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ABSTRACT

Gender Based Taxation and the Division of Family Chores*

Gender Based Taxation (GBT) satisfies Ramsey’s optimal criterion by taxing less the more elastic labor supply of (married) women. This holds when different elasticities between men and women are taken as exogenous and primitive. But in this paper we also explore differences in gender elasticities which emerge endogenously in a model in which spouses bargain over the allocation of home duties. GBT changes spouses’ implicit bargaining power and induces a more balanced allocation of house work and working opportunities between males and females. Because of decreasing returns to specialization in home and market work, social welfare improves by taxing conditional on gender. When income sharing within the family is substantial, both spouses may gain from GBT.

JEL Classification: D13, H21, J16, J20

Keywords: optimal taxation, economics of gender, family economics, elasticity of labor supply

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1 Introduction

According to optimal taxation theory a benevolent government should tax less the goods and services which have a more elastic supply. Women labor supply is more elastic than men’s. Therefore, tax rates on labor income should be lower for women than for men.

This argument is well known in the literature, but it is not taken seriously as a policy proposal. This is surprising since a host of other gender based policies are routinely discussed, and often implemented, such as gender based affirmative action, quotas, different retirement policies for men and women, and also indirect gender based policies like child care subsidies, and maternal leaves.\(^1\) Many of these gender based interventions become even more puzzling in light of the basic economic principle that society should prefer policies interfering with “prices” (such as the tax rate) rather than “quantities” (such as affirmative action or quotas) in the market.\(^2\)

The optimality of GBT hinges on different elasticities of the labor supply between men and women. If the labor supply elasticity is taken as a primitive, exogenous parameter that differentiates genders, then the argument is quite straightforward. GBT provides substantial welfare and GDP gains because it minimizes the aggregate social loss from labor market distortions. As we discuss in more detail later, this argument is robust to perturbations in the modeling framework (Ramsey or Mirrlees), and goes through in extentions of the model that consider cross elasticities, heterogenous households and household production.

However, differences in labor supply functions of men and women, in-

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\(^1\)For instance, gender based affirmative action is common in the US, Spain has recently introduced stringent quotas for female employment in many sectors and public support for child care is common in many European countries. Sweden has recently reformed paternal leave policies with the goal of inducing males to stay more at home with children and females to participate more continuously in the labor market.

\(^2\)In international trade, for instance, a sort of “folk theorem” states that tariffs are weakly superior to import quotas as a trade policy. Taxing polluting activities is generally considered superior to controlling them with quantitative restrictions.
cluding their elasticities may not only depend on innate characteristics or preferences but emerge endogenously from the internal organization of the family. In fact, as documented for instance by Goldin (2006), Blau and Kahn (2007) and Albanesi and Olivetti (2007), both women participation rate and the elasticity of their labor supply, may evolve over time as a result of technologically induced or culturally induced changes in the organization of the family. Therefore, we also explore the case in which men and women are identical in terms of innate abilities, preferences and predispositions, but men have more explicit bargaining power at home (possibly for historical reasons). In this case, a gendered allocation of work at home and gender differences in market participation and labor elasticities derive exclusively from the intra-family bargaining. If men have a stronger bargaining power, then they assume fewer unpleasant, tiring home duties. Hence, they participate more in the market, exercise more effort, and earn more than their female spouses. The possibility to avoid home duties allows men to engage in careers that offer “upside potential” in terms of wages and promotions. For women, it is the opposite: they basically work only for their wage. As a result, men are less sensitive to changes in the wage since what matters for them, relative to women, is also the intrinsic expected pleasure they derive from careers and market activity. We note that the implied positive correlation between the amount of home duties and the elasticity of labor supply in our model accords well with recent empirical evidence. Aguiar and Hurst (2007) and Blau and Kahn (2007) document a decline both in the ratio of female over male home duty and the ratio of female over male elasticity of labor supply in the last 50 years.

To the extent that the division of family chores is unbalanced, GBT improves welfare. In addition to satisfying the Ramsey principle of optimal

3Alesina and Giuliano (2007) study the effect of different cultural traits on family values and ties as a determinant of women participation in the labor force. Ichino and Moretti (2006) show instead how more persistent biological gender differences may affect the absenteeism of men and women and, indirectly, labor market equilibria.
taxation, GBT generates a more equitable allocation of house versus market work. Because of decreasing returns to scale, reallocating “the last hour that the mother spends with the children to the father” is welfare improving for the family as a whole, and under certain conditions it can be welfare improving for both spouses (as well as for children). Our numerical simulations show that given a difference in the labor supply elasticities calibrated on US estimates (which in the model maps into a corresponding difference in the bargained allocation of home duties), GBT implies rather different tax rates for husbands and wives and can substantially improve welfare, as well as increase GDP and total employment.

We can also interpret our result in terms of a difference between short versus long run effects of GBT. The case in which labor supply functions and their different elasticities across gender are exogenous can be interpreted as the short run, namely an horizon in which the family organization and the allocation of home duties is not likely to change. In the long run, instead, the family responds to government policies and evolves to a new equilibrium with a different organization of allocation of home duties.

We take a different approach from the literature in modeling household production. The traditional approach builds on the Beckerian theory of the allocation of time (1965), and assumes that household duty is an input to the family production function for the production of a household good. In our model with endogenous gender differences in elasticities we start by a woman and a man who form a family and receive a collection of shocks that must be allocated between the two spouses. With this assumption we intend to capture the fact that there are features of the daily household routine, for example a sick child or a broken dishwasher, that are easy to conceptualize as exogenous but negotiable jobs to be done but not as the output of an intra-household process that transforms time input into a household good. Obviously the two approaches are not mutually exclusive and a more general model of household allocation of time and shocks could capture both aspects
of family life.

We further illustrate the link between our model and the literature in Section 2. Section 3 discusses GBT in the short run, that is when gender differences in labor supply elasticities are held constant. In Section 4 we endogenize the allocation of household chores and in Section 5 we show how family bargaining implies an intra-household division of duties, market participation and elasticities. In what we call the long run the government sets taxes anticipating the family’s reaction to fiscal pressure. This is analyzed in Section 6. Section 7 concludes.

2 Related Literature

The present paper lies at the intersection of three strands of research. The first is concerned with the structure of the family.\(^4\) The traditional “uni-
tary” approach, in the spirit of Samuelson (1956) and Becker (1974), treats the household as a single decision making unit. Although this approach is closely linked with the traditional consumer’s theory, it is at odds with the notion of individualism, and, most importantly for our purposes, lacks the proper foundations to conduct intrahousehold welfare analysis.\(^5\) The “collective approach” to family modeling, initiated by Chiappori (1988, 1992) and Apps and Rees (1988), builds instead on the premise that every person has well defined individual preferences and only postulates that collective decisions lie on the Pareto frontier.\(^6\) In fact the (long run) model that we consider is in the spirit of the collective approach with (asymmetric) Nash-bargained


\(^5\)Two notable empirical failures of the unitary model are the restrictions that arise from the income pooling hypothesis and the symmetry of the Slutsky matrix. See, for example, Thomas (1990), Browning, Bourguignon, Chiappori and Lechene (1994), Lundberg, Pollak and Wales (1997), and Browning and Chiappori (1998).

\(^6\)A more specific approach, taken first by Manser and Brown (1980) and McElroy and Horney (1981), “selects” a specific point on the Pareto frontier by assuming that members of the family Nash-bargain over the allocation of commodities. Lundberg and Pollak (1993), instead, argue that threat points are internal to the marriage and can be seen as (possibly inefficient) non-cooperative equilibria of the marriage game.
household allocations. The difference with the above models is that the bargaining is not on the allocation of consumption, income and labor supply per se, but on the allocation of home duties.

The second relevant strand of literature refers to the taxation of couples. The “conventional wisdom” says that under specific assumptions, we should tax at a lower rate goods that are supplied inelastically as suggested by Ramsey (1927). The application of the Ramsey “inverse elasticity” rule in a model of labor supply implies that males should be taxed on a higher tax schedule than females because they have a less elastic labor supply function. This point was made by Rosen (1977) and Boskin and Sheshinski (1983).\footnote{The argument was raised using variants of the Diamond and Mirrlees (1971a and 1971b) and Atkinson and Stiglitz (1972) frameworks, also adopted in this paper. Using the Mirrlees (1971) approach, the elasticity of labor supply reappears in the optimal tax schedule, albeit in a less clear way. For an ambitious paper that takes the latter approach see Kleven, Kreiner and Saez (2006), or Kremer (2003) within an application to the problem of age based taxation. We also note that in a Mirrleesian application of our model, there is one more factor in favor of GBT: since the female distribution of income has more mass concentrated towards the low income levels, its hazard rate is typically higher and therefore marginal tax rates for females should be lower.}

Since gender is inelastically supplied, this proposition relates also to the insight that taxes should be conditioned on non-modifiable characteristics as in Akerlof (1978) and Kremer (2003).\footnote{See Mankiw and Weinzierl (2007) for a recent application of this idea aimed at discussing the validity of the welfarist approach to optimal taxation.}

This conventional wisdom regarding lower taxes for women can be challenged or reinforced in at least three ways. First, it might be the case that the female’s tax rate is a better policy instrument when considering across household redistribution. Boskin and Sheshinski (1983) show that this is not the case in their numerical calculations. Recently, Apps and Rees (2007) place the conventional wisdom on a firmer basis and give intuitive and empirically plausible conditions under which it is optimal to tax males at a higher rate even with heterogeneous households. Second, Piggott and Whalley (1996) raise the issue of intrahousehold distortion of efficiency in models...
with household production. Since the optimal tax schedule must maintain productive efficiency (Diamond and Mirrlees 1971a), imposing differential tax treatment distorts the intrahousehold allocation of resources and raises a further cost for the society. Although the Piggott and Whalley argument is intuitive, Apps and Rees (1999b) and Gottfried and Richter (1999) show that the cost of distorting the intra-household allocation of resources cannot offset the gains from taxing on an individual basis according to the standard Ramsey principle. We are interested in exploring the optimality of individual taxes in a model where within household redistribution is explicitly taken into account. In that respect our (long run) model is in line and reinforces the conventional wisdom.9

The third strand of literature attempts to explain gender differences in labor markets. For example, Albanesi and Olivetti (2006) propose that gender differences can be supported by firms’ expectations that the economy is on a gendered equilibrium in a model with incentive constraints. More traditional theories assume that females have a comparative advantage in home production and males in market production, but Albanesi and Olivetti (2007) show that improved medical capital and the introduction of the infant formula has reduced the importance of this factor. In Becker (1985) gender differences in earnings arise from the fact that females undertake tiring activities that reduce work effort. So, workers with the same level of human capital, earn wages that are inversely related to their housework commitment. The substitutability between home duties and market earnings also arises in our model, although we consider the effect of an investment in costly effort as well.

