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## The Partitioning of Constraints

by

HORST SIEBERT

Constraints define the permissible set of values of variables in the economic system. Such restrictions arise for a number of causes. *Technological* constraints are given by technological relationships such as the production function. A *resource* restriction describes the limited availability of factors of production for the existing competing uses. *Financial* constraints relate to consistencies in received income and spending such as cash-flow considerations or the budget constraints for the individual household, the government or the economy (balance of payments equilibrium). *Institutional* constraints originate from organizational structures and rules of procedures in formal organizations. *Normative* constraints may be introduced by informal “ethical” value judgments, by political decisions and the creation of rules by the judicial system. Finally, *consistency* conditions require the consistency of autonomous decisions in a complex system such as the equality of supply and demand.

The above distinction of types of constraints is not clear-cut. Technological constraints such as the production function are influenced by institutional rules (labor regulations) and normative considerations (environmental regulation). Consistency conditions may be affected by institutional rules (disequilibrium in the labor market). The constraints may relate to different agents of the economic system so that we may envision the economic system as a net of hierarchical constraints with each layer of constraints applying to a more disaggregated level of economic agents.

Each economic system is confronted with the problem of transforming higher-level types of constraints into constraints for lower-level agents. For instance, the balance-of-payments equilibrium must be transformed into a set of constraints for all agents involved in international transactions. One approach to transforming higher-level types of constraints into restrictions for individual agents is *ad hoc* partitioning which will be studied in Section 1. We will show that *ad hoc* partitioning makes the constraints “more powerful”; that is, it will increase the opportunity costs of constraints. In Section 2, some examples of the *ad hoc* partitioning of constraints are given. The final section reverses the argumentation and addresses the problem of aggregating constraints. Some policy conclusions are pointed out.

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### 1. *Ad Hoc Partitioning*

Ad hoc partitioning can be defined as the breaking down of a higher-level constraint into a set of lower-level constraints by such criteria as historical experience, given patterns of use, technological conditions, a "first-come, first-served" rule, equity considerations, or administrative practicability. Note that ad hoc partitioning is here understood as the result of a political process or of an administrative decision. We distinguish two types of partitioning, namely a quantitative partitioning and a qualitative partitioning.

*Quantitative Partitioning.* In this case, a quantitative constraint is given that is broken down into quantitative constraints for lower-level agents. Consider, for example, an economic system being confronted with a new constraint  $z \leq \bar{z}$  where  $\bar{z}$  may either be a reduced availability of a natural resource such as oil or a new normative restraint being introduced by legislation such as that defining a minimum environmental quality (or maximum amount of tolerable emissions). Then this new constraint may be broken down into a set of constraints for the individual agents according to some ad hoc criteria such as past use of the resource or a percentage reduction of emissions starting from a given pollution level. The constraint is broken down in such a fashion that  $\sum_i z_i = z \leq \bar{z}$  so that all the individual constraints satisfy the higher-level constraint<sup>1</sup>.

*Qualitative Partitioning.* In the above case, lower-level constraints add up to the overall constraint. Another type of partitioning arises when the constraint variable  $z$  is determined by a set of other variables  $x, y$  and the functional relationship may not be known:

$$z = f(x, y) \leq \bar{z}.$$

Assume for instance a constraint that economic welfare should not be too different for the regions of a nation. Since the welfare functions are not known, this constraint cannot be specified. In this case, the equity constraint may be defined in terms of the independent variables. For instance, the constitution of the Federal Republic of Germany requires the "similarity of living conditions between regions" which applies to the provision of public goods. Another example is the overall normative constraint "freedom" which implies that protected spheres (basic rights) have to be assigned to individuals. This constraint cannot be considered as being reached by a quantitative partitioning of an overall constraint.

Whereas in the case of quantitative constraints the problem of consistency of partitioning does not arise<sup>2</sup>, qualitative partitioning may lead to inconsistencies. For instance, the allocations of rights may not be feasible if the protected sphere of one individual contradicts the protection of another individual. We

<sup>1</sup> Cf. "point" rationing (SAMUELSON [1947], p. 165).

