^i and Y and hence observes the budget equation

$$z_i = p_{xi}x_i + p_y y_i. \quad (4)$$

The definitions of z_i, g_{xi} and g_{yi} combined with (1) and (4) yield country i 's trade balance

$$p_y(y_{si} - y_i) + \pi_e(c_{yi} - e_{yi}) - p_e(e_{xi} + e_{yi}) = 0. \quad (5)$$

In the n -country economy described above a policy consists of a choice of instruments⁷ $\mathbf{t}_x := (t_{x1}, \dots, t_{xn}) \in \mathbb{R}_+^n, \mathbf{t}_y := (t_{y1}, \dots, t_{yn}) \in \mathbb{R}_+^n, \mathbf{c} := \{c_1, \dots, c_n\} \in C := \left\{ \mathbf{c} \in \mathbb{R}_+^n \mid \sum_j c_j = \bar{c} \right\}$ and $\mathbf{c}_y(\mathbf{c}) := [c_{y1}(\mathbf{c}), \dots, c_{yn}(\mathbf{c})] \in C_y(\mathbf{c}) := [0, c_1] \times [0, c_2] \times \dots \times [0, c_n]$. With $\mathbf{c} \in C$ and $\mathbf{c}_y(\mathbf{c}) \in C_y(\mathbf{c})$ being fixed, the emissions ceiling (3) for country i 's non-ETS sector is also determined. As noted above, the sectoral emissions cap $c_i - c_{yi}$ is implemented through an appropriate choice of t_{xi} . Hence if we take $\mathbf{c}, \mathbf{c}_y(\mathbf{c})$ and \mathbf{t}_y as policy decision variables, as we will do, the tax rates \mathbf{t}_x are endogenous variables rather than independent policy parameters.

Having introduced the necessary notation and described the structure of the model we now define the competitive equilibrium as follows:

*Let the world market prices p_e and p_y , and some policy $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ be given. The prices π_e, \mathbf{p}_x , the tax rates \mathbf{t}_x , and the allocation $(\mathbf{x}_s, \mathbf{e}_x, \mathbf{y}_s, \mathbf{e}_y, \mathbf{x}, \mathbf{y})$ constitute a competitive equilibrium of the n -country economy, if (1), (2) and (3) hold and if for $i = 1, \dots, n$:*⁸

- (x_{si}, e_{xi}) satisfies $e_{xi} = \operatorname{argmax} [p_{xi}X^i(\tilde{e}_{yi}) - (p_e + t_{xi})\tilde{e}_{xi}]$ and $x_{si} = X^i(e_{xi})$,
- (y_{si}, e_{yi}) satisfies $e_{yi} = \operatorname{argmax} [p_y Y^i(\tilde{e}_{yi}) - \pi_e(\tilde{e}_{yi} - c_{yi}) - (p_e + t_{yi})\tilde{e}_{yi}]$ and $y_{si} = Y^i(e_{yi})$,
- (x_i, y_i) satisfies $(x_i, y_i) = \operatorname{argmax} [U^i(\tilde{x}_i, \tilde{y}_i) \text{ s.t. } (4)]$.

⁷Throughout the paper bold letters denote row vectors.

⁸The variables in the next three lines that are marked by a wiggle (like " \tilde{e}_{xi} ") are meant to vary over \mathbb{R}_+ . In contrast, when there is no wiggle we deal with an equilibrium value of the respective variable.

Assuming that production functions are concave and utility functions are quasi-concave, it can be shown that for given p_e, p_y and policy $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ (with appropriate upper bounds on the tax rates \mathbf{t}_y) a competitive equilibrium exists and is unique. However, such equilibria are not cost effective, in general, and the equilibrium distribution of welfares will crucially depend on the policy $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ chosen. To obtain a clear focus on distribution, in the next Section 3 we disentangle distribution from allocative inefficiency.

3 Distributional impacts of cost-ineffective policies

Eichner and Pethig (2009) show that cost effectiveness is attained if and only if

$$t_{xi} = t_x \quad \text{and} \quad t_{yi} = t_y \quad \text{for all} \quad i = 1, \dots, n \quad \text{and} \quad t_x = \pi_e + t_y. \quad (6)$$

According to (6), cost effectiveness requires marginal abatement costs, and hence producer prices of emissions, to be the same across sectors and countries. For every $\mathbf{c} \in C$ there is one and only one vector of national permit caps, denoted $\mathbf{c}_y(\mathbf{c}) = \mathbf{c}_y^*(\mathbf{c})$, that secures the equalization of marginal abatement costs. Hence the permit caps are no independent policy parameters anymore. Unfortunately, the policy that is currently applied in the EU and that will be applied in the event of more countries joining an EU-type policy is cost-ineffective for various reasons. Two major reasons for inefficiency are inefficiently fixed permit caps, $\mathbf{c}_y(\mathbf{c}) \neq \mathbf{c}_y^*(\mathbf{c})$, and tax rates $\mathbf{t}_y = (t_{y1}, \dots, t_{yn})$ that differ across countries.⁹ These constellations of policy parameters generate not only excess costs but also distributional effects. Since our focus in the present paper is on distribution, we now aim to decompose the total policy effect into an efficiency effect and a distributional effect which will then enable us to separate the distributional effects from the overall impacts of mixed policies.

To establish that decomposition it is convenient to introduce the following notation. We restrict attention to policies $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$, $\mathbf{c} \in C$, for which an equilibrium exists. Also, we denote

⁹Since the national tax rates \mathbf{t}_y are not uniform in the EU, it is not second best, in general, to choose the allocation of national permit caps $\mathbf{c}_y(\mathbf{c})$, such that marginal abatement costs are the same across sectors and countries, as is optimal in case of cost-effective policies (Eichner and Pethig 2008). It is unlikely that the permit caps $\mathbf{c}_y(\mathbf{c})$ laid down in the national allocation plans of all member states are the second-best permit caps because there are no indications that the governments of the EU member states have (appropriately) accounted for the preexisting tax rates \mathbf{t}_y in calculating those caps.

by D^n the set of cost-ineffective policies, by D^e the set of those cost-effective policies which do without overlapping taxes ($\mathbf{t}_y = \mathbf{0}$) and write $u_i[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ for the welfare of country i in the equilibrium associated with policy $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$.

Proposition 1.

There exist functions $Z : D^n \rightarrow D^e$ and $B : D^n \rightarrow]1, \infty[$ such that

$$u_i [Z[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]] = B [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] \cdot u_i [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] \quad \text{for } i = 1, \dots, n. \quad (7)$$

Proof. For $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] \in D^n$ define

$$\rho_i [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] := \frac{u_i [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]}{u_1 [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]} \quad \text{for } i = 1, \dots, n$$

and define the policy $[\hat{\mathbf{c}}, \mathbf{c}_y^*(\hat{\mathbf{c}}), \mathbf{0}] \in D^e$ by the equations

$$u_i [\hat{\mathbf{c}}, \mathbf{c}_y^*(\hat{\mathbf{c}}), \mathbf{0}] = \rho_i [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] \cdot u_1 [\hat{\mathbf{c}}, \mathbf{c}_y^*(\hat{\mathbf{c}}), \mathbf{0}] \quad \text{for } i = 1, \dots, n. \quad (8)$$

A distribution of national caps $\hat{\mathbf{c}} \in C$ satisfying (8) clearly exists and is located on the welfare possibility frontier generated by the set of welfare distributions $\mathbf{u} [\mathbf{c}, \mathbf{c}_y^*(\mathbf{c}), \mathbf{0}]$ with $[\mathbf{c}, \mathbf{c}_y^*(\mathbf{c}), \mathbf{0}] \in D^e$. Hence (8) defines the mapping $Z : D^n \rightarrow D^e$ such that $[\hat{\mathbf{c}}, \mathbf{c}_y^*, \mathbf{0}] = Z [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$. From the definition of $\rho_i [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ above combined with (8) and $[\hat{\mathbf{c}}, \mathbf{c}_y^*, \mathbf{0}] = Z [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ follows

$$\frac{u_i [Z [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]]}{u_i [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]} = \frac{u_j [Z [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]]}{u_j [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]} =: B [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] \quad \text{for } i, j = 1, \dots, n.$$

It remains to show that the ratio of utilities that we have defined above as $B [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ is greater than one. Since the equilibrium associated to $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$ is not cost effective (because the tax rates t_{y_i} are presupposed to differ across countries) the welfare distribution of that equilibrium is clearly located below the welfare distribution frontier for the policies $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] \in D^e$. As a consequence, $B [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] > 1$. ■

The message of Proposition 1 is straightforward. For each cost-ineffective policy $[\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] \in D^n$ there is a cost-effective policy without overlapping taxes, $Z [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] \in D^e$, such that the equilibrium welfare distribution of the former is linked to the latter via the equation

$$\mathbf{u} [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y] = \frac{\mathbf{u} [Z [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]]}{B [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]}.$$

In that way we are able to specify the distributional impact of the cost-ineffective policy $Z [\mathbf{c}, \mathbf{c}_y(\mathbf{c}), \mathbf{t}_y]$. In other words we can take advantage of Proposition 1 by restricting our attention to comparisons of cost-effective policies in the remainder of the paper.