Regarding the elasticity of labor supply, Goldin (2006) documents that the fast rise of female’s labor supply elasticity in the 1930-1970 period was the result of a declining income effect and a rising, due to part time employment, 

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9Earlier models have emphasized that intrahousehold redistributional factors are important. However, these papers are either concerned with the across household heterogeneity (Apps and Rees, 2007), or follow a policy reform approach (Brett, 1998, and Apps and Rees, 1999a), or focus on the positive effects of the taxation of couples (Gugl, 2004).
substitution effect. During the last thirty years, she argues, females started viewing employment as a long term career rather than as a job, and this caused a decline in the substitution effect and the labor supply elasticity. This interpretation is consistent with how we model, in our long run setting, the elasticity effect of a commitment to stay in the labor market in order to take advantage of the opportunities offered by it. Blau and Khan (2007) also document and quantify the reduction in the labor elasticity of married women in the US, which however remains well above that of men, at a ratio of about 4 to 1. Even in Sweden, where gender differences in labor market outcomes are arguably less dramatic than elsewhere, Gelber (2007) estimates that the elasticity of women is twice that of men.10

3 Exogenous Elasticities

A family consists of a male and a female who participate in market and home activities. A costly investment in training makes a person more productive for the market. Husband and wife share a fraction of the income they produce with market work. For the moment we let household activities in the background and treat them as exogenous.

3.1 Setup of the Model

The index \( j = m, f \) identifies the gender. The utility function of gender \( j \) is simply given by

\[
U_j = C_j - \frac{1}{a_j} L_{aj} - \frac{1}{2} \tau_{j}^2
\]  

where \( C_j \) is consumption, \( \frac{1}{a_j} L_{aj} \) represents the disutility cost of supplying \( L_{j} \) units of labor, \( a_j > 2 \) and \( \frac{1}{2} \tau_{j}^2 \) is the cost of training. Each person is endowed with one unit of time for work, so \( L_{j} \leq 1 \). The quasi-linearity with

\footnote{Gelber’s (2007) results are also important because they analyze the responses to the very large Swedish Tax Reform of 1991 and therefore represent a rare example of causal identification and estimation of labor supply elasticities of household members.}
respect to consumption allows us to obtain closed form solutions at least up to a point. We discuss below the effect of this assumption on our numerical results.

The timing is as follows. First, the government sets labor income taxes. Then, the male and the female take as given the tax rates and decide individually the amount of consumption, labor supply and training to maximize their utilities. A perfectly competitive, constant-returns to scale firm pays workers their marginal productivity and makes zero profits. The price of the consumption good is one and the production function for worker $j$ is

$$Q_j = \tau_j L_j$$

(2)

Therefore, the wage rate $W_j$ equals $\tau_j$. Spouse $j$ maximizes utility taking as given the labor income tax rate $t_j$ and the other spouse's decisions

$$\max_{C_j,L_j,\tau_j} U = C_j - \frac{1}{a_j}L_j^{a_j} - \frac{1}{2}\tau_j^2$$

(3)

subject to

$$C_j = s(1 - t_j)W_j L_j + (1 - s)(1 - t_k)W_k L_k$$

(4)

$$W_j = \tau_j$$

(5)

where $k$ is the other spouse and $1/2 \leq s \leq 1$ is the sharing parameter. Within a single tax regime, $s$ has the interpretation of an intrahousehold inequality parameter. When $s = 1/2$, then the family fully pools its resources and the ratio of consumption levels $C_j/C_k$ equals 1. When $s = 1$, the ratio of consumption levels is pinned down by the ratio of gross incomes, $C_j/C_k = W_j L_j / W_k L_k$, and there is no sharing of resources. We rationalize the sharing parameter as a technological externality that captures the non-excludable and non-rivalrous, at least to some extent, nature of the common consumption of goods within the family.\footnote{For example, once the family purchases an electric appliance such as a refrigerator or a dishwasher it is difficult to imagine how a spouse can be excluded from its consumption. Or, the consumption of cable television from one family member does not restrict the consumption of the good by other members of the family.} Finally, note that in deciding the level of training, workers
internalize that a higher level of investment increases their productivity and therefore their wage rate.

The solution to the above maximization problem yields the labor supply and the training decision functions (see the Appendix to Section 3.1 for details)

\[
L_j = (s(1 - t_j))^{\frac{2}{\sigma_j}} = (s(1 - t_j))^{\frac{2\sigma_j}{1-\sigma_j}} \quad (6)
\]

\[
\tau_j = (s(1 - t_j))^{\frac{a_j}{2(1-\sigma_j)}} = (s(1 - t_j))^{\frac{1+\sigma_j}{1-\sigma_j}} \quad (6)
\]

where

\[
\sigma_j = \frac{\partial L_j}{\partial W_j} \frac{W_j}{L_j} = \frac{1}{a_j - 1} \quad (7)
\]

is the own elasticity of labor supply with respect to an exogenous variation in the wage rate. For this Section, cross elasticities are zero because we have assumed quasi-linear preferences. In Section 5.4 we also discuss non zero cross elasticities.

Suppose now that for exogenous reasons we have \(a_m > a_f\). For the moment we take this difference in preferences as primitive and do not explain it as it may come from innate gender characteristics or more likely historically induced gender roles which are especially strong in certain cultures (Alesina and Giuliano, 2007). Under \(a_m > a_f\) the prediction of the model is that males

- work more in the market: \(L_m > L_f\);
- have a lower elasticity of labor supply: \(\sigma_m < \sigma_f\);
- invest more in training: \(\tau_m > \tau_f\);
- receive a higher wage: \(W_m > W_f\).

These predictions are in line with what we observe in real life labor markets. In Figures 1 and 2 we depict the labor market equilibrium. Assuming
that $a_m > a_f$, Figure 1 describes a situation in which males supply more labor than females. This happens for two reasons. First, given an exogenous wage rate, male participate more in the market. Second, they also invest more in training. In turn, investment in training endogenously shifts the labor demand curve up and increases the wage rate $W$. As a result the gender differential in labor market participation and earnings expands. In Figure 2 we describe an exogenous shift in the tax rate $t_j$ for spouse $j$. Taxation distorts both the labor-consumption margin and the decision to invest in training, so that both the labor supply and the labor demand curve shift. The final equilibrium is characterized by lower participation in the labor market and lower pre-tax wage rate.

3.2 Gender Based Taxation

The planner sets taxes for the male and the female in order to raise revenues and finance a public good $G$. The public good does not provide utility to anyone and the proceedings are not rebated back.\footnote{This is without loss in generality since the nature of the results (throughout the paper) does not change when we allow for revenues to be distributed in a lump sum way. See Lundberg, Pollak and Wales (1997) for an natural experiment with intrahousehold lump sum transfers.} In doing so, the planner anticipates the private market equilibrium. Let $U_m(t_m, t_f; a_m, a_f, s)$ and $U_f(t_m, t_f; a_m, a_f, s)$ denote the indirect utility function for the male and the female respectively. In this Section we assume that the planner weights people uniformly, but we revisit this issue in Section 6.1 where it matters more.\footnote{Under $\Omega = \frac{1}{1-e}(U_m^{1-e} + U_f^{1-e})$ with inequality aversion ($e > 0$), the difference in the resulting tax rates is even more profound. The same holds for the rest of the paper.} Then, the planner solves

$$\max_{t_m, t_f} \Omega = U_m(t_m, t_f; a_m, a_f, s) + U_f(t_f, t_m; a_m, a_f, s)$$ (8)

subject to the constraint

$$t_m W_m(t_m; a_m, s)L_m(t_m; a_m, s) + t_f W_f(t_f; a_f, s)L_f(t_f; a_f, s) \geq G$$ (9)
**Proposition 1** If $\sigma_m \leq \sigma_f$, then $t_m \geq t_f$.

The proof of Proposition 1 and the intermediate derivations are presented in the Appendix to Section 3.2. Here we just mention that (8) and (9) is not a concave program and we have to establish sufficient conditions for the existence of an interior global optimum with $t_m \geq t_f$.

This proposition is an application of the Ramsey (1927) rule. It is welfare enhancing to tax less the “commodity” which is supplied with higher elasticity. If $a_m > a_f$, then females are more elastic and distorting their labor and training decisions results in a greater excess burden for the society. In Table 1 we present the welfare gains when moving from a single tax to differentiated taxes by gender. Gender Based Taxation (GBT) generates more equality in labor market outcomes. For conservative values of the elasticity ratio such as $\frac{\sigma_m}{\sigma_f} = \frac{1}{2}$, which was recently estimated by Gelber (2007) for Sweden, GBT raises welfare and GDP by more than 1%. For an elasticity ratio of $\frac{\sigma_m}{\sigma_f} = \frac{1}{3}$ the gains from GBT exceed the 4% of GDP. Naturally, GBT is more efficient the higher is the level of distortions (i.e. the higher is public expenditure $G$) and the lower is the ratio of elasticities $\frac{\sigma_m}{\sigma_f}$. We defer the discussion of how the resource sharing parameter $s$ affects the gender taxes for Section 6.2.