<sup>2</sup> When more than one agent undertakes the partitioning and when more than one overall constraint is considered, the problem of consistency may very well arise even in the case of quantitative partitioning.

here neglected this aspect of feasibility or consistency as discussed in the literature on the allocation of rights<sup>3</sup>.

The ad hoc partitioning of constraints seems to be a widely used procedure in government activity. If a country is faced with a reduced availability of oil, quantitative rationing according to some criteria such as past use may be proposed. A shortage of housing may lead to some governmental allocation mechanism allocating housing according to the number of members in the family, age, merit or other criteria. If there is a shortage of foreign currency, i.e. a deficit in the balance of payments, the convertibility of a currency may be reduced or import restrictions may be implemented. Environmental protection may be undertaken by a permit system with emission norms for each facility and abatement technology specified according to the state of art. Permits to open up a well for ground water often are given on a "first-come, first-served" basis, for instance in West Germany. Limitations on the number or size of ships or gear for fishing, on the fishing grounds and seasons, and on the volume to be fished are another example. Permits to enter a market (pharmacies in Europe) may be defended by an overall objective such as protecting the consumer. Zoning laws and prohibitive measures to locate in an agglomerated area are further examples. Partitioning has been proposed for creating an Anti-OPEC demand cartel where the limited total quantity of permissible demand must be partitioned to the members of the cartel. Finally, in the case of overproduction for the European agricultural market, quantity restrictions for the individual producer are now being discussed.

Constraints express the phenomenon of scarcity. There are competing uses for a good (in a broad interpretation), and ad hoc partitioning represents one approach to the allocation of a limited quantity to the competing uses. We now can establish an important property of ad hoc partitioning.

*Proposition:* Ad hoc partitioning of a constraint into a set of lower-level constraints makes the constraint more powerful. Alternatively we can state: Ad hoc partitioning of a constraint will increase the opportunity costs of the constraint.

## 2. Three Examples of Ad Hoc Partitioning

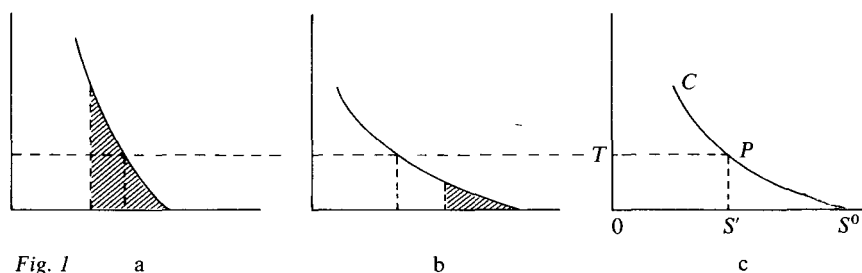
We will prove the above proposition intuitively by considering three examples<sup>4</sup>.

(1) *Air Quality Regulations.* Assume that a quality standard for an environ-

<sup>3</sup> Cf. SEN [1976, 1979].

<sup>4</sup> A more formal proof of our proposition would have to determine the ramifications of ad hoc partitioning in an optimization model or in a general equilibrium framework. In such an analysis we would have to study the effects of partitioning on the individual household, on firms and on market processes. For this analysis, however, a static equilibrium model would not be sufficient since partitioning is especially interesting in a dynamic setting. On the effects of partitioning on the household cf. SAMUELSON [1947], TOBIN and HOUTHAKKER [1950-51], HOWARD [1977], LATHAM [1980], MACKAY and WHITNEY [1980].

mental medium such as air is introduced so that the tolerable quantity of emissions is fixed by a political decision at  $OS'$  in Figure 1c. The initial level of emissions is denoted by  $OS^0$ . The quantity  $S^0S'$  has to be abated. The curve  $S^0C$  denotes marginal abatement costs measured from  $S^0$ . The marginal abatement costs for the economy are aggregated from the marginal abatement cost curve of the firms, in our example of two firms only.