4 Distributional impacts of variations in cost-effective carbon control

Recall our observation at the beginning of the previous section that if policies are cost-effective, permit caps are no independent policy parameters anymore. It suffices, therefore, to describe cost-effective policies simply by (\mathbf{c}, t_y) , where $\mathbf{c} \in C$ and $t_y \in \mathbb{R}_+$. For convenience of notation, we will write $\pi_e(\mathbf{c}, t_y)$, $p_{xi}(\mathbf{c}, t_y)$, $x_i(\mathbf{c}, t_y)$ etc. when referring to the values of variables belonging to the cost-effective competitive equilibrium associated with the policy (\mathbf{c}, t_y) . Moreover, we will denote the entire equilibrium as $E(\mathbf{c}, t_y) := [P(\mathbf{c}, t_y), A(\mathbf{c}, t_y)]$, where $P(\mathbf{c}, t_y) := [\pi_e(\mathbf{c}, t_y), \mathbf{p}_x(\mathbf{c}, t_y)]$ are the equilibrium prices and where $A(\mathbf{c}, t_y) := [\mathbf{x}_s(\mathbf{c}, t_y), \mathbf{e}_x(\mathbf{c}, t_y), \mathbf{y}_s(\mathbf{c}, t_y), \mathbf{e}_y(\mathbf{c}, t_y), \mathbf{y}(\mathbf{c}, t_y)]$ is the equilibrium allocation for the group of countries.

Next we specify the domain of all (\mathbf{c}, t_y) , $\mathbf{c} \in C$, for which a cost-effective equilibrium exists. Observe that raising t_y reduces aggregate emissions, ceteris paribus. The regular reaction of the equilibrium permit price π_e is to decline (but see below). In any case, for sufficiently large t_y the permit price π_e is eventually driven down to zero, and with further increases in t_y total emissions would fall short of the overall cap \bar{c} . To capture the borderline case let $\mathbf{c} \in C$ be given and consider $\bar{t}_y > 0$ defined by

$$\begin{aligned} \sum_j [e_{xj}(\mathbf{c}, \bar{t}_y) + e_{yj}(\mathbf{c}, \bar{t}_y)] &= \bar{c} \quad \text{and} \quad \pi_e(\mathbf{c}, \bar{t}_y) = 0 \\ \text{and} \quad \sum_j [e_{xj}(\mathbf{c}, t_y) + e_{yj}(\mathbf{c}, t_y)] &< \bar{c} \quad \text{and} \quad \pi_e(\mathbf{c}, t_y) = 0 \quad \text{for all} \quad t_y > \bar{t}_y. \end{aligned} \quad (9)$$

Although the polar case of the tax rate which crowds out the permit price is not too interesting in itself, it is useful for later reference and we therefore investigate it in more detail in

Proposition 2.

- (i) Suppose an ETS is in operation and consider the policy $[\mathbf{c}, \bar{t}_y(\mathbf{c})]$, where $\bar{t}_y(\mathbf{c})$ is the tax rate that crowds out the permit price, $\pi_e[\mathbf{c}, \bar{t}_y(\mathbf{c})] = 0$. Comparing the equilibria for alternative distributions of national emissions caps yields $\bar{t}_y(\mathbf{c}) = \bar{t}_y(\mathbf{c}') =: \bar{t}_y$ and hence $A(\mathbf{c}, \bar{t}_y) = A(\mathbf{c}', \bar{t}_y) =: A(\bar{t}_y)$ for all $\mathbf{c}, \mathbf{c}' \in C$.
- (ii) Suppose in the absence of an ETS the uniform emissions tax rate $\bar{t} = \bar{t}_{xi} = \bar{t}_{yi}$, all i , implements the group emissions cap \bar{c} , and denote the resultant equilibrium allocation by $A(\bar{t})$. Comparing $A(\bar{t})$ and $A(\bar{t}_y)$ from Proposition 2(i) yields $A(\bar{t}) = A(\bar{t}_y)$.

(iii) Suppose an ETS is in operation and consider the policy $(\hat{\mathbf{c}}, 0)$, where $\hat{\mathbf{c}} = \mathbf{e}(\bar{t})$ and $\mathbf{e}(\bar{t}) := [(e_{x1} + e_{y1}), \dots, (e_{xn} + e_{yn})]$ are the values of emissions in the tax-only equilibrium of Proposition 2(ii). Comparing $A(\hat{\mathbf{c}}, 0)$ and $A(\bar{t})$ from Proposition 2(ii) yields $A(\hat{\mathbf{c}}, 0) = A(\bar{t})$.

Proof.

Ad (i). Consider the equilibrium $E[\mathbf{c}, \bar{t}_y]$ for $\mathbf{c} \in C$ and switch to $\mathbf{c}' \in C$, $\mathbf{c}' \neq \mathbf{c}$ while keeping $t_y = t_y(\mathbf{c})$ and $P[\mathbf{c}, \bar{t}_y(\mathbf{c})]$ unchanged. The only effect of moving from \mathbf{c} to \mathbf{c}' is a change in exports and imports of permits. But since $\pi_e = 0$ producer profits and consumer incomes remain unchanged. That proves $E[\mathbf{c}, \bar{t}_y(\mathbf{c})] = E[\mathbf{c}', \bar{t}_y(\mathbf{c})]$.

Ad (ii). Consider an equilibrium $E(\mathbf{c}, \bar{t}_y)$ of Proposition 2(i) where an ETS is in operation. Since $t_{yi} = \bar{t}_y$, all i , is presupposed, $t_{xi} = \bar{t}_y$, all i , and $\pi_e(\mathbf{c}, \bar{t}_y) = 0$ follow from (6). We conclude that operating the ETS at zero permit price is equivalent to implementing the cap \bar{c} by means of an emissions tax at rate $t = \bar{t}_y$ that is uniform across all sectors and countries.

Ad (iii). Suppose an ETS is in operation and consider a situation where $\hat{\mathbf{c}} = \mathbf{e}(\bar{t})$ (as defined above), $t_y = 0$, $\pi_e = \bar{t}$, and $t_{xi} = \pi_e$, all i . In that case the profits of all firms are as in the tax-only equilibrium of Proposition 2(ii), because the producer price for emissions, $\pi_e + t_y$ and t_{xi} , are the same as under the tax-only regime and permit exports and imports do not take place. Therefore, the equilibrium associated with the policy $(\hat{\mathbf{c}}, 0)$ is such that $A(\hat{\mathbf{c}}, 0) = A(\bar{t})$.

■

Proposition 2(i) states that once the overlapping emissions tax is so high as to push the permit price to zero, variations in the distribution of caps, $\mathbf{c}, \mathbf{c}' \in C$, $\mathbf{c} \neq \mathbf{c}'$, have no distributional potential anymore. That potential is positive for all $t_y < \bar{t}_y$ and it is the greater, the lower is the rate of the overlapping tax. Proposition 2(ii) supplements Proposition 1(i) by highlighting that if $t_y(\mathbf{c}) = \bar{t}_y$ in an ETS, the ETS can be abolished without any displacement effects because owing to $\pi_e(\mathbf{c}, \bar{t}_y) = 0$ the ETS neither creates nor transfers any wealth. Thus we can switch from a limiting case of mixed policy instruments (formally belonging to box 2 in Table 1) to a tax-only policy (box 3 in Table 1). Proposition 2(iii) establishes the equivalence of the tax-only policy (box 3 in Table 1) (with no ETS in operation) and a particular ETS policy in the absence of overlapping taxes (box 1 in Table 1). The equivalent policy in box 1 is characterized by the distribution of national emissions caps, for which all equilibrium exports and imports are zero.

From combining all parts of Proposition 2 also follows that there is some policy $(\hat{\mathbf{c}}, 0)$ in box 1 of Table 1 that generates the same welfare distribution as the policies (\mathbf{c}, \bar{t}_y) which belong to box 2 of Table 1.

