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by Christopher P. Reicher and Johannes Utlaut

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Christopher P. Reicher

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Keywords: Oil shocks, interest rates, inflation.

JEL classification: E31, E58, N50.
The relationship between oil prices and long-term interest rates*

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Chris Reicher and Johannes Utlaut, IfW-Kiel

1. Introduction and previous literature

On the face of it oil price hikes imply a dilemma for policy makers in terms of a trade-off between lower real output and higher inflation. However, this problem is only in effect if there is indeed a significant impact of oil prices on macroeconomic variables. It is a commonly expressed view that the oil price shocks of the 1970s were responsible for the macroeconomic performance in the U.S. at that time. Another view is that the Fed’s monetary policy reaction to oil price shocks actually induced macroeconomic turbulence. We would like to contribute to this ongoing debate by uncovering a long-run empirical relationship between oil prices and interest rates and discussing whether this relationship holds in theory. It does. We note that the 1970s and 1980s saw an unanchoring of monetary policy, with long-run interest rates showing particularly large fluctuations at the same time as real oil prices. We find a very strong relationship between these two things throughout the entire postwar era, even when we split the period into subsamples. We argue that this relationship is robust to VAR ordering assumptions. Furthermore, we take an off-the-shelf model and show that this model predicts that monetary policy, particularly when it becomes unanchored, should in fact have strong effects on oil prices. Previous studies which ignore the effects of monetary policy on oil prices have missed out on this important fact.

Within the empirical literature on the oil price-macroeconomy relationship there are different strands of research. Some studies, setting a clear focus on inflation, implement augmented Phillips curve frameworks to estimate pass-through from oil prices into general prices, in a reduced form sense. Hooker (2002), LeBlanc and Chinn (2004), De Gregorio, Landretertche and Neilson (2007), Van den Noord and André (2007) and Chen (2009) do this. Recognizing that pass-through is a general equilibrium phenomenon, others have adopted a different methodological approach and estimated vector autoregressions (VARs). Hamilton (1983), Burbidge and Harrison (1984), Mork (1989), Hooker (1996), Federer (1996), Edelstein and Kilian (2007) and Gronwald (2009) do this. However, we conduct a VAR analysis focusing on the relationship between oil prices and nominal variables. Not surprisingly, a number of authors have also done this.
Blanchard and Galí (2008) apply structural VAR techniques in order to evaluate the difference between the effects of oil price shocks on GDP growth and inflation in the 2000s and in the 1970s. They estimate multivariate VARs for the U.S., France, Germany, the U.K., Italy and Japan and rolling bivariate VARs for a more detailed analysis of the U.S. They find a significant difference for effects on both inflation and output which, they conclude, could plausibly be explained by a decrease in real wage rigidities, the increased credibility of monetary policy and the decrease in the share of oil in consumption and production. In addition the effects of oil price shocks are found to have coincided in time with large shocks of some different nature. De Gregorio, Landerretche and Neilson (2007) estimate rolling VARs for 12 countries. They find that the effect of oil shocks on inflation has weakened for most of the countries in the sample. Examining a number of factors that could explain this decline they do not find strong evidence pointing to any single specific explanation. However, among the most important factors that might help to explain the decline, they argue, are a more favorable inflation environment, and the fact the 2007 oil price shock was largely the result of strong world demand. Clark and Terry (2009) conduct Bayesian estimates of a VAR, which allows for both coefficient drift and stochastic volatility, to examine the pass through of energy price inflation to core inflation in the U.S. The estimates yield a pronounced reduction in the pass through from approximately 1975 onwards. They further argue that this decline has been sustained through a recent period of markedly higher volatility of shocks to energy prices. On the basis of reduced form and structural VAR evidence, they also find that monetary policy has been less responsive to energy price inflation since approximately 1985.

