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Escaping the Unemployment Trap: The Case of East Germany

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September 2008
Escaping the Unemployment Trap: The Case of East Germany

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ABSTRACT

Escaping the Unemployment Trap: The Case of East Germany

This paper addresses the question of why high unemployment rates tend to persist even after their proximate causes have been reversed (e.g., after wages relative to productivity have fallen). We suggest that the longer people are unemployed, the greater is their cumulative likelihood of falling into a low-productivity “trap,” through the attrition of skills and work habits. We develop a model along these lines, which allows us to bridge the gap between high macroeconomic employment persistence versus relatively high microeconomic labor market flow numbers. We calibrate the model for East Germany and examine the effectiveness of three employment policies in this context: (i) a weakening of workers’ position in wage negotiations due to a drop in the replacement rate or firing costs, leading to a fall in wages, (ii) hiring subsidies, and (iii) training subsidies. We show that the employment effects of these policies depend crucially on whether low-productivity traps are present.

JEL Classification: E24, J30, J31, J64

Keywords: labor market, labor market trap, East Germany

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1 Introduction

The persistence of high unemployment rates in the large continental European economies - France, Germany, Italy and Spain - remains a challenge to economists, despite a prodigious literature on the subject (e.g. Decressin and Fatás, 1995, Elhorst, 2005, Faini et al., 1997, Gray, 2004, Sinn and Westermann, 2001, Taylor and Bradley, 1997). The mystery is not how these high unemployment rates arose, for usually regions of relatively high unemployment are generally ones in which labor costs have been relatively high in relation to productivity. Rather, the mystery is why the high unemployment rates in some regions have far outlived their original causes. Specifically, why do these persistent unemployment rates not disappear, even after labor costs fall relatively to productivity?

East Germany is a good case example. After German reunification, East German real wages rose dramatically relative to productivity and unemployment jumped upwards in response. With the social and monetary union in October 1990, East German labor costs jumped from 7% to about one half of the West German level in 1991\(^1\) (see e.g., Franz and Steiner, 2000, Sinn, 2002). Since then, however, labor costs have fallen steadily in relation to productivity, but the employment rate has remained stubbornly low, hovering near 20 percent for the past decade (see figure 1\(^2\)). Traditional labor market analysis has trouble accounting for this experience.\(^3\)

This paper suggests a simple explanation\(^4\): Once people remain unemployed for a long time, they tend to fall into a "trap" representing a contraction of their employment opportunities. Snower and Merkl (2006) describe several such traps, but do not model them. Immediately after German reunification, East German wage bargaining was conducted primarily by West German unions and employers, and these had strong incentives to push East German wages up, in order to reduce migration of East German workers to West Germany and of West German firms to the East. Given the low short-run elasticity of labor demand, this "bargaining by proxy" was not only in the interests of West German unions, but also West German firms who feared the entry of new firms sparked by the new migration flows. The upward wage pressure was reinforced through generous unemployment benefits and associated welfare entitlements. The resulting East German wage hike led to a sharp fall in East German employment, and this effect was prolonged through the introduction of generous job security provisions and costly hiring regulations, which raised the persistence of employment (i.e. made current employment depend more heavily on past employment). The persistently low employment was mirrored in long-term unemployment.\(^5\)

This is where possibility of traps arises. The longer workers are unemployed, the more prone they are to attrition of skills and work habits and they are of course unable to get on-the-job training. As their productivity falls, it is more difficult for them to find a new job, even if labor costs fall relative to the average productivity of the employed workforce.

Naturally, if these "efficiency labor costs," i.e. labor costs deflated by average productivity,

---

\(^1\)These numbers are based on Sinn (2002) who uses the informal exchange rate before German unification as reference point. The cost pressure is also evident if the exchange rate is left completely aside. The East German wage increased from about one third of the West German level after unification to about 70 percent in 1994 (see, e.g., Sinn, 2002).

\(^2\)Sources: Bundesagentur für Arbeit (2006a, b) and Statistische Ämter des Bundes und der Ländern (2006), own calculations.

\(^3\)There is of course an efficiency wage argument to be made in favor of similar wages in East and West Germany (see, e.g., Akerlof and Yellen, 1988): Substantially higher wages in the West than in the East could lead workers in the East to reduce their work effort, or it might lead high-productivity Eastern workers to migrate to the West. For the sake of analytical simplicity, we have not included these effects in our model. It is worth noting, however, that the high unemployment rates induced through the rise of Eastern wages toward Western levels has also lead to migration, and the threat of future unemployment may also reduce work effort.

\(^4\)For an alternative explanation see Uhlig (2006).

\(^5\)The share of long-term unemployed (with a duration of more than one year) has increased from one quarter in 1992 to roughly one half today (Sachverständigenrat, 2004).
Figure 1: East German labor cost normalized by productivity and the employment rate for dependently employed workers.

fell sufficiently to more than compensate for the drop in the productivity of affected workers, then their employment opportunities would improve; but the data appear to suggest that these costs did not fall enough.\(^6\)

This paper models such a trap, and examines its implications for labor market activity and employment policy. We build an analytical model of the low-productivity trap as well as its connection with the rest of the economy and calibrate it for the East German labor market.\(^7\)

Our paper contributes to the unemployment literature by examining the implications of labor market traps for employment and unemployment dynamics. As noted, our model describes a labor market where workers with primary jobs who become unemployed risk losing their access to high-productivity jobs (for instance, because they become stigmatized and demotivated through their unemployment spell). The longer they are unemployed, the greater this cumulative risk becomes.\(^8\) On the other hand, workers with "trapped jobs" may gain skills or access to high-productivity jobs (e.g., by using their jobs to gain information and contact to other employment opportunities), and thereby obtain a primary job. The longer they remain employed, the greater is the cumulative likelihood of becoming a primary worker. In short, unemployment is the road to bad jobs and lengthy unemployment spells, whereas employment

\(^6\)Furthermore, the massive East German investment subsidies that were granted in the aftermath of reunification - often paid to prevent uncompetitive firms to lay off their employees - resulted in the creation of capital that was relatively unproductive and prone to underutilization (see, for example, Sinn, 1995). The labor cooperating with this capital became similarly unproductive and underutilized, even if efficiency labor costs subsequently fall. What these traps have in common is that they are both associated with low productivity. The long-term unemployed are prone to become less productive and this traps them in unemployment. See Fuchs-Schündeln and Izem (2007) and Ragnitz (2007) for a thorough analysis of the low labor productivity in East Germany. See Burda (2006) for a neo-classical model of economic integration with adjustment costs, which explains the "capital deepening" and the "labor thinning" in the East.

\(^7\)For simplicity, we ignore the interrelation with the West German labor market. Such a connection of two regional labor markets can be found in the context of a simpler modeling strategy in Snower and Merkl (2006).

\(^8\)Note that for simplicity, in the model we keep the period probability of losing access to high productivity jobs constant over time. However, if workers remain unemployed for several periods their cumulative probability increases of being downgraded increases (compared to employed workers who do not face any risk).
is the road to good jobs and shorter unemployment spells.

The trap highlights a major, often ignored, cost of unemployment. A specific rise in efficiency labor costs sends employees into short-term unemployment; but should this state persist and thus turn into lengthy unemployment, then an equal and opposite fall in efficiency labor costs may be insufficient to bring these workers back into employment. Our notion of a labor market "trap" is related to the literature on segmented labor markets, for example, models that divide the labor market into a high-wage "primary jobs" and low-wage "secondary jobs."9

Our paper sheds light on an important micro-macro puzzle. While employment seems to be highly persistent on the macroeconomic level10, the microeconometric evidence on job flow numbers seems to suggest a more fluid labor market.11 Our assumption of "primary jobs" and "trapped jobs" suggests a solution to this puzzle. For given microeconomic flow levels, movements between different job types (primary and trapped) make employment much more persistent than it would otherwise have been. Thereby the apparent disagreement between the microeconomic evidence of large job flows and the macro evidence of high employment persistence is accounted for.

The partitioning of the labor market into primary and trapped workers also has important implications for the effectiveness of labor market policies. Specifically, we show the following:

- The existence of low-productivity traps implies that reductions in the wages associated with trapped jobs (induced, say, by cuts in unemployment benefits or firing costs), on their own, are relatively ineffective in raising the employment rate (both in relation to the primary jobs and to an economy without low-productivity traps).

- Hiring subsidies for the trapped unemployed have a relatively strong positive influence on employment, i.e. for a given subsidy size (both absolute and relative to the wage) they are more cost-effective12 than hiring subsidies for primary unemployed. There are two driving forces: The presence of traps reduces the deadweight effects of hiring subsidies and hiring subsidies enable more trapped workers to move to primary jobs.

- Training subsidies and programs that raise the productivity of workers with trapped jobs, thereby improving their chances of obtaining a primary job, may also have a relatively strong employment long-run effect, but this effect takes a long time to manifest itself.

The paper is organized as follows. Section 2 presents our model. In Section 3 this model is calibrated for the East German labor market. Section 4 considers the policy implications. Finally, Section 5 concludes.

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9 See, for example, Bulow and Summers (1986), McDonald and Solow (1985), Weitzman (1989), Dickens and Lang (1988) for the early foundations of this literature and Kleven and Sorensen (2004) and Lommerud et al. (2004) for more recent contributions. For the empirical literature see, for example, Dickens and Lang (1985), Saint-Paul (1996) for a survey and Ghilarducci and Lee (2005) for a recent contribution.

10 Blanchard and Summers (1986) argue that European unemployment show hysteresis ($n_t = \rho n_{t-1} + \varepsilon_t$, with $\rho = 1$). While this is a very extreme point of view (because it would make unemployment non-stationary), most economists would agree that the European (un-)employment rates are highly persistent, i.e. $\rho$ is relatively close to 1.