These GDP gains are very large, possibly unreasonably so. We also have explored other examples which eliminate the quasi-linearity with respect to consumption. For reasonable parameters and functional forms we find that with a ratio of elasticities $\frac{\sigma_m}{\sigma_f} = \frac{1}{4}$ (as in Blau and Kahn (2007) for the US), GBT raises GDP by approximately 1.24%.\(^\text{15}\)

\(^{14}\)For cross-country evidence on the gender differential on labor supply elasticities see Alesina, Glaeser and Sacerdote (2005), Blau and Kahn (2007), Blundell and MacCurdy (1999).

\(^{15}\)For this exercise we use the standard CRRA/power expression for the subutility of consumption which induces both substitution and income effects on labor supply. In this case, the concavity of the subutility function for consumption mutes the welfare gains from the reallocation of resources. For more details see the Appendix to Section 3.2.
4 The Organization of the Family

Thus far we have assumed that different labor market behavior of men and
cwomen derive from exogenous differences in preferences and attitudes. That
is, we have taken the key parameters $a_m$ and $a_f$ as our primitives. In what
follows we propose a possible formalization of the household allocation of
home duties which derives these parameters endogenously.

A family has to undertake $2A$ family duties, or chores. Each duty is per-
formed by one spouse. When a spouse performs one home duty she/he gets
nothing while the other spouse gets a positive shock in the labor market. The
argument is similar to that of Becker (1985) who posits that the spouse who
does more homework has fewer “energy units” to allocate into the market.

Therefore, there are $2A$ corresponding labor market shocks that hit the
family. The shocks are assumed to be i.i.d. and denoted as $x_i$. Each random
variable $x_i$ is distributed as a chi-squared with one degree of freedom, i.e.
$x_i \sim \chi^2_1$. Let $2a_m$ be the number of $x_i$ shocks that the male absorbs; each
shock corresponds to one unit ”off-duty” that he gets. $2a_f = 2(A - a_m)$ is
the amount of home duties that the male gets, and therefore it is also the
number of labor market shocks that the female absorbs. By the properties
of the $\chi^2$ distribution we can define an ”aggregate shock” for the male as
$\omega_m = \sum_{i=1}^{2a_m} x_i$, with support in $[0, \infty)$ and expected value $E(\omega_m) = 2a_m$.
Similarly for the female we have that $\omega_f = \sum_{i=2a_m+1}^{2A} x_i$, with support in
$[0, \infty)$ and $E(\omega_f) = 2a_f$. Ex post utility for spouse $j = m, f$ is defined over
bundles of consumption, labor and training and given by

$$V_j = C_j - \frac{1}{a_j} v(L_j) \omega_j - \frac{1}{2} \tau_j^2$$

(10)

where $C$ is consumption, $L$ is labor supply in the market, and $\tau$ is amount
of training. The subutility of labor is given by $v(L_j) = \frac{1}{2} \left(1 - \frac{1}{\tau_j^2} \right) < 0,$
with $v' > 0$, $v'' < 0$ and $L_j < 1$. This specific ”$\chi^2$ shocks - CARA utility”
environment is adopted to obtain the more familiar CRRA representation of
the (ex ante) utility function that we used in Section 3.

To fix ideas about the nature of the shocks, consider the situation where the male and the female decide how to allocate home duties over a period of two weeks. Specifically, for each weekday, one of the two spouses must be in “charge of the kids” (i.e. take them to school, make sure that they have their time after school organized etc.). This hypothetical situation can be mapped in our notation as follows. 2A = 10 is the total number of days in which one parent has to take the kids to school while the other is exempted from these home duties. 2a_m is the number of days that the male is not in charge of the kids and therefore 2a_f is the total number of days where the male is in charge of the kids. For each of the i = 1, ..., 2a_m days where the father is not in charge of the kids and works in the market, there is a positive shock x_i that affects his utility of working in the market. To put it differently (and with a slight abuse of language), in the days in which a spouse is not in charge of kids, she/he has more energy and can make “things happen” at work and get a positive utility reward. There are also days in which the spouse is in charge of the children and work provides only the basic wage with no upside options.

The ex post utility of working in the market for spouse j is given by the term \(-\frac{1}{a_j}e^{\omega(L_j)}< 0\). Given a realization of \(\omega_j\), a higher amount of labor supply decreases utility. For given amount of labor supply, a favorable realization of \(\omega_j\) increases the utility of working in the market (or decreases the disutility of working). Since the shock \(\omega_j\) has not been realized when spouses decide how much to consume, supply labor and invest in costly training, we need to work with the ex ante utility function. Using the moment generating function of a chi-squared random variable with 2a_f degrees of freedom we

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16 In this sense one cannot "quit a child" and home duty in our model is intrinsically different from having a second job.

17 The abuse of language is that we do not model energy explicitly. Instead, taking less home duties directly implies the possibility or receiving a higher labor market shock.
obtain

\[ U_j = E_\omega_j V_j = C_j - \frac{1}{a_j} L_j^{a_j} - \frac{1}{2} \tau_j^2 \]  

(11)

The “\( \chi^2\)-CARA” ex post representation of preferences in (10) allows us to work with the familiar CRRA-power expression for labor supply in (11), which is the utility function used in Section 3. With this derivation we intend to provide a rationale for the key parameter \( a_j \). While in the previous section gender differences were “innately” built in preferences (so that \( a_m \) and \( a_f \) were “genetically” or “culturally” fixed in a permanent way), in Section 5 we develop a bargaining game which delivers the equilibrium division of chores between the two spouses and ultimately determines endogenously their market participation and elasticity.

The expected marginal utility of working is given by

\[ U_{L_j} = -L_j^{a_j-1} \]  

with \( a_j > 2 \) and \( L_j < 1 \)  

(12)

so that fewer home duties (higher \( a_j \)) increase the expected marginal utility of working for spouse \( j \). Because the latter expects a higher realization of the labor market shock \( \omega_j \), he or she works more, invests more in training and earns a higher wage rate. This means that home duties and participation in the market are substitutes. At the same time, assuming fewer home duties implies a higher elasticity of the expected marginal utility of working with respect to labor supply

\[ \varepsilon_{U_{LL_j},L_j} = U_{LLL} U_L = (a_j - 1) = \frac{1}{\sigma_j} \]  

(13)

Since the lower the amount of home duties the more sensitive is the marginal utility of working to movements in the supply of labor, a given change in the wage rate \( W_j \) meets with a smaller movement in labor supply \( L_j \) in order to restore the first order condition for labor supply. This implies that spouse \( j \) has a less elastic labor supply.

\[ ^{18}\text{We have that for a random variable } \omega_j \sim \chi^2_{2a_j} \text{ the moment generating function evaluated at some } q < 1/2 \text{ is given by } M_\omega(q) = E_{\omega} (e^{q\omega_j}) = \left( \frac{1}{1-2q} \right)^{a_j}. \]
Thus, the gender gap in labor supply elasticities can be traced back to the attitudes of the two spouses towards risk and to the differences in the access to labor market shocks which is determined by the bargained allocation of home duties. For spouse \( j \) and given a specific realization of the labor market shock \( \omega \), we define \( u = -\frac{1}{c}e^{v(L)\omega} \) to be the expost disutility from labor supply.

We also define the curvature functions \( r_\omega = -\frac{u_{\omega\omega}}{u_\omega} \) and \( r_L = \frac{u_{LL}}{u_L} \) as measures of the attitude towards risky realizations of \( \omega \) and \( L \) respectively. Then we can show that

\[
\frac{\partial r_\omega}{\partial L} = -\frac{\partial r_L}{\partial \omega} = -v'(L) < 0 \tag{14}
\]

The first part of the symmetry condition (14) states that a spouse who participates more in the labor market is less risk averse to stochastic realizations of \( \omega \). This third-order cross partial effect is a diversification motive. High realizations of \( L \) cause spouse \( j \) to be less averse to \( \omega \)-uncertainty since uncertainty “per unit” of labor decreases. The second part of equation (14) states that a spouse getting a good realization of \( \omega \) is more risk averse to stochastic realizations of participating in the market \( L \).

The intuition for the gender gap in elasticities is that if men get fewer home duties (\( a_m > a_f \)), then they get a higher number of positive shocks to the utility of working. For men, this expectation of more favorable labor market opportunities (a high realization of \( \omega_m \)) is an expected intrinsic benefit from working. So men prefer to commit to a larger amount of labor ex ante, which is also more stable because it is calibrated not only on the wage but also on the intrinsic expected benefit of working. When the wage changes, this committment makes them less willing to adjust their labor supply. The reverse happens for women.

\[19\]We don’t have a minus sign in the definition of \( r_L \) because labor is a “dis-commodity”, i.e. \( u_L < 0 \). See Appendix to Section 4 for more details.
5 Household Bargaining

5.1 Timing

The timing is as follows. First the government sets the tax rate(s). Then the family members bargain over the allocation of home duties, which results in equilibrium values for $a_j$. Next, labor supply decisions are taken, wages paid, shocks realized and consumption shared. A commitment technology makes it impossible for the government to change the tax rates after family bargaining decisions are made or after the realization of labor market shocks.

5.2 Bargaining over Home Duties

At the second stage of the game the couple decides whether to marry or not. If the male and the female decide to marry, then they bargain over the allocation of home duties, $A = a_m + a_f$. In doing so, they both rationally anticipate the resulting labor market equilibrium. The utility of a spouse $j$ when married is given by the indirect utility function at stage 3, as described by the maximization of (3) subject to the constraints (4) and (5). We assume that the autarky utility level of each spouse (the threat point), is given by the value function of the following program

$$
\max_{C_j, L_j, \tau_j} T_j = C_j - \frac{1}{\phi} L_j^\phi - \frac{1}{2} \tau_j^2 - z
$$

subject to

$$
C_j = (1 - t_j) W_j L_j \quad \text{and} \quad W_j = \tau_j
$$

The disutility of being alone is $z$. A single does not share resources so he or she gets a “full share of a smaller pie”. Importantly, a single has a shock $\omega_s \sim \chi^2_{2\phi}$ with $\phi = A$, which means that singles take less home duties than a married person, for instance because they have no children.\footnote{This assumption can be relaxed. Even when a single has the same amount of home duty as a married person \textit{on the equilibrium path}, the results do not change. See the Appendix to Section 5.3.} Translated
into the words of the example in Section 4, a single person never has to drive
the kids to school.

