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Choice Over Time

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Forschungsinstitut
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Choice Over Time

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ABSTRACT

Choice Over Time*

In the last twenty years a growing body of experimental evidence has posed a challenge to the standard Exponential Discounting Model of choice over time. Attention has focused on some specific 'anomalies', notably preference reversal and declining discount rates, leading to the formulation of the model of hyperbolic discounting which is finding increasing favour in the literature. In this paper we provide a survey of both the theoretical modelling and the experimental evidence relating to choice over time. As we will show, a careful analysis of the mapping between theoretical models and experimental investigations raises questions as to whether some of the most focused upon anomalies should be indeed classified as such, or whether they are really the most challenging ones for conventional theory. New developments are emerging both at the theoretical and empirical level, opening up new exciting avenues for investigation

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1 Introduction

Many economic decisions have a time dimension. Hence the need to describe how outcomes available at future dates are evaluated by individual agents. The history of the search for a 'rational' model of preferences over (and choices between) dated outcomes bears some interesting resemblances and dissimilarities with the corresponding search in the field of risky outcomes. First, a standard and widely accepted model was settled upon. This is the exponential discounting model (EDM) (Samuelson [50]), for which the utility from a future prospect is equal to the present discounted value of the utility of the prospect. That is, an outcome x available at time t is evaluated now, at time $t = 0$, as $\delta^t u(x)$, with δ a constant discount factor and u an (undated) utility function on outcomes. So according to the EDM, x at time t is preferred now to y at time s if

$$\delta^t u(x) > \delta^s u(y)$$

Similarly a sequence of timed outcomes x_1, x_2, ... x_T is preferred to another sequence y_1, y_2, ... y_T if

$$\sum_{i=1}^{T} \delta^{i-1} u(x_i) > \sum_{i=1}^{T} \delta^{i-1} u(y_i)$$

Subsequently, an increasing number of systematic 'anomalies' were demonstrated in experimental settings. This spurred the formulation of more descriptively adequate 'non-exponential' models of time preferences.

This mirrors the events for the standard model of decision under risk, the Expected Utility model, in which case observed experimental anomalies led to the formulation of Non-Expected Utility models. However, unlike the case of choice between risky outcomes, for choice over time no normative axioms of 'rationality' were formulated which had the same force as, say, the von Neumann-Morgenstern Independence axiom of utility theory. Perhaps for this reason, economists have been readier to accept one specific alternative model, that of hyperbolic discounting.

In this chapter we review both the theoretical modelling and the experimental evidence relating to choice over time. Most of the space is devoted to choices between outcome-date pairs, which has been better studied, especially experimentally, but in section 4 we also discuss choices between time sequences of outcomes. In the next section we examine the axiomatic foundation for models based on discounting, exponential or otherwise. In section 3 we review the 'new breed' of models emerged as a response to experimental observations. Section 5 looks in more detail at the empirical evidence, while section 6 is devoted to evaluating the explanatory power of the various theories. Section 7 concludes.

2 Axiomatics of exponential discounting for outcome-date pairs

We begin by describing a basic axiomatisation of exponential discounting for outcome-date pairs due to Fishburn and Rubinstein [11]. This will help us in giving a sense of the type of EDM violations that one may expect to observe in practice.

We should make clear at the outset that we follow the standard economic approach of taking *preferences (as revealed by binary choices),* as the primitives of the analysis. Any 'utility' emerging from the analysis will simply describe the primitive preferences in a numerical form. We are not, therefore, considering 'experience' utility (i.e. the psychological benefit one gets from experience) as a primitive, an approach which is more typical in the psychology literature. Also, we focus on time preferences as if the agent can *commit* to them: this is in order to avoid a discussion of the thorny issue of time-consistency[1], which would deserve a treatment on its own.

Let $X \subseteq \mathcal{R}_+$, with $0 \in X$, represent the set of possible outcomes (interpreted as gains, with 0 representing the *status quo)*, and denote by $T \subseteq \mathcal{R}_+$ the set of times at which an outcome can occur (with $t = 0 \in T$ representing the "present"). Unless specified, T can be either an interval or a discrete set of consecutive dates.

A time-dependent outcome is denoted as (x, t): this is a promise, with no risk attached, to receive outcome $x \in X$ at date $t \in T$. Let $\succcurlyeq$ be a preference ordering on $X \times T$. The interpretation is that $\succcurlyeq$ is the preference expressed by an agent who deliberates in the present about the promised receipts of certain benefits at certain future dates.

As usual, let $\succ$ and $\sim$ represent the symmetric and asymmetric components, respectively, of $\succcurlyeq$. Fishburn and Rubinstein [11]'s characterisation uses the following axioms:[2]

Order: $\succcurlyeq$ is reflexive, complete, and transitive.
Monotonicity: If $x > y$ then $(x, t) \succ (y, t)$.
Continuity: $\{(x, t) : (x, t) \succcurlyeq (y, s)\}$ and $\{(x, t) : (y, s) \succcurlyeq (x, t)\}$ are closed sets.
Impatience: Let $s < t$. If $x > 0$ then $(x, s) \succ (x, t)$, and if $x = 0$ then $(x, s) \sim (x, t)$.
Stationarity: If $(x, t) \sim (y, t + t')$ then $(x, s) \sim (y, s + t')$, for all $s, t \in T$ and $t' \in \mathcal{R}$ such that $s + t', t + t' \in T$.

The first four axioms alone guarantee that preferences can be represented by a real-valued 'utility' function u on $X \times T$ with the natural continuity and monotonicity property

[1] Initiated by Strotz [54].

[2] Fishburn and Rubinstein [11] consider the general case where the outcome can involve a loss as well as a gain, that is $x < 0$, and they do not require that $0 \in X$. Here we focus on the special case only to simplify the exposition.

(that is, u is increasing in x and decreasing in t, and it is continuous in both arguments when T is an interval). The addition of Stationarity allows the following restrictions:

Theorem 1 ***(Fishburn and Rubinstein, 1982)*** *If Order, Monotonicity, Continuity, Impatience and Stationarity hold, then, given any $\delta \in (0,1)$, there exists a continuous and increasing real-valued function u on X such that*

$$(x,t) \succcurlyeq (y,s) \Leftrightarrow \delta^t u(x) \geq \delta^s u(t)$$

In addition, $u(0) = 0$, and if X is an interval, then u is unique (for a given δ) up to multiplication by a positive constant.

The representation coincides formally with exponential discounting, but note well the wording of the statement. One may fix the 'discount factor' δ arbitrarily to represent a given preference relation that satisfies the axioms, provided the 'utility function' u is calibrated accordingly. In other words, for any two discount factors δ and δ', there exist two utility functions u and v such that (u,δ) preferences in the representation of theorem 1 are identical to (v,δ') preferences in the same type of representation. In order to interpret δ as a uniquely determined parameter expressing 'impatience' one would need an *external* method to fix u. This is an important observation, often neglected in applications, which naturally raises the question about what then exactly is *impatience* here. Benoit and Ok [3] deal with this question by proposing a natural method to compare the delay aversions of time preferences, analogous to methods to compare the risk aversion of preferences over lotteries. As they show, in the EDM it is possible that the delay aversion of a preference represented by (u,δ) is greater than that represented by (v,δ') even though $\delta > \delta'$.

Moreover, given the uniqueness of u only up to multiplication by constants, and the positivity of u for positive outcomes, an additive representation (at least for strictly positive outcomes) is as good as the exponential discounting representation. That is, taking logs and rescaling utilities by dividing by $-\log\delta$, one could write instead

$$(x,t) \succcurlyeq (y,s) \Leftrightarrow u(x) - t \geq u(y) - s$$

Coming back to the axioms, Continuity is a standard technical axiom. Order is a rationality property deeply rooted in the economic theory of choice. Cyclical preferences for example are traditionally banned from economic models. Monotonicity and impatience are also universally assumed in economic models, which are populated by agents for whom more of a good thing is better, and especially for whom a good thing is better if it comes sooner: certainly these are reasonable assumptions in several contexts, though as we shall see, not in others.

Stationarity, however, does not appear to have a very strong justification, either from the normative or from the positive viewpoint. So it should not be too surprising to observe in practice violations of this axiom and in fact, as we shall see later in some detail, plenty of them have been recorded. What is surprising is rather the willingness of economists to have relied unquestionably for so many years on a model, the EDM, which takes stationarity for granted. Indeed Fishburn and Rubinstein themselves explicitly state that 'we know of no persuasive argument for stationarity as a psychologically viable assumption' ([11], p. 681). This led them to consider alternative separable representations that do not rely on Stationarity. One assumption (which is popular in the theory of measurement) is the following:

Thomsen separability: If $(x,t) \sim (y,s)$ and $(y,r) \sim (z,t)$ then $(x,r) \sim (z,s)$.

