Environmental policy, allocation of resources, sector structure and comparative price advantage
Environmental Policy, Allocation of Resources, Sector Structure and Comparative Price Advantage

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The paper analyzes how environmental policy affects sector structure, the allocation of resources and comparative price advantage in a two-sector equilibrium model.

Environmental quality can be considered as a public good being influenced by emissions from consumption and production processes. Since the assimilative capacity of the environment has been used so far as a free production input, the common-property-resource has been heavily overused. Environmental policy attempts to decide which of the competing uses of the environment should have priority and by which policy instruments environmental scarcity can be internalized into the decisions of the subsystems of the economy.

In this paper we analyze to what extent environmental policy will affect sector structure, the allocation of resources and relative price. Since relative price in a closed economy determines comparative advantage (relative to the foreign country), the paper also answers the question to what extent environmental policy will affect the comparative price advantage of a country. In former models (Siebert 1974, 1976) only partial equilibrium models were developed neglecting the demand side or not closing the model with respect to receipts from the emission tax. In this paper, the model is closed with respect to the demand side, and demand conditions are explicitly taken into consideration.

The frame of reference is a two-sector-model in which production generates pollutants as a joint product. The government levies an emission tax per unit of pollutant emitted to the environment. We treat the environment as a national public good and ignore international, transnational and regional environmental systems1. Section I presents the assumptions, Section II develops the model, and in Section III the implications of the model are discussed. Section IV points out possible extensions.

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1 On these problems compare Walter (1975).
I. Assumptions

A 1. In order to keep the model as simple as possible we assume only one type of resource, \( R \), and a production function

\[ Q_i = F_i(R_i) \quad \text{with} \quad F'_i > 0, \quad F''_i < 0 \]

A 2. The production of commodities \( i = 1, 2 \) generates pollutants \( S_i^p \) as a joint product. For simplifying purposes there is only one type of pollutants. It is assumed that pollutants emitted rise proportionally or progressively with resources used, i.e.

\[ S_i^p = H_i(Q_i) = H_i[F_i(R_i)] = Z_i(R_i) \quad \text{with} \quad Z'_i > 0, \quad Z''_i \geq 0; \quad H_i > 0 \]

The convex emission function \( Z_i \) is suggested intuitively by engineering production functions. With the activity level of an engine reaching or exceeding capacity, it is realistic to assume that inputs have to be increased progressively for an additional unit of output. This suggests that emissions rise progressively. A more precise explanation for the convexity of the emission function follows from the application of the mass balance concept to production functions. In terms of weight a mass balance exists between input and output. Since mass cannot be lost in a production process, regular output and emissions must be identical to inputs, in weight terms. Define the production function in weight terms. For a given technology this function must be concave, if the regular production function is concave. Then it follows that the emission function \( Z_i \) must be convex.

A 3. Resources may also be used for abatement purposes. Let \( S_i^a \) indicate the quantity of pollutants reduced in Sector \( i \). The abatement function is given by

\[ S_i^a = F^{-1}_a(R_i) \quad \text{with} \quad F'_a > 0, \quad F''_a < 0 \]

The abatement function describes a technology that prevents pollutants from entering the environment. Additionally it could be assumed that a technology exists to reduce pollutants ambient in the environment (water treatment).

A 4. Net emissions or pollutants ambient in the environment are defined as emissions produced minus emissions abated. A diffusion function is not explicitly introduced.

\[ S_i = S_i^p - S_i^a \]

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\(^2\) Gutenberg (1972), S. 326.
\(^3\) Compare Sontheimer (1975).
A 5. Firms maximize profits and regard commodity prices, factor prices and the emission tax as given.

A 6. The resource can be used for production and abatement and is given

\[ R_t + R_2 + R'_1 + R'_2 = \bar{R} \]

A 7. Commodity demand is given by

\[ Q_i^D = D_i(p, Y) \]

with \( p = \tilde{p}_1/\tilde{p}_2 \) where \( \tilde{p}_i \) indicate nominal prices and \( p \) is the relative price.

