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Helping as Relationship Building: Organizational Design and Team Performance*

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Abstract

Helping colleagues builds up workplace relationships which have both productive and private benefits, at the cost of time that could otherwise be spent on one's own work. We model this trade-off to examine how the private value of workplace relationships affects reward design, team size, and leadership. Under individual rewards, helping is insufficient when the private value of relationships is low and excessive when it is high. More productive relationships can lead members to spend too much time helping, reducing team output even under an optimal reward scheme. When individual and team rewards are combined, the optimal weight on team performance is lower in teams whose members place greater private value on relationships, and higher in teams with greater common ability. Under individual rewards, expanding a team can curb excessive helping by weakening its mutual reinforcement, but also reduces the productive benefits of relationships. The optimal team size balances these effects and is larger when members place a higher private value on relationships. By committing to an effort allocation first, the leader induces more helping from the follower and strengthens social ties. This form of leadership can improve output when relationships are insufficiently developed, but lowers output when relationship formation is already at or beyond the output-maximizing level. Both members nevertheless gain in utility.

Keywords: Social relationships; Helping; Team production; Incentive design; Effort allocation; Strategic complementarity
JEL Classification: D23; M52; M54

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1 Introduction

Making workplace relationships more productive need not increase team output. Helping colleagues advances their tasks and builds social relationships that facilitate coordination and information sharing. These relationships also provide private benefits, such as companionship and recognition. Because helping takes time away from one's own task, the same relationship can improve production directly while altering effort allocation in ways that reduce output.

The tension between helping and own-task work has empirical counterparts. Cross et al. (2016) report that growing demands for collaboration leave employees less time for their own work. In a nine-month field study of a software engineering team, Perlow (1999) describes how repeated interruptions made it difficult for engineers to complete their individual work. An engineer whose expertise was needed by colleagues reported difficulty balancing their requests for assistance with his own deliverables. Barnes et al. (2008) find that backing-up behavior benefits recipients but can lead helpers to neglect their own tasks, especially when workloads are evenly distributed across team members. These studies illustrate how assistance that advances a colleague's task can compete with the provider's own work. They motivate our question: how do the productive and private benefits of workplace relationships affect members' effort allocation, and how should organizations respond through rewards and team organization?

We consider a team of two members, each with a fixed amount of time to divide between their own task and helping the other member. Helping directly increases the colleague's task output and builds a shared social relationship. The relationship improves productivity in both tasks and provides private benefits to both members. Its formation can be complementary: helping by one member raises the contribution of the other's helping to the relationship. As a result, members' incentives to help are interdependent. The organization evaluates total task output, excluding private relationship benefits, while members choose how to allocate their time in response to their own rewards and the private value of the relationship.

Under individual performance rewards, the privately chosen level of helping need not maximize team output. Members do not receive the direct output benefits of helping a colleague, so they may help too little when they place a low private value on the relationship. As this value rises, they help more, initially improving the allocation of time and increasing team output. Eventually, however, the private benefits induce them to help beyond the output-maximizing level. Additional helping then reduces own-task production by more than it increases output elsewhere in the team. Team output therefore follows an inverted-U pattern as the private value of the relationship increases.

More surprisingly, improving the productivity of the relationship itself can reduce equilibrium team output. At a given allocation of time, a more productive relationship raises output from both tasks. Yet the improvement also makes helping more attractive to each member. More time is devoted to helping, and complementarity in relationship formation can reinforce this response across members. When helping is already excessive, the resulting loss of own-task production can outweigh the direct productivity gain. We characterize when this reversal occurs. The result shows that improving a productive feature of the workplace need not improve performance once members adjust how they use their time.

We next ask whether reward design can correct these distortions. The organization chooses a mixture of individual performance rewards and equal shares of team output. Team rewards encourage helping because members receive part of the additional output their assistance creates for colleagues. Private benefits from the relationship provide a similar incentive, so the output-maximizing weight on team rewards decreases as the private value of the relationship increases. When helping is already excessive under individual rewards, however, none of the available mixtures can reduce it sufficiently. The output decline caused by greater relationship productivity can therefore persist even under optimally chosen rewards.

