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Quality Distortions in Vertical Relations

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Quality Distortions in Vertical Relations*

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January 2010

Abstract

This paper examines how delivery tariffs and private quality standards are determined in vertical relations that are subject to asymmetric information. We consider an infinitely repeated game where an upstream firm sells a product to a downstream firm. In each period, the firms negotiate a delivery contract comprising the quality of the good as well as a nonlinear tariff. Assuming asymmetric information about the actual quality of the product and focusing on incentive compatible contracts, we show that delivery contracts are more efficient the lower the firms’ outside options, i.e. the higher their mutual dependency. Buyer power driven by a reduced outside option of the upstream firm enhances the efficiency of vertical relations, while buyer power due to an improved outside option of the downstream firm implies less efficient outcomes.

JEL-Classification: D82, L14, L15

Keywords: Quality Uncertainty, Private Standards, Vertical Relations, Buyer Power

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1 Introduction

Before consumer goods reach final consumer markets, they typically undergo an extended transformation process based on the use of numerous inputs. The quality of consumer goods, thus, relies on the quality of all inputs that are used within the value chain. Besides faulty inputs inappropriate handling at each transformation stage can also cause severe product failures. Firms that can not effectively control for their suppliers’ quality run the risk of litigation\(^1\) and damage to their brand names and reputations. Moreover, various recent events show that the costs for late failure elimination can be substantial: DaimlerChrysler recalled about 1.3 million cars in order to check the battery control unit software for the electrical and braking systems as well as the voltage regulator in the alternator in 2005.\(^2\) Two years later, Mattel recalled about 18 million toys that were produced in China because of small dislodgeable magnets as well as toxic lead paint. More recently, the Chinese Melamine-scandal forced manufacturers such as Arla, Nestle and Cadbury to recall their products in a worldwide action.\(^3\) Quality agreements between the trading partners along the chain help to mitigate potential risks by complementing more basic public standards. Inter alia, they clarify quality specifications and how they are met and define each trading partners’ responsibilities. Though quality agreements are made in all manufacturing industries, they are particularly common in the food sector. The so called private quality standards can be either developed on a business-to-business (e.g. Global GAP, SQF100) or a business-to-consumer basis (e.g. Tesco’s Nature’s Choice, Carrefour’s Filière Qualité), whereas business-to-consumer standards play an increasingly important role.\(^4\)

Against this background, we examine how delivery tariffs and private quality standards are determined in vertical relations that are subject to asymmetric information. That is, the buyer

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\(^1\) Several product safety regulations like the Consumer Product Safety Act in the U.S. or the European Directive on General Product Safety (2001/95/EC) have imposed the duty on manufacturers and retailers to sell only products that are safe. Furthermore, there are also specific requirements that apply to specific types of products (e.g. food, pharmaceuticals, and medical devices).

\(^2\) http://www.dw-world.de/dw/article/0,"1543346,00.html


\(^4\) Tesco’s Nature Choice—the quality program of the UK’s largest retailer—was implemented in 1992 in order to ensure that fruits and vegetables are grown to high safety, quality and environmental standards. Likewise, Carrefour, Europe’s largest retailer, has developed a quality supply chain label guaranteeing that products meet a specified level of quality at each production stage. Meanwhile, this program has been applied to about 250 supply chains, covering more than 35,000 producers.
can neither observe the actual quality of the supplier’s product nor can she monitor the supplier’s production process itself. In order to capture the implied incentive problems in a long-term buyer-seller relationship, we consider an infinitely repeated game. At the beginning of each period, an upstream seller negotiates with a downstream buyer about a delivery contract that comprises delivery tariffs as well as quality requirements. The latter may refer to a quality agreement, which can be determined either jointly by both firms or unilaterally by the downstream firm. The product quality realizes anew in each period. Its value depends on various external factors like the workers’ mood, weather conditions or the infestation by pests in agricultural production. Thus, the actual quality is supposed to be random in each period.

By spending effort, the upstream firm reduces the danger of product damage. The more the upstream firm invests in smart and careful monitoring systems, the higher the probability of meeting the quality requirements specified in the delivery contract. In other words, despite the implementation of a monitoring system, the production process at the manufacturers’ level can run into failures. We assume that the actual quality can only be observed by the upstream firm. The downstream firm, however, neither observes the upstream firm’s effort in monitoring systems nor can she verify the actual quality of the product. This is due to the fact, that it may be prohibitively expensive to fully control the complete batch even though new developed testing technologies will provide better information at lower cost and in a shorter time.

Given this framework of hidden action and hidden information, we analyze incentive compatible delivery contracts which guarantee that the upstream firm truthfully reveals the quality of her product and that firms continue their business relationship. We show that both the delivery tariffs and the quality negotiated between the firms are efficient if and only if the firms’ outside options, i.e. the profits they could earn with alternative trading partners, are sufficiently low. Hence, the efficiency of delivery tariffs and quality decisions is only ensured if the mutual dependency between the vertically related firms is high enough. The higher the firms’ outside options, the more delivery tariffs and the target quality have to be distorted in order to satisfy the upstream firm’s incentive constraints. While wholesale prices are monotonically increasing in the firms’ outside options, the negotiated quality is a u-shaped function of the firms’ outside options. Key for these results is that the upstream firm’s incentive constraint prevents efficient risk sharing between the firms. To reduce the implied efficiency losses and to increase the up-
stream firm’s investment incentives, wholesale prices have to be distorted upwards. The higher the wholesale price the upstream firm gets if she complies with the quality requirements, the more the upstream firm will invest. Furthermore, the lower the quality requirements the more likely the upstream firm can meet the requirements by increasing her investment. Hence, as long as wholesale prices are only little distorted, the quality requirements are inefficiently low. However, highly distorted wholesale prices may well induce the firms to increase the quality requirements as the upstream firm’s investment incentives are increasing in the wholesale price. Correspondingly, high investment incentives implied by high wholesale prices allow the firms to increase the quality requirements without reducing the probability of meeting these requirements too much. Additionally, the higher the quality the higher are the downstream firm’s expected revenues. Hence, relatively high outside options can lead to upward distortions of both the wholesale prices and the negotiated qualities.

These results contribute to the large literature on contracting with imperfect information. Our assumption that the upstream firm’s investment are not observable corresponds to the classical principal agent models with moral hazard (see for example Holmstrom 1979 and the comprehensive analysis in Bolton and Dewatripont 2005). Whereas most of these models assume that outputs are observable and focus on optimal contracts with risk averse agents, our model builds on risk-neutral agents and repeated interactions. Furthermore, we assume that the downstream firm can not observe the actual quality of the upstream firm’s product. Delivery contracts have therefore to be based on the upstream firm’s announcement about the realized quality of the good. The implied incentive constraint for truthful revelation rests on repeated interactions and differs from the incentive constraints analyzed in hidden information models with heterogenous agents (see for example Baron and Myerson 1982). Considering the dynamic structure of our model, we assume that periods are independent and focus on stationary contracts (see Fudenberg et al. 1990). Our model thus combines the classical moral hazard problem with deviation incentives analyzed in the literature on reputation and collusion (see for example Milgrom and Roberts 1982, Schmidt 1993, and Feuerstein 2005).