Given this specification of the utilities in marriage and in autarky, for
any pair of taxes \((t_m, t_f)\), the maximization of the asymmetric Nash-product
delivers the allocation of home duties:

\[
[U_m(a_m; t_m, t_f, s) - T_m(t_m, \phi, z)]^\gamma [U_f(a_m; t_f, t_m, s) - T_f(t_f, \phi, z)]^{1-\gamma} \quad (17)
\]

where \(\gamma\) is the bargaining power of the husband.

We study the case of \(\gamma > 1/2\), which can be justified as the historical in-
heritance from a time in which physical power mattered, with cultural forces
persistently affecting family formation.\(^{21}\) The fact that men have stronger
bargaining power, seems consistent with survey evidence. Friedberg and
Webb (2006) use data from the Health and Retirement Study and document
that nearly 31% of males believe that “they have the final say in major deci-
sions” while only 12% believe that their spouse is in the same condition. At
the same time, approximately 31% of the females admit that their husband
has the final say while only 16% believe to have the final say in major deci-
sions. A biased allocation of home duties in favor of the male accords well
with the existent empirical evidence. Aguiar and Hurst (2007) show that
although the difference between male and female house work has decreased
during the last 50 years, females still perform nearly twice as much homework
as males.

Our marriage specification is, admittedly, simplified, but note that it
assumes that only the explicit bargaining power \(\gamma\) is exogenous, while the
effective implicit bargaining power, that derives from the combined effects of
\(\gamma\) and the threat points, is endogenous and indeed depends on GBT. There

\(^{21}\)The effects and causes of different family structures with specific reference to the role
of women and allocation of home duties has been the subject of empirical cross country
research by Alesina and Giuliano (2007), and Fernandez (2007). Their results suggest
that one should be cautious in applying to different countries and cultures the same set of
preferences on the issue of gender roles.
is a feedback effect from government policy to the intra-household allocation of bargaining power because the outside option of a spouse $j$ depends on the tax rate $t_j$. For example when the tax rate decreases, spouse $j$ acquires more implicit bargaining power through increased training, wage rate and market participation.\footnote{Pollak (2007) argues convincingly that the wage rate and implicitly the level of human capital should determine the outside option of a spouse. Our specification addresses, at least partly, this concern because taxes distort the training decision and endogenously shift the labor demand curve. However, as we discuss in the concluding section, our specification does not recognise dynamic elements of acquiring bargaining power such as investment in human capital.}

5.3 Properties of the Bargaining Solution

We consider the properties of the solution mapping $a_m(t_m, t_f) : [0, 1]^2 \rightarrow (2, A - 2)$. The bargaining solution prescribes how the family allocates home duties for any pair of tax rates, given parameters $\gamma$, $s$, $A$ and $z$. We cannot derive closed-form expressions for the solution $a_m(t_m, t_f)$ and its comparative statics, but we can discuss intuitively (and establish numerically) two important properties of the bargaining solution. For more details see the Appendix to Section 5.3.

First, the sharing parameter affects the allocation of home duties. Specifically for given $(t_m, t_f)$, an increase in $s$, i.e. less resource sharing, makes the allocation of shocks more unbalanced, $\frac{\partial a_m}{\partial s} > 0$. We call this the sharing effect and depict it in Figure 3. This Figure plots the male’s indirect utility function $U_m(a_m)$ as a function of home duties. When the male makes take it or leave it offers to the female (equivalent to $\gamma = 1$) and there is no income sharing, he chooses the maximum feasible level of $a_m$, that is he chooses not to take any home duties. As the sharing of resources becomes important ($s$ decreases) the male decides to take some amount of homework, even though he has the maximum level of bargaining power. When income is shared, it is never individually optimal for the male to have the female not
working in the market. The same intuition applies for any level of bargaining weights (i.e. \(1/2 < \gamma < 1\)). As income pooling becomes more important the intra-household allocation process becomes more balanced. At some level \(s_E\), resource sharing is so important that the allocation of shocks is completely balanced, even without GBT.

The sharing of resources implies that there is an externality in the model. Inspection of the solution (6) suggests that as resource sharing increases, both spouses participate less in market activities because they lose part of their individual claims over the market product.\(^{23}\) For extreme levels of sharing, \(s < s_E\), the male with the bargaining power is better off by staying at home and having the female working and sharing her income with him. This prediction is not realistic and from now on we restrict attention to \(s \geq s_E\).\(^{24}\)

By increasing the tax rate for the male \(t_m\) and keeping fixed the female’s tax rate \(t_f\) and the level of sharing \(s\) we can examine the second property of the bargaining solution. Three are the relevant effects:

- **Redistribution Effect**: \(\frac{\partial U_m}{\partial t_m} < 0\). When \(t_m\) increases, the male is worse off inside the marriage and demands a lower amount of home duties (higher \(a_m\)) in order to “stay in the contract”.

- **Threat Effect**: \(\frac{\partial T_m}{\partial t_m} < 0\). When \(t_m\) increases, the male is worse off outside the marriage and his implicit bargaining power decreases. This means that he is willing to accept a higher amount of home duties (lower \(a_m\)) in order to “stay in the contract”.

\(^{23}\)In that sense this model is a little more individualistic that the collective family model of Chiappori (1988, 1992).

\(^{24}\)Even though, for given allocation of home duties, a spouse works and invests less the greater is the sharing of resources, the intra-household allocation process in the bargaining stage of the game is always efficient because the Nash bargaining process is Paretian. Referring to Figure 5, note that the allocation of resources always lies on the Pareto frontier because we cannot make one spouse better off without worsening the position of the other (see also equation (20)).
**Cross Redistribution Effect:** $\frac{\partial U_f}{\partial t_m} < 0$. Because spouses share resources inside the marriage, a higher $t_m$ makes the female worse off inside the marriage. In order to “stay in the contract” she must be compensated with less home duties ($a_m$).

We can show (see the Appendix to Section 5.3) that the threat effect always dominates the redistribution effect. That is, a higher tax rate brings a more balanced allocation, $a_m < 0$ because $25$

$$\frac{\partial U_m}{\partial t_m} - \frac{\partial T_m}{\partial t_m} > 0$$

which holds if (but not only if) $s < 1$ and $a_m < \phi$. Similar reasoning (but not symmetric because $\gamma > 1/2$) holds for varying the female’s tax rate and $\frac{\partial a_m}{\partial t_f} > 0$.

We sum up this discussion in Figure 4 which depicts the solution to the bargaining program as a function of the sharing of resources $s$ and the ratio of taxes $t_m / t_f$.

### 5.4 Cross Elasticities

With an endogenous allocation of home duties the cross elasticities of labor supply are not zero as in Section 3. We can write for spouse $k$

$$c_{L_k,t_j} = \frac{\partial L_k}{\partial t_j} t_j = \left( \frac{\partial L_k(a_k)}{\partial t_j} + \frac{\partial L_k}{\partial a_k} \frac{\partial a_k}{\partial t_j} \right) \frac{t_j}{L_k}$$

The term $\frac{\partial L_k(a_k)}{\partial t_j}$ in (19) is the response of $k$’s labor supply to $j$’s tax rate for a given allocation of home duties. This is zero as in Section 3 because preferences are quasi-linear and the budget constraint is separable in spouses’ net incomes. The term $\frac{\partial L_k}{\partial a_k} \frac{\partial a_k}{\partial t_j}$ appears because the allocation of home duties is endogenous and responds to variations in the tax rate. For instance, a

\footnote{See the Appendix to Section 5.3 for the robustness of this result after considering the second order effects.}
higher tax rate for the male $t_m$, increases the relative bargaining power of the female. As a result, the female takes less home duties ($a_f$ increases), and the cross elasticity of labor supply with respect to her spouse’s tax rate is positive.\footnote{This is in line with the empirical evidence, see for example Aaberge and Colombino (2006) for negative cross wage elasticities in Norway and Blau and Kahn (2007) for the US.}

6 Gender Based Taxation

6.1 Government Objectives

As we discussed in Section 3, GBT is optimal if the labor supply of men is less elastic than than that of women. Based on our derivation in Section 4 this happens when males assume fewer home duties than women and women have no comparative advantages in home duties. Obviously then with $\gamma = 1/2$, the market and non-market behavior between spouses is identical and there is no need for GBT. However, as discussed before, there is ample evidence for gender differences in labor market participation rates and elasticities and for a biased allocation of home duties and decision making power within the family, suggesting that $\gamma$ must be greater than 1/2 for our model to be a good description of the real world. But in a world where $\gamma > 1/2$ how should a social planner evaluate the utility of men and women? A natural premise is that the social planner evaluates people equally, that is we adopt the utilitarian welfare function, $\Omega = U_m + U_f$. Thus we have a sort of “social dissonance” (Apps and Rees 1988) between the preferences of society (as for example implied by the utilitarian function $\Omega$) and the equilibrium result of an intrafamily game in which one party has a disproportionate share of power. If this the case, then, there is a “distortion” that could justify government’s intervention which, in addition to financing the public good efficiently according to the Ramsey rule, affects intra-family bargaining as
well.

