Belke, Ansgar; Klose, Jens

Working Paper

(How) do the ECB and the Fed react to financial market uncertainty? The Taylor rule in times of crisis

DIW Discussion Papers, No. 972

Provided in Cooperation with:
German Institute for Economic Research (DIW Berlin)

Suggested Citation: Belke, Ansgar; Klose, Jens (2010) : (How) do the ECB and the Fed react to financial market uncertainty? The Taylor rule in times of crisis, DIW Discussion Papers, No. 972

This Version is available at:
http://hdl.handle.net/10419/36736

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes. You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Discussion Papers

972

Ansgar Belke • Jens Klose

(How) Do the ECB and the Fed React to Financial Market Uncertainty? The Taylor Rule in Times of Crisis

Berlin, February 2010
Opinions expressed in this paper are those of the author(s) and do not necessarily reflect views of the institute.
(How) Do the ECB and the Fed React to Financial Market Uncertainty?

The Taylor Rule in Times of Crisis

by

Ansgar Belke
(University of Duisburg-Essen, DIW Berlin & IZA Bonn)

and

Jens Klose
(University of Duisburg-Essen)

Abstract

We assess differences that emerge in Taylor rule estimations for the Fed and the ECB before and after the start of the subprime crisis. For this purpose, we apply an explicit estimate of the equilibrium real interest rate and of potential output in order to account for variations within these variables over time. We argue that measures of money and credit growth, interest rate spreads and asset price inflation should be added to the classical Taylor rule because these variables are proxies of a change in the equilibrium interest rate and are, thus, also likely to have played a major role in setting policy rates during the crisis. Our empirical results gained from a state-space model and GMM estimations reveal that, as far as the Fed is concerned, the impact of consumer price inflation, and money and credit growth turns negative during the crisis while the sign of the asset price inflation coefficient turns positive. Thus we are able to establish significant differences in the parameters of the reaction functions of the Fed before and after the start of the subprime crisis. In case of the ECB, there is no evidence of a change in signs. Instead, the positive reaction to credit growth, consumer and house price inflation becomes even stronger than before. Moreover we find evidence of a less inertial policy of both the Fed and the ECB during the crisis.

**JEL code**: E43, E52, E58

**Keywords**: Subprime crisis, Federal Reserve, European Central Bank, equilibrium real interest rate, Taylor rule

**Corresponding authors**:

Ansgar Belke  
University of Duisburg-Essen, DIW Berlin and IZA Bonn  
Universitätsstr. 12,  
D-45117 Essen, Germany  
Phone: +49 201 183 2277  
E-mail: ansgar.belke@uni-due.de

Jens Klose  
University of Duisburg-Essen  
Universitätsstr. 12,  
D-45117 Essen, Germany  
Phone: +49 201 183 3218  
E-mail: jens.klose@ibes.uni-due.de
1. Introduction

Since the start of the subprime crisis in August 2007, central banks all over the world have cut interest rates at a rapid pace trying to overcome the negative effects to the economy. Even though interest rates were lowered everywhere, the timing of the cuts differed considerably among central banks. The probably most pronounced difference can be established if one compares the monetary policies of the US Federal Reserve (Fed) and the European Central Bank (ECB). While the Fed started to cut rates already in August 2007 the ECB did not lower the interest rate until October 2008.¹ At the time of writing, the Fed policy rates have reached their lower bound of 0 - 0.25% while the ECB has still some ample room to cut rates further since the policy rate currently amounts to one percent.² The different mandates of both central banks might be one explanation of these diverging policies. Whereas the goal of the ECB is to maintain price stability, the Fed also has to promote maximum employment and moderate long-term interest rates. So it does not come as a surprise that the Fed puts a larger weight on output stabilization than the ECB which, in turn, explains the more aggressive response of the Fed to the crisis.

However, both central banks appear to have adjusted their policy with the beginning of the crisis. In this context, it becomes important to check empirically which factors were the driving forces behind their interest rate decisions. Hence, we estimate Taylor reaction functions (i.e. Taylor rules (Taylor 1993)) for both central banks separately for the period before the start of the financial turmoil and the period thereafter in order to test whether there are significant differences in the response coefficients in both periods.³ What is more, we include “additional” factors driving interest rate decisions in augmented Taylor rules and test

¹ In fact, the ECB even increased the policy rate in 2008M7 in response to upcoming risks to price-stability (ECB (2008), p.5).
² Lowering rates might probably not be an option in reality since the ECB already started its exit from unconventional monetary policies at its most recent council meeting on December 3rd, 2009. However, from an econometric perspective this might be an important precondition for a “balanced” estimation.
³ It might be argued that using Taylor rules in this context might not be appropriate because during the crisis central banks have started to apply unconventional monetary policies. Thus, they did not confine themselves to target the short-term interest rate throughout our sample period as it is supposed by the Taylor rule. But we argue in line with Jobst (2009) that we need to distinguish between the adjustment of the liquidity implementation framework and the target of monetary policy. While there have been many programs to alter the implementation especially in the case of the Fed (see Federal Reserve Bank of New York 2008 and 2009), there is up to date no clear evidence whether the target of the Fed has changed (for example to target inflation expectations in the presence of a zero nominal rate as Reis (2010) suggests it). Moreover, Aït-Sahalia et al. (2009) find in their analysis that interest rate policy within the crisis was able to reduce interest rate spreads while measures of unconventional monetary policy appear to play only a minor role. Thus, we feel legitimized to (still) use the interest rate as the dependent variable.
whether central banks get indeed less inertial within a crisis as proposed by Mishkin (2008 and 2009). As “additional” explanatory variables and in the spirit of Tucker (2008) we insert different measures of credit and money growth, an interest rate spread variable and asset price inflation variables, the latter being represented by stock and real estate prices. We suspect some of these factors to have become more important during the crisis - for instance, because they might proxy some changes in the equilibrium interest rate - while the response to others is likely to have decreased.

We decided to include an explicit measure of the equilibrium real interest rate in our Taylor rule estimate since this measure is also likely to have changed alongside the start of the crisis (see McCulley/Toloui (2008) and Tucker (2008)). That is exactly why we explicitly estimate the equilibrium real interest rate using the state-space approach introduced by Laubach/Williams (2003).

The remainder of this paper is organized as follows. In section 2, we introduce our approach to estimate the equilibrium real interest rate and present our empirical results. In section 3, we explain the Taylor rule framework, also addressing our adjustments of it which became necessary with an eye on the financial crisis. The results of our Taylor rule estimations are represented in section 4. Section 5 concludes.

2. The equilibrium real interest rate – conceptual and empirical issues

The concept of the equilibrium real interest rate which can be traced back to Wicksell (1898) has only on a few occasions played a decisive role when estimating Taylor rules. Instead, it has simply been held constant in most of the studies. This comes quite as a surprise as there is a large strand of literature dealing with variations of this variable over time. We start from the basic insight that changes of the equilibrium interest rate can turn out to be very large in times of a financial crisis because the standard factors influencing this rate tend to be subject to extensive variations as well. That is exactly why we do not feel justified to generate the equilibrium real interest rate simply as an trend measure like it was recently done, for instance, by Belke/Klose (2009) for normal times because this “leads to substantial biases when output or inflation varies significantly” (Wu (2005) p. 1).

---

4 One noticeable exception is Leigh (2008) who used a framework similar to ours.
5 See e.g. Bomfim (2001), Cuaresma/Gnan/Ritzerberger-Gruenwald (2004) and Horváth (2009) for other approaches of models with a time varying equilibrium real interest rate.
In order to generate a time varying series of the equilibrium real interest rate, we thus decided to strictly follow the approach introduced by Laubach/Williams (2003) who used a state-space model to estimate this unobservable variable. One additional advantage of this approach is that it also treats the potential level of output as an unobserved variable which gets estimated within the model. So we do not have to rely on sometimes arbitrary de-trending methods to generate a time series of potential output to be used in our Taylor rule estimations. We feel legitimized to do so because the state-space approach has been applied in several other studies to estimate the equilibrium real interest rate for different industrialized countries. Up to now, however, according to our knowledge, the sample period underlying the respective estimations do in no case include the episode of the subprime crisis.

