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Is Euro Area Money Demand (Still) Stable? – Cointegrated VAR versus Single Equation Techniques

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Is Euro Area Money Demand (Still) Stable?
Cointegrated VAR versus Single Equation Techniques

Ansgar Belke* Robert Czudaj†

Abstract
In this paper we present an empirically stable euro area money demand model. Using a sample period until 2009:2 shows that the current financial and economic crisis that started in 2007 does not appear to have any noticeable impact on the stability of the euro area money demand function. We also compare single equation methods like the ARDL approach, FM-OLS, CCR and DOLS with the commonly used cointegrated Johansen VAR framework and show that the former are under certain circumstances more appropriate than the latter. What is more, they deliver results that are more in line with the economic theory. Hence, FM-OLS, CCR and DOLS are useful in estimating standard money demand as well, although they have only been rarely applied for this purpose in previous studies.

JEL Codes: C12, C22, C32, E41, E43, E58
Keywords: ARDL model, cointegration, euro area, financial crisis, money demand

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1. Introduction

Estimating a reliable money demand function is essential for the European Central Bank (ECB) with respect to its target of price stability (ECB (2004a), p. 49). Money plays a crucial role within the ECB’s monetary policy strategy since the reference value of money growth is seen as a benchmark for evaluating monetary developments in the euro area. Potential output growth has been estimated at around 2 to 2.5 percent, price stability implies consumer price inflation of below 2 percent, and a negative trend in velocity results in an increase of monetary growth in a range between 0.5 and 1 percent. Under these assumptions, the reference value for monetary growth has been set at 4.5 percent per year and has not been revised any more since (ECB (2004b), p. 64 and Dreger and Wolters (2008), p. 2). This reference value corresponds to one of the two pillars in the ECB’s two-pillar framework and is oriented at the widespread but often discussed view that money is a long-run but in some cases also a medium-run indicator of future inflation.\(^1\)\(^2\) Hence, if, and only if, the demand for money turns out to be verifiable stable, changes in the stock of money can be expected to have a predictable impact on inflation (Belke and Polleit (2009), pp. 677, 681, and Issing et al. (2001), p. 6). However, different inflation forecast approaches such as the p-star model require a reliable estimation of a money demand function.\(^3\) Therefore the stability issue has received particular attention in the literature since M3 growth started to accelerate in 2001 due to the terrorist attacks of September 2001 and the burst of the new economy bubble. Instability emerged at that time since large funds were reallocated into safe and liquid assets that are part of M3 during that time (Belke and Polleit (2009), p. 136). Currently, in the global financial and economic crisis that started in 2007, the stability issue gains even more interest.

Hence, the main purpose of our paper is to check empirically whether the money demand function in the euro area is (still) stable in its parameters. Moreover, we would like to assess which estimation technique is the preferred one to deal with this issue. Finally, it is of interest whether the financial crisis and the efforts spent by the ECB to manage the former by means of lowering its policy rates and conducting unorthodox monetary policies has had any effect on the magnitude of the coefficients and/or the stability of the long-run money demand function. We feel legitimized to stress again that the stability of the money demand equation is an important prerequisite of a sound conduct of the monetary pillar of the ECB’s monetary policy strategy.

\(^1\)\textit{Inflation is always and everywhere a monetary phenomenon.} (Milton Friedman, Wincott Memorial Lecture, London, September 16\(^{th}\) 1970)

\(^2\)This is in line with ECB (2004b), p. 47 and Benati (2009).

\(^3\)See, for example, Nicoletti-Altimari (2001), p. 13.
In the following, we take the theoretical and empirical findings in the previous literature as a starting point of our analysis and use commonly accepted econometric methods. Taking into account exogeneity issues, focus on the search for a cointegration relation between money aggregates and related variables. Moreover, apply a variety of different econometric techniques to compare their estimation results and their usefulness in identifying and determining the euro area money demand function. Accordingly, we apply the cointegrated VAR approach by Johansen (1995), the ARDL approach by Pesaran, Shin and Smith (1999, 2001) and other single equation methods such as fully-modified ordinary least squares (FM-OLS), canonical cointegration regression (CCR) and dynamic OLS (DOLS), which all will be discussed later. A further aim of this paper is to check the impact of the sample size in finding a stable cointegrating relation.

The remainder of the paper is organized as follows. In the next section we present the underlying theory, which draws the connection between money and the other variables of interest. In section 3 we highlight the available recent studies on euro area money demand in order to compare our settings and results with theirs. The main part of our study, the econometric analysis, is presented in section 4 where we discuss the data used and check the variables for stationarity and for cointegration sequentially within the Johansen framework and the ARDL approach. Furthermore, we look into the constancy of our estimated parameters. We check our estimation results for robustness in section 5. For this purpose, we apply the other three above-mentioned single equation methods and different lag lengths. Section 6 concludes and gives an outlook on further research.

2. Theory of money demand

In the corresponding theoretical and empirical literature, different specifications of money demand functions are used. These studies have generally in common that a money demand equation contains a scale variable to describe the economy’s level of transactions and a variable reflecting the opportunity costs of holding money. Whereas there is general agreement in the literature in the case of choosing a scale variable, the selection of the opportunity cost variable repeatedly results in some controversy. Hence, the literature tends to use different interest rates, interest rate differentials or the inflation rate as a proxy for the opportunity costs. According to Fase and Winder (1998), a common specification of the money demand function has

the following form:

\[(m - p)t = \beta_0 + \beta_1 y_t + \beta_2 i^l_t + \beta_3 i^s_t + \beta_4 \pi_t + u_t, \tag{1}\]

where \(m\) denotes a monetary aggregate (M1, M2 or M3), \(p\) represents a measure of the prices level, \(y\) stands for a measure of real income as the scale variable, \(i^l\) and \(i^s\) are the long-run and the short-run interest rate, respectively, \(\pi\) represents the inflation rate and \(u\) the error term. The short-term interest rate is a proxy of the M3 "own rate" and the long-term interest rate characterizes the return on alternative assets (Coenen and Vega (2001), pp. 729-730). Except for both interest rates, we take all variables in logs. We expect the following signs of the coefficients: \(\beta_1 > 0\), \(\beta_2 < 0\), \(\beta_3 > 0\) and \(\beta_4 < 0\). The Baumol-Tobin model and the quantity theory predict the income elasticity to be \(\beta_1 = 0.5\) and \(\beta_1 = 1\), respectively (Baumol (1952), Tobin (1956) and Friedman (1956)). However, in the empirical literature one tends to find empirical realizations of income elasticities for a broad money aggregate which are slightly beyond unity. This pattern is often said to be due to the neglect of the wealth effect (Coenen and Vega (2001), p. 728, and Boone and van den Noord (2008), p. 531). The estimated coefficients of both interest rates represent semi-elasticities given that both variables have not been taken as logarithms as suggested by Fair (1987), p. 473.

3. Recent studies on euro area money demand - A survey

This section examines several important critical specification issues and procedures when empirically modelling money demand in the euro area. In order to allude to the more relevant choices, we define the long-run equation as proposed in the preceding section. Table 1 shows the different specifications of the money demand equation in the euro area taken from earlier studies and their coefficient estimates. The estimated income elasticity turns out to take values between 1.1 to 1.5 in most of the studies. This range is broadly in line with predictions from theory and the short deviation from unity is often explained within the standard portfolio approach by the neglect of a wealth variable. Some authors try to cope with this issue by using different proxies of wealth such as financial wealth or stock prices, respectively (Fase and Winder (1998), p. 509, Bruggeman et al. (2003), pp. 30-32, and Beyer (2009), pp. 11-22).

- Table 1 about here -

\({\text{Fase and Winder (1998), p. 512, use a wealth variable in addition to our specification.}}\)
3.1. Opportunity costs of holding money

In the empirical literature the operationalization of the opportunity costs of holding money is often discussed rather controversially. Our choice of taking the short-term and long-term interest rates as proxy of the M3 "own rate" and the return on alternative assets respectively goes back to Coenen and Vega (2001), pp. 729-730. Brand and Cassola (2004), p. 821, confine themselves to use only the long-run interest rate to seize the opportunity costs, since, from their perspective, the long-term interest is able to sufficiently capture the dynamics of the interest rate differential between the long-run interest rate and the M3 "own rate" and this simplification reduces the complexity of the model. Calza et al. (2001) apply two different specifications of the opportunity cost variable in their model. They insert the difference between the short-term interest and the M3 "own rate" as well as the difference between the long-term interest and the M3 "own rate". Moreover, they show that this specification only has a slight impact on the estimation results. However, they do not include the inflation rate in their model because they do not attach any additional explanatory content with respect to money demand to it - at least compared to the nominal long-term interest rate (Calza et al. (2001), p. 8). Nevertheless, inflation is usually interpreted, for instance by Dreger and Wolters (2009), as a part of the opportunity costs, as it represents the costs of holding money in spite of holding real assets (Ericsson (1999), p. 38, and Dreger and Wolters (2009), pp. 112-113). Bruggeman et al. (2003), p. 37, demonstrate how to construct the "own rate of return" of euro area M3, which is used in a variety of other recent studies as well.