A 8. Income \( Y \) is defined from the production side. There are no savings. In order to close the model, we assume that the government spends the tax income received in form of transfers to the households. Consequently disposable income of the households is identical to net national income at market prices and is defined as

\[ Y = pQ_1 + Q_2 \]

Observe that \( Y \) includes transfers not explicitly shown and that \( p \) is consumers price and not producers price. If \( Y \) would be defined with respect to producers price \( p^* \), emissions taxes (and transfers) would appear explicitly on the right side of (7).

A 9. Commodity markets must be in equilibrium so that

\[ Q_i = Q_i^D \]

A 10. The government levies an emission tax \( \tilde{z} \) (in nominal terms) on net emissions \( S_i \), with \( \tilde{z} \) being changed parametrically\(^4\).

II. The model

1. Factor demand by the profit maximizing firm is given by maximizing

\[
L_i = \tilde{p}_i Q_i - \tilde{r} (R_i + R'_i) - \tilde{z} S_i
\]

s. t. \( Q_i - F_i(R_j) \leq 0 \)

\(^4\) Additionally, a damage function may be introduced indicating that environmental quality is affected by net emissions

\[ U = G(S) \quad \text{with} \quad G' < 0, \quad G'' < 0. \]
where \( r \) denotes the resource price in nominal terms. Assuming that production takes place in both sectors, i.e., that we have a meaningful problem, and defining \( \tau = \frac{r}{p_2} \) and \( z = \frac{z}{p_2} \), the conditions for profit maximizing factor demand are given as

\[
\tau = (p - H_1) F'_1 (R_1)
\]

(9)

\[
\tau = (1 - z H_2) F'_2 (R_2)
\]

\[
\tau = z F''_2 (R_2)
\]

2. The system of equations (1) - (9) has the 17 variables \( S_{fi}, S_i, S_i', Q_f, R_i, R_i', Q_i'', p, Y \) and \( r \) and 18 equations. The definition of \( Y \) in equation 7 states that total demand is equal to income, so that in a two-sector model the equilibrium condition for one of the product markets is redundant (Walras Law) and should be omitted. By substitution the system can be simplified to

\[
F_1 (R_1) = D_1 (p, p F_1 (R_1) + F_2 (R_2)) \quad (i)
\]

\[
\tau = z F''_1 (R_1) \quad (i-iii)
\]

(10)

\[
\tau = (p - z H'_1) F'_1 (R_1) \quad (iv)
\]

\[
\tau = (1 - z H'_2) F'_2 (R_2) \quad (v)
\]

\[
\bar{R} = R_1 + R_2 + R_i' + R_i'' \quad (vi)
\]

Total differentiation and substitution of (10 vi) into (i) - (v) yields

\[
\begin{bmatrix}
a_1 & 0 & 0 & 1 & -F'_1 \\
0 & a_2 & 0 & 1 & 0 \\
0 & 0 & -z F''_1 & 1 & 0 \\
z F''_1 & z F''_2 & z F''_2 & 1 & 0 \\
b_1 F_1 - D_1 F_2 & 0 & 0 & -b_2
dR_1 & dR_2 & dR_1' & dr & dp
\end{bmatrix}
\begin{bmatrix}
dR_1 \\
dR_2 \\
dR_1' \\
dr \\
0
\end{bmatrix}
= \begin{bmatrix}
-H'_1 F'_1 dz \\
-H'_2 F'_2 dz \\
F''_1 dz \\
F''_2 dz \\
0
\end{bmatrix}
The coefficients are defined as follows:

\[ a_i = z_{H_i} F_i^{i2} - (p_i - z_{H_i}) F_i^{ii} > 0 \] (i)

\[ b_1 = 1 - D_{1V} p = D_{2V} \] (ii)

\[ b_2 = D_{1p} + D_{1V} F_1 < 0 \] (iii)

In (12i) we have \( p_i = p \) for \( i = 1 \) and \( p_i = 1 \) for \( i = 2 \).