If the government could choose directly the allocations of home duties and then set taxes to raise a pre-specified amount of revenues, it would choose the *ungendered* equilibrium \( (a_m = a_f, t_m = t_f) \) and there would be no need for GBT; this would be the first best. In Figure 5, we depict this Edgeworth’s (1897) “egalitarian” solution: starting from a gendered equilibrium \( (a_m > a_f) \), we can allocate one more unit of home duty to the male from the female and increase social welfare because there are “decreasing returns to specialization”. In other words, the first hour that the father spends with his children is more productive than the female’s last hour. This is true because starting from \( a_m > a_f \) we have

\[
\frac{\partial \Omega}{\partial a_m} = \frac{\partial U_m}{\partial a_m} + \frac{\partial U_f}{\partial a_m} < 0
\]  

(20)

Although the government cannot dictatorially impose a balanced intrahousehold allocation of duties, gender based taxes affect the allocation of chores between spouses and can bring the society closer to the first best.

### 6.2 The Organization of the Family

The planning program is

\[
\max_{t_m, t_f} \Omega = U_m(t_m, t_f; a_m, a_f, s) + U_f(t_f, t_m; a_f, a_m, s)
\]  

(21)

subject to the constraint

\[
t_mW_mL_m + t_fW_fL_f \geq G
\]  

(22)

The difference with respect to Section 3.2 is that now the allocation of home duties is endogenous and the government anticipates it. That is:

\[
a_m = a_m(t_m, t_f; \gamma, s, z)
\]

\[
a_f = a_f(t_f, t_m; \gamma, s, z).
\]
\[ W_j = W_j(t_j, a_j(t_j, t_f)) \]
\[ L_j = L_j(t_j, a_j(t_j, t_f)) \]
for \( j = m, f \).

Starting from a single tax rate, the government can induce a more balanced allocation by differentiating taxes and setting \( t_m > t_f \). As long as labor supply elasticities remain different (\( \sigma_f > \sigma_m \)), GBT also reduces fiscal distortions as in Section 3.2. There is an implicit cost, however, of taxing the male at a higher rate: not only we distort his labor supply and training decisions (as in Section 3.2) but also we force him endogenously to take more home duties (lower \( a_m \)) which further reduces the government’s ability to extract revenues from the primary earner. This “Laffer curve” effect appears in the first order conditions and increases the ratio of the female over the male marginal revenue (see Appendix to Section 6.2 for further elaboration). It can be inspected by looking at the bliss point of spouse \( j \) under exogenous and endogenous bargaining. For the former case, the peak of the Laffer curve is given at the point where the elasticity of earnings with respect to the tax rate equals -1
\[
t_j^b = \frac{E_j}{\frac{dE_j}{dt_j}} = \frac{a_j - 2}{2a_j} = \frac{1 - \sigma_j}{2(1 + \sigma_j)}
\]
where \( E_j = W_j L_j \) are pre-tax earnings. Notice that if a higher \( t_j \) reduces \( a_j \), then the peak of the Laffer curve shifts to the left. Then, for the endogenous bargaining case we have that
\[
\hat{t}_j^b = \frac{E_j}{\frac{\partial E_j}{\partial t_j} - \frac{\partial E_j}{\partial a_j} \frac{\partial a_j}{\partial t_j}}
\]
with \( \hat{t}_j^b < t_j^b \) as long as \( \frac{\partial E_j}{\partial a_j} > 0 \) and \( \frac{\partial a_j}{\partial t_j} < 0 \) as it is the case for the male.

In Figure 6 we depict the solution for the ratio of optimal gender based taxes \( \frac{t_m}{t_f} \) as a function of the sharing parameter \( s \).\(^27\) There are three areas of interest. In Area I the externality is so high (\( s < s_E \)) that the male decides to

\(^{27}\)See Appendix to Section 6 for details on the calibration and the solution.
stay at home. The female works more, earns more, is less elastic and the male enjoys resources mainly from his spouse’s income. As mentioned before, this case does not accord with real life labor markets and we can safely dismiss it.\footnote{See also the references for the empirical failure of the income pooling hypothesis given in Section 2.} In Area II, the male has the bargaining power and without extreme sharing of resources he prefers to assume fewer home duties. As a result he works, invests and earns more than the female. The analysis of Section 4 applies, so the male is also less elastic. In Figure 6 we depict the ratio of labor supply elasticities (that move in the opposite direction of the ratio of home duties) under a single and gender based taxes together with the ratio of optimal taxes, $t_m/t_f$. Gender based taxes induce a more balanced allocation of home duties and bring closer to 1 the ratio of elasticities because they increase the implicit bargaining power of the female. Moreover, as long as $\sigma_f > \sigma_m$, the conventional Ramsey principle applies and GBT reduces fiscal distortions. Note that with endogenous bargaining and starting from $\gamma > 1/2$, it is relatively more costly for society to tax females than it is in the exogenous bargaining case. The reason is that every extra unit of tax revenues that the government raises from the female further deteriorates her implicit bargaining power and results in a more gendered allocation (see Appendix to Section 6.2 for this argument).

In Area III, $t_m > t_f$ is still optimal. In this region, with less resource sharing and given the intuition of the sharing effect in Section 5.3, the ratio of home duties and the ratio of elasticities diverge even more. However, the ratio of tax rates starts to decline. The intuition for this result is given in Figure 7. This Figure depicts the ratio of wage rates (or training levels) $W_m/W_f$ as a function of the sharing parameter $s$ under a single and gender based taxes. Note that the ratio of optimal taxes $t_m/t_f$ in Figure 6 traces the interhousehold inequality $I = W_m/W_f$ that prevails under a single tax rate in Figure 7. The reason is that GBT reacts to the male over female wage ratio under a single
tax rate, which is a measure for the misallocation of home duties without government intervention.\textsuperscript{29} At the same time, GBT targets the wage ratio under differentiated taxes, because this is correlated with spouses’ relative decision making power.\textsuperscript{30} Since GBT reallocates efficiently the bargaining power between spouses, the ratio of wage rates under GBT, and therefore the relative decision power, shifts down relative to the single tax rate, as shown in Figure 7. At some level of sharing $s_M$, however, the household by its own reduces inequality in earnings and since it is costly for the government to further increase the elasticity of the primary earner, there is no reason why the ratio of taxes should continue to diverge for $s > s_M = .92$.

The reason why resource sharing and inequality under a single tax rate exhibit an inverted U-shaped relationship is the following. Under a high level of resource sharing, both the male and the female participate less in the market, and the inequality ratio is low. As resource sharing declines ($s$ increases) both partners participate more, but the male at an increasing rate and therefore inequality starts to rise. Under extremely low levels of income pooling the female starts to participate at an increasing rate, so inequality begins to fall. Even for no resource sharing, i.e. $s = 1$, we always have $W_m > W_f$, so the government would always set $t_m > t_f$. See Appendix to Section 6.2 for more details.

In Figure 8 we depict the gains in welfare, GDP and employment when moving from a single to gender based taxes as a function of $s$. The gains are highest when pre-gender based inequality is at its maximum and begin to fall when GBT becomes less beneficial as in Area III.

Finally, in Figure 9 we depict the possibility that both spouses gain under GBT. If resource sharing is important, both spouses gain when moving from a single to gender based taxes because the female starts to work, train and

\textsuperscript{29}Under a single tax rate the ratio of wages $\frac{W_m}{W_f}$ is higher the more gendered is the allocation of home duties. See equation (29) in the Appendix to Section 6.2.

\textsuperscript{30}In particular, we can write autarky utilities as $T_j = \frac{2}{25}W_j^2(t_j) - z.$
earn more, a decision which is not internalized by the family when spouses individually decide how much to participate in the market.

7 Conclusions

In this paper we begin to analyze the effects of Gender Based Taxation as a potential tax policy. If the intra-family bargaining process favors the husband, GBT with lower tax rates for females is superior to an ungendered tax rate. In what one could label the “short run”, namely before the family organization adjusts to the new tax regime, GBT reduces tax distortion because of the Ramsey principle according to which one should tax less commodities with higher supply elasticities. When the spouses react to GBT by re-bargaining over household duties, GBT leads to a more equitable distribution of household chores and market activities. To the extent that this reallocation does not produce complete equity between male and female and therefore the supply elasticities remain different, GBT is optimal. The reallocation towards more equality of household duties is an additional welfare improving effect if society evaluates the welfare of males and females equally. In the “long run”, the welfare gains of GBT derive both from the Ramsey principle and from a more “efficient” organization of the family that takes into account the decreasing marginal benefits in home versus market activities. In our model GBT is optimal for the couple with both members weighted equally and, for some parameter values, for both members of the couple individually.

Rather than reviewing in more details our results it is worth discussing several important avenues for future research. First, we have not considered the possibility of a comparative advantage of females in home production. Albanesi and Olivetti (2007) point out that technological improvements have certainly reduced women’s comparative advantage in household production.
and duties, nevertheless comparative advantage may still exist. In this case there would be forces going in opposite directions. On the one side, the government does not want to impose lower taxes on women and encourage female market participation because this would oppose possible increasing returns that the household enjoys when spouses specialize in market and non-market activities. On the other side, imposing higher taxes on females, as discussed in Section 5.3, results in a further deterioration of their implicit bargaining power and opens up the gap in the labor supply elasticities. When elasticities diverge, we expect the Ramsey effect to become stronger and counterbalance the effect of comparative advantage. Which of these two effects prevails is an open issue that requires more theoretical and empirical work.

Second, our model does not allow for a realistic marriage market since it considers a society in which marriage is optimal for everybody along the equilibrium path. A proper discussion of the marriage market would require the introduction of some heterogeneity within the pool of men and women and the consideration of a matching or a searching model.

Third, in the present model the word “training” can be interchanged with “effort”. The training decision is taken when the couple is already formed. Therefore, we cannot analyze a situation in which a man or a woman, when unmarried, invest in training as a commitment to gain bargaining power. This interesting extension could be discussed in an even more general model in which the marriage market is also endogenized. A key question that this analysis could help answering is whether or not GBT should refer to only married couples or to males and females regardless of their marriage status. Alternatively, if we allowed for different tax rates not only across genders but also within genders, our model would suggest that taxing

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31 Note that Ichino and Moretti (2006) find that biological differences explain a large part of the gender differential in absenteeism which translates in a 12% fraction of the earning gap.
single men at a higher rate might be a superior policy because it reduces directly the autarky utility of men, inducing them to accept more home duties in order to marry. An evaluation of these more complicated tax structures would depend undoubtedly on their redistributive properties in a world of heterogeneous households.