The model proposed by Laubach/Williams (2003) which we use in order to generate a measure of the equilibrium real interest rate consists of the following six equations:

\[ y_t - \bar{y}_t = b_{y1}(y_{t-1} - \bar{y}_{t-1}) + b_{y2}(y_{t-2} - \bar{y}_{t-2}) + \frac{b_{r}}{2} \sum_{i=1}^{2}(r_{t-i} - \bar{r}_{t-i}) + \epsilon_{1,t} \]

\[ \pi_t = c_{\pi1}\pi_{t-1} + c_{\pi2}(\pi_{t-2} + \pi_{t-3} + \pi_{t-4}) + c_{\pi3}(\pi_{t-5} + \pi_{t-6} + \pi_{t-7} + \pi_{t-8}) + c_{\pi4}(\pi_{t}^{N} - \pi_{t}) + c_{\pi5}(\pi_{t-1}^{D} - \pi_{t-1}) + c_{\pi6}(y_{t-1} - \bar{y}_{t-1}) + \epsilon_{2,t} \]

\[ \bar{y}_t = \bar{y}_{t-1} + g_{t-1}^{L} + \epsilon_{3,t} \]

\[ g_t = g_{t-1} + \epsilon_{4,t} \]

\[ z_t = z_{t-1} + \epsilon_{5,t} \]

\[ \bar{r}_t = g_t + z_t \]

where \( y_t / \bar{y}_t \) stands for the output and its potential, \( r_t / \bar{r}_t \) is the real interest rate and its equilibrium value, \( \pi_t \) is the inflation rate, \( \pi_t^{N} \) symbols the import price inflation while \( \pi_t^{O} \) displays oil price inflation, \( g_t \) is the annualized growth rate of potential output and \( z_t \) stands for additional factors that influence \( r_t \) such as the time preference of the consumers or the population growth rate. In this model, equations (1) and (2) represent the measurement or signal equations while (3) to (5) are the state equations. Equation (6) describes the

---

6 A state-space model consists of signal and state equations. The state equations show how the unobservable variables in the system are specified while the signal equations tell us how these variables along with other exogenous variables help us to estimate the fitted values of a known variable.

7 For the US, see also Clark/Kozicki (2005), Trehan/Wu (2006), for the euro area see Wintr/Guarda/Rouabah (2005), Mesonnier/Renne (2007), Garnier/Wihelmsen (2008) and for the United Kingdom Larsen/McKeown (2004).

8 The source of the data and their construction are explained in detail in Appendix A.
construction of the equilibrium real interest rate which is essentially derived out of the two random walk variables $g_t$ and $z_t$.\footnote{In fact, Laubach and Williams did not restrict the coefficient of the annualized growth rate to unity as we do here. But as the estimates of this coefficient in other papers generally turn out to be quite close to one, we feel legitimized to do so.}

We choose the lag structure in line with Laubach/Williams (2003) and in a way such that the IS-equation (1) consists of two lags of the output gap and two lags of the real interest rate gap, assuming equal weights $\frac{b_z}{2}$ of the latter. For the Phillips curve representation in equation (2) we apply eight lags of inflation and one lag of the output gap. The responses to the second up to the fourth lag and to the fifth up to the eighth lag of inflation are both supposed to be identical and amount to $\frac{c_{ns_2}}{3}$, and $\frac{c_{ns_3}}{4}$, respectively. Moreover, the inflation coefficients sum up to unity. Our sample period ranges from 1971Q1 to 2009Q2.\footnote{Since data for the most variables of the euro area are only available from the 1990s onwards, we use the Area Wide Model (AWM) database by Fagan/Henry/Mestre (2005) which is available from the website of the European Business Cycle Network and goes back to 1970Q1. The start of our sample period not earlier than in 1971 is thus due to data construction of the different inflation rates.} Hence, the former also includes data covering parts of the period of the subprime crisis.

Since the standard deviations of the trend growth rate $g_t$ and $z_t$ might be biased towards zero, due to the so called pile-up-problem\footnote{In our context, the pile-up-problem occurs since in maximum likelihood estimations the standard deviations of $g_t$ and $z_t$ are likely to be biased towards zero. The median unbiased estimator corrects for this.} (Stock 1994), we cannot estimate the above model in a straightforward fashion. Hence, we correct for this by using the median unbiased estimator as discussed by Stock/Watson (1998). What is more, we proceed in four steps, strictly in line with the suggestions of Laubach/Williams (2003). First, we estimate the signal equations separately by OLS using the Hodrick-Prescott-filter (Hodrick/Prescott 1997) to generate a series of potential output. In the IS-equation, we omit the real interest rate gap. As a second step, we use the Kalman-filter to estimate these signal equations, assuming that the trend growth rate is constant. Taking this as a starting point, we are able to compute the median unbiased estimate $\lambda_g$ which is equal to $\frac{\sigma_g}{\sigma_z}$. We use this relationship in a third step and add the real interest rate gap to equation (1). We also relax our assumption of a constant trend growth rate.

Taking these considerations as a starting point, we can estimate equations (1) to (5), assuming that $z_t$ being constant. $g_t$ and $z_t$ enter the IS-equation by inserting (6) in (1). What is more and strictly in line with Trehan/Wu (2006), we assume the Fisher equation with rational expectations ($r_t = i_t - \pi_{t+1}$) to hold. This has the positive side-effect that the real
interest rate can be calculated within the model since inflation expectations do not need to be specified separately. Having estimated these equations, we can compute the median unbiased estimator as \( \lambda_z = \frac{\sigma_x}{\sigma_z} \cdot \frac{b_x}{\sqrt{2}} \). As a final step, we include this relationship in equation (5) and estimate the whole system by means of the maximum likelihood estimation method.\(^{12}\)

We report the corresponding results in Table 1, together with the results of Trehan/Wu (2006) for the US and Garnier/Wilhelmsen (2009) for the euro area which both serve as comparisons and benchmarks for our estimate.

**Table 1 - Estimates of the state-space model: first results**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>USA</th>
<th>Trehan/Wu</th>
<th>Garnier/Wilhelmsen</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_{y_1} )</td>
<td>1.60 (15.21)</td>
<td>1.16 (9.42)</td>
<td>1.65 (16.29)</td>
</tr>
<tr>
<td>( b_{y_2} )</td>
<td>-0.67 (-6.18)</td>
<td>-0.24 (-2.03)</td>
<td>-0.79 (-8.72)</td>
</tr>
<tr>
<td>( b_r )</td>
<td>-0.06 (-2.94)</td>
<td>-0.13 (-5.37)</td>
<td>-0.02 (-0.61)</td>
</tr>
<tr>
<td>( c_{\pi_1} )</td>
<td>1.16 (19.91)</td>
<td>0.49 (8.25)</td>
<td>0.82 (13.61)</td>
</tr>
<tr>
<td>( c_{\pi_2} )</td>
<td>-0.30 (-4.35)</td>
<td>0.37 (4.94)</td>
<td>-0.05 (-0.71)</td>
</tr>
<tr>
<td>( c_{\pi_3} )</td>
<td>0.14</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>( =1-c_{\pi_1}-c_{\pi_2} )</td>
<td>0.026</td>
<td>0.004</td>
<td>0.070</td>
</tr>
<tr>
<td>( c_{\pi^N} )</td>
<td>(3.34)</td>
<td>(3.91)</td>
<td>(6.88)</td>
</tr>
<tr>
<td>( c_{\pi^0} )</td>
<td>0.001</td>
<td>0.05</td>
<td>-0.003</td>
</tr>
<tr>
<td>( c_y )</td>
<td>(1.72)</td>
<td>(4.16)</td>
<td>(-1.92)</td>
</tr>
<tr>
<td>( \lambda_g )</td>
<td>0.023</td>
<td>-</td>
<td>0.055</td>
</tr>
<tr>
<td>( \lambda_z )</td>
<td>0.073</td>
<td>-</td>
<td>0.157</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>0.390</td>
<td>0.57</td>
<td>0.118</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>0.251</td>
<td>0.80</td>
<td>0.339</td>
</tr>
<tr>
<td>( \sigma_3 )</td>
<td>0.559</td>
<td>0.46</td>
<td>0.478</td>
</tr>
<tr>
<td>( \sigma_4 )</td>
<td>0.085</td>
<td>0.20</td>
<td>0.112</td>
</tr>
<tr>
<td>( \sigma_5 )</td>
<td>0.715</td>
<td>0.22</td>
<td>1.283</td>
</tr>
<tr>
<td>( log-likelihood )</td>
<td>-170.25</td>
<td>-415.56</td>
<td>-196.64</td>
</tr>
</tbody>
</table>

**Notes:** t-statistics in parentheses.

According to Table 1, it turns out that our estimates are generally in line with those gained by a couple of benchmark studies. Additionally, we have generated a time series for

\(^{12}\) We display the whole system written in matrix language in Appendix B.
potential output $\bar{y}_t$, its growth rate $g_t$ and the additional factors $z_t$. This enables us to calculate a series for the output gap and the equilibrium real interest rate via equation (6). The corresponding series for the US and the euro area are displayed in Figure 1.a and 1.b for the period from 1998M7 to 2009M6. We have chosen this sample period with an eye on data availability and will definitely use it in the next section to estimate the different Taylor rule specifications.