Reward design also depends on members' common ability. An improvement that makes own-task work and direct assistance more productive, holding relationship productivity fixed, calls for a larger team reward weight when the optimal mix is interior. Under individual rewards, members receive the direct return to their own-task work but not to the help they provide. Higher ability therefore induces less helping. The two productivity improvements thus induce opposite reallocations of time: greater relationship productivity can lower output through excessive helping, whereas higher common ability can do so through insufficient helping. With optimally chosen rewards, higher common ability always raises team output, whereas greater relationship productivity can still lower it. Team rewards can correct insufficient helping, but cannot reduce helping below the level induced by individual rewards.

Team formation provides another means of shaping helping incentives. We extend the model to an organization with a fixed workforce divided into equal-sized teams. Each member distributes helping time equally among colleagues. In larger teams, the mutual reinforcement of members' helping choices becomes weaker, reducing the incentive to help. This can improve output when a small team helps excessively, but a sufficiently large team may instead provide too little help. We characterize when the optimal team size is finite and interior. When it is, the output-maximizing team size increases with the private value of the relationship.

Leadership also changes how relationships form. The leader first commits to an allocation of effort between own-task work and helping. The follower observes this allocation before choosing their own, and both members help more than under simultaneous choice, strengthening their relationship. This can improve output when relationships are insufficiently developed for production, but lowers it when relationship formation already draws too much time from own-task work. Both members nevertheless gain in utility, so a leadership structure they prefer can reduce team output.

We also examine differences in members' private relationship values under individual rewards, holding ability equal. Such differences make helping choices unequal and weaken their mutual reinforcement. This can improve team output when helping is initially excessive. The member who values the relationship more helps more, yet produces less on their own task. Individual task output can therefore obscure contributions made through helping.

Section 2 reviews the related literature. Section 3 presents the model and the output effects of relationship productivity and common ability. Section 4 studies reward design. Section 5 examines team size, leadership, and differences in private relationship values. Section 6 discusses implications, and Section 7 concludes. The appendices contain proofs and extensions.

2 Related Literature

This paper relates to the literature on helping and reward design in organizations. In a closely related analysis, Wang and He (2008) study effort allocation between departmental work and cross-functional teamwork, incorporating private social rewards and social capital valued by the firm. Whereas Wang and He (2008) model private social rewards as functions of each worker's own teamwork effort, alongside jointly generated firm-level social capital, we model both private and productive benefits as arising from a common relationship jointly formed through helping, whose complementarity can lead to excessive helping. Itoh (1991a) analyzes incentive contracts that induce productive assistance among agents. Auriol et al. (2002) show that limited commitment to future wages creates career-related incentives to withhold help, prompting more collectively oriented rewards. In our model, fixed time budgets and private relationship benefits can generate excessive helping even when rewards are fixed in advance. Greater weight on team rewards cannot correct this distortion. Siemsen et al. (2007) distinguish helping from outcome and knowledge linkages and characterize the associated individual and group incentives. Their help-linkage model abstracts from the displacement of own-task effort by helping. Crama et al. (2019) examine a formal process for helping across projects and show that encouraging help need not be profitable because of free riding. Drago and Garvey (1998) find that stronger promotion incentives are associated with less helping. Rob and Zemsky (2002) examine how nonpecuniary motives for cooperation interact with incentive intensity.

The allocation of time between individual and collaborative activities also appears in research on team coordination. Roels and Corbett (2024) examine alternative rules for scheduling coordination, which takes time away from individual production, and show that their effectiveness depends on team size. Our analysis considers a different organizational decision. Given a fixed workforce, the organization chooses how many members to place in each team, while members independently determine their helping time. Team size affects these choices through complementarity in relationship formation, which can generate an interior optimal team size.

Our focus on relationship formation connects this analysis to research on social relations and incentives in the workplace. Itoh (1991b) studies social interaction that affects the disutility of work and shows that a firm may prefer to restrict interaction even when workers derive private benefits from it. Rotemberg (1994) and Dur and Sol (2010) analyze the formation of altruism among coworkers and its relationship with incentives. Hattori and Yamada (2025) study a different channel: social interaction reduces differences in altruism between team members, which improves team performance when their productive efforts are complementary. In our model, members allocate time between own-task effort and helping, taking their private values of the relationship as given. Complementarity operates in relationship formation: helping by one member makes the other member's helping more effective in improving the relationship. This mechanism does not require altruism to change, and the private value of the relationship is distinct from concern for a colleague's payoff.