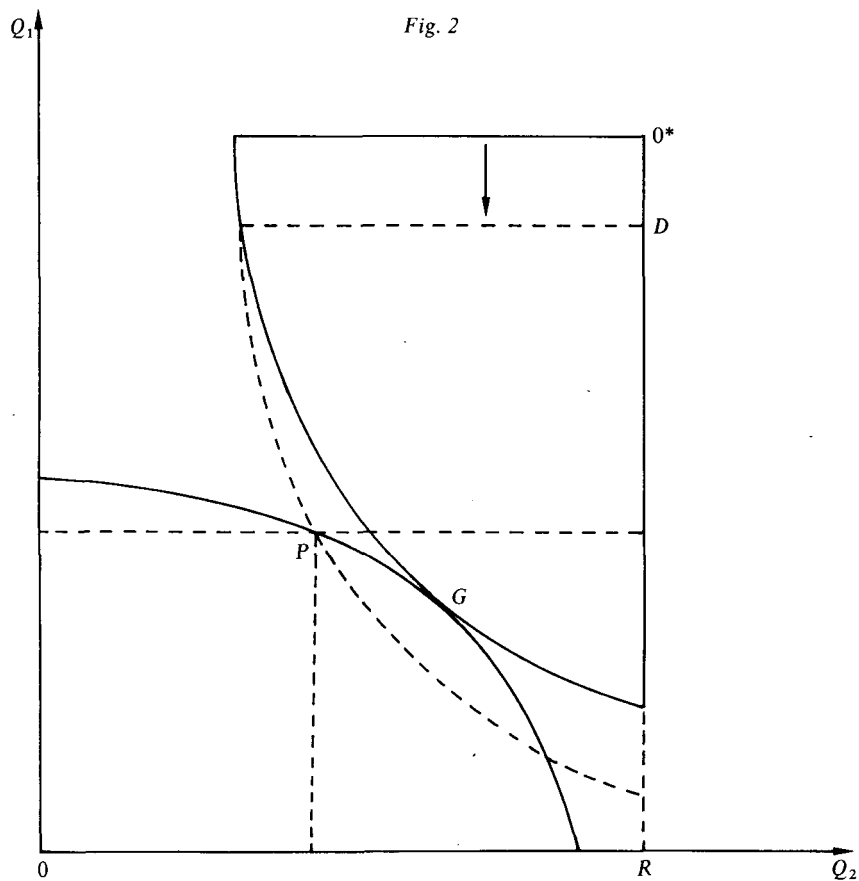


Assume now that the quantity of emissions to be abated is allocated to the firms according to their level of pollution in the initial situation. Then the shaded areas indicate the abatement costs of the economy. Note that these shaded areas are larger than the triangle  $PS'S^0$ . Ad hoc partitioning has increased the costs of abatement. By ad hoc partitioning, the economy loses more resources for other opportunities than are required to satisfy the overall environmental constraint.

If one envisions the environmental allocation problem as being illustrated by a three-dimensional transformation space (SIEBERT [1981a]), the ad hoc partitioning will move the allocation point inside the transformation space for a given environmental quality.

Environmental quality laws (Bundesimmissionsschutzgesetz and TA-Luft in West Germany, the Clean Air Act in the United States) use a more complicated ad hoc partitioning, having, however, a similar effect of unnecessarily augmenting the opportunity costs of environmental protection. The quality target is transformed via a permit system where permits have to be granted if the state of art is used in abatement and if the quality target is not violated. The costs of this "first-come, first-served" principle can be seen when no additional permits can be given out in non-attainment regions.

(2) *Guaranteeing Minimum Output.* Assume a world of two countries and a scenario where the two countries plan the international division of labor as in the Comecon. Let the constraint be that in both countries together  $d:0R$  units of commodity 2 must be produced. We know that the Koopmans-efficient allocation is given by point  $G$ , that is, the tangential solution to the two transformation curves.



Assume now that this solution is not known to the planners and that they split up the overall constraint into two separate constraints  $a$  and  $a^*$  so that

$$Q_2 = a$$

$$Q_2^* = a^*$$

$$a + a^* = d$$

where  $Q_2$  is the quantity of commodity 2 produced in country I and  $Q_2^*$  in country II. Let  $Q_2^W$  indicate the quantity produced in both countries together. This partitioning guarantees that the overall constraint  $Q_2^W = d$  is satisfied. But by partitioning, the constraint has become more powerful: The production of commodity 1 is reduced by  $0^*D^5$ .

<sup>5</sup> A more realistic case of partitioning arises when we assume that each nation wants to protect its import substitute sector, for instance by guaranteeing a minimum amount of production (and employment).