Furthermore, our paper contributes to the literature on quality standards and buyer power. Quality standards have received growing attention in politics as well as in economics. So far,
however, most of the economic literature refers to public minimum quality standards. Only a small strand of literature focuses on private standards. Analyzing product differentiation and ignoring asymmetric information as well as quality uncertainties, Giraud-Héraud et al. (2003) show that the retailer’s incentive to differentiate their businesses via private labels are the higher the lower the public minimum standard. Taking asymmetric information into account, potential efficiency gains of private standards are discussed in OECD (2006). While private standards can improve the efficiency of the food system, they may also establish entry barriers and may thus lead to exclusionary effects. The wide literature on buyer power analyzes the sources of buyer power and its impact on the overall efficiency of vertical relations. Considering the sources of buyer power, credible threats to vertically integrate or to support market entry at the upstream level are analyzed by Katz (1987) and Sheffman and Spiller (1992). Inderst and Shaffer (2007) focus on potential delisting strategies after downstream mergers. More closely related to our paper are those articles tackling the efficiency effects of buyer power. Inderst and Wey (2003, 2007) point out that the formation of large buyers and thus the emergence of buyer power may increase consumer surplus as well as overall welfare since suppliers’ investment incentives increase. Montez (2008) shows that an upstream firm may choose higher capacities when buyers merge as long as the cost of capacity are sufficiently low. Negative welfare effects due to increased buyer power are analyzed by Inderst and Shaffer (2007). They show that a retail merger can induce the manufacturers to reduce the variety of their products in order to comply with "average" preferences (see also Chen 2004). Battigalli et al. (2007) find that buyer power weakens a supplier’s incentive to invest in quality improvements. Our results point out that the efficiency effects of increased buyer power strongly depend on the sources of buyer power. Buyer power due to lower outside options of the suppliers leads to more efficient contracts, while buyer power based on credible threats to vertically integrate or based on global sourcing strategies induces less efficient contracts.

5 Ronnen (1991) shows that a minimum quality standard which has been set by the government reduces differentiation among competitors and, thus, leads to more intense competition. Crampes and Hollander (1995) approve these results for variable costs of differentiation. These results are, however, only robust for price competition. If firms compete in quantities, minimum quality standards decrease overall welfare (Valletti, 2000). Furthermore, Ecchia and Lambertini (1997) show that minimum quality standards impede collusion.

6 It is, however, heavily debated whether private standards may harm suppliers, in particular smallholders in developing countries. For detailed case studies, see Balsevich et al. (2003) and Boselie et al. (2003).

7 For a survey of the sources and consequences of buyer power, see Inderst and Mazarotto (2008) as well as Inderst and Shaffer (2008).
The remainder of the paper is organized as follows: In Section 2 we specify our model. Section 3 focuses on the downstream firm’s pricing decisions as well as the upstream firm’s incentives to invest and to truthfully announce the quality of her product. In Section 4, we analyze the bargaining process in the intermediate good market and consider the quality choice of the downstream firm. Section 5 presents an example that illustrates our results. Finally, we conclude and discuss our findings.

2 The Model

We analyze an infinitely repeated game where an upstream firm $U$ sells a product $x$ to a downstream firm $D$ that distributes the product to final consumers. Both firms are supposed to be risk-neutral. At the beginning of each period firms $U$ and $D$ negotiate about a delivery contract that is supposed to be binding. The contract comprises a target quality $\bar{\theta}$ for product $x$ as well as a two-part delivery tariff $T$ consisting of a wholesale price $w$ and a fixed fee $F$. Focusing on quality uncertainty, we assume that the good’s quality $\theta$ is stochastically determined in each period. The upstream firm $U$, however, can increase the probability of complying with the target quality by investing some effort $e$. We further assume that neither the realization of $\theta$ nor the effort spent by the upstream firm can be observed by the downstream firm $D$. In order to avoid potential efficiency losses due to this kind of asymmetric information, firms have to rely on incentive compatible delivery contracts.

**Demand and Quality.** Good $x$ is assumed to be an experience good. For simplicity, we assume that consumers learn the product’s quality immediately after it is offered and adjust their demand accordingly. Alternatively, consumers may learn the product’s quality after having consumed it. Assuming that consumers buy sequentially and that they share their information, we obtain essentially the same results as long as each period is long enough.

We denote the demand for good $x$ by $X(p, \theta)$ where $p$ indicates the price. Demand is increasing in $\theta$, decreasing in $p$ and concave in both arguments, i.e. $X_p < 0 < X_\theta$ and $X_{pp}, X_{\theta\theta} < 0$.\(^8\) In each period the product’s quality $\theta$ is stochastically determined, where $\theta$ can be either high or low, i.e. $\theta \in \{\underline{\theta}, \bar{\theta}\}$ with $\underline{\theta} < \bar{\theta}$. The probability that good $x$ is of quality $\bar{\theta}$ is decreasing in

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\(^8\)Subscripts denote partial derivatives. In order to simplify the notation, we omit the arguments of the functions where this does not lead to any ambiguity.
$\bar{\theta}$, while it is increasing in the effort $e$ firm $U$ invests in careful production techniques, monitoring or quality assurance systems. Denoting $\rho(e, \bar{\theta})$ the probability that $\bar{\theta}$ realizes, we have

$$
\Pr(\theta = \bar{\theta} | e) = \rho(e, \bar{\theta}) \quad \text{and} \quad \Pr(\theta = \bar{\theta} | e) = 1 - \rho(e, \bar{\theta})
$$

with $\rho(0, \bar{\theta}) = 0$ and $\rho e < 0 < \rho e$, $\rho e < 0$ for all $e > 0$.

Additionally, we assume that effort induces increasing and convex costs of $c(e)$ per period with $c', c'' > 0$. After exerting effort, the upstream firm privately learns the realization of $\theta$.