The opportunity cost of cooperation is also emphasized in organizational research. Bergeron (2007) argues that allocating limited time to organizational citizenship behavior can reduce performance on one's assigned tasks. Ellington et al. (2014) document diminishing returns in the association between the frequency of interpersonal helping and individual task performance. Bergeron et al. (2013) find that,

in a professional services firm using an outcome-based control system, more time spent on citizenship behavior is associated with smaller salary increases and slower advancement, controlling for time spent on assigned tasks. Roos et al. (2022) show that group rewards and endogenous social norms can generate excessive cooperation in an agent-based model with heterogeneous values. Our contribution concerns how such an allocation responds to a productivity improvement. When a relationship becomes more productive, members increase helping, and complementarity in relationship formation amplifies their responses. If helping is already excessive, the resulting loss of own-task effort can outweigh the direct productivity gain. This output decline can persist after rewards are optimized within the class we study. The same complementarity also explains why differences in the private value of the relationship can mitigate excessive helping.

Empirical and experimental studies motivate these mechanisms. Bowler and Brass (2006) document associations between friendship ties and the provision and receipt of helping, while Flynn (2003) relates the generosity and frequency of favor exchange to social status and individual productivity. Methot et al. (2016) find that relationships combining work and friendship involve both benefits associated with trust and costs of relationship maintenance. In a field experiment, Bandiera et al. (2013) show that changes in team incentives affect friendship-based team formation and productivity. Belot and Hakimov (2026) find that promoting social activities improves workplace relations and raises performance in the firm's team competition during the final month of the intervention. Particularly close to our setting, Hattori and Yamada (2025) conduct a laboratory experiment with previously unacquainted pairs. Pre-task social interaction improves team performance and reduces differences in altruism, with mediation analysis supporting the convergence channel. Our analysis complements this evidence by identifying conditions under which helping motivated by the private benefits of the relationship instead becomes excessive and lowers team output.

3 The Model and Baseline Results

3.1 Environment

We consider a team with two members, indexed by $i \in \{1, 2\}$, and write $j \neq i$ for the other member. Each member has one unit of time to allocate between own-task effort, e_i, and helping the other member, h_i. The time constraint is

$$e_i + h_i = 1, \qquad e_i, h_i \geq 0. \tag{1}$$

Thus, choosing $h_i \in [0, 1]$ determines $e_i = 1 - h_i$. Helping includes advice, reviewing a colleague's work, and other assistance that advances the colleague's task. Total working time is fixed. The cost of helping is the output forgone on one's own task.[1]

Helping also improves the social relationship between the members. Let R denote relationship quality and let $\rho \geq 0$ measure complementarity in relationship formation. We specify

$$R = h_1 + h_2 + \rho h_1 h_2. \tag{2}$$

[1]We do not model the recipient's time spent responding to assistance. The time constraint records the time each member devotes to their own task and to helping the other member. Appendix B relaxes this assumption.

Unilateral helping improves the relationship. When $\rho > 0$, the marginal contribution of one member's helping to relationship quality is greater when the other member also helps:

$$\frac{\partial R}{\partial h_i} = 1 + \rho h_j, \qquad \frac{\partial^2 R}{\partial h_i \partial h_j} = \rho.$$

This complementarity concerns how the relationship is formed. It does not impose a preference for repaying past assistance. The case $\rho = 0$ retains relationship formation but removes the complementarity.

Both own-task effort and helping have diminishing returns. Let $a > 0$ denote the members' common ability, $c > 0$ the productivity coefficient for own-task effort, $b > 0$ that for direct helping, and $\gamma \geq 0$ relationship productivity. Member i's task output is

$$y_i = ac\left(e_i - \frac{e_i^2}{2}\right) + ab\left(h_j - \frac{h_j^2}{2}\right) + \gamma R. \tag{3}$$

Ability a raises the productivity of both own-task effort and direct helping in the same proportion. It captures skills useful for both activities, holding relationship productivity fixed. The first term is output generated by the member's own-task effort. The second is the direct contribution of assistance received from the other member. The third term, γR, represents the productive benefits of the relationship for member i's task, such as easier information sharing and coordination. Relationship productivity γ measures these benefits per unit of relationship quality. These benefits accrue equally to both tasks.

