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# How Child Costs and Survival Shaped the Industrial Revolution and the Demographic Transition: A Theoretical Inquiry

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**Abstract.** This study provides a unified growth theory to correctly predict the initially negative and subsequently positive relationship between child mortality and net reproduction observed in industrialized countries over the course of their demographic transitions. The model captures the intricate interplay between technological progress, mortality, fertility and economic growth in the transition from Malthusian stagnation to modern growth. Not only does it provide an explanation for the demographic observation that fertility rates response with a delay to lower child mortality. It also identifies a number of turning points over the course of development, suggesting a high degree of complexity regarding the relationships between various economic and demographic variables.

*Keywords:* Economic Growth, Mortality, Fertility, Structural Change, Industrial Revolution.

*JEL:* O11, O14, J10, J13.

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## 1. INTRODUCTION

In recent years, several so-called *unified growth theories* have been forwarded to try to motivate the historical shift from economic stagnation to modern growth. Following seminal work by Galor and Weil (2000), these include Boucekkine et al. (2002), Doepke (2004), Galor and Moav (2002), Galor and Mountford (2008), Jones (2001), Kögel and Prskawetz (2001), Lucas (2002), Strulik and Weisdorf (2008), Cervellati and Sunde (2005, 2007) and Tamura (2002). A part of this exercise consists of providing a micro foundation for the fundamental links between economic and demographic variables, from pre-industrial times to the present day. One particular issue that scholars have been struggling with for a long time (although not only in the context of unified growth theory) is the impact of lower child mortality on fertility and net reproduction.<sup>1</sup> Most macroeconomic models can replicate the fact that a lower death risk of children leads to fewer births. However, since falling child mortality reduces the cost of producing surviving children, which means that surviving children become cheaper, net reproduction in these models ends up increasing in response to lower child mortality. This consists of a contrast to the experience of most industrialized countries in the later part of their demographic transitions (Doepke 2005).

This study provides a model where the relationship between child mortality and net reproduction is positive during early stages of development, but turns negative during later, more advanced stages. This result relies on the notion that fertility, and thus net reproduction, is affected not only by child mortality, but also by the cost of raising children. The theory arises from combining two existing contributions: Strulik (2008) and Weisdorf (2008). Like in Strulik (2008), we assume that parents care, not only about surviving offspring, but also their nutritional status. For a given level of nutritional input, a drop in child mortality brings parents to reduce their fertility, as more children now survive. At the same time, higher survival probability causes parents to nourish their children more (a quantity-quality substitution effect). That further improves the offsprings' survival probability, which again lowers fertility, and so on and so forth.<sup>2</sup>

Second, as in Weisdorf (2008), we use the fact that in a closed economy the real cost of food—and thus the cost of raising children—is affected by productivity growth in agriculture *as well as* industry. The effect from industry works through the labor market equilibrium condition: when

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<sup>1</sup>Net reproduction measures the number of offspring (normally women) living through to the end of their fertile age.

<sup>2</sup>This is consistent with empirical evidence which suggests that nutrition played a key role in Britain's mortality transition (Harris 2004), and that malnutrition has severe effects on child mortality (see Rice et al., 2000, Pelletier et al., 2003, and Caulfield et al., 2004)

productivity in industry grows, industrial workers receive higher wages. In a closed economy with free labor mobility, agricultural terms of trade (the relative cost of food) will then have to adjust to insure that laborers remain in agriculture. Therefore, if the relative costs of food is decisive for fertility, productivity growth in agriculture and industry both impact on the number of offspring that parents choose to have.

Hence, the key to understanding why the relationship between mortality and net reproduction changes over time lies in the fact that fertility is determined *both* by changes in mortality *and* by technological advancements in agriculture and industry. Below, we use these features to build a unified growth theory that correctly predicts the relationship between child mortality and net reproduction over the course of the demographic transition. The theory accounts for the intricate interplay between technology, mortality, fertility and income per capita in the process from stagnation to growth. The theory is also in line with Engel's law, replicating the stylized fact that, when income rises, total food expenditures increase while the share of food expenditures to income falls. When calibrated, the model points to several structural breaks, i.e. incidents where the correlation between various variables, such as fertility and population growth, or population growth and TFP growth shifts sign. This explains why empirical scholars may have a hard time identifying the relationship between economic and demographic variables over the long run.

The paper continues as follows. Section 2 provides a brief introduction to the stylized facts for Western Europe (particularly England) regarding the evolution of mortality, fertility and net reproduction over the long run. It also offers a summary of the theoretical literature related to the present work. Section 3 details the theoretical framework, and Section 4 explores the its balanced growth dynamics. Section 5 calibrates the model to analyze its adjustment dynamics, while Section 6 concludes.

## 2. EMPIRICAL EVIDENCE AND THE RELATED LITERATURE

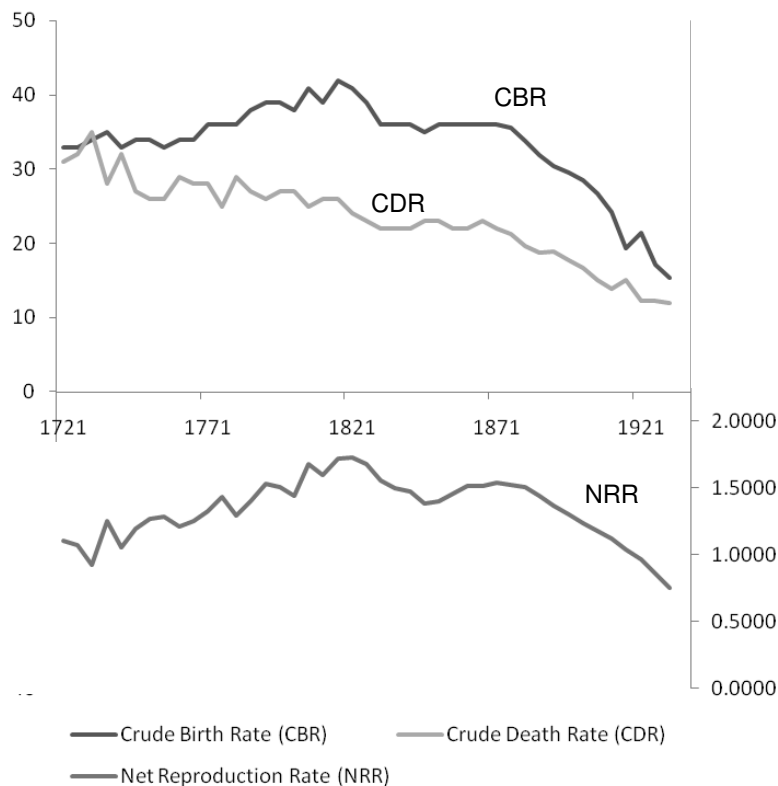
In most Western Europe countries, the demographic transition occurred in the later half of the 19th century.<sup>3</sup> After a peak in the 1870s, birth rates drop roughly one-third over the subsequent 50 years. In England, total fertility rates decline by close to 50 percent, from nearly five children per women in 1875 to 2.4 children by 1920. Crude birth rates follow a similar pattern, declining by 44 percent from 36 per thousand inhabitants in 1875 to 20 in 1920 (Figure 1).