This allows a different representation result:

Theorem 2 ***(Fishburn and Rubinstein, 1982)*** *If Order, Monotonicity, Continuity, Impatience and Thomsen separability hold, and X is an interval, then there are continuous real-valued functions u on X and δ on T such that*

$$(x,t) \succcurlyeq (y,s) \Leftrightarrow \delta(t)\, u(x) \geq \delta(s)\, u(y)$$

In addition, $u(0) = 0$ and u is increasing while δ is decreasing and positive.

This is therefore an axiomatisation of a discounting model, in which the discount factor is not constant. However, while Thomsen's separability is logically much weaker than stationarity and it is useful to gauge the additional strength needed to obtain a constant discount factor, one may wonder how intuitive or reasonable a condition it is itself. One might not implausibly argue, for example, that if exactly $(y-x)$ is needed to compensate for the delay of $(s-t)$ in receiving x, and if exactly $(z-y)$ is needed to compensate for the delay of $(t-r)$ in receiving y, then exactly $(z-x)$ is needed to compensate for the delay of $(s-r)$ in receiving x. This argument does not seem to us introspectively much more cogent than stationarity,[3] though it permits the elegant and flexible representation of theorem 2.

It should be clear from the above results and discussion that the EDM for outcome-date pairs is best justified on the basis of its simplicity and usefulness in applications. Violations especially of the stationarity aspect of it are to be expected, and while they have captured most of the attention, it is perhaps violations of other properties, such as Order, which would appear to be more intriguing, striking as they do more directly at the core of traditional thinking about economic rationality.

[3] Fishburn and Rubinstein [11] also provide a different argument for Thomsen separability, based on an independence condition when the domain of outcomes is enriched to include gambles.

3 Recent models for outcome-date pairs

3.1 Hyperbolic discounting

As we mentioned already, over the past twenty years or so a body of empirical evidence has emerged documenting that actual behaviour consistently and systematically contradicts the predictions of the standard model. As we discuss more fully in section 5, various exponential discounting 'anomalies' have been identified.[4] As we explain further in section 6, in a sense some of these are not anomalies at all: they do not violate any of the axioms in the theorems above, but only make specific demands on the shape of the utility function. Among those that do violate the axioms in the representations, one particular effect has captured the limelight: preferences are rarely stationary, and people often exhibit a strict preference for "immediacy". Decision makers may be indifferent between some immediate outcome and a delayed one, but in case they are both brought forward in time, the formerly immediate outcome loses completely its attractiveness. More formally, if x and y are two possible outcomes, situations of the type described above can be summarised as

$$(x,0) \succ (y,t) \text{ and } (y,t+\tau) \succ (x,\tau)$$

Note that this violates jointly four of the five axioms in the characterization of theorem 1, with the exception of Impatience. Let $x' \neq x$ be such that $(x',0) \sim (y,t)$ (such an x' exists by Continuity). It must be that $x' < x$ (for otherwise if $x' > x$ then by Monotonicity $(x',0) \succ (x,0) \succ (y,t)$ and by Order $(x',0) \succ (y,t)$). By Stationarity $(x',\tau) \sim (y,t+\tau)$. Then by Monotonicity again $(x,\tau) \succ (y,t+\tau)$, a contradiction with the observed preference.

However this is commonly interpreted as a straight violation of Stationarity, since the latter is sometimes defined in terms of strict preference as well as indifference. It is, however, compatible with the weaker requirement of Thomsen separability.

As a matter of fact, many researchers observing these phenomena do not pay attention to any axiomatic system at all, preferring rather to concentrate directly on the EDM representation itself (sometimes implicitly assuming a linear utility). In the EDM representation the displayed preferences are written as

$$u(x) > \delta^t u(y) \text{ and } \delta^\tau u(x) < \delta^{t+\tau} u(y)$$

which is impossible for any utility function u and fixed δ.

[4] For a survey of these violations, see Loewenstein and Thaler [29] or Loewenstein and Prelec [26]; for a thorough treatement of issues concerning choice over time see Elster and Loewenstein [9].

This present time bias (immediacy effect) is a special case of what is known as '*preference reversal*' (or sometimes 'common ratio effect' in analogy with Expected Utility anomalies in the theory of choice under risk), expressed by the pattern:

$$(x,t) \succ (y,s) \text{ and } (y,t+\tau) \succ (x,s+\tau)$$

Strictly speaking, as the agent is expressing preferences at one point in time (the present), nothing is really 'reversed': the agent simply expresses preferences over different objects, and these preferences happen not to be constrained by the property of stationarity. The reason for the 'reversal' terminology betrays the fact that often, especially in the evaluation of empirical evidence, it is implicitly assumed that there is a coincidence between the *current* preferences over future receipts (so far denoted $\succcurlyeq$) and the *future* preferences over the same receipts to be obtained at the same dates. In other words, now dating preferences explicitly, $(x,t) \succcurlyeq_0 (y,s)$ is assumed to be equivalent to $(x,t) \succcurlyeq_\tau (y,s)$, where $\succcurlyeq_\tau$ with $\tau \leq s,t$ is the preference at date τ. If today you prefer one apple in one year to two apples in one year and one day, in one year you also prefer one apple immediately to two apples the day after. It is far from clear that this is a good assumption. In this way, the displayed observed pattern can be taken as a 'reversal' of preferences during the passage of time from now to date τ. Whether this is a justified interpretation or not, the displayed pattern does contradict the EDM. This is, though, a somewhat 'soft' anomaly, in the sense that it does not contradict basic tenets of economic theory, and it can be addressed simply by changes in the functional form of the objective function which agents are supposed to maximize. Notably, it can be explained by the now popular model of *hyperbolic discounting* (HDM)[5] (as well as by other models). In the HDM it is assumed that the discount factor is a hyperbolic function of time.[6] In its general form, $\delta : T \to \mathcal{R}$ is given as

$$\delta(t) = (1+at)^{-\frac{b}{a}} \text{ with } a,b > 0$$

In the continuous time case, in the limit as a approaches zero, the model approaches the EDM, that is

$$\lim_{a \to 0} (1+at)^{-\frac{b}{a}} = e^{-bt}$$

[5] See for instance Phelps and Pollack [39], Loewenstein and Prelec [26], Laibson [22], Frederick, Loewenstein and O'Donoghue [12].

[6] For evidence documenting behaviour compatible with this functional form see for instance Ainslie [1], Benzion, Rapoport and Yagil [4], Laibson [22], Loewenstein and Prelec [26] and Thaler [55]. It is important to stress that Harrison and Lau [18] have argued against the reliability of the elicitation methods used to obtain this empirical evidence. They argue that this evidence is a direct product of the lack of control for credibility in experimental setting with delayed payment.

For any given b (which can be interpreted as the discount rate), a determines the departure of the discounting function from constant discounting, and is inversely proportional to the curvature of the hyperbolic function.

Hyperbolic discount functions imply that discount rates decrease over time. The hyperbolic functional form captures in an analytically convenient way the idea that the rate of time preference between alternatives is not constant but varies, and in particular decreases as delay increases. So people are assumed to be more impatient for tradeoffs (between money and delay) near the present than for the same tradeoffs pushed further away in time. It can account for preference reversals.

This model fits in the representation of theorem 2 in section 2. Preference reversal can be easily reconciled within an extension of the EDM, in which the requirement of stationarity has been weakened to Thomsen separability.

The present time bias can be captured even more simply in the most widely used form of declining discount model, the *quasi-hyperbolic* model or '(β, δ) model'. In it, the rate of time preference between a present alternative and one available in the next period is $\beta\delta$, whereas the rate of time preference between two consecutive future alternatives is δ. Therefore (x, t) is evaluated now as $u(x)$ if $t = 0$ and as $\beta\delta^t u(x)$ if $t > 0$, where $\beta \in (0, 1]$ (the case of $\beta = 1$ corresponds to exponential discounting). So we may have

$$u(x) > \beta\delta u(y) \text{ and } \beta\delta^{t+\tau} u(y) > \beta\delta^{\tau} u(x)$$

'rationalising' the present time bias. As we expand on further below, this same approach can be applied in the case of sequences of outcomes (see section 4).