Figure 1.a shows how different the equilibrium real interest rates evolved over time on both sides of the Atlantic. In case of the euro area, the equilibrium real interest rate moves along the two percentage point benchmark before the crisis. Then it decreases sharply after the start of the crisis and becomes even negative in 2008. For the US, in contrast, we find an almost steady decrease of the equilibrium real interest rates which is probably a bit more pronounced in the first phase of the crisis. However, both the euro area and the US equilibrium real interest rate seem to have reached their floor since the end of 2008. Rates are no longer declining from this point onwards on both sides of the Atlantic.

Figure 1.a - Equilibrium real interest rates, state-space calculation

![Equilibrium real interest rates, state-space calculation](image)

Figure 1.b - Output gap, state-space calculation

![Output gap, state-space calculation](image)

A closer inspection of Figure 1.b which displays the euro area and US output gap time series reveals that both variables move remarkably in parallel with each other, with the US output gap throughout taking higher values than its euro area counterpart. However, both series display the expected crisis-caused decline in output towards the end of the sample.

Having found adequate measures of the equilibrium real interest rate variable and output gap we are now able to estimate Taylor rules separately for the periods before and after the start of the subprime crisis. For this purpose, we briefly explain the basic concept of the
Taylor rule in the next section, jointly with possible extensions that might describe the interest rate setting of both central banks more accurately in times of financial crisis.

3. The Taylor Rule and extensions necessitated by the financial crisis

In 1993 John B. Taylor proposed a new specification of the monetary policy reaction function which arguably covered the interest rate setting behavior of the Fed during the period 1987-1992 quite well. According to his generalized rule, the Fed reacts to deviations of the inflation rate from its target and to deviations of the output from its potential, the so-called output gap. Hence, we can write the basic Taylor reaction function as follows:

\[ i_t = \bar{r}_t + \pi_t + \alpha_{\pi}(\pi_t - \pi^*) + \alpha_y Y_t \]

where \( i_t \) is the interest rate set by the Fed, \( \bar{r}_t \) is the equilibrium real interest rate, \( \pi_t \) and \( \pi^* \) represent the inflation rate and its target, \( Y_t \) is the output gap we calculated above and \( \alpha_{\pi}, \alpha_y \) are the reaction coefficients to the inflation and output gap respectively. In his seminal paper, John B. Taylor sets \( \bar{r}_t \) and \( \pi^* \) equal to two and the reaction coefficients equal to 0.5 each. With this device, he is able to mimic the interest rate setting of the Fed in the above mentioned period.

But there is certainly a set of additional variables (later on called “additional” variables) beyond the inflation rate and the output gap which might drive the interest rate setting behavior of the Fed and the ECB in times of an exceptional crisis. Following the literature, we identified four groups of variables which may be of higher/reduced interest in this period.

The first one is an almost common extension of the Taylor rule not only in times of a financial turmoil. It consists of expanding the rule by the growth of a target monetary aggregate which as usual is M2 in case of the US and is represented by M3 in case of the ECB.\(^{13}\) We expect the sign of this coefficient to be positive, implying that the central banks cut rates in response to an increase in money growth because an expansion of the monetary base in the long run leads to inflation according to the seminal Friedman (1963) view.\(^{14}\) However, during a financial crisis the importance of the money growth in an ordinary Taylor reaction function should be decreasing since output stabilization typically gets more important.

\(^{13}\) See for a complimentary analysis before the subprime crisis Ullrich (2003) or Belke/Polleit (2007).
\(^{14}\) For the euro area this view is supported by the recent empirical analysis of Hall et al. (2009).
than fighting inflation in such a scenario. This should hold especially for the Fed since stable prices are just one out of three targets the Fed has to reach in contrast to the ECB where a low inflation rate is the primary target.

The second group of variables covers different measures of credit growth which takes into account the hypothesis that within this crisis a credit crunch/rationing is likely to occur.\footnote{See Borio/Lowe (2004) for a discussion on the role of credit before the subprime crisis. Christiano et al. (2008) and Curdia/Woodford (2009) show that adding an aggregate credit variable tends to improve the goodness-of-fit of Taylor rule estimates.} A credit crunch/rationing is a scenario in which commercial banks cut the amount lent to individuals. There are many different reasons for a credit crunch/rationing one can think of. But the most pertinent ones in the ongoing crisis are surely that the value of collateral decreases and the equity of the banks declines with the decline in asset prices. By the amount of credit offered, the capital markets are linked with the real economy. So the rationale behind the implementation of this variable is that central banks are trying to overcome the credit crunch/rationing by endowing the banks with more liquidity so that they could again raise credit and, by this, promote investment and consumption. This is done by lowering interest rates, thus pouring additional money into the market to make credit lending work again.

We expect the estimated coefficient to increase during the crisis as it gets more important than before to provide the economy with the liquidity so urgently needed. In order to check whether this fits with US and euro area data, we decided to estimate Taylor rules using three different credit measures. First we use overall credit supplied by banks. Second, we impose commercial and industrial credit for the Fed and industrial credit for the ECB, since these measures are most likely influenced by a potential credit crunch or credit rationing. As a third alternative, we insert real estate credit because the crisis has its roots in the housing sector and it is reasonable to assume that in this sector a credit crunch/rationing occurred.

Measuring increased risk in capital markets and the associated change in the equilibrium interest rate is the goal of a third category of variables inserted by us, i.e. interest rate spreads.\footnote{See Martin/Milas (2009) for a survey of the usefulness of applying interest rate spreads for an assessment of optimal monetary policy in the UK during the subprime crisis.} During the current crisis the focus in this context switched towards the Libor/overnight indexed swap (OIS) spread.\footnote{See, for instance, Taylor (2008), Armatier/Krieger/McAndrews (2008) and Michaud/Upper (2008).} But unfortunately this spread displayed only little variation before the subprime crisis so that the coefficients are estimated imprecisely. That is why we do not use this measure in the Taylor rule framework.
Another interest rate spread which exhibits variation before and after August 2007 is the long-term/short-term spread. As short-term rate the 3-month rate is favored and the long-run rate is those for ten-year treasury/governmental securities. A rising spread signals rising risk within the capital market for long-term credits which drive investment decisions. Since central banks are generally expected to lower their interest rates in response to a rise in the interest rate spread, the coefficient of the spread should be negative (Tucker, 2008). In addition, we should expect an even stronger monetary policy reaction throughout the ongoing crisis, since reducing the risk in the markets have explicitly been addressed by the authorities as a main goal of both the Fed and the ECB policy during this time.\textsuperscript{18}

The fourth and last group of potential variables which might be influencing central banks’ interest rate setting during the crisis comprises asset price inflation.\textsuperscript{19} Here, we focus on the two main asset classes housing and stock. As the sharp decline in house prices has commonly been regarded as the main trigger of the crisis it seems quite natural to include it into our regression analysis. But how does (and should) monetary policy react if house prices decline? The answer can be divided into two parts. The first one covers wealth effects by the house owners of the houses. The second effect was surely more pronounced in this crisis. It is related to the effects on the value of collateral underlying loans and mortgages. If house prices fall, the collateral is worth less and higher interest rates would have to be paid in order to have access to mortgage financing. This finally results in less credit given to individuals and indicates that central banks should lower interest rates in order to offset the effects induced by the loss of collateral. In the same vein, the wealth effect which leads to less consumption should be countered by a rate cut in order to enhance consumption. Thus we expect a positive sign for this coefficient.

Considering stock prices, there is also a wealth and collateral effect acting in the same way as for house prices. But in addition, there is an effect on the companies issuing stock according to Tobin’s \( q \) (Tobin 1969) which relates the market capitalization of a firm to its replacement costs. If the market capitalization expressed by the cumulative value of stock falls in response to a drop in stock prices, \( q \) falls and a firm would thus cut investment. The response of the central banks to this would be the same as for the other two effects. So we

\[ \text{\textsuperscript{18} See, for instance, Bernanke (2008), Mishkin (2009) for the US and Trichet (2009) referring to the ECB.} \]

expect the interest rate to fall if stock prices are decreasing thus leading to a positive
coefficient of this variable in Taylor rule estimations. Additionally we expect the influence of
this parameter to have increased in the crisis because it is more likely that the Fed reacts more
aggressively to a downturn in stock prices than to a steady increase as it was mainly the case
before the subprime crisis. In the case of the ECB, the effects should be less pronounced since
financing of the firms in mainly done by receiving credit from banks and much less via the
capital market (Stark 2009).

4. Empirical evidence - Is there a crisis effect on the Taylor rule?

In this section we present the results of our estimations of the Taylor rule for the Fed
and the ECB. But, prior to this, we explain our estimation procedure.