Having clarified the allocation in some polar cases we now turn to more relevant intermediate cases addressing the question as to how equilibria differ when alternative policies $(\mathbf{c}, t_y) \in C \times [0, \bar{t}_y]$ are pursued. Primarily we are interested in the distribution of national welfares associated with different intermediate policies. For convenience of notation let us refer to that distribution as $\mathbf{u}(\mathbf{c}, t_y) := [u_1(\mathbf{c}, t_y), \dots, u_n(\mathbf{c}, t_y)]$, where $u_i(\mathbf{c}, t_y)$ is country i 's welfare associated with the equilibrium $E(\mathbf{c}, t_y)$. We seek to answer the question as to what the impact of country i 's welfare is of variations

- in the distribution of national emissions caps when the overlapping tax rate remains constant ($dc_i = -\sum_{j \neq i} dc_j \neq 0$ and $dt_y = 0$) and
- in the overlapping tax rate when the distribution of national emissions caps remains constant ($dc_j = 0$ for all j and $dt_y \neq 0$).

Consider first policies (\mathbf{c}, t_y) with t_y being fixed. If we start from an equilibrium $E(\mathbf{c}, t_y)$ and consider small changes dc_i in country i 's cap ($i = 1, \dots, n$) under the constraint $\sum_j dc_j = 0$, the comparative static effects of dc_i (Appendix) are¹⁰

$$\frac{du_i}{\lambda_i dc_i} = \left[\frac{t_y(\alpha_i \delta_i - \beta_i \gamma_i + \alpha_i D_z^i \Delta e_{yi}) + \gamma_i \Delta e_{yi}}{\gamma_i} \right] \frac{d\pi_e}{dc_i} + \frac{\alpha_i t_y D_z^i \pi_e}{\gamma_i} + \pi_e, \quad (10a)$$

$$\frac{d\pi_e}{dc_i} = \frac{\frac{\alpha_i D_z^i \pi_e}{\gamma_i}}{\sum_j \frac{\beta_j \gamma_j - \alpha_j \delta_j - D_z^j \Delta e_{yj}}{\gamma_j}}, \quad (10b)$$

$$\frac{dp_{xi}}{dc_i} = \frac{(\delta_i + D_z^i \Delta e_{yi})}{\gamma_i} \frac{d\pi_e}{dc_i} + \frac{D_z^i \pi_e}{\gamma_i}. \quad (10c)$$

Although the sign of the effects of increasing c_i are ambiguous in general, the terms (10a) - (10c) simplify considerably if we restrict our attention to quasi-linear utility functions $U^i(x_i, y_i) = V^i(x_i) + y_i$ with V^i being increasing and strictly concave.¹¹ For this special functional form the

¹⁰ $\alpha_i, \beta_i, \delta_i, \gamma_i$ are defined in the Appendix and $\Delta e_{yi} := c_i - e_{xi} - e_{yi}$ is the amount of permits exported or imported by country i .

¹¹It may be possible to derive from (10a)-(10c) more informative results for functional forms that are less restrictive than quasi-linear utility functions. However, we consider the latter sufficient for the purpose of the present paper because our focus is on distributional *equivalence* of policies' rather than on a full characterization of the distributional *impacts* of those policies.

income effect of the demand for good X^i is zero ($D_z^i = 0$) which turns the equations (10a) - (10c) into

$$\frac{du_i}{\lambda_i dc_i} = \pi_e > 0, \quad \frac{d\pi_e}{dc_i} = \frac{dp_{xi}}{dc_i} = 0. \quad (10d)$$

The results in (10d) are as expected. Increasing country i 's cap increases private income in country i with the straightforward consequence that its residents are better off. In addition, (10d) reveals that quasi-linear utility functions eliminate spillover effects on the market of good X^i .

Suppose next that $\mathbf{c} \in C$ is fixed and that starting from $t_y = 0$, the tax rate t_y is successively raised. Eichner and Pethig (2008) determine the comparative static effects of dt_y as

$$\frac{du_i}{\lambda_i dt_y} = t_y \left(\frac{\alpha_i \delta_i - \beta_i \gamma_i}{\gamma_i} \right) \left(\frac{d\pi_e}{dt_y} + 1 \right) + \left(\frac{\alpha_i t_y D_z^i + \gamma_i}{\gamma_i} \right) \Delta e_{yi} \frac{d\pi_e}{dt_y}, \quad (11a)$$

$$\frac{d\pi_e}{dt_y} = - \frac{1}{1 + \frac{\sum_j \frac{\alpha_j D_z^j}{\gamma_j} \Delta e_{yj}}{\sum_j \frac{\alpha_j \delta_j - \beta_j \gamma_j}{\gamma_j}}}, \quad (11b)$$

$$\frac{dp_{xi}}{dt_y} = \frac{\delta_i + \Delta e_{yi} D_z^i}{\gamma_i} \frac{d\pi_e}{dt_y} + \frac{\delta_i}{\gamma_i}. \quad (11c)$$

Again, for quasi-linear utility functions there are no interdependence effects on the market of good X^i such that the equations (11a) - (11c) simplify to¹²

$$\frac{du_i}{dt_y} \begin{cases} > 0, & \text{if country } i \text{ imports permits,} \\ < 0, & \text{if country } i \text{ exports permits,} \end{cases} \quad (11d)$$

$$\frac{d\pi_e}{dt_y} = -1 \quad \text{and} \quad \frac{dp_{xi}}{dt_y} = 0. \quad (11e)$$

We summarize the preceding results in

Proposition 3.

Every policy $(\mathbf{c}, t_y) \in C \times [0, \bar{t}_y]$ has an impact on the welfare distribution $\mathbf{u}(\mathbf{c}, t_y)$ via the policy parameter $\mathbf{c} \in C$ as well as via the policy parameter $t_y \in [0, \bar{t}_y]$. In the case of quasi-linear preferences enlarging a country's emissions cap (at the expense of the other countries' caps) always enhances its welfare, whereas a country gains [loses] from increasing the overlapping

¹²Eichner and Pethig (2008) show that these results do not hold, in general, when utility functions are not quasi-linear.

tax, if it imports [exports] permits. Under more general assumptions on preferences, market interdependence effects render ambiguous the distributional effect of both policy parameters.

In the next section we show that for all mixed policies (box 2 in Table 1) as well as for the tax-only policy (box 3 in Table 1) there is a policy $(\mathbf{c}, 0) \in C \times \{0\}$ (box 1 in Table 1) that leaves the welfare distribution unchanged.

5 Equivalence of ETS-only policies and mixed policies

We have demonstrated in the previous section that changes in the utility profile $\mathbf{u}(\mathbf{c}, t_y)$ can be brought about either by varying t_y while keeping \mathbf{c} constant or by varying \mathbf{c} while keeping t_y constant (setting perhaps $t_y = 0$). This observation suggests to examine the possibility of neutralizing the welfare effects of an exogenous change in t_y by an appropriate change in \mathbf{c} . In other words, we want to answer the question whether for some given policy $(\mathbf{c}, t_y) \in C \times]0, \bar{t}_y[$ one can find $\tilde{\mathbf{c}} \in C, \tilde{\mathbf{c}} \neq \mathbf{c}$, such that $\mathbf{u}(\tilde{\mathbf{c}}, 0) = \mathbf{u}(\mathbf{c}, t_y)$. We have shown that result already in Proposition 2 for the polar policies $(\mathbf{c}, \bar{t}_y), \mathbf{c} \in C$. Now we will show such a mapping for the more relevant intermediate policies. As in Proposition 1 we denote by D^e the set of cost-effective policies (\mathbf{c}, t_y) satisfying $\mathbf{c} \in C$ and $t_y = 0$ and further define D^t to be the set of cost-effective policies satisfying $\mathbf{c} \in C$ and $t_y \in]0, \bar{t}_y]$. Clearly, $D^e \cup D^t$ is the full set of cost-effective policies. Using that notation we establish

Proposition 4.