3.2 Relative Discounting

Ok and Masatlioglu [36] have recently proposed an interesting and challenging axiomatic model which, though retaining a certain notion of discounting, dispenses with the usual notion of evaluating future outcomes in terms of their present value. In their 'relative' discounting model (RDM), in other words, it is not possible in general to attribute a certain value to outcome-date pairs (x, t) and state that the outcome-date pair with the higher value is preferred. More precisely, their representation (axiomatized for the case where the set of outcomes X is an open interval) is of the following type: there exists a positive, real valued and increasing utility function u on outcomes and a 'relative discount' function $\delta : T \times T \to \mathcal{R}$ defined on date *pairs* such that

$$(x, t) \succsim (y, s) \Leftrightarrow u(x) \geq \delta(s, t)\, u(y)$$

The relative discount function δ is positive, continuous, decreasing in its first argument for any fixed value of the second argument (with $\delta(\infty, t) = 0$), and $\delta(s, t) = 1/\delta(t, s)$.

The model is axiomatised in terms of a set of axioms which includes some weak (but rather involved) separability conditions.

The authors' own interpretation of the preference $(x,t) \succsim (y,s)$ is that 'the worth at time t of the utility of y that is to be obtained at time s is strictly less than the worth at time t of the utility of x that is to be obtained at time t'. They argue that one of the the main novelties of the RDM is that the comparison between the values of (x,t) and (y,s) is not made in the present but at time t or s. However it seems hard to tell when a comparison between atemporal utilities is made. When comparing *outcome-date pairs*, and not utilities, it is certainly at time 0 that the agent is making the comparison. So one could as naturally say that the comparison between the utilities $u(x)$ and $u(y)$ is also made at time 0, but instead of discounting the utility of the later outcome by the entire delay with which it is to be received, it is only discounted by a measure of its relative delay with the earlier outcome, whose utility is not discounted at all (psychologically this corresponds to 'projecting' the future into the present, which seems reasonable). While this might appear a little like splitting hairs, the issue might become important if the present agent were allowed to disagree with his later selves on the atemporal evaluation of outcomes, that is on the function u to be used (in the existing model this disagreement between current and future selves cannot happen, by an explicit assumption made on preferences). A final, and in our opinion appealing, interpretation of the model is as a threshold model with an additive time-dependent threshold in which the term $\delta(s,t)$ is not seen as a multiplicative relative discount factor but just as a 'utility fee' to be incurred for an additional delay. In fact, just as we did for the EDM representation in section 2, here, too, we can apply a logarithmic transformation to obtain a representation of the type

$$(x,t) \succsim (y,s) \Leftrightarrow u(y) \geq u(x) + \delta(s,t)$$

Whatever the interpretation, one virtue of the RDM is that it can explain some 'hard' anomalies, notably particular types of preference intransitivities (although no cycle within a given time t is allowed - contrast this with the 'vague time preference' model discussed below). The relative discounting representation includes as special cases both exponential and hyperbolic discounting. Therefore, beside intransitivities, it can also account for every soft anomaly for which the HDM can account. In this sense the model is successful. On the flip side of the coin, one might argue that it is almost too general, and many other special cases are also included in it. For example, the subadditive discounting or similarity ideas discussed in the next section can also be formulated in this framework.

A similar model has been studied independently by Scholten and Read [52], who call it the 'discounting by interval' model. Their interpretation, motivation and analysis is however quite different from that of Ok and Masatlioglu [36]. In their model, the discount

function is defined on intervals of time, which is equivalent to defining it on pairs of dates as for the RDM. But the authors argue for comparisons between alternatives to be made by means of usual *present values*, for which the later outcome is first discounted to the date of the earlier outcome (using the discount factor which is appropriate for the relevant interval) and then discounted again to the present (using the appropriate discount factor which is appropriate for this different interval). So, formally: for $s > t$,

$$(x,t) \succsim (y,s) \Leftrightarrow \delta(0,s)\, u(y) \geq \delta(0,s)\, \delta(s,t)\, u(x) \Leftrightarrow u(y) \geq \delta(s,t)\, u(x)$$

Scholten and Read do not axiomatise their model, but focus on interesting experimental evidence suggesting some possible restrictions of the discounting function.

3.3 Similarities and subadditivity

While not proposing fully fledged-models, two contributions by Read [42] and Rubinstein ([47], [48]), put forth some analytical ideas to interpret certain types of anomalies. We consider the contributions by these two authors in turn.

Subadditivity. Read [42] suggest that a model of *subadditive discounting* might apply. This means that the average discount rate for a period of time might be lower than the rate resulting from compounding the average rates of different subperiods. Furthermore, he suggests that the finer the partition into sub-periods, the more pronounced this effect should be. Formally, if $[0,T]$ is a time period divided into the intervals $[t_0,t_1],...,[t_{k-1},T]$. Let $\delta_T = \exp^{-r_T T}$ be the average discount factor for the period $[0,T]$ (where r_T is the discount rate for that period), and $\delta_i = \exp^{-r_i T}$ the average discount factor that applies to the sub-period beginning at i (where r_i is the discount rate for that period). Then, if there is subadditivity, for any amount x available at time t_k, and letting u denote an atemporal utility function, we have that

$$u(x)\, \delta_T > u(x)\, \delta_0 \delta_1 \cdot ... \cdot \delta_{k-1}$$

More abstractly, this general idea could even be defined independently of the existence of an atemporal utility function. Given preferences $\succcurlyeq$ on outcome-date pairs, if

$$\begin{aligned} (x,t_k) &\sim (x_{k-1},t_{k-1}) \sim ... \sim (x_0,0) \\ \text{and } (x,t_k) &\sim (x_0',0) \end{aligned}$$

subadditivity could be taken as implying that

$$x_0' > x_0$$

It is important to note, though, that in the absence of further assumptions on preferences the existence of a separable discount function is not guaranteed. The RDM discussed in the previous section characterises subadditive discounting by $\delta(t,r) > \delta(t,s)\,\delta(s,r)$.

This is reminiscent of some empirical evidence for decisions under risk, according to which the total compound subjective probability of an event is higher the higher the number of sub-events into which the event is partitioned (e.g. Tversky and Koehler [58]). Preferences for which discounting is subadditive may not be compatible with hyperbolic discounting, that is, discount rates may be constant or increasing in time, contradicting the HDM, while implying subadditivity. This is precisely the evidence found by Read [42].

Similarity. Rubinstein ([47], [48]) argues that similarity judgements may play an important role when making choices over time (or under risk). He also shifts attention to the *procedural* aspects of decision making. He suggests that a decision procedure he originally defined for choices under risk (in Rubinstein [46]) can be adapted to model choices over time, too. Let $\approx_{time}$ and $\approx_{outcome}$ be similarity relations (reflexive and symmetric binary relations) on times and outcomes respectively. So $s \approx_{time} t$ reads 'date s is similar to date t' and $x \approx_{outcome} y$ reads 'outcome x is similar to outcome y'. Rubinstein examines the following procedure to compare any outcome-date pairs (x,t) and (y,s):

Step 1) If $x \geq y$ and $t \leq s$, with at least one strict inequality, then $(x,t) \succ (y,s)$. Otherwise, move to step 2.

Step 2) If $t \approx_{time} s$, *not* $(x \approx_{outcome} y)$ and $x > y$ then $(x,t) \succ (y,s)$. If $x \approx_{outcome} y$, *not* $(t \approx_{outcome} s)$ and $t < s$ then $(x,t) \succ (y,s)$.

If neither the premise in step 1 nor the premise in step 2 applies, the procedure is left unspecified. Rubinstein used this idea to show how it serves well to explain some anomalies, some of which run counter to the HDM as well as to the EDM.

Of course, once the broad idea has been accepted, many variations of this procedure seem also plausible. For example Tversky [56] had suggested a '*lexicographic semiorder*' procedure according to which agents rely on their ranking of the attributes of an alternative in a lexicographic way when choosing between different alternatives. The first attribute of each alternative is compared. If, and only if, the difference exceeds some fixed threshold value then a choice is made accordingly. Otherwise, the agent compares the second attribute of each alternative, and so on. Yet another procedure reminiscent of Tversky's lexicographic semiorder is described in the next section.[7]

[7] Kahneman and Tversky [21], too, discuss the intransitivities possibly resulting from the 'editing' phase of prospect theory, in which small differences between gambles may be ignored.

Finally, Rubinstein's [47] experiments show that precisely the same type of decision situations that creates a difficulty for the EDM may also be problematic for the HDM, while they may be easily and convincingly accounted for by similarity-based reasoning. He argues that, in this sense, the change to hyperbolic discounting is not radical enough.

3.4 Vague time preferences

Manzini and Mariotti [30] introduce the notion of 'vague time preferences' as an application of their general two-stage model of decision-making[8]. The starting consideration is that the perception of events distant in time is in general 'blurred'. Even when a decision maker is able to choose between, say, an amount x of money now and an amount y of money at time t, it may be more difficult to confront the same type of alternatives once these are both distant in time. This difficulty in comparing alternatives available in the future may blur the differences between them in the decision maker's perception. In other words, the passage of time weakens not only the perception of the alternatives (which are perceived, in Pigou's famous phrase[9] 'on a diminished scale' because of the defectiveness of our 'telescopic faculty'), but the very *ability to compare* alternatives with one another.