### III. Implications

The problem is to determine how the allocation of resources is affected by environmental policy. We first study how factor demand of firms changes with an emission tax for a given relative price \( p \). We then analyze how relative price changes, and how the allocation of resources and national income are affected in this case.

1. Assuming a given commodity price and considering only the factor market conditions and the resource constraint, i.e. (10 ii - vi), the change in factor use in response to an increase in the emission tax is given by the subsystem of equation 11 not containing the fifth row of the system and not containing the fifth column of the matrix of derivatives. From appendix I we have the following results

\[ \frac{dR_1'}{dz} > 0 \] (13 i)

\[ \frac{dR_1}{dz} < 0; \quad H_1' F_1' > H_2' F_2' \] (13 ii)

\[ \sum \frac{dR_j}{dz} < 0 \] (13 iii)

\( H_1' F_1' \) indicates the marginal tendency to pollute (per unit of resource), \( H_1' F_1' > H_2' F_2' \) specifies that Sector 1 is pollution intensive. For \( b_2 < 0 \) follows from Slutsky's rule. Let \( D_{iP_{comp}} \) denote the pure substitution effect. We have

\[ D_{1p} = D_{iP_{comp}} - D_{1V} D_1 \text{ or } D_{iP_{comp}} = D_{1p} + D_{1V} F_1 (R_1) = b_2 < 0, \]

since the pure substitution effect is always negative.

If \( Z_1 (R_1) > Z_2 (R_2) \) for \( R_1 = R_2 \), \( H_1' F_1' > H_2' F_2' \) can be specified as \( S_{1p} > S_{2p} \) for \( R_1 \geq R_2 \). This transformation also holds for \( R_1 < R_2 \) if either Sector 1 is not "very" small compared to Sector 2 or if \( Z_2 \) has a sufficiently weaker curvature than \( Z_1 \). Observe that \( H_1' F_1' > H_2' F_2' \) demands that the \( Z_{r} \)-function should not cross, i.e. emission intensities should not be reversed.
given commodity prices, an emission tax will reduce the output of the pollution-intensively produced commodity. Resource use in the pollution abatement activity of the pollution-intensively producing sector will increase. Also we know that resources will be withdrawn from production and will be used in abatement activities. This effect does not depend on differences in the production intensity of the two sectors.

2. Allowing the commodity price to vary we have the following results from appendix II (as sufficient conditions):

\[(14 \text{i})\]
\[A > 0: \begin{cases} \frac{D'_{2Y}}{D'_{1Y}} \geq 0 \\ F'^{2}_{1} + D'_{1Y} F'^{2}_{2} \geq 0 \end{cases} \]

\[(14 \text{ii})\]
\[\frac{dR'_{i}}{dz} > 0: \quad D'_{1Y} F'^{2}_{2} + D'_{2Y} F'^{2}_{1} \geq 0 \]

Assuming (14 i) is given we have

\[(14 \text{iii})\]
\[\sum \frac{dR_{i}}{dz} < 0 \]

\[(14 \text{iv})\]
\[\frac{dR_{i}}{dz} < 0: \begin{cases} H_{1} F'_{1} > H_{2} F_{2} \\ D'_{1Y} F'^{2}_{2} \geq a_{1} D'_{1Y} F'^{2}_{2} \end{cases} \]

\[(14 \text{v})\]
\[\frac{dp}{dz} > 0: \begin{cases} H_{1} F'_{1} > H_{2} F_{2} \\ a_{2} D'_{2Y} F'^{2}_{2} \geq a_{4} D'_{1Y} F'^{2}_{2} \end{cases} \]

\[(14 \text{vi})\]
\[\frac{dY}{dz} < 0: \begin{cases} H_{1} F'_{1} > H_{2} F_{2} \\ - \eta^{2}_{1} > \alpha > 1 \quad \text{with } \alpha = \frac{1 - \frac{F'^{2}_{2}}{pF'^{2}_{1}}}{1 - \frac{F'^{2}_{2}}{pF'^{2}_{1}}} \end{cases} \]

In (14 vi) \(\eta_{1}\) denotes the direct price elasticity of demand.