A number of authors have looked at the particular issue of monetary policy in more detail. Bernanke, Gertler and Watson (1997) employ standard and modified VAR systems. They find that an important part of the effect of oil price shocks on the economy does not result from the change in oil prices, per se, but from monetary policy reacting to increased inflation. Hamilton and Herrera (2004) directly challenge this result by working out shortcomings in the methodology. Herrera and Pesavento (2009) use a modified VAR system for the U.S. economy in order to investigate if the impact of oil price changes on different macroeconomic variables has changed since the mid-1980s. They find that the magnitude and duration of the response of output, and especially prices, has diminished. They further find that since the beginning of the Great Moderation monetary policy has played a reduced role
in dampening the effects of oil price shocks. Kilian and Lewis (2009) modify the VAR model used by Bernanke, Gertler and Watson (1997). They find that there is no evidence of systematic monetary policy responses to oil price shocks after 1987, but that this finding is unlikely to be explained by reduced real wage rigidities. Furthermore their results imply that there is no evidence that the Fed’s policy response to oil price shocks prior to 1987 was responsible for substantial fluctuations in real output. Barsky and Kilian (2002) and Kilian (2009) support the view of Bernanke, Gertler and Watson (1997) that instead of oil supply shocks, monetary policy is the primary explanation of the stagflation of the 1970s. However, they do not apply a VAR or look at long term interest rates. Doing so, as we do, adds strong support to their view.

In comparison with these authors, we add long-run restrictions to our VAR model and we treat inflation and nominal interest rates as sharing a common long-run trend. This allows us to specifically investigate if there is a linkage between oil price changes and changes in the long-run nominal interest rate, which serves as an indicator of long-term inflation expectations. It also allows us to cleanly deal with nonstationarity. We find a strong relationship between long-run interest rates and oil prices throughout the entire postwar period, especially if one is willing to adopt the view that monetary policy choices unrelated to oil prices mainly drive long-run inflation rates. We find that a simple model where some prices are sticky and others are flexible actually predicts such a strong relationship, though the model underpredicts the true magnitude of this relationship. The magnitude of the relationship, but not its existence, is a bit of a puzzle. Both theory and the data seem to indicate that oil prices should be endogenous, and this calls into question any identification scheme (such as that of Blanchard and Galí) which takes them as exogenous.

2. Data used for our analysis

To perform the VAR analysis, we use seven series which are similar to those used by Blanchard and Galí (2008). We calculate the PCE deflator ex energy as a Tornqvist index from the NIPA PCE and PCE energy deflators, with the PCE gasoline deflator used to interpolate the PCE energy deflator in the 1955-59 period (a period of relative calm in energy prices). We use the following series in the analysis; they run from 1955.I through 2009.III in order to avoid disruptions in the series caused by the Korean War:
1. The change in real oil price (deflated by the PCE deflator ex energy). We use the West Texas Intermediate price which can be obtained from the FRED database.

2. The change in long-run nominal interest rates (measured as the 10-20 year treasury constant maturity forward rate), the latter of which we interpolate using the 10-year and 30-year treasury rates when 20-year rates were not available.

3. The change in log productivity, from the BLS’s business productivity statistics.

4. The civilian unemployment rate, from the CPS.

5. The log nonfinancial corporate labor share, from the BEA.

6. The Fed Funds Rate minus the long-run nominal interest rate.

7. The PCE inflation rate ex energy, minus the long-run nominal interest rate. For 1955-58, we interpolate the energy component of PCE using the gasoline component of energy.

The transformed series are stationary and it is both simple and efficient to run the system as an ordinary VAR. Using this VAR we can estimate the long-relationships between these things and put confidence bands on the estimates.

3. Econometric methodology

We estimate the VAR using OLS, while we identify the shocks in two steps. First, we recover the long-run residuals from the short-run residuals using the companion matrix. Then we recover the structural shocks by ordering these shocks in a sensible way (such as in a lower triangular ordering). The VAR representation itself follows its usual form, with the stationary observables \( x \) forming a \( k \times T \) matrix:

\[
x_t = c + \sum_{p=1}^{P} A_p x_{t-p} + \varepsilon_t.
\]  

(1)
We can easily estimate the coefficients c and A through OLS, and the resulting residual matrix \( \varepsilon \) is of full rank and iid over time. The eigenvalues of the sum of the A matrices and of their estimates must all lie below one in absolute value.