3.2. The aggregation problem

A further problem often encountered when estimating the money demand function for the euro area is the availability of aggregated euro area data prior to 1995. Some authors try to construct euro data by aggregating the national data of M3, GDP, consumer prices and interest rates. Scale variables are denominated in national currencies and should first be converted into euro by using the irrevocably fixed exchange rates and then summed up to the euro area aggregate time series, as it is done, for instance, by Bruggeman et al. (2003), pp. 10-11. An advantage of this method is that it is consistent with the method that is used by the ECB since the start of Stage Three of EMU. Nevertheless, Fagan and Henry (1998), p. 495, present an alternative aggregation method, the so-called index method. According to this

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6The ECB has announced the irrevocably fixed exchange rates on December 31, 1998 (and determined on June 19, 2000, in the case of Greece, on July 11, 2006 for Slovenia, on July 10, 2007 for Malta and Cyprus and on July 7, 2008 for Slovakia).
method the log-level index for the euro area series is defined as the weighted sum of the log-levels of the national series, where the weights are the shares of the national GDP in euro area GDP in 2001 measured at PPP exchange rates. Applying this method avoids possible spurious correlation among the euro area series, which are merely due to changes in the exchange rates. In contrast to the so-called exchange rate method the index method is also applicable to interest rates. In this context it is possible to use the M3 shares instead of the GDP shares of each country as weights (Bruggeman et al. (2003), p. 12). Fase and Winder (1998) discuss the effects based on alternative aggregation methods. Winder (1997) and Beyer et al. (2001) provide a general discussion on aggregation issues underlying the construction of historical euro area data. To apply any aggregation method, one requires a quite large sample of national data of the 16 euro area countries.

3.3. Appropriate econometric techniques and the issue of weak exogeneity

Most of the studies we refer to in the context of euro area money demand estimation apply the Johansen framework to identify cointegration relations among the variables and to estimate the coefficients of the cointegrating vectors and the error-correction parameters, which indicate the speed of adjustment from short-run deviations to the long-run equilibrium. Generally, the empirical literature we survey in this paper (see Table 1) uses quarterly data. Thus, one should to estimate a VAR model with four lags, but the consensus is the estimation with two lags to avoid an overly strong decrease in degrees of freedom. To estimate a VAR model in the vector error-correction representation, one important step is to select a trend specification (Johansen (1995), pp. 80-84).

The empirical literature usually applies an unrestricted constant, allowing for a linear trend in the variables but not in the cointegrating relations. Although, Brand and Cassola (2004), p. 823, test the model with a restricted constant as well, allowing for no linear trend in the variables and the cointegrating equations. If the found cointegration rank is larger than one, identifying restrictions have to be imposed and then to be tested. The literature delivers different versions of model restrictions.

After estimating the coefficients, the variables are usually tested for their weak exogeneity vis-à-vis the system. Intuitively, weak exogeneity of right-hand side variables allows us to condition on these variables without specifying their generating process with no loss of useful information (Engle et al. (1983) pp. 282-283, and Urbain (1992) pp. 188-189). Regarding this property, the available studies arrive at

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7See, for example, Coenen and Vega (2001), p. 733.
conflicting results, while, for instance, Brand and Cassola (2004), p. 824, come up with the striking result that no variable included in their approach can be considered to be weakly exogenous, Coenen and Vega (2001), p. 735, find weak exogeneity of the GDP variable, both interest rates and the inflation rate.

According to Enders (2010) who refers to an example originally introduced by Johansen and Juselius (1990) it might be possible to argue that GDP should be weakly exogenous (Enders (2010), p. 407, and Johansen and Juselius (1990), pp. 181-206). However, some authors claim that the Johansen procedure could be in certain circumstances susceptible to spurious cointegration (Gonzalo and Lee (1998), p. 149). Taking this potential flaw into account, several authors estimate the cointegrating equation by means of some alternative techniques such as the Engle and Granger (1987) procedure, the autoregressive distributed lag (ARDL) model proposed by Pesaran and Shin (1999) and the fully-modified OLS approach by Phillips and Hansen (1990) as well. Applying this wide array of techniques might well serve to check for robustness of the results. By doing so, we follow Calza et al. (2001), who show that their results originally produced within the Johansen framework are quite robust to the use of alternative techniques (Calza et al. (2001), pp. 10, and 30).

3.4. Stability tests

In a variety of studies stability tests such as the recursive estimated coefficients or the Chow test are used to check for stability of the estimated long-run parameters and to identify possible structural breaks (Chow (1960)). For instance, Brand and Cassola (2004), Coenen and Vega (2001) and Calza et al. (2001) do not find any evidence of a structural break in their sample periods ranging from 1980:1 to 1999:3, from 1980:4 to 1998:4 and from 1980:1 to 1999:4, respectively (Brand and Cassola (2004), pp. 826-828, Coenen and Vega (2001), pp. 738-739, and Calza et al. (2001), pp. 13, and 22-24). Calza and Sousa (2007) provide three reasons why M3 money demand has been more stable in the euro area than in other large economies. First, they argue that instability in the money demand in some economies is country-specific. Second, from their point of view financial innovation appears to have had a weaker impact on money demand in the euro area than, for instance, in the US. Third, they make a more technical point, i.e. the aggregation of the money demand of the individual euro area countries could be held responsible for the stability of the money demand in the euro area.

Coenen and Vega (2001) and Funkel (2001) use a dummy variable to model a structural break in 1986, which should, corresponding their view, take into account special developments in German data according to which some debt securities were subjected to reserve requirements (Coenen and Vega (2001), p. 733). Bruggeman
et al. (2003) criticize the above-mentioned stability tests, arguing that these tests only make use of a part of the sample and apply alternative techniques to check for stability. However, they get no conflicting results compared to the commonly used techniques for their sample from 1980:4 to 2001:4 (Bruggeman et al. (2003), pp. 19-25). Hall et al. (2008) re-estimated the Calza et al. (2001) model over the period 1980:1 to 2006:3 and found a structural break in 2001:2 (Hall et al. (2008), p. 13, and De Santis et al. (2008), pp. 9-10, and 41-42). Dreger and Wolters (2008), pp. 7, and 19, use a dummy variable referring to a structural break in 2002:1.

Although many researchers face the stability issue, there is still a gap in analyzing the impact of the financial crisis on euro area money demand, since the observation period of the majority of the studies ends before 2007. Exceptionally, Beyer (2009), pp. 22-24, 27-39, and 46-53, shows that euro area money demand is still stable at the start of the crisis by applying the same methods as Bruggeman et al. (2003).

3.5. Further modifications

Besides the implementation of a wealth effect variable into the model further amendments are proposed in the literature. For example, Calza et al. (2001) extend their basic model with the world oil price index as a proxy of import prices in order to correct for potential difficulties arising from the application of the GDP deflator as a measure of the general price level for relatively open economies (Calza et al. (2001), pp. 14, 34, and Lütkepohl and Wolters (1999b), p. 109). Greiber and Setzer (2007), pp. 8, and 13, augment their money demand specification by the real residential property price index and the housing wealth indicator, respectively.

Beyer (2009) as one of the most recent empirical studies on the euro area money demand function adds housing wealth to the model as well to account for the wealth effect. What is more, Greiber and Lemke (2005) implement uncertainty as an additional variable in their money demand model. Hamori and Hamori (2008) estimate the money demand for M1, M2 and M3 based on a panel analysis, which includes eleven different countries. Finally, Carstensen et al. (2009) estimate euro area money demand for four countries (Germany, France, Spain and Italy) individually and aggregated (Beyer (2009), pp. 11-24, Greiber and Lemke (2005), pp. 6, 16, Hamori and Hamori (2008), pp. 275-282, and Carstensen et al. (2009), pp. 77-82).

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9They use the fluctuation test suggested by Hansen and Johansen (1999), examine the constancy of β using the Nyblom (1989) tests studied by Hansen and Johansen (1999) and the constancy of the Φ, Γ1, and α parameters using the fluctuation test developed by Ploberger et al. (1989).
4. Empirical analysis

We start our empirical analysis with the description of the underlying data. Further, we assess the order of integration of the variables and the lag length which appears adequate to properly apply the Johansen framework. We use the latter to come up with some basic estimation results. What is more, additional estimations which apply the ARDL approach complete the whole picture and serve as a robustness check of the estimation results gained with the Johansen procedure. Moreover, we check our parameter estimates for constancy. We do so applying two different and suitable data sets which are described in the following.

4.1. The data

In our study we use quarterly data ranging from 1995:1 to 2009:2. The underlying time series are taken from the ECB Statistical Data Warehouse. As our measure of the money aggregate, we apply quarterly averages of the seasonally adjusted month-end stocks of M3. We take the seasonally adjusted real gross domestic product (GDP) as a proxy of the scale variable\(^{10}\) and rely on the the GDP deflator (1995 = 100) as our measure of the price level. The inflation variable is constructed by the annualised quarterly changes in the GDP deflator. The three-month Euribor money market rate (short-term interest rate) measures the return on assets included in the definition of M3 and the 10-year treasury bond yield (long-term interest rate) characterizes the return on assets excluded from the monetary aggregate.