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<sup>3</sup> For a more detailed description of the Western Europe's economic and demographic patterns, see Galor (2005).

With the exception of France and the US, substantial mortality decline precedes the fall in fertility. In England, mortality rates begin to fall roughly one and a half century prior to the drop in fertility. During early phases of England's so-called *mortality revolution*, it was primarily lower child mortality that gave momentum to the drop in death rates. As the fall in child mortality appeared before the fertility decline set in, falling mortality initially gave rise to an *increase* in net reproduction. After the onset of the fertility decline, however, the relationship changes. From then on, falling birth rates go hand in hand with declining rates of net reproduction. Eventually, falling child mortality is outpaced by the decline in fertility, as reflected in a reduction in the net rate of reproduction.<sup>4</sup>

Figure 1: Crude Birth Rates, Crude Death rates, and Net Reproduction Rates for England, 1721-1931



Sources: Wrigley and Schofield (1984), Wrigley (1969) and Reher (2004).

A unified growth theory that wants to capture the long-run evolution of mortality, fertility and net reproduction must thus account not only for the increase of net reproduction and the spike in

<sup>4</sup> Estimating the demo-economic interaction for Sweden, Eckstein et al. (1999) find that declining child mortality explains about 80% of the historical fertility decline while increasing real wages explain less than one-third.

birth rates observed prior to the fertility decline. It must also be able to predict the subsequent decrease of fertility and net reproduction once the fertility transition is running its course.

Studies on the relationship between child mortality and fertility goes back at least to Becker and Barro (1988, 1989). In the Barro-Becker model, parents derive utility from surviving offspring. Their fertility decision is affected by the costs of producing surviving offspring. Lower child mortality reduces the costs of surviving offspring. So when child mortality falls, parents give birth to more children. Jointly, falling child mortality and rising fertility lead to higher net reproduction. But evidence points in the other direction: at the onset of the demographic transition, net reproduction was falling. The Barro-Becker type models, therefore, search for other explanations behind the fertility decline than falling mortality (Doepke, 2005). Among recent studies, where mortality plays a role in the process of development, a shift from child quality to child quantity is thus generated through parents' investments in education and human capital accumulation (Azarnert 2006; Ehrlich and Kim 2005; Kalemli-Ozcan, Ryder, and Weil 2000; Soares 2005).

A refinement of these models, a shift from exogenous to endogenous mortality has been invoked to capture the long run trends in economic and demographic variables (Doepke 2005; Jones 2001; Kalemli-Ozcan 2002; Lagerlöf 2003; Weisdorf 2004). Jones (2001), Lagerlöf (2003) and Weisdorf (2004) compare directly to our work in that they analyze the effect of mortality on fertility in the context of unified growth theory. Remarkably, they all share the common feature that falling death rates have no impact on parents' fertility decision. In Jones (2001), parents' preferences imply that the elasticity of substitution between consumption and children is always greater than one, an assumption that ultimately generates a drop in fertility as income grows. In Lagerlöf (2003) and Weisdorf (2004), the decline of fertility is a result of human capital accumulation and a parental trade off between child quantity and quality.

Here, we set out to explore the direct role of child mortality for parents' fertility decision, extending the model offered by Strulik and Weisdorf (2008). While the previous work neglects the role of death in development, mortality in the present setup is made endogenous, partly to child nutrition, provided by the parents, partly to general health factors (availability of medicine, provision of sewage etc), which are exogenous to parents. When general health factors improve, made possibly in the model by technological progress in industry, parents find it advantageous to spend more on nutrition of their children. This, in turn, induces a child quantity-quality trade-off, and explains how changes in economic conditions shape demographic patterns (Lee 2002).

This result compares to Cervellati and Sunde (2007), who also investigate the link between mortality and fertility over the very long run. They focus on the interaction between education and adult longevity as a driver of growth, emphasizing the skill premium (as opposed to agricultural terms of trade in the present study) as a key factor in the transition from stagnation to growth. It is particularly comparable to the current work in the sense that it highlights the role of relative prices in development.

### 3. THE MODEL

**3.1. Fertility, mortality, and net reproduction.** We consider a two-period overlapping generations economy with children and adults. Let  $L_t$  denote the number of adults in period  $t$ , and  $n_t$  the number of births per adult.<sup>5</sup> The birth rate (referred to also as *total fertility*) is determined endogenously below. The variable  $\pi_t \in [0, 1]$  measures a child's survival probability (implicitly, its mortality), and is synonymous to the fraction of children, born in period  $t$ , who are still alive at period  $t + 1$ . It follows that the net reproduction rate – the number of offspring living through their fertile age – is  $\pi_t n_t$ . Changes in the size of the labor force (the adult population) between any two periods can thus be expressed as

$$L_{t+1} = \pi_t n_t L_t. \tag{1}$$

We consider two types of child survival probabilities: an extrinsic and an intrinsic survival rate. The extrinsic rate, denoted  $\bar{\pi}_t \in [0, 1]$ , is exogenous to a parent, but is affected by general-purpose hygiene or health improving factors, such as sewerage, water toilets, central heating, clinical devices, vaccines, pharmaceuticals and medical knowledge in general. We take advances in industrial knowledge, measured by  $M_t$ , to be a good proxy for this. Specifically, it is assumed that  $\bar{\pi}_t = \bar{\pi}(M_t)$ , where  $\bar{\pi}'_M > 0$  with  $\lim_{M_t \rightarrow \infty} \bar{\pi}_t = a < 1$ . The variable  $M_t$ , and thus  $\bar{\pi}_t$ , are determined endogenously below.