The identification of the shocks proceeds in two steps. First we convert the short-run residuals into long-run residuals using the companion matrix. Then we orthogonalize the long-run residuals in some manner by making assumptions about the ordering of shocks. To obtain the long-run residuals, one can first represent the short-run residuals as a function of the long-run residuals using the equation \( \varepsilon_t = B \eta_t \). In this case, it is simple to obtain B. To get a unit long-run effect of the long-run residuals, the following equation must hold:

\[
I_k = \left( I_k - \sum_{p=1}^{p} A_p \right)^{-1} B,
\]

which yields the coefficient matrix B given the restrictions placed on the eigenvalues of A:

\[
B = I_k - \sum_{p=1}^{p} A_p.
\]  \hspace{0.5cm} (2)

Then one can recover the long-run residuals by transforming the estimated short-run residuals using the equation \( \eta_t = B^{-1} \varepsilon_t \), which is possible since B is nonsingular.

We recover the structural shocks \( \zeta_t \) from the long-run shocks \( \eta_t \) through a more judgmental ordering procedure. First we represent \( \eta_t = C \zeta_t \), where C reflects the ordering of the different variables. If things are ordered from top to bottom (i.e. real oil prices are exogenous, followed by long-run interest rates, etc), then C would be a lower-triangular matrix which can be obtained by iteratively regressing each residual on the previous set of residuals. The diagonal terms each equal one, and the off-diagonal terms equal the effects of each of the previous structural shocks \( \zeta_t \) on the composite residual \( \eta_t \). If long-run interest rates come before oil prices in the ordering, then this will change the first two rows of C. It does not matter the order that things come in afterwards, so long as one is not interested in
the effects of the other structural shocks. These are just arbitrary reduced-form shocks which fill out the shock matrix.

Upon obtaining the matrix C, one can then create impulse responses. The impact response equals the matrix product $BC\zeta_t$, and one can iterate through equation (1) to get the impulse responses $x_{t}^{IRF}$.

$$x_{t}^{IRF} = \sum_{p=1}^{P} A_p x_{t-p}^{IRF} + BC \zeta_t.$$  \tag{3}

We calculate the impulse response of each variable this way. The long-run cumulative response is just given by the C matrix itself.

4. Results

4.1. Full sample, 1955-2009

In order to compute the impulse responses, we must first determine the VAR lag order based on consistent information criteria. The Hannan-Quinn Criterion continuously yields a lag order of two. The Schwarz Criterion continuously yields an order of one. As a one-quarter lag seems relatively short we present our results for two lags.

We first estimate the model based on the whole sample, assuming time invariance of the model. The eigenvalues of the sum of the two $A$ matrices are all below one in absolute value, which is a necessary condition to do the proposed inference. The persistent long-run effects on all variables of a unit structural shock in the log real price of oil, with oil ordered first in the VAR, are obtained from the first column of the C matrix. Figure 1 shows the impulse responses to this shock.¹ The long-run nominal interest rate increases by 0.53 percent on a quarterly basis, or 2.1 percent on an annual basis. Unemployment in fact falls somewhat. Productivity perhaps surprisingly, does not change much, and short-run inflation and interest rates rise ever so slightly above trend.

¹ Dashed lines denote bootstrapped 95% confidence intervals taken from 10,000 draws of length 219.
For the long-run nominal interest rate we also consider a structural shock of one percent on a quarterly basis, with the interest rate shock ordered first in the VAR. The log real oil price increases by 0.85. Productivity and the unemployment rate, as well as inflation and interest rates, react in the same way as to the unit oil price shock. As the impulse responses show, we cannot easily distinguish the effects of an interest rate shock from those of an oil price shock. The estimated long-run residuals for oil prices and interest rates have a correlation of +0.67 in the data. The bootstrap results suggest a t-statistic of about 2.4 on this correlation against the null that the correlation is zero and a one-sided p-value of 0.017. Clearly, there is some long-run relationship between interest rates and oil prices which shows up in the data, and it is difficult to distinguish oil shocks from shocks to long-run interest rates.