Task output y_i is distinct from member i's total contribution to the team. In particular, the direct output generated by h_i is recorded in y_j. This distinction matters because baseline rewards depend on individual task output. We normalize the reward per unit of individual task output to one. Let $s \geq 0$ be the common private value of the relationship, measuring the private benefits of the relationship per unit of relationship quality. Member i's utility is

$$u_i = y_i + sR.$$

The term sR represents the private benefits of workplace social relationships, such as companionship and approval, that each member values independently of task output. These benefits do not enter the production function directly, although they affect members' helping choices. The parameter s is a preference weight: a higher s raises the private benefits of the relationship at a given relationship quality. It is distinct from the amount of social interaction and from relationship quality itself.

All parameters and technologies are common knowledge. The members choose their time allocations simultaneously, after which output and utility are realized. We study Nash equilibrium in this one-period game. To evaluate the allocation of time, we compare equilibrium with the allocation that maximizes team output, defined as $Y = y_1 + y_2$. This is a criterion of team production rather than aggregate member utility or firm profit under fully flexible contracting. We do not model participation constraints or wage adjustments through which an employer might capture employees' private benefits from their relationship. Those benefits affect members' choices but are not part of the organization's output measure.

We restrict parameters so that output maximization leaves time for both activities. Raising h_i

reduces own-task output at rate ach_i and increases the productive relationship benefits across both tasks at rate $2\gamma(1+\rho h_j)$. At $h_i = h_j = 1$, these rates are ac and $2\gamma(1+\rho)$; the direct marginal output from additional help is then zero. We impose the following restriction.

Assumption 1.

$$ac > 2\gamma(1+\rho).$$

Section 3.3 shows that Assumption 1 also makes team output strictly concave. It does not restrict s, so equilibrium can still assign all time to helping. Unless otherwise stated, each comparative static holds the other parameters fixed and preserves Assumption 1.

3.2 Equilibrium

Given h_j, member i solves

$$\max_{h_i \in [0,1]} \left\{ \frac{ac}{2}(1-h_i^2) + ab\left(h_j - \frac{h_j^2}{2}\right) + (\gamma+s)(h_i + h_j + \rho h_i h_j) \right\}.$$

The objective is strictly concave in h_i. At an interior optimum, the first-order condition is

$$ach_i = (\gamma+s)(1+\rho h_j). \tag{4}$$

The left-hand side is the marginal output forgone on one's own task. The right-hand side sums the marginal increases in the productive benefits and private benefits of the relationship accruing to member i. The direct benefit of helping the other member, $ab(h_i - h_i^2/2)$, does not enter this condition because it accrues to the other member's task.

Write $h_i^{\mathrm{BR}}(h_j)$ for member i's best response to h_j, where the superscript BR denotes a best response. The solution, including the time constraint, is

$$h_i^{\mathrm{BR}}(h_j) = \min\left\{1, \frac{(\gamma+s)(1+\rho h_j)}{ac}\right\}. \tag{5}$$

For an interior best response, its slope is $\rho(\gamma+s)/(ac)$. Helping choices are therefore strict strategic complements when $\rho > 0$ and $\gamma + s > 0$. More helping by the other member raises the return to relationship formation and hence the incentive to help. This response operates through the relationship; assistance received does not change the marginal product of own-task effort in (3).

We denote equilibrium values with an asterisk. Let h^* and e^* denote the common helping and own-task effort levels in a symmetric equilibrium. The private value of the relationship at which both members devote all their time to helping is denoted by s_{sat}, where the subscript indicates saturation of the time constraint. Setting $h_i = h_j = 1$ in (4) gives

$$s_{\mathrm{sat}} = \frac{ac}{1+\rho} - \gamma > 0. \tag{6}$$

The inequality follows from Assumption 1.

Lemma 1. *The game has a unique Nash equilibrium. It is symmetric, with*

$$h^* = \begin{cases} \dfrac{\gamma + s}{ac - \rho(\gamma + s)}, & 0 \le s < s_{\text{sat}}, \\ 1, & s \ge s_{\text{sat}}, \end{cases} \qquad e^* = 1 - h^*. \tag{7}$$

The equilibrium is interior if and only if $0 < \gamma + s < ac/(1 + \rho)$.