11 Microeconometric hazard rates for Germany suggest yearly job finding rates ($\eta$) of around 40 to 80 percent (see, e.g., Hunt, 2004, Wilke, 2005), depending on personal criteria such as gender or education. This would make the employment rate relatively unpersistent, given the standard employment dynamics curve ($n_t = (1 - \phi_1 - \eta_1) n_{t-1} + \eta_1$, where $n$ is the employment rate and $\phi$ is the firing rate).

12 We call a policy more "cost effective" than another policy when it generates more employment, for a given net government expenditure outlay.
Our labor market has primary jobs and trapped jobs. The average productivity per worker is assumed to be lower for trapped \((a_T)\) than for primary workers \((a_P)\).\(^{13}\) Moreover, firms face a random cost, \(\varepsilon_t\), iid across workers and time, with a constant cumulative distribution \(\Gamma (\varepsilon_t)\). This cost may be interpreted as an operating cost or as a negative productivity shock.

Decisions in the labor market are made in the following sequence: First, workers move between the two job types. Specifically, each unemployed worker who previously had a primary job has an exogenously given probability \(\nu\) of losing productivity and thereby losing access to primary jobs (due either to skill attrition or loss of access to good jobs); and each employed worker with a trapped job has an exogenously given probability \(\varpi\) of gaining productivity and thereby gaining access to a primary job.\(^{14}\) Second, the wage is determined through bargaining. Third, the value of the random cost, \(\varepsilon_t\), is revealed. Finally, firms make their hiring and firing decisions.

Let the hiring rates of primary and trapped workers be \(\eta_P\) and \(\eta_T\), respectively, and let their firing rates be \(\varphi_P\) and \(\varphi_T\), respectively. (These hiring and firing rates will be derived choice-theoretically below.) The transitions between the various economic states are pictured in figure 2. Each employed primary and trapped worker remains employed with probability \((1 - \varphi_P)\) and \((1 - \varphi_T)\), respectively; she becomes unemployed with probability \(\varphi_P\) and \(\varphi_T\), respectively. Each unemployed primary and trapped worker remains unemployed with probability \((1 - \eta_P)\) and \((1 - \eta_T)\), respectively; she becomes employed with probability \(\eta_P\) and \(\eta_T\), respectively.

\(^{13}\)In our model we abstract from capital accumulation. While East Germany indeed had a lack of capital at the beginning of the nineties, the massive capital subsidies have reversed this situation in certain sectors. Gerling (2002) and the Sachverständigenrat (2004) show, for example, that nowadays the East German industrial capital intensity (defined as capital stock per worker) is higher than in West Germany.

\(^{14}\)Thus the cumulative probability of that an unemployed primary worker falls into the low-productivity trap rises with the duration of unemployment, and the cumulative probability of an employed trapped worker to escape from the trap rises with employment duration.
2.1 Wage Determination

We assume that the wage is the outcome of a Nash bargain between the median insider and her firm for primary and trapped jobs. The median insider faces no risk of dismissal at the negotiated wage. There are constant returns to labor. Under bargaining agreement, the insider receives the wage $w_{T,t}$ and the firm receives the expected profit $(a_T - w_{T,t})$ in each period $t$. The expected present value of returns to a trapped insider under bargaining agreement $(V_{T,t}^I)$ is

$$V_{T,t}^I = w_{T,t} + \delta \left( (1 - \varpi) \left( 1 - \phi_{T,t+1} \right) V_{T,t+1}^I + (1 - \varpi) \phi_{T,t+1} V_{T,t+1}^I \right),$$

where $\delta$ is the discount factor and $V_{T,t+1}^I$ is the expected present value of returns of an employed trapped (primary) worker and $V_{T,t+1}^I$ is the expected present value of returns of an employed trapped (primary) worker, respectively. Note that with probability $\varpi$ a trapped worker is upgraded to a primary job and thus has a higher future present value. The expected present value of returns to the firm under bargaining agreement is

$$\widetilde{\Pi}_{T,t} = a_{T,t} - w_{T,t} - \varepsilon_T^{MI} + \delta \left( (1 - \varpi) \left( 1 - \phi_{T,t+1} \right) \widetilde{\Pi}_{T,t+1} - (1 - \varpi) \phi_{T,t+1} f_{T,t+1} \right),$$

where $\widetilde{\Pi}_{T,t+1}$ is the future profit of a trapped (primary) worker, weighted by the probability that the worker stays in the respective job (as a trapped or primary worker) and $\varepsilon_T^{MI}$ is the operating cost of the trapped median insider. Under disagreement, the insider’s fallback income is $b_{T,t}$, assumed equal to the unemployment benefit. The firm’s fallback profit is $-f_{T,t}$, which is the firing cost per employee for trapped workers. In words, during disagreement the insider imposes the maximal cost on the firm (e.g. through strike, work-to-rule, sabotage) short of inducing dismissal. Assuming that disagreement in the current period does not affect future returns, the present values of insider’s returns under disagreement is

$$V_{T,t}^{II} = b_{T,t} + \delta \left( (1 - \varpi) \left( 1 - \phi_{T,t+1} \right) V_{T,t+1}^{II} + (1 - \varpi) \phi_{T,t+1} V_{T,t+1}^{II} \right),$$

and the present value of the firm’s agreement under disagreement is

$$\widetilde{\Pi}_{T,t}^{II} = -f_{T,t} + \delta \left( (1 - \varpi) \left( 1 - \phi_{T,t+1} \right) \widetilde{\Pi}_{T,t+1}^{II} - (1 - \varpi) \phi_{T,t+1} f_{T,t+1} \right).$$

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15The critical reader may object that insider power has been seriously eroded in East Germany due to the fall in union membership since reunification. The first response to this objection is that we should not confuse our insider bargaining with union bargaining, since our Nash bargaining problem could be interpreted as the individual median insider bargaining with her firm. Second, much of the erosion of East German insider power since reunification has resulted from the replacement of bargaining by proxy (in which West German unions and firms had dominant influence on negotiations about East German wages) by self-sufficient bargaining (in which East German workers and firms have taken control of East German wage determination). In our model, we assume that East German wage determination is entirely self-sufficient in this sense. And finally, although union membership has dropped in East Germany, union wage agreements still have very broad coverage. For example, in 2003 firms that were covered by a firm level or sectoral wage agreement employed 54 percent of all workers in East Germany. A large share of the other firms followed existing wage agreements voluntarily, covering 52 percent of the remaining employees (Schnabel, 2005).

16This assumption is made merely for analytical convenience; various other assumptions would lead to similar results. The wage could e.g. be the outcome of a bargain between the firm and the marginal worker, or between the firm and a union representing all employees. In this last case, the insiders’ objective in the bargain will depend on their retention rate.

17In what follows, only those variables have time subscripts that, for given parameter values, actually vary through time in our model. $j$ is the index for the productivity level. It can either be $P$ (primary worker) or $T$ (trapped worker).
Thus the insider’s bargaining surplus is
\[ V_I^{T,t} - V_{0,I}^{T,t} = w_{T,t} - b_{T,t}, \] (5)
and the firm’s bargaining surplus is
\[ \tilde{\Pi}_{T,t} - \tilde{\Pi}_{0,I}^{T,t} = a_{T,t} - w_{T,t} - \varepsilon^{MI} + f_{T,t}. \] (6)

The negotiated wage maximizes the Nash product (\( \Lambda \))
\[ \Lambda = (w_{T,t} - b_{T,t})^\gamma (a_{T,t} - w_{T,t} - \varepsilon^{MI} + f_{T,t})^{1-\gamma}, \] (7)
where \( \gamma \) represents the bargaining strength of the insider relative to the firm. Thus the negotiated wage is
\[ w_{T,t} = (1 - \gamma) b_{T,t} + \gamma (a_{T,t} - \varepsilon^{MI} + f_{T,t}). \] (8)

The bargaining problem is analogous for primary workers (see Appendix), so that the negotiated primary wage is
\[ w_{P,t} = (1 - \gamma) b_{P,t} + \gamma (a_{P,t} - \varepsilon^{MI} + f_{P,t}). \] (9)

### 2.2 Employment Decision

Having determined the wage, we now proceed to derive the hiring and firing rates for primary and trapped jobs.

#### 2.2.1 Primary Workers

Given the realized value of the random cost variable \( \varepsilon_t \), which is iid across individuals and time and whose mean is normalized to zero, an insider generates the following present value of expected profit:\(^{18}\)
\[ \Pi_t = -\varepsilon_t + \sum_{t=0}^{\infty} \delta^t (1 - \phi_P)^t (a_P - w_P) - \delta \phi_P f_P \sum_{t=0}^{\infty} \delta^t (1 - \phi_P)^t. \] (10)
i.e. with probability \((1 - \phi_P)\) the insider is retained and generates profit \((a_P - w_P)\), whereas with probability \(\phi_P\) is fired and generates the firing cost \(f_P\) (constant per employee).\(^{19}\)

The insider is fired when her generated profit is less than the firing cost: \( \Pi_t < -f_P \), so that
\[ \varepsilon_t > (a_p - w_P + (1 - \delta) f_P) / (1 - \delta (1 - \phi_P)). \]
Recalling that \( \Gamma (\varepsilon_t) \) is the cumulative density of the random cost \( \varepsilon_t \), the firing rate is given by the following implicit function:\(^{20}\)
\[ \phi_P = 1 - \Gamma \left( \frac{a_P - w_P + (1 - \delta) f_P}{1 - \delta (1 - \phi_P)} \right). \] (11)
The firm faces a hiring cost of \( h \), constant per worker. An entrant is hired when his generated profit exceeds this hiring cost: \( \Pi > h_P \). Thus the hiring rate is
\[ \eta_P = \Gamma \left( \frac{a_P - w_P - \delta \phi_P f_P}{1 - \delta (1 - \phi_P)} - h_P \right). \] (12)

---

\(^{18}\)In what follows, only those variables have time subscripts that, for given parameter values, actually vary through time in our model.

\(^{19}\)The expected operating cost conditional on being retained or hired is normalized to zero and its cumulative distribution \( \Gamma (\varepsilon) \) is time-invariant.