4.2. Testing for stability and structural change

To explore how the behavior of the system has changed over time and in particular to see if the relationship between interest rates and oil prices has changed over time, we conduct a multitude of Chow Tests. We consider the results of the break-point test statistic in particular because the results of the sample-split statistic do not define reasonable subsamples in a historical context. The break-point statistic tests for a change in the interrelationships of the variables as well as for a change in the variance of the shocks. The sample-split statistic on the contrary only tests for a change in the VAR coefficients, thus assuming a constant white noise covariance matrix.

Our tests allow for an abrupt change at some date as well as for intermediate adjustment periods of any length. To ensure stable estimates we have to impose some critical minimum length on both subsamples. That is to say, if the minimum length of the subsamples is too small, the test statistic will be maximized for at least one subsample having exactly its minimum length. The reason is simply that there tends to be less variation in short samples. In our case, the critical minimum lengths of the first and second subsamples are 22 and 23 quarters. This implies that the estimated structural change may have only taken place between 1961-I and 2003-IV. For this setting the break-point statistic is maximized for the first subsample ending in 1973-IV and the second subsample beginning in 1985-III. The corresponding value is 1,548.7. Accordingly a continuous change in our seven variable VAR seems to have taken place between the breakdown of the Bretton Woods System and the beginning of the Great Moderation. However, if we constrain the first subsample to contain at
least one additional quarter (1974-I), then we find it most likely to see an instantaneous
change between 1985-II and 1985-III. The corresponding value of the break-point test
statistic is 1,506.9. This indicates an immediate change in the levels of our VAR coefficients
and variances with the beginning of the great moderation. We therefore estimate our VAR
model for both subsamples according to the estimated structural break dates, the latter of
which Clark and Terry (2009) also estimate. The resulting eigenvalues yield that the
estimated coefficient-matrices are again stable.

We also look more closely at the 1970s. A structural break in our VAR may also have
occurred with the breakdown of the Bretton Woods System in 1973. In order to evaluate this,
we employ the same kind of Chow Tests as above. We reset the minimum length of the first
subsample to its critical value of 22 quarters, implying that the first subsample may not have
ended until 1960-IV. Now, new minimum lengths are appointed to the second subsample
successively, starting with its length according to the beginning of the great moderation plus
one additional quarter. That would be the second subsample beginning in 1985-II at the latest.
As soon as the restriction is imposed that the change in the structure of the VAR has to be
completed in 1982-III at the latest, the second subsample is found to be longer than its
minimum length. The break-point test statistic is maximized for the first subsample ending in
1973-IV and the second subsample beginning in 1976-IV. The value is 1,508.2. Similarly
Clark and Terry (2009) find a reduced pass-through into inflation from approximately 1975
onwards. This subsample coincides with the 1979 oil shock and the Volcker disinflation but
it misses the 1973 shocks.

4.3. Results for the subsamples

The enduring effects on both interest rates due to an oil price shock in the 1955-1985
subsample (Figure 2) do not significantly differ from the full sample estimation; the effect
on long-run interest rates in particular is almost the same, with a 2.0 percent long-run effect
of log real oil prices on annualized interest rates. Productivity now drops by 4.7 percent. The
effects on unemployment, short-run interest rates, and inflation are slightly amplified, but
they are still small and in the case of unemployment expansionary. A one percent increase in
the quarterly long-run interest rate (with interest rates ordered first) induces a long-run rise in

2 Dashed lines denote bootstrapped 95% confidence intervals taken from 10,000 draws of length 122.
the log real price of oil of 0.72, which is almost as large as the full-sample 0.85. Productivity drops by 3.3 percent. Similarly as in the full sample, this portion of the sample shows that it is not easy to distinguish oil shocks from long-run interest rate shocks.

For the period after 1985 (Figure 3), a unity structural log oil price shock (ordered first) raises the long-run interest rate by 0.84% annually;\(^3\) the response has been dampened by more than half. Instead of falling, productivity rises by 7.0 percent. A 1.0 percent structural shock in the quarterly long-run interest rate has a much stronger impact on the log real oil price, which rises by 1.65. Productivity now rises by 9.4 percent. Under both shocks unemployment still falls. In comparing Figures 2 and 3, a few things stand out. The long-run relationship between oil prices and interest rates remains strong, while the effects of oil shocks on other variables have become somewhat dampened. The positive bumps to short-run inflation and labor’s share have become much more muted, almost entirely dampened in the case of short-run inflation.