Fourth, our model does not distinguish between the intensive and extensive margins of labor supply decisions. There is instead an important discontinuity between starting to work from inactivity and increasing working time if someone is already active in the market.

Finally, we believe that a comparison of Gender Based Taxation with other gender and family policies, such as quotas, affirmative action, forced parental leave and public supply of services to the families, is necessary within a unified theoretical framework in order to draw policy conclusions. We see no reason why GBT should not be an excellent “horse” in a race with all these alternative policies. In fact our basic economic intuition regarding the superiority of price incentives versus quantity restrictions or regulations would make GBT a favorite in the race, but we still have to run it.
References


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Appendix

Appendix to Section 3.1

Equation (6) is derived by substituting constraints (4) and (5) into the objective function (3), taking the first order conditions with respect to \( L \) and \( \tau \) and solving the resulting system of equations. The second order sufficient conditions for this maximization problem hold, as \( U_{LL} < 0, U_{\tau \tau} < 0, U_{LL}U_{\tau \tau} - U_{L\tau}^2 > 0 \) (\( U \) is globally strictly concave in \( (L, \tau) \)). Equation (7) gives the elasticity of labor supply with respect to an exogenous variation in the wage rate. This is derived from the first order condition with respect to labor supply (for given amount of training), \( s(1-t_j)W_j - L_j^{a_j-1} = 0 \). Starting from \( a_m > a_f \), the comparative statics on wages, labor supply, elasticities and training, follow directly from inspection of (6) and (7).

Appendix to Section 3.2

We first substitute the solution (6) and the constraints (4) and (5) into the objective function (3) and derive the indirect utility function for gender \( j \). We denote the revenues collected from gender \( j \) evaluated at the solution (6) as \( R_j = t_jL_j(t_j; a_j, s)W_j(t_j; a_j, s) \). Then we can write the planning program as

\[
\max_{t_m, t_f} \Omega = U_m(t_m, t_f; a, s) + U_f(t_f, t_m; a, s) \text{ subject to } R = R_m + R_f \geq G.
\]

A standard complication in public economics (Diamond and Mirrlees 1971b, Myles 1995, pp 113-114) arises from the fact that the above maximization problem is not sufficiently "concave". In the dual approach to public finance, the problem arises because the consumer’s indirect utility function is quasi-convex in prices (and income). The program can be turned into a concave problem for a social welfare function of the form \( \Omega(U_m, U_f) \), with \( \Omega \) being sufficiently concave (high inequality aversion), but in general the transformation of a concave with a convex function is not guaranteed to be concave. In our case, with quasi-linear preferences and the utilitarian welfare (i.e. \( \Omega_{U_m} = \Omega_{U_f} = 1 \)), welfare is strictly convex in the tax rates. This means that we cannot invoke standard sufficient conditions from the theory of concave programming.

To establish the existence of the solution and the sufficiency of the first order condition for the above problem we follow fairly standard steps.\(^{32}\) First, from the definition of the indirect utilities \( U_m \) and \( U_f \), it is straightforward to show that the welfare function \( \Omega = U_m + U_f \) is strictly decreasing in \( t_m \), strictly decreasing in

\(^{32}\)We also can show that this is true for any welfare function that is more concave than the utilitarian case (which is the least concave welfare function).
that if a contour set of the revenue isolevel $R$ increases welfare and still satisfies the budget constraint. This is shown in Figure 10. The reason is that then we can decrease.

We also have that the maximum revenue the solution is

$$m_{\text{max},G} = \frac{\partial R_i}{\partial t_j} = sL_j^2[1-t_j-t_j(a_j+2)]$$

The Laffer curve for spouse $j$ peaks at the tax rate where the elasticity of earnings with respect to the tax rate is minus unity, so that $t^b_j = \frac{a_j-2}{2a_j}$. We also have that

$$\frac{\partial^2 R_i}{\partial t_j^2} = sL_j^2[-1-\frac{a_j+2}{a_j-2}] + 2sL_j \frac{\partial L_j}{\partial t_j} [1-t_j-t_j^2(a_j+2)]$$

The first term is negative while the second term is negative if $\frac{\partial R_j}{\partial t_j} > 0$. So, the revenue function for spouse $j$ is concave if (but not only if) we are at the upwards sloping part of the Laffer curve. Given the properties of $R_m$ and $R_f$, total revenues $R = R_m + R_f$ are strictly increasing in each of $t_m$ and $t_f$ and strictly concave in $(t_m, t_f)$ if (but not only if) $(t_m, t_f) < (t^b_m, t^b_f)$. This means that in the $(t_m, t_f)$ space the gradient vector of the revenue function $\nabla R$ points towards the bliss point and the upper contour set of the revenue isolevel $R = G$ is strictly convex in that region. Note that if $a_m > a_f$, then the bliss point lies above the 45 degree line which signals that $t_m > t_f$ holds in the solution. So, if the government wants to extract the maximum revenue the solution is $t_m = t^b_m > t_f = t^b_f$. We define $G_{\text{max}}$ to be the maximum sustainable level of public expenditure with $G_{\text{max}} = R(t^b_m, t^b_f)$.

Next, it is easy to show that $t_m > t^b_m$ or $t_f > t^b_f$ can never solve the program. If this was not the case, then we could increase both welfare and revenues which contradicts optimality. Therefore, without loss in generality we now restrict attention to the set $D = \{(t_m, t_f) : t_m \in [0, t^b_m], t_f \in [0, t^b_f], G_{\text{max}} \geq R \geq G\}$. Since $D$ is a compact set and $\Omega$ is a continuous function, by Weierstrass Theorem, a global maximum exists in $D$. Finally, it must be the case the constraint always binds at the optimum. If this was not the case, then we could increase welfare by decreasing some tax rate, while still satisfying the constraint.

Fix an arbitrary level of public expenditure. Since we know that $t_m > t_f$ if $G = G_{\text{max}}$ we now restrict to $G < G_{\text{max}}$. As we showed before, in this area, the revenue function is strictly increasing and strictly concave. The next step is to establish that for $a_m > a_f$, i.e. $\sigma_f > \sigma_m$, $t_f > t_m$ is never an optimal solution. To show that this cannot be an optimum, it suffices to show that the slope of the welfare function in the $(t_m, t_f)$ space is always greater in absolute value than the slope of the revenue function at every point along the $R = G$ level where $t_f > t_m$. This is shown in Figure 10. The reason is that then we can decrease $t_f$, increase $t_m$, increase welfare and still satisfy the budget constraint. $t_f > t_m$ is never optimal because the relative marginal cost of taxing a female is higher than the relative

\[\text{Because labor supply and training are always in the interior all inequalities are strict.}\]
marginal revenue that the government extracts.

The slope of the revenue function is given by 
\[
\frac{R_{ij}}{R_{im}} = -\frac{sL_j^2(1-t_f-t_f^{a_f+2})}{sL_m^2(1-t_m-t_m^{a_m+2})},
\]
and the slope of the welfare indifference curve by 
\[
\frac{\Omega_{ij}}{\Omega_{im}} = -\frac{sL_j^2(1-t_f)[s+(1-s)(\frac{2a_f}{a_f+2})]}{sL_m^2(1-t_m)[s+(1-s)(\frac{2a_m}{a_m+2})]}
\]
Now starting from \(a_m > a_f (\sigma_f > \sigma_m)\) and \(t_f > t_m\) we have that 
\[
\frac{s+(1-s)(\frac{2a_m}{a_m+2})}{s+(1-s)(\frac{2a_f}{a_f+2})}
\]
is larger than one larger than \(\frac{1-t_f}{1-t_m}(\frac{a_f+2}{a_f+2})\), for all \(s\), so that the welfare indifference curve is steeper than the revenue level at any point where \(t_f > t_m\) holds.

Similarly, we can establish that the only point where we cannot increase welfare without violating the constraint is the tangency point (notice however that we had to go through this argument first). In that point the welfare indifference curve is less convex than the budget constraint and the optimal taxes satisfy the condition 
\[
s + (1-s)\left(\frac{2a_f}{a_f+2}\right) = 1 - \frac{tf}{1-t_f}\left(\frac{a_f+2}{a_f+2}\right)
\]
Equation (25) establishes that if \(\sigma_f > \sigma_m\) then \(t_m > t_f\). The tangency condition is unique because the utility function is strictly concave. This ensures that the objective function (8) is strictly convex and the constraint (9) is strictly concave in the tax rates.

For the version of the model without quasi-linear preferences, the utility function for spouse \(j\) is given by 
\[
U_j = X_j - \frac{1}{a_j}L^{a_j} - \frac{r_f^2}{2}
\]
where \(X_j\) is a “composite commodity” given by 
\[
X_j = s\left(\frac{CL^{1-\theta}}{1-\theta}\right) + (1-s)\left(\frac{CL^{1-\theta}}{1-\theta}\right)\]
and the budget constraint is simply 
\[
C_j = (1-t_f)W_jL_j.
\]
In Table 1, we set \(\frac{G}{GDP} = 20\%\) and \(\theta = 0.5\).