In the 'vague' time preferences model, the central point is that the evaluation of a time dependent alternative is made up of two main components: the pure time preference (it is better for an alternative to be available sooner rather than later, and there exists a limited ability to trade off outcome for time); and *vagueness*: when comparing different alternatives, the farther they are in time, the more difficult it is to distinguish between them.

For (x,t) to be preferred to (y,s) on the basis of a time-outcome tradeoff, the utility of x may exceed the utility of y *by an amount which is large enough so that the individual can tell the two utilities* apart. The amount by which utilities must differ in order for the decision maker to perceive the two alternatives as distinct is measured by a the positive vagueness function σ, a real valued function on outcomes. When the utilities differ by more than σ, then we say that the decision maker prefers the alternative yielding the larger utility *by the primary criterion.* Formally the primary criterion consists of a possibly incomplete preference relation on outcome-date pairs, represented by an interval order as follows:

$$(x,t) \succ (y,s) \Leftrightarrow u(x,t) > u(y,s) + \sigma(y,s)$$

where u is monotonic increasing in outcomes and decreasing in time. When neither alter-

[8] See Manzini and Mariotti [31].

[9] See Pigou [40].

native yields a sufficiently high utility, the decision maker is assumed to resort to some additional heuristic in order to make his choice (*secondary criterion*). Since each alternative has a time and an outcome component, two natural heuristics are distinguished. In the '*outcome prominence*' version, the decision maker will first try and base his choice on which of the two available ones is the greater outcome; and only if this comparison is not decisive will he resolve his choice by selecting the earlier alternative. On the contrary, in the '*time prominence*' version of the model, the decision maker first compares the two alternatives by the time dimension. If one comes earlier, then that is his choice; otherwise he looks at the other dimension, the outcome, and selects based on which is higher.

Formally, let $\succ$ be defined as in the display above, and let $a \sim b$ if and only if neither $a \succ b$ nor $b \succ a$. Assume that P and I are the asymmetric and symmetric parts, respectively, of a complete order on the set of pure outcomes X. Finally, let $\succcurlyeq^*$ (with $\succ^*$ and $\sim^*$ the corresponding symmetric and asymmetric parts, respectively) denote a complete preference relation (not necessarily transitive) on the set of alternatives (i.e. outcome-date pairs) $X \times T$, and let $i = (x_i, t_i) \in X \times T$ for $i \in \{a, b\}$. Then the two alternative models are as follows:

Outcome Prominence Model (OPM):

1. $a \succ^* b \Leftrightarrow$

   (a) $a \succ b$ (Primary Criterion), or

   (b) $(a \sim b, x_a P x_b)$ or $(a \sim b, x_a I x_b, t_a < t_b)$ (Secondary Criterion)

2. $a \sim^* b \Leftrightarrow (a \sim b, x_a I x_b, t_a = t_b)$

Time Prominence Model (TPM):

1'. $a \succ^* b \Leftrightarrow$

   (a) $a \succ b$ (Primary Criterion), or

   (b) $(a \sim b, t_a < t_b)$ or $(a \sim b, t_a = t_b, x_a P x_b)$ (Secondary Criterion)

2'. $a \sim^* b \Leftrightarrow (a \sim b, x_a I x_b, t_a = t_b)$

In its simplest specification, the (σ, δ)-model, there are just two parameters, with δ taken as the individual's discount factor (which embodies the 'pure time preference' component of preference), σ a positive constant measuring the individual's vagueness, and u assumed linear in outcome.

4 Preferences over sequences of outcomes

When it comes to sequences of outcomes available at given times, the standard exponential discounting model still widely used is that introduced by Samuelson [50], whereby sequence $((x_1, t_1), (x_2, t_2), ... (x_T, T))$ is preferred to sequence $((y_1, t_1), (y_2, t_2), ... (y_T, T))$ whenever the present discounted utility of the former is greater than the present discounted utility of the latter:

$$\sum_{t=1}^{T} u(x_t) \delta^{t-1} > \sum_{t=1}^{T} u(y_t) \delta^{t-1}$$

Similarly to the case of outcome-date pairs, Loewenstein and Prelec [26] highlighted that there exists a number of anomalies which cannot be accommodated within the standard framework. We will discuss these anomalies in greater detail in section 5, while here we limit ourselves to present the functional form that Loewenstein and Prelec [26] introduce to account for these phenomena. They propose that the utility of some sequence $x = ((x_1, t_1), (x_2, t_2), ... (x_T, T))$ should be represented by

$$U(x) = \sum_{i=1}^{T} v(x_i) \delta(t_i)$$

where δ is a discount function assumed to be a generalised hyperbola, $\delta(t) = \frac{1}{(1+at)^{\frac{b}{a}}}$, as in the general case of hyperbolic discounting we saw earlier, and v is a value function on which the following restrictions are imposed:

V1: the value function is steeper in the loss than in the gain domain:

$$v(x) < -v(-x)$$

V2: the value function is more elastic for losses than for gains:

$$\varepsilon_v(x) < \varepsilon_v(-x) \text{ for } x > 0, \text{ where } \varepsilon_v \equiv \frac{x \partial v(x)}{v(x)}$$

V3: the value function is more elastic for outcomes that are larger in absolute magnitude:

$$\varepsilon_v(x) < \varepsilon_v(y) \text{ for } 0 < x < y \text{ or } y < x < 0$$

Manzini, Mariotti and Mittone [32] pursue a different approach, where building on Manzini and Mariotti [30] they postulate a theoretical model which extends the one for outcome-date pairs to sequences. In order to rank monetary reward sequences, the decision maker looks first at the standard exponential discounting criterion. However, preferences are incomplete, so that sequences are only partially ordered by the criterion. Here too they are completed by relying on a secondary criterion. Sequence x is preferred

over another sequence y if the discounted utility of x exceeds the discounted utility of y by at least $\sigma(y)$. When sequences cannot be compared by means of discounted utilities, the decision-maker is assumed to focus on one prominent attribute of the sequences. This prominent attribute ranks (maybe partially) the sequences and allows a specific choice to be made. This latter aspect of the model is in the spirit of Tversky, Sattath and Slovic [60]'s *prominence hypothesis*. The attribute may be context dependent, so that for instance in the case of outcome-date pairs case, as we saw above, each alternative has two obvious attributes that may become prominent, the date and the outcome.

We stress that, at a fundamental level, the only departure from the standard choice theoretic approach is that the decision maker's behavior is described by combining sequentially two possibly incomplete preference orderings, instead of using directly a complete preference ordering. In the case of monetary sequences we use the following representation for preferences. Let $\succ^*$ denote the strict binary preference relation on the set of alternatives (sequences) A, where a typical sequence has the form $i = (i_1, i_2, ..., i_T)$. For given u, σ, δ with the usual meaning, and secondary criterion P_2:

For all $a, b \in A$, we have $a \succ^* b$ if and only if

1. either $\sum_{t=1}^{T} u(a_t)\,\delta^{t-1} > \sum_{t=1}^{T} u(b_t)\,\delta^{t-1} + \sigma(b)$, or
2. $\sum_{t=1}^{T} u(a_t)\,\delta^{t-1} \leq \sum_{t=1}^{T} u(b_t)\,\delta^{t-1} + \sigma(b)$, $\sum_{t=1}^{T} u(b_t)\,\delta^{t-1} \leq \sum_{t=1}^{T} u(a_t)\,\delta^{t-1} + \sigma(a)$, and aP_2b

The above obviously begs the question of which secondary criterion one should use. This can be suggested by the empirical evidence available so we postpone examining this issue further, to explore suggestions from data (see sections 5 and 6).

We should finally note that although positive discounting of some form or the other is deeply ingrained in much economic thinking and in virtually all economic policy, the issue of whether this is a justified assumption is open. Fishburn and Edwards [10] axiomatise, in a discrete time framework, a 'discount neutral' model of preferences over sequences that differ at a finite number of periods. Their general representation takes the following form:

$$a \succcurlyeq b \Leftrightarrow \sum_{\{t:a_t \neq b_t\}} u_t(a_t) \geq \sum_{\{t:a_t \neq b_t\}} u_t(b_t)$$

where the u_t are real-valued functions on an outcome sets X_t that may possibly vary with the date. The axioms they use for this model express conditions of order, continuity sensitivity (every period can affect preference) and of course (given the additive form) independence across periods. When it is also assumed that the outcome sets X_t are the same, further separability assumptions of a measure-theoretic nature allow the following specialisation of the model:

$$a \succcurlyeq b \Leftrightarrow \sum_{\{t:a_t \neq b_t\}} \delta(t)\, u(a_t) \geq \sum_{\{t:a_t \neq b_t\}} \delta(t)\, u(b_t)$$

where $\delta(t)$ is a positive number for any period t. It is not required to be included in the interval $(0,1)$, and therefore it is consistent with 'negative discount rates'. Finally, a form of stationarity yields a constant, but possibly negative discount rate model:

$$a \succcurlyeq b \Leftrightarrow \sum_{\{t:a_t \neq b_t\}} \delta^{t-1} u(a_t) \geq \sum_{\{t:a_t \neq b_t\}} \delta^{t-1} u(b_t)$$

where δ is a uniquely defined positive number.