One problem in estimating a reliable money demand function for the euro area is the still relatively small sample size, resulting of the fact that the European Monetary Union (EMU) is still a quite young institution. The only way to arrive at a larger sample is to aggregate the national data prior to 1995 on a euro area level. For instance, Coenen and Vega\(^{11}\) followed this way. Hence, we use their aggregated time series ranging from 1980:1 up to 1994:4 to extend our sample and to check the impact of the sample size by estimating our model for the longer time horizon again.

4.2. Stationary tests

To test for the usual time series properties of the data we first conduct some unit root tests. To check for cointegration between the variables within the Johansen framework we have to establish first that the variables of interest are integrated of order one \((I(1))\). As usual, we apply the augmented Dickey-Fuller test (ADF test)
and the Phillips-Perron test (PP test) to test the null of a unit root in the time series and refer to the critical values tabulated by MacKinnon (Dickey and Fuller (1979), Phillips and Perron (1988), MacKinnon (1991) pp. 271-274 and MacKinnon (1996) pp. 603-616). We present the resulting empirical realizations of the $t$-statistics and the $p$-values for the levels and the first differences for both samples in Table 2.

- Table 2 about here -

The results for the small sample (1995:1-2009:2) indicate that, except for $\pi_t$, all variables of interest seem to be non-stationary, i.e. $I(1)$. However, the rate of inflation appears to be integrated of order zero ($I(0)$), probably because it is constructed of the first difference of the GDP deflator. Hence, $\pi_t$ and the GDP deflator $p_t$ are again tested for stationarity with the ADF and the PP test dropping the intercept from the auxiliary test equation. The corresponding results convey some evidence that the inflation rate $\pi_t$ is $I(1)$ and that $p_t$ inevitably amounts to $I(2)$ as it is often also the result in various other studies such as Johansen and Juselius (1990). However, our results are often highly dependent on the number of lags used in the specifications of the test equation and also on the number of observations used. Furthermore, it is well-known that the univariate stationary tests have a rather low power. Hence, it appears useful in our context to check the stationarity properties with multivariate tests as well (DeJong et al. (1992) and Coenen and Vega (2001), p. 732). Hence, we assume all variables to be $I(1)$ just in line with Wesche (1998), p. 21, and others. Correspondingly, we feel legitimized to argue that the Johansen framework is applicable in our context.

The results for the larger sample (1980:1-2009:2), which are displayed in the bottom part of Table 2, tell us the same story, but turn out to be even more significant especially for $i^d_t$, which indicates following Schwert (1989) that our first and basic sample period might be too short.

### 4.3. Empirical VAR model

Consider a vector autoregressive (VAR) model of order $p$ following Sims (1980), pp. 15-33:

$$Y_t = A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + \Phi X_t + \epsilon_t,$$

(2)

where $Y_t$ is a $k$-dimensional vector of non-stationary $I(1)$ variables, $X_t$ is a $d$-dimensional vector of deterministic variables, and $\epsilon_t$ is a vector of innovations with zero mean and covariance matrix $\Omega$. In the following, we estimate a VAR($p = 2$)
model with \( Y_t = \left[ (m - p)_t, y_t, i_t^*, i_t^*, \pi_t \right]' \), \( k = 5 \) and \( d = 0 \) in order to test for cointegration. We do not a priori include dummy variables and, thus, let the dynamic structure absorb potential shocks, because a thorough visual inspection of the series does not reveal a permanent break in the cointegrating relation among the used variables.

First of all, we check for the adequate lag length to be used for the VAR model and test the usual properties of the residuals. Table 3 shows that the likelihood ratio (LR), the final prediction error (FPE) and Hannan-Quinn (HQ) information criterion all recommend, in case of the small sample, a lag length of \( p = 2 \), while applying an unrestricted VAR model up to a maximum lag length of 5. Only the Akaike (AIC) and the Schwarz (SC) criterion indicate a lag length of \( p \neq 2 \). Our results of the lag length selection criteria for the large sample are not tabulated here. Nevertheless, their application leads to the same empirical pattern with even more unambiguity than in the smaller sample period. Later on in section 5.2 we additionally apply different lag lengths to check the robustness of our results.

Table 4 displays the results of our misspecification tests for both samples. The modified White test by Kelejian (1982) lets us suspect the residuals to be heteroskedastic. According to the Jarque-Bera test with Cholesky orthogonalization as suggested by Lütkepohl (2007), pp. 174-181, the assumption that the residuals are normally distributed appears to be violated in case of the larger sample. However, the null of serial correlation of order 1 and 2 is rejected in case of both samples.

- Tables 3 and 4 about here -

### 4.4. Cointegration analysis

Since we cannot reject that our macro time series contain a unit root, we feel legitimized to apply the ordinary VAR-based cointegration test which sticks to the methodology developed by Johansen. Let us consider our VAR(\( p \)) model adapted to the VEC representation:

\[
\Delta Y_t = \Pi Y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \Phi X_t + \epsilon_t. \tag{3}
\]

Granger’s representation theorem asserts that if the coefficient matrix \( \Pi \) has reduced rank \( r < k \), then there exist \( k \times r \) matrices \( \alpha \) and \( \beta \) each with rank \( r \) such that

13 All the criteria are discussed by Lütkepohl (2007), pp. 146-157.
14 The results are available upon request.
\[ \Pi = \alpha \beta' \text{ and } \beta' Y_t \] is \( I(0) \) (Engle and Granger (1987), pp. 255-258). \( r \) is the number of cointegrating relations (the cointegrating rank) which we identify using the trace statistics. Moreover, each column of \( \beta \) is a cointegrating vector. The elements of \( \alpha \) represent the adjustment parameters in the VECM. \( \Gamma_j \) capture the short-run effects of the time series. Johansen’s method consists of estimating the matrix \( \Pi \) from an unrestricted VAR with a maximum likelihood technique and testing whether we can reject the restrictions implied by the reduced rank of \( \Pi \).

The upper part of Table 5 displays our results of the unrestricted cointegration rank test for the short sample. The trace test rejects the hypothesis of no cointegration relation and the hypothesis of one cointegration relation between the variables and accepts the hypothesis of a cointegration rank of a maximum of 2 at a 5% significance level. Accordingly, we feel legitimized to assume two cointegration relations \( (r = 2) \) among the variables and to estimate the cointegrating equations with the maximum likelihood procedure, as suggested by Johansen.

However, before estimating \( \hat{\Pi} \) one has to make an assumption about the deterministic trend specification of the used time series. One can choose between five trend specifications (Johansen (1995), pp. 80-84). As stated above, the two specifications usually recommended in the empirical literature are the one including levels \( Y_t \) without deterministic trends and cointegrating equations with restricted intercepts and the other one containing levels \( Y_t \) with linear deterministic trends and cointegrating equations with unrestricted intercepts. Following Coenen and Vega (2001), we choose the second specification and estimate \( \hat{\Pi} \) assuming linear trends in the level data and an unrestricted constant in the cointegrating equations.

Estimating the parameters \( \alpha \) and \( \beta \) within

\[ \Delta Y_t = \alpha \beta' Y_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta Y_{t-j} + \Phi X_t + \epsilon_t, \tag{4} \]

using \( Y_t = \left[ (m-p)_t \ y_t \ i_t^* \ i_t \ \pi_t \right]' \) from the basic sample (1995:1-2009:2), two lags, \( r = 2 \) and the trend specification mentioned yields two cointegrating equations \( \hat{\beta}' Y_t \) which have the below-stated form:

\[ (m-p)_t = -9.80 + 1.45*** y_t - 12.32*** \pi_t, \tag{5} \]

\[ \text{The test is applied using } p\text{-values taken from MacKinnon et al. (1999), pp. 570-576.} \]

\[ \text{See, for example, Brand and Cassola (2004), p. 823 and Coenen and Vega (2001), p. 733, respectively.} \]
\[ i_t^* = -0.06 + 1.61***i_t^* + 2.88***\pi_t. \] (6)

The first cointegration equation can be interpreted as the money demand function and the second one as an equation combining the term structure of interest rates and the Fisher inflation parity. Conditional on the choice of cointegration rank, the LR test does not reject the imposed restrictions at conventional levels based on the \(\chi^2\)-statistic of 3.82 with \(df = 2\) and a \(p\)-value = 0.15. The coefficients have the predicted signs and turn out to be highly significant. Especially, the estimated income elasticity nearly reveals the same magnitude as proposed by the standard literature (1.1 to 1.5). The estimated empirical realizations of the adjustment parameters to the long-run equilibrium \(\hat{\alpha}\) take the following values:

\[
\hat{\alpha} = \begin{bmatrix}
-0.07*** & -0.25*** \\
-0.02 & 0.07 \\
0.01 & 0.10*** \\
-0.04*** & -0.07 \\
-0.12*** & -0.17
\end{bmatrix},
\] (7)

where the first element of the first column represents the error-correction parameter of the estimated money demand function. Accordingly, the speed of adjustment to the long-run equilibrium seem to be very low.