Appendix to Section 4

The term \(\frac{1}{\theta}\) in front of the disutility of labor in (10) is just a normalizing constant and it is easy to verify that cross gender differences hold as before even without this constant. \(U = E\omega V = C - \frac{1}{\theta}L^\theta - \frac{1}{2}r^2\) is the expected utility function which is derived under the properties of the chi-squared distribution. Then, equations (12)-(13) are obvious. \(u = -\frac{1}{\theta}v'(L)\omega\) is the expost disutility from labor supply (in \(V = C + u - \frac{1}{2}r^2\)), where \(v(L) = \frac{1}{2}(1 - \frac{1}{\theta})\), with \(v' > 0, v'' < 0, v''' > 0\). We also define the curvature functions 
\[
r_\omega = -\frac{u}{\omega} = -v(L) \quad \text{and} \quad r_L = \frac{uL}{\omega} = \frac{v'(L)}{v(L)} + v'(L)\omega.
\]
While we have that \(r_\omega > 0\), so spouses are always risk averse in \(\omega\)-variations, \(r_L > 0\) only for \(\omega > 4L\). However, every spouse expects to be expost averse to \(L\)-variations because 
\(E_\omega r_L = \frac{v'(L)}{v(L)} + 2av'(L) > 0\). \(r_\omega\) is constant in \(\omega\) (hence the terminology
"CARA") but changes with $L$. $r_L$ is not constant but depends on $L$ and $\omega$. For the third order effect $\frac{\partial^3 E}{\partial \omega^3}$, a sufficient, but not necessary, condition for this to be negative ("risk prudence") is that $a_1 > 3$.

What matters for our results is the, in expectation, variation of $r_L$ with the expected opportunity offered by the labor market, $E(\omega)$. More specifically, $E_\omega r_L$ is positively correlated with the $E(\omega)$ because

$$\frac{\partial E_\omega(r_L)}{\partial E(\omega)} = \frac{1}{2L^2} > 0$$

(26) states that as the expectation of favorable labor market opportunities increases, a spouse expects to be less willing to adjust labor supply $L$ ex post. This flattens the (exante) utility contours (in the $(C, L)$ space) and lowers the elasticity of labor supply.

Appendix to Section 5.3

Denoting $a = a_m$, the derivative of the indirect utility function for the male with respect to $a$ is given by $\frac{\partial U_m}{\partial a} = \frac{1}{2}L^2 \frac{1}{a} \ln L + (1-s)(1-t_f) \frac{\partial E_f}{\partial a}$, where $E_f = W f L_f$ are female’s earnings. In the absence of sharing we have $\frac{\partial U_m}{\partial a} > 0$. In the presence of sharing, the second term tends to lower $\frac{\partial U_m}{\partial a}$ because the male loses consumption by forcing the female to stay at home. For extreme levels of sharing, $\frac{\partial U_m}{\partial a} < 0$. This is Area I in Figure 6. Similarly for the female. From now on we restrict the discussion in Areas II and III, with $\frac{\partial U_m}{\partial a} > 0$ and $\frac{\partial U_f}{\partial a} < 0$. In all Pareto efficient allocations, $\frac{\partial U_m}{\partial a}$ and $\frac{\partial U_f}{\partial a}$ have the opposite sign.

Write the first order condition for the maximization of (17) as $F(a, t_m, t_f, s, \gamma) = \gamma \frac{\partial U_m}{\partial a} \rho - (1-\gamma) \frac{\partial U_f}{\partial a} \rho = 0$. Differentiating this identity with respect to $a$ we get that $-\frac{\partial F}{\partial a} = \gamma \frac{\partial^2 U_m}{\partial a^2} - (1-\gamma) \frac{\partial^2 U_f}{\partial a^2}$. Since strong individual rationality holds ($U_m > T_m$ and $U_f > T_f$), a sufficient but not necessary condition for $-\frac{\partial F}{\partial a} > 0$ is that $U_m$ and $U_f$ are concave in $a$, which is true in the Pareto area (see Appendix to Section 6.1) - the second order condition holds.

Differentiate the first order condition with respect to $\gamma$ and get that $\frac{\partial F}{\partial \gamma} = \frac{\partial U_m}{U_m - T_m} - \frac{\partial U_f}{U_f - T_f} > 0$. Therefore, using the second order condition, we have $\frac{\partial a}{\partial \gamma} = \frac{\partial a}{\partial \gamma} > 0$, and naturally the male gets less home duties the larger is his bargaining power.

For the sharing effect the second-order effects are too complicated to yield a meaningful comparative static. However, in all our results the first order effect, i.e. that as sharing increases, the male wants to induce work effort from the female
and takes more home duties, dominates (see Figures 3 and 4 for an example) and for reasonable perturbation of parameters we have $\frac{\partial a}{\partial t} > 0$.

What matters for the argument that gender based taxes change the implicit bargaining power is that $\frac{\partial a}{\partial m} < 0$. However, the intuition may well be inspected by changing one tax rate at the time.

The redistribution, threat and cross redistribution effects follow from simple inspection of the utilities under marriage and under autarky. That the threat effect dominates the redistribution effect can be established by differentiating $U_m$ and $T_m$ to obtain $\frac{\partial U_m}{\partial m} - \frac{\partial T_m}{\partial m} = -(s(1-t_m))^{\frac{m+2}{m-2}} - (-t_m)^{\frac{2m+2}{m-2}} > 0$ which holds if (but not only if) $s < 1$ and $a_m < \phi$. A weaker sufficient condition is that a single person takes less home duties than a married person, which we believe is a reasonable condition. This condition becomes sufficient and necessary for no resource sharing, $s = 1$.

That $\frac{\partial U_m}{\partial m} - \frac{\partial T_m}{\partial m} > 0$ is "almost" sufficient for $\frac{\partial a_m}{\partial m} < 0$ can be established as follows. We want to show that $\frac{\partial E}{\partial m} < 0$. For this write $\frac{\partial E}{\partial m} = \gamma \frac{-\partial U_m}{\partial a} \frac{\partial a_m}{\partial m} - \frac{\partial T_m}{\partial m} + \gamma \frac{\partial^2 U_m}{\partial a \partial t_m} \frac{\partial a_m}{\partial m} - (1-\gamma) \frac{\partial^2 U_f}{\partial a \partial t_f} \frac{\partial a_m}{\partial m}$. We have that $\frac{\partial U_m}{\partial a} > 0$ and $\frac{\partial U_f}{\partial a} < 0$. Now the term $\frac{\partial U_m}{\partial m} - \frac{\partial T_m}{\partial m}$ is positive because the threat effect dominates the redistribution effect. The term $\frac{\partial U_f}{\partial m}$ is the cross redistribution effect and it is negative. The term $\frac{\partial^2 U_m}{\partial a \partial t_m}$ by virtue of the Envelope and Young’s Theorems can be written as $\frac{\partial^2 U_m}{\partial a \partial t_m} = -s \frac{\partial E_m}{\partial a}$ and it is negative as earnings decrease with home duties. Finally, the term $\frac{\partial^2 U_f}{\partial a \partial t_f} = -(1-s)\frac{\partial E_m}{\partial a} - (1-t_m)\frac{\partial^2 E_f}{\partial a \partial t_f}$ may be either positive or negative, depending on the elasticity of earnings with respect to home duties. If it is negative, which holds when $s$ is not too high (say in Area II), then $\frac{\partial E}{\partial m} < 0$ as wanted. If it is positive, then $\frac{\partial E}{\partial m} < 0$ cannot be established analytically, but in our numerical results the last effect never dominates the three first effects. The reason is that for $s$ very large which is necessary for $\frac{\partial E_m}{\partial a} - (1-t_m)\frac{\partial^2 E_f}{\partial a \partial t_f} < 0$ to hold, $(1-s)$ times $\frac{\partial E_m}{\partial a} - (1-t_m)\frac{\partial^2 E_f}{\partial a \partial t_f}$ becomes negligible. In the absence of sharing, $s = 1$, $\frac{\partial E}{\partial m} < 0$ always holds because the first and the second terms are always negative, and therefore $\frac{\partial a}{\partial m} < 0$. $\frac{\partial a}{\partial t} > 0$ can be examined similarly.

Appendix to Section 6

We denote $a_m = a$ and $a_f = A - a$. We calibrate the total number of home duties to be 20 ($A = 10$). This delivers elasticities of labor supply around .2 for the male and .3 for the female under no resource sharing. The bargaining power $\gamma$ is set at 3/4 because (i) with resource sharing and (ii) spouses being ”risk averse in a”, the allocation of resources is quite balanced. For example, with $s = 1$ (i.e. the
male willing to take as less home duties as possible) and \( z = .2 \), the male extracts around 60% of the marriage surplus. For \( s < 1 \) this is even smaller and changing the bargaining power does not create significant variation in the results. Similarly for \( z \) (subject to maintaining strong individual rationality).

In drawing Figures 6-9 we keep constant public expenditure as a percentage of GDP. The reason is that GDP falls quickly with a declining \( s \) (both spouses work less), and therefore holding constant the level of public expenditure \( G \) results in unmeaningful comparisons. \( G/GDP \) is set at 20%.

Even though \( 2a_m \) and \( 2a_f \) can take only integer values, for expository reasons we discretize the total number of shocks \( A \) into a more continuous grid and treat them as continuous variables when conducting comparative statics. Alternatively, we could increase \( A \) to create meaningful variations in \( a_m \) and \( a_f \), but at the expense of calibrating the elasticities and burdening the notation.

To solve the model we specify a grid for \( a \) and two grids on \( t_m \) and \( t_f \). After deriving the Nash-bargaining solution as a function of the tax rates, we substitute \( a(t_m, t_f) \) in the social welfare function and search for the optimum set of taxes that satisfy the government’s budget constraint.

**Appendix to Section 6.1**

We don’t have an analytic expression for the solution of the bargaining program. Working numerically and intuitively, the first point is that \( U_m \) is not globally concave in \( a \). Taking the second derivative with respect to \( a \) we have, for example for the male, that

\[
\frac{\partial^2 U_m}{\partial a^2} = L^a \ln L\left[\frac{1}{a} - \frac{a a_f}{a_m} + L^a \left[ -\frac{2}{a^2} + \ln L\left[\frac{1}{a_m} + \frac{1}{a_m - 2}\right]\right] \right] + (1 - s)(1 - t_f) \frac{\partial^2 E_f}{\partial a^2},
\]

where \( E_f = W_f L_f \) are the female’s earnings. While the first and the second terms are negative, the third term is ambiguous and depends among other things on the level of sharing. However, except for extreme levels of \( a \) (close to 2 or close to A-2) and extreme levels of sharing (\( s \) close to 1/2), concavity is ensured. In particular, in the absence of sharing (\( s = 1 \)), the first two terms yield an unambiguous concave indirect utility \( U_m \). In our numerical results, \( U_m \) is concave in \( a \) everywhere in the Pareto efficient area (i.e. when (20) holds, see Figure 5). Similarly for the concavity of \( U_f \). Since the Nash-bargained allocations are by assumption Pareto efficient, concavity in the area of interest is assured, and the bargaining solution is well defined.