\(^{20}\)We assume that \((\partial \Gamma / \partial \phi) > -1\), so that a rise in \((a - w)\) or \( f \) both reduce the firing rate.
2.2.2 Trapped Workers

As noted, each worker with a trapped job is assumed to have an average productivity $a_T$ that is lower than the one of his counterpart with a primary job. Furthermore, trapped workers have a probability $\pi$ of moving into primary jobs. Thus, the present value of the profit generated by an entrant for a trapped job is $^{21}$

$$\Pi_{t,T} = -\varepsilon_t + \frac{a_T - w_T - \delta (1 - \pi) \phi_T f_T}{1 - \delta (1 - \phi_T)(1 - \pi)} - \phi_p \delta \pi \frac{f_p}{(1 - \delta(1 - \phi_T)(1 - \pi))} + (1 - \phi_p) \delta \pi \left( \frac{a_p - w_p - \delta \phi_p f_p}{(1 - \delta(1 - \phi_p)(1 - \pi))} \right). \quad (13)$$

Along the same lines as before, a worker is fired if her expected profits are smaller than minus the firing costs ($\pi_t < -f_T$):

$$\phi_T = 1 - \Gamma \left( \frac{a_T - w_T - \delta (1 - \pi) \phi_T f_T}{1 - \delta(1 - \phi_T)(1 - \pi)} + \frac{f_T - \phi_p \delta \pi \pi \frac{f_p}{(1 - \delta(1 - \phi_T)(1 - \pi))}}{\phi_p - w_p - \delta \phi_p f_p} \left( \frac{1 - \delta(1 - \phi_p)(1 - \pi)}{1 - \delta(1 - \phi_T)(1 - \pi)} \right) \right). \quad (14)$$

And she is hired if the expected profits are bigger than the hiring costs ($\pi_t > h_T$).

$$\eta_T = \Gamma \left( \frac{a_T - w_T - \delta (1 - \pi) \phi_T f_T}{1 - \delta(1 - \phi_T)(1 - \pi)} - h_T - \phi_p \delta \pi \frac{f_p}{(1 - \delta(1 - \phi_T)(1 - \pi))} + \frac{f_p - \phi_p \delta \pi \pi \frac{f_p}{(1 - \delta(1 - \phi_T)(1 - \pi))}}{\phi_p - w_p - \delta \phi_p f_p} \left( \frac{1 - \delta(1 - \phi_p)(1 - \pi)}{1 - \delta(1 - \phi_T)(1 - \pi)} \right) \right). \quad (15)$$

2.3 Employment Dynamics

We allow for the possibility that the employed workers with trapped jobs may raise their productivity - through learning-by-doing, improved work motivation, better work habits and so forth - and then move into primary jobs. Specifically, we also allow for the possibility that unemployed workers who previously had primary jobs may lose productivity - through attrition of human capital, reduced work motivation, lost work habits, etc. - and then have access only to trapped jobs. In particular, we assume that, in each period, a constant proportion $\pi$ of the employed workers with trapped jobs ascend to primary jobs, and a constant proportion $\nu$ of the unemployed primary workers lose access to primary jobs.

Thus, we obtain the following employment equation for primary jobs: $^{22}$

$$N_{P,t} = (1 - \phi_p) N_{P,t-1} + (1 - \phi_p) \pi N_{T,t-1} + \eta_T (1 - \nu) U_{P,t-1}. \quad (16)$$

The number of employed with primary jobs ($N_{P,t}$) consists of workers who are retained from the previous period $^{23}$ plus the newly hired workers ($\eta_T (1 - \nu) U_{P,t-1}$).

For trapped workers the employment dynamics equation is:

$$N_{T,t} = (1 - \phi_T) (1 - \pi) N_{T,t-1} + \eta_T (U_{T,t-1} + \nu U_{P,t-1}). \quad (17)$$

The number of employed trapped workers is equal to those who are retained and have not received a human capital upgrade ($(1 - \phi_T) (1 - \pi) N_{T,t-1}$) plus the newly hired workers ($\eta_T (U_{T,t-1} + \nu U_{P,t-1})$). $^{24}$

$^{21}$See the Appendix for a detailed derivation.

$^{22}$Note that capital letters ($N$, $U$) refer to levels, while small letters ($n$, $u$) are (un-)employment rates.

$^{23}(1 - \phi_p) N_{P,t-1}$ are the primary employees carried forward from the previous period and $(1 - \phi_p) \pi N_{T,t-1}$ are the previously trapped workers who received a human capital upgrade.

$^{24}$Note that the pool of potential recruits is enlarged by those who moved from primary to trapped jobs ($\nu U_{t-1, P}$).
After some re-formulations (see Appendix), we obtain an employment dynamics equation describing the employment rate of primary workers:

\[
n_{P,t} = \frac{1}{g_{t,P}} [(1 - \phi_P) n_{P,t-1} + (\eta_P (1 - \nu)) (1 - n_{P,t-1})] + (1 - \phi_P) \varpi \frac{L_{T,t-1}}{L_{P,t}} n_{T,t-1}.
\]

(18)

and the employment rate of trapped workers:

\[
n_{T,t} = \frac{1}{g_{t,T}} [(1 - \phi_T) (1 - \varpi) n_{T,t-1} + \eta_T (1 - n_{T,t-1})] + \eta_T \nu (1 - n_{P,t-1}) \frac{L_{P,t-1}}{L_{T,t}}.
\]

(19)

and the employment rate of trapped workers:

\[
n_{T,t} = \frac{1}{g_{t,T}} [(1 - \phi_T) (1 - \varpi) n_{T,t-1} + \eta_T (1 - n_{T,t-1})] + \eta_T \nu (1 - n_{P,t-1}) \frac{L_{P,t-1}}{L_{T,t}}.
\]

(18)

where \( L_P \) and \( L_T \) are the labor forces of workers with primary and secondary jobs. \( g_{t,P} = L_{P,t}/L_{P,t-1} \) and \( g_{t,T} = L_{T,t}/L_{T,t-1} \) are the labor force growth for primary and trapped workers. The labor force in the respective sector is equal to the previous period’s labor force plus the net movement of workers who obtain/lose access to primary jobs:

\[
L_{P,t} = L_{P,t-1} - \nu u_{P,t-1} L_{P,t-1} + \varpi n_{T,t-1} L_{T,t-1},
\]

(20)

and

\[
L_{T,t} = L_{T,t-1} + \nu u_{P,t-1} L_{P,t-1} - \varpi n_{T,t-1} L_{T,t-1}.
\]

(21)

Setting the labor force growth rate to zero and omitting time subscripts, we obtain the following steady state value for the employment rate of primary workers:

\[
n_P = \frac{\eta_P (1 - \nu) + (1 - \phi_P) \varpi \frac{L_{P,t}}{L_{T,t}} + \eta_T \nu (1 - n_{P,t-1}) \frac{L_{P,t-1}}{L_{T,t}}}{\phi_P + (\eta_P (1 - \nu)) + (1 - \phi_P) \varpi (1 - [(1 - \phi_T) (1 - \varpi)] + \eta_T)},
\]

(22)

and the employment rate of trapped workers:

\[
n_T = \frac{\eta_T \nu (1 - n_P) \frac{L_{P,t}}{L_{T,t}}}{(1 - [(1 - \phi_T) (1 - \varpi)] + \eta_T)}.
\]

(23)

Logically, if we set \( \nu = \varpi = 0 \), we have two entirely separated types of workers in this economy and the above formula delivers the well-known formula:

\[
n_P = \frac{\eta_P}{\phi_P + \eta_P} \quad \text{and} \quad n_T = \frac{\eta_T}{\phi_T + \eta_T}.
\]

(24)

3 Calibration of the Model

In 2004, 17.2 percent of the East German full time employed workers were below the low wage income threshold, which is defined a two thirds of the East German median income, i.e. they earned below 7.36 € per hour (Rhein and Stamm, 2006). The jobs of these workers are our proxy for the trapped jobs. From Hunt (2004) we know that about 60 to 80 percent of unemployed in East Germany do not "survive" their first year of unemployment, i.e. they leave unemployment within one year, which we interpret as hiring. During the second year of unemployment the non-survival rate drops to much smaller numbers, roughly ranging in the magnitude of 20 to 50 percent (very much dependent on gender and observation period), with even smaller non-survival rates thereafter. It can be assumed that trapped workers represent
a large share of the long-term unemployed since they have lower hiring rates and higher firing rates than primary workers. However, they do not do so exclusively, since primary workers in our model can stay unemployed for several periods without becoming employed and trapped (although the probability is decreasing over time). For simplicity, we set the steady state (indicated by the subscript 0) hiring rate for trapped workers ($\eta_{T,0}$) to 30 percent and the one for primary workers to 80 percent ($\eta_{P,0}$), roughly corresponding to Hunt’s (2004) non-survival rates for long-term and short-term unemployed respectively. In accordance with a transition table for the European Union (one year transition probability from "low pay" to "no pay", see European Commission, 2004), we set the steady state firing rate for trapped workers equal to $\phi_{T,0} = 0.18$. To obtain an aggregate employment rate of 80 percent\textsuperscript{25}, we set the steady state firing rate of primary workers ($\phi_{P,0}$) to 12 percent.

Furthermore, we have to choose an exogenous probability of an employed trapped worker to move to a primary job ($\varpi$). According to Rhein et al. (2005) the probability for German low wage income earners to move beyond the low income threshold after 5 years is 32.5 percent.\textsuperscript{26} The European Commission (2004) calculates a probability of 50 percent for a low-pay worker to move to a higher pay within seven years.\textsuperscript{27} In line with these two pieces of evidence, we set $\varpi = 0.08$.

Note that our model is highly stylized. For simplicity, we assume a constant transition rate over time (both for the movement from the primary to the trapped jobs and vice versa). However, this appears to be reasonable, since it implicitly means that the cumulative probability for an unemployed worker with a primary job of moving to a trapped job increases over time.