By contrast to the extrinsic child survival probability, the intrinsic survival rate is affected by parents' nutritional investment in their offspring. We allow for diminishing marginal productivity to nutritional investments, so that an additional unit of nutrition is more effectively hedging against child mortality at low levels of extrinsic survival rates than at high ones. More specifically, we assume that the overall child survival probability (intrinsic as well as extrinsic) is given by

$$\pi_t = \bar{\pi}_t + [1 - \bar{\pi}_t] \cdot \lambda \cdot h_t, \quad \lambda > 0, \tag{2}$$

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<sup>5</sup> To keep the model tractable, we assume that  $n_t$  is continuous, and that reproduction is asexual.

where  $\lambda$  is a productivity parameter and where  $h_t$  measures the nutritional status of an offspring, the level of which is determined endogenously below.<sup>6</sup>

**3.2. Preferences and optimization.** Adult individuals (parents) maximize utility, which they derive from three sources: surviving offspring,  $\pi_t n_t$ , the nutritional status of their offspring,  $h_t$ , and number of manufactured goods consumed, measured by  $m_t$ . We assume that preferences are described by a utility function where the elasticity of marginal utility is higher for  $h_t$  and  $n_t$  than for  $q_t$ . This is to capture the fact that, in times of crisis, parents will try to smooth fertility and nutritional status of their offsprings more than their consumption other, less vital (i.e. manufactured) goods (Livi-Bacci, 2006). The simplest utility function that captures such a ‘hierarchy of needs’ is of quasi-linear form:

$$u_t = m_t + \beta \log(h_t) + \gamma \log(\pi_t n_t), \quad \beta, \gamma > 0. \quad (3)$$

Similar to Andreoni (1989) and Becker (1960), the parameter  $\beta$  measures the extent to which parents care about the nutritional status of their offspring. We assume that  $\gamma > \beta$ , implying that parents without children do not allocate income to child nutrition.

To make the model tractable, we assume that nutritional goods are demanded only during childhood, and some of it then stored for adulthood.<sup>7</sup> The price of manufactured goods is set to one, so that we can let  $p_t$  denote the price of one unit of nutrition (food), measured in terms of manufactured goods. Each offspring consumes  $h_t$  units of nutrition. This means that the total costs of raising  $n_t$  children, measured in terms of manufactured goods, is  $p_t h_t n_t$ . The budget constraint of an individual adult thus reads

$$w_t = p_t h_t n_t + m_t, \quad (4)$$

where  $w_t$  is parental income, also measured in terms of manufactured goods.

The optimization problem of an adult individual involves maximizing (3) subject to (2) and (4). The solutions—the optimal number of births and the level of nutritional input per children—are

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<sup>6</sup> We furthermore assume that  $a \leq \beta/\gamma$ . This ensures, as will become obvious below – when (6) is inserted in (2) – that  $\pi \in (0, 1)$ .

<sup>7</sup> It will not affect the qualitative nature of the results if, instead, the individual’s nutritional demand were to be divided over two periods.



given by

$$n_t = \frac{\beta\gamma\lambda(1 - \bar{\pi}_t)}{p_t\bar{\pi}_t(\gamma - \beta)}, \quad (5)$$

$$h_t = \frac{(\gamma - \beta)\bar{\pi}_t}{\beta\lambda(1 - \bar{\pi}_t)}. \quad (6)$$

Based on (5) and (6) the following should be noted. First, children are normal goods, so when the price of children rises, demand (fertility) falls. Second, due to the specific preference function, there is no *direct* income-effect on the demand for children. However, as will become apparent below, an *indirect* income-effect will enter through the price of nutrition ( $p_t$ ). Third, it follows from (5) that an improvement of the child survival probability reduces fertility ( $\partial n_t / \partial \bar{\pi}_t < 0$ ), meaning that lower child mortality reduces birth rates, a conclusion perfectly consistent with evidence (Galloway et al., 1998, Eckstein et al., 1999). Fourth, equation (6) reveals that a higher extrinsic probability of child survival leads to a higher nutritional status of the offspring ( $\partial h_t / \partial \bar{\pi}_t > 0$ ). This follows from quantity-quality substitution. As more children survive, parents reduce fertility, putting emphasis on nutrition instead. In turn, more nutrition improves the intrinsic survival probability. This generates a feedback mechanism, running from higher (extrinsic) child survival over more nutrition per child to higher (intrinsic) child survival.

By combining (5) and (6) with (2), we get the net rate of reproduction, which is given by

$$\pi_t n_t = \bar{\pi}_t n_t + (1 - \bar{\pi}_t)\lambda h_t n_t = \frac{\lambda\gamma^2(1 - \bar{\pi}_t)}{p_t(\gamma - \beta)} \quad (7)$$

Equation (7) captures a main feature of the current framework: that higher child survival probability—corresponding to lower child mortality—reduces net fertility, an empirical regularity lacking in previous contributions (cf. the discussion in Doepke 2005).

**3.3. Production.** We consider a dual-sector economy with agriculture and industry. In both sectors, new technology arises from learning-by-doing. More specifically, output, as well as new knowledge, occurs according to the following production functions:

$$Y_t^A = \mu A_t^\varepsilon (L_t^A)^\alpha = A_{t+1} - A_t, \quad 0 < \alpha, \varepsilon < 1 \quad (8)$$

$$Y_t^M = \delta M_t^\phi L_t^M = M_{t+1} - M_t, \quad 0 < \phi < 1. \quad (9)$$

The variable  $A_t$  measures TFP in agriculture, whereas  $M_t$  measures TFP in industry (manufacturing). We allow for diminishing returns to new knowledge in both sectors by assuming that

$0 < \varepsilon, \phi < 1$ . Agricultural production is subject to constant returns to labor and land. Land is assumed to be in fixed supply, and the total amount is normalized to one. With  $0 < \alpha < 1$  there is thus diminishing returns to labor in agriculture. Industrial production, by contrast, is subject to constant returns to labor, implying that land is not an important factor in industrial production. As is standard in the related literature, we abstract throughout from the use of physical capital in production.