(i) There exists a function $F : D^t \longrightarrow D^e$ such that

$$\mathbf{u}[F(\mathbf{c}, t_y)] = \mathbf{u}(\mathbf{c}, t_y).$$

(ii) There exists a correspondence $\bar{F} : D^e \longrightarrow D^t$ such that

$$\mathbf{u}(\mathbf{c}, t_y) = \mathbf{u}(\tilde{\mathbf{c}}, t_y) \quad \forall (\mathbf{c}, t_y) \in \bar{F}(\tilde{\mathbf{c}}, 0) \subset D^t.$$

$\bar{F}(\tilde{\mathbf{c}}, 0)$ has the property that for every $t_y \in]0, t_y(\tilde{\mathbf{c}}[$ there exists $\mathbf{c}' \in C$ such that $(\mathbf{c}', t_y) \in \bar{F}(\tilde{\mathbf{c}}, 0)$. $t_y(\tilde{\mathbf{c}})$ is defined as

$$t_y(\tilde{\mathbf{c}}) := \min_i \frac{\tilde{\pi}_e + \tilde{c}_i}{\tilde{e}_{xi} + \tilde{e}_{yi}} \in]0, \tilde{\pi}_e[$$

where $\tilde{\pi}_e, \tilde{e}_{xi}$ and \tilde{e}_{yi} are the equilibrium values under the ETS-only policy $(\tilde{\mathbf{c}}, 0)$.

Proof.

Ad (i). Consider $(\mathbf{c}, t_y) \in D^t$ and define $\tilde{\mathbf{c}}$ by

$$\tilde{c}_i := \frac{\pi_e c_i + t_y(e_{xi} + e_{yi})}{\pi_e + t_y}, \quad i = 1, \dots, n. \quad (12)$$

Also denote the equilibrium values associated to $(\tilde{\mathbf{c}}, 0)$ by $\tilde{\pi}_e, \tilde{p}_{xi}$ etc. Proposition 4(i) follows from

Lemma 1.

(a) If $(\mathbf{c}, t_y) \in D^t$ and $\tilde{\mathbf{c}}$ satisfies (12), then $(\tilde{\mathbf{c}}, 0) \in D^e$.

(b) If $(\mathbf{c}, t_y) \in D^t$ and $\tilde{\mathbf{c}}$ satisfies (12), then it holds $E(\tilde{\mathbf{c}}, 0) = E(\mathbf{c}, t_y)$.

Ad (a). Summation of (12) and invoking (2) and (3) immediately yields

$$\sum_j \tilde{c}_j = \frac{\pi_e \sum_j c_j + t_y \sum_j (e_{xj} + e_{yj})}{\pi_e + t_y} = \frac{(\pi_e + t_y) \sum_j c_j}{\pi_e + t_y} = \bar{c}.$$

Ad (b). Consider $\tilde{\mathbf{c}}$ as defined in (12) and observe that prices satisfy

$$\tilde{\pi}_e = \pi_e + t_y \quad \text{and} \quad \tilde{p}_{xi} = p_{xi} \quad i = 1, \dots, n. \quad (13)$$

In the sequel we compare the equilibrium allocations $A(\tilde{\mathbf{c}}, 0)$ and $A(\mathbf{c}, t_y)$. In equilibrium the first-order conditions of profit maximization under policies (\mathbf{c}, t_y) and $(\tilde{\mathbf{c}}, 0)$, respectively, are

$$p_y Y^i(e_{yi}) = p_e + \pi_e + t_y \quad \text{and} \quad p_{xi} X^i(e_{xi}) = p_e + t_x = p_e + \pi_e + t_y, \quad (14)$$

$$p_y Y^i(\tilde{e}_{yi}) = p_e + \tilde{\pi}_e \quad \text{and} \quad \tilde{p}_{xi} \tilde{X}^i(\tilde{e}_{xi}) = p_e + \tilde{\pi}_e. \quad (15)$$

Combining (13), (14) and (15) immediately yields

$$\tilde{e}_{hi} = e_{hi} \quad \text{and} \quad \tilde{h}_{si} = h_{si} \quad \text{for} \quad h = x, y. \quad (16)$$

Next, we wish to show that $\tilde{h}_i = h_i(\cdot)$ for $h = x, y$. To that end invoke (13) to transform (12) as follows:

$$\begin{aligned} \tilde{\pi}_e \tilde{c}_i = \pi_e c_i + t_y(e_{xi} + e_{yi}) & \iff \tilde{\pi}_e \tilde{c}_i - \tilde{\pi}_e(e_{xi} + e_{yi}) = \pi_e c_i + t_y(e_{xi} + e_{yi}) - \tilde{\pi}_e(e_{xi} + e_{yi}), \\ & \tilde{\pi}_e(\tilde{c}_i - e_{xi} - e_{yi}) = \pi_e(c_i - e_{xi} + e_{yi}). \end{aligned} \quad (17)$$

If policy (\mathbf{c}, t_y) is given, the consumer's income is

$$z_i = p_{xi}x_{si} + p_y y_{si} - p_e(e_{xi} + e_{yi}) - \pi_e(c_i - e_{xi} - e_{yi}). \quad (18)$$

If policy $(\tilde{\mathbf{c}}, 0)$ is given, the income is

$$\tilde{z}_i = \tilde{p}_{xi}\tilde{x}_{si} + p_y\tilde{y}_{si} - p_e(\tilde{e}_{xi} + \tilde{e}_{yi}) - \tilde{\pi}_e(\tilde{c}_i - \tilde{e}_{xi} - \tilde{e}_{yi}).$$

From (13), (16), (17) and (18) follows $\tilde{z}_i = z_i$. Consequently the consumer's budget constraint under policy (\mathbf{c}, t_y) is the same as under policy $(\tilde{\mathbf{c}}, 0)$ since prices satisfy (13). The straightforward conclusion is $\tilde{x}_i = x_i$ and $\tilde{y}_i = y_i$. Thus we have shown that $A(\tilde{\mathbf{c}}, 0) = A(\mathbf{c}, t_y)$ and $E(\tilde{\mathbf{c}}, 0) = E(\mathbf{c}, t_y)$.

Ad (ii). Consider $(\tilde{\mathbf{c}}, 0) \in D^e$, denote the corresponding equilibrium values by $\tilde{\pi}_e, \tilde{p}_{xi}$, etc. and define a policy (\mathbf{c}, t_y) by

$$c_i = \frac{\tilde{\pi}_e \tilde{c}_i - (\tilde{e}_{xi} + \tilde{e}_{yi})t_y}{\tilde{\pi}_e - t_y}, \quad i = 1, \dots, n. \quad (19)$$

We prove the following

Lemma 2.

(a) If $(\tilde{\mathbf{c}}, 0) \in D^e$ and (\mathbf{c}, t_y) satisfies (19), then $(\mathbf{c}, t_y) \in D^t$.

(b) If $(\tilde{\mathbf{c}}, 0) \in D^e$ and (\mathbf{c}, t_y) satisfies (19), then it holds $E(\mathbf{c}, t_y) = E(\tilde{\mathbf{c}}, 0)$.

Ad (a). Summation of (19) and invoking (2) and (3) proves Lemma 2(a).

Ad (b). First we infer from (19) that there is some country i for which $c_i < 0$ if and only if $t_y \geq t_y(\tilde{\mathbf{c}})$. Hence $t_y \in]0, t_y(\tilde{\mathbf{c}})]$ is a necessary equilibrium condition. Next we determine the allocation $(\mathbf{x}_s, \mathbf{e}_x, \mathbf{y}_s, \mathbf{e}_y, \mathbf{x}, \mathbf{y})$ if

$$\pi_e = \tilde{\pi}_e - t_y, \quad p_{xi} = \tilde{p}_{xi} \quad \text{and} \quad t_{xi} = \pi_e + t_y \quad (20)$$

and $t_y \leq t_y(\tilde{\mathbf{c}})$ is given. It is easy to show that all profit-maximizing inputs and outputs are the same: $\mathbf{x}_s = \tilde{\mathbf{x}}_s$, $\mathbf{e}_x = \tilde{\mathbf{e}}_x$, $\mathbf{y}_s = \tilde{\mathbf{y}}_s$, and $\mathbf{e}_y = \tilde{\mathbf{e}}_y$. Since consumer prices are the same, $p_y \equiv 1$ and $\mathbf{p}_x = \tilde{\mathbf{p}}_x$, we can conclude that $\mathbf{x} = \tilde{\mathbf{x}}$ and $\mathbf{y} = \tilde{\mathbf{y}}$, iff the consumers incomes are the same, $\mathbf{z} = \tilde{\mathbf{z}}$, i.e. iff for all i

$$p_{xi}x_{si} + y_{si} - p_e(e_{xi} + e_{yi}) - \pi_e(c_i - e_{xi} - e_{yi}) = \tilde{p}_{xi}\tilde{x}_{si} + \tilde{y}_{si} - p_e(\tilde{e}_{xi} + \tilde{e}_{yi}) - \tilde{\pi}_e(\tilde{c}_i - \tilde{e}_{xi} - \tilde{e}_{yi}).$$