4.1. Estimation issue

To estimate our different Taylor rule specifications for the US and the euro area, we
use the GMM procedure. The latter appears highly adequate for our purposes because at the
time of its interest rate setting decision, the central banks cannot observe the ex-post realized
right hand side variables. That is why the central banks have to base their decisions on lagged
values only (Belke/Polleit 2006). We decided to use the first six lags of inflation and the
output gap and - whenever it is added to the regression equation - the first six lags of the
“additional” variable as instruments. Moreover, we perform a J-test to test for the validity of
over-identifying restrictions to check for the appropriateness of our selected set of
instruments. As the relevant weighting matrix we choose, as usual, the heteroskedasticity and
autocorrelation consistent HAC matrix by Newey and West (1987).

In order to dispose of enough data-points to catch the dynamics of the interest rate
setting process during the period of the crisis we decided to use monthly instead of quarterly
data. Since the equilibrium real interest rate and the output gap estimated in part 2 are of a
quarterly frequency, they need to be transformed into monthly data. This is done by using a
cubic spline commonly used to interpolate variables to a higher frequency. To The source and
construction of the transformed variables are explained in Appendix A.

For estimation purposes we rearrange equation (7) as follows:

\[ i_t = a_0 + \bar{r}_t - (a_\pi - 1)\pi^* + a_\pi \pi_t + a_y Y_t + a_x X_t + \varepsilon_t \]  

20 See for an application of the cubic spline in the context of the Taylor rule Gerdesmeier/Roffia (2005).
with $a_\pi = 1 + \alpha_\pi$ such that, according to the Taylor principle, $a_\pi > 1$. We assume the inflation target $\pi^*$ to be 2% as suggested by Taylor as well.\(^{21}\) $a_0$ is a constant term added to the Taylor rule which should be zero cause all factors influencing this term (namely the equilibrium real interest rate and the inflation target) are explicitly modeled within this equation. The variable $X_t$ added in (8) covers the “additional” variables we include in the Taylor rule, namely: Money growth of M2 or M3 (M), overall credit growth (CR), (commercial and) industrial credit growth (CR_(COMM)_IND), real estate credit growth (CR_HOUSE), the interest rate gap (I), stock price inflation (S) and house price inflation (HP). $\varepsilon_t$ covers the error term.

In order to account for the claim raised by Mishkin (2008 and 2009) that monetary policy is less inertial during a financial crisis we also implemented an interest rate smoothing term into equation (8). Hence, the specification of the monetary policy reaction function changes to:

\[
(9) \quad i_t = \rho \cdot i_{t-1} + (1 - \rho) \cdot \left[ a_0 + \bar{r}_t - (a_\pi - 1)\pi^* + a_\pi \pi_t + a_y Y_t + a_x X_t \right] + \varepsilon_t,
\]

where $\rho$ is the smoothing parameter taking reasonable values between zero (the interest rate is only driven by the fundamentals in the Taylor rule and not by the lagged interest rate) to one (the lagged interest rate is the best predictor for the contemporaneous interest rate). If Mishkin is right, we expect the interest rate smoothing parameter to fall significantly within the crisis.

### 4.2. Estimation Results

In order to estimate whether there is a change in the Fed and the ECB estimated reaction function coefficients with the beginning of the subprime crisis, we divide our sample period into two subsamples. The first one covers the period before the subprime crisis started. As the “pre-crisis” period we choose 1999M1-2007M1. Although the start is normally dated to August 2007\(^{22}\) we did not make use of the data ranging from 2007M2 to 2007M7 in order to avoid that any economic indicators which already indicate an upcoming crisis are included.

\(^{21}\) Although an inflation target of 2% percent is not explicitly announced by the Fed like the ECB does, it is widely accepted as an inflation target corresponding to price stability without running the risk of deflation.

\(^{22}\) See e.g. Cecchetti (2008) pp. 12-17, Taylor/Williams (2009) p. 60. For a detailed schedule what happened around that time and the decisions made by the most important central banks as a reaction to this see Bank for International Settlement (2008) pp. 56-74. They should not be repeated here.
in the estimations for the “pre-crisis” sample. For our “subprime” estimation we use the period 2007M8 to 2009M6, due to data constraints.\textsuperscript{23}

4.2.1 Results for basic specifications

The Fed Taylor rule

In Table 2 we present the results of equation (8) for the Fed, i.e. the Taylor rule without interest rate smoothing based on a sample period which excludes the subprime crisis.

Table 2 - Taylor rule estimates for the Fed - the “pre-crisis” period

<table>
<thead>
<tr>
<th>2.1</th>
<th>2.2</th>
<th>2.3</th>
<th>2.4</th>
<th>2.5</th>
<th>2.6</th>
<th>2.7</th>
<th>2.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>-4.20***</td>
<td>-5.93***</td>
<td>-4.10***</td>
<td>-1.82***</td>
<td>-3.72***</td>
<td>-1.26***</td>
<td>-4.82***</td>
</tr>
<tr>
<td>(0.18)</td>
<td>(0.41)</td>
<td>(0.29)</td>
<td>(0.38)</td>
<td>(0.32)</td>
<td>(0.43)</td>
<td>(0.17)</td>
<td>(0.45)</td>
</tr>
<tr>
<td>$a_\pi$</td>
<td>0.62***</td>
<td>0.81***</td>
<td>0.78***</td>
<td>-0.20</td>
<td>0.92***</td>
<td>0.23*</td>
<td>0.51***</td>
</tr>
<tr>
<td>(0.17)</td>
<td>(0.14)</td>
<td>(0.19)</td>
<td>(0.14)</td>
<td>(0.14)</td>
<td>(0.13)</td>
<td>(0.15)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>$a_y$</td>
<td>1.18***</td>
<td>1.32***</td>
<td>1.24***</td>
<td>-0.10</td>
<td>1.07***</td>
<td>0.42***</td>
<td>1.54***</td>
</tr>
<tr>
<td>(0.12)</td>
<td>(0.11)</td>
<td>(0.13)</td>
<td>(0.20)</td>
<td>(0.11)</td>
<td>(0.10)</td>
<td>(0.13)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>$a_M$</td>
<td>0.16***</td>
<td>-0.03</td>
<td>0.22***</td>
<td>0.04*</td>
<td>-0.85***</td>
<td>-0.021***</td>
<td>-0.18***</td>
</tr>
<tr>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.09)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>$a_{cr, comm_ind}$</td>
<td>0.22***</td>
<td>0.04*</td>
<td>-0.85***</td>
<td>0.021***</td>
<td>-0.18***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.09)</td>
<td>(0.004)</td>
<td>(0.03)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{cr, house}$</td>
<td>-0.021***</td>
<td>0.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.004)</td>
<td>(0.11)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_s$</td>
<td>0.61</td>
<td>0.60</td>
<td>0.61</td>
<td>0.61</td>
<td>0.82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{hp}$</td>
<td>0.09</td>
<td>0.10</td>
<td>0.12</td>
<td>0.08</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$adj\ R^2$</td>
<td>0.63</td>
<td>0.54</td>
<td>0.61</td>
<td>0.60</td>
<td>0.87</td>
<td>0.61</td>
<td>0.82</td>
</tr>
<tr>
<td>J-stat</td>
<td>0.10</td>
<td>0.09</td>
<td>0.10</td>
<td>0.11</td>
<td>0.11</td>
<td>0.08</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Notes: GMM estimates, ***/*** indicates significance at the 10%/5%/1% level, standard errors in parentheses, for the J-statistic we put p-value in parentheses; abbreviations of the regressors follow those of Appendix A; Sample 1999M1-2007M1.

Table 2 reveals that the Taylor principle, i.e. the assumption of an inflation coefficient $a_\pi$ above unity, is violated in all but one (column 2.8) cases considered. Thus, we do not find evidence supporting the hypothesis that an increase in the inflation rate is offset by an even larger increase in the nominal interest rate (which would then lead to a rise in the real rate). The estimated output gap coefficient $a_y$ is on average larger than the proposed 0.5, often even exceeding unity. However, there are two exceptions, i.e. an output gap coefficient below

\textsuperscript{23} For the euro area house price inflation only data up to 2008M12 are available, which were provided by Andreas Rees. So the results rely on this shorter period. Additionally we use only the first four lags of the variables as instruments in order to have enough degrees of freedom left.
unity: when adding consumer and industrial credit growth (column 2.4) and the interest rate gap (column 2.6) to the previous specification. In these estimations both estimated coefficients of the output gap turn out to be much smaller and are insignificant in one case.