**3.4. Equilibrium.** The variables  $L_t^A$  and  $L_t^M$  measure total labor input in agriculture and industry, respectively. Together, they make up the entire labor force, i.e.

$$L_t^A + L_t^M = L_t. \quad (10)$$

The share of total labor devoted to agriculture,  $L_t^A/L_t$ , is determined by the market equilibrium condition for nutritional (i.e. food or agricultural) goods. This condition says that the total supply of nutrition,  $Y_t^A$ , equals total demand, which—given that each child demands  $h_t$  units of food—is  $h_t n_t L_t$ . Using (8), the market equilibrium condition for nutrition thus implies that the fraction of workers engaged in agriculture is given by

$$\theta_t \equiv \frac{L_t^A}{L_t} = \left( \frac{h_t n_t L_t^{1-\alpha}}{\mu A_t^\varepsilon} \right)^{\frac{1}{\alpha}}. \quad (11)$$

Note that agricultural TFP growth releases labor from agriculture, whereas population growth and higher levels of child nutrition has the opposite effect.

Suppose that there are no property rights over land, meaning that the land rent is zero, and thus that a representative adult individual receives the average product of the sector in which it is employed. The labor market equilibrium condition then implies that the real price of nutrition adjusts, so that farmers and manufacturers earn the same income, i.e. so that  $w_t = p_t Y_t^A / L_t^A = Y_t^M / L_t^M$ . By the use of (5)-(6) and (7)-(11), this means that the price of one unit of nutrition, measured in terms of manufactured goods, is

$$p_t = \frac{(\delta M_t^\phi)^\alpha (\gamma L_t)^{1-\alpha}}{\mu A_t^\varepsilon}. \quad (12)$$

It follows that the price of nutrition increases with TFP growth in industry, as well as with the size of the population, whereas TFP growth in agriculture has the opposite effect.

Inserting (12) into (7) gives us the net reproduction rate in a general equilibrium, which reads

$$\pi_t n_t = \frac{\mu A_t^{\bar{\epsilon}}}{\left(\delta M_t^{\phi}\right)^{\alpha} (\gamma L_t)^{1-\alpha}} \cdot \frac{\gamma^2 \lambda (1 - \bar{\pi}_t)}{(\gamma - \beta)}. \quad (13)$$

The two main forces affecting net reproduction are clearly visible in (13). The first term captures the negative effect of a higher price of nutrition. The second term contains the negative effect of lower child mortality.

#### 4. BALANCED AND UNBALANCED GROWTH IN THE LONG RUN

In the following, we explore the balanced growth dynamics of the model. Along a balanced growth path, all variables are constant or grow at constant rates. Let a balanced growth rate of a variable  $x$  be denoted by  $g^x$  (to be identified by a missing time index). According to (8), the gross rate of TFP growth in agriculture is  $g_t^A = (A_{t+1} - A_t)/A_t = \mu(L^A_t)^{\alpha}/A_t^{1-\epsilon}$ . Along a balanced growth path, the left hand side is constant by definition, so that the right hand side must be constant as well. Furthermore, the share of labor in agriculture must be constant, implying that  $L^A$  grows at the same rate as  $L$ . Thus, constant TFP growth in agriculture requires that

$$1 + g^A = (1 + g^L)^{\alpha/(1-\epsilon)}. \quad (14)$$

Similarly, we get from (9) that a constant rate of growth of TFP in industry requires that

$$1 + g^M = (1 + g^L)^{1/(1-\phi)}. \quad (15)$$

Combining (13) with (14) and (15), the gross rate of growth of net reproduction can then be written as

$$1 + g_{t+1}^L = \frac{\pi_t n_t}{\pi_{t+1} n_{t+1}} = \frac{1 - \bar{\pi}_{t+1}}{1 - \bar{\pi}_t} \cdot \frac{(1 + g_t^A)^{\epsilon} (1 + g_t^L)^{\alpha}}{(1 + g_t^M)^{\alpha\phi}}. \quad (16)$$

Along a balanced growth path, the level of TFP in industry is either constant, or is growing at a constant rate. In either case,  $\bar{\pi}$  will eventually assume a constant value, meaning that, along a balanced growth path, the first term on the right-hand side is equal to one. Using this information, and inserting (14) and (15) into (16), we find that the equilibrium law of motion for population growth is given by

$$1 + g_{t+1}^L = (1 + g_t^L)^{\eta}, \quad \eta \equiv \alpha + \frac{\alpha\epsilon}{1-\epsilon} - \frac{\phi\alpha}{1-\phi}. \quad (17)$$

Along a balanced growth path, the population level grows at constant rate meaning that  $g_{t+1}^L = g_t^L = g^L$ . This leaves two possibilities for balanced growth. Either there is no population growth ( $g^L = 0$ ) or – assuming the knife-edge condition that  $\eta = 1$  – the population level is growing or shrinking at a constant rate. However, it follows from (14) that  $|\eta| < 1$  is required for stability reasons. Therefore, growth on the knife-edge not only demands a very specific parameter constellation; it also implies that the economy starts off *on* a balanced growth path (with suitable initial values) and remains there forever. This essentially eliminates the possibility of having balanced growth together with population growth. The implication—that there is no population growth on a balanced growth path—means that there is also no TFP growth in steady state (as can be verified by looking at (14) and (15)). This conclusion is summarized in the following proposition.

PROPOSITION 1. *There exists a unique balanced growth path with zero population growth and zero (exponential) economic growth. A sufficiently small knowledge elasticity in agriculture,*

$$\epsilon < \frac{1 - \phi - \alpha + 2\alpha\phi}{1 - \phi + \alpha\phi}, \quad (18)$$

*prevents unbalanced growth in the long-run.*

*Proof.* The proof is found in the Appendix.<sup>8</sup> □

## 5. ADJUSTMENT DYNAMICS AND CALIBRATION

The aim of this section is to see whether the model can replicate the stylized development pattern observed among industrialized countries from their pre-industrial era to the present-day and beyond.<sup>9</sup> Therefore, in the following, we explore the model’s adjustment dynamics towards balanced growth.

Suppose we start off with an economy in which the population level is relatively small; the share of labor employed in agriculture is relatively high; and the level of income per capita is relatively close to subsistence. Furthermore, suppose that birth rates, as well as rates of child mortality, are relatively high, meaning that the net rate of reproduction is close to that of replacement. Roughly speaking, these are the characteristics of a pre-industrial, agricultural society.