Since all inputs and outputs are the same, this equation simplifies to

$$\pi_e(c_i - e_{xi} - e_{yi}) = \tilde{\pi}_e(\tilde{c}_i - \tilde{e}_{xi} - \tilde{e}_{yi})$$

which can readily be turned into (19) after making use of $\pi_e = \tilde{\pi}_e - t_y$ from (20). This observation completes the proof of $E(\mathbf{c}, t_y) = E(\tilde{\mathbf{c}}, 0)$ and Lemma 2. Since Lemma 2 holds for every $t_y \in]0, t_y(\tilde{\mathbf{c}}[$ the second part of Proposition 4(ii) is also proved. ■

According to Proposition 4(i), for each mixed policy $(\mathbf{c}, t_y) \in D^t$ there exists a unique ETS-only policy, $(\tilde{\mathbf{c}}, 0) \in D^e$, yielding the same welfare distribution as the mixed policy. The governments of all countries are indifferent with respect to these policies because each policy produces the same resource allocation: $A(\tilde{\mathbf{c}}, 0) = A(\mathbf{c}, t_y)$ and hence switching policies leaves all countries' welfare positions unchanged. In view of $e_{hi}(\mathbf{c}, t_y) = e_{hi}(\tilde{\mathbf{c}}, 0)$ for $h = x, y$ and $i = 1, \dots, n$ equation (12) implies, in fact, that the values of permits imported or exported are the same under the policies (\mathbf{c}, t_y) and $(\tilde{\mathbf{c}}, 0)$. As a consequence, country i 's income remains unchanged which leaves the representative consumer's demand for consumption goods unaffected when $p_{xi}(\tilde{\mathbf{c}}, 0) = p_{xi}(\mathbf{c}, t_y)$.

Proposition 4(ii) provides, first of all, the information that we can turn Proposition 4(i) around in the sense that if one starts from the ETS-only policy $F(\mathbf{c}, t_y)$ and chooses the mixed policy $(\mathbf{c}, t_y) \in D^t$ the associated equilibrium allocations remain unchanged. More importantly, for every policy $(\mathbf{c}, 0) \in D^e$ there is a large set $\bar{F}(\mathbf{c}, 0)$ of mixed policies that are equivalent to the ETS-only policy.¹³

Figure 1 illustrates both parts of Proposition 4 for the case of two countries. The line 0_10_2 is equal to $\bar{\mathbf{c}}$ such that each point on 0_10_2 (e.g. B or E) represents a partition (c_1, c_2) of \bar{c} . Furthermore, $0_1G = 0_2H = \bar{t}_y$ and all curves in Figure 1, e.g. BD , represent welfare indifference curves on which both countries' welfare levels are constant. The ratio u_1/u_2 increases when moving from left to right (e.g. when moving on 0_10_2 toward 0_2 or on CK toward K). By construction of Figure 1 the point E on 0_10_2 is special because in the associated equilibrium

¹³Proposition 4(ii) also covers the special case $t = \bar{t}_y$, since $\tilde{c}_i(\bar{t}_y)$ follows immediately from (12) for $\pi_e = 0$ and thus reproduces the result we have already established in Proposition 1 (with much less effort). The important extension of Proposition 2 is that for every policy (\mathbf{c}, t_y) in box 2 of Table 1 there is some policy $(\mathbf{c}', 0)$ in box 1 of Table 1 such that $\mathbf{u}(\mathbf{c}', 0) = \mathbf{u}(\mathbf{c}, t_y)$.

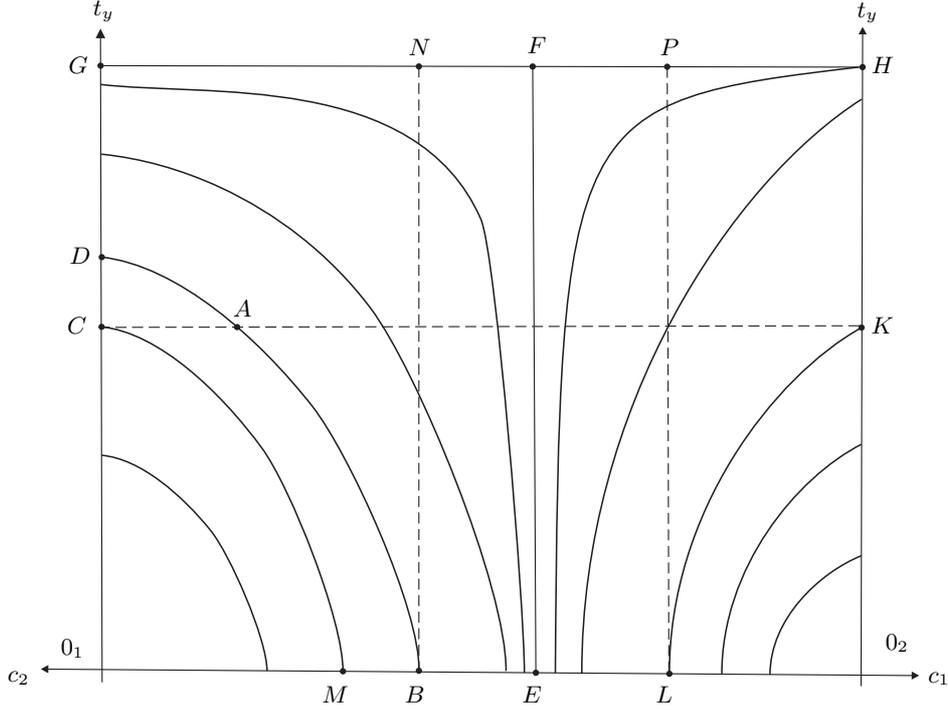


Figure 1: Welfare implications of cost-effective mixed policies

no permit exports and imports take place. This is why the equilibrium allocations and welfare distributions are the same along the lines EF and GH .

To illustrate Proposition 4(i) in Figure 1 suppose that the point A represents the initial mixed policy $(\mathbf{c}, t_y) \in D^t$. Then $F(\mathbf{c}, t_y)$ corresponds to the point B . It is clear that one can also move backward from B to A . But more importantly, equivalent to the ETS-only policy B are all mixed policies that correspond to a point on the curve BD . Hence if we associate the policy $(\tilde{\mathbf{c}}, 0)$ with the point B , the set $\bar{F}(\tilde{\mathbf{c}}, 0)$ corresponds to the set of all points on the curve BD .

Figure 1 can also be used to illustrate the range of possible welfare (re)distributions depending on variations in t_y for constant \mathbf{c} , or in \mathbf{c} for constant t_y . Obviously, if $t_y = 0$, one can achieve any feasible welfare ratio u_1/u_2 by the choice of $\mathbf{c} \in C$ (i.e. by moving along 0_10_2). However, the greater is the tax rate t_y , the smaller becomes the range of u_1/u_2 that can be attained by changes in $\mathbf{c} \in C$. For example, if $t_y = 0_1C$ in Figure 1, the welfare distributions that can be generated by moving from C to K are those which are generated by ETS-only policies when moving from M to L . If $t_y = 0_1G = \bar{t}_y$, variations of $\mathbf{c} \in C$, do not change the welfare distribution at all.

Consider next the scenario in which $\mathbf{c} \in C$ is kept constant and t_y is successively raised from $t_y = 0$ to \bar{t}_y . In Figure 1 that scenario would correspond to starting at some initial ETS-only policy, such as B or L , and moving toward N and P , respectively. Clearly, the move from B to N [L to P] implies a change in welfare distribution equal to that which is implied by the move from B to E [L to E]. In other words, the less permit exports and imports occur in some given initial ETS-only policy the smaller is the potential of the overlapping emissions tax to bring about changes in the welfare distribution.

Given the mapping from $(\mathbf{c}, t_y) \in D^t$ to $[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0] \in D^e$ it is natural to ask what the sign and the magnitude are of the differences $c_i - \tilde{c}_i(\mathbf{c}, t_y)$ and $c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)$ and how these differences vary with c_i and t_y , respectively. The answers are provided in

Proposition 5.

Suppose the policy $(\mathbf{c}, t_y) \in D^t$ is applied.

(i) $c_i < \tilde{c}_i(\mathbf{c}, t_y)$ [$c_i > \tilde{c}_i(\mathbf{c}, t_y)$], if country i imports [exports] permits.