(3) *Welfare Constraints.* In Figure 3 ad hoc partitioning relates to a qualitative constraint. Consider two regions I and II and assume regional transformation functions between the private goods 2 and the public good 1. Assume that the regional transformation curves differ for a number of reasons such as production conditions, resource availability with some immobility of resources, etc. Let  $AA$  represent the transformation curve of region I and let  $BB$  denote the transformation curve of region II. Assume for simplifying purposes that both regions have an identical welfare function. Then the target of providing the same level of welfare for both regions would require the solutions  $P_I$  and  $P_{II}$ .

In reality, however, the welfare function is not known. Therefore, a constraint is formulated with respect to one of the determinants of welfare such as public good 1. This approach has been followed in the constitution of the Federal Republic of Germany where the "similarity of living conditions" is required relating to the provision of public goods. Relative to a position of perfect information on a welfare function, the partitioning of the constraint clearly reduces welfare. In Figure 3 a situation is shown where welfare in both regions is reduced.

A similar problem arises in the case of the regionalization of environmental policy. If the overall equity constraint requires identical welfare in both regions, the constraint does not have to be broken down. Then one region may have a

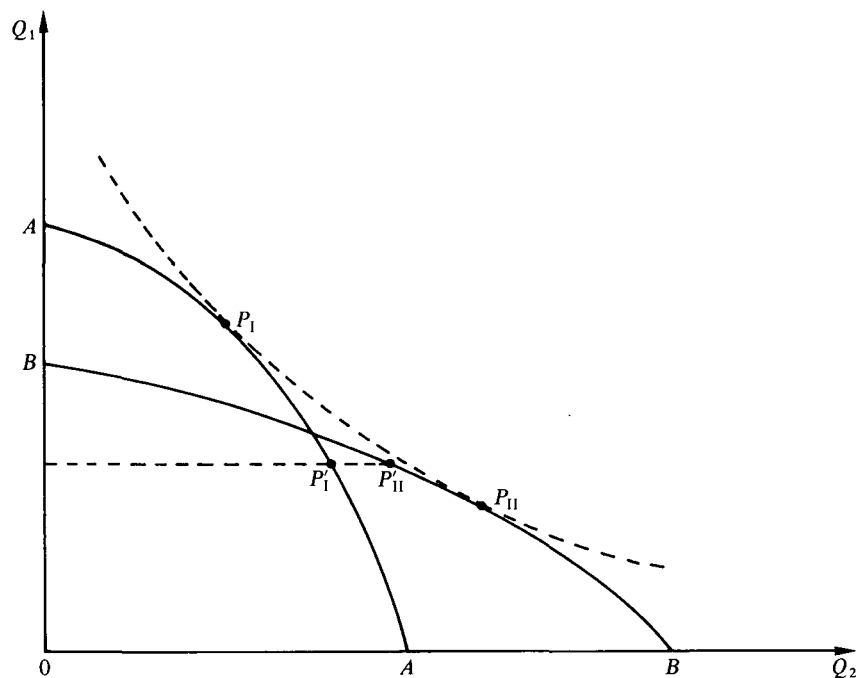


Fig. 3

higher environmental quality and a lower regional income compared to another area but still enjoy the same welfare. If a constraint requiring an identical environmental quality for both regions is introduced, both regions may reach a lower welfare level.

### 3. Aggregating Partitioned Constraints

The shortcomings of ad hoc partitioning consist in the fact that ad hoc partitioning does not introduce the implicit value that the overall constraint has for the economic system. Assume that a stricter environmental constraint is introduced into an economic system. Then we know from optimization theory that the change in the constraint can be evaluated in terms of losses in the overall objective function of the economic system. Note that once a normative constraint such as an environmental norm is introduced, this implicit or imputed value of the variable in the constraint is given. Ad hoc partitioning does not transfer this implicit value of the overall constraint to the lower-level constraints. This is the basic reason why the ad hoc partitioning of a constraint makes the constraint more powerful.