Diagram 1 summarizes the sufficient conditions of equation 14.

We have the following results:

3. Assume both commodities are not inferior, so that their marginal propensities to consume \((D'_{iY} \geq 0)\) are nonnegative. Then the determinant \(\Delta > 0\). Assume Sector 1 is the pollution-intensively producing sector. Then we have:

Resource use in the pollution-intensively producing sector will decline. Resource use in each abatement activity will increase; resource in production \((\sum R_{i})\) will be reduced.
Environmental policy will shift the sector structure of the economy in favor of the abatement activities with production being affected negatively. We can establish that production in the pollution-intensively producing sector will decline. In the less pollution-intensively producing Sector 2, resource use may increase or decrease. The model allows both cases. Thus, in one case Sector 1 and Sector 2 lose resources to the abatement activity, whereas in the other case, Sector 1 loses resources to Sector 2 and the abatement activity. Under the assumptions made, emissions will be reduced and environmental quality will improve. This follows from $\frac{dR_i}{dz} < 0$ and $\frac{dR_j}{dz} > 0$.

4. Condition (14 v) for a rise in the relative price can be split into two sufficient conditions.

i) $D'_{3Y} \geq pD'_{1Y}$. Under the conditions specified below, national income will decline as a consequence of environmental policy. Then $D'_{3Y} \geq pD'_{1Y}$ guarantees that demand for the pollution-intensively produced commodity 1 is reduced less than demand for the commodity 2. This difference in the income effect of the two commodities makes sure that

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$\frac{dY}{dx} > 0$ follows from $H'_{i1} > H'_{i2}$. Note that $F'_{1} p > F'_{2}$ follows from $H'_{i1} F'_{1} > H'_{i2} F'_{2}$.

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the relative price of the pollution-intensively produced commodity must rise.

\[ a_2 \geq a_1: \left| \frac{dR_1}{dr} \right| > \left| \frac{dR_2}{dr} \right| \] specifies that (for given \( p \) and \( z \)), Sector 1 is more sensitive to changes in resource price than Sector 2; we may also say that Sector 1 is more dependent on the resource \( R \). This condition can be interpreted as a rudimentary form of a factor-intensity condition in a one-factor model. We can expect that this condition unfolds into a set of factor intensity conditions in a multi-factor model. As a result we have:

The relative price of the pollution-intensively produced commodity will rise if the marginal propensity to consume for this commodity is lower than the less pollution-intensively produced commodity and if the pollution-intensively producing sector depends heavily on the resource \( R \).

Sufficient conditions for a rise in the relative price can partly substitute each other. Assume Sector 1 is "very" pollution-intensive\(^{10} \). Then the relative price of commodity 1 may rise even if it has a high income elasticity of demand and loses demand quantities with a decline in income. Or for identical pollution-intensities of both sectors, the relative price will rise, if Sector 1 has a sufficiently smaller propensity to consume and thus loses a smaller quantity in demand requiring a higher adjustment in relative price. Finally, assume Sector 2 heavily depends on resource \( R \). Then \( p \) can rise nevertheless, if Sector 1 is sufficiently more pollution-intensive or if Sector 1 has a sufficiently lower income elasticity.

\( \frac{dp}{dz} > 0 \) indicates that environmental policy will under the conditions indicated affect the comparative price advantage of a country. Assume a country exports the pollution-intensively produced commodity 1. Then environmental policy will reduce the comparative price advantage of that country\(^{11} \). Consider two countries with a different abundance of environmental services and assume that \( z > z^* \) reflects the difference in endowment with \( z^* \) indicating the emission tax of the

\(^9\) Differentiate the factor demand conditions (9) for given \( p \) and \( z \) with respect to \( r \)

\[ \frac{dR_1}{dr} = \left( \frac{P_1}{P_2} \right) \frac{1}{F_i^r - zH_i^r F_i^r} = - \frac{1}{a_1} \quad \text{and} \quad \left| \frac{dR_1}{dz} \right| > \left| \frac{dR_2}{dz} \right| : a_2 > a_1 \]

\(^{10}\) Assume for instance Sector 2 does not pollute at all with \( H_2^r = 0 \).