As explained in the model section, there are economies-of-scale to population. Since the initial population level is low, learning-by-doing effects, to begin with, are relatively modest. Hence, TFP

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<sup>8</sup> Note that the term ‘unbalanced growth’ refers to an equilibrium growth path along which growth rates are exploding or imploding (and should not be confused with ‘unstable’, i.e. off-equilibrium growth).

<sup>9</sup> For a detailed description of the development course of industrialized countries, see Galor (2005).

growth in agriculture is slow, yet faster than in industry, where labor resources, and thus learning-by-doing effects, are even smaller.

TFP growth in agriculture has two effects on development. On the one hand, because it releases labor from agriculture, agricultural TFP growth increases the share of labor allocated to industrial activities. On the other hand, higher TFP growth in agriculture *relative to industry* makes nutrition, and therefore children, relatively less expensive. According to (5), this raises fertility, which tends to increase the net reproduction rate.

At the same time, with economies-of-scale at work in both agriculture and industry, the transfer of labor out of agriculture gradually speeds up TFP growth in industry. As industrial knowledge gains momentum, extrinsic child survival begins to increase. This leads parents to spend more on nutrition per child, which further improves the survival probability of offspring through a reduction in intrinsic child mortality. As follows from (13), this drop in mortality lowers fertility, which tends to slow down the net reproduction rate.

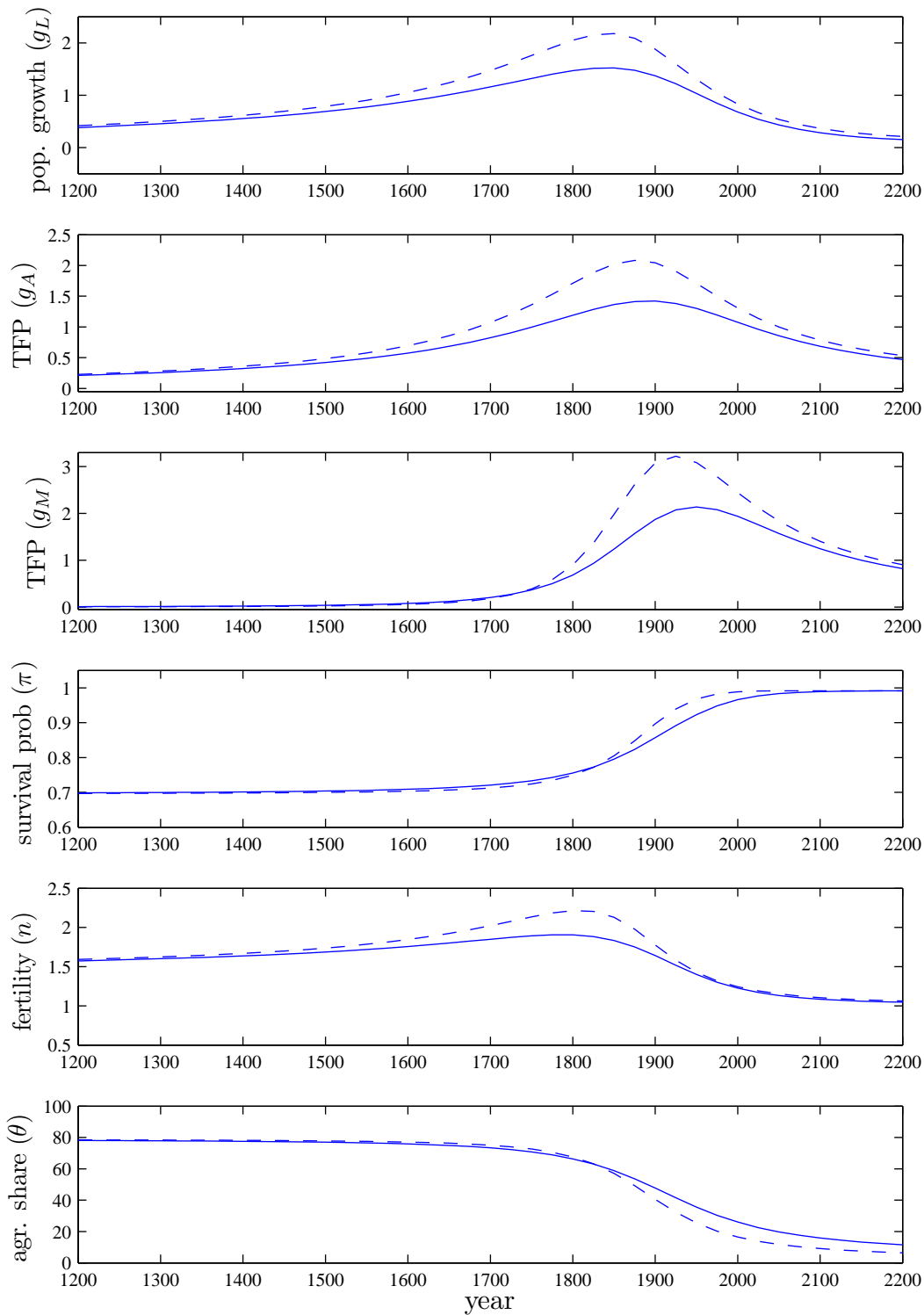
When advances in industrial knowledge, and thus child mortality decline, are relatively slow, the 'cheaper nutrition' effect dominates the 'mortality decline' effect. Hence, during early stages of development, the net reproduction rate goes up. In this period, therefore, declining child mortality is accompanied by rising rates of birth and net reproduction.

Since the transfer of labor out of agriculture gradually accelerates TFP growth in industry, the industrial sector's TFP growth rate eventually surpasses that of agriculture. Henceforth, the price of nutrition gradually increases, and children thus become relatively more expensive. All else equal, this leads to lower birth rates and therefore falling net reproduction. At the same time, advances in industrial knowledge creates further decline in extrinsic child mortality, leading to more investment in children's nutritional status, and therefore to lower intrinsic mortality. Hence, by contrast to previous periods, declining child mortality is now accompanied by falling rates of birth and net reproduction.

Eventually, the child survival rate reaches its maximum. Due to rising prices of nutrition, however, birth rates are still falling. Sooner or later, therefore, the net rate of reproduction reaches that of replacement, and population growth ultimately (and endogenously) comes to a halt.