Proof. See Appendix A.1 for the uniqueness argument, including boundary choices. □

For an interior equilibrium,

$$\frac{\partial h^*}{\partial s} = \frac{\partial h^*}{\partial \gamma} = \frac{ac}{[ac - \rho(\gamma + s)]^2} > 0, \qquad \frac{\partial h^*}{\partial \rho} = \frac{(\gamma + s)^2}{[ac - \rho(\gamma + s)]^2} > 0. \tag{8}$$

A greater private value of the relationship or greater relationship productivity increases helping and reduces own-task effort. Complementarity amplifies these responses through the other member's helping choice. By contrast, equilibrium helping does not depend on b. Under individual performance rewards, the member providing assistance does not receive its direct output benefit.

3.3 The Output-Maximizing Allocation

To assess whether equilibrium helping is too low or too high for team output, we first derive the allocation that maximizes output. Substituting (1)–(3) into $Y = y_1 + y_2$ gives the output function

$$Y(h_1, h_2) = ac + (ab + 2\gamma)(h_1 + h_2) - \frac{a(b + c)}{2}(h_1^2 + h_2^2) + 2\gamma\rho h_1 h_2. \tag{9}$$

We suppress parameter arguments and display the helping choices as the arguments of Y. The allocation problem is

$$\max_{(h_1, h_2) \in [0,1]^2} Y(h_1, h_2). \tag{10}$$

At an interior solution, the first-order condition for member i's helping is

$$ach_i = ab(1 - h_i) + 2\gamma(1 + \rho h_j). \tag{11}$$

Helping sacrifices output on the provider's own task. It also generates a direct benefit on the recipient's task and increases the productive benefits of the relationship for both tasks. The latter two effects appear on the right-hand side.

Assumption 1 makes the Hessian of Y negative definite: its eigenvalues are $-a(b + c) + 2\gamma\rho$ and $-a(b + c) - 2\gamma\rho$. The marginal effect of helping is positive at $h_i = 0$ and negative at $h_i = 1$ for every feasible h_j. The unique maximizer is therefore interior and, by symmetry, assigns equal helping to the two members. We denote output-maximizing helping by h^Y, using the superscript Y for this output criterion. Solving (11) gives

$$h^Y = \frac{ab + 2\gamma}{a(b + c) - 2\gamma\rho} \in (0, 1). \tag{12}$$

Comparing (4) and (11) reveals two sources of divergence. Each member omits both the direct output contribution of helping to the other task and the increase in the productive benefits of the relationship for that task. Each member also values the private benefits of the relationship, which are excluded from team output. A higher private value of the relationship can offset the omitted marginal output contributions, but it can also induce more helping than maximizes output.

For a symmetric allocation with common helping h, output is

$$Y(h,h) = ac + 2(ab + 2\gamma)h - (a(b+c) - 2\gamma\rho)h^2. \tag{13}$$

Let $Y^Y = Y(h^Y, h^Y)$ denote maximum team output. Completing the square yields

$$Y^Y - Y(h,h) = (a(b+c) - 2\gamma\rho)(h - h^Y)^2, \tag{14}$$

where $Y^Y = ac + (ab+2\gamma)^2/(a(b+c) - 2\gamma\rho)$. Along symmetric allocations, output falls as helping moves away from h^Y in either direction.

3.4 The Private Value of the Relationship and Team Output

We now vary the private value of the relationship s while holding the production technology fixed. Write $Y^* = Y(h^*, h^*)$ for equilibrium team output. We call helping insufficient for team output when $h^* < h^Y$ and excessive when $h^* > h^Y$. These comparisons concern the allocation of time, rather than whether helping produces a positive benefit for its recipient.

The value of s that implements the output-maximizing allocation is denoted by $\bar{s}$. To obtain it, we impose $h^* = h^Y$ in the individual first-order condition. This gives

$$\bar{s} = \frac{ach^Y}{1 + \rho h^Y} - \gamma = \frac{c(ab + 2\gamma)}{c + b(1+\rho)} - \gamma. \tag{15}$$

At this allocation, the private benefits of the relationship offset the marginal output benefits omitted from each member's decision. We have $0 < \bar{s} < s_{\text{sat}}$, because equilibrium helping at $s = 0$ is below $h^Y < 1$ and increases to one at saturation.

Proposition 1. *Equilibrium helping and team output satisfy the following properties:*

(i) For $0 \le s < \bar{s}$, $h^* < h^Y$, and Y^* is strictly increasing in s.

(ii) At $s = \bar{s}$, $h^* = h^Y$ and $Y^* = Y^Y$.

(iii) For $\bar{s} < s < s_{\text{sat}}$, $h^* > h^Y$, and Y^* is strictly decreasing in s.