Additionally, the null of stationarity given the cointegration space is tested applying the likelihood-based procedure (LR test), which characterizes a multivariate version of the Dickey-Fuller test, against the alternative of non-stationarity. What is more, the null of weak exogeneity of the variables is checked with the LR test as well. The results are stated in Table 6.

- Table 6 about here -

According to our results, all series seem to be stationary. However, on the one hand this results is highly dependent on the chosen cointegration rank \(r\). On the other hand, our results could be interpreted in a way that the ARDL approach is in this case a more appropriate technique than the Johansen framework, because it does not require a priori knowledge that all variables are \(I(1)\) (Pesaran and Shin (1999), Belke and Polleit (2006a), Belke and Polleit (2006b) and Belke and Polleit (2006c)).

According to Enders (2010), it would not at all be surprising if \(y_t\) would be the only weakly exogenous variable in our variable set (Enders (2010), p. 407). Exactly this result is already indicated by the second element of the first column of \(\hat{\alpha}\), which takes an empirical value around zero.

We enacted the same procedure also for the larger sample period (1980:1 to 2009:2)
to make our findings more robust. Again, the results of the trace test let us accept
the hypothesis of at most two cointegration ranks (see lower part of Table 5).
Using the same restrictions as applied for the short sample our estimation of the
cointegrating equations yields the following results:

\[(m - p)_t = -3.68 + 1.01^{**} y_t - 1.77^{**} \pi_t, \quad (8)\]

\[i_t^i = 0.01 + 0.82^{**} i_t^s + 0.22^{**} \pi_t, \quad (9)\]

Whereas the estimated coefficients also have the same signs as in case of the smaller
sample period and are again highly significant, their magnitudes seem to be even
more striking according to the theoretical predictions and the previous empirical
studies. Conditional on the existence of three cointegrating relations, the LR test
hardly accepts the imposed restriction at a 1% significance level based on the \(\chi^2\)-
statistic of 9.41 with \(df = 2\) and a \(p\)-value = 0.01.

\[
\hat{\alpha} = \begin{bmatrix}
0.15^{**} & 0.64^{**} \\
0.14^{**} & 0.98^{**} \\
0.00 & 0.04 \\
-0.00 & -0.08 \\
-0.74^{**} & -3.18^{**}
\end{bmatrix}
\quad (10)
\]

The speed of adjustment to the long-run equilibrium given above seems to be a
bit larger and the error-correction term of the money demand has a positive sign
now.

4.5. ARDL approach

One popular method to check how robust the estimated money demand equation
is to apply additional estimation techniques. The autoregressive distributed lag
(ARDL) model by Pesaran, Shin and Smith (1999, 2001) is usually applied in our
context as well (for example, Coenen and Vega (2001), pp. 736-741 and Calza et al.
(2001), pp. 12-13, 30). The ARDL approach has the advantage that it does not
require all variables to be \(I(1)\) as the Johansen framework and it is still applicable
if we have \(I(0)\)- and \(I(1)\)-variables in our set. As stated above and closely following,
for instance, Belke (2010) and Narayan and Smyth (2004), it is potentially at least as appropriate in such a case as the Johansen framework. The estimated error-correction representation of the ARDL model with unrestricted intercept and without trend has the following general specification with \( p \) and \( q_i \) lags:

\[
\Delta(m - p)_t = \alpha_0 + \sum_{j=1}^{p-1} \psi_j \Delta(m - p)_{t-j} + \sum_{j=0}^{q_1-1} \alpha_{1j} \Delta y_{t-j} + \sum_{j=0}^{q_2-1} \alpha_{2j} \Delta i^a_{t-j}
\]

\[+ \sum_{j=0}^{q_3-1} \alpha_{3j} \Delta i^d_{t-j} + \sum_{j=0}^{q_4-1} \alpha_{4j} \Delta \pi_{t-j} + \gamma_0(m - p)_{t-1} + \gamma_1 y_{t-1} + \gamma_2 i^a_{t-1} + \gamma_3 i^d_{t-1} + \gamma_4 \pi_{t-1} + u_t, \]

where the \( \gamma_i \) denote the long-run parameters and \( \psi_j \) and \( \alpha_{ij} \) the short-run dynamic coefficients of the model. \( u_t \) is uncorrelated with the lagged endogenous and exogenous regressors and the first differences of the exogenous regressors and their lags.

First, we estimate an ARDL model with \( p = q_1 = \ldots = q_4 = 2 \) lags according to the SC as proposed by Pesaran and Shin (1999), p. 374, using the short sample (1995:1-2009:2) and second, we normalize coefficients by imposing \( \beta_i = \frac{\gamma_i}{\gamma_0} \). The corresponding cointegration equation has the following form:

\[
(m - p)_t = 1.96 y_t + 6.86^* i^a_t - 2.80^* i^d_t - 2.23 \pi_t.
\]

Hence, the coefficients have the same predicted signs as the ones estimated before with the Johansen approach and again the income elasticity is of nearly the same magnitude, which is totally in line with the recent work of Beyer (2009), p. 19.

Third, we test the null of no cointegration defined by \( H_0 : \gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0 \) against the alternative of \( H_1 : \gamma_0 \neq \gamma_1 \neq \gamma_2 \neq \gamma_3 \neq \gamma_4 \neq 0 \) by means of the Wald F-test. Pesaran et al. (2001) provide two sets of asymptotic critical values: one set assuming that all the regressors are \( I(1) \); and another set assuming that they are all \( I(0) \). These two sets of critical values refer to two polar cases but actually provide a band covering all possible classifications of the regressors into \( I(0), I(1) \) (fractionally integrated or even mutually cointegrated).

As a next step, we use the appropriate bounds testing procedure. The test is consistent. For a sequence of local alternatives, it follows a non-central \( \chi^2 \)-distribution asymptotically. This is valid irrespective of whether the underlying regressors are \( I(0), I(1) \) or mutually cointegrated. The recommended proceedings based on the

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\(^{18}\)See Belke (2010), pp. 9-10 and Narayan and Smyth (2004), p. 5 or further such as Bahmani-Oskooee and Ng (2002), Halicioglu (2004), Morley (2007), Payne (2003), Faria and León-Ledesma (2003) and Baharumshah et al. (2009), in which the ARDL approach is also used in comparable cases.
The $F$-statistic are as follows. One has to compare the $F$-statistic computed in the second step with the upper and lower 90, 95 or 99 percent critical value bounds ($F_U$ and $F_L$). As a result, three cases can emerge. If $F > F_U$, one has to reject $\gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$ and hence to conclude that there is a long-term relationship between $(m - p)_t$ and the vector of exogenous regressors. However, if $F < F_L$, one cannot reject $\gamma_0 = \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$. In this case, a long-run relationship does not seem to exist. Finally, if $F_L < F < F_U$ the inference has to be regarded as inconclusive and the order of integration of the underlying variables has to be investigated more deeply. The 90, 95 and 99 percent lower and upper critical value bounds of the $F$-test statistic are dependent on the number of regressors, on which kind of intercept is included and on whether a linear trend is included or not. They are usefully summarised by Pesaran et al. (2001), pp. 300-301. They list the critical value bounds for the application with unrestricted intercept, without trend and four exogenous regressors in their Table CI(iii). At the 90 percent level the critical value bound amounts to 2.45 to 3.52, at the 95 percent level the band ranges from 2.86 to 4.01 and at the 99 percent level it ranges from 3.74 to 5.06.

The $F$-statistic of 3.27 with $df = (5, 40)$ for the small sample turns not out to be significant and indicates the third case since the $F$-statistic is within the band $(F_L < F < F_U)$ at the 90 and 95 percent critical value bounds and thus inference depends on the integration order of the used variables.

We conducted the same procedure also for the larger sample (1980:1-2009:2) which yields the estimated cointegration equation:

$$(m - p)_t = 1.29^{***} y_t + 5.07^{***} i^*_t - 3.38^{*} i^*_t - 2.34^{***} \pi_t. \quad (13)$$

Hence, the estimated long-run coefficients are significant and again seem to be even more unambiguous using the extended sample. The joint effect of the five variables on $\Delta(m - p)_t$ is highly significant, according to the $F$-statistic of 4.31 with $df = (5, 100)$ and a $p$-value = 0.00. Since $F > F_U$ at the 90 and 95 percent critical value bounds, one has to conclude that there is a long-term cointegrating relationship between $(m - p)_t$ and the other used variables.

As a necessary complement, we also conduct residual tests for both samples to check the adequacy of our specification applying the Breusch-Godfrey serial correlation LM test, Harvey’s heteroscedasticity test, Jarque-Bera’s normality test and Ramsey’s RESET test (Godfrey (1978a), Godfrey (1978b), Harvey (1976), Bera and Jarque (1981) and Ramsey (1969)). The results are displayed in Table 7.

In the case of both sample periods, our specification seems to fit the data well except for the rejection of the normality assumption for the large sample and some
indication of functional inadequacy for the short sample which might result of omitted or irrelevant variables.