**Tariffs.** Using the revelation principle, we focus on delivery tariffs which are contingent on the quality level $\hat{\theta}$ which the upstream firm $U$ announces after having observed the realized quality of the product. The delivery tariff $T(w, F, \hat{\theta})$ is given by

$$
T(w, F, \hat{\theta}) = \begin{cases} 
(w, F) & \text{if } \hat{\theta} = \bar{\theta} \\
(w, F) & \text{if } \hat{\theta} = \bar{\theta}
\end{cases}
$$

The downstream firm cannot directly observe the actual quality of good $x$ as she can not fully control the complete batch ex ante due to prohibitively high costs. However, the downstream firm is able to infer the actual quality from her demand as consumers learn the quality and adjust their demand accordingly. Thus, the downstream firm is able to detect untruthful announcements by observing her actual demand. Nevertheless she cannot verify untruthful announcements ex-post, as low qualities can also be caused by misconduct such as improper shipping or handling at the downstream level. While this implies that untruthful announcements cannot be punished contractually, the downstream firm is supposed to refrain from continuing the relation with the upstream firm $U$ once untruthful announcements has been detected. In this case the game continues such that both firms get their outside options.

**Expected profits.** To simplify the analysis, we normalize production costs at the upstream level as well as distribution and transformation costs at the downstream level to zero. Employing $T(w, F, \hat{\theta})$ and assuming truthful announcement, the expected per period profit of the upstream
The expected profit of the upstream firm $E\pi^U$ is given by

$$E\pi^U = \rho(e, \bar{\theta})\pi^U + (1 - \rho(e, \bar{\theta}))\bar{\pi}^U - c(e)$$  \hspace{1cm} (3)

with:

$$\pi^U = wX(p, \bar{\theta}) + F$$

$$\bar{\pi}^U = wX(p, \theta) + F.$$ 

In turn, the downstream firm’s expected per period profit $E\pi^D$ can be written as

$$E\pi^D = \rho(e, \bar{\theta})\pi^D + (1 - \rho(e, \bar{\theta}))\bar{\pi}^D$$  \hspace{1cm} (4)

with:

$$\pi^D = (p - w)X(p, \bar{\theta}) - F$$

$$\bar{\pi}^D = (p - w)X(p, \theta) - F.$$ 

**Outside options.** Let denote $\Gamma^U$ and $\Gamma^D$ the outside options of the upstream and the downstream firm, respectively. We assume that $\Gamma^U$, $\Gamma^D \geq 0$ and that $\Gamma^U$ as well as $\Gamma^D$ are exogenously given. The value of the outside option reflects the alternatives a firm has in the case of negotiation break-down. A decrease in the number of alternative trading partners at the upstream level reduces the downstream firm’s outside option, i.e. $\Gamma^D$, while a lower number of trading partners at the downstream level reduces the outside option of the upstream firm, i.e. $\Gamma^U$. The smaller $\Gamma^U$, the more the upstream firm depends on the downstream firm in order to get her products distributed to final consumers. This comes along with a strong gatekeeper position of the downstream firm. Note further, that $\Gamma^U$ does not depend on the realized quality. This is due to our assumptions that delivery contracts are negotiated before the quality realizes and that contracts are binding. We exclude the possibility that the upstream or the downstream firm can quit the contract after the quality has been realized.\(^9\)

In summary, we analyze an infinitely repeated game where the following four-stage game takes place in every period: First, firms $U$ and $D$ negotiate a target quality $\bar{\theta}$ and a menu of two-part tariffs which are contingent on the quality $\tilde{\theta}$ the upstream firm will announce. The upstream firm decides how much effort to spend and observes the realized quality in the second stage.

---

\(^9\)We consider explicit contracts and do not analyze the possibility that the negotiating parties can break the contractual agreement on the (contingent) delivery tariffs. This setting is in line with the traditional literature on incentive contracts but differs from relational incentive contracts in repeated games with moral hazard and adverse selection as analyzed in Levin (2003).
stage. Subsequently, she decides whether or not to report the actual quality truthfully. In the last stage of the game, given the upstream firm’s announcement, the appropriate delivery tariffs are selected. The downstream firm chooses her prices and finally demand as well as profits realize. The interaction between firms $D$ and $U$ ends if the downstream firm detects an untruthful announcement by observing that actual demand differs from expected demand. In this case, both firms get their outside option in all future periods. We focus on stationary contracts and solve the game by backward induction.

3 Prices, Announcement, and Effort

We begin our analysis by characterizing the optimal downstream prices for given two-part tariffs and an announced quality $\hat{\theta}$. We then solve the third stage of the game, where we determine the upstream firm’s incentives to announce the true realization of $\theta$. Subsequently, we consider the effort the upstream firm spends in order to enhance the probability of achieving the target quality. Delivery tariffs as well as target qualities negotiated by the firms will be analyzed in the next section.

**Downstream Prices.** In the final stage of the game, the downstream firm sets the price $p$ for product $x$. This decision is based on the quality the upstream firm has announced and the implied delivery tariff. Though the downstream firm can infer the actual quality from her demand ex post, we assume that she can not adjust her prices in the downstream market.\(^\text{10}\) Accordingly, the downstream firm’s optimal prices are determined by

\[
\max_p \left[ (p - w) \cdot X(\cdot, \hat{\theta}) - F \right]_{T(w,F,\bar{\theta})}. \tag{5}
\]

Let $\bar{p}(w, \bar{\theta})$ and $p(w, \theta)$ denote the solutions of (5) for $\hat{\theta} = \bar{\theta}$ and $\hat{\theta} = \theta$ respectively.

**Announcement.** After having observed the realized quality, the upstream firm announces the quality $\hat{\theta}$, which also determines the actual delivery tariff. Deciding whether or not to announce the realized quality truthfully, the upstream firm trades off her potential gains from deviating in the current period against the losses resulting from not trading with the downstream

\(^{10}\)Due to price announcements in leaflets or advertising brochures the downstream firm is committed to the price she has set initially. Thus, price adjustments are not possible in the short-run.
firm in all future periods. Denoting $\bar{E}\pi^U$ firm $U$’s expected continuation profit and $\delta > 0$ the discount factor, firm $U$’s incentive constraints for truthful announcements can be written as

$$IC_1 : \ wX + F + \frac{1}{\delta} \bar{E}\pi^U \geq \bar{w}X(\bar{p}, \bar{\theta}) + \bar{F} + \frac{1}{\delta} \Gamma^U \quad (6)$$

$$IC_2 : \ \bar{w}X + F + \frac{1}{\delta} E\pi^U \geq \bar{w}X(p, \bar{\theta}) + F + \frac{1}{\delta} \Gamma^U \quad (7)$$

with $\ : \ X := X(\bar{p}, \bar{\theta}), \ \bar{X} := X(p, \bar{\theta})$.

The upstream firm reports the actual quality of product $x$ truthfully if the incentive constraints (6) and (7) are satisfied.