5 Assessing empirical evidence

Our starting point has been to underline how some observed patterns of choice are irreconcilable with the standard theoretical model. So far, in assessing the theories, we have taken the empirical evidence at face value. However, a more rigorous assessment of the reliability of the empirical evidence itself is called for.

Indeed, assessing time preferences is a non trivial matter. A common theme emerging from the huge literature is that their reliable elicitation poses several methodological problems, and it results in vastly different ranges for discount factors estimates.[10] Although a plethora of studies exist which elicit time preferences, these hardly have proceeded in a highly standardised way. Many confounding factors occur from one study to another, which hamper systematic comparisons to determine to what extent these differences depend on the elicitation methods themselves as opposed to other differences in experimental design. Moreover, as we shall explain, some recent empirical advances even put into serious question certain results of the 'traditional' evidence.

5.1 Psychological effects

To begin with, there are two families of possible psychological effects which act as confounding factors in the evaluations of time preferences: 'hypothetical bias' and 'affective response'. The first term refers to the fact that a substantial proportion of experimental subjects makes different choices when answering hypothetical questions as compared with situations where the answer determines the reward of the responder. For instance, one thing is to ask a subject how much he is prepared to pay for a cleaner environment in the abstract, and quite another is to ask the same question as part of a policy document

[10] See e.g. Table 1 in Frederick, Loewenstein and O'Donoghue [12].

that is going to determine the amount of taxation.[11] Because of this, it would seem reasonable to want to rely on experimental evidence arising from designs which are *incentive compatible*, that is such as that the respondent's reward for participation depends on the answer he or she has given.

By affective response we refer to the emotive states that might be evoked when experimental subjects have to evaluate the delayed receipt of a good or a service, as compared to money. For instance, Loewenstein and Prelec [27] explain by a 'preference for improving sequences' the behaviour of a consistent proportions of decision makers that, after having chosen a fancy French restaurant over a local Greek one, and prefer it sooner rather than later, also choose the sequence (Greek dinner in one month and French dinner in two months) over the opposite sequence (French dinner in one month and Greek dinner in two months). This preference for increasingness can be motivated by 'savouring': a decision maker might like to postpone a pleasant activity so as to enjoy the 'build up' to it. As a flip side to this, 'dread' would be reduced by anticipating an unpleasant task and reducing the time spent in contemplation of this unsavouring activity.[12] More generally one can think of a plethora of potential relevant 'attributes' of a sequence which might influence choice (see e.g. Read and Powell [44], who study subjects' stated verbal motivation for their choices). These affective responses do not only involve sequences, of course, e.g. in the case of the choice of the optimal timing of a kiss to your favourite movie star. Goods and services may possess characteristics which make them idiosyncratically attractive or repulsive to respondents, and evoke feelings quite other than pure time preferences.

5.2 Soft anomalies

In addition to the psychological effects mentioned earlier, framing effects may be rather substantial, too. Loewenstein [24] observed a "delay/speed up" asymmetry, i.e. a difference in the willingness to pay for anticipating receipt of a good and that to postpone it. He showed that when subjects were asked to imagine that they owned a good (a video recorder in the experiment) available in one year, they would be prepared to pay only \$54 on average in order to anticipate receipt, and obtain it now. On the other hand, when asked to imagine they actually owned the videorecorder, subjects were asking on average a compensation of \$126 in order to delay receiving it for a year. Loewenstein

[11] The literature on whether or not the payment of experimental subjects has an effect on response is huge, see e.g. Plott and Zeiler [41], Read [43], Hertwig and Ortmann [20] and Ortmann and Hertwig [37], just to cite a few. Cummings, Harrison and Rutström [8] have examined this in the context of the type of dichotomous choices that are asked in time preferences elicitation, though in a different domain. Manzini, Mariotti and Mittone [32] instead deal with the time domain.

[12] An early formal model of this kind of effects has been proposed by Loewenstein [23].

interpreted this as a framing effect,[13] a purely psychological phenomenon. He conjectured that if prompted to imagine that he owns a good that is immediately available, when asked how much he would have to be paid to delay receipt of the good, a decision maker frames the delay as a loss. If instead the decision maker is prompted to imagine that he owns a good available at a later date, when asked how much he would be willing to pay in order to anticipate collection he would frame this last occurrence as a gain. Note that this type of results were found in both purely hypothetical scenarios as well as an incentive compatible one. If, as in prospect theory,[14] losses count more than gains, then there is an asymmetry in discount rates elicited from the two choice frames. Agents are less willing to anticipate the gain than to postpone a loss, i.e. they are more patient for speeding up than for delaying an outcome. As we explain more in detail in section 6, however, these phenomena are not really a violation of standard discounting theorems, as they only impose restrictions on the shape of the utility function.

While the delay/speed-up asymmetry refers to differences in the implied discount rates depending on the time when the good is available, the so called 'magnitude effect' refers to differences in the implied discount rates between large and small outcomes. It was first reported by Thaler [55], who found that, in an hypothetical setting, on average subjects were indifferent between receiving \$15 immediately and \$60 in a year, and at the same time indifferent between receiving $\$3,000$ immediately and $\$4,000$ in a year. While the first choice (assuming linear utility) implies a 25% discount factor, the second implies a much larger implicit discount factor, of 75%. Shelley [53] carried out a study of both the delay/speed up asymmetry and the magnitude effect. She carried out a test for the possible combinations of gain, loss and neutral frames with either a receipt or a payment. For receipts, she found that implied discount rates are higher for small amounts (\$40 and \$200) than for large amounts of money (\$1000 and \$5000), and for speed up than for delay (time horizons considered were of 6 months and one year for the small amounts, and 2 and 4 years for the large amounts). From an economist's perspective, the problem is that all these experiments were based on hypothetical choices, without real payments. However, they have been replicated also with real monetary payments (see e.g. Pender [38]). Similarly to the previous anomaly, this can be reconciled with the EDM.

We have already discussed one of the main phenomena that violates one of the axioms (stationarity), namely preference reversal. Intriguingly, in addition to the 'direct' preference reversal we have considered, recently Sayman and Öncüler [51], have found evidence of what they dub 'reverse time inconsistency', whereby subjects who prefer a smaller-sooner reward when both options are in the future switch to the larger-later reward when

[13] See Tversky and Kahneman [57].

[14] See Kahneman and Tversky [21]

the smaller option becomes imminent.

Thaler [55] also observes evidence consistent with *discount rates declining with the time horizon*. That is, subjects were asked questions of the following type: What is the amount of money to be received at dates $t_1, t_2, ..., t_K$ that would make you indifferent to receiving x now? The implied discount rates (assuming linear utility) were declining as the dates increased (for example, they were 345% over a one-month horizon and 19% over a ten-year horizon).[15] There is a certain air of unreality about these values, and we shall say more about this aspect later, when we consider the issue of risk-aversion and of field, as opposed to hypothetical experiment, data. However, we emphasize now that even within the realm of experimental observations within an assumed linear utility model, Read [42] uncovers contrary evidence. Discount rates appear to be constant across three consecutive eight-month periods. Rather, his evidence is consistent with subadditive discounting, as discussed in section 3.3.

5.3 Source of data and other elicitation issues

Since our focus is on the rationality or otherwise of decision makers, we ought to consider whether it is possible to reconcile economic theory with either experimental evidence arising from experimental designs which are *incentive compatible*, or with empirical evidence from *field data* (which, using real-life choices, automatically avoid any worry about incentive compatibility); and with data that involve only monetary outcomes.

While the discrepancies between observations, and the unrealistic values found, suggest that some problems must be addressed in the elicitation procedures, the point is that paying subjects is in itself not necessarily enough to produce reliable data. What an incentive compatible elicitation mechanism must do to be dependable is to induce people to reveal (what they perceive to be) their true evaluation of the good in question. Various methods have been used in domains different from time. In fact, the literature on the elicitation of 'home grown values' for all sorts of goods is vast. Traditionally, experimenters *induced* preferences (i.e. valuations of specific goods) in experimental subjects in order to assess the validity or otherwise of a given theoretical model. As the interest has moved towards assessing and eliciting subject preferences in choices among different goods, or in their valuation for some goods, various mechanisms have been introduced to tease out 'home grown' preferences from experimental subjects.