\(^{11}\) If a country exports the less pollution-intensive commodity and if it undertakes environmental policy, its comparative price advantage will be improved.
foreign country. If all other factors influencing the autarky prices are identical in the two countries we have \( p(z) > p^*(z^*) \) which implies that the country abundant in environmental services will have a comparative price advantage and will export the pollution-intensively produced commodity.

5. Net national product at market prices \( Y \) is affected by environmental policy in two ways:

- Resource use in production will decrease, so that for a given price \( p \) national income will fall (withdrawal effect).
- The pollution-intensively produced commodity has to be revalued since market price must include the social cost of production. The revaluation-effect runs counter to the withdrawal effect.

National income will fall if the withdrawal effect outweighs the revaluation effect. This is the case, if the price elasticity of demand for the pollution-intensively produced commodity is sufficiently large, i.e. \(-\eta_1 > \alpha > 1\). (Condition 14 vi). A high price elasticity of demand for commodity 1 makes sure that the pollution-intensively producing sector will lose large demand quantities, so that the revaluation effect will not be too high.

From the definition of \( \alpha \) we have an interesting interrelation between demand conditions and the condition of emission intensity.

i) Assume Sector 1 is strongly pollution-intensive\( ^{12} \) so that \( \alpha \) is close to unity. Then the price elasticity does not have to be too high if the withdrawal effect is to be strong. A high pollution-intensity of Sector 1 means that production costs rise strongly in Sector 1, relative price will rise, and Sector 1 will lose demand quantities, even if the price elasticity of demand is not too high.

ii) If Sector 1 is only "weakly" pollution-intensive compared to Sector 2, \( \alpha \) is higher than unity, and the demand for the pollution intensively produced commodity must be very elastic for demand quantities to decline. In other words, in condition (14 v) a high price elasticity of demand for the pollution-intensively produced commodity may be substituted by a strong pollution-intensity of Sector 1.

6. Structural policy will become an important issue in Western European countries in the future. Therefore it is of interest to determine how environmental policy will influence sectoral structure. We have

\[ 12 \text{ Note that } pF'_1 = r + zH'_1 F'_1 \text{ and } F'_2 = r + zH'_2 F'_2, \text{ so that } H'_1 F'_1 > H'_2 F'_2 \implies \frac{F'_2}{pF'_1} < 1. \]
already shown that the production sectors will be reduced and the abatement activities will increase. Measuring sector structure \( s \) in terms of resources used in the two sectors, i.e. \( s = \frac{R_1}{R_2} \), we know that \( \frac{ds}{dz} < 0 \) if \( \frac{dR_2}{dz} > 0 \), since \( \frac{dR_1}{dz} < 0 \). This means that environmental policy will change sector structure to the disadvantage of the pollution-intensive sector. Since \( \frac{dR_2}{dz} < 0 \) cannot be ruled out, we are not sure, however, whether demand conditions are such that the pollution-intensively producing Sector 1 may expand relatively with environmental policy. From equation (I.6) in the appendix it can be seen that sector structure measured in resource units will change in favor of Sector 2 (for given \( p \)), if

\[
R_2 F_2^p D_2^p > R_1 F_1^p D_1^p
\]

Assuming identical sector weights in the initial situation, i.e. \( R_1 = R_2 \), we have as a sufficient condition for Sector 1 to decline relatively that \( pD_1^p > D_2^p \), where the difference must be large enough to offset \( pF_1^p > F_2^p \). Condition (15) also is sufficient for the case of a variable price as can be seen from equation (II.7) in the appendix. Additionally sector structure can be defined as \( s^* = \frac{Q_1}{Q_2} \).

### IV. Extensions

The foregoing analysis can be easily expanded. The following questions seem to be of interest.

1. If more than one resource is introduced, we can expect that the \( a_2 \geq a_1 \)-condition will unfold into a set of factor-intensity conditions.