But this does not come as a surprise as a brief look at the data reveals that consumer/industrial credit growth and the interest rate gap exhibit the closest (positive and negative) correlation with the Fed Funds Rate. Hence, it appears rather unlikely that inflation and the output gap variables included in our specification of the Taylor rule are estimated with the necessary precision. However, both “additional” variables are highly significant and the goodness-of-fit as measured by the adjusted $R^2$ is larger than for the “normal” Taylor rule. What is more, an equally good fit can also be found for the specification adding house price inflation (2.8) where the inflation and output gap parameter does not fall.

Seen on the whole, we consistently find significant results for the “additional” variables with the exception of overall credit growth (column 2.3). This result might be driven be the opposing signs of commercial and industrial (column 2.4) to housing credit growth (2.5). While the prior shows the expected positive sign the latter points to a weakly significant negative reaction. Also the signs of the estimated coefficients of money growth (column 2.2) and of the interest rate gap (column 2.6) are in line with theory and significantly different from zero. In contrast to that we find a significantly negative response of the interest rate to asset price inflation expressed in terms of stock price inflation (column 2.7) and house price inflation (column 2.8). This suggests that the Fed even triggered bubbles in these sectors which in the end fueled the financial crisis.

After having assessed the goodness-of-fit of the Taylor rule in the “pre-crisis” period, we now turn to an estimation of the coefficients of this rule across the financial crisis and compare these coefficients with those gained from the estimation for the “pre-crisis” period. The corresponding results for the Fed are displayed in Table 3.

Table 3 reveals that the inflation coefficient which turns out to be rather low already in the pre-crisis era even decreases in magnitude further now. In most cases, the estimated coefficient stays significant but turns negative, indicating that the Fed reacts to a rise in inflation by a reduction in the policy rate which is clearly at odds with the recommendations of the “normal” Taylor rule. However, this pattern might be driven by the period up to the mid-2008 in which inflation increased sharply in response to the worldwide rise in oil prices while the interest rate was already declining. Afterwards, inflation declined at a rapid pace as
Nevertheless it is evident that at least in the first year of the crisis the Fed did the opposite of what it should have done according to the guiding lines of the Taylor rule regarding the inflation response.

Table 3 - Taylor rule estimates for the Fed – the “subprime” period

<table>
<thead>
<tr>
<th></th>
<th>3.1</th>
<th>3.2</th>
<th>3.3</th>
<th>3.4</th>
<th>3.5</th>
<th>3.6</th>
<th>3.7</th>
<th>3.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>0.62***</td>
<td>5.51***</td>
<td>3.04***</td>
<td>2.07***</td>
<td>2.03***</td>
<td>2.01***</td>
<td>0.96***</td>
<td>2.91***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.26)</td>
<td>(0.19)</td>
<td>(0.02)</td>
<td>(0.15)</td>
<td>(0.04)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$a_\pi$</td>
<td>-0.28***</td>
<td>-0.39***</td>
<td>-0.21***</td>
<td>0.18***</td>
<td>-0.47***</td>
<td>0.01***</td>
<td>-0.07***</td>
<td>0.17***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(0.02)</td>
<td>(0.00)</td>
<td>(0.01)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>$a_y$</td>
<td>1.05***</td>
<td>0.63***</td>
<td>1.54***</td>
<td>1.08***</td>
<td>1.42***</td>
<td>0.34***</td>
<td>0.30***</td>
<td>0.18***</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.02)</td>
<td>(0.05)</td>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$a_M$</td>
<td>-0.72***</td>
<td>(0.03)</td>
<td>-0.40***</td>
<td>(0.03)</td>
<td>-0.15***</td>
<td>(0.00)</td>
<td>-0.23***</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$a_{cr}$</td>
<td>-0.93***</td>
<td>(0.01)</td>
<td>0.055***</td>
<td>(0.001)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{cr, comm, ind}$</td>
<td>-0.15***</td>
<td>(0.00)</td>
<td>-0.23***</td>
<td>(0.02)</td>
<td>-0.93***</td>
<td>(0.01)</td>
<td>0.055***</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$a_{cr, house}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_g$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{hp}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$adj R^2$</td>
<td>0.85</td>
<td>0.94</td>
<td>0.89</td>
<td>0.99</td>
<td>0.88</td>
<td>0.95</td>
<td>0.94</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(0.87)</td>
<td>(0.94)</td>
<td>(0.91)</td>
<td>(0.96)</td>
<td>(0.94)</td>
<td>(0.90)</td>
<td>(0.93)</td>
<td>(0.95)</td>
</tr>
</tbody>
</table>

Notes: GMM estimates, */**/*** denotes significance at the 10%/5%/1% level, standard errors in parentheses, for the J-statistic we put the p-value in parentheses; abbreviations of the regressors follow those of Appendix A; Sample 2007M8-2009M6.

Remarkably, the estimated reaction of the interest rate to the output gap remains positive and highly significant across all specifications. However, the coefficient estimates differ in magnitude by almost 1.5, depending on the specification chosen. Compared to the “pre-crisis” estimates, no clear-cut picture emerges with respect to the question whether the impact of the output gap has really changed. In most cases the magnitude of the estimated output gap coefficient decreases. However, only for the specifications including money growth (just comparing column 2.2 to column 3.2), all credit growth measures (columns 2.3 to 2.5 with columns 3.3 to 3.5) or asset price inflation (columns 2.7 to 2.8 with columns 3.7 to 3.8) we find a significant difference between the coefficients estimated in both subsamples.\textsuperscript{24}

While the influence of the output gap decreases in case of the specifications including money growth and asset price inflation (columns 3.2, 3.7, 3.8), the opposite is true in case of the

\textsuperscript{24} We used the Wald-test in order to assess the significance of the differences between both coefficients.
Taylor rule specifications including commercial and industrial and real estate credit growth (columns 3.3 to 3.5).

Turning now to the “additional” variables, some remarkable differences are worth mentioning. Except for the interest rate spread-adjusted rule (column 3.6), where the estimated coefficient $\alpha_i$ stays almost unchanged after the start of the financial crisis period (indicating that the Fed did and does give equal weight to interest rate gaps), the estimated coefficients of all other variables change their sign and remain significant.\(^{25}\) The impact of money growth (column 3.2) turns negative which corresponds with theory since in a crisis stabilizing the economy is more important than fighting future inflation.

The negative sign of our estimated credit growth coefficients (columns 3.3 to 3.5) is somehow surprising. Strictly speaking, this means that the Fed has responded to a credit growth decline with a rise in interest rates. However, such a result appears counter-intuitive since the literature seems to claim that the sign should be positive. For instance, some have suggested that because of imperfections in financial intermediation, it is more important for central banks to monitor and respond to variations in the volume of bank lending than would be the case if the frictionless financial markets of Arrow-Debreu theory were more nearly descriptive of reality. A common recommendation in this vein is that monetary policy should be used to help to stabilize aggregate private credit, by tightening policy when credit is observed to grow unusually strongly and loosening policy when credit is observed to contract.

For example, Christiano et al. (2007) propose that a Taylor rule that is adjusted in response to variations in aggregate credit may represent an improvement upon an unadjusted Taylor rule. But according to Curdia/Woodford (2009), pp. 39ff., the optimal degree of response to a reduction in credit is much less strong than would be required to fully stabilize aggregate credit at some target level, especially if there are financial frictions as in the current crisis. Like us, they find an optimally negative response to their calibrated aggregate credit coefficient for example to a governmental debt shock.

Moreover, we have to differentiate between credit rationing and a credit crunch. Whereas under a credit crunch the market mechanism still works in the sense that interest rate adjustments bring supply of and demand for loans in equilibrium, credit rationing implies an

\(^{25}\) However, there are two exceptions. The specification including overall and housing credit growth had an (in-) significantly negative influence before Subprime. Within the crisis it turns significantly negative and the Wald test indicates a significantly drop in the response coefficient, thus the picture for credit growth measures is not changed by this result.
equilibrium in which there is an excess demand for loans over credit supply (Green/Oh 1991). If there is credit rationing, the interest rate has to increase in order to reduce credit demand and expand credit supply to a new market equilibrium. Thus, our results imply that the Fed judges the reduced credit growth as a malfunctioning of the market (credit rationing).

Finally, the negative coefficient can simply be explained by the minor importance of US credit markets in refinancing of companies. Since it is most likely that the focus of the Fed is to avoid bankruptcy of companies, it becomes evident that this is not achieved via the credit market but through other means for example through stock prices.

In contrast to the estimations for the “pre-crisis” period, the estimated coefficients of asset price inflation (columns 3.7 to 3.8) display the expected positive sign. This is compatible with the view that during the financial crisis the Fed reacted to the downturn in asset prices with a reduction in the policy rate. This, in turn, suggests that the Fed reacts to movements in asset prices in an asymmetric fashion, i.e. stronger to a downturn than to an increase of asset prices, since we find an accommodative policy towards asset price inflation in our estimations for the “pre-crisis” period (columns 2.7 to 2.8).