(ii) Consider an economy with quasi-linear utility functions. The impact of changes in c_i and t_y on the differences $c_i - \tilde{c}_i(\mathbf{c}, t_y)$ and $c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)$ are, respectively,¹⁴

$$\frac{d[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{dc_i} = \frac{t_y}{(\pi_e + t_y)^2} \quad \text{and} \quad \frac{d[c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)]}{dc_i} = \frac{t_y}{(\pi_e + t_y)^2}, \quad (21a)$$

$$\frac{d[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{dt_y} = \frac{\Delta e_{yi}}{\pi_e + t_y} \quad \text{and} \quad \frac{d[c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)]}{dt_y} = \frac{\Delta e_{yi}}{\pi_e + t_y}, \quad (21b)$$

where $\Delta e_{yi} := c_i - e_{xi}(\mathbf{c}, t_y) - e_{yi}(\mathbf{c}, t_y)$.

Proof.

Ad (i). Proposition 5(i) is straightforward from rewriting (12) as

$$\tilde{c}_i(\mathbf{c}, t_y) = c_i - \frac{t_y \Delta e_{yi}}{\pi_e + t_y}.$$

Ad (ii). Differentiation of $\tilde{c}_i(\mathbf{c}, t_y)$ with respect to c_i and t_y yields, after some rearrangement of terms,

$$\frac{d\tilde{c}_i}{dc_i} = 1 - \frac{t_y \frac{d\Delta e_{yi}}{dc_i} - t_y \frac{d\pi_e}{dc_i} \Delta e_{yi}}{(\pi_e + t_y)^2}, \quad (22a)$$

$$\frac{d\tilde{c}_i}{dt_y} = -\frac{\pi_e \left(1 - \frac{d\pi_e}{dt_y} \cdot \frac{t_y}{\pi_e}\right)}{(\pi_e + t_y)^2} \Delta e_{yi} - \frac{t_y}{\pi_e + t_y} \cdot \frac{d\Delta e_{yi}}{dt_y}. \quad (22b)$$

¹⁴The variation dc_i is carried out under the constraint $\sum_j dc_j = 0$.

Differentiate $c_i - \tilde{c}_i(\mathbf{c}, t_y)$ with respect to c_i and t_y , respectively, and make use of (22a) and (22b) to get

$$\frac{d[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{dc_i} = \frac{t_y \frac{d\Delta e_{yi}}{dc_i} - t_y \frac{d\pi_e}{dc_i} \Delta e_{yi}}{(\pi_e + t_y)^2}, \quad (23a)$$

$$\frac{d[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{dt_y} = \frac{\pi_e \left(1 - \frac{d\pi_e}{dt_y} \cdot \frac{t_y}{\pi_e}\right)}{(\pi_e + t_y)^2} \Delta e_{yi} + \frac{t_y}{\pi_e + t_y} \frac{d\Delta e_{yi}}{dt_y}. \quad (23b)$$

Consider first (23a). From (10a) and (10c) we infer that $\frac{dp_{xi}}{dc_i} = \frac{d\pi_e}{dc_i} = 0$ if utility functions are quasi-linear ($D_z^i = 0$). In addition, (A23) in the Appendix implies $\frac{de_{xi} + de_{yi}}{dc_i} = 0$ for $\frac{dp_{xi}}{dc_i} = \frac{d\pi_e}{dc_i} = 0$ and hence $\frac{d\Delta e_{yi}}{dc_i} = 1$ follows. Making use of this information in (23a) we get $\frac{d[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{dc_i} = \frac{t_y}{(\pi_e + t_y)^2}$. Next, we differentiate $c_{yi}(\mathbf{c}, t_y) - \tilde{c}_{yi}(\mathbf{c}, t_y)$ with respect to c_i to obtain $\frac{d[c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)]}{dc_i} = \frac{dc_{yi}}{dc_i} - \frac{d\tilde{c}_{yi}}{dc_i}$. Accounting for $c_{yi} = c_i - e_{xi}$, $\tilde{c}_{yi} = \tilde{c}_i - \tilde{e}_{xi}$ and $\frac{de_{xi}}{dc_i} = \frac{d\tilde{e}_{xi}}{dc_i} = 0$ from (A22) yields $\frac{d[c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)]}{dc_i} = \frac{d[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{dc_i}$ which in turn establishes (21a).

Now we turn to (23b). Implicit in (14) the demand for fossil fuel (and for emissions permits) of country i is given by the functions $E^{xi}(p_{xi}, \pi_e + t_y)$ and $E^{yi}(\pi_e + t_y)$. Totally differentiating these functions E^{xi} and E^{yi} with respect to t_y gives us

$$\frac{dE^{xi}(p_{xi}, \pi_e + t_y)}{dt_y} = \frac{\frac{d\pi_e}{dt_y} + 1}{p_{xi} X_{ee}^i} - \frac{x_{si}}{p_{xi} X_{ee}^i} \cdot \frac{dp_{xi}}{dt_y} \quad \text{and} \quad \frac{dE^{yi}(\pi_e + t_y)}{dt_y} = \frac{\frac{d\pi_e}{dt_y} + 1}{Y_{ee}^i}. \quad (24)$$

Obviously, in view of (11e), i.e. $\frac{dp_{xi}}{dt_y} = 0$ and $\frac{d\pi_e}{dt_y} = -1$ for $i = 1, \dots, n$, the equations (24) imply

$$\frac{dE^{xi}(p_{xi}, \pi_e + t_y)}{dt_y} = \frac{dE^{yi}(\pi_e + t_y)}{dt_y} = \frac{d(e_{xi} + e_{yi})}{dt_y} = 0. \quad (25)$$

Next, we differentiate $\Delta e_{yi} = c_i - e_{xi}(\mathbf{c}, t_y) - e_{yi}(\mathbf{c}, t_y)$ with respect to t_y to obtain $\frac{d\Delta e_{yi}}{dt_y} = 0$. Using this information and $\frac{d\pi_e}{dt_y} = -1$ in (23b) establishes $\frac{d[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{dt_y} = \frac{\Delta e_{yi}}{\pi_e + t_y}$. Invoking the same arguments as above straightforwardly leads to $\frac{d[c_{yi} - \tilde{c}_{yi}(\mathbf{c}, t_y)]}{dt_y} = \frac{d[c_i - \tilde{c}_i(\mathbf{c}, t_y)]}{dt_y}$. ■

According to Proposition 5(i), replacing policy (\mathbf{c}, t_y) by the policy $[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$ implies that country i 's emissions cap under the new policy $[\tilde{\mathbf{c}}(\mathbf{c}, t_y), 0]$ is greater [smaller] than under the old policy (\mathbf{c}, t_y) , if country i imports [exports] permits under the old policy. Moreover, the gap $|c_i - \tilde{c}_{yi}(\mathbf{c}, t_y)|$ is greater for a permit-exporting country i and smaller for a permit-importing country i , the greater is country i 's initial emissions cap, c_i . However, raising the tax rate t_y in the initial policy (\mathbf{c}, t_y) widens the gap $|c_i - \tilde{c}_{yi}(\mathbf{c}, t_y)|$ for both permit-exporting and permit-importing countries.

It should be noted, however, that if the assumption of quasi-linear utility functions is relaxed, the distributional impact of changes in c_i and t_y will be less clear-cut. Since markets are interdependent, an exogenous change in t_y must be expected to trigger repercussions in other markets so that the crucial presupposition of Proposition 5(ii), $(dp_{xi}/dc_i) = (dp_{xi}/dt_y) = 0$ for all i , is not satisfied, in general. However, if interdependence effects are present, general information cannot be gained from (22a) and (22b) neither on the sign nor on the magnitude of the differential quotients. In particular, the results (21a) and (21b) that changes in dc_i and dt_{yi} fully translate into a change in the permit cap \tilde{c}_{yi} must be considered special cases.

To highlight the relevance of Proposition 5 regarding the distributional impact of fixing \mathbf{c} and t_y in policies (\mathbf{c}, t_y) , suppose the group of countries has agreed on some distribution of emissions caps, $\mathbf{c} \in C$, satisfying certain equity criteria as in case of the EU burden sharing agreement. If the countries should have determined their "fair" distribution \mathbf{c} without accounting for the preexisting overlapping tax(es), the true distributional impact of the policy (\mathbf{c}, t_y) is unfair according to the equity criteria chosen.

6 Methods of measuring the (re)distributional impact of carbon emissions control

This section focuses on cost-effective policies again and aims at *measuring* the distributional impact of those policies $(\mathbf{c}, t_y) \in D^t$. Our finding that the equilibrium associated to all policies (\mathbf{c}, \bar{t}_y) is independent of \mathbf{c} suggests taking the distributional impact of the tax-only policy as a benchmark for assigning national emissions caps. Recall that, according to Proposition 2, a given policy $(\mathbf{c}, t_y) \in D^t$ is equivalent to a pure ETS with $\tilde{\mathbf{c}}(\mathbf{c}, t_y) \in D^e$, and that the tax-only policy \bar{t}_y is equivalent to a pure ETS with $\tilde{\mathbf{c}}(\bar{t}_y) \in D^e$. From these observations the following measure of distribution is straightforward:

Measure I of Distribution.