**Effort.** Turning to the effort chosen by the upstream firm, we focus on the case where the incentive constraints (6) and (7) are satisfied.\(^{11}\) Employing (3), the supplier’s optimal effort $e^*(\cdot)$ is implicitly given by

$$\rho_e \pi^U - \rho_e \pi^U = c'(e) \iff \rho_e = \frac{c'(e)}{\Delta \pi^U} \ with \ \Delta \pi^U := \pi^U - \bar{\pi}. \quad (8)$$

For later reference, note further that (8) and simple comparative statics lead to

$$\text{sign} \frac{\partial e^*}{\partial \bar{w}} = \text{sign} \frac{d \Delta \pi^U}{d \bar{w}}. \quad (9)$$

Inspection of (9) shows that for given $\bar{F}$ and $\bar{F}$ as well as $\bar{w}$ sufficiently low, the effort level chosen by the upstream firm will increase in $\bar{w}$, i.e. $\partial e^*/\partial \bar{w} > 0$ as long as $\bar{w} < \bar{w}^k := \arg \max \bar{w}X$. In turn, the effort level reacts ambiguously in the target quality level $\bar{\theta}$. Since we have

$$\text{sign} \frac{\partial e^*}{\partial \bar{\theta}} = \text{sign} \left[ \rho_{e\bar{\theta}} \Delta \pi^U + \rho_e \frac{d \Delta \pi^U}{d \bar{\theta}} \right] \quad (10)$$

with $\rho_{e\bar{\theta}} < 0$, (10) implies that $\partial e^*/\partial \bar{\theta}$ is negative as long as $\Delta \pi^U$ is sufficiently high.

\(^{11}\)Since the equilibrium delivery conditions will be such that the incentive constraints are satisfied, we do not analyze the optimal effort when either $IC_1$ or $IC_2$ are violated.
4 Delivery Tariffs and Quality

In the first stage of the game both firms negotiate about a non-linear delivery tariff and a target quality. The firms aim at maximizing their joint profit when determining the delivery tariff. The joint profit is divided such that each party gets her disagreement payoff plus a share of the incremental gains from trade. Formalizing this approach, we apply the symmetric Nash bargaining solution.\(^\text{12}\) The Nash bargaining solution in each period is then given by

\[
\max_{T(w,F,\theta),\bar{\theta}} N = \left[ E\pi^D - \Gamma^D \right] \left[ E\pi^U - \Gamma^U \right].
\]

Wholesale prices as well as the target quality \(\bar{\theta}\) are determined in order to maximize the joint profit of both firms. Incremental gains from trade are shared by the fixed fees. Additionally, (11) implies that the upstream (downstream) firm’s bargaining position is the better the higher her outside option \(\Gamma^U\) (\(\Gamma^D\)).

In order to analyze both the negotiated target quality \(\bar{\theta}\) and the delivery tariff \(T(w,F,\theta)\), we first assume that the incentive constraints (6) and (7) are not binding. We will use this solution as a benchmark for the more complicated case where contracts have to ensure truthful announcement.

4.1 Unconstrained Bargaining

Assuming that (6) and (7) are not binding, we maximize (11) with respect to the tariffs \((F,\overline{w})\) and \((F,w)\) as well as \(\bar{\theta}\). The optimal tariffs \((w^*, F^*)\) and \((\overline{w}^*, \overline{F}^*)\) satisfy \(^\text{13}\)

\[
\begin{align*}
w^* &= \overline{w}^* = 0; \quad \overline{F}^* - F^* = \Delta \quad \text{and} \quad E\pi^D - \Gamma^D = E\pi^U - \Gamma^U, \\
\end{align*}
\]

where \(\Delta\) is defined as \(\overline{p}\overline{X} - \overline{p}\overline{X}\). Furthermore, (12) and the envelope theorem imply \(d\Delta/d\theta = \overline{p}\overline{X}_\theta\). Hence, the first order condition for the optimal target quality \(\overline{\theta}^*\) can be written as

\[
\frac{\partial N}{\partial \theta} = \rho \Delta + \rho \frac{d\Delta}{d\theta} = \rho \overline{p}\overline{X}_\theta + \rho \Delta = 0.
\]

\(^{12}\)For a non-cooperative foundation of the generalized Nash bargaining solution, see Binmore et al. (1986).

\(^{13}\)These results are derived in the proof of Proposition 1.
As expected, wholesale prices equal marginal costs implying undistorted monopoly prices in the downstream market. In turn, the fixed fees are used to divide the joint profit and to efficiently allocate the risk of getting a low quality. Since (12) leads to $\pi^D - \pi^U = 0$, any risk is fully borne by the upstream firm, which also implies that the upstream firm’s decision with respect to $e$ maximizes the expected joint profit of both firms (see (8)). Using these results, the optimal target quality $\bar{q}^*$, implicitly given by (13), maximizes the overall expected profits of both firms.

Thus, we get:

**Proposition 1** If the incentive constraints are not binding, the bargaining outcome is efficient.

**Proof.** See Appendix.

Proposition 1 confirms the well-known result that non-linear tariffs ensure an efficient outcome in a bilateral bargaining framework. The negotiating firms maximize their joint surplus by determining the delivery tariffs in order to guarantee efficient investment and pricing decisions. However, taking imperfect information about the upstream firm’s investments and the actual quality into account, efficiency can only be ensured if the upstream firm’s incentives constraints are satisfied. Employing (12), note first that (7) is satisfied as long as (6) holds. Moreover, focusing on the stationary solution, i.e. on $E\pi^U = E\pi^U$ and $E\pi^D = E\pi^D$, it is easy to show that (6) is equivalent to

\[ \Gamma \leq (\rho - 2\delta)\Delta + p\bar{X} - c(e^*) \]  

with \( \Gamma := \Gamma^U + \Gamma^D \).

Note that (14) does not rely on $\Gamma^U$ only. Given that total surplus is divided among both firms, incentive compatibility requires that the sum of both firms’ outside options is low enough. Efficient bargaining outcomes, thus, require that the total gains from trade, i.e. $E\pi^U + E\pi^D - (\Gamma^U + \Gamma^D) = \rho\Delta + p\bar{X} - c(e^*) - (\Gamma^U + \Gamma^D)$, are high enough while the discount rate $\delta$ must be low enough.

---

14 See (28) and (29) in the Appendix.
4.2 Constrained Bargaining

When (14) is binding, the negotiated delivery tariffs as well as the target quality $\theta$ have to be distorted in order to meet the upstream firm’s incentive constraints. More precisely, delivery tariffs and the target quality have to induce the upstream firm to reveal the actual quality of her product.

Analyzing the constrained bargaining problem, we get that the wholesale price $\bar{w}$ is strictly positive. Furthermore, the target quality is distorted in order to reduce efficiency losses due to inefficient effort decisions by the upstream firm.