The ECB Taylor rule

Turning now to the Taylor rule estimations for the ECB we display our results for the pre-crisis episode in Table 4. The results show that - as in the case of the Fed - the Taylor principle is never fulfilled. Additionally, the reaction coefficients of the output gap all take values of around one, thus being in line with those found for the Fed in Table 2. Hence, as far as the Taylor rule coefficients of inflation and the output gap are concerned both central banks seem to have followed a rather similar policy before the crisis hit.

However, we find some differences when looking at the “additional” variables. The coefficients of money growth (column 4.2) and stock price inflation (column 4.7) are insignificant in contrast to the Fed estimates (columns 2.2 and 2.7). For money growth this result is surprising since the ECB explicitly announced a prominent role to this measure. But we get a clearer picture of the ECB’s reaction to credit growth (columns 4.3 to 4.5) as compared to the Fed since all three measures show a small but significantly positive reaction. This result is not surprising since credit markets in the euro area play a much bigger role in financing corporations than in the US (Stark 2009). The interest rate gap (column 4.6) and the house price inflation variable (column 4.8) exhibit a negative response coefficient as is also the case for the Fed (columns 2.6 and 2.8).
### Table 4 - Taylor rule estimates for the ECB - the “pre-crisis” period

<table>
<thead>
<tr>
<th></th>
<th>4.1</th>
<th>4.2</th>
<th>4.3</th>
<th>4.4</th>
<th>4.5</th>
<th>4.6</th>
<th>4.7</th>
<th>4.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>-1.76***</td>
<td>-1.65***</td>
<td>-2.00***</td>
<td>-1.95***</td>
<td>-2.85***</td>
<td>-1.05***</td>
<td>-1.77***</td>
<td>-0.79***</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.24)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.19)</td>
<td>(0.04)</td>
<td>(0.06)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>$a_\pi$</td>
<td>0.61***</td>
<td>0.59***</td>
<td>0.27*</td>
<td>0.05</td>
<td>0.27***</td>
<td>0.23***</td>
<td>0.76***</td>
<td>0.42***</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.14)</td>
<td>(0.14)</td>
<td>(0.06)</td>
<td>(0.12)</td>
<td>(0.06)</td>
<td>(0.14)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>$a_y$</td>
<td>0.97***</td>
<td>0.88***</td>
<td>1.10***</td>
<td>1.04***</td>
<td>1.20***</td>
<td>0.88***</td>
<td>0.81***</td>
<td>0.99***</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.11)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>$a_M$</td>
<td>-0.01</td>
<td>0.03***</td>
<td>(0.00)</td>
<td>0.06***</td>
<td>(0.01)</td>
<td>0.12***</td>
<td>(0.02)</td>
<td>-0.44***</td>
</tr>
<tr>
<td>$a_{cr}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{cr, ind}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{cr, house}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_s$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{np}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$adj\ R^2$</td>
<td>0.82</td>
<td>0.82</td>
<td>0.93</td>
<td>0.95</td>
<td>0.94</td>
<td>0.96</td>
<td>0.78</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td>(0.79)</td>
<td>(0.86)</td>
<td>(0.87)</td>
<td>(0.81)</td>
<td>(0.73)</td>
<td>(0.76)</td>
<td>(0.35)</td>
</tr>
<tr>
<td>$J$-stat</td>
<td>0.09</td>
<td>0.10</td>
<td>0.11</td>
<td>0.11</td>
<td>0.12</td>
<td>0.11</td>
<td>0.10</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Notes: GMM estimates, *** denotes significance at the 10%/5%/1% level, standard errors in parentheses, for the $J$-statistic we put p-value in parentheses; abbreviations of the regressors follow those of Appendix A; Sample 1999M1-2007M1.

In order to check whether there has been a change in behavior of the ECB in the wake of the financial crisis, we have to compare the results of Table 4 to those of the estimates for the time after 2007M8. For this purpose, we display the results for the ECB during the subprime crisis in Table 5.

We find a significant increase vis-à-vis the “pre-crisis” period in the response to inflation in all specifications except for column 5.8 where the inflation coefficient also grows but the difference to column 4.8 is insignificant according to our Wald-tests. Whereas the inflation coefficient rises, the output gap coefficient drops within the crisis period. There is evidence that the ECB now even puts a significant negative weight on the output gap. For instance, this pattern even might indicate that in times of a crisis the ECB goes back to its mandate of fighting inflation as its preponderant goal.

Referring now to the “additional” variables, we find always highly significant estimates, as was also the case for the Fed in the “crisis period”. However, the sign and the magnitude of the responses differ in some cases from their empirical realizations before the crisis. For the credit measures (columns 4.3 to 4.5/5.3 to 5.5) we now find a significant increase, implying that the ECB reacts to the reduced credit growth with even stronger cuts in
the interest rate in contrast to the pre-crisis era where this reaction was less pronounced. This empirical finding is compatible with the view that the ECB tends to classify the reduced credit growth as a credit crunch rather than credit rationing. Remember that, on the contrary and according to our results in Table 3, the Fed appears to follow the interpretation of a credit rationing (columns 3.3 to 3.5). This finding is supported by the quote of ECB-president Trichet (2009) that “results do not point to a severe rationing of credit, although […] surveys of banks indicate that credit standards have been tightened”.

Table 5 - Taylor rule estimates for the ECB – the “subprime” period

<table>
<thead>
<tr>
<th></th>
<th>5.1</th>
<th>5.2</th>
<th>5.3</th>
<th>5.4</th>
<th>5.5</th>
<th>5.6</th>
<th>5.7</th>
<th>5.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>0.52*** (0.05)</td>
<td>1.61*** (0.44)</td>
<td>0.13 (0.25)</td>
<td>-2.31*** (0.12)</td>
<td>-0.59*** (0.14)</td>
<td>1.17*** (0.01)</td>
<td>0.40*** (0.01)</td>
<td>0.61 (0.03)</td>
</tr>
<tr>
<td>$a_\pi$</td>
<td>1.23*** (0.07)</td>
<td>1.10*** (0.02)</td>
<td>0.77*** (0.03)</td>
<td>0.83*** (0.03)</td>
<td>1.15*** (0.05)</td>
<td>0.66*** (0.02)</td>
<td>1.28*** (0.02)</td>
<td>0.57*** (0.05)</td>
</tr>
<tr>
<td>$a_y$</td>
<td>-0.35*** (0.05)</td>
<td>-0.12* (0.06)</td>
<td>-0.19*** (0.03)</td>
<td>-0.55*** (0.03)</td>
<td>-0.55*** (0.05)</td>
<td>-0.37*** (0.01)</td>
<td>-0.30*** (0.01)</td>
<td>-0.88*** (0.03)</td>
</tr>
<tr>
<td>$a_M$</td>
<td>(0.04)</td>
<td>0.05*** (0.01)</td>
<td>0.27*** (0.01)</td>
<td>0.24*** (0.02)</td>
<td>0.63*** (0.02)</td>
<td>-0.011*** (0.001)</td>
<td>0.26*** (0.02)</td>
<td>0.26*** (0.02)</td>
</tr>
<tr>
<td>$a_{cr}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{cr, ind}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{cr, house}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_i$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_s$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{np}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$adj R^2$</td>
<td>0.91</td>
<td>0.92</td>
<td>0.91</td>
<td>0.95</td>
<td>0.92</td>
<td>0.97</td>
<td>0.87</td>
<td>0.45</td>
</tr>
<tr>
<td>$J$-stat</td>
<td>0.28</td>
<td>0.32</td>
<td>0.29</td>
<td>0.31</td>
<td>0.32</td>
<td>0.30</td>
<td>0.26</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Notes: GMM estimates, ***/*** denotes significance at the 10%/5%/1% level, standard errors in parentheses, for the J-statistic we put the p-value in parentheses; abbreviations of the regressors follow those of Appendix A; Sample 2007M8-2009M6.

For stock price inflation (columns 4.7 and 5.7) we find a significantly negative estimated response coefficient. Hence, within the crisis the ECB seems to react to lower stock price inflation by increasing the policy rate. This result may be explained by the minor importance of the stock market in the euro area compared to the US where we found indeed a significantly positive coefficient of stock price inflation in the “crisis period” as opposed to a negative coefficient in the “pre-crisis” period (columns 2.7 and 3.7).

In contrast to stock prices, including house price inflation (columns 4.8 and 5.8) leads to the same conclusion as for the Fed, i.e. a significantly negative impact in the “pre-crisis” period and a significantly positive impact in the “crisis” period (columns 2.8 and 3.8). While
the ECB even accommodated house price inflation before the crisis, they actively tackle this measure in the downturn.