The assumption that employed trapped workers have a higher probability of moving to better paid jobs than unemployed trapped workers has strong empirical underpinnings (the probability of moving from low pay to medium pay is 5 times bigger than the probability of moving from no pay to medium pay\textsuperscript{28}), although for simplicity we model this in highly stylized manner (i.e. unemployed trapped workers have zero probability of obtaining a primary job).

Low pay is only loosely connected to workers’ educational background, although it is of course somewhat more common among workers with low qualification. 15.2 percent of the workers in the low-wage sector do not have a completed apprenticeship (versus a share of 11.5 percent of all employed workers), while 60 percent do have an apprenticeship, but not higher educational degree (versus 63.3 percent of all employed workers).\textsuperscript{29} This shows that factors other than the educational background play a very important role in determining the wage level. In our model, endogenous human capital appreciation (which can be interpreted as access to good jobs) and depreciation play this role.

By setting the labor share of primary workers to 76 percent, about 17 percent of all employed workers have trapped jobs; thus corresponding to the numbers by Rhein and Stamm (2006). To obtain a stable initial equilibrium, we set the probability of an unemployed primary worker to lose access to primary jobs ($\upsilon$) to 11.2 percent.\textsuperscript{30} In our initial equilibrium the unemployment rate for primary workers is 12 percent, whereas it amounts to 35 percent for trapped workers.

We set the replacement rates for primary and trapped workers to 65 and 80 percent, respectively.\textsuperscript{31} Aggregate real productivity ($a$, gross value added per worker) in 2005 was about

\textsuperscript{25}This corresponds to the employment rate of dependently employed in East Germany (see Bundesagentur für Arbeit, 2006a, b).

\textsuperscript{26}Corresponding to an average yearly probability of 7.6 percent.

\textsuperscript{27}Corresponding to an average yearly probability of 9.4 percent.

\textsuperscript{28}The numbers are for the European Union in 2000 (See European Commission (2004)). Low pay is defined as an hourly gross wage below 2/3 of the median and medium pay between 2/3 and 4/3 of the median hourly gross wage.

\textsuperscript{29}The numbers are taken from Rhein et al. (2005, p.3).

\textsuperscript{30}This is necessary to guarantee that the condition $\upsilon U_{NT} = \varpi N_T$ holds, i.e. in the old steady state the number of people moving from trapped to non-trapped jobs equals those moving into the other direction.

\textsuperscript{31}The net replacement ratios (unweighted average across six family types) of workers with 67, 100, and 150
€38,000 and real wages \( (w, \text{measured as real labor costs}) \) were about €22,000 in East Germany.\(^{32}\) (All estimates are divided by the German GDP deflator, base year 1991.\(^{33}\)). We set the productivity for trapped workers to 50 percent of the economy’s average, while setting the one of primary workers to 110 percent of the average productivity.

Furthermore, we assume that in the long-run the productivity and all real costs (the wage, the hiring and firing costs and the operating cost \( \varepsilon \)) grow at the same rate of two percent \((\alpha = 1.02)\). All future values are discounted \((\delta)\) at rate 3\%.\(^{34}\)

In the literature firing costs \((f_i)\) and hiring costs \((h_i)\) which amount to 60 percent and 10 percent of labor costs, respectively, are proposed (Chen and Funke, 2003). It is well known that the employment duration is one of the most important determinants of firing costs\(^{35}\). Thus, we set them to 40 percent for trapped workers, whose employment duration is shorter due to higher firing rates, and to 70 percent for primary workers. We assume that all workers have the same bargaining power. It is set equally \((\gamma = 0.195)\) in order to match the aggregate labor costs in East Germany.

We simulate our model in a linearized form, choosing first derivatives of the cumulative function that replicate the employment path from 1991 to 2004 as closely as possible in the homogeneous model. (For the derivation of the linearized equations see Appendix.)

4 Policy Exercises

We now consider the effects of various labor policies in the context of our calibrated model of the East German labor market. We first examine the employment effects of policies targeted at trapped workers, and then investigate untargeted policies. In both cases, we explore the influence of (i) a reduction of the ratio of the firing costs to the wage ("firing cost ratio") together with a fall in the replacement ratio\(^{36}\), (ii) hiring subsidies, (iii) training subsidies that raise the probability of moving from trapped to the primary jobs. For the training subsidies the policy can of course only be targeted at trapped employees.

4.1 Policies Targeted at Trapped Workers

4.1.1 Lower Replacement Rate and Firing Costs

Figure 3 shows the effects of a 5, 10 and 20 percent reduction of both the firing cost ratio (the ratio of firing costs to the wage) and the replacement ratio (the ratio of unemployment benefits to the wage) for trapped workers, which both take place in period 0:

Steady state effects: A lower replacement ratio and a lower firing cost ratio for trapped workers affect the wage bargaining process. They change the fall-back position of both bargaining parties. As a consequence, insiders bid for lower wages. This improves firms’ incentives to hire and retain more of the less productive workers and thus to increase their long-run employment percent of average productivity are 78.25, 68.25, and 64.67 percent, respectively (OECD, 2006).

\(^{32}\)Source: Statistische Ämter des Bundes und der Länder (2006).

\(^{33}\)This is done to make numbers comparable to Snower and Merkl (2006).

\(^{34}\)This is the average real interest rate over last 15 years, calculated as the yearly money market interest rate minus the inflation rate (using the GDP deflator). Source: International Financial Statistics, International Monetary Fund.

\(^{35}\)See e.g. Grund (2006).

\(^{36}\)In Snower and Merkl (2006) we have done several ex-post policy exercises with a model that did not contain traps. Especially during the last years of the observation period (1991-2004), our prediction was more optimistic than the real outcome, suggesting the existence of labor market traps. The first policy exercise is the same as in Snower and Merkl (2006), but the innovation of this paper over Snower and Merkl (2006) is that it models the effects of labor market traps. It turns out that they have far-reaching implications for the effectiveness of employment policies, as shown below.
rate for trapped workers. A 20 percent reduction of the replacement ratio and firing cost ratio makes wages fall to about two thirds of their initial steady state value. But this considerable reduction lifts the trapped workers’ employment rate only from 58 percent to 65 percent. The reason can be found in the microfounded hiring and firing equations. Since trapped workers face a higher steady state firing rate, the expected future profits of an employed trapped worker is smaller than for primary workers. For given operating costs this leads to smaller hiring and firing sensitivities with respect to wage changes.

There are two reasons why the effects on the overall employment rate are quite moderate: (i) Trapped workers cover only a small share of all workers (24 percent). (ii) Only some of the newly hired workers obtain a human capital upgrade which leads to a higher employment rate, while most of the newly hired trapped workers face a high risk of being fired (compared to primary workers). In the long-run a 20 percent reduction of the replacement ratio and firing cost ratio for trapped workers only reduces the share of trapped workers from 24 to 22 percent.

As a consequence, a 20 percent reduction of the replacement ratio and firing cost ratio (inducing a wage reduction to two thirds of the initial value) for trapped workers increases the overall long-run employment rate only by 2 percentage points. This very insensitive reaction may explain why the recent reduction of the wages in East Germany (compared to the productivity) did not have much of an effect on the employment rate (see figure 1). 

Adjustment dynamics: The increased hiring rate and reduced firing rates do not only lift the employment rate for trapped workers. With more employed people and an exogenously given probability to move from trapped to the primary jobs, the upward movement increases. It takes a long time until this development shows its full effects: For a 20 percent reduction of the replacement ratio and the firing cost ratio, 90 percent of the convergence to the new steady state are realized only after 10 years.

37 Note that trapped workers’ wages react more sensitively to cuts in the replacement rate and firings costs than primary workers’ wages.

38 Note that the reduction of the employment rate at the beginning and middle of the nineties can easily be explained by the initial wage shock. However, it is more difficult to explain the development during the last ten years.
Figure 4: Effects of a hiring subsidy for trapped workers

If the replacement ratio of the most unemployment-prone group is reduced (the trapped unemployed), the described policy comes at the price of increased income inequality (between high income and low income earners). While this policy may help some trapped workers who would not have found a job otherwise and who get a chance to move to a primary job, it hurts the trapped insiders who obtain a lower wage and the trapped workers who remain unemployed and receive lower unemployment benefits (due to lower unemployment benefits). 39

4.1.2 Hiring Subsidies

Figure 4 shows the employment effects of a hiring subsidy which is targeted at trapped workers, viz., a subsidy of 50, 75 and 100 percent of the respective wage.

Steady state effects: A hiring subsidy for trapped workers increases the firms’ incentive to hire more workers with lower productivity. Other than in a homogenous economy, hiring subsidies deliver a double dividend. Besides the immediate hiring effects, there is a longer lasting "transition effect," caused by the movement from trapped to primary jobs. The increased employment rate strengthens the upward mobility to primary jobs. A hiring subsidy of 100 percent would for example reduce the share of trapped workers (of the overall workforce) from 24 to 22.5 percent.

Adjustment dynamics: The after effects of the increased movement to primary jobs take some time to work themselves out: for a 100 percent hiring subsidies, 90 percent of the distance to the new steady state is reached after 12 years.

If hiring subsidies are targeted at trapped workers only (as done in the simulation), they are much more cost-effective40 than an untargeted strategy: (i) the deadweight is much lower since the initial steady state hiring rates for trapped workers are below those for primary workers, (ii) the replacement ratio of trapped workers is above those of primary workers and thus the savings (in terms of the respective wage) generated by the job creation are much bigger, (iii) the aforementioned "transition effect." strengthens the overall outcome.

39 See Brown, Merkl and Snower (2006) for a more detailed analysis of the inequality effects of different policies.
40 Defined as employment effect for a given additional government expenditure.
Hiring subsidies need to be financed. According to our simulation, long-run net expenditures caused by a 100 percent hiring subsidy for all trapped workers are about the same as the long-run net savings generated by a 7 percent reduction of the firing cost ratio and replacement ratio.41

Hiring subsidies increase employment, without worsening the living standard of the poorest workers, namely, the unemployed trapped workers (since they continue to receive the same benefits as before). As a consequence, it may be easier from a political economy point of view to implement hiring subsidies than reducing the replacement ratio, which makes the unemployed workers worse off.