Starting with (6), $F$ leads to truthful announcements as long as

$$F = wX + F - \bar{w}\hat{X} + \frac{1}{\delta} \left( \frac{\Delta \pi^U - \Gamma^U}{\delta} \right) \text{ with } \hat{X} := X(\bar{p}, \bar{q}).$$  \hspace{1cm} (15)

Employing (15), maximizing (11) with respect to $\bar{w}$, $w$ and $F$ and focusing again on the stationary solution, i.e. on $E\pi^D(\cdot) = E\pi^D$ and $E\pi^U(\cdot) = E\pi^U$, the optimal tariffs $(w^c, F^c)$ and $(\bar{w}^c, \bar{F}^c)$ satisfy $w^c = 0$, $E\pi^D = E\pi^U - \Gamma$ and

$$\rho \frac{d}{d\bar{w}} [\bar{p}X] + \rho_e \frac{d\pi^*}{d\bar{w}} (\Delta - \Delta \pi^U) = 0. \hspace{1cm} (16)$$

The optimal target quality $\bar{\theta}^c$ is implicitly given by

$$\left[ \rho \Delta + \rho \frac{d\Delta}{d\bar{\theta}} \right] + \rho_e \frac{d\pi^*}{d\bar{\theta}} (\Delta - \Delta \pi^U) = 0. \hspace{1cm} (17)$$

Compared to (12) and (13), (16) and (17) show that the optimal choice of $\bar{w}$ and $\bar{\theta}$ balances the efficiency losses due to inefficient wholesale prices and target qualities and their impact on the upstream firm’s investment decision. The implications for the upstream firm’s investment are captured by the second term on the RHS of (16) and (17), respectively, where $\rho_e (\Delta - \Delta \pi^U)$ measures the differences between joint marginal profits and the upstream firm’s marginal profits from increasing $e$. Furthermore, we get that the optimal delivery contract is such that the

\hspace{1cm} 15These results are derived in the proof of the next proposition.
following inequalities
\[
[\Delta - \Delta \pi^U] > 0, \quad \left[ \rho \sigma \Delta + \rho \frac{d \Delta}{d \bar{\theta}} \right] > 0 \quad \text{and} \quad \frac{\partial e^*}{\partial \bar{\theta}} < 0
\]  
(18)
hold. Combining (16)—(18) and restricting the analysis to \( \bar{w}^c < \bar{w}^k \), we obtain:

**Proposition 2**  *If the incentive constraint is binding, the bargaining solution is characterized by inefficiently low upstream investments and \( \bar{w}^c = 0 < \bar{w}^c \). Furthermore, for given \( \bar{w}^c \) and \( e^* \), the optimal target quality \( \bar{\theta}^c \) is inefficiently low.*

**Proof.** See Appendix. ■

Under unconstrained bargaining wholesale prices as well as the target quality maximize joint surplus, while the fixed fees are used to divide the joint surplus between the firms and to ensure efficient effort decisions by shifting all risk to the upstream firm. If the upstream firm’s incentive constraint is binding, one of these instruments has to be used to ensure truthful announcement. Hence, delivery tariffs, the allocation of risk and, thus, the upstream firm’s effort decisions become inefficient. In order to alleviate the implied inefficiencies and to increase the upstream firm’s effort, the optimal \( \bar{w}^c \) as well as the optimal target quality \( \bar{\theta}^c \) are distorted. While a positive \( \bar{w}^c \) induces higher effort (see (9)), the optimal target quality \( \bar{\theta}^c \) must be distorted downwards for given \( \bar{w}^c \) and \( e^* \) (see (10) and (18)). Note that \( \bar{w}^c > 0 \) causes a double mark-up problem inducing further efficiency losses.

Though Proposition 2 shows that both \( \bar{w}^c \) and \( \bar{\theta}^c \) are inefficient, the overall effect of the incentive constraint (14) on the target quality \( \bar{\theta}^c \) can be ambiguous. Restricting the analysis to linear demand \( X(p, \theta) \) with \( X_{pp} = X_{p\theta} = 0 \) and defining \( \Gamma^c \) as the highest \( \Gamma \) where the upstream firm’s incentive constraint is not binding, we obtain:

**Proposition 3**  *Assuming linear demand and starting with \( \Gamma = \Gamma^c \), an increase in \( \Gamma \) leads to a lower target quality, i.e. \( \bar{\theta}^c < \bar{\theta}^* \), as long as \( \rho_0 / \rho \) is decreasing in \( e \). With \( \Gamma > \Gamma^c \) an increase in \( \Gamma \) is the more likely to increase \( \bar{\theta}^c \) the more the wholesale price \( \bar{w}^c \) increases in \( \Gamma \).*

**Proof.** See Appendix. ■

The first part of Proposition 3 confirms Proposition 2 as \( \Gamma \) close to \( \Gamma^c \) leads to relatively low distortions with respect to \( \bar{w}^c \) and \( e^* \). Thus, in order to increase the upstream firm’s effort, the
target quality must be lower than in the case of unconstrained bargaining. A further increase in the firms’ outside options, i.e. $\Gamma > \Gamma^c$, results in higher distortions of the wholesale price and higher effort spent by the upstream firm. This allows the firms to increase the target quality $\overline{c}$ without reducing the probability of getting $\overline{c}$ too much. In fact, the example analyzed in the next section shows that a high outside option $\Gamma$ may well lead to $\overline{c} > \overline{c}^*$. So far, we have limited our analysis to the case where the downstream and the upstream firm decide jointly about the quality requirements. However, our results do not change if either the downstream firm or the upstream firm sets the quality requirements unilaterally. When deciding about the target quality, either firm anticipates that delivery tariffs are negotiated in order to maximize total industry profit. As this profit is split between the firms according to the Nash bargaining solution, each firm has an incentive to choose the target quality such that total industry profit is maximized. Hence, we have:

Corollary 1 *The choice of the target quality is the same irrespective of whether firms negotiate the target quality or whether the target quality is chosen unilaterally by one of the firms before negotiations about delivery tariffs take place.*

Corollary 1 shows that our analysis can be directly applied to private standards chosen by either the downstream or the upstream firm. In particular, private standards implemented by downstream retailers in order to ensure the quality assurance along the value chain tend to be more efficient the lower the upstream firm’s outside option. Note that this may be caused by the ongoing consolidation process in the retail markets and thus the increasing retailer’s gatekeeper control towards final consumer markets. Conversely, private standards tend to be inefficiently high if upstream firms have various alternative trading possibilities.

5 Example

In order to illustrate our results and to characterize the potential inefficiencies due to imperfect information in more detail, we now turn to a simple example. We use a standard quasi-linear utility function and focus on the comparative statics with respect to the firms’ outside options.
Consumers’ utility is given by

$$U(x, \theta) = \left(1 + \frac{1}{4}\sqrt{\theta}\right)x - \frac{1}{2}x^2 - px. \quad (19)$$

Differentiating $U(x, \theta)$ with respect to $x$, we obtain

$$X(p, \theta) = \begin{cases} 1 + \sqrt{\theta}/4 - p & \text{for } p \leq 1 + \sqrt{\theta}/4, \\ 0 & \text{otherwise}. \end{cases} \quad (20)$$

Furthermore, we normalize $\theta$ to zero and assume that the probability $\rho(e, \theta)$, the upstream firm’s costs $c(e)$ and the discount factor $\delta$ are given by

$$\rho(e, \theta) = \min \left\{ \frac{e}{1 + \theta}, 1 \right\}, \quad c(e) = \frac{e^2}{2} \text{ and } \delta = 0.1. \quad (21)$$

Note that (21) implies that $\rho_{\theta}/\rho$ does not depend on $e$.