Relative to the policy of implementing \bar{c} with an emissions tax only that is uniform across all sectors and all countries the redistributive implication of policy $(\mathbf{c}, t_y) \in D^t$ is measured by

$$[\tilde{\mathbf{c}}(\mathbf{c}, t_y) - \tilde{\mathbf{c}}(\bar{t}_y)] \in \mathbb{R}^n. \quad (26)$$

Under conditions specified in Section 3 we know that switching from the tax-only policy to the policy $(\mathbf{c}, t_y) \in D^t$ makes country i better [worse] off if and only if $\tilde{c}_i(\mathbf{c}, t_y) > \tilde{c}_i(\bar{t}_y)$ [$\tilde{c}_i(\mathbf{c}, t_y) < \tilde{c}_i(\bar{t}_y)$]. The advantage of Measure I is to translate tax policies and policies mixing taxes and emissions trading into shares of permit endowments. Its downside is, however, that its link to the utility distribution is not unambiguous under general forms of utility functions and that it is not a monetary measure.

These limitations are overcome, however, by another straightforward measure that takes as benchmark the welfare associated to the tax-only policy. To construct that measure we first introduce a vector of transfer payments $\boldsymbol{\theta} := (\theta_{y1}, \dots, \theta_{yn}) \in \mathbb{R}^n$ in an equilibrium with policy (\mathbf{c}, t_y) . As a result, the welfare of country i becomes equal to

$$u_i(\mathbf{c}, t_y; \theta_i) := U^i [D^i(\cdot), z_i(\mathbf{c}, t_y) + \theta_i - p_{xi}(\mathbf{c}, t_y)D^i(\cdot)], \quad (27)$$

when it receives the positive or negative transfer θ_i . In (27) $D^i(\cdot) := D^i [p_{xi}(\mathbf{c}, t_y), z_i(\mathbf{c}, t_y) + \theta_i]$ is the demand for good X^i .

Measure II of Distribution.

Relative to the policy of implementing \bar{c} with an emissions tax only that is uniform across all sectors and all countries the redistributational implication of policy $(\mathbf{c}, t_y) \in D^t$ is measured by the monetary transfer $\boldsymbol{\theta}(\mathbf{c}, t_y) := [\theta_1(\mathbf{c}, t_y), \dots, \theta_n(\mathbf{c}, t_y)] \in \mathbb{R}^n$, where for all i the monetary transfer $\theta_i = \theta_i(\mathbf{c}, t_y)$ is defined by¹⁵

$$u_i[\tilde{\mathbf{c}}(\bar{t}_y), 0; \theta_i] = u_i(\mathbf{c}, t_y; 0). \quad (28)$$

According to (28) $\theta_i(\mathbf{c}, t_y)$ is the amount of money country i needs to receive or to pay in order to shift its utility from the level $u_i[\tilde{\mathbf{c}}(\bar{t}_y), 0; \theta_i]$ to the level $u_i(\mathbf{c}, t_y; 0) = u_i(\mathbf{c}, t_y)$ which it actually enjoys in the equilibrium attained under the policy (\mathbf{c}, t_y) . Switching from $[\tilde{\mathbf{c}}(\bar{t}_y), 0]$ to (\mathbf{c}, t_y) creates winners and losers. If $\theta_i(\mathbf{c}, t_y) > 0$, country i loses through that policy switch because it needs the compensation $\theta_i(\mathbf{c}, t_y) > 0$ in order to be indifferent between both policy schemes. Conversely, if $\theta_i(\mathbf{c}, t_y) < 0$ it gains through that policy switch because its income under policy (\mathbf{c}, t_y) needs to be reduced by $\theta_i(\mathbf{c}, t_y)$ to make its utility level under policy (\mathbf{c}, t_y) equal to the level it enjoys under the tax-only policy.

¹⁵ θ_i is in spirit analogous to the Hicksian equivalent variation.

7 Concluding Remarks

Burden sharing is well known to be a crucial precondition for successful international carbon emissions control within the EU as well as world wide. In the present paper we do not address fairness in burden sharing but focus, instead, on the questions preceding the fairness issue, namely what the true national burdens are in hybrid emissions control policies and how to measure them. We show that when an ETS covering only part of all participating countries' economies is combined with an overlapping emissions tax the net impact on national welfares results from an integrated account of the partial welfare effects of both instruments. Our equivalence result allows expressing each country's net burden carried in a mixed policy as the net burden it carries in a hypothetical but equivalent ETS-only policy. In other words, the distributional impact of a uniform overlapping tax is thus 'translated' into changes in national emissions caps. The national net burdens are shown to be measurable as deviations from the burdens implied by the tax-only policy.

Our paper provides a message for parties involved in negotiations about an agreement on the distribution of national emissions caps in the context of a joint ETS. When major emissions taxes overlapping with the ETS exist, the negotiated national emissions caps are distorted indicators of national burdens, when the burdens implicit in the overlapping taxes are not taken into account. Rational burden sharing negotiations need to consider the 'burden impact' of both instruments. There are reasons to doubt whether the parties in the EU burden sharing agreement had at their disposal all the information about the incidence of their agreed-upon national caps that is needed to share the burden according to their own fairness criteria. The parties are advised to calculate their 'true' net burdens invoking the equivalence result established above and the associated measures.

In the major part of the paper we assume cost-effective mixed policies to avoid blurring distributional and efficiency effects. Yet the hybrid EU policy is not cost effective because, among other things, the extant national overlapping taxes are not uniform across countries. We were able to show that our procedure of specifying burdens for cost-effective mixed policies can be extended to the empirically relevant scenario of non-uniform taxes. In this case distributional equivalence is combined with an overall efficiency gain that may be distributed by increasing the welfare of all countries at a uniform rate. The economist's recommendation would be, of

course, to eliminate the inefficiency through tax harmonization in the first place.

References

- Babiker, M., Metcalf, G.E. and J. Reilly (2003), Tax distortions and global climate policy, *Journal of Environmental Economics and Management* 46, 269-287.
- Bento, A.M. and M. Jacobsen (2007), Ricardian rents, environmental policy and the 'double-dividend' hypothesis, *Journal of Environmental Economics and Management* 53, 17-31.
- Böhringer, C., Koschel, H. and U. Moslener (2008), Efficiency losses from overlapping economic instruments in European carbon emissions regulation, *Journal of Regulatory Economics* 33, 299-317.
- Bovenberg, A.L. and R. de Mooij (1994), Environmental levies and distortionary taxation, *American Economic Review* 94, 1085-1089.
- Chichilnisky, G. and G. Heal (1994), Who should abate carbon emissions? An international viewpoint. *Economics Letters* 44, 443-449.
- Eichner, T. and R. Pethig (2009): Efficient CO₂ emissions control with emissions taxes and international emissions trading, *European Economic Review* 53, 625-635.
- Eichner, T. and R. Pethig (2008): EU-type carbon emissions trade, overlapping emissions taxes and the (re)distribution of welfare, manuscript.
- EU (2000): Green paper on greenhouse gas emissions trading within the European Union, COM (2000) 87 final (presented by the Commission).
- EU (2003a), 'Directive establishing a scheme for greenhouse gas emission allowances within the Community and amending Council Directive 96/61/EC', *Council Directive 2003/87/EC*, European Commission, Brussels.
- EU (2003b), 'Directive restructuring the Community framework for the taxation of energy products and electricity', *Council Directive 2003/96/EC*, European Commission, Brussels.

- International Energy Agency (2007): Energy prices and taxes. *Quarterly Statistics: Third Quarter 2007*.
- Johnstone, (2003), 'The use of tradable permits in combination with other environmental policy instruments', Report ENV/EPOC/WPNEP (2002)28/FINAL, OECD, Paris.
- Marklund, P.-O. and E. Samakovlis (2007): What is driving the EU burden-sharing agreement: Efficiency or equity?, *Journal of Environmental Management* 85, 317-329.
- Nordhaus, W.D. (2006): Life after Kyoto: Alternative approaches to global warming policies. *American Economic Review. Papers and Proceedings* 96, 31-34.
- Peterson, S. and G. Klepper (2007): Distribution matters - Taxes vs. emissions trading in post Kyoto climate regimes, Kiel Working Paper No. 1380, Kiel Institute for the World Economy.
- Phylipsen, G.J.M., Bodem JW. and K. Blok (1998): A Triptych sectoral approach to burden differentiation; GHG emissions in the European bubble, *Energy Policy* 26, 929-943.
- Rosendahl, K.E. (2008): Incentives and prices in an emissions trading scheme with updating, *Journal of Environmental Economics and Management* 56, 69-82.
- Shiell, L. (2003): Equity and efficiency in international markets for pollution permits, *Journal of Environmental Economics and Management* 46, 38-51.
- Sorell, S. and J. Sijm (2003), 'Carbon trading in the policy mix', *Oxford Review of Economic Policy* 19, 420-437.