**5.1. Calibration.** In order to investigate adjustment dynamics quantitatively, we now calibrate the model. Parameter values are chosen, so that the peak of the demographic transition matches that of 19th-century England and so that the maximum rate of industrial TFP growth (and subsequent

Figure 2: Long-Run Dynamics



Solid lines: benchmark case, dashed lines: higher higher extrinsic mortality:  $b = 0.15$  and less initial endowment  $L_0$  and  $A=0$ , otherwise benchmark parameters. From top to bottom the diagrams show population growth, productivity growth in agriculture, productivity growth in manufacturing, child survival rate, fertility rate, and the labor share in agriculture.

slowdown) appears in the late 20th century. Specifically, the rate of net reproduction reaches a peak of 1.5 percent per year in 1875, and industrial TFP growth begins to slow down around 1975, after a maximum of 2 percent per year.

For comparative purposes, we use as many parameter values as possible from the benchmark run calibrated in Strulik and Weisdorf (2008). Hence, the following parameters are chosen:  $\alpha = 0.8$ ,  $\epsilon = 0.45$ ,  $\phi = 0.3$ ,  $\mu = 0.5$ ,  $\delta = 0.32$ , and  $\gamma = 3.4$ . Start values are  $\theta_0 = L_0^A/L_0 = 0.8$ ,  $A_0 = 100$ , and  $L(0) = 100$ . The value for  $M_0$  is obtained endogenously, and is given by  $(\gamma/(\delta\theta_0))^{1/\phi}$ . Extrinsic child mortality is parameterized by the logistic function  $\bar{\pi}_t = a(1 - e^{-bMt})$ . The parameters  $a$ ,  $b$ ,  $\lambda$  and  $\beta$  are chosen, so that the child survival probability is 70 percent in the High Middle Ages, and its maximum is set just below one hundred percent. The parameter values are thus  $a = 0.35$ ,  $b = 0.28$ ,  $\beta = 1.2$ , and  $\lambda = 0.14$ . For better readability of the results, one generation is set to 25 years, or approximately the length of the fecundity period.

Figure 2 shows the calibrated adjustment path. The period analyzed runs from year 1200 to year 2200. Solid lines show the path of the benchmark economy; dashed lines concern an alternative economy to be discussed further below. Initially, labor is predominately allocated to agriculture. In line with numbers provided by Galor (2005), industrial TFP growth is almost absent during early stages of development. Productivity growth in agriculture is around 0.2 percent per year during the high middle ages, supporting a population growth rate of 0.2 percent per year.

Increasing knowledge in agriculture, which to begin with manifests itself in a slowly decreasing price of nutrition, translates almost entirely into population growth, meaning that standards of living are hardly affected by technological progress. A slow growth of population slowly furthers agricultural development, and little by little agricultural TFP growth builds up to reach 1.4 percent at the end of the 19th century. By then, agriculture TFP growth has made possible a substantial transfer of labor into industry, causing an upsurge in industrial TFP growth. This substantiates significant decline in extrinsic mortality, leading parents to increase their spending on child nutrition. With decreasing rates of extrinsic and intrinsic mortality, child survival probability is on a fast rise.

During the mid 18th century TFP growth in manufacturing surpasses TFP growth in agriculture and the growth rate of food prices begins to rise  $\partial g_P/\partial t > 0$ . The sign of the growth rate, however, remains negative for almost another century, indicating that the economic system has enough momentum for food prices to continue to fall. Around 1825 the relative price of food begins to increase, inducing people to spend more on industrial goods and reduce fertility. Because of declining child

mortality, population growth continues to grow for another 2 generations peaking in 1875. From then on, falling child mortality and higher costs of nutrition work in the same direction, leading to substantial fertility decline.

By the end of the 20th century, the demographic transition is almost complete. The child survival probability assumes its maximum value close to one hundred percent, and net reproduction is close to its replacement level. TFP growth in industry reaches its peak around 1975, and then begins to slow down. This decline, however, is a gradual process, leaving enough momentum for industrial TFP growth to exceed one percent per year far into the 22st century.

Note how the introduction of endogenous child mortality modifies the results obtained in Strulik and Weisdorf (2008). In the latter study, the peak of population growth coincides with that of fertility, which in turn takes place after agricultural TFP growth is surpassed by TFP growth in industry. Here, by contrast, the breakup of the population growth rate into a fertility and a mortality rate permits us to track down separately the occurrence of important economic and demographic events. For example, in the present calibration exercise, fertility peaks around 1825, whereas population growth reaches its maximum in 1875. Agricultural TFP growth peaks around 1850, whereas industrial TFP growth in manufacturing arrives at its summit in 1975. What is more, consistent with empirical evidence (with the exception of France), falling child mortality in the present framework precedes the drop in fertility. The current model thus correctly predicts the positive relationship between child mortality and net reproduction observed during most industrialized countries' demographic transitions, a feature that seems to be missing in previous contributions.

The model predicts that demographic transition and economic take-off are accompanied by a complete structural change ( $\theta \rightarrow 0$  for  $t \rightarrow \infty$ ). The speed of structural change during industrialization, however, is underestimated for the basic set of parameters. Since in the present model everything is fully endogenous (i.e. no time series is imputed, as in some of the related literature), it is impossible to correct this result without accepting a less good approximation of the data elsewhere. In order to demonstrate this trade-off by example, we set  $\mu = 0.6$  instead of 0.5 for an alternative economy, indicating that at any level of economy activity, agricultural productivity is higher. All other parameters are taken from the basic run. For comparison,  $A(0)$  has been adjusted such that the alternative run also predicts population growth to peak in 1875. Adjustment dynamics are represented by dotted lines in Figure 2.