4.1.3 Training Measures

Training subsidies or other measures that improve job-related training (e.g. on the job training, qualification courses, training measures, etc.), could improve trapped workers’ productivity and consequently their access to primary jobs. In our model, better training measures can be captured in terms of an increase in the exogenously given probability of moving from trapped to primary jobs ($\pi$). Figure 5 shows what happens if the probability of moving from trapped to primary jobs increases from 8 to 16 percent. The latter number roughly corresponds to a rate found in many other European Union countries, such as Belgium, Denmark, France, Italy the Netherlands or Spain (European Commission, 2004).

Steady state effects: The training measures above raise the economy’s overall steady state employment rate by generating more people primary workers who face higher employment rates. Naturally, the steady state employment rate of trapped workers does not increase, as only the mobility between job types is affected but not the hiring and firing rates. Thus, better training measures change the share of primary relative to trapped workers. The aforementioned policy would increase the share of primary workers from 74 to 86.5 percent.

Adjustment dynamics: It takes a very long time until such a policy shows its full effects.
4.2 Untargeted Policies

4.2.1 Reduction of Unemployment Benefits and Firing Cost Ratio

If the unemployment benefits and firing cost ratio are reduced for all workers (not just for trapped workers), the employment effects will be modified as follows (see figure 6):

(i) The hiring rate for primary workers increases and the firing rate decreases, as firms’ obtain an incentive to hire/retain more of the less productive workers.

(ii) While a higher employment rate for primary workers is reached quickly, there are long-lasting after-effects through the movement of labor from trapped to primary jobs. A lower unemployment rate for primary workers means that fewer people lose access to primary jobs and thus the number of trapped workers shrinks compared to primary workers. While a 20 percent cut in unemployment benefits and firing cost ratio for trapped workers only would increase the primary workers’ share labor share from 76 to 78 percent, extending the policy to the entire economy would increase the primary workers’ labor share from 78 to 88 percent.

(iii) If the firing rate for primary workers goes down, there is a positive spillover effect on the hiring and firing rates for trapped workers (see equations (14) and (15)). Since trapped workers have a constant probability of getting a human capital upgrade in the future, higher retention rates for primary workers increase these workers’ profitability, giving an incentive to firms to retain/hire more of the less productive workers.
4.2.2 Hiring Subsidies

In this section we compare untargeted hiring subsidies (provided to all workers) to those targeted at trapped unemployed (as described in the previous section). Providing a 100 percent hiring subsidy\(^{43}\) to all workers (instead of trapped workers only) would roughly double the employment effects which are shown in the exercise with targeted hiring subsidies. However, untargeted subsidies would come at a substantial cost to the government. Specifically, the net costs\(^{44}\) of such an untargeted strategy would be about 9 times higher than those for a 100 percent hiring subsidy targeted at trapped unemployed. The main reason is the very substantial deadweight effect because the steady state hiring rates for primary workers are substantially larger than for trapped workers.

4.3 Summary of Calibration Results

4.3.1 Kick-Starting East Germany

Our calibration exercise shows that even very significant wage reductions for trapped workers (induced by reductions in the respective replacement ratio and the firing cost ratio) would not be sufficient to bring East Germany to employment levels comparable to West Germany.\(^{45}\) If the replacement ratio and firing cost ratio are reduced for primary workers as well, this does not only make primary workers more profitable for firms, but also improves the average profitability of the trapped workers (each of them receives a human capital upgrade with a certain probability). Consequently, the employment rate for trapped workers will rise. Furthermore, the lower unemployment rate for primary workers will reduce the number of workers who move to trapped jobs, thus increasing the economy’s ratio of primary to trapped workers. Our calibration shows that these spillover effects are very important. Reductions of the replacement ratio and firing cost ratio for all workers can improve the employment rate for trapped workers and in the economy as a whole much more than a policy that is focused on trapped workers.

While an untargeted strategy is more effective for the reduction of the replacement ratio and firing cost ratio, the opposite is true for hiring subsidies. If they are targeted at trapped workers, they turn out to be more cost effective than untargeted hiring subsidies, for the following reasons. In the presence of traps, hiring subsidies yield a double dividend of increased hiring and transition to get access to primary jobs. Furthermore, the associated deadweight for trapped jobs is much smaller than for primary jobs. As shown in our calibration, the net budgetary outlay for an targeted subsidy is one ninth as high as the one for an untargeted hiring subsidy, while it delivers one half of the overall employment effects.

Training measures improve the prospects of trapped workers and thus lift the economy’s employment rate in the long-run. But it takes a long time until they show their full effects.

As shown above, a moderate cut in the replacement ratio and a reduction of the firing cost ratio could be combined with a substantial hiring subsidy in a self-financing policy package. Together with improved training measures these labor market policies would help the East to become somewhat more independent of the "caring hand that cripples"\(^{46}\).

\(^{43}\)Measured in terms of the respective wage.

\(^{44}\)Defined as the costs for the hiring subsidy minus the increased revenue from higher employment (via higher tax revenues with an assumed tax rate of 20 percent and lower costs for unemployment benefits) in the new steady state.

\(^{45}\)This result differs very much from Snower and Merkl (2006) who show in a labor market model without traps that very moderate reforms at the beginning of the nineties would have had substantial positive effects.

\(^{46}\)Snower and Merkl, 2006.
4.3.2 General Lessons for Regional Unemployment Problems

The behavior of the dual labor market, with primary and trapped jobs differs in two substantial respects from a homogenous labor market:

(i) As shown above, even very substantial reductions in the replacement ratio and the firing cost ratio for trapped workers are not sufficient to reduce the unemployment ratio to rates which can usually be observed in continental European countries, say around 10 percent.

(ii) The effects of different labor market policies are much more persistent under a dual labor market than under a homogenous labor market. We illustrate this phenomenon in figure 7. It takes at least a decade for policies like the reduction of the replacement ratio and firing cost ratio or hiring subsidies to show 90 percent of their after effects. Training subsidies need even more time to show 90 percent of their full after effects. For a comparison: In an economy which only consists of primary workers, almost the whole effects of labor market reforms would already be visible after one year ("Primary Workers Only").

5 Robustness of the Numerical Results

Our quantitative results should be treated with caution. Although the parameter values have been chosen with great care, they are subject to uncertainties. On this account it is worth noting that our qualitative conclusions are remarkably robust over the relevant ranges of the calibrated parameters, although of course the quantitative outcomes change somewhat. To give the reader an impression of this property, we provide three robustness checks: (i) a labor market with higher transition probabilities between primary and trapped jobs. (ii) an economy with lower initial steady state firing rates, (iii) an economy with higher steady state hiring rates.

(i) In response to the critique that the mobility between the two types of jobs is too low in the model above, we double the yearly probability of moving from a trapped to a primary job from $\varpi = 0.08$ to $\varpi = 0.16$.\footnote{To keep the share of primary and trapped workers constant (see Rhein and Stamm, 2006), we have to increase the human capital downgrade probability to $v = 0.21$} While the qualitative outcomes are the same, the newly calibrated economy reacts somewhat more sensitively to policies such as changes of the unemployment...
benefits than under our baseline calibration. The reason is straightforward. When trapped workers have a higher probability of becoming primary workers, their present value of profits is more strongly related to those of primary workers (see equation (13)). As shown above, the hiring and firing elasticities for primary workers are bigger than for trapped workers.\footnote{The intuition is that trapped workers have a shorter average job tenure and therefore a lower present value of profits.} Therefore, this higher sensitivity is transmitted from primary to trapped jobs, since the latter have a bigger probability of increasing their human capital. This means that the economy’s employment persistence decreases somewhat. After a 5% reduction of both the firing costs and the replacement rate for both types of workers, it takes 14 years for 90 percent of the after-effects to have worked themselves out in our baseline calibration. The economy with a twice as big upward transition probability reaches the same percentage already after only 9 years. However, even the latter is a remarkably persistent reaction.

(ii) A lower initial steady state firing rate\footnote{Note that both a lower firing rate and a higher hiring rate lead of course to a steady state with a higher employment rate.} increases workers’ average job tenure. Therefore, lower wages (which are, for example, induced by lower unemployment benefits) have a more substantial effect on workers’ average present value of profits, i.e. the hiring and firing decisions are more sensitive to policy changes than with lower initial firing rates. When we decrease primary workers’ steady state firing rate from $\phi_p = 0.12$ to $\phi_p = 0.1$ and when we reduce the unemployment benefits and firing costs by 5 percent for all workers, the employment rate increases by about one half more than under the baseline calibration,\footnote{To keep the shares of the two types of workers constant (both for exercise (ii) and (iii)), we have to increase the human capital downgrade probability $\nu$.} implying a higher overall labor demand elasticity. Note, however, that this reaction would be driven almost entirely by primary workers (where the average present value is much bigger), while the labor demand elasticity for trapped workers would remain almost unaffected. Even when we decrease the initial steady state firing rate for trapped workers from $\phi_T = 0.18$ to $\phi_T = 0.16$, for a 5% reduction of the firing costs and the unemployment benefits (for trapped workers), the overall employment effects are only about one twentieth larger than in the baseline calibration.

Thus, even under different firing rates, the labor demand for trapped workers continues to react very sluggishly. Our conclusion that changes in unemployment benefits and firing costs are most effective if they are untargeted also holds under different initial firing rates.

(iii) Different hiring rates have much less of an effect compared to different firing rates, because the hiring rate has no influence on the present value of firms (see, e.g., equation (15)). When we increase the primary workers’ hiring rate from $\eta_p = 0.8$ to $\eta_p = 0.85$, the labor market reaction after changes in the unemployment benefits and firing costs stays roughly the same as under the baseline calibration.