Together with the results of the Chow Tests from above the following pattern is very likely: Our VAR system is stable until 1973-IV when nominal oil prices and long-term interest rates do not move much. The breakdown of the Bretton Woods System then initiates a three year adjustment process. From 1976-IV onwards the system is stable again, until 1985-II. Based on these results we reestimate the model for the time before the Great Moderation, based on the shorter subsample ranging from 1976-IV to 1985-II. The impulse responses are shown in Figure 4.\(^4\) The resulting eigenvalues imply stable coefficient matrices. For three variables there are significant differences in the effects of an oil price shock compared to the estimations based on the longer pre-break subsample. The long-run interest rate rises by 4.7 percent annually instead of 2.0 percent, while the other impulse responses look much like those in the longer pre-1985 sample. The increase in the log real oil price due to a 1.0 percent structural shock in the quarterly long-run interest rate is now 0.53 instead of 0.72. Under both specifications the responses of unemployment, detrended interest rates, and detrended inflation are dampened as well, and productivity does not move by much.

In short, the results of the split subsamples indicate that there is a robust positive statistical relationship between long-run nominal interest rates and long-run real oil prices, and this

\(^3\) Dashed lines denote bootstrapped 95% confidence intervals taken from 10,000 draws of length 99.

\(^4\) Dashed lines denote bootstrapped 95% confidence intervals taken from 10,000 draws of length 37.
relationship has remained positive throughout time. This relationship is robust to two
different VAR orderings, and the responses to long-run interest rate shocks and long-run oil
price shocks are remarkably similar. The magnitude of the effect of oil prices on interest
rates, when oil prices are ordered first, has fallen substantially. If one thinks of monetary
policy as determining long-run interest rates, then the effect of long-run interest rates and
inflation on real oil prices has if anything increased from a large value to a very large value.

5. The predictions of a theoretical model

Traditional monetary theory holds that monetary policy determines long-term interest rates
through its effect on inflation expectations. There is already a large literature on the
relationship between monetary policy and oil prices, mostly having to do with policy
responses to oil supply shocks. We throw this analysis into reverse. We set up a simple
model of oil prices based on that of Aoki (2001) and ask how well it predicts the rise in real
oil prices observed after a large permanent increase in inflation and nominal interest rates.5
In Aoki’s model, some goods are continuously traded and have flexible prices, while others
have sticky prices which are reset at a predetermined rate. We analyze the behavior of
relative prices in general equilibrium at steady state, assuming that long-run inflation (and by
extension the long-run nominal interest rate) is a choice variable for the central bank.

Consumers seek to maximize the present value of the utility from consumption (which equals
output) minus the disutility of labor:

$$
E \sum_{t=0}^{\infty} \beta^t \left[ \ln(C_t) - \frac{1}{1 + \chi} N^{1+\chi} \right],
$$

subject to the budget constraint

$$
C_t + B_{t+1} = Y_t + (1 + r_{t-1})B_t.
$$

Economywide output is aggregated according to the CES aggregator

---

5 Aoki’s model puts the earlier work of Gordon (1975, 1984) and Phelps (1978) into a modern forward-looking context.
\[ Y_t = \left[ \frac{1}{\alpha} (Y_t^F)^{\frac{\gamma-1}{\gamma}} + (1 - \alpha) \frac{1}{\alpha} (Y_t^S)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \]

with the budget constraint

\[ P_t Y_t = P_t^F Y_t^F + P_t^S Y_t^S. \]

Quantities and prices with an ‘F’ in the superscript refer to the flexible-price good, and those with an ‘S’ superscript refer to the sticky-price good. Price-taking behavior by demanders yields the demand curves

\[ Y_t^F = \alpha \left( \frac{P_t}{P_t^F} \right)^{\gamma} Y_t, \]

and

\[ Y_t^S = (1 - \alpha) \left( \frac{P_t}{P_t^S} \right)^{\gamma} Y_t. \]