Calculating $\overline{\omega}^c$ and $\overline{\theta}^c$ (see (12) and (13)) and analyzing the upstream firm’s incentive constraint (6), it is easy to show that there exists a critical value $\Gamma^c \approx 0.22$ such that (6) is binding for all $\Gamma = \Gamma^U + \Gamma^D > \Gamma^c$. We also find a value of $\Gamma$ above which total industry surplus is lower than the firms’ outside options. This corresponds to $\Gamma^m \approx 0.25$ indicating that no trade occurs for all $\Gamma = \Gamma^U + \Gamma^D > \Gamma^m$. Using (15)—(17), we obtain the graphs shown in Figure 1. If the incentive constraint is binding, i.e. $\Gamma > \Gamma^c$, a higher $\Gamma$ unambiguously increases the optimal wholesale price $\overline{\omega}^c$. However, the relation between $\Gamma$ and the optimal target quality $\overline{\theta}^c$ is not monotone. Starting from $\Gamma = \Gamma^c$, an increase in $\Gamma$ first reduces $\overline{\theta}^c$, while for higher values of $\Gamma$ the target quality $\overline{\theta}^c$ is finally increasing in $\Gamma$.

![Figure 1: Optimal $\overline{\omega}^c(\Gamma)$ and $\overline{\theta}^c(\Gamma)$](image)

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This non-monotonic relation confirms Proposition 3. It also shows that rather high outside options may well lead to inefficiently high target qualities, i.e. \( \theta^c > \theta^r \). Additionally, while higher outside options lead to more severe distortions and thus to lower overall industry profits, it is easy to show that each firm’s expected profit is increasing in her own outside option but decreasing in the other firm’s outside option. Finally, extending our results to the analysis of social welfare, we define expected social welfare \( EW \) as the sum of expected consumer surplus and firms’ profits, i.e.

\[
EW = \rho \left[ U(X, \theta) + pX \right] + (1 - \rho) \left[ U(X, \theta) + pX \right] - c(e^*). \tag{22}
\]

Figure 2 shows that \( EW \) is unambiguously decreasing in \( \Gamma \) for all \( \Gamma > \Gamma^c \). Obviously, positive wholesale prices \( \bar{w}^c \) as well as low target qualities \( \bar{\theta}^c \) reduce the firms’ expected profits as well as the expected consumer surplus. Although relatively high values of \( \Gamma \) may lead to higher target qualities, the implied distortions due to a high wholesale price \( \bar{w}^c \) and an inefficiently low effort level lead to lower expected social welfare. Thus, business strategies that enhance the firms’ trading alternatives may well reduce social welfare. This holds for global sourcing strategies which allow buyers to better replace their (established) suppliers as well as for certification decisions of suppliers. The latter is true as certification may increase the outside option a supplier has vis-à-vis her buyers by lowering her transactions costs when delivering alternative trading partners.

![Figure 2: Expected Welfare in \( \Gamma \) (EW(\( \Gamma \)))](image)

Our model also allows us to evaluate the welfare consequences of retailers’ increasing buyer power (OECD 1998, EC 1999). Relating retailer’s buyer power to her share in overall profits, buyer power in our model can be either caused by a high value of the buyer’s outside option or
by a low value of the supplier’s outside option.\textsuperscript{16} We find that the welfare effects of increasing buyer power crucially depend on its sources. If higher buyer power is caused by a diminished outside option of the supplier, contracts become more efficient and social welfare raises. Thus, as long as downstream consolidation reduces the supplier’s outside option, i.e. $\Gamma^U$, by limiting her trading alternatives, downstream consolidation and the implied increase in buyer power lead to more efficient contracts. In turn, if higher buyer power is based on an improved outside option of the retailer, i.e. $\Gamma^D$, contracts become less efficient and social welfare decreases. Additionally, buyer power tracing back to an improved outside option of the retailer may well lead to more stringent quality requirements as observed in the retail industry (OECD 2006).

So far, our discussion has neglected potential interactions between markets and, thus, competition effects on the different layers of the vertical structure. For example, globalization does generally not only affect the outside option of the downstream firms. It can also affect the upstream firms’ outside options as global sourcing may lead to stronger competition between upstream firms and may thus decrease their outside options. A priori, it is, therefore, not clear which effects dominate and how globalization and downstream consolidation affects negotiations in intermediate goods markets in terms of quality and tariffs.

6 Conclusion

We have analyzed a simple vertical structure with one upstream firm selling a good to a downstream firm over an infinite number of periods. Considering a framework of hidden action and hidden information and using the Nash bargaining solution, we have shown that high gains from trade lead to efficient qualities and delivery tariffs. With high outside options of the firms, incentive compatible contracts have to be distorted. While wholesale prices are inefficiently high, the negotiated target qualities depend non-monotonically on the firms’ outside options. Inefficiently high qualities can result whenever the firms’ outside options are high enough.

Applying these results to the analysis of buyer power, we find that large buyer power resulting from low outside options of suppliers leads to reduced wholesale and retail prices as well as to more efficient qualities. Similarly, relation specific investments by upstream firms can not only enhance the bargaining position of the downstream firm, it can also increase the efficiency of

\textsuperscript{16}We do not consider the exogenously given bargaining power of the negotiating parties.
the firms’ interaction. Conversely, a better outside option of the retailer—for example due to global sourcing (or private label) strategies—implies less efficient contracts in intermediate good markets. Thus, the impact of buyer power on the efficiency of delivery contracts crucially depends on the sources of buyer power.

We have limited our analysis to the case of binding contracts where firms are committed to adhere to the negotiated delivery tariffs. Turning to informal or relational contracts, the upstream as well as the downstream firm would have the possibility to renegotiate the delivery tariffs after the product’s quality has been realized. Potential renegotiation of delivery tariffs rules out any delivery tariff which is ex post inefficient. Correspondingly, delivery tariffs with wholesale prices above marginal costs can not be part of a stationary equilibrium contract. Combining these observations, we get that optimal informal or relational contracts may lead to even more distorted quality decisions. While endogenous outside options and high qualities increase the upstream firm’s incentive to deviate from the relational contract, ex post efficiency of delivery tariffs reduces the number of instruments the firms can use to provide the upstream firm with the right incentives to invest. These observations and our results concerning the relation between the optimal marginal delivery tariff and the target quality lead to the conjecture that relational contracts imply a monotone negative relation between the negotiated (target) qualities and the firms’ outside options.