(iv) For $s \ge s_{\text{sat}}$, $h^* = 1$ and $Y^* = ab + 2\gamma(2+\rho)$.

Proof. At $s = 0$, the individual condition omits the positive marginal benefit $ab(1-h^*) + \gamma(1 + \rho h^*)$, so $h^* < h^Y$. Equation (7) increases continuously to one as s reaches s_{sat}. It crosses h^Y once, at (15). The output comparisons follow from (14), and substituting $h = 1$ into (13) gives part (iv). □

The private value of the relationship initially encourages assistance whose benefits are not fully rewarded. Beyond $\bar{s}$, members devote too little time to their own tasks. Helping remains productive for the recipient: its marginal effect on that task is $ab(1-h_i)+\gamma(1+\rho h_j)>0$ in an interior equilibrium. However, the total output gain from additional helping is smaller than the output forgone on the provider's own task.

For $s < s_{\text{sat}}$, the chain rule makes the change in sign explicit:

$$\frac{dY^*}{ds} = 2(a(b+c)-2\gamma\rho)(h^Y - h^*)\frac{\partial h^*}{\partial s}.$$

The derivative at $s=0$ is interpreted from the right. Appendix C extends the inverted-U pattern to general concave production and supermodular relationship technologies under appropriate regularity conditions.

Complementarity shifts the threshold at which helping becomes excessive:

$$\frac{\partial \bar{s}}{\partial \rho} = -\frac{bc(ab+2\gamma)}{[c+b(1+\rho)]^2} < 0.$$

Although complementarity also increases output-maximizing helping when $\gamma > 0$, it lowers the private value of the relationship needed to implement that allocation. The mutual response in helping makes excessive helping possible at a smaller s. In contrast, greater direct helping productivity b raises h^Y without changing h^*, so a larger private value of the relationship is needed to reach the output-maximizing allocation.

The output criterion must be distinguished from aggregate utility. For a fixed s, define $W = u_1 + u_2 = Y + 2sR$. At a nonsaturated equilibrium, the marginal effect of either member's helping on W is

$$\left.\frac{\partial W}{\partial h_i}\right|_{h_1=h_2=h^*} = ab(1-h^*) + (\gamma+s)(1+\rho h^*) > 0,$$

where we have used the individual first-order condition. Thus, even when helping is excessive for output, increasing it can raise aggregate utility. We use excessive helping only with reference to team output and do not interpret an output loss as a loss of aggregate utility. If an employer could fully capitalize the private relationship benefits into wage adjustments, the relevant contracting objective could differ from the output criterion used here. Appendix D considers an organizational objective that also includes a share of these private benefits.

3.5 Relationship Productivity and Team Output

We next hold the private value of the relationship s fixed and increase relationship productivity γ. In the individual first-order condition (4), s and γ enter only through their sum. Hence an increase in either parameter has the same effect on equilibrium helping. Unlike an increase in s, however, an increase in γ also raises team output at fixed helping choices, by $2R \geq 0$. We examine whether the resulting increase in helping can outweigh this direct productivity benefit.

Let $R^* = 2h^* + \rho(h^*)^2$ denote relationship quality at the symmetric equilibrium. For a nonsaturated

equilibrium, the total effect on output can be decomposed as

$$\frac{dY^*}{d\gamma} = \underbrace{2R^*}_{\text{direct productivity effect}} + \underbrace{2(a(b+c) - 2\gamma\rho)(h^Y - h^*)\frac{\partial h^*}{\partial \gamma}}_{\text{effect of the change in time allocation}}. \tag{16}$$

The response $\partial h^*/\partial\gamma$ is given in (8); the same expression applies as a right derivative at $\gamma = s = 0$. Excessive helping is necessary, but not sufficient, for a productivity reversal. Substituting the equilibrium response into (16) gives the exact condition

$$\begin{aligned} \frac{dY^*}{d\gamma} < 0 \quad \Longleftrightarrow \quad & (1+\rho h^*)^2\big[a(b+c)h^* - ab - 2\gamma(1+\rho h^*)\big] \\ & > ac\big[2h^* + \rho(h^*)^2\big]. \end{aligned} \tag{17}$$

The inequality compares the output lost through additional helping with the direct gain from greater relationship productivity. It applies throughout the nonsaturated region, not only near the helping constraint.