The CES price index is given by

\[ P_t^{\gamma / \gamma} = \alpha (P_t^F)^{1 - \gamma} + (1 - \alpha) (P_t^S)^{1 - \gamma}. \]

Firm-level output within the sticky price sector is aggregated by aggregators according to the CES production function:

\[ Y_t^S = \left[ \int_0^{\theta} y_t^{\eta} \eta \, dj \right]^{\theta / \theta - 1}, \]

with the budget constraint
\[ P_t^S Y_t^S = \int_0^1 p_{j\beta} y_{j\beta} dj. \]

Price-taking behavior by aggregators yields the demand curve

\[ y_{j\beta} = \left( \frac{p_{j\beta}}{P_t^S} \right)^{-\theta} Y_t^S, \]

and the CES price index is given by:

\[ P_t^S = \left[ \int_0^1 p_{j\beta}^{-\theta} dj \right]^{-\frac{1}{\theta}}. \]

Individual prices in the sticky sector are sticky with a probability \( \omega \) and producers produce according to the production function \( y_{it} = z_t N_{it} \). Firms seek to reset prices such that they maximize expected profits conditional on not changing prices and on satisfying whatever demand comes along as a function of those prices. This is reflected by their objective function:

\[
E_i \sum_{i=0}^{\infty} \omega^i D_{t+i} \left[ p_i^* \frac{1}{P_{t+i}} y_{jt+i} (p_t^*) - W_{t+i} N_{it+i} (p_t^*) \right] = E_i \sum_{i=0}^{\infty} \omega^i D_{t+i} \left[ p_i^* \left( \frac{p_i}{P_{t+i}} \right)^{-\theta} Y_{t+i}^S - \frac{W_{t+i}}{Z_{t+i}} \left( \frac{p_i}{P_{t+i}} \right)^{-\theta} Y_{t+i}^S \right].
\]

Taking first derivatives gives the first order condition:

\[
E_i \sum_{i=0}^{\infty} \omega^i D_{t+i} \left[ (1 - \theta) \frac{p_i^*}{P_{t+i}} \left( \frac{p_i}{P_{t+i}} \right)^{-\theta} Y_{t+i}^S + \theta \frac{W_{t+i}}{Z_{t+i}} \left( \frac{p_i}{P_{t+i}} \right)^{-\theta} Y_{t+i}^S \right] = 0,
\]

so that
\[
\frac{(\theta - 1)}{\theta} \frac{p_{t+1}^s}{p_t} E_t \sum_{i=0}^{\infty} \omega^i D_{t+i,4} \left[ \frac{p_t}{p_t^{\mu}} \left( \frac{p_{t+1}^s}{p_t^{\mu}} \right)^{-\theta} Y_{t+i}^s \right] = E_t \sum_{i=0}^{\infty} \omega^i D_{t+i,4} \left[ \frac{w_{t+i}}{z_{t+i}} \left( \frac{p_t^s}{p_{t+i}} \right)^{-\theta} Y_{t+i}^s \right].
\]

Output aggregates to:

\[
N_t^S = \int_{j=0}^{1} \left( \frac{p_{t+j}^s}{p_t^s} \right)^{-\theta} \frac{Y_{t+j}^s}{z_t},
\]

so that production in the sticky sector aggregates according to:

\[
N_t^S = \left( \omega \left( \frac{p_{t-1}^s}{p_t^s} \right)^{-\theta} + (1 - \omega) \left( \frac{p_t^s}{p_t^s} \right)^{-\theta} \right) \frac{Y_t^S}{z_t}.
\]

The price index for the sticky-price goods follows the law of motion:

\[
(p_t^s)^{1-\theta} = (1 - \omega)(p_t^s)^{1-\theta} + \omega(p_{t-1}^s)^{1-\theta}.
\]

Firms within the flexible price sector have a decreasing-returns Cobb-Douglas production function in labor and maximize profits which equal

\[
P_t^F x_t (N_t^F)^{\varphi} - W_t N_t^F,
\]

which gives the first order condition:

\[
P_t^F \varphi x_t (N_t^F)^{\varphi-1} = W_t.
\]

Household labor supply has the first order condition

\[
\frac{W_t}{Y_t} = \ln N_t^F.
\]
and markets clear so that consumption equals output; total bondholdings are zero; and household budget constraints bind in equilibrium.