**Appendix to Section 6.2**

The first order necessary condition for interior local optimum for the program described in Section 6.2 is given by

\[
39
\]
The term $\partial a$ in the brackets $\partial U$ because the government desires to affect the allocation of home duties. The term $f$ to the male’s social marginal cost of taxation $\gamma$ endogenous bargaining and starting from $\gamma > \frac{1}{2}$ this term results in a more gendered allocation. This intuition tax revenues that the government raises from the female further deteriorates her to tax the female than it is in the exogenous bargaining case. Every unit of area). From the analysis in Section 5.3 and in this Appendix the term is negative because of decreasing returns of specialization (at least, in the Pareto $a > A$ directly. Starting from $a > A - a$ (i.e. the male getting less home duties) this term is negative because of decreasing returns of specialization (at least, in the Pareto area). From the analysis in Section 5.3 and in this Appendix the term $\partial a$ in the numerator is positive and the term $\partial a$ in the denominator is negative.

While certainly not sufficient this condition can shed some light in the workings of the solution. In the left hand side, the first terms in the numerator and the denominator are the same as in the case of the exogenous bargaining problem (as in equation (25)). The second terms in the numerator and the denominator appear of the solution. In the left hand side, the first terms in the numerator and the denominator are the same as in the case of the exogenous bargaining problem (as in equation (25)). The second terms in the numerator and the denominator appear because the government desires to affect the allocation of home duties. The term $\frac{1}{1-t_f}$ in the numerator and the denominator is common in the numerator and the denominator. This would have been the first order condition if the government could affect $a$ directly. Starting from $a > A - a$ (i.e. the male getting less home duties) this term is negative because of decreasing returns of specialization (at least, in the Pareto area). From the analysis in Section 5.3 and in this Appendix the term $\frac{\partial a}{\partial t_f}$ in the numerator is positive and the term $\frac{\partial a}{\partial t_m}$ in the denominator is negative.

Therefore, relative to the case with exogenous bargaining, the ratio of the female’s to the male’s social marginal cost of taxation $\frac{\partial m}{\partial t_f}, \frac{\partial m}{\partial t_m}$ increases. With endogenous bargaining and starting from $\gamma > \frac{1}{2}$ it is relatively more costly to tax the female than it is in the exogenous bargaining case. Every unit of tax revenues that the government raises from the female further deteriorates her implicit bargaining power and results in a more gendered allocation. This intuition lies in the heart of the $t_m > t_f$ result in Section 6.2.

Things however are complicated by the fact that the ratio of marginal revenues also changes relative to the exogenous bargaining case. The difference stems from the last terms in the numerator and the denominator of the right hand side of (28). The term $\frac{\partial E_m}{\partial t_m}$ measures the shift in the peak of the Laffer curve for spouse $j$ due to the shift in the intrahousehold allocation of resources. For example, increasing the male’s tax rate results in less bargaining power for the male who has to ”settle in” with a smaller $a$. Then the male participates less in the labor market and becomes less risk averse, per the intuition of Section 4. This increases his labor
supply elasticity, which poses an extra cost for the society since the government wants to tax the male. Relative to the exogenous bargaining case, the last terms in the numerator and the denominator in general raise the female over the male ratio of marginal revenues. The reason why this appears to be true is that for \( a_m > a_f \) we have that \( \frac{\partial E_f}{\partial a} \) is greater than \( \frac{\partial E_m}{\partial a} \) in absolute value because earnings are concave in \( a \). Also in our simulations \( \frac{\partial a}{\partial t_m} \) seems to be less responsive than \( \frac{\partial a}{\partial t_f} \) due to the bargaining power of the male. If the ratio of the marginal revenues increases, then it is less easy to extract revenues from the male in the endogenous bargaining case. The simultaneous increase of the ratio of marginal costs and the ratio of marginal revenues under endogenous bargaining, prohibits us from comparing the optimal solution \( \frac{t_f}{t_m} \) under the two regimes.

Finally, the relationship between pre-gender based taxation inequality and the sharing parameter \( s \) can be examined by writing the inequality ratio as

\[
I = \left( s(1-t) \right) \frac{a_m}{a_m-2} \frac{A-a_m}{A-a_m-2} - \frac{A}{A-a_m-2}
\]  

(29)

The first point is that since for \( s = 1 \) and \( \gamma > 1/2 \) we always have \( a_m > A-a_m \), we get that \( I(s=1) > 1 \). Second, for a given level of \( t \) that raises revenues equal to \( G \), let’s call \( K(s) = \frac{a_m(s)}{a_m(s)-2} - \frac{A-a_m(s)}{A-a_m(s)-2} \). Since \( a_m'(s) > 0 \), we have that \( K'(s) < 0 \). The two opposite forces of \( s \) on \( I \) can been illustrated as follows. For given \( K(s) < 0 \), a higher \( s \) decreases \( I \) because the female participates more in order to balance the less sharing of resources that takes place in the family. For given \( s(1-t) \), a higher \( s \) causes \( K(s) \) to become more negative and this tends to increase inequality \( I \). This is because the male shares less resources with the female and “exerts” his bargaining power by choosing an even more unbalanced home duties ratio. The two forces exactly cancel out at point \( s_M = .92 \) in Figures 6 and 7.
Table 1: Welfare effects of Gender Based Taxation with exogenous bargaining

<table>
<thead>
<tr>
<th>Focus</th>
<th>Tax regime</th>
<th>Parameter values</th>
<th>Endogenous ratios</th>
<th>Gains (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Parameter values</td>
<td>Endogenous ratios</td>
<td>Gains (in %)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$G_{GBT}$</td>
<td>$\frac{a_m}{a_f}$</td>
<td>$\frac{\sigma_m}{\sigma_f}$</td>
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<td>G</td>
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<td>1.05 0.98 1.16 1.35</td>
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<tr>
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<td>GBT</td>
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<td></td>
</tr>
<tr>
<td>$\frac{\sigma_m}{\sigma_f}$</td>
<td>GBT</td>
<td>20% 1.83 0.50 0.95</td>
<td>1.05 0.98 1.16 1.34</td>
<td>0.65 0.56 0.84 1.23</td>
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<td></td>
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<tr>
<td></td>
<td>GBT</td>
<td>20% 2.58 0.33 0.95</td>
<td>1.09 0.97 1.31 1.66</td>
<td>2.42 1.96 2.96 4.30</td>
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<tr>
<td></td>
<td>single</td>
<td>1.16 1.16 1.72 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: In the first three rows elasticities are $\sigma_m = 0.1$ for the male and $\sigma_f = 0.2$ for the female. For the last row we have $\sigma_m = 0.089$ and $\sigma_f = 0.267$ respectively. For later reference, we note that in all cases we keep the total number of "shocks" $A = a_f + a_m$ constant. For the version of the model with CRRA subutility for consumption, for elasticities $\sigma_m = 0.07$ and $\sigma_f = 0.28$ we find the ratio of optimal taxes to be $\frac{t_m}{t_f} = 1.62$, welfare gains of approximately 0.26%, employment gains of around 0.93%, gains in training of 0.42%, and GDP gains of 1.24%.
Figure 1: The Labor Market

The graph shows the wage rate for males and females as a function of labor. The solid line represents males, and the dashed line represents females. The wage rate for males increases as labor increases, while the wage rate for females increases more sharply. The wage rates for both males and females are indicated by horizontal lines at certain wage rates.
Figure 2: The Effects of Taxes on the Labor Market

The graph illustrates the wage rate as a function of labor under low and high tax scenarios. The x-axis represents labor, ranging from 0 to 1, and the y-axis represents the wage rate, ranging from 0 to 1.

- **Low Tax**: The solid line represents the wage rate under a low tax scenario.
- **High Tax**: The dashed line represents the wage rate under a high tax scenario.

The graph shows how taxes affect the wage rate and labor participation in the labor market.
Figure 3: Sharing of Resources and Allocation of Shocks

![Graph showing utility (male) as a function of\( a (m) \) for different values of\( s \).]

- Solid line: \( s = 1 \)
- Dashed line: \( s = 0.9 \)
- Dotted line: \( s = 0.8 \)
Figure 4: The Bargaining Solution

Fraction of Home Duties to Female

$tm/tf = 0.85$
$tm/tf = 1$
$tm/tf = 1.15$
Figure 5: Ungendered Equilibrium is the First Best

Pareto Efficient Allocations

Utility

U (m)

U (f)
Figure 6: $\frac{t_m}{t_f}$, $\frac{\sigma_m}{\sigma_f}$ - $s$; $\gamma = 3/4$, $z = 0.2$, $GDP = 20\%$
Figure 7: Wage Ratios - \( s; \gamma = \frac{3}{4}, z = 0.2, \frac{G}{GDP} = 20\% \)
Figure 8: Gains - $s, \gamma = 3/4, z = 0.2, \frac{G}{\text{GDP}} = 20\%$. 

The diagram shows the gains for different values of $s$ with $\gamma = 3/4, z = 0.2$ and $\frac{G}{\text{GDP}} = 20\%$. The X-axis represents the variable $s$, and the Y-axis represents the gain percentage. The diagram is divided into three regions labeled I, II, and III, each representing different gain percentages: Welfare Gain %, GDP Gain %, and Employment %.
Figure 9: Both Spouses May Be Better Off with GBT
Figure 10: Social Welfare Indifference Curves and Revenue Constraint