Finally, our model does not reproduce quality failures and thus food scandals in equilibrium. Although there is a positive probability that the product’s quality undercut the target quality level, product recalls never occur as the upstream firm always informs the downstream firm truthfully about the actual quality. However, assuming that the firms are not fully informed about each other’s outside option or the discount factor, the upstream firm may have an incentive to deviate if her actual quality is low. In this case, false announcements by the upstream firm and, correspondingly, product failures become possible.

**Appendix**

**Proof of Proposition 1** Maximizing (11) with respect to the tariff \( w, F \) and \( w, F \), using the envelope theorem with respect to \( e^*(\cdot) \) and analyzing the respective first order conditions, we
first obtain

\[
\frac{\partial N}{\partial F} = 0 \iff \frac{\rho e \partial e^*}{\rho} \Delta \pi^D - \rho = -E \pi^D(\cdot) - \Gamma^D \quad (23)
\]

\[
\frac{\partial N}{\partial \bar{F}} = 0 \iff -\rho e \partial e^* \Delta \pi^D - (1 - \rho) = -E \pi^D(\cdot) - \Gamma^D \quad (24)
\]

with \( \Delta \pi^D := \pi^D - \bar{\pi}^D \). (23), (24) and \( \partial e^*/\partial F = -\partial e^*/\partial \bar{F} \) imply that we must have

\[
\Delta \pi^D = 0 \quad \text{and} \quad E \pi^D(\cdot) - \Gamma^D = E \pi^U(\cdot) - \Gamma^U. \quad (25)
\]

Using (25), the first order conditions with respect to \( \bar{w} \) and \( \underline{w} \) can be written as

\[
\frac{\partial N}{\partial \bar{w}} = \rho\bar{w} \frac{\partial \bar{X}}{\partial \bar{p}} \frac{\partial \bar{p}}{\partial \bar{w}} = 0; \quad \frac{\partial N}{\partial \underline{w}} = (1 - \rho)\underline{w} \frac{\partial \underline{X}}{\partial \underline{p}} \frac{\partial \underline{p}}{\partial \underline{w}} = 0, \quad (26)
\]

which leads to \( \bar{w}^* = \underline{w}^* = 0 \). Employing these results, we also get

\[
\frac{\partial N}{\partial \bar{\theta}} = \rho \frac{d\Delta}{d\bar{\theta}} + \rho \sigma \Delta = \rho \bar{p} \bar{X} + \rho \sigma \Delta = 0 \quad (27)
\]

where the last equality follows from using the envelope theorem with respect to \( \bar{p} \). Finally, solving for \( \bar{F}^* \) and \( \underline{F}^* \), we obtain

\[
\bar{F}^* = \frac{1}{2} \left[ (2 - \rho)\bar{p} \bar{X} - (1 - \rho)p \underline{X} + c(e^*) + \Gamma^U - \Gamma^D \right] \quad (28)
\]

\[
\underline{F}^* = \frac{1}{2} \left[ (1 + \rho)p \bar{X} - \rho \bar{p} \bar{X} + c(e^*) + \Gamma^U - \Gamma^D \right]. \quad (29)
\]

**Proof of Proposition 2** Using (15) note first that the firms’ profits for \( \theta = \bar{\theta} \) and \( \theta = \underline{\theta} \) can be written as

\[
\bar{\pi}^D = (\bar{p} - \bar{w})\bar{X} - \bar{w} \bar{X} - \bar{F} + \bar{w} \bar{X} - \frac{1}{\delta} \left( E \bar{\pi}^U - \Gamma^U \right) \quad (30)
\]

\[
\underline{\pi}^D = (\underline{p} - \underline{w})\underline{X} - \underline{F} \quad (31)
\]

\[
\bar{\pi}^U = \bar{w} \bar{X} + \bar{w} \bar{X} + \bar{F} - \bar{w} \bar{X} + \frac{1}{\delta} \left( E \bar{\pi}^U - \Gamma^U \right) \quad (32)
\]

\[
\underline{\pi}^U = \underline{w} \underline{X} + \underline{F}. \quad (33)
\]
Employing (30)—(33) and considering first $F$ and $w$, we obtain

\[
\frac{\partial N}{\partial F} = 0 \iff E\pi^D(\cdot) - \Gamma^D = E\pi^U(\cdot) - \Gamma^U \quad (34)
\]

\[
\frac{\partial N}{\partial w} = (1 - \rho) \frac{d}{dw} [pX] = 0 \Rightarrow w^* = 0. \quad (35)
\]

Turning to $w$, solving (34) for $F^c$, using $E\pi^D(\cdot) = E\pi^D$ and $E\pi^U(\cdot) = E\pi^U$ as well as (8), the first order condition for $w^c$ can be written as

\[
\frac{\partial N}{\partial w} = \rho_e \frac{\partial e^*}{\partial w} [\Delta - \Delta\pi^U] + \rho \frac{d}{dw} [pX] = 0, \quad (36)
\]

where $\Delta\pi^U$ is given by (recall $\Gamma := \Gamma^U - \Gamma^D$)

\[
\Delta\pi^U = w(X - \hat{X}) + \frac{1}{2\delta} [\rho\Delta + pX - c(e^*) - \Gamma]. \quad (37)
\]

To prove the proposition, note first that $d/dw [pX] = 0$ for $w = 0$ and $d/dw [pX] < 0$ for all $w > 0$. Furthermore, using (9), we obtain

\[
sign \frac{\partial e^*}{\partial w} = sign \frac{d}{dw} \left[ w(X - \hat{X}) \right] > 0 \text{ as long as } w < w^k. \quad (38)
\]

Considering the sign of $\Delta - \Delta\pi^U$ and assuming $\Delta - \Delta\pi^U \leq 0$, (36) and (38) imply $w = 0$. Additionally, we get

\[
\Delta - \Delta\pi^U \leq 0 \iff \Gamma \leq (\rho - 2\delta)\Delta + pX - c(e^*) \text{ for } w = 0, \quad (39)
\]

which contradicts the assumption that (14) is binding. Hence we must have $\Delta - \Delta\pi^U > 0$ and therefore inefficient risk sharing as well as $w^c > 0$.