6. Calibration and comparative analysis of steady states

In order to deliver quantitative predictions we must first calibrate the model. The quarterly gross real interest rate from the data equals 1.0081 and gross inflation equals 1.0080. Growth equals 1.0078, bringing growth nearly into line with interest rates. All prices are initially normalized to one, as are real consumption of the sticky and flexible goods. The share of output of agriculture and mining in the US economy, from the BEA’s industry accounts, is 4.1%. This should be regarded as a lower bound for the share of flexible-price commodities in expenditure since the US is a net importer of oil, and this excludes the flexible-price production of other sectors. Prices in the sticky sector are updated 25% of the time for a value of $\omega$ of 0.75. The demand parameter $\theta$ equals 10 to ensure a markup of about 10%. The inverse labor supply elasticity $\chi$ equals 0, corresponding with indivisible labor.

We derive an exact solution to the model for steady-state analysis; the equations used to do so are described in an appendix. A permanent increase in quarterly inflation by 1% per quarter will feed through to steady-state real prices by altering the pattern of expenditure between sticky-price and flexible-price goods. Since it is easier to increase nominal spending on the flexible price goods, increases in nominal spending will show up there more quickly than they will in the sticky-price sector, and this will push up demand for the flexible price good. Since the flexible price good is not elastically supplied, this will push up its real price too.

We first look at an inelastic case, seeing how well we can match the 85% increase in oil prices observed after a 1% rise in long-run quarterly interest rates and inflation. With supply and demand elasticities for the flexible price good both set to 0.1 (that is, $\phi$ set to 0.1/1.1 and $\gamma$ set to 0.1), the ratio $P^F/P^S$ rises by 30.6% in log terms. Raising the initial expenditure share $\alpha$ to 8% reduces this price effect to 26.4%, and raising the inverse labor supply elasticity $\chi$ to 5 reduces this effect drastically to 6.5%. An inelastic labor supply is clearly important if long-run inflation is to have substantial effects on relative prices between sectors, while expenditure shares do not matter much.
Not surprisingly, supply and demand elasticities at the product level are crucial. Raising the long-run demand elasticity \( \gamma \) to 0.25 reduces the long-run relative price effect to 19.3%, while raising the long-run supply elasticity to 0.5 reduces the effect to 15.8%. Doing both of these things simultaneously as suggested by Hamilton (2008) lowers the long-run relative price effect to 12.9%. Further raising the inverse labor supply elasticity \( \chi \) to 5 reduces this effect even more drastically to 5.4%. The more inelastic the economy, the greater the effect of monetary policy on real prices.

In short, theory does in fact predict a substantial rise in real oil prices in response to a permanent rise in the central bank’s inflation target, with an upper bound somewhere near 30% and a more realistic guess somewhere near 13%. The 85% increase in oil prices that we observe in our VAR, however, exceeds this upper bound by a wide margin, though the confidence interval on this estimate is fairly wide. In this sense the magnitude of the observed comovement between interest rates and oil prices is a bit of a puzzle. Our results nonetheless indicate that the feedback from monetary policy into oil prices themselves is worth exploring, and such a feedback will contaminate identification if one assumes that real oil prices are exogenous.

7. Conclusions

By including long-run interest rates into a VAR and taking proper account of nonstationarity, we have uncovered an interesting relationship in the data which deserves further attention. Throughout the entire postwar period, there has been a strong positive relationship between long-run interest rates and real oil prices that has not gone away, though the relationship has weakened somewhat in the post-1985 period. Our results further indicate that the Fed’s short-run response to oil price shocks has changed over time; short-run inflation and interest rates have become much more stable in response to a unit oil shock. The relationship between oil prices and productivity has completely disappeared, while the negative relationship with unemployment has not changed much at all. Statistically, it is almost impossible to distinguish shocks to oil prices from shocks to long-run nominal interest rates, and if one thinks that long-run nominal interest rates are determined by monetary policy, this means that monetary policy has large effects on oil prices in the long run.
Looking at a simple theoretical model, we conclude that it is reasonable to expect strong feedback from monetary policy into oil prices. As nominal demand rises, spending on sticky-price goods takes longer to adjust, so new spending flows first into flexible-price goods such as oil and agricultural products. Nonetheless, the theoretical model still underpredicts the magnitude of the relationship that we actually find, which is a bit of a puzzle.
References