The above procedure should be repeated for ARDL regressions of each element of the vector of exogenous regressors on the remaining relevant variables in order to select the so called "forcing variables". However, this is not the objective in our case.

4.6. Parameter stability tests

The performance and goodness-of-fit of all estimations conducted by us in this paper indicates that our estimation results for the euro area money demand are quite robust. This is especially the case with respect to the estimated empirical realization of the income elasticity. Hence, a remaining critical issue that arises in our context is related to the stability of the estimated coefficients over time. In the ideal case, the stability tests should be conducted applying different methods in order to yield robust results.

Accordingly, we start with the Johansen procedure and apply it in the same way as before in section 4.4, but this time sequentially letting the sample end after each year from 2001 on in order to check whether the coefficient estimates stay the same. We limit this exercise to the larger sample period, because we like to identify possible effects of events such as the burst of the so called dotcom-bubble in 2000, September 11th in 2001 or the emergence of the global financial crisis in 2007. The short sample unfortunately does not include enough observations to subdivide the sample period and to still arrive at a sufficient number of degrees of freedom.

Our results in Table 8 indicate that there is some reason to consider the estimated coefficients of both regressors as stable over time. The results of the trace test and the maximum eigenvalue test show that we can establish one or two cointegrating relations between the five time series.

Figures 1 and 2 display the identified cointegrating relations estimated with the Johansen approach alternatively for both sample periods. If the larger sample period is used, coefficient stability turns out to be higher except for the change over (i.e., after 1994:4) from the Coenen and Vega (1999) sample to ours.

To check for stability of the coefficients, the latter are usually estimated in a recursive fashion. Hence, we proceed like this in case of our estimated ARDL model and

\[ \text{Table 8 about here} \]

\[ \text{Figures 1 and 2 about here} \]
furthermore apply the cumulative sum (CUSUM) of squares test to both sample periods (Brown et al. (1975)).

- Figures 3 and 4 about here -

According to Figures 3 and 4 the coefficients estimated based on the larger sample period seem to be much more stable, except for the very beginning of the sample in the early eighties.

- Figures 5 and 6 about here -

However, the CUSUM of squares test surprisingly draws a quite different picture and indicates some structural change in the larger sample. The empirical realization of the CUSUM of squares test statistic falls outside the 5% significance bands for more than ten years (see Figures 5 and 6). Since previous tests do not support this finding, we could not definitely assume a structural break. There is particularly no indication of a structural break in the most interesting periods (2001 or 2007).

5. Robustness checks

To check for robustness of the estimated coefficients, three other cointegrating regression methods are applied as well as the Johansen framework and the ARDL approach are used again while modifying the lag length. The findings of the latter are summarised in Table 9. Additionally, different model specifications are estimated using the cointegrated VAR approach with two lags for both samples as done before in section 4.4.

5.1. Other cointegrating regression methods

Hence, we also estimate the same specification by means of three other fully-efficient single equation cointegrating regression methods, namely the fully-modified OLS (FM-OLS) by Phillips and Hansen (1990), the canonical cointegration regression (CCR) by Park (1992) and the dynamic OLS (DOLS) by Saikkonen (1992) and Stock and Watson (1993). They have already been used by earlier studies for the purpose of estimating money demand functions, and will possibly be used in further studies for the euro area to put the achieved results of the above-mentioned techniques under scrutiny.

20The Coenen and Vega (2001) (see pp. 728-729, 733-736) and the Brand and Cassola (2004) specification (see pp. 818-819) are used. The results confirm our findings without improving the coefficient estimates. Nevertheless, they are not listed to save space, but they are available upon request.

21See, for example, Calza et al. (2001), p. 30, Norrbin and Reffett (1997) and Mark and Sul (2003) in case of FM-OLS, CCR and DOLS, respectively.
5.1.1. Fully-modified OLS

The FM-OLS estimation procedure works with the standard triangular representation of a regression specification and assumes the existence of a single cointegrating vector (Phillips and Hansen (1990) and Hansen (1992)). Taking this as a starting point, we estimate the following cointegration equation based on a \((d + 1)\)-dimensional time series vector process \([(m - p)_t, X'_t]^{n\times1}:

\[
(m - p)_t = X'_t \beta + D'_t \gamma_1 + u_{1t},
\]

where \(D_t = [D'_1t, D'_2t]^{n\times1}\) are deterministic trend regressors and \(X'_t = [y_t, i'_t, \ddot{i}'_t, \pi_t]\) represents \(d = 4\) stochastic regressors, which are governed by the system of equations:

\[
X_t = \Gamma'_1 D_{1t} + \Gamma'_2 D_{2t} + \epsilon_{2t},
\]

with

\[
\Delta \epsilon_{2t} = u_{2t}.
\]

In our case, the deterministic trend regressors \(D_{1t}\) only contain a constant and enter both the cointegrating equation and the regressors equations, while we include any additional deterministic trend regressors \(D_{2t}\) in the regressors equations \([15]\), but not in the cointegrating equation \([14]\).

The FM-OLS estimator employs a semi-parametric correction to avoid estimation problems caused by the long-run correlation between the cointegrating equation and stochastic regressors innovations. The resulting estimator is asymptotically unbiased and has fully efficient mixture normal asymptotics allowing for standard Wald tests using asymptotic \(\chi^2\) statistical inference.

Hence, the FM-OLS estimator is given by

\[
\hat{\theta}_{FMOLS} = \begin{bmatrix} \hat{\beta} \\ \hat{\gamma}_1 \end{bmatrix} = \begin{pmatrix} \sum_{t=1}^{T} Z_t Z'_t \end{pmatrix}^{-1} \begin{pmatrix} \sum_{t=1}^{T} Z_t (m - p)_t + T \begin{bmatrix} \hat{\lambda}_{12}' \\ 0 \end{bmatrix} \end{pmatrix},
\]

where \(Z_t = (X'_t, D'_t)\),

\[
(m - p)_t^+ = (m - p)_t - \check{\omega}_{12} \hat{\Omega}_{22}^{-1} \hat{u}_2
\]

represents the transformed data and

\[
\hat{\lambda}_{12}^+ = \hat{\lambda}_{12} - \check{\omega}_{12} \hat{\Omega}_{22}^{-1} \hat{\Lambda}_{22}
\]
represents the estimated bias correction term with the long-run covariance matrices \( \hat{\Omega} \) and \( \hat{\Lambda} \) (with their elements \( \hat{\omega}_{12}, \hat{\Omega}_{22}, \hat{\lambda}_{12} \) and \( \hat{\Lambda}_{22} \)), which are computed using the residuals \( \hat{u}_t = (\hat{u}_{1t}', \hat{u}_{2t}')' \) (Hamilton (1994), pp. 613-618).

Our FM-OLS estimation based on the short sample period with the long-run covariance estimates \( \hat{\Omega} \) and \( \hat{\Lambda} \) applying a prewhitened kernel approach (with one lag according to SC) with a Bartlett kernel and Newey-West fixed bandwidth = 4.0000 leads to the following result:

\[
(m - p)_t = -5.78^{***} + 1.16^{***} y_t - 3.08^{***} i_t^t + 2.18^{***} i_t^s - 2.08^{***} \pi_t. \quad (20)
\]

What is more, using the longer sample period, the FM-OLS approach with long-run covariance estimates using a non-prewhitened Bartlett kernel and a Newey-West fixed bandwidth = 4.0000 yields the estimated cointegrating equation:

\[
(m - p)_t = -5.16^{***} + 1.11^{***} y_t - 1.91^{**} i_t^t + 1.80^{***} i_t^s - 0.15^{*} \pi_t. \quad (21)
\]

### 5.1.2. Canonical cointegration regression

The CCR estimation procedure is in principle closely related to FM-OLS, but instead employs stationary transformations of the data to eliminate the long-run correlation between the cointegrating equation and stochastic regressors innovations. Hence, the CCR estimator is defined by

\[
\hat{\theta}_{CCR} = \begin{bmatrix} \hat{\beta} \\ \hat{\gamma}_1 \end{bmatrix} = \left( \sum_{t=1}^{T} Z_t^* Z_t'^* \right)^{-1} \sum_{t=1}^{T} Z_t^* (m - p)_t^*, \quad (22)
\]

where

\[
Z_t^* = (X_t'^*, D_t^*'),
\]

\[
X_t^* = X_t - \left( \hat{\Sigma}^{-1} \hat{\Lambda}_2 \right)' \hat{u}_t \quad (23)
\]

and

\[
(m - p)_t^* = (m - p)_t - \left( \hat{\Sigma}^{-1} \hat{\Lambda}_2 \tilde{\beta} + \begin{bmatrix} 0 \\ \hat{\Omega}_{22}^{-1} \tilde{\omega}_{21} \end{bmatrix} \right)' \hat{u}_t \quad (24)
\]

represents the transformed data. The \( \tilde{\beta} \)'s are estimates of the cointegrating equation coefficients applying static OLS, \( \hat{\Lambda}_2 \) is the second column of \( \hat{\Lambda} \) and \( \hat{\Sigma} \) represents the estimated contemporaneous covariance matrix of the residuals (Hamilton (1994), pp. 618-625).