A sufficient condition for a reversal near the saturation threshold is

$$\rho > \frac{\sqrt{5}-1}{2}, \qquad 0 \le \gamma < \frac{ac(\rho^2+\rho-1)}{2(1+\rho)^3}. \tag{18}$$

This condition is compatible with Assumption 1 for every $b > 0$. At $\gamma = 0$, derivatives are from the right.

Proposition 2. *At a nonsaturated equilibrium:*

(i) If $h^* \le h^Y$, then $dY^*/d\gamma > 0$.

(ii) The derivative $dY^*/d\gamma$ is negative if and only if (17) *holds. In particular, condition* (18) *ensures a negative derivative for all s sufficiently close to s_{sat} from below.*

(iii) If $\rho = 0$, then $dY^*/d\gamma > 0$ for every $s < s_{\text{sat}}$.

Proof. See Appendix A.2 for the comparisons and the existence claim. □

Part (iii) makes complementarity necessary *in the quadratic benchmark*.[2] Without complementarity, helping increases by $1/(ac)$ per unit increase in relationship productivity, with no amplification through the colleague's response. The resulting allocation loss cannot outweigh the direct gain.

Figure 1 illustrates the reversal. Helping becomes excessive at $\gamma = 0.012$, but output continues to rise until $\gamma \simeq 0.0482$. It then falls by approximately 1.66% up to $\gamma = 0.07$, as the allocation loss outweighs the direct gain. All allocations are interior. The figure distinguishes excessive helping from a negative output response; the magnitude is specific to these parameters.

[2] Appendix C shows that complementarity can amplify the helping response but is not necessary for a reversal under general concave production technologies.

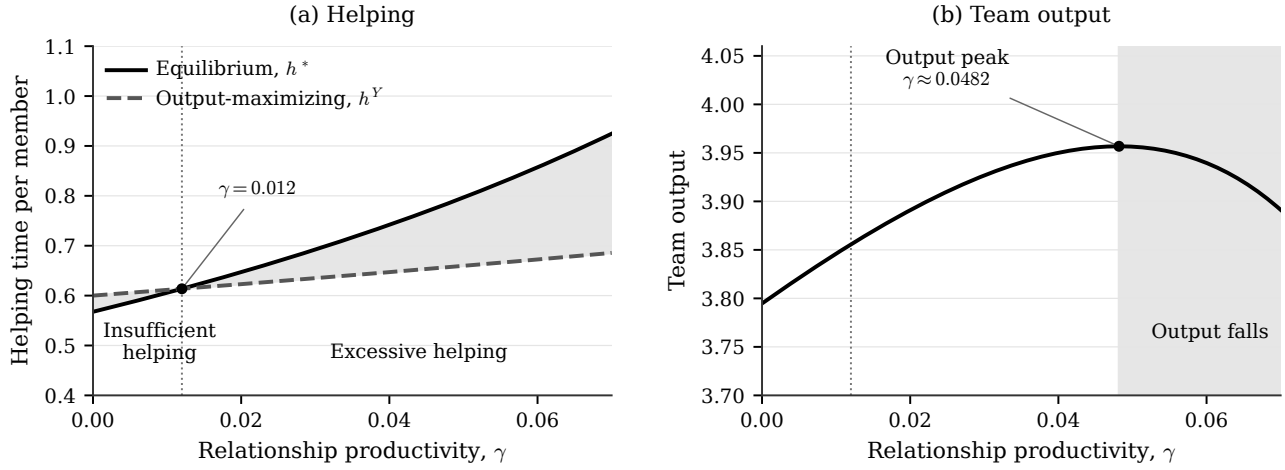


Figure 1: Relationship productivity, helping, and team output

Notes: $a = 1$, $c = 2$, $b = 3$, $s = 0.42$, and $\rho = 3$. Individual rewards are held fixed while γ varies over $[0, 0.07]$. Panel (a) compares equilibrium helping with output-maximizing helping. The dotted vertical line in both panels marks their equality at $\gamma = 0.012$. Panel (b) plots equilibrium team output; the shaded interval has $dY^*/d\gamma < 0$. Assumption 1 holds throughout, and all displayed allocations are interior.