Appendix: The comparative statics of changing the permit cap

c_i

The cost-effective competitive equilibrium is determined by the equations

$$\sum_j c_j = \sum_j (e_{xj} + e_{yj}), \quad (\text{A1})$$

$$x_{si} = x_i \quad i = 1, \dots, n, \quad (\text{A2})$$

$$x_{si} = X^i(e_{xi}), \quad i = 1, \dots, n, \quad (\text{A3})$$

$$x_i = D^i(p_{xi}, z_i), \quad i = 1, \dots, n, \quad (\text{A4})$$

$$z_i = p_{xi} + y_{si} - p_e(e_{xi} + e_{yi}) + \pi_e(c_i - e_{xi} - e_{yi}), \quad i = 1, \dots, n, \quad (\text{A5})$$

$$y_{si} = Y^i(e_{yi}), \quad i = 1, \dots, n, \quad (\text{A6})$$

$$z_i = p_{xi}x_i + y_i, \quad i = 1, \dots, n, \quad (\text{A7})$$

$$p_{xi}X_e^i(e_{xi}) = p_e + t_x, \quad i = 1, \dots, n, \quad (\text{A8})$$

$$Y_e^i = p_e + \pi_e + t_y, \quad i = 1, \dots, n, \quad (\text{A9})$$

$$t_x = \pi_e + t_y. \quad (\text{A10})$$

In (A1) - (A10) good Y is chosen as numeraire. The demand function (A4) follows from the first-order condition for utility maximization. It is convenient to compress the system of equations (A1) - (A10) as follows

$$\sum_j c_j = \sum_j (e_{xj} + e_{yj}), \quad (\text{A11})$$

$$X^i(e_{xi}) = D^i(p_{xi}, z_i), \quad (\text{A12})$$

$$z_i = p_{xi}X^i(e_{xi}) + Y^i(e_{yi}) - p_e(e_{xi} + e_{yi}) + \pi_e\Delta e_{yi}, \quad (\text{A13})$$

$$p_{xi}X_e^i(e_{xi}) = Y_e^i(e_{yi}), \quad (\text{A14})$$

$$Y_e^i(e_{yi}) = p_e + \pi_e + t_y, \quad (\text{A15})$$

$$y_i = Y^i(e_{yi}) - p_e(e_{xi} + e_{yi}) + \pi_e\Delta e_{yi}, \quad (\text{A16})$$

where $\Delta e_{yi} := c_i - e_{xi} - e_{yi}$. Our aim is to determine through a comparative static analysis the impact of exogenous variations in the caps c_i **subject to the constraint** $\sum_j dc_j = 0$. To that

end (A11) - (A15) are totally differentiated.

$$\sum_j (de_{xj} + de_{yj}) = 0, \quad (\text{A17})$$

$$X_e^i de_{xi} - D_p^i dp_{xi} - D_z^i dz_i = 0, \quad (\text{A18})$$

$$dz_i - x_i dp_{xi} - t_y (de_{xi} + de_{yi}) - \Delta e_{yi} d\pi_e - \pi_e dc_i = 0, \quad (\text{A19})$$

$$X_e^i dp_{xi} + p_{xi} X_{ee}^i de_{xi} - Y_{ee}^i de_{yi} = 0, \quad (\text{A20})$$

$$Y_{ee}^i de_{yi} - d\pi_e = 0. \quad (\text{A21})$$

Inserting $de_{yi} = \frac{d\pi_e}{Y_{ee}^i}$ from (A21) in (A20) yields

$$de_{xi} = \frac{d\pi_e}{p_{xi} X_{ee}^i} - \frac{X_e^i dp_{xi}}{p_{xi} X_{ee}^i}. \quad (\text{A22})$$

Summation of de_{xi} from (A22) and de_{yi} from (A21) gives

$$de_{xi} + de_{yi} = \alpha_i dp_{xi} - \beta_i d\pi_e, \quad (\text{A23})$$

where $\alpha_i := -\frac{X_e^i}{p_{xi} X_{ee}^i} > 0$ and $\beta_i := -\left(\frac{1}{Y_{ee}^i} + \frac{1}{p_{xi} X_{ee}^i}\right) > 0$. Inserting (A23) in (A17) we obtain

$$\frac{\sum_j \alpha_j dp_{xj}}{\sum_j \beta_j} = d\pi_e. \quad (\text{A24})$$

Next, we take advantage of (A23) to turn (A19) into

$$dz_i = (x_i + \alpha_i t_y) dp_{xi} + (\Delta e_{yi} - \beta_i t_y) d\pi_e + \pi_e dc_i. \quad (\text{A25})$$

We make use of (A22) and (A25) to transform (A18) into

$$dp_{xi} = \frac{(\delta_i + D_z^i \Delta e_{yi})}{\gamma_i} d\pi_e + \frac{D_z^i \pi_e}{\gamma_i} dc_i, \quad (\text{A26})$$

where $\delta_i := \alpha_i - \beta_i t_y D_z^i$ and $\gamma_i := \alpha_i X_e^i - D_p^i - (x_i + \alpha_i t_y) D_z^i$. We insert (A26) into (A24) to obtain, after some rearrangement of terms,

$$d\pi_e \left[\sum_j \left(\frac{\beta_j \gamma_j - \alpha_j \delta_j - D_z^j \Delta e_{yj}}{\gamma_j} \right) \right] = \sum_j \frac{\alpha_j D_z^j \pi_e}{\gamma_j} dc_j. \quad (\text{A27})$$

Next, we differentiate the utility function $u_i = U^i(x_i, y_i)$ and use the first-order condition of the consumer's utility maximization problem to get

$$\frac{du_i}{\lambda_i} = p_{xi} dc_i + dy_i, \quad (\text{A28})$$

where λ_i is the marginal utility of income. From (A3), (A8) and (A10) we infer

$$dx_i = X_{ee}^i de_{xi} = \frac{p_e + \pi_e + t_y}{p_{xi}} de_{xi}. \quad (\text{A29})$$

From (A16) we obtain with the help of (A15)

$$dy_i = t_y de_{yi} - (p_e + \pi_e) de_{xi} + \Delta e_{yi} d\pi_e + \pi_e dc_i. \quad (\text{A30})$$

Inserting (A30) and (A29) in (A28) yields after some rearrangement of terms

$$\frac{du_i}{\lambda_i dc_i} = t_y \frac{de_{xi} + de_{yi}}{dc_i} + \Delta e_{yi} \frac{d\pi_e}{dc_i} + \pi_e. \quad (\text{A31})$$

From (A23) it follows that

$$\frac{de_{xi} + de_{yi}}{dc_i} = \alpha_i \frac{dp_{xi}}{dc_i} - \beta_i \frac{d\pi_e}{dc_i}. \quad (\text{A32})$$

(A26) yields

$$\frac{dp_{xi}}{dc_i} = \frac{(\delta_i + D_z^i \Delta e_{yi})}{\gamma_i} \frac{d\pi_e}{dc_i} + \frac{D_z^i \pi_e}{\gamma_i}. \quad (\text{A33})$$

Making use of (A33) in (A32) yields

$$\frac{de_{xi} + de_{yi}}{dc_i} = \left(\frac{\alpha_i \delta_i - \beta_i \gamma_i + \alpha_i D_z^i \Delta e_{yi}}{\gamma_i} \right) \frac{d\pi_e}{dc_i} + \frac{\alpha_i D_z^i \pi_e}{\gamma_i}. \quad (\text{A34})$$

Finally, taking advantage of (A34) in (A31) establishes

$$\frac{du_i}{\lambda_i dc_i} = \left[\frac{t_y (\alpha_i \delta_i - \beta_i \gamma_i + \alpha_i D_z^i \Delta e_{yi}) + \gamma_i \Delta e_{yi}}{\gamma_i} \right] \frac{d\pi_e}{dc_i} + \frac{\alpha_i t_y D_z^i \pi_e}{\gamma_i} + \pi_e. \quad (\text{A35})$$