---

\(^{22}\)Lower case letters indicate in this case that elements are vectors and capital letters denote matrices.
Hence, applying the CCR procedure for the short sample period with long-run covariance estimates using the same kernel and Newey-West fixed bandwidth as applied in the FM-OLS case yields the estimated money demand equation:

\[(m - p)_t = -5.66^{***} + 1.15^{***} y_t - 3.07^{***} \Delta i^d_t + 2.4^{***} \Delta i^s_t - 1.45^{**} \pi_t. \quad (25)\]

In contrast, the CCR estimation using the large sample period with long-run covariance estimates applying a non-prewhitening Bartlett kernel and a Newey-West fixed bandwidth = 4.0000 results in the following specification of the euro area money demand:

\[(m - p)_t = -5.16^{***} + 1.11^{***} y_t - 1.84^{**} \Delta i^d_t + 1.82^{***} \Delta i^s_t - 0.34^{*} \pi_t. \quad (26)\]

Seen on the whole, using both methods we come up with roughly the same coefficient estimates. Moreover, the latter are also in line with the predictions from theory and the estimates within the Johansen and the ARDL framework. Furthermore, all coefficients are significant and the estimates derived from the larger sample period seem to be more appropriate since their magnitudes are even more striking according to the theoretical predictions and the previous empirical studies. Especially, the cointegrating equations which we derived from the larger data set explain the money demand in the euro area very well.

The estimated income elasticity turns out to be positive and is nearly approaching unity. The estimated coefficients of the interest rate spread \((i^d_t - i^s_t)\) and the rate of inflation which both approximate the opportunity costs of holding money prove to be negative and relatively small.

### 5.1.3. Dynamic OLS

The DOLS estimation technique involves augmenting the cointegrating regression \((14)\) with \(q\) lags and \(r\) leads of \(\Delta X_t\) such that the new cointegrating equation error term is orthogonal to the entire history of the stochastic regressor innovations:

\[
(m - p)_t = X'_t \beta + D'_t \gamma_1 + \sum_{j=-q}^{r} \Delta X'_{t+j} \delta + v_{1t}.
\]

However, the DOLS estimation procedure works under the assumption that the added lags and leads of \(\Delta X_t\) completely eliminate the long-run correlation among \(u_{1t}\) and \(u_{2t}\). Hence, the resulting estimator is then given by \(\hat{\theta}_{DOLS} = (\hat{\beta}', \hat{\gamma}_1)\) and displays the same asymptotic distribution as those derived with the FM-OLS and the CCR estimation procedure.

The DOLS approach using both sample periods with one lag and one lead and long-
run covariance estimates applying Bartlett kernel and Newey-West fixed bandwidth \( = 4.0000 \) delivers the following results:

\[
(m - \rho)_t = -8.69^{***} + 1.35^{***} y_t + 0.06 i_t^l + 1.74^{***} i_t^* - 3.21^{***} \pi_t, \tag{28}
\]

and

\[
(m - \rho)_t = -5.19^{***} + 1.11^{***} y_t - 1.62^{***} i_t^l + 2.00^{***} i_t^* - 1.06^{***} \pi_t, \tag{29}
\]

respectively. Hence, the only difference in estimation results in comparison to the other methods used by us is the estimated semi-elasticity of the long-term interest rate. The latter does not turn out to be significant and displays the wrong sign for the shorter sample period. Nevertheless, according to the Wald \( F \)-test, the joint effect of the four variables on \((m - \rho)_t\) is highly significant in the case of all three cointegrating regression methods.\(^{23}\)

5.2. Different lag lengths

Choosing the adequate lag length is always of interest when estimating VAR models since the underlying structure is often said to be atheoretic. As mentioned above, it is recommended in the literature to apply two lags as the adequate specification in contexts like ours while using quarterly data. In our case, this choice is clearly supported by the likelihood ratio, the final prediction error and the Hannan-Quinn information criterion while setting the maximum lag length at a value of five. Nevertheless, the Akaike information criterion recommends five lags and the Schwarz information criterion points in favour of just one lag for the short sample period. However, the extension of the maximum lag length leads to different recommended lag lengths. With an eye on the frequency of the data, it is also conceivable to apply four lags. Hence, from this perspective it may be useful to check the estimation results of our cointegrated VAR and ARDL specification dependent on using different lag lengths for the large sample, whereas the small sample does not include enough observations for such kind of an exercise. In the following, we proceed like that and present the corresponding results in Table 9. Another possibility would be to estimate the ARDL model with different lag lengths for the endogenous and each exogenous variable.

- Table 9 about here -

So, where do we get from here? First, most of the results in this section confirm the above-mentioned findings especially with respect to the income elasticity, so

\(^{23}\)The results are available upon request.
that one could conclude by comparing the results of Table 9 and section 5.1 with the ones in the previous sections 4.4 and 4.5 that our results are robust. Second, quite unsurprisingly, the larger sample period provides even more striking findings according to related above-mentioned theoretical and empirical literature. Third, the modifications of the lag length do not produce better results than the initial models presented in the previous sections 4.4, 4.5 and 5.1.

6. Conclusion

In this contribution, we have shown that euro area money demand can be considered as stable across different time periods. This result proves to be surprisingly robust to a battery of robustness checks. In fact, we have used two different sample sizes which allowed the conclusion that - just in line with Beyer (2009) - the ongoing financial and economic crisis has had no noticeable impact on the stability of euro area money demand. Especially with respect to the income elasticity we come up with very robust results independent of the specific estimation method applied.

In a nutshell, we have pointed out that other efficient single equation methods beyond the commonly used cointegration techniques such as the Johansen framework or the ARDL approach like the above-mentioned FM-OLS, CCR and DOLS could be helpful in estimating a statistically reliable money demand function. In view of the remaining uncertainty surrounding the integration properties of the involved variables (I(0) or I(1)), single-equation techniques should be preferred. However, the amount of available euro area data might still be considered to be quite limited - at least as measured by the standards of asymptotic theory. Hence, one should still make use of aggregated national data but inference should be drawn with adequate caution. This is the more valid since we have shown by comparing the results of both samples used for each technique applied that the more data we use in our estimation exercises, the more the results are in line with the theoretical predictions and the findings of previous empirical investigations.

Finally, it should be also of interest to test for and estimate the cointegrating relation while augmenting the money demand equation with additional explanatory endogenous and/or exogenous variables and introducing the identifying restrictions for that augmented variable set as well. We leave this task to future research.
References