Therefore, we conclude that our main conclusions are remarkably robust. Since the dual labor market structure makes employment very persistent, it takes a long time until policies show their full effects and it is particularly difficult to reduce the unemployment for trapped workers.\footnote{The same is true regarding the results which policies should be done in untargeted manner and which not.}

6 Concluding Thoughts

The paper explains a puzzling aspect of the European unemployment problem, namely, that unemployment rates are very persistent despite changes in wages relative to productivity. For this purpose, we have developed a dual labor market model with primary and trapped jobs. We calibrate this model with reference to East German data and we show numerically that the labor demand for trapped workers reacts very sluggishly to changes in the unemployment
benefits and firing costs. We propose additional measures to enable workers to escape from the unemployment trap, namely, hiring subsidies and better training schemes.

East Germany is simply an extreme example of this "unemployment trap" phenomenon, which also exists in France, Italy, Spain and elsewhere. The phenomenon makes the inequality across regions especially persistent and policy makers have been at a loss about how to treat this problem. Our paper provides new insights about which policies are useful and effective under these circumstances and on potential trade-offs which policy makers face.

7 References


8 Appendix

8.1 Sequencing of Decisions

<table>
<thead>
<tr>
<th>Decisions:</th>
<th>Time Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Human Capital</td>
<td>Employment Decision</td>
</tr>
<tr>
<td>Upgrading / Depreciation</td>
<td>Bargaining</td>
</tr>
<tr>
<td>Insider Bargaining</td>
<td>2 is Revealed</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Period t</td>
<td>Period t+1</td>
</tr>
</tbody>
</table>

Figure 8: Sequencing of Decisions

8.2 Wage Bargaining

8.2.1 Bargaining: Primary Workers

The expected present value of returns to a primary insider under bargaining agreement \( V^I_{P,t} \) is

\[
V^I_{P,t} = w_{P,t} + \delta \left( (1 - \phi_{P,t+1}) V^I_{P,t+1} + (1 - \phi_{P,t+1}) V^U_{P,t+1} \right), \tag{25}
\]

where \( \delta \) is the discount factor and \( V^U_{P,t+1} \) is the expected present value of returns of an unemployed primary worker and \( V^I_{P,t+1} \) is the expected present value of returns of an employed primary worker. The expected present value of returns to the firm under bargaining agreement is

\[
\Pi^I_{P,t} = (a_{P,t} - w_{P,t}) + \delta \left( (1 - \phi_{P,t+1}) \Pi^I_{P,t+1} - \phi_{P,t+1} f_{P,t+1} \right), \tag{26}
\]

where \( \Pi^I_{P,t+1} \) is the future profit for primary workers.

Under disagreement, the insider’s fallback income is \( b_{P,t} \), assumed equal to the unemployment benefit. The firm’s fallback profit is \( -f_{P,t} \), which is the firing cost per primary employee. Assuming that disagreement in the current period does not affect future returns, the present values of insider’s returns under disagreement is

\[
V'^I_{P,t} = b_{P,t} + \delta \left( (1 - \phi_{P,t+1}) V'^I_{P,t+1} + (1 - \phi_{P,t+1}) V'^U_{P,t+1} \right), \tag{27}
\]

and the present value of the firm’s agreement under disagreement is

\[
\Pi'^I_{P,t} = -f_{P,t} + \delta \left( (1 - \phi_{P,t+1}) \Pi'^I_{P,t+1} - \phi_{P,t+1} f_{P,t+1} \right). \tag{28}
\]

Thus the insider’s bargaining surplus is

\[
V^I_{P,t} - V'^I_{P,t} = w_{P,t} - b_{P,t}, \tag{29}
\]
and the firm’s bargaining surplus is
\[ \Pi_{P,t} - \Pi_{P,t}^I = a_{P,t} - w_{P,t} + f_{P,t}. \]  

The negotiated wage maximizes the Nash product (\( \Lambda \))
\[ \Lambda = (w_{P,t} - b_{P,t})^\gamma (a_{P,t} - w_{P,t} + f_{P,t})^{1-\gamma}. \]  

Thus:
\[ w_{P,t} = (1 - \gamma) b_{P,t} + \gamma (a_{P,t} + f_{P,t}). \]  

8.2.2 Further Assumptions
We assume that the firm costs are proportional to the wage
\[ f_{i,t} = \rho_{i,t} w_{i,t} \]  
where \( i \) is the index for primary (\( P \)) and trapped (\( T \)) workers with the "firm cost ratio" \( \rho_{i,t} \) for the respective worker type and that the unemployment benefit in our model is given by \( b_{i,t} = \beta_{i,t} w_{i,t} \) with the net replacement ratio \( \beta_{i,t} \) in the respective sectors. Thus, the negotiated wage is
\[ w_{i,t} = \frac{\gamma}{(1 - \beta_{i,t}(1 - \gamma) - \rho\gamma)} a_{i,t}. \]  

8.3 Model Derivation
8.3.1 Trapped Workers’ Profit
Trapped workers have an average productivity \( a_T \) and there is an exogenously given probability \( \varpi \) for employed workers to move to become primary workers. Firms take the regime switch into account (upgrade of trapped to primary workers), which increases the profitability.

<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Human Capital Upgrading</td>
<td>( \varpi )</td>
</tr>
<tr>
<td>No Upgrading + Firing</td>
<td>( (1 - \varpi) \phi_T )</td>
</tr>
<tr>
<td>No Upgrading + Retention</td>
<td>( (1 - \varpi)(1 - \phi_T) )</td>
</tr>
</tbody>
</table>

The profit function below \( (\Pi_{t,\text{regime}1}) \) corresponds to the first regime (average profits weighted with the probability that workers stay trapped):

\[ \Pi_{t,\text{regime}1} = -\varepsilon_t + \sum_{t=0}^{\infty} \delta^t (1 - \phi_T)^t (1 - \varpi)^t (a_T - w_T) - \\
(1 - \varpi) \delta \phi_T \sum_{t=0}^{\infty} \delta^t (1 - \varpi)^t (1 - \phi_T)^t \]

\[ \Pi_{t,\text{regime}1} = -\varepsilon_t + \frac{a_T - w_T - \delta (1 - \varpi) \phi_T f_T}{1 - \delta (1 - \phi_T)(1 - \varpi)}. \]  

In each subsequent period a worker moves with probability \( \varpi \) from a trapped to a primary job. The profit function below \( (\Pi_{t,\text{regime}2}) \) corresponds to the second regime:

\[ \Pi_{t,\text{regime}2} = \delta \varpi \sum_{t=0}^{\infty} \delta^t (1 - \varpi)^t (1 - \phi_T)^t \]

\[ = \left[ \frac{-\phi_P f_P + (1 - \phi_P) \left( \sum_{t=0}^{\infty} \delta^t (1 - \phi_P)^t (a_P - w_P) - \right)}{\delta \phi_P f_P \sum_{t=0}^{\infty} \delta^t (1 - \phi_P)^t} \right]. \]
The second line of the formula describes the present value of a worker if she is upgraded to a primary job. An upgraded primary worker has the probability \( \phi_P \) of not being fired immediately and a probability \( (1 - \phi_P) \) of being retained. If the latter is the case, she has the same expected profit stream as a primary workers: \( \sum_{t=0}^{\infty} \delta^t (1 - \phi_P)^t (a_P - w_P - \delta \phi_P f_P) \sum_{t=0}^{\infty} \delta^t (1 - \phi_P)^t \). Since every period a certain fraction of workers moves to primary jobs, we have to write a double sum. A fraction \( \varpi \) among those who have not been fired \( (1 - \varpi)^t (1 - \phi_T)^t \) moves to primary jobs.

\[
\Pi_{t,\text{regime2}} = \delta \varpi \sum_{t=0}^{\infty} \delta^t (1 - \varpi)^t (1 - \phi_T)^t \left[ -\phi_P f_P + (1 - \phi_P) \left( \frac{a_P - w_P - \delta \phi_P f_P}{1 - \delta (1 - \phi_P)} \right) \right].
\]

(37)

\[
\Pi_{t,\text{regime2}} = -\phi_P \delta \varpi \left( \frac{f_P}{1 - \delta (1 - \phi_T)} \right) + (1 - \phi_P) \delta \varpi \left( \frac{a_P - w_P - \delta \phi_P f_P}{(1 - \delta (1 - \phi_P))(1 - \delta (1 - \varpi)(1 - \phi_T))} \right).
\]

(38)

Thus, the overall expected profit \( \Pi_t = \Pi_{t,\text{regime1}} + \Pi_{t,\text{regime2}} \) is:

\[
\Pi_t = -\varepsilon_t + \frac{a_P - w_P - \delta (1 - \varpi) \phi_T f_T}{1 - \delta (1 - \phi_T)(1 - \varpi)} - \phi_P \delta \varpi \left( \frac{f_P}{1 - \delta (1 - \phi_T)} \right) + (1 - \phi_P) \delta \varpi \left( \frac{a_P - w_P - \delta \phi_P f_P}{(1 - \delta (1 - \phi_P))(1 - \delta (1 - \varpi)(1 - \phi_T))} \right).
\]

(39)

### 8.3.2 Employment Dynamics

**Primary Workers:** The (primary) employment in period \( t \) is equal to the people who are retained, both from the pool of employed \( (N_{Pt-1}) \) and from the human capital upgrades \( (\varpi N_{T,t-1}) \). The two groups have the same retention probability \( 1 - \phi_P \). A proportion \( \eta_P \) of the unemployed primary workers is hired. The pool of primary unemployed workers is reduced by a share \( v \) (workers who move to trapped jobs).

\[
N_{Pt} = (1 - \phi_P) N_{Pt-1} + (1 - \phi_P) \varpi N_{T,t-1} + \eta_P U_{Pt-1} - \eta_P v U_{Pt-1}.
\]

(40)

\[
N_{Pt} = (1 - \phi_P) N_{Pt-1} + (1 - \phi_P) \varpi N_{T,t-1} + (\eta_P (1 - v)) U_{Pt-1}.
\]

(41)

Next, we introduce \( g_{t,P} \) which is the growth rate of the primary workforce from period \( t - 1 \) to \( t \) \( (g_{t,P} = L_{Pt}/L_{Pt-1}) \).