The negative reaction to the interest rate spread increases significantly within the crisis as we have expected it from theory (columns 4.6 and 5.6). However, the reaction is less pronounced than the one of the Fed in both periods (columns 2.6 and 3.6). Concerning money growth (columns 4.1 and 5.1) we are not able to find any significant difference of the estimates between the “pre-crisis” and the “crisis” period.

4.2.2 Testing for different degrees of interest rate smoothing before and after the start of the crisis

A final comparison enacted by us refers to the question whether the Fed and the ECB react less inertially during a crisis such as the subprime crisis as, for instance, suggested by Mishkin (2008, 2009). Therefore we insert an interest rate smoothing term into the regression equations and compare the interest rate smoothing parameter ($\rho$) in equation (9) for the two sample periods and central banks. We display our results in Table 6. However, in contrast to the preceding sections we refrain from drawing any conclusions with respect to the Taylor rule coefficients from these estimates since comparisons of these estimates might be biased by different degrees of interest rate smoothing. That is why we do not explicitly display the estimated Taylor rule coefficients (except the smoothing term) in Table 6.\(^{26}\)

Both central banks are characterized by a high degree of interest rate smoothing in the pre-crisis era. For the Fed there is only one specification where we find a smoothing coefficient of less than 0.95. Although the coefficients are on average not that large for the ECB there are still only three specifications with a smoothing coefficient of even less than 0.94. However, in the case of the ECB there is one outlier (0.38 in column 3, row 6). Nevertheless, we can conclude that before the subprime crisis the past interest rate was the best predictor for the present interest rate.

When looking at the crisis period, we find a significant decrease for the Fed in four out of eight specifications. This decrease is especially pronounced for the Taylor rules including commercial and industrial credit growth and house price inflation. But we are unable to detect a significant decrease in the Taylor rule without any “additional” variables which appears to be against the intuition of Mishkin (2008, 2009).

\(^{26}\) However, the results are available from the authors upon request.
Table 6 - Estimated interest rate smoothing terms in Taylor rules for the Fed and the ECB

<table>
<thead>
<tr>
<th></th>
<th>Fed</th>
<th>ECB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-Crisis</td>
<td>Crisis</td>
</tr>
<tr>
<td>TR</td>
<td>1.00***</td>
<td>0.92***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>$a_M$</td>
<td>0.98***</td>
<td>0.75***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>$a_{cr}$</td>
<td>0.96***</td>
<td>0.94***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$a_{cr,(comm),ind}$</td>
<td>0.86***</td>
<td>0.32***</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>$a_{cr,house}$</td>
<td>0.96***</td>
<td>0.94***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$a_i$</td>
<td>0.99***</td>
<td>0.86***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$a_s$</td>
<td>0.99***</td>
<td>0.79***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$a_{hp}$</td>
<td>0.98***</td>
<td>0.17***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.04)</td>
</tr>
</tbody>
</table>

Notes: GMM estimates, */**/*** denote significance at the 10%/5%/1% level, standard errors in parenthesis; TR = standard Taylor rule, remaining abbreviations indicate additional included variables in the Taylor rule as they are introduced in Appendix A.

For the ECB we even find in six out of eight specifications a significant decrease in the interest rate smoothing term as compared to the “pre-crisis” period. However, we also estimated a significant increase in the specification including industrial credit growth in which the coefficient within the crisis even exceeds unity. But since all other specifications point to a decrease in the degree of interest rate smoothing, our results support the view that the ECB and the Fed behaved according to the Mishkin principle, i.e. that they should behave less inertially during the financial crisis.

5. Conclusions

In this paper we have shown that the response of the Fed and the ECB to economic fundamentals, which are likely to have changed within the crisis starting in 2007, differs considerably. We use a Taylor rule specification including two explicitly modeled unobserved variables, i.e. the equilibrium real interest rate and potential output, which are estimated with a state-space approach. By means of this specific approach we are able to show that reaction coefficients for the two central banks differed only to a little extent before the crisis started. However, within the crisis the ECB tends to go back to its mandate of fighting consumer price inflation at the cost of some output losses. The Fed acts the other way round by promoting output and putting a smaller weight on suppressing consumer price inflation.
When it comes to the inclusion of “additional” variables we find a significant decrease in the Fed reaction to money growth in line with the less pronounced role of fighting inflation, while the reaction to this measure has not changed significantly for the ECB which is surprising because this result also leads to a less forward looking behavior of ECB concerning future inflation.

For the credit measures we observe a complete different reaction of both central banks. While the Fed reacts significantly negative to credit growth which is rational when there is credit rationing rather than a credit crunch the reverse is true for the ECB. This can also be explained by the larger importance of credit markets in the euro area than in the US. In the euro area credits are the most common refinancing opportunity for companies thus a credit crunch would have larger effects in these economy than in one where refinancing is done to a large extend by issuing stock like in the US.

This differing reactions can also be seen when looking at the change in the reaction to stock price inflation. While the Fed formerly even accommodated stock price inflation in the downturn stock price disflation is tackled actively while the ECB in the pre-crisis era did not react to this measure at all and within the crisis responds even negative. However, for the other asset price class observed within this paper, house price inflation, we find the same significant reaction for both central banks. They seem to have accommodated inflation of this kind before the crisis hit and fight it since the beginning of the subprime crisis actively.

For the last “additional” variable, the interest rate gap, we find for both central banks the expected negative response in all periods. Thus both have ever since reacted actively to risks in capital markets.

Besides all the differences mentioned above we were also able to show that both central banks react less inertial to fundamentals within the crisis according to the Mishkin principle (Mishkin 2008 and 2009).
References


Stark, J. (2009): Monetary Policy Before, During and After the Financial Crisis, Speech given at the University of Tübingen (Germany), November 11 2009.


Appendix A: Data sources

A.1.a: Real equilibrium interest rate (USA)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Measure</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>GDP</td>
<td>Bureau of Economic Analysis</td>
</tr>
<tr>
<td>interest rate</td>
<td>federal funds rate</td>
<td>OECD</td>
</tr>
<tr>
<td>inflation rate</td>
<td>year-on-year change in the personal consumption expenditure (PCE) price index (excluding food and energy)</td>
<td>Bureau of Economic Analysis</td>
</tr>
<tr>
<td>import price inflation (nonpetroleum)</td>
<td>year-on-year change of the price index for imports of nonpetroleum products</td>
<td>Bureau of Economic Analysis</td>
</tr>
<tr>
<td>import price inflation (petroleum)</td>
<td>year-on-year change in the price index for imports of petroleum and petroleum products</td>
<td>Bureau of Economic Analysis</td>
</tr>
</tbody>
</table>

A.2.a: Taylor rules (USA)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Measure</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>interest rate (i_us)</td>
<td>federal funds rate</td>
<td>OECD</td>
</tr>
<tr>
<td>inflation (π_us)</td>
<td>year-on-year change in consumer price index</td>
<td>OECD</td>
</tr>
<tr>
<td>output-gap (Y_us)</td>
<td>GDP subtracted by potential GDP calculated by state-space estimate</td>
<td>Bureau of Economic Analysis, OECD</td>
</tr>
<tr>
<td>money growth (M_us)</td>
<td>year-on-year change in monetary aggregate M2</td>
<td>OECD</td>
</tr>
<tr>
<td>credit growth (CR_us)</td>
<td>year-on-year change in overall bank credit</td>
<td>Federal Reserve</td>
</tr>
<tr>
<td>commercial and industrial credit growth (CR_COMM_IND_us)</td>
<td>year-on-year change in commercial and industrial loans</td>
<td>Federal Reserve</td>
</tr>
<tr>
<td>house credit growth (CR_HOUSE_us)</td>
<td>year-on-year change in real estate loans</td>
<td>Federal Reserve</td>
</tr>
<tr>
<td>interest rate spread (I_us)</td>
<td>difference 10 year treasury securities yields and 3-months yields</td>
<td>Federal Reserve</td>
</tr>
<tr>
<td>stock price growth (S_us)</td>
<td>year-on-year change in an index of all common stock listed on the NYSE</td>
<td>OECD</td>
</tr>
<tr>
<td>house price growth (HP_us)</td>
<td>year-on-year growth of the S&amp;P/Case-Shiller home price index</td>
<td>Standard and Poor’s</td>
</tr>
</tbody>
</table>
A.1.b: Real equilibrium interest rate (euro area)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Measure</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>GDP</td>
<td>ECB, AWM</td>
</tr>
<tr>
<td>interest rate</td>
<td>Eonia</td>
<td>ECB, AWM</td>
</tr>
<tr>
<td>inflation rate</td>
<td>year-on-year change in the</td>
<td>ECB, AWM</td>
</tr>
<tr>
<td></td>
<td>HICP</td>
<td></td>
</tr>
<tr>
<td>import price inflation</td>
<td>year-on-year change in the</td>
<td>ECB, AWM</td>
</tr>
<tr>
<td></td>
<td>import price deflator</td>
<td></td>
</tr>
<tr>
<td>oil price inflation</td>
<td>year-on-year change in oil</td>
<td>ECB, AWM</td>
</tr>
<tr>
<td></td>
<td>prices</td>
<td></td>
</tr>
</tbody>
</table>