The most popular methods relied upon in the literature on the elicitation of preferences other than time preferences are English auction, second price auction and Becker-De

[15] See also Benzion, Rapoport and Yagil [4] for an example in the case of hypothetical choices and Pender [38] for actual choices.

Groot- Marschak procedure (BDM). For each of them bidding one's true value is a dominant strategy, and in many experimental settings instructions encourage bidders explicitly to understand and learn the dominant strategy (see e.g. Rutström [49]).

Let us consider them in turn:

- English (or ascending) auction: agents compete for obtaining a good. With the so called 'clock' implementation of the auction, the price of the good increases steadily over time. As time passes participants can withdraw. When only one is left, he 'wins' the object and he alone pays the price at which he won.

- Vickrey (i.e. second price sealed bid) auction: subjects submit a single bid, secretly from all other participants. The one with the highest bid wins the object, but pays only the second highest price. This is why it is strategically equivalent to the English auction, since in the latter the winner is the one who stays when the second highest bidder gives in.

- BDM: this is also equivalent to the two previous auctions, although here bidders play 'against' a probability distribution, rather than other subjects. Because of this the BDM procedure has the objectionable difficulty that it introduces a probability dimension to the problem. Subjects have to declare their willingness to pay for a good. Then a price is drawn from a uniform distribution, and if this price is higher than the willingness to pay, the agent gets nothing, whereas if it is lower, the agent pays the price drawn (so for a winning bidder it is as if he put forward the highest bid in a second price auction, with the price drawn playing the role of the second highest bid from a fictitious bidder).

All the above are strategically equivalent: so would it make any difference which one is used in the lab?

In auctions with induced preferences (i.e. where subjects are told what their valuation for a good is) Noussair, Roibin and Ruffieux [35] find that Vickrey auctions are more reliable in eliciting preferences than the BDM procedure. Again with induced values, Garratt, Walker and Wooders [13] find that in the comparison with the usual student population, when using experienced e-bay bidders as experimental subjects, the difference between over and under bidding is no longer significant (while the proportion of agents bidding their value is indistinguishable from standard lab implementation with students).

On the other hand, when preferences are not induced (i.e. they are 'home grown'), Rutström [49] finds that (average) bids are higher in the second price auction than in either BDM or first price auctions. Moreover, as noted by Harrison [16], these elicitation methods suffer from serious incentive properties in the neighbourhood of the truth telling

dominant strategy: deviations may be 'cheap' enough that experimental subjects do not select the dominant strategy.

Although none of these auction methods has been applied until recently (see below) to time preferences, the systematic discrepancies between alternative methods to elicit preferences for goods suggest that different elicitation methods might also produce different estimates when applied to the time domain. The most relied upon elicitation technique for time preferences at the moment consists in asking a series of questions, in table format, of the type "Do you prefer: A) X today or B) $X + x$ at time T", where x is some additional monetary amount which increases steadily (from a starting value of zero) as the subject considers the sequence of questions (see Coller and Williams [6] and Harrison, Lau and Williams [19]). A decision maker would start switching from selecting A to selecting B from one specific choice onwards, making it possible to infer the discount factor.[16] This table method has been used with additional variations, namely an additional piece of information (e.g. giving for each choice the implicit annual discount/interest rate implied by each choice and the prevalent market rate in the real economy) in order to reduce the extent to which subjects anchor their choices to their own experience outside the lab and unknown to the experimenters. Coller and Williams [6] found discount rates to be much lower than previously found once this kind of censoring is taken into account.

A very recent experimental study by Manzini, Mariotti and Mittone [33] has made the first comparative analysis of the table method, the BDM and the Vickrey auction in a choice over time setting. Preliminary results show a similarity of elicited values between the latter two methods, but a marked difference between them and the table method.

However, one must be aware that all choice experiments involving questions about money-date pairs only reveal discount factors for *money* in an unequivocal way. It is often implicitly assumed that such experiments also reveal the discount factor for *consumption*, but this interpretation requires the assumption that the money offered in the experiment is consumed immediately: subjects do not use capital markets to reallocate their consumption over time. This assumption is not outrageous (especially for small amounts, it may not be implausible that capital markets considerations are ignored) but certainly it cannot be taken for granted without further study. Coller and Williams [6] were the first to point out the possible censoring effects of capital markets on experimental subjects' responses. Cubitt and Read [7] explore in great detail what exactly can be inferred from responses to the standard laboratory tasks on choice over time once it is admitted that subjects are able and willing to access imperfect[17] capital markets, so that

[16] To be precise, one can only infer a *range* for the discount factor, whose width depends on the size of the progressive increments of the additional monetary increments x.

[17] I.e., with the borrowing interest rate possibly differing from the lending rate.

the implicit laboratory rate of interest competes with the market ones. They point out that the choice between two money-date pairs in the presence of capital markets is not really the choice between two points in the standard Fischer diagram[18] but rather the choice between two whole *consumption frontiers*. As is intuitive, this fact greatly reduces the possibility of inference about discount factors for consumption.

A different but conceptually related reservation about the correct inferences to be drawn from experimental results comes from the recent work by Noor [34]. He observes that nothing excludes that experimental subjects integrate the laboratory rewards with the anticipated future levels of wealth. The striking implication is that if such future wealth levels are expected to change, all main documented soft anomalies, including preference reversal, turn out to be compatible with the EDM. Intuitively, if the subject is more cash constrained now than he expects to be at a later date, so that his marginal utility for money is higher now than it is expected to be in the future, he may well choose according to the pattern of the preference reversal phenomenon, while still making his choices on the basis of a constant, and not declining, discount factor. In a precise sense, the EDM is shown in this work to have no empirical content unless the integration with expected future wealth is excluded a priori. However, Noor also suggests an experimental design including risky prospects as outcomes, which is sufficiently rich to test the EDM.

Furthermore, from *field data*, Harrison Lau and Williams [19] find that, unlike previous claims of non-constant discount factors, although discount factors do depend on household characteristics, within each homogeneous group discount factors over one and three years horizon are indeed constant. But this is not all: as we mentioned earlier, the concavity of the utility function may explain apparent anomalies, thus calling for *both* the time preferences *and* the preference for the good whose receipt is delayed to be elicited simultaneously. If a single utility value $u(x,t)$ is elicited, rather then a separate assessment of the time and outcome components, it seems fair to argue that the concavity of the utility function might conflate into the estimate of discount factors. Starting from this consideration, Andersen, Harrison, Lau and Rutström [2] show that the implausibly high estimates of discount factors previously uncovered fall substantially once the concavity of the utility function is taken into account in the estimation, and both risk and time preferences are elicited from experimental subjects. The difference is quite dramatic: whereas under the assumption of risk neutrality the point estimate of the yearly discount factor is roughly 25%, it falls six-fold to about 4% once risk aversion is (correctly) accounted for.[19]

[18] In which consumption levels at two distinct dates are represented on each axis on the plane.

[19] Field data from the retiriment options offered to retired military personnel in the US (see Warner and Pleeter [61]) suggest higher than expected discount rates. However see Harrison and List [17] for a critique of the 'heroic' extrapolation method used.

Summing up, then, when it comes to preferences over outcome-date pairs, once the correct estimation techniques are used and concavity of the utility function is allowed for, the wildly varying discount factor estimates fall to more manageable ranges of variation and 'realistic' values.

5.4 Hard anomalies

There are other observed violations of the EDM which are more fundamental in the sense that, unlike preference reversal, they seem to contradict the basic assumption of maximization of any economically reasonable objective function. One notable instance is that human decision-makers have been shown to make *intransitive* choices. Although most data in this direction come from choices under risk, the evidence available for time preferences though limited is clear in suggesting that violations of transitivity are more frequent in this domain. Tversky, Slovic and Khaneman [59] show that a substantial 15% of subjects exhibited cyclical patterns of choice that could not be explained by 'framing effects' - and in an experiment which was not designed to uncover cycles. When the issues of cycles in choice is addressed directly, the evidence is even more striking: Roelofsma and Read [45] found that the *majority* of intertemporal choices were intransitive. Cyclical choice is thus one 'solid' anomaly that cannot be accommodated within any discounting model.

While incentive compatible experimental investigations of choices over outcome-date pairs form a small but non-negligible literature, experimental investigations of choices over reward *sequences* are extremely thin on the ground in the economics literature, especially with financially motivated subjects. Arguably, this is because the difficulties highlighted above are exacerbated by payment of experimental subjects having to take place over weeks if not months. The unreliability of data from experiments based on hypothetical choices seems to be driving the recent increase in incentive compatible experimental designs.