2. The model can easily be extended to an open economy. Then the market equilibrium conditions of equation (6) have to be substituted by the definition of excess demand functions

\[
E_i = C_i - Q_i
\]

and similarly for the foreign country. World markets must be in equilibrium, \( E_1 + E_2^* = 0 \), and the balance of payment condition of the home country must be fulfilled

\[
pE_1 + E_2 = 0.
\]

3. In this case such questions as the effect of environmental policy on trade equilibrium and the term of trade may be analyzed. Also,

\[\text{Note that if this condition is not fulfilled, Sector 1 may decline relatively less since the first term in equation (I.6) in the appendix is negative.}\]
if the model is suitably extended, the effects of environmental policy on the balance of payments or the exchange rate may be studied. Finally the interesting question arises that the traditional gains from trade will be reduced, if a country specializes in the production of the pollution-intensive produced commodity. This is due to the fact that in a situation where no environmental policy is undertaken, comparative cost advantages are distorted and consequently the gains from trade are not specified correctly.

Appendix

I.

\[ D = a_1 \left\{ a_2 z \left( - F'_1 F'_2 + z^2 F'_{1} F'_{2} \right) + a_2 z^2 F'_{1} F'_{2} \right\} + 0 \]

Define \( Z_1 \) and \( Z_2 \) as

\[ Z_1 = \left( F''_1 + F''_2 \right) a_2 p F_1 - z^2 F'_{1} F'_{2} \left( H'_1 F'_1 - H'_2 F'_2 \right) \]

\[ Z_2 = - \left( F''_1 + F''_2 \right) a_2 F_2 - z^2 F'_{1} F'_{2} \left( H'_1 F'_1 - H'_2 F'_2 \right) \]

\[ \frac{dR_1}{dz} = \frac{Z_1}{D} \]

\[ \frac{dR_2}{dz} = - \frac{Z_2}{D} \]

\[ \frac{dR'_1}{dz} = \frac{Z_2}{D} \left( F'_1 a_1 F'_2 + a_2 p F'_1 \right) \]

\[ \frac{dR'_2}{dz} = \frac{z F'_1}{z} + z F'_1 \frac{dR'_1}{dz} \]

\[ \frac{d \left( R_1/R_2 \right)}{dz} = \frac{1}{R_2 D} \left( \left( F''_1 + F''_2 \right) a_2 p F_1 R_2 + a_1 F'_2 R_1 + \right) + \]

\[ + z^2 F'_{1} F'_{2} \left( H'_1 F'_1 - H'_2 F'_2 \right) \left( R_1 - R_2 \right) \]

II.

\[ \Delta = - b_2 D - a_2 D_2 F'_1 z \left( F''_1 + F''_2 \right) + z^2 F'_1 F''_2 \left( D'_1 F'_1 + D'_2 F'_2 \right) \]

Define \( \Delta_1 = - b_2 Z_1 + F'_1 F''_2 D'_1 \left( F''_1 + F''_2 \right) \)

\[ \frac{dR_1}{dz} = \frac{\Delta_1}{\Delta} \]
The paper analyzes the problem how environmental policy affects sector structure, the allocation of resources, relative price (and comparative advantage) and national income. The frame of reference is a two-sector model in which production generates pollutants as a joint product. The model is closed with respect to the demand side and with respect to the receipts from the emission tax. Conditions with respect to production, emission and demand are specified under which resource use in the pollution-intensively producing sector will fall, resource use in the abatement activity will increase, the relative price of the pollution-intensively produced commodity will rise and national income will fall. Possible extensions are indicated.

Summary

The paper analyzes the problem how environmental policy affects sector structure, the allocation of resources, relative price (and comparative advantage) and national income. The frame of reference is a two-sector model in which production generates pollutants as a joint product. The model is closed with respect to the demand side and with respect to the receipts from the emission tax. Conditions with respect to production, emission and demand are specified under which resource use in the pollution-intensively producing sector will fall, resource use in the abatement activity will increase, the relative price of the pollution-intensively produced commodity will rise and national income will fall. Possible extensions are indicated.

Zusammenfassung

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