Not transferring the implicit value of a constraint to the individual agents of a society is not only detrimental in a static condition; it also creates some additional shortcomings in a dynamic setting where adjustments have to take place. Quite a few constraints in economic policy relate to availability of resources, technologies, patterns of demand and supply, etc. All these factors change over time, with the partitioned constraints very often remaining constant for some periods. With the partitioning failing to indicate the implicit value of the constraint, incentives to overcome some long-run causes of the overall constraint are substantially reduced. As an example consider the incentive effects of an ad hoc partitioning of a reduced oil supply or of reduced housing. In these cases, ad hoc partitioning does not provide very many incentives that change the underlying causes of the overall constraint.

Note that quantitative ad hoc partitioning may take different forms and that the opportunity costs of partitioning may vary accordingly. For instance, a limited quantity may be allocated to all users; or we may only concentrate on new users (pollution permits for new facilities). Black markets or hidden prices (lump-sum payments for rent controlled housing declared as reimbursements for the investment of the landlord or former tenants<sup>6</sup>) may affect the partitioning and, consequently, its opportunity costs.

The policy question is whether we can find other approaches to the partitioning of constraints, namely a breaking down that also transfers the information on the implicit value of the constraints to all the agents. Consider the environmental constraint discussed in example 1. Here an emission tax  $OT$

<sup>6</sup> Consider the standing practice of the so-called "Ablöse" in Austria.



would generate exactly the abatement behavior that minimizes total abatement costs and that ensures that the costs  $PS'S^0$  are not surpassed. This follows from the construction of the aggregated abatement costs function. Also, marketable discharge permits for the tolerable quantity of emissions could be auctioned off, yielding a price  $OT$  per unit of emission. Applying performance standards for a set of firms or allowing the banking or offsets of pollution permits (the "bubble concept") would also reduce the rigidity of the detailed compliance requirements<sup>7,8</sup>.

In example 2, the overall interest of providing  $OR$  units of commodity 2 for the two nations together could be reached by the allocation  $G$  instead of  $P$ . By moving from  $P$  to  $G$ , the constraint that  $OR$  units of commodity 2 must be produced is still satisfied, but at the same time, more of commodity 1 can be supplied. The opportunity costs of the partitioned constraint can be reduced.

In example 3, the overall constraint is regional welfare; this is broken down into a constraint on the provision of public goods. The overall constraint is broken down without knowing how the partitioned constraints affect the target variable that the constraint is supposed to protect. The implicit value of the overall constraint cannot be transferred into the partitioned constraints.

The policy conclusion is that it may be worthwhile to attempt an aggregation of partitioned constraints by using price tags for the constrained variable or other institutional mechanisms such as auctions and the definition of property rights that eventually will transform the scarcity expressed by a constraint into price signals<sup>9</sup>. It should be noted, however, that some constraints cannot be aggregated without losing their characteristics. Protecting personal liberty or other basic rights must be defined with respect to the individual. To these qualitative constraints our reasoning does not apply.

### *Zusammenfassung*

#### *Zur Zerlegung von Restriktionen*

In der praktischen Politik werden aggregative Beschränkungen gemeinhin ad hoc in eine Anzahl von Beschränkungen aufgespalten, die auf niedrigerer Ebene wirken. In diesem Beitrag wird argumentiert, daß eine solche ad-hoc-Aufspaltung die Beschränkungen wirkungsvoller macht und ihre Opportunitätskosten unnötig erhöht. Dies gilt besonders in einer nicht-statischen Welt mit sich

<sup>7</sup> On the "bubble concept", cf. for instance SIEBERT [1981 b].

<sup>8</sup> Samuelson points out that in the case of point rationing, making stamps interchangeable, "is in a certain sense optimal" (SAMUELSON [1947], p. 171).

<sup>9</sup> Also consider cost sharing in an institutional setting such as the water associations of the Ruhr area where the cost of fulfilling a constraint is shared by the members of the association and where the attribution of the costs to the individual polluters creates incentives to abate pollutants.

wandelnden ökonomischen Bedingungen. Die Ineffizienz der ad-hoc-Aufspaltung resultiert daraus, daß der implizite Wert der Gesamtrestriktion sich nicht auf die Restriktionen der niedrigeren Ebene überträgt. Die wirtschaftspolitische Implikation dieser Überlegung ist, daß aufgespaltene Restriktionen wo immer möglich aggregiert werden sollten und daß geeignete Aggregationsmechanismen gefunden werden müssen.

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