Turning to $\theta$ and using the envelope theorem with respect to $e^*$, the optimal target quality $\bar{\theta}^c$ is implicitly given by

\[
\frac{dN}{d\theta} = \rho_{\theta} \Delta + \rho_e \frac{\partial e^*}{\partial \theta} [\Delta - \Delta\pi^U] + \rho \frac{d\Delta}{d\theta} = 0. \quad (40)
\]
Employing (40) and $\Delta - \Delta \pi^U > 0$, we get
\[
\text{sign} \frac{\partial e^*}{\partial \bar{\theta}} = -\text{sign} \left( \rho_0 \Delta + \rho \frac{d\Delta}{d\bar{\theta}} \right). \tag{41}
\]
To determine the sign of the RHS of (41), note first that (40) leads to
\[
\frac{\partial e^*}{\partial \bar{\theta}} = -\frac{\rho_0 \Delta + \rho \frac{d\Delta}{d\bar{\theta}}}{\rho_e (\Delta - \Delta \pi^U)}. \tag{42}
\]
Substituting $\partial e^*/\partial \bar{\theta}$ and solving (42) for $\rho_0$, we get
\[
\rho_0 \leq 0 \Rightarrow \rho_e \leq \Psi := -\frac{1}{\rho_e \Delta \pi^U} \left[ -\frac{\rho \frac{d\Delta}{d\bar{\theta}}}{\Delta - \Delta \pi^U} \Theta + \rho_e^2 \frac{\bar{X}}{p} \bar{\pi} \right] \tag{43}
\]
with $\Theta := \frac{d}{de} \left[ \rho_e \pi^U - \rho_e \Delta \pi^U - c'(e) \right] < 0$.

Furthermore, differentiating $\partial e^*/\partial \bar{\theta}$ partially with respect to $\rho_e$, we get
\[
\frac{\partial}{\partial \rho_e} \left[ \frac{\partial e^*}{\partial \bar{\theta}} \right] = -\frac{\Delta \pi^U}{\Theta} > 0. \tag{44}
\]
Finally, evaluating $\partial e^*/\partial \bar{\theta}$ at the critical level $\Psi$, we have
\[
\left. \frac{\partial e^*}{\partial \bar{\theta}} \right|_{\rho_e = \Psi} = -\frac{\rho \frac{d\Delta}{d\bar{\theta}}}{\rho_e (\Delta - \Delta \pi^U)} < 0. \tag{45}
\]
Turning back to (41), we thus have $\rho_0 \Delta + \rho \frac{d\Delta}{d\bar{\theta}} > 0$, which completes the proof.

**Proof of Proposition 3** To prove the first part of the proposition, note first that $\Gamma = \Gamma^c$ implies $\bar{w}^c = 0$ and that (17) reduces to (13). Furthermore, we have
\[
\frac{\partial^2 N}{\partial \bar{\theta} \partial \Gamma} = \frac{\rho_e \partial e^*}{2\delta \partial \bar{\theta}} < 0 < \frac{\rho_e \partial e^*}{2\delta \partial \bar{w}} = \frac{\partial^2 N}{\partial \bar{w} \partial \Gamma}, \tag{46}
\]
where the first inequality in (46) follows from the continuity of $\partial e^*/\partial \bar{\theta}$ and (43)–(45). Using
(46) and linear demand, simple but tedious calculations yield

\[
\begin{align*}
\text{sign} \frac{\partial \theta^c}{\partial \Gamma} \bigg|_{\Gamma = \Gamma^c} &= \text{sign} \left[ \frac{\partial^2 N}{\partial \bar{\theta} \partial \bar{\theta}} \frac{\partial^2 N}{\partial \bar{\omega} \partial \bar{\Gamma}} - \frac{\partial^2 N}{\partial \bar{\theta} \partial \bar{\Gamma}} \frac{\partial^2 N}{\partial \bar{\omega}^2} \right]_{\Gamma = \Gamma^c} \quad (47) \\
&= \text{sign} \left[ 2 \rho_1 \bar{X} \left( \frac{\partial \epsilon^*}{\partial \bar{\omega}} \right)^2 + \frac{\partial \epsilon^*}{\partial \bar{\theta}} \left( 2 \frac{\partial \epsilon^*}{\partial \bar{\omega}} \left[ \bar{X} - \bar{\tilde{X}} \right] - \rho \bar{X}_p \right) \right]_{\Gamma = \Gamma^c}. \quad (48)
\end{align*}
\]

Substituting \( \partial \epsilon^*/\partial \bar{\theta} \) and \( \partial \epsilon^*/\partial \bar{\omega} \) into (48) and evaluating at \( \Gamma = \Gamma^c \), we obtain

\[
\rho_{e\theta} \Delta + \rho_{e \Delta} \leq 0 \Rightarrow \text{sign} \frac{\partial \theta^c}{\partial \Gamma} \bigg|_{\Gamma = \Gamma^c} < 0, \quad (49)
\]

which together with \( \rho_{e\Delta} + \rho \Delta \theta = 0 \) leads to \( \bar{\theta}^c < \bar{\theta}^* \) for \( \Gamma > \Gamma^c \) but \( \Gamma \) small enough. To prove the second part, assume that the incentive constraint (14) is binding. Then we must have

\[
\Gamma = \rho \Delta + \rho \bar{X} + 2 \delta \bar{\omega}^c(\bar{X} - \bar{\tilde{X}}) - c(\epsilon^*) - 2 \delta \frac{c'(\epsilon^*)}{\rho_e}. \quad (50)
\]

Defining

\[
\Phi(\bar{\theta}^c(\Gamma), \bar{\omega}^c(\Gamma), \Gamma) := \rho \Delta + \rho \bar{X} + 2 \delta \bar{\omega}^c(\bar{X} - \bar{\tilde{X}}) - c(\epsilon^*) - 2 \delta \frac{c'(\epsilon^*)}{\rho_e}, \quad (51)
\]

applying the implicit function theorem, differentiating (51) totally with respect to \( \Gamma \), substituting the equilibrium values of \( \partial \epsilon^*/\partial \bar{\omega} \) and \( \partial \epsilon^*/\partial \bar{\theta} \) (see (40) and (36)), we obtain

\[
\frac{d \bar{\theta}^c}{d \Gamma} = \frac{1}{\Phi} - \frac{\rho}{2} \left[ \frac{d \bar{X}}{d \bar{\theta}} \right] \frac{d \bar{\omega}^c}{d \Gamma} \quad \text{with} \quad \Phi := \frac{\partial \Phi}{\partial \bar{\theta}} + \frac{\partial \Phi}{\partial \epsilon^*} \frac{\partial \epsilon^*}{\partial \bar{\theta}}, 
\]

where we have again used \( X_{pp} = X_{p \theta} = 0 \). Since \( \frac{d \bar{X}}{d \bar{\theta}} \left[ \frac{d \bar{X}}{d \bar{\theta}} \right] / d \bar{\theta} < 0 < \rho \Delta + \rho \Delta / d \bar{\theta} \), (52) shows that \( \bar{\theta}^c \) is more likely to increase in \( \Gamma \) the higher \( d \bar{\omega}^c / d \Gamma \).

References