Appendix: Steady state of the two-sector model

Solving for the steady state of the model involves solving the following system of equations:

\[ p^* \sum_{i=0}^{\infty} \left( \frac{\omega \Gamma}{R^i \Gamma} \right)^i [1 - \theta] + E \sum_{i=0}^{\infty} \left( \frac{\omega \Gamma}{R^i \Gamma} \right)^i \left[ \frac{\theta W}{z} \right] = 0, \]

or in closed form,

\[ p^* \left( \frac{1}{\Pi^\omega R - \Pi \omega^i} \right) (1 - \theta) + \frac{1}{\Pi^\omega R - \omega^i} \left[ \frac{\theta W}{z} \right] = 0; \quad \text{(sticky price setting)} \]

\[ (\Pi)^{1-\theta} = (1 - \omega) \left( \frac{p^* \Pi}{P^S} \right)^{1-\theta} + \omega; \quad \text{(sticky price aggregation)} \]

\[ zN^S = \omega \Pi^\theta + (1 - \omega) \left( \frac{P^* \Pi}{P^S} \right)^\theta Y^S; \quad \text{(sticky sector production)} \]

\[ xz(N^F)^\theta = Y^F; \quad \text{(flexible sector production)} \]

\[ P^F \omega xz(N^F)^{\theta-1} = W; \quad \text{(flexible sector wages)} \]

\[ \frac{W}{Y} = \ln \frac{z}{\xi}; \quad \text{(total labor supply)} \]

\[ N = N^F + N^S; \quad \text{(labor market clearing)} \]

\[ Y = \left[ \frac{1}{\alpha^Y (Y^F)^{\gamma-1} + (1 - \alpha^Y) (Y^S)^{\gamma-1}} \right]^{\frac{\gamma}{\gamma-1}}; \quad \text{(aggregate output)} \]

\[ Y^F = \alpha (P^F)^\gamma Y; \quad \text{(flexible price demand)} \]
\[ Y^s = (1 - \alpha)(P^s)^{\gamma} Y; \]  

(sticky price demand)

and

\[ 1 = \alpha(P^r)^{1-\gamma} + (1 - \alpha)(P^s)^{1-\gamma}. \]  

(aggregate price level)

Real interest rates are stationary and unrelated to inflation, so their values remain the same as before. Long-run steady states are well behaved as a function of inflation, so simple numerical search algorithms work well. Finding the response to a rise in inflation involves recalculating the entire steady state of the economy after raising the inflation rate.
Figure 1: Impulse Responses to an Oil Price Shock and an Interest Rate Shock, 1955:1 – 2009:III

Shocks are a unit shock to log oil prices and a unit shock to quarterly nominal interest rates, each ordered first. See Section 4 for further details.
(Figure 1 continued)
Figure 2: Impulse Responses to an Oil Price Shock and an Interest Rate Shock, 1955:I – 1985:II

Shocks are a unit shock to log oil prices and a unit shock to quarterly nominal interest rates, each ordered first. See Section 4 for further details.
(Figure 2 continued)

Oil Price Shock

Interest Rate Shock

Unemployment rate

Log corporate labor share

Negative interest rate spread

Inflation minus long-run interest rate
Figure 3: Impulse Responses to an Oil Price Shock and an Interest Rate Shock, 1985:III – 2009:III

Shocks are a unit shock to log oil prices and a unit shock to quarterly nominal interest rates, each ordered first. See Section 4 for further details.
(Figure 3 continued)
Figure 4: Impulse Responses to an Oil Price Shock and an Interest Rate Shock, 1976:IV – 1985:II

Shocks are a unit shock to log oil prices and a unit shock to quarterly nominal interest rates, each ordered first. See Section 4 for further details.
(Figure 4 continued)