The first experimental paper on preferences over *monetary* sequences of outcomes is due to Loewenstein and Sicherman [28]. In a survey of members of the public entering a museum, interviewers asked participants to choose among hypothetical alternative profiles of either wages of savings plan over a number of years. Loewenstein and Sicherman [28] found evidence of preference for sequences of increasing monetary payments (versus constant or decreasing ones). They explain this finding by pointing to a preference for maintaining the 'current' consumption level, so that wages should be non-decreasing. Admittedly, though, these were hypothetical questions, and some respondents motivated their preference for increasing sequences with inflation. In addition, the framing of these

questions as salary profiles might evoke an improvement in one's career, or just be what one would generally expect (i.e. affective response). However other authors have found evidence of preferences for constant and even decreasing sequences of outcomes over time (e.g. Chapman [5], Gigliotti and Sopher [14], Guyse, Keller and Epple [15]). The domain of choice seems also to be important (e.g. there are differences in observed choices depending on whether or not the sequences are of money, or health or environmental outcomes see e.g. Chapman [5] and Guyse, Keller and Epple [15]).

Manzini, Mariotti and Mittone [32] asked subjects to make binary choices among all possible pairs of monetary sequences, with an increasing, constant, decreasing or 'jump' (i.e. end effect) pattern, both in a paid condition (where subjects do indeed receive the sums corresponding to the sequence chosen) and an unpaid condition (where choices are hypothetical). Previous experimental evidence on reward sequences suggests that the general trend of the sequence (increasing or decreasing) is relevant to make decisions. However, in this case the data provide much weaker evidence than Loewenstein and Prelec's [25] in support of their view that 'sequences of outcomes that decline in value are greatly disliked' (p. 351). It is found that, even in the simple decision problems studied, where monetary sequences can be clearly ordered according to their trends, simply choosing according to the heuristics that favours the 'increasingness' of the trend does a rather poor job at explaining the data. The modal subject and choice is 'rational', in the sense of being compatible with positive time preference combined with preference for income smoothing (concave utility function). Therefore while choice incompatible with EDM is observed, it is not to the extent that the existing literature suggests. When there are no affective factors involved (such as, for example, the sense of dread for choices relating to health, or the sense of failure involved in a decreasing wage profile), some theory of positive discounting can provide a rough approximation of the choice patterns. However a non negligible proportion of our subjects (around 30%) choose in ways that are incompatible with any form of positive discounting (exponential, hyperbolic or otherwise). These subjects violate the basic economic assumption that for a good, the sooner the better, suggesting that other mechanisms beyond discounting are at work. That is, Loewenstein and Prelec's pioneering findings do capture, beside affective factors, some of the heuristic considerations that people use when evaluating 'neutrally' (without affects) money sequences. Moreover the study finds that 'irrational' choices present a systematic pattern, not encountered previously. Of these, the most striking are the association between certain types of rational choices and irrational choices (those who prefer a decreasing to a constant sequence are disproportionately concentrated among those who also prefer a constant to an increasing sequence); and the association between irrational choices of a different type (choosing an increasing over a decreasing sequence is very strongly as-

sociated with choosing an increasing over a constant sequence). Such patterns cannot be generated by any discounting model, nor by such a model augmented with random independent mistakes.

One last puzzling experimental finding that we wish to highlight is due to Rubinstein [48]. In a hypothetical setting he finds a 'single outcome/sequence of outcomes' type of preference reversal of the kind highlighted by Loewenstein and Prelec [27] in a different domain: in a classroom experiment a majority of students preferred to receive a payment of 997 monetary units at a later date t^* than 1000 at an even later date t^*+1, but when choosing between sequences of four payments of a constant amount of either 997 starting at t' or 1000 starting at $t'+1$, the latter sequence was now preferred, contradicting the theory of hyperbolic discounting.

That is, subjects exhibited the following type of behavior: they chose x to be received at some date t^* versus the larger sum $x+z$ to be received at date t^*+1 (they were impatient and preferred smaller reward earlier rather than larger reward later) but chose the sequence

$$a = ((x+z, t'+1), (x+z, t'+2), (x+z, t'+3), (x+z, t'+4))$$

over the sequence

$$b = ((x, t'), (x, t'+1), (x, t'+2), (x, t'+3))$$

where $t'+4 \leq t^*+1$. This contradicts not only the EDM but also the HDM and in fact any model of discounting based on diminishing impatience (declining discount rates): if the subject were impatient at the late date t^* and not willing to trade off one unit of delay for an additional reward z, he should have been unwilling to perform all four trade-offs of this type involved in the comparisons between sequences. Rubinstein's explanation is based on similarity: given choices between sequences of alternatives with two distinct attributes, the decision maker uses a procedure whereby first of all he tries and rank alternatives based on dominance (i.e. greater outcome and earlier time of receipt); if this is not decisive, he looks for similarities between the two dimensions, trying to discriminate based on evident differences. If none can be found, then he chooses based on some different criterion.[20] As we will see in section 6, however, other theories, too, can explain this phenomenon.

6 Empirical 'anomalies' and theory

As we have already made clear, some of the anomalous types of behaviour discussed in the empirical literature are not 'hard' anomalies after all, in the sense that they can be

[20] See also Rubinstein [46].

easily accommodated within the axiom set of theorem 2 in section 2. This is the case for the main phenomenon of *preference reversal* described in section 3.1. So we begin by tackling soft anomalies first (delay/speed-up asymmetry, magnitude effect and inverse preference reversal), and then move to the 'hard anomalies' (cycles in choice and the 'single outcome/sequence of outcomes' preference reversal).

The *delay/speed-up asymmetry* does not even violate by itself any of the Fishburn-Rubinstein axioms for exponential discounting and in this sense it is not an anomaly. Imagine for notational simplicity that the object whose receipt is to be delayed or anticipated is an amount of money x. So the effect can be written within the EDM, in a non-contradictory way, as:

$$\begin{aligned} u(x-K) &= \delta^t u(x) \\ u(x) &= \delta^t u(x+P) \end{aligned}$$

where K and P are, respectively, the amount of money that the agent is willing to pay to anticipate the receipt of the money which he is entitled to receive at date t, and P is the amount of money that the agent requires to delay to date t the receipt of the amount of money to which he is entitled now.

The *magnitude effect,* like the delay/speed-up asymmetry, also does not violate any of the Fishburn-Rubinstein axioms for exponential discounting. As noted by Ok and Masatlioglu [36],[21] for example, the exact Thaler's numbers reported before are compatible with the EDM with a $\delta = 0.95$ and a concave utility function $u(x) = x^{.42} + 45.9$ defined on the positive real line.

However, although formally this phenomenon is indeed compatible with exponential discounting, some observations are in order. One might argue that although *some* utility function can necessarily be found (given the non-violation of the axioms) that fits the (few) observations, additional constraints might be desirable for the utility function, and these constraints might create an incompatibility with the observed effects. For example, it is often informally argued (and often simply taken for granted) that for small amounts the utility function ought to be linear. In this case, the EDM is incompatible with the magnitude effect (using Thaler's numbers, it would require for example $\delta = \frac{15}{60} = \frac{250}{350} = \frac{3000}{4000}$). However, in this case the magnitude effect (which involves only two dates) is also incompatible with the HDM, and indeed with *any* separable discounting model axiomatised in Theorem 2 of Fishburn and Rubinstein: we would still obtain a contradiction of the type

$$\frac{x}{y} = \delta(t) = \frac{x'}{y'} \text{ but also } \frac{x'}{y'} < \frac{x}{y}$$

[21] See Ok and Masatlioglu [36] in the (2003) version.

So it seems to us that the real point about such effects is that either they do not constitute an EDM anomaly, or if they do (because of the linearity of utility) they also constitute an anomaly for (much) more general discounting models, notably including HDM. It is incompatible with the linear version of relative discounting, too, and it can be made compatible with a linear utility version of Manzini and Mariotti's (σ, δ) −model only in a variant in which the vagueness term σ is made to depend on the outcome (see Manzini and Mariotti [30] for details).