Dividing by \( L_{Pt} \), we obtain:

\[
n_{Pt} = \frac{1}{g_{t,P}} (1 - \phi_P) n_{Pt-1} + (1 - \phi_P) \varpi \frac{N_{T,t-1}}{L_{Pt}} + \frac{1}{g_{t,P}} (\eta_P (1 - v)) (1 - n_{Pt-1}).
\]

(42)
\[
N_{P,t} = \frac{1}{g_{t,P}} [(1 - \phi_P) n_{P,t-1} + (\eta_P (1 - \nu)) (1 - n_{P,t-1})] \\
+ (1 - \phi_P) \omega \frac{L_{T,t-1}}{L_{P,t}} n_{T,t-1}.
\] (43)

The primary labor force is equal to the previous period’s labor force plus the net movement from trapped jobs:
\[
L_{P,t} = L_{P,t-1} - \nu u_{P,t-1} L_{P,t-1} + \omega n_{T,t-1} L_{T,t-1}. 
\] (44)

In the steady state, the growth rate of the labor force is equal to 0 \((g_{t,P} = L_{P,t}/L_{P,t-1} = 1)\) and all time indices can be dropped. Thus, the following equation holds:
\[
n_P (\phi_P + (\eta_P (1 - \nu))) = (1 - \nu) \eta_P + \omega (1 - \phi_P) n_T \frac{L_T}{L_P}.
\] (45)

And the following constraint (human capital upgrades must equal downgrades) has to hold in the steady state:
\[
\nu U_P = \omega N_T.
\] (46)

**Trapped Workers:** The trapped employed equal the retained workers from the previous period (who did not receive a human capital upgrade: \((1 - \omega) N_{T,t-1}\)) plus the hired trapped unemployed (their number has been enlarged by the human capital depreciation: \(\eta_T U_{T,t-1} + \eta_T \nu U_{P,t-1}\)):
\[
N_{T,t} = (1 - \phi_T) (1 - \omega) N_{T,t-1} + \eta_T U_{T,t-1} + \eta_T \nu U_{P,t-1}.
\] (47)

Dividing by \(L_{T,t}\):
\[
n_{T,t} = \frac{1}{g_{t,T}} [(1 - \phi_T) (1 - \omega) n_{T,t-1} + \eta_T (1 - n_{T,t-1})] + \eta_T \nu \frac{U_{P,t-1}}{L_{T,t}}.
\] (48)

\[
n_{T,t} = \frac{1}{g_{t,T}} [(1 - \phi_T) (1 - \omega) n_{T,t-1} + \eta_T (1 - n_{T,t-1})] + \\
\eta_T \nu (1 - n_{P,t-1}) \frac{L_{P,t-1} \frac{L_T}{L_P}}{L_{T,t}}.
\] (49)

The trapped labor force is equal to the previous period’s labor force plus the net movement from primary unemployed:
\[
L_{T,t} = L_{T,t-1} + \nu u_{P,t-1} L_{P,t-1} - \omega n_{T,t-1} L_{T,t-1}.
\] (50)

In the steady state the following relationship holds:
\[
n_T = (1 - \phi_T) (1 - \omega) n_T + \eta_T (1 - n_T) + \eta_T \nu (1 - n_P) \frac{L_T}{L_P}.
\] (51)

\[
n_{T,t} = \frac{\eta_T + \eta_T \nu (1 - n_P) \frac{L_T}{L_P}}{(1 - (1 - \phi_T) (1 - \omega) + \eta_T)}.
\] (52)

Inserting (52) into (45), we obtain the following steady state relationship:
\[
\eta_P (1 - v) + (1 - \phi_P) \varpi \frac{\eta_I + \eta_T (1 - n_P) \frac{L_P}{L_T}}{(1 - (1 - \phi_T)(1 - \varpi) + \eta_T} L_T, \tag{53}
\]

\[
n_P = \frac{\eta_P (1 - v) + (1 - \phi_P) \varpi \frac{\eta_I + \eta_T (1 - n_P) \frac{L_P}{L_T}}{(1 - (1 - \phi_T)(1 - \varpi) + \eta_T} L_T}{\phi_P + (\eta_P (1 - v) + (1 - \phi_P) \varpi \frac{\eta_I + \eta_T (1 - n_P) \frac{L_P}{L_T}}{(1 - (1 - \phi_T)(1 - \varpi) + \eta_T} L_T}, \tag{54}
\]

If \( \nu = \varpi = 0 \), we have two entirely separated sectors in this economy and we obtain the following steady state relationship:

\[
n_P = \frac{\eta_P}{\phi_P + \eta_P}. \tag{55}
\]

8.4 Derivations for the Calibration

8.4.1 Non-Trapped Workers

The detailed derivations of the steady state firing and hiring rates under different policy exercises is analogous to Snower and Merkl (2006)\textsuperscript{52}, providing the following linearized equations:

\[
\phi_{P,\text{new}} = \phi_{P,0} - A_P [(a_{P,\text{new}} - w_{P,\text{new}}) - (a_{P,0} - w_{P,0})]
- C_P (f_{P,\text{new}} - f_{P,0}), \tag{56}
\]

and

\[
\eta_{P,\text{new}} = \eta_{P,0} + G_P [(a_{P,\text{new}} - w_{P,\text{new}}) - (a_{P,0} - w_{P,0})]
- I_P (f_{P,\text{new}} - f_{P,0}) - K_P (h_{P,\text{new}} - h_{P,0}) - L_P (\phi_{P,\text{new}} - \phi_{P,0}), \tag{57}
\]

where all coefficients \( A_P \) to \( L_P \) have a positive sign.

8.4.2 Trapped Workers

Firing Rate: A worker is fired if \( \pi_t < -f_T \).

\[
\phi_T = 1 - \Gamma \left( \frac{a_T - w_T - \delta (1 - \varpi) \phi_T f_T}{1 - \delta (1 - \phi_T)(1 - \varpi)} + f_T - \phi_P \delta \varpi \frac{f_T}{(1 - \delta (1 - \phi_T)(1 - \varpi))} + (1 - \phi_P) \delta \varpi \frac{a_T - w_T - \delta (1 - \varpi) \phi_T f_T}{1 - \delta (1 - \phi_T)(1 - \varpi)} \right). \tag{58}
\]

For the calibration we deflate all variables to their 1991 real value (using German GDP deflator\textsuperscript{53}) and take into account a 2\% (\( \alpha = 1.02 \)) growth rate of all variables (\( a, w, f \)) and the operating costs to make the calibration more realistic and comparable to Snower and Merkl (2006).

\[
\phi_T = 1 - \Gamma \left( \frac{1}{\alpha^{15}} \left( \frac{1}{\alpha^{15}} \frac{a_T - w_T - \delta (1 - \varpi) \phi_T f_T}{1 - \delta (1 - \phi_T)(1 - \varpi)} + f_T - \phi_P \delta \varpi \frac{f_T}{(1 - \delta (1 - \phi_T)(1 - \varpi))} + (1 - \phi_P) \delta \varpi \frac{a_T - w_T - \delta (1 - \varpi) \phi_T f_T}{1 - \delta (1 - \phi_T)(1 - \varpi)} \right) \right). \tag{59}
\]

\textsuperscript{52}See page 39 of the detailed version.

\textsuperscript{53}Source: International Financial Statistics, International Monetary Fund.
Next, we take a first order Taylor approximation for the firing rate (where the subscript "0" refers to old steady state values and the subscript "new" refers to new steady state values). Therefore, we need the first derivatives at the old steady state with respect to the following variables:

\[
\frac{\partial \phi_{T,0}}{\partial (a_T - w_T)} = - \frac{1}{\alpha^15} \Gamma'_{f,0} \left[ \frac{1}{1 - \alpha \delta (1 - \phi_T) (1 - w)} \right],
\]

\[
\frac{\partial \phi_{T,0}}{\partial (a_P - w_P)} = - \frac{1}{\alpha^15} \Gamma'_{f,0} \left[ \frac{(1 - \phi_P) \delta \alpha w}{(1 - \delta \alpha (1 - \phi_P)) (1 - \delta \alpha (1 - w) (1 - \phi_T))} \right],
\]

\[
\frac{\partial \phi_{T,0}}{\partial \phi_T} = - \frac{1}{\alpha^15} \Gamma'_{f,0} \left[ \frac{-\delta \alpha \phi_T (1 - w)}{1 - \alpha \delta (1 - \phi_T) (1 - w)} + 1 \right],
\]

\[
\frac{\partial \phi_{T,0}}{\partial \phi_P} = - \frac{1}{\alpha^15} \Gamma'_{f,0} \left[ \frac{-\delta \alpha^2 \phi_P}{(1 - \delta \alpha (1 - \phi_P))(1 - \delta \alpha (1 - w) (1 - \phi_T))} \right],
\]

\[
\frac{\partial \phi_{T,0}}{\partial \phi_T} = - \frac{1}{\alpha^15} \Gamma'_{f,0} \left[ \frac{[(1 - \delta \alpha (1 - \phi_P))(1 - \delta \alpha (1 - w) (1 - \phi_T))]}{[(1 - \delta \alpha (1 - \phi_P))(1 - \delta \alpha (1 - w)(1 - \phi_T))]} - \frac{-\delta \alpha^2 \phi_P}{(1 - \delta \alpha (1 - \phi_P))(1 - \delta \alpha (1 - w)(1 - \phi_T))} \right],
\]

\[
\frac{\partial \phi_{T,0}}{\partial \phi_P} = - \frac{1}{\alpha^15} \Gamma'_{f,0} \left[ \frac{[1 - \delta \alpha (1 - \phi_P)] [1 - \delta \alpha (1 - w) (1 - \phi_T)]}{[1 - \delta \alpha (1 - \phi_P)(1 - \delta \alpha (1 - w) (1 - \phi_T))]} \right]
\]