A.2.b: Taylor rules (euro area)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Measure</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>interest rate (i_ea)</td>
<td>Eonia</td>
<td>ECB</td>
</tr>
<tr>
<td>inflation (π_ea)</td>
<td>year-on-year change in the HICP</td>
<td>ECB</td>
</tr>
<tr>
<td>output gap (Y_ea)</td>
<td>GDP subtracted by potential GDP calculated by state-space estimate</td>
<td>ECB</td>
</tr>
<tr>
<td>money growth (M_ea)</td>
<td>year-on-year change in monetary aggregate M3</td>
<td>ECB</td>
</tr>
<tr>
<td>credit growth (CR_ea)</td>
<td>year-on-year change in overall bank credit</td>
<td>ECB</td>
</tr>
<tr>
<td>industrial credit growth</td>
<td>year-on-year change in industrial loans</td>
<td>ECB</td>
</tr>
<tr>
<td>(CR_ IND_ea)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>house credit growth</td>
<td>year-on-year change in real estate loans</td>
<td>ECB</td>
</tr>
<tr>
<td>(CR_HOUSE_ea)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>interest rate spread (I_ea)</td>
<td>difference 10 year government bond yields and 3-month Euribor</td>
<td>ECB</td>
</tr>
<tr>
<td>stock price growth (S_ea)</td>
<td>year-on-year change in the Dow Jones EURO STOXX index</td>
<td>OECD</td>
</tr>
<tr>
<td>house price growth (HP_ea)</td>
<td>year-on-year growth of the house price index</td>
<td>Rees, Datastream</td>
</tr>
</tbody>
</table>
Appendix B: State-Space model

The signal equations are given by (see (1) and (2)):

\begin{align}
\pi_t &= c_\pi t_\pi t_{-1} + \frac{c_{\pi z}}{3} (\pi_{t-2} + \pi_{t-3} + \pi_{t-4}) + \frac{c_{\pi s}}{4} (\pi_{t-5} + \pi_{t-6} + \pi_{t-7} + \pi_{t-8}) \\
&\quad + c_{\pi N} (\pi_{t-1}^N - \pi_t) + c_{\pi o} (\pi_{t-1} - \pi_{t-1}) + c_y (y_{t-1} - \bar{y}_t) + \varepsilon_{2,t} \\
\end{align}

\begin{align}
y_t - \bar{y}_t &= b_{y_1} (y_{t-1} - \bar{y}_{t-1}) + b_{y_2} (y_{t-2} - \bar{y}_{t-2}) + \frac{b_r}{2} \sum_{i=1}^{2} (r_{t-i} - \bar{r}_{t-i}) + \varepsilon_{1,t}
\end{align}

Inserting (6) and using the Fisher equation with rational expectations yields:

\begin{align}
y_t - \bar{y}_t &= b_{y_1} (y_{t-1} - \bar{y}_{t-1}) + b_{y_2} (y_{t-2} - \bar{y}_{t-2}) \\
&\quad + \frac{b_r}{2} (i_{t-1} + i_{t-2} - \pi_t - \pi_{t-1} - g_{t-1} - g_{t-2} - z_{t-1} - z_{t-2}) + \varepsilon_{1,t}
\end{align}

Using the Phillips curve representation (A.1) in the IS-equation (A.2.1) and some rearranging leads to:

\begin{align}
y_t &= \bar{y}_t + \left( b_{y_1} + \frac{b_r}{2} c_y \right) y_{t-1} + \left( b_{y_2} + \frac{b_r}{2} c_y \right) y_{t-2} + \frac{b_r}{2} i_{t-1} + \frac{b_r}{2} i_{t-2} - \frac{b_r}{2} c_{\pi t} \pi_{t-1} \\
&\quad - \frac{b_r}{2} \left( \frac{c_{\pi z}}{3} + c_{\pi t} \right) \pi_{t-2} - \frac{b_r}{3} c_{\pi z} \pi_{t-3} - \frac{b_r}{3} c_{\pi z} \pi_{t-4} - \frac{b_r}{2} \left( \frac{c_{\pi s}}{4} + \frac{c_{\pi z}}{3} \right) \pi_{t-5} \\
&\quad - \frac{b_r}{4} c_{\pi s} \pi_{t-6} - \frac{b_r}{4} c_{\pi s} \pi_{t-7} - \frac{b_r}{4} c_{\pi s} \pi_{t-8} - \frac{b_r}{8} c_{\pi s} \pi_{t-9} - \frac{b_r}{2} c_{\pi N} (\pi_{t-1}^N - \pi_{t-1}) \\
&\quad - \frac{b_r}{2} c_{\pi N} (\pi_{t-2}^N - \pi_{t-2}) - \frac{b_r}{2} c_{\pi o} (\pi_{t-2} - \pi_{t-2}) - \frac{b_r}{2} c_{\pi o} (\pi_{t-3} - \pi_{t-3}) \\
&\quad - \left( b_{y_1} + \frac{b_r}{2} c_y \right) \bar{y}_{t-1} - \left( b_{y_2} + \frac{b_r}{2} c_y \right) \bar{y}_{t-2} - \frac{b_r}{2} g_{t-1} - \frac{b_r}{2} g_{t-2} - \frac{b_r}{2} z_{t-1} \\
&\quad - \frac{b_r}{2} z_{t-2} + \varepsilon_{1,t}
\end{align}
So the Model can be written as:

\[
\begin{bmatrix}
\gamma_t \\
\pi_t
\end{bmatrix}
= 
\begin{bmatrix}
\frac{b_y}{2} c_y & b_y + \frac{br}{2} c_y & b_r - \frac{br}{2} c_{\pi 1} - \frac{br}{4} c_{\pi 2} \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\gamma_t \\
\pi_t
\end{bmatrix}
\begin{bmatrix}
\frac{c_{\pi 3}}{3} - \frac{br}{3} c_{\pi 2} - \frac{br}{2} c_{\pi 3} - \frac{br}{2} c_{\pi 4} - \frac{br}{4} c_{\pi 3} - \frac{br}{4} c_{\pi 4} - \frac{br}{8} c_{\pi 3} \\
\frac{c_{\pi 4}}{4} - \frac{br}{2} c_{\pi N} - \frac{br}{2} c_{\pi N} - \frac{br}{2} c_{\pi N} - \frac{br}{2} c_{\pi N} - \frac{br}{2} c_{\pi N} - \frac{br}{2} c_{\pi N} - \frac{br}{2} c_{\pi N}
\end{bmatrix}
\cdot X_t
\]

\[
+ \begin{bmatrix}
1 & -b_y + \frac{br}{2} c_y & -b_y + \frac{br}{2} c_y & 0 & -\frac{br}{2} & 0 & -\frac{br}{2} & 0
\\
0 & -c_y & 0 & -\frac{br}{2} & 0 & 0 & 0 & 0
\end{bmatrix}
\cdot S_t
+ \begin{bmatrix}
\epsilon_{1,t} \\
\epsilon_{2,t}
\end{bmatrix}
\]

\[
= A'X_t + B'S_t
\]

With:

\[
X_t = 
\begin{bmatrix}
\gamma_{t-1} \\
\gamma_{t-2} \\
i_{t-1} \\
i_{t-2} \\
\pi_{t-1} \\
\pi_{t-2} \\
\pi_{t-3} \\
\pi_{t-4} \\
\pi_{t-5} \\
\pi_{t-6} \\
\pi_{t-7} \\
\pi_{t-8} \\
\pi_{t-9} \\
(\pi_{t-1}^N - \pi_{t-1}) \\
(\pi_{t-2}^N - \pi_{t-2}) \\
(\pi_{t-2}^O - \pi_{t-2}) \\
(\pi_{t-3}^O - \pi_{t-3})
\end{bmatrix}
\]

\[
\text{and} \quad S_t = 
\begin{bmatrix}
\tilde{\gamma}_t \\
\tilde{\gamma}_{t-1} \\
\tilde{\gamma}_{t-2} \\
g_{t-1} \\
g_{t-2} \\
\zeta_t \\
\zeta_{t-1} \\
\zeta_{t-2}
\end{bmatrix}
\]