A. Tables

Table 1: Literature survey

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Sample</th>
<th>Money demand function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beyer (2009)</td>
<td>80:1-07:4</td>
<td>$(m - p)_t = 1.7y_t - 4.11hpi_t$</td>
</tr>
<tr>
<td>Brand and Cassola (2004)</td>
<td>80:1-09:3</td>
<td>$(m - p)_t = 1.331y_t - 1.61i_t^p$</td>
</tr>
<tr>
<td>Brand et al. (2002)</td>
<td>80:1-01:2</td>
<td>$(m - p)_t = 1.34y_t - 0.45i_t^p$</td>
</tr>
<tr>
<td>Bruggeman et al. (2003)</td>
<td>81:3-01:4</td>
<td>$(m - p)_t = 1.38y_t - 0.81i_t^p + 1.31i_t^q$</td>
</tr>
<tr>
<td>Calza et al. (2001)</td>
<td>80:1-09:4</td>
<td>$(m - p)_t = 1.34y_t - 0.86(i_t^p - i_t^q)$</td>
</tr>
<tr>
<td>Carstensen et al. (2009)</td>
<td>79:4-04:4</td>
<td>$(m - p)_t = 0.994y_t - 0.03i_t^p$</td>
</tr>
<tr>
<td>Clausen (1998)</td>
<td>80:1-06:4</td>
<td>$(m - p)_t = 0.98y_t - 4.15i_t^p + 2.08i_t^q$</td>
</tr>
<tr>
<td>Coenen and Vega (2001)</td>
<td>80:4-09:4</td>
<td>$(m - p)_t = 1.125y_t - 0.865(i_t^p - i_t^q) - 1.512\pi_t$</td>
</tr>
<tr>
<td>Dedola et al. (2001)</td>
<td>82:1-09:4</td>
<td>$(m - p)_t = 1.38y_t + 0.41(i_t^p - i_t^q) - 1.71(i_t^q - i_t^p)$</td>
</tr>
<tr>
<td>De Santis et al. (2008)</td>
<td>80:1-07:3</td>
<td>$(m - p)_t = 1.34y_t - 0.76(i_t^p - i_t^q)$</td>
</tr>
<tr>
<td>Dreger and Wolters (2009)</td>
<td>83:1-04:4</td>
<td>$(m - p)_t = 1.238y_t - 5.162\pi_t$</td>
</tr>
<tr>
<td>Dreger and Wolters (2009)</td>
<td>83:1-06:4</td>
<td>$(m - p)_t = 0.955y_t + 0.031y_t \cdot D02Q1 - 6.743\pi_t$</td>
</tr>
<tr>
<td>Fagan and Henry (1998)</td>
<td>80:3-09:4</td>
<td>$(m - p)_t = 1.59y_t - 0.70i_t^p + 0.60i_t^q$</td>
</tr>
<tr>
<td>Fase and Winder (1998)</td>
<td>72:1-09:4</td>
<td>$(m - p)_t = 0.66y_t - 1.33i_t^p + 1.07i_t^q - 1.33\pi_t + 0.341w_t$</td>
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<tr>
<td>Funke (2001)</td>
<td>80:3-09:4</td>
<td>$(m - p)_t = 1.21y_t - 0.3i_t^p + 0.06D86</td>
</tr>
<tr>
<td>Golinelli/Pastorello (2002)</td>
<td>80:3-07:4</td>
<td>$(m - p)_t = 1.373y_t - 0.68i_t^p$</td>
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<tr>
<td>Greiber and Lemke (2005)</td>
<td>80:1-04:4</td>
<td>$(m - p)_t = -8.18 + 1.10y_t - 2.43(i_t^q - i_t^p) + 0.68unc_t$</td>
</tr>
<tr>
<td>Greiber and Setzer (2007)</td>
<td>81:1-06:4</td>
<td>$(m - p)_t = 0.32y_t - 2.55i_t^q + 0.84hpi_t$</td>
</tr>
<tr>
<td>Holtemöller (2004)</td>
<td>84:1-01:4</td>
<td>$(m - p)_t = 1.275y_t - 0.75i_t^p$</td>
</tr>
<tr>
<td>Knetolems (2002)</td>
<td>80:1-01:3</td>
<td>$(m - p)_t = y_t - 1.45i_t^q$</td>
</tr>
<tr>
<td>Müller (2003)</td>
<td>84:1-00:4</td>
<td>$(m - p)_t = 1.57y_t - 2.22i_t^q + 1.87i_t^q$</td>
</tr>
<tr>
<td>Warne (2006)</td>
<td>80:2-04:4</td>
<td>$(m - p)_t = 1.38y_t - 0.26(i_t^p - i_t^q)$</td>
</tr>
<tr>
<td>Wesche (1997)</td>
<td>73:3-93:4</td>
<td>$(m - p)_t = 1.56y_t - 2.40i_t^q$</td>
</tr>
</tbody>
</table>
Table 2: Unit root tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>(m - p)_t</th>
<th>y_t</th>
<th>( i_t^* )</th>
<th>( i_t' )</th>
<th>( \pi_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1995:1-2009:2</td>
<td>level t-stat.</td>
<td>3.54</td>
<td>-1.29</td>
<td>-2.92</td>
<td>-3.29</td>
</tr>
<tr>
<td></td>
<td>level p-values</td>
<td>1.00</td>
<td>0.63</td>
<td>0.05*</td>
<td>0.02**</td>
</tr>
<tr>
<td></td>
<td>( \Delta ) t-stat.</td>
<td>-4.76</td>
<td>-3.38</td>
<td>-3.99</td>
<td>-5.17</td>
</tr>
<tr>
<td></td>
<td>( \Delta ) p-values</td>
<td>0.00***</td>
<td>0.02**</td>
<td>0.00***</td>
<td>0.00***</td>
</tr>
<tr>
<td>PP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>test</td>
<td>level t-stat.</td>
<td>3.06</td>
<td>-1.51</td>
<td>-2.11</td>
<td>-3.79</td>
</tr>
<tr>
<td></td>
<td>level p-values</td>
<td>1.00</td>
<td>0.52</td>
<td>0.24</td>
<td>0.01**</td>
</tr>
<tr>
<td></td>
<td>( \Delta ) t-stat.</td>
<td>-4.78</td>
<td>-3.37</td>
<td>-4.04</td>
<td>-5.10</td>
</tr>
<tr>
<td></td>
<td>( \Delta ) p-values</td>
<td>0.00***</td>
<td>0.02**</td>
<td>0.00***</td>
<td>0.00***</td>
</tr>
</tbody>
</table>

Note: * Statistical significance at the 10% level, ** at the 5% level, *** at the 1% level. For both tests the series contain a unit root under the null. Critical values are taken from MacKinnon (1996): 10% –2.57, 5% –2.87, 1% –3.46. The test equation is estimated including an intercept. For the ADF test the number of lag is chosen by using the SC. Maximum lag number is 10 or 12 for the samples 1995:1-2009:2 and 1980:1-2009:2, respectively. For the PP test autocovariances are weighted by the Bartlett kernel.

Table 3: Lag length selection criteria - empirical realizations (1995:1-2009:2)

<table>
<thead>
<tr>
<th>Lag</th>
<th>LogL</th>
<th>LR</th>
<th>FPE</th>
<th>AIC</th>
<th>SC</th>
<th>HQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>694.86</td>
<td>NA</td>
<td>2.06e – 18</td>
<td>-26.53</td>
<td>-26.35</td>
<td>-26.46</td>
</tr>
<tr>
<td>1</td>
<td>1076.21</td>
<td>674.70</td>
<td>2.31e – 24</td>
<td>-40.24</td>
<td>-39.11*</td>
<td>-39.81</td>
</tr>
<tr>
<td>2</td>
<td>1112.81</td>
<td>57.72*</td>
<td>1.52e – 24*</td>
<td>-40.69</td>
<td>-38.62</td>
<td>-39.90*</td>
</tr>
<tr>
<td>3</td>
<td>1134.27</td>
<td>29.71</td>
<td>1.87e – 24</td>
<td>-40.55</td>
<td>-37.55</td>
<td>-39.40</td>
</tr>
<tr>
<td>4</td>
<td>1162.54</td>
<td>33.71</td>
<td>1.90e – 24</td>
<td>-40.68</td>
<td>-36.74</td>
<td>-39.16</td>
</tr>
<tr>
<td>5</td>
<td>1194.29</td>
<td>31.75</td>
<td>1.88e – 24</td>
<td>-40.93*</td>
<td>-36.06</td>
<td>-39.06</td>
</tr>
</tbody>
</table>

Note: An asterisk * indicates the selected lag from each column criterion.
Table 4: Misspecification tests

<table>
<thead>
<tr>
<th>Sample</th>
<th>Test</th>
<th>Stat.</th>
<th>df</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995:1-</td>
<td>LM(1)</td>
<td>$\chi^2 = 36.46$</td>
<td>25</td>
<td>0.07*</td>
</tr>
<tr>
<td>2009:2</td>
<td>LM(2)</td>
<td>$\chi^2 = 27.82$</td>
<td>25</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>Jarque-Bera</td>
<td>$\chi^2 = 15.16$</td>
<td>10</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>White</td>
<td>$\chi^2 = 365.97$</td>
<td>300</td>
<td>0.01***</td>
</tr>
</tbody>
</table>

Note: * Statistical significance at the 10% level, ** at the 5% level, *** at the 1% level.

Table 5: Unrestricted cointegration rank test

<table>
<thead>
<tr>
<th>Sample</th>
<th>Hypothesis</th>
<th>Eigenvalues</th>
<th>Trace-stat.</th>
<th>95% crit. v.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995:1-</td>
<td>$r = 0$</td>
<td>0.47</td>
<td>91.80</td>
<td>69.82</td>
<td>0.00***</td>
</tr>
<tr>
<td></td>
<td>$r \leq 1$</td>
<td>0.40</td>
<td>57.20</td>
<td>47.86</td>
<td>0.01**</td>
</tr>
<tr>
<td></td>
<td>$r \leq 2$</td>
<td>0.27</td>
<td>29.69</td>
<td>29.80</td>
<td>0.05*</td>
</tr>
<tr>
<td></td>
<td>$r \leq 3$</td>
<td>0.17</td>
<td>12.74</td>
<td>15.49</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>$r \leq 4$</td>
<td>0.05</td>
<td>2.98</td>
<td>3.84</td>
<td>0.08</td>
</tr>
<tr>
<td>1980:1-</td>
<td>$r = 0$</td>
<td>0.23</td>
<td>82.59</td>
<td>69.82</td>
<td>0.00***</td>
</tr>
<tr>
<td></td>
<td>$r \leq 1$</td>
<td>0.22</td>
<td>52.57</td>
<td>47.86</td>
<td>0.02**</td>
</tr>
<tr>
<td></td>
<td>$r \leq 2$</td>
<td>0.10</td>
<td>24.49</td>
<td>29.80</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>$r \leq 3$</td>
<td>0.08</td>
<td>12.89</td>
<td>15.49</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>$r \leq 4$</td>
<td>0.03</td>
<td>3.63</td>
<td>3.84</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Note: * Statistical significance at the 10% level, ** at the 5% level, *** at the 1% level.
Table 6: Likelihood ratio test of variable stationarity and weak exogeneity (95:1-09:2)

<table>
<thead>
<tr>
<th>Test</th>
<th>(m - p)_t</th>
<th>y_t</th>
<th>i_t</th>
<th>i_t</th>
<th>π_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable stationarity</td>
<td>χ²-stat.</td>
<td>4.40</td>
<td>5.09</td>
<td>5.04</td>
<td>5.17</td>
</tr>
<tr>
<td></td>
<td>df</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>95% critical value</td>
<td>7.82</td>
<td>7.82</td>
<td>7.82</td>
<td>7.82</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>0.22</td>
<td>0.17</td>
<td>0.17</td>
<td>0.16</td>
</tr>
<tr>
<td>Weak exogeneity</td>
<td>χ²-stat.</td>
<td>5.70</td>
<td>3.54</td>
<td>7.11</td>
<td>6.49</td>
</tr>
<tr>
<td></td>
<td>df</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>95% critical value</td>
<td>5.99</td>
<td>5.99</td>
<td>5.99</td>
<td>5.99</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>0.06*</td>
<td>0.17</td>
<td>0.03**</td>
<td>0.04**</td>
</tr>
</tbody>
</table>

Note: * Statistical significance at the 10% level, ** at the 5% level, *** at the 1% level.