Thus, we obtain the following expression:
\[ \phi_{T,\text{new}} = \phi_{T,0} + \frac{\partial \phi_{T,0}}{\partial (a_T - w_T)} \left[ (a_{T,\text{new}} - w_{T,\text{new}}) - (a_{T,0} - w_{T,0}) \right] + \frac{\partial \phi_{T,0}}{\partial (a_P - w_P)} \left[ (a_{P,\text{new}} - w_{P,\text{new}}) - (a_{P,0} - w_{P,0}) \right] + \frac{\partial \phi_{T,0}}{\partial f_T} (f_{T,\text{new}} - f_{T,0}) + \frac{\partial \phi_{T,0}}{\partial f_P} (f_{P,\text{new}} - f_{P,0}) + \frac{\partial \phi_{T,0}}{\partial f_P} (\phi_{T,\text{new}} - \phi_{T,0}) + \frac{\partial \phi_{T,0}}{\partial f_P} (\phi_{P,\text{new}} - \phi_{P,0}) \]
\[
\phi_{T,\text{new}} = \phi_{T,0} - A_T [(a_{T,\text{new}} - w_{T,\text{new}}) - (a_{T,0} - w_{T,0})] \\
-B_T [(a_{P,\text{new}} - w_{P,\text{new}}) - (a_{P,0} - w_{P,0})] - C_T (f_{T,\text{new}} - f_{T,0}) \\
+D_T (f_{P,\text{new}} - f_{P,0}) + E_T (\phi_{T,\text{new}} - \phi_{T,0}) + F_T (\varpi_{\text{new}} - \varpi_0). 
\]

where \( A_T \) to \( F_T \) are all positive constants.

Thus, higher productivity and lower wages lead to a reduction of the firing rate. Higher firing costs for trapped workers reduce firing (not taking their indirect effect via the wage formation into account which outweighs the direct effect), whereas higher firing costs for primary workers increase firing for trapped workers. There is a positive spillover effect from the firing rate for primary to the trapped workers, i.e. if the firing rate for primary workers is reduced, the same is true for the firing rate for trapped workers. Furthermore, a higher intersectoral mobility reduces firing for trapped workers (as the average profitability of trapped workers increases).

**Hiring Rate:** A worker is hired if \( \pi_t > h_T \). Thus:

\[
\eta_T = \Gamma \left( \frac{a_T - w_T - \delta (1 - \varpi) \phi_T f_T}{1 - \delta (1 - \phi_T)(1 - \varpi)} + h_T + \frac{\delta (1 - \phi_P)(1 - \varpi)}{1 - \delta (1 - \phi_P)(1 - \varpi)} \right). 
\]

Analogous to the firing rate the hiring rate is re-written as:

\[
\eta_T = \Gamma \left( \frac{1}{\alpha^{15}} \left( \frac{a_T - w_T - \delta \alpha (1 - \varpi) \phi_T f_T}{1 - \delta \alpha (1 - \phi_T)(1 - \varpi)} + h_T + \frac{\delta \alpha (1 - \phi_P)(1 - \varpi)}{1 - \delta \alpha (1 - \phi_P)(1 - \varpi)} \right) \right). 
\]

To obtain the first order Taylor approximation, we need to calculate the first partial derivatives:

\[
\frac{\partial \eta_{T,0}}{\partial (a_T - w_T)} = \frac{1}{\alpha^{15}} \Gamma_{h,0} \left[ \frac{1}{1 - \alpha \delta (1 - \phi_T)(1 - \varpi)} \right], 
\]

\[
\frac{\partial \eta_{T,0}}{\partial (a_P - w_P)} = \frac{1}{\alpha^{15}} \Gamma_{h,0} \left[ \frac{(1 - \phi_P) \delta \alpha \varpi}{(1 - \delta \alpha (1 - \phi_P))(1 - \delta \alpha (1 - \varpi)(1 - \phi_T))} \right], 
\]

\[
\frac{\partial \eta_{T,0}}{\partial f_T} = \frac{1}{\alpha^{15}} \Gamma_{h,0} \left[ \frac{-\delta \alpha \phi_T (1 - \varpi)}{1 - \alpha \delta (1 - \phi_T)(1 - \varpi)} \right], 
\]

\[
\frac{\partial \eta_{T,0}}{\partial f_P} = \frac{1}{\alpha^{15}} \Gamma_{h,0} \left[ \frac{-\delta \alpha \phi_P (1 - \varpi)}{(1 - \delta \alpha (1 - \phi_P)(1 - \varpi))} \right], 
\]

\[
\frac{\partial \eta_{T,0}}{\partial h} = \frac{1}{\alpha^{15}} \Gamma_{h,0}. 
\]

\[
\frac{\partial \eta_{T,0}}{\partial \varpi} = \frac{1}{\alpha^{15}} \Gamma_{h,0} \left[ \left( \frac{-\delta \alpha f_T (1 - \varpi)(1 - \delta \alpha (1 - \phi_T)(1 - \varpi)) - \delta \alpha (1 - \varpi) [a_T - w_T - \delta \alpha (1 - \varpi) \phi_T f_T]}{(1 - \delta \alpha (1 - \phi_T)(1 - \varpi))^2} \right) \right], 
\]

\[
\frac{\partial \eta_{T,0}}{\partial \phi_T} = \frac{1}{\alpha^{15}} \Gamma_{h,0} \left[ \left( \frac{-\delta \alpha f_T (1 - \varpi)(1 - \delta \alpha (1 - \phi_T)(1 - \varpi)) - \delta \alpha (1 - \varpi) [a_T - w_T - \delta \alpha (1 - \varpi) \phi_T f_T]}{(1 - \delta \alpha (1 - \phi_T)(1 - \varpi))^2} \right) \right], 
\]

\[
\frac{\partial \eta_{T,0}}{\partial \phi_P} = \frac{1}{\alpha^{15}} \Gamma_{h,0} \left[ \left( \frac{-\delta \alpha f_T (1 - \varpi)(1 - \delta \alpha (1 - \phi_T)(1 - \varpi)) - \delta \alpha (1 - \varpi) [a_T - w_T - \delta \alpha (1 - \varpi) \phi_T f_T]}{(1 - \delta \alpha (1 - \phi_T)(1 - \varpi))^2} \right) \right], 
\]
\[
\frac{\partial \eta_{T,0}}{\partial \phi_P} = \frac{1}{\alpha^{15} \Gamma'_{h,0}} [ (1 - \delta \alpha (1 - \phi_P)) (1 - \delta \alpha (1 - \varpi) (1 - \phi_T)) ] \\
\frac{\partial \eta_{T,0}}{\partial \varpi} = \frac{1}{\alpha^{15} \Gamma'_{h,0}} [ (1 - \delta \alpha (1 - \phi_P)) \frac{\delta \alpha \varpi (a_P - w_P) - \delta^2 \alpha^2 \varpi f_P (1 - 2 \phi_P) - \delta^2 \alpha^2 \varpi f_P (1 - \phi_P) (\delta \alpha (1 - \varpi)) + \delta^2 \alpha^2 \varpi f_P (1 - \phi_P)}{(1 - \delta \alpha (1 - \varpi) (1 - \phi_P))^{\frac{1}{2}}} ] (79)
\]

Thus, the first order Taylor approximation is

\[
\eta_{T,new} = \eta_{T,0} + \frac{\partial \eta_{T,0}}{\partial (a_T - w_T)} [(a_{T,new} - w_{T,new}) - (a_{T,0} - w_{T,0})] \\
+ \frac{\partial \eta_{T,0}}{\partial (a_P - w_P)} [(a_{P,new} - w_{P,new}) - (a_{P,0} - w_{P,0})] \\
+ \frac{\partial \eta_{T,0}}{\partial f_T} (f_{T,new} - f_{T,0}) + \frac{\partial \eta_{T,0}}{\partial f_P} (f_{P,new} - f_{P,0}) \\
+ \frac{\partial \eta_{T,0}}{\partial \phi_T} (\phi_{T,new} - \phi_{T,0}) + \frac{\partial \eta_{T,0}}{\partial \phi_P} (\phi_{P,new} - \phi_{P,0}) \\
+ \frac{\partial \eta_{T,0}}{\partial \varpi} (\varpi_{new} - \varpi_0). (81a)
\]

Or by substituting the coefficients

\[
\eta_{T,new} = \eta_{T,0} + G_T [(a_{T,new} - w_{T,new}) - (a_{T,0} - w_{T,0})] \\
+ H_T \begin{pmatrix} a_{P,new} \\ -w_{P,new} \\ a_{P,0} \\ -w_{P,0} \end{pmatrix} - I_T \begin{pmatrix} f_{T,new} \\ -f_{T,0} \end{pmatrix} - J_T \begin{pmatrix} f_{P,new} \\ -f_{P,0} \end{pmatrix} \\
- K_T \begin{pmatrix} h_{T,new} \\ -h_{T,0} \end{pmatrix} - L_T \begin{pmatrix} \phi_{T,new} \\ -\phi_{T,0} \end{pmatrix} - M_T \begin{pmatrix} \phi_{P,new} \\ -\phi_{P,0} \end{pmatrix} + N_T \left( \varpi_{new} - \varpi_0 \right). 
\]

where \( G_T \) to \( N_T \) are all positive coefficients. The rationale for the signs of the coefficients is the same as for the linearized firing rate for trapped workers.\(^{54}\)

\(^{54}\)For the linearized model a value for the first derivative of the cumulative function has to be chosen (\( \Gamma' \)). Snower and Merkl (2006) set the same values for the firing and hiring rate, while we choose (\( \Gamma'_h = 6 \times 10^{-7} \)) and (\( \Gamma'_h = 6 \times 10^{-6} \)), where \( f \) and \( n \) stand for the firing and hiring rate respectively. In the homogenous model, this provides us with a similar labor demand elasticity and thus a similar employment path, but is more in line with the empirical evidence on hiring and firing elasticities (for a summary, see Orszag and Snower, 1999).