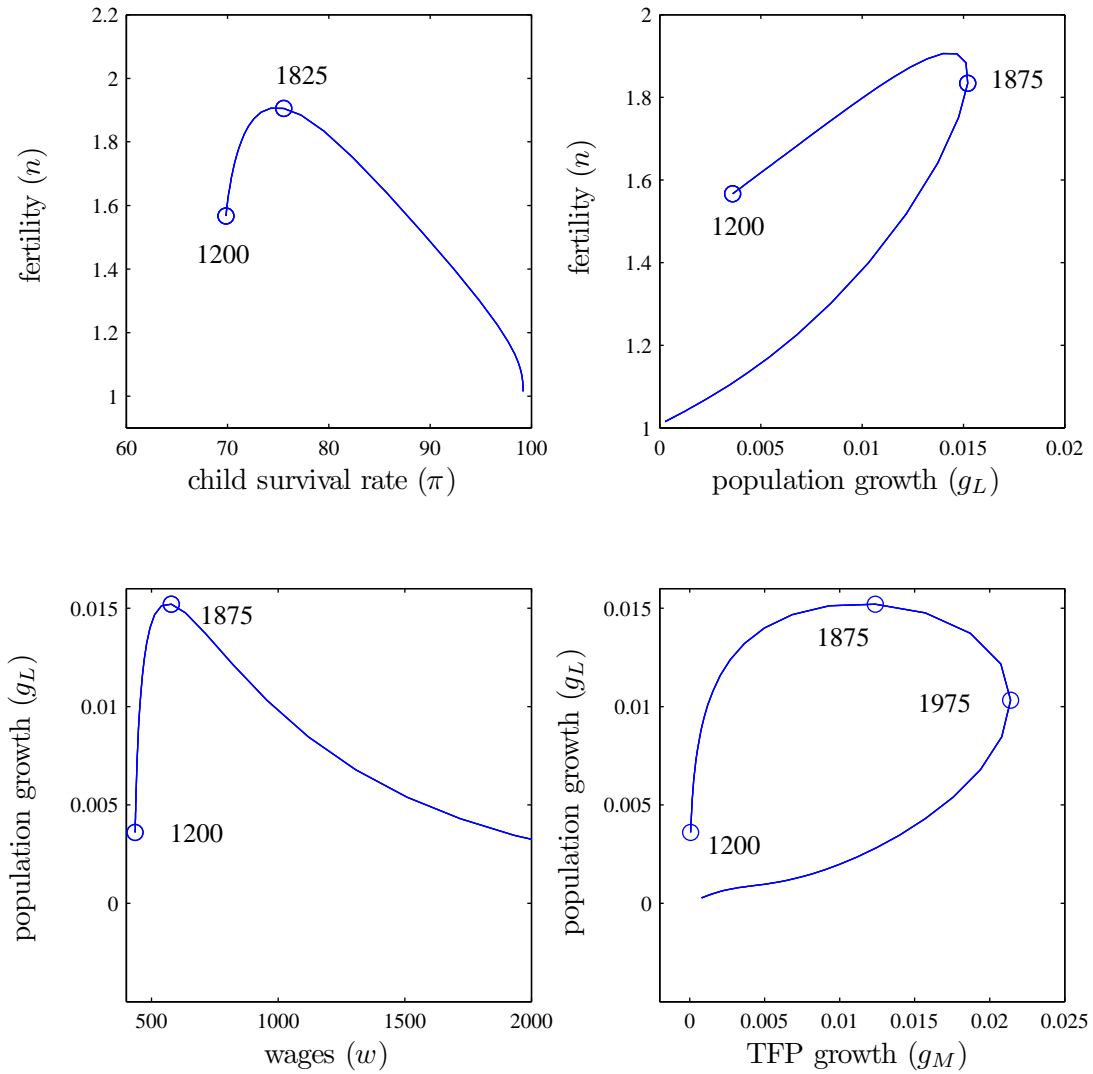
In line with standard Malthusian reasoning the model predicts that higher agricultural productivity is chiefly translated in higher population growth. With respect to income, structural change, and child survival both economies behave almost equally, i.e. all these variables show almost no improvement during the Middle Ages. Yet when industrialization sets in and the agricultural sector visibly sets free labor for manufacturing, the population dividend (Bloom et al. 2001) pays off. The high- $\mu$ -economy where population was growing at higher rate in the past is able to shift an absolutely larger number of worker to the production of non-food goods. Because there are economies of scale at work, the larger economy learns faster and generates higher growth of productivity in manufacturing *and* in agriculture so that structural change happens more quickly. Because of higher productivity growth in agriculture, the larger economy can sustain a higher fertility rate before food prices start to rise. While the alternative model specification outperforms the basic run by generating more momentum of structural change, it also predicts, counterfactually for England, that population growth peaks at a higher rate of about 2 percent annually. Naturally, by highlighting the dynamics between demographic change and economic growth, the model has neglected other important drivers of structural change as, for example, capital accumulation, international trade (globalization) and R&D-based growth.

The model also generates some interesting non-linearities along the adjustment path analyzed above. Based on Figure 3, which shows some of the relevant non-monotonous relationships (for the benchmark economy), the following observations can be made. First, during early stages of development, the model predicts a positive correlation between child survival probability and fertility, while, during later stages, the sign of the correlation is reversed (panel 1). Specifically, lower mortality goes together with rising fertility when the effect of falling food prices on fertility dominates the quantity-quality substitution effect. By contrast, lower mortality appears together with lower fertility when quantity-quality substitution effect dominates that of falling food prices.

Second, due to a strong preventive-check mechanism during early phases of development, there is almost no correlation between wages and population growth in the beginning of the analyzed period. At later stages, however, such as during the demographic transition, there is a strong negative relationship between the two, followed by a weaker negative correlation after the ending of the transition (panel 2).

Third, the overall relationship between birth rates (total fertility) and net reproduction (net fertility) is a slightly positive one (panel 3). That is, during early stages of development, birth rates