Table 7: Residual tests

<table>
<thead>
<tr>
<th>Sample</th>
<th>Test</th>
<th>Test-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995:1-</td>
<td>Breusch-Godfrey</td>
<td>F(2,38) = 1.17</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>Harvey</td>
<td>F(14, 40) = 0.43</td>
<td>0.95</td>
</tr>
<tr>
<td>2009:2</td>
<td>Jarque-Bera</td>
<td>χ²(2) = 2.50</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>Ramsey</td>
<td>F(1,39) = 3.63</td>
<td>0.06*</td>
</tr>
<tr>
<td>1980:1-</td>
<td>Breusch-Godfrey</td>
<td>F(2,98) = 1.47</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>Harvey</td>
<td>F(14,100) = 0.43</td>
<td>0.13</td>
</tr>
<tr>
<td>2009:2</td>
<td>Jarque-Bera</td>
<td>χ²(2) = 54.04</td>
<td>0.00***</td>
</tr>
<tr>
<td></td>
<td>Ramsey</td>
<td>F(1,99) = 0.22</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Note: * Statistical significance at the 10% level, ** at the 5% level, *** at the 1% level. Ramsey’s RESET test is applied using squared residuals.
Table 8: Johansen procedure with different sample upper limits

<table>
<thead>
<tr>
<th>Upper limit</th>
<th>$\beta_y$</th>
<th>$\beta_x$</th>
<th>Trace</th>
<th>Max. eigenv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001:4</td>
<td>0.98***</td>
<td>-2.36***</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2002:4</td>
<td>0.98***</td>
<td>-2.38***</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2003:4</td>
<td>0.98***</td>
<td>-2.33***</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2004:4</td>
<td>0.98***</td>
<td>-2.38***</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2005:4</td>
<td>1.00***</td>
<td>-2.35***</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2006:4</td>
<td>1.01***</td>
<td>-2.69***</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2007:4</td>
<td>1.00***</td>
<td>-2.46***</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2008:4</td>
<td>1.00***</td>
<td>-2.09***</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: * Statistical significance at the 10% level, ** at the 5% level, *** at the 1% level. The left part of the table shows the estimated coefficients based on the Johansen procedure with different sample upper limits. The right part displays the associated unrestricted cointegration rank tests.
Table 9: Johansen procedure and ARDL approach with different lag lengths

<table>
<thead>
<tr>
<th>Lags</th>
<th>Money demand function</th>
<th>$\chi^2(2)$-statistic</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(m - p)_t = -3.32 + 0.99^{<em><strong>}y_t - 3.02^{</strong></em>}\pi_t$</td>
<td>14.67</td>
<td>0.00***</td>
</tr>
<tr>
<td>3</td>
<td>$(m - p)_t = -3.83 + 1.02^{<em><strong>}y_t - 1.12^{</strong></em>}\pi_t$</td>
<td>12.06</td>
<td>0.00***</td>
</tr>
<tr>
<td>4</td>
<td>$(m - p)_t = -3.64 + 1.01^{<em><strong>}y_t - 1.28^{</strong></em>}\pi_t$</td>
<td>12.63</td>
<td>0.00***</td>
</tr>
<tr>
<td>5</td>
<td>$(m - p)_t = -3.65 + 1.01^{<em><strong>}y_t - 1.37^{</strong></em>}\pi_t$</td>
<td>13.89</td>
<td>0.00***</td>
</tr>
<tr>
<td>6</td>
<td>$(m - p)_t = -3.97 + 1.03^{<em><strong>}y_t - 1.07^{</strong></em>}\pi_t$</td>
<td>5.02</td>
<td>0.08*</td>
</tr>
<tr>
<td>7</td>
<td>$(m - p)_t = -4.06 + 1.03^{<em><strong>}y_t - 0.93^{</strong></em>}\pi_t$</td>
<td>13.57</td>
<td>0.00***</td>
</tr>
<tr>
<td>8</td>
<td>$(m - p)_t = -6.47 + 1.19^{<em><strong>}y_t + 8.53^{</strong></em>}\pi_t$</td>
<td>10.69</td>
<td>0.01***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lags</th>
<th>Money demand function</th>
<th>$F$-stat. (df)</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$(m - p)_t = 1.25^{<strong>}y_t + 5.78^{</strong>*}i_t^s - 4.82^{**}i_t^d - 1.52\pi_t$</td>
<td>2.67 (5, 94)</td>
<td>0.03***</td>
</tr>
<tr>
<td>4</td>
<td>$(m - p)_t = 1.21^{<em>}y_t + 4.93^{</em><strong>}i_t^s - 4.27^{</strong>}i_t^d - 0.98\pi_t$</td>
<td>1.54 (5, 88)</td>
<td>0.19</td>
</tr>
<tr>
<td>5</td>
<td>$(m - p)_t = 1.19y_t + 8.10^{<em>}i_t^s - 9.28^{</em>}i_t^d + 0.47\pi_t$</td>
<td>1.38 (5, 82)</td>
<td>0.24</td>
</tr>
<tr>
<td>6</td>
<td>$(m - p)_t = 1.95y_t + 25.39i_t^s - 26.19i_t^d + 3.88\pi_t$</td>
<td>0.93 (5, 76)</td>
<td>0.47</td>
</tr>
<tr>
<td>7</td>
<td>$(m - p)_t = 1.09y_t + 7.70^{*}i_t^s - 11.28i_t^d + 2.77\pi_t$</td>
<td>1.08 (5, 70)</td>
<td>0.38</td>
</tr>
<tr>
<td>8</td>
<td>$(m - p)_t = 0.83y_t + 12.91i_t^s - 22.45^{*}i_t^d + 7.27\pi_t$</td>
<td>1.76 (5, 64)</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Note: * Statistical significance at the 10% level, ** at the 5% level, *** at the 1% level. The upper part of the table shows the estimated money demand functions based on the Johansen procedure with different lag lengths and the lower part the same based on the ARDL approach. The Johansen procedure is applied with $r = 2$. The LR test is used to check within the Johansen framework whether to reject the imposed restrictions or not. In case of the estimations based on the ARDL approach, the null of no cointegration is tested by means of the Wald $F$-test.
B. Figures

Figure 1: Cointegration relations (1995:1-2009:2)

Note: The figure shows the cointegrating equations (5) and (6) for the euro area estimated based on the Johansen procedure. They can be interpreted as the euro area money demand equation (left-hand side) and an equation combining the term structure of interest rates and the Fisher inflation parity (right-hand side), respectively, for the short sample (1995:1-2009:2).

Figure 2: Cointegration relations (1980:1-2009:2)

Note: The figure shows the cointegrating equations (8) and (9) for the euro area estimated based on the Johansen procedure. They characterize the money demand (left-hand side) and an equation combining the term structure of interest rates and the Fisher inflation parity (right-hand side) for the large sample (1980:1-2009:2).
Figure 3: Recursive estimates of the coefficients of the ARDL model (1995:1-2009:2)

Note: The figure shows one after the other the recursive estimates of the coefficients of the ARDL model $\gamma_0$, $\gamma_1$, $\gamma_2$, $\gamma_3$ and $\gamma_4$ (see equation (11)) for the short sample (1995:1-2009:2).
Figure 4: Recursive estimates of the coefficients of the ARDL model (1980:1-2009:2)

Note: The figure shows one after the other the recursive estimates of the coefficients of the ARDL model $\gamma_0$, $\gamma_1$, $\gamma_2$, $\gamma_3$ and $\gamma_4$ (see equation (11)) for the large sample (1980:1-2009:2).
Figure 5: CUSUM of Squares of the ARDL model (1995:1-2009:2)

Note: The figure shows the cumulative sum (CUSUM) of squares test for the ARDL model with the associated 5% significance bands for the short sample (1995:1-2009:2).

Figure 6: CUSUM of Squares of the ARDL model (1980:1-2009:2)

Note: The figure shows the cumulative sum (CUSUM) of squares test for the ARDL model with the associated 5% significance bands for the large sample (1980:1-2009:2).