In addition, it is fair to say that the evidence is still too scant. In order to fit a utility function and check its 'reasonableness' (or compatibility with independent data on concavity-convexity) one would need many more observations in different regions of the time and outcome space. At best the existing observations might be simply suggestive of the fact that human decision makers use certain yet to be discovered 'heuristic' procedures when judging outcome-date pairs. Certainly the magnitude effect, even together with the assumption of exponential discounting, is not necessarily and intrinsically related to diminishing marginal utility. For example, suppose $u(x,t) = \delta^t x^\alpha$, where $\alpha \in (0,1)$. Then let J and K be the compensations required by a decision maker to delay by one period from now the receipt of x and y, respectively, that is $\delta(x+J)^\alpha = x^\alpha$ and $\delta(y+K)^\alpha = y^\alpha$. This implies

$$\frac{\delta(y+K)^\alpha}{\delta(x+J)^\alpha} = \frac{y^\alpha}{x^\alpha} \Leftrightarrow \frac{x}{x+J} = \frac{y}{y+K}$$

so that the magnitude effect (which requires that, if say $y < x$, then $\frac{y}{y+K} < \frac{x}{x+J}$) *never* obtains. It seems unlikely that simple changes of functional forms with respect to the EDM will be descriptively adequate in general. However, as we noted already, a new literature is emerging which attempts to estimate both the shape of the utility function and discount factors at the same time (see Andersen, Harrison, Mortensen and Rutström [2]): this type of research may in due course shed additional light on the issue of magnitude effects.

Sayman and Öncüler [51]'s *negative preference reversal* is a soft anomaly that cannot be accommodated within the class of HDM. Indeed, if $x < y$, the preferences $(x,\tau) \succ (y, t+\tau)$ and $(y,t) \succ (x,0)$ correspond to $\delta(\tau)u(x) > \delta(t+\tau)u(y)$ and $\delta(t)u(y) > u(x)$ (in a separable discounting model). This is obviously incompatible with HDM since it requires decreasing (rather than increasing) discount rates. To the contrary, this type of preferences can be accommodated within the model of vague time preferences.

For instance in the simple (σ, δ) representation with the TPM we need

$$x\delta^\tau \leq y\delta^{t+\tau} + \sigma \text{ and } y\delta^{t+\tau} \leq x\delta^\tau + \sigma$$
$$y\delta^t > x + \sigma$$

so that $(x,\tau) \succ^* (y, t+\tau)$ by the Secondary Criterion while $(y,t) \succ^* (x,0)$ by the Primary

Criterion. Negative preference reversal is also compatible with Ok and Masatlioglu's RDM. Here we need

$$\delta(t,0) = \frac{1}{\delta(0,t)} > \frac{u(x)}{u(y)} > \delta(t+\tau,\tau)$$

When interpreted as revealed preferences, observed *cycles in binary choice* are a harder anomaly to deal with because they violate the fundamental axiom of Order. For outcome-date pairs, the HDM cannot explain cycles, whereas alternative theories (most notably Ok and Masatlioglu's RDM and our own theory of vague time preferences) can. Similarly Read's subadditive discounting and Rubinstein's similarity based decision making are also consistent with the phenomenon.

Consider three alternatives (x,r), (y,s) and (z,t), with $x<y<z$ and $r<s<t$. In the RDM it is perfectly possible to have that $u(x) > \delta(t,r)\,u(z)$, $u(z) > \delta(s,t)\,u(y)$, but $u(y) > \delta(r,s)\,u(x)$. This could happen for example if there is very little perceived difference between t and s and between s and r, but the difference between t and r is perceived as significant (imagine $r<s<t$ and $x<y<z$). Then the latter two inequalities attest to the fact that the differences $z-y$ and $y-x$ are enough to compensate for the the small delays, but the difference $z-x$ is not enough to compensate for the 'large' delay. Note however that in the model this 'compounding of small differences' effect is not allowed to hold for the outcome dimension, but just for the time dimension. With these assumptions it is also easy to see how Rubinstein's similarities might work: if t and s are similar, and so are r and s, while all other comparisons are perceived as different, then (z,t) is preferred over (y,s), (y,s) is preferred over (x,r) and because r and t are not similar, nor are x and z, it is enough for the unspecified completing criterion to pick (x,r) over (z,t) to complete the cycle.

Subadditive discounting, too, can explain cyclical choices. Consider the three time intervals $[0,r]$, $[r,s]$ and $[s,t]$, and let the discount factors over an interval $[a,b]$ be denoted by δ_{ab}. In the presence of subadditivty we have that $\delta_{0t} > \delta_{0r}\delta_{rs}\delta_{st}$. So to generate the cycle in choice where (y,s) is chosen over (z,t) which is chosen over (x,r) which is chosen over (y,s), assume that the decision maker splits the time intervals involved in each binary comparisons taking into account the common delay. Then we need:

$$\begin{aligned} u(y)\,\delta_{0s} &> u(z)\,\delta_{0s}\delta_{st} \\ u(z)\,\delta_{0r}\delta_{rt} &> u(x)\,\delta_{0r} \\ u(x)\,\delta_{0r} &> u(y)\,\delta_{0r}\delta_{rs} \end{aligned}$$

which simplifies to

$$\begin{aligned} u(y) &> u(z)\,\delta_{st} \\ u(z)\,\delta_{rt} &> u(x) \\ u(x) &> u(y)\,\delta_{rs} \end{aligned}$$

the last two inequalities imply $u(z)\,\delta_{rt} > u(y)\,\delta_{rs}$, which is compatible with the first inequality as long as $u(z)\,\delta_{rt} > u(z)\,\delta_{st}\delta_{rs}$, that is if $\delta_{rt} > \delta_{rs}\delta_{st}$, which is precisely what subadditive discounting entails.

The vague theory of time preferences is also consistent with intransitivities. For instance in the OPM we need

$$\begin{aligned} &x\delta^r > z\delta^t + \sigma \\ &y\delta^s \leq z\delta^t + \sigma \text{ and } z\delta^t \leq y\delta^s + \sigma \\ &y\delta^s \leq x\delta^r + \sigma \text{ and } x\delta^r \leq y\delta^s + \sigma \end{aligned}$$

so that $(x,r) \succ^* (z,t)$ but $(x,r) \sim (y,s)$ and $(y,s) \sim (z,t)$ by the Primary Criterion. However by the Secondary Criterion $(y,s) \succ^* (x,r)$ and $(z,t) \succ^* (y,s)$, thereby producing a cycle. Obviously any theory that can cope with cycles can cope with 'direct' preference reversal, so we skip the details.

Next, consider the *'outcome-sequence' type of preference reversal* presented by Rubinstein [48] for the case of monetary sequences, and Loewenstein and Prelec [27] in a different domain. Here once again the caveat must be that both these papers present results from hypothetical questions. This notwithstanding, we report it here because as we discussed earlier, these pose a harder challenge to conventional theories. Obviously Rubinstein's own similarity considerations provide an explanation to this observed phenomenon. In addition, the model of vague time preferences for sequences can provide an alternative explanation. For the sequences described in the previous section suppose for example that the secondary criterion is the natural one proposed by Rubinstein himself, namely 'Pareto dominance' between the outcome sequences, and that

$$\begin{aligned} \delta^{t^*}u(x) &> \delta^{t^*+1}u(x+z) + \sigma(x+z, t^*+1) \\ \sum_{i=0}^{3}\delta^{t'+i}u(x) &\leq \sum_{i=1}^{3}\delta^{t'+i}u(x+z) + \sigma(a) \end{aligned}$$

In this case the preference between outcome-date pairs can be explained by present discounted utility (primary criterion) and the preference between sequences can be explained by the secondary criterion.

Finally, the evidence of moderate preference for increasing sequences discussed in Manzini, Mariotti and Mittone [32] is also consistent with the model proposed therein, as well as obviously with the model by Fishburn and Edwards [10].

7 Concluding remarks

In the last twenty years a growing body of experimental evidence has posed a challenge to the standard Exponential Discounting Model of choice over time. Attention has focused on some specific 'anomalies', notably preference reversal and declining discount rates, leading to the formulation of the model of hyperbolic discounting which is finding increasing favour in the literature. As we have seen, it is debatable whether some of the most focused upon anomalies should be indeed classified as such, or whether they are really the most challenging ones for conventional theory. If they violate any axiomatic property, this is Stationarity, which is not strongly defendable even on normative, let alone descriptive, grounds. A group of theoretical ideas is beginning to emerge which can address not only violations of Stationarity, but even more challenging observed phenomena.

At the same time, at the empirical level much progress is being made on two fronts: 'sophisticated' estimation of discount factors (e.g. considering censoring factors) and simultaneous presence of discounting and risk aversion, a traditionally much neglected issue until very recently. The results are quite stunning, implying as they do a serious reconsideration of the previous estimates. On the other hand, other recent work challenges the conventional interpretation given to responses to standard experimental choice tasks.

While in the theory of choice under risk there exist rationality axioms that exert a strong normative appeal, this is less clearly the case for choice over time. Thus, it is natural to step towards models of *procedural* rationality as opposed to normative rationality, given that the latter lacks a clear notion. Some work already exists in this direction, but much more remains to be done in this fascinating area, as we are still quite far from a clear-cut 'best' theory.

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