Figure 3: Correlations Along the Transition: 1200 to 2200

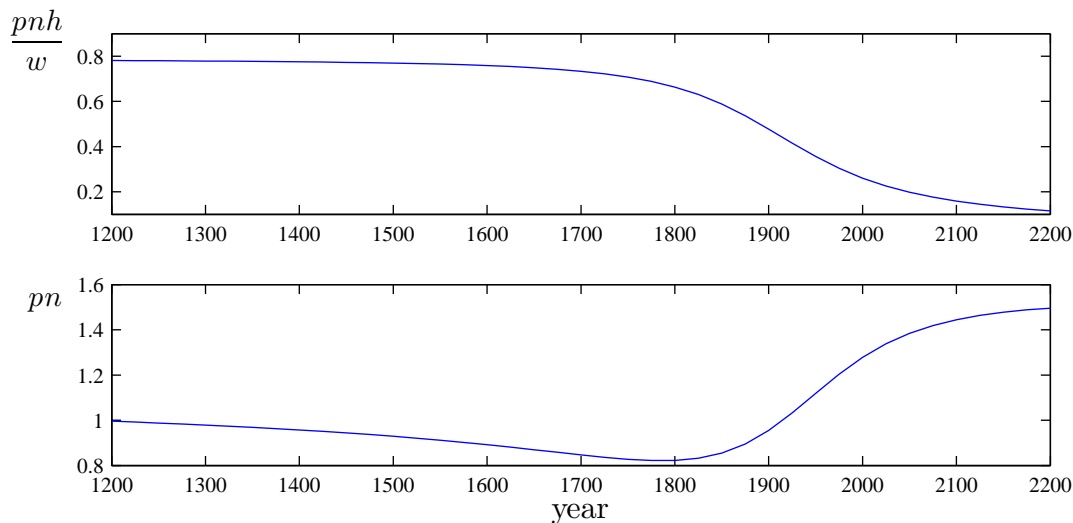


and net reproduction are both rising, while, during late stages, both are falling. For a short period in between, however, the birth rate is declining while net reproduction still grows.

Finally, the relationship between net reproduction and industrial TFP growth forms an almost perfect orbit (panel 4). During early stages of development, there is a positive relationship between TFP growth and growth of population. However, during the 'industrial revolution', i.e. the sharp rise in industrial TFP growth rates, the sign of the correlation turns negative. In the post-industrial period, i.e. after the peak in industrial TFP growth, the sign turns positive once again, as rates of productivity and population growth are both decreasing. The non-monotonic relationships depicted in Figure 3 should give some ideas as to why empirical studies are having a hard time reaching

consensus regarding the correlation between economic and demographic variables (e.g. Brander and Dowrick, 1994).

Figure 4: Engel’s Law and Expenditure per Child



A final observation to be made based on the model’s predictions concerns its consistency with Engel’s Law. Engel’s Law states that nutrition expenditure’s share of income, measured by  $(p_t n_t h_t)/w_t$ , will decline along with the process of growth. This is illustrated in Figure 4, which also shows that nutritional expenditure per child, measured by  $p_t h_t$ , gradually drop until the mid-19th century, then increase steeply throughout the 20th century, after which it continues to increase, but at a somewhat slower pace (for comparison, results are shown relative to initial expenditure, i.e. expenditure in 1200 has been normalized to one). These results reveal another important role for endogenous mortality. Because more children survive at high levels of  $M$ , parents are more inclined to spend much on child nutrition irrespective of the rising relative price for food.

## 6. CONCLUSION

This study provides the first unified growth model to endogenize mortality, fertility and growth, while not relying on human capital accumulation to generate a demographic transition. The result is based on the insight that parents care not only about surviving offspring, but also about their offspring’s nutritional status, and that the price of nutritional goods, on which parents’ fertility decision depends, responds to structural transformation in productive sectors. The model provides an economic rationale for the observable demographic fact of delayed fertility decline in response to

improving child survival. Another major implication of the study, as demonstrated by the use of calibration, is the predicted non-monotonic correlations between various economic and demographic variables, a heads-up for empirical scholars who try to make sense of such relationships.

The model is robust to three important modifications. First, constant returns-to-labor in industrial production can be replaced by diminishing returns without affecting the model's qualitative conclusions. Second, the current preference function implies that the income-effect on the demand for children is zero. It is possible to show, however, that adding a positive (and direct) effect of income on children will not affect the qualitative results of the model. Finally, the long-run predictions of the model—that there is no economic growth in steady state—can be modified without damage to the qualitative results if we permit a positive rate of growth of population on the balanced growth path. All three modifications, however, come at a cost to simplicity.

**Appendix: Proof of Proposition 1** While a *balanced* growth path involves stagnant levels of population and income, an *unbalanced* growth path, characterized by imploding or exploding growth, may in principle exist. In the following, we explore the two cases of unbalanced growth, starting with the case of imploding growth. Imploding growth implies perpetually negative population growth, i.e.  $n_t$  is smaller than one and  $L_t$  is decreasing. It is easy to see that imploding growth is not an option since  $g_t^A$  and  $g_t^M$  are bound to be non-negative. There is no forgetting-by-doing. With  $\lim_{L \rightarrow 0} g^A = 0$  and  $\lim_{L \rightarrow 0} g^M = 0$ , we have  $\lim_{L \rightarrow 0} n = \text{const.}/L^{1-\alpha}$  from (11). As  $L_t$  converges to zero,  $n_t$  goes to infinity. A contradiction to the initial assumption of  $n_t$  being smaller than one. There is no imploding growth. Intuitively, decreasing marginal returns of labor in agriculture ( $\alpha < 1$ ) prevent implosion. As population size decreases agricultural productivity goes up and prices go down so that fertility and thus next period's population increases.

Explosive growth, on the other hand, cannot be ruled out if  $\eta > 1$ , In this case the relative price of food ultimately goes to zero and fertility to infinity. With growing population growth, productivity growth in both sectors grows hyper-exponentially until the economy reaches infinite fertility in finite time. We can solve the stability condition  $\eta < 1$  for the critical  $\epsilon$ . A sufficient condition for stability of balanced growth is (18). Thus, the learning elasticity in agriculture must not be too large. Otherwise agricultural productivity rises so steeply with population growth that it can sustain further falling food and triggers further population growth. Inspection of (14) shows that the critical  $\epsilon$  is decreasing in  $\alpha$  and increasing in  $\phi$ . Intuitively, the larger the counterbalancing forces of limited land (i.e. the lower  $\alpha$ ) and of learning in the manufacturing sector (the larger  $\phi$ ) are, the higher can learning in the agricultural sector be without leading to explosion. The following proposition summarizes the considerations made above about balanced and unbalanced growth.

Note that  $\epsilon < \epsilon_{crit}$  is a *sufficient* condition of stability since it holds for any path along which there is constant population growth. It could be relaxed for the only existing balanced growth path according to which, as shown, population growth is not only constant but also zero. For an intuition of the result inspect (16) again and imagine adjustment dynamics along which an “agricultural revolution” occurs before an “industrial revolution”, i.e. a path along which  $g^A$  is – because of decreasing returns – already declining whereas  $g^M$  is still on the rise. The fact that  $g^A$  and thus  $gL$  are faster approaching to zero than  $g^M$  relaxes the stability condition obtained from analytical considerations, which implicitly assume that all growth rates are in the neighborhood of a balanced growth path.

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