The reversal is not an argument for keeping helping fixed after a productivity improvement. The output-maximizing response is also positive:

$$\frac{\partial h^Y}{\partial \gamma} = \frac{2a[c + b(1+\rho)]}{[a(b+c) - 2\gamma\rho]^2} > 0.$$

The problem is the equilibrium allocation of time relative to this changing benchmark. Nor does the reversal extend into the interior of the saturated region. When both members already devote all their time to helping, the allocation is locally fixed and $dY^*/d\gamma = 2(2+\rho) > 0$. At the saturation boundary, the two one-sided derivatives must be distinguished.

3.6 Common Ability and Team Output

We now increase common ability a, holding individual rewards and the other parameters fixed. This raises the productivity of own-task effort and direct helping while leaving relationship productivity γ unchanged. For $\gamma + s > 0$, an interior equilibrium satisfies

$$\frac{\partial h^*}{\partial a} = -\frac{c(\gamma + s)}{[ac - \rho(\gamma + s)]^2} < 0. \tag{19}$$

Higher ability raises the opportunity cost of helping, while its direct benefit still accrues to the colleague. Both members therefore help less. Their choices depend on the return to the relationship relative to ability, $(\gamma+s)/a$, so increases in a and γ move helping in opposite directions. Complementarity reinforces both responses.

The threshold separating insufficient and excessive helping also rises with ability. Differentiating (15)

gives

$$\frac{\partial \bar{s}}{\partial a} = \frac{bc}{c + b(1+\rho)} > 0. \tag{20}$$

Thus, higher ability expands the range of private relationship values for which helping is insufficient, accounting for the change in the output-maximizing allocation.

At a nonsaturated equilibrium, the total output effect is

$$\begin{aligned} \frac{dY^*}{da} = & \underbrace{c[1-(h^*)^2] + b[2h^* - (h^*)^2]}_{\text{direct productivity effect}} \\ & + \underbrace{2[a(b+c) - 2\gamma\rho](h^Y - h^*)\frac{\partial h^*}{\partial a}}_{\text{effect of the change in time allocation}}. \end{aligned} \tag{21}$$

The direct effect is strictly positive. Reducing helping adds to this gain when helping is excessive, but can offset it when helping is insufficient.

Proposition 3. *At a nonsaturated equilibrium under individual rewards, an increase in common ability a has the following effects:*

(i) If $\gamma + s > 0$, equilibrium helping and relationship quality strictly decrease.

(ii) If $h^* \geq h^Y$, then $dY^*/da > 0$. There is a nonempty open set of parameters with $\rho > 0$ and $h^* < h^Y$ for which $dY^*/da < 0$.

(iii) If $\rho = 0$, then $dY^*/da > 0$.

Proof. See Appendix A.3 for the output comparisons and an exact calculation establishing the negative effect. □

This is the same time-allocation mechanism as in Proposition 2, operating on the other side of the output-maximizing allocation. Greater relationship productivity can reduce output by increasing excessive helping; greater common ability can reduce output by decreasing insufficient helping. Insufficient helping alone does not imply an output decline: the allocation loss must exceed the direct gain. Figure 2(a) in Section 4.3 illustrates this output decline. The next section examines how reward design can address these distortions.

4 Individual and Team Rewards

The organization chooses the mix of individual and team rewards to maximize team output. The baseline technology, time constraints, and Assumption 1 remain unchanged.

4.1 Reward Design and Equilibrium Helping

Let $\lambda \in [0,1]$ denote the weight on team rewards.[3] Member i's utility is now

$$u_i = (1-\lambda)y_i + \lambda\frac{y_i + y_j}{2} + sR. \tag{22}$$

At $\lambda = 0$, rewards depend on individual task output; at $\lambda = 1$, each member receives half of team output. Total material rewards equal Y for every mixture.

The organization first commits to λ to maximize team output within this reward class. Members observe the weight and choose their time allocations simultaneously. We solve by backward induction, holding s exogenous.

Given λ and h_j, member i maximizes (22) over $h_i \in [0,1]$. Material rewards place weights $1-\lambda/2$ and $\lambda/2$ on own and colleague output, while the common term γR retains weight one. The objective is strictly concave, with interior first-order condition

$$\left(1 - \frac{\lambda}{2}\right) ach_i = \frac{\lambda ab}{2}(1-h_i) + (\gamma + s)(1 + \rho h_j). \tag{23}$$

Team rewards give the provider a share of the direct output from helping and reduce the share of forgone own-task output. The reward weight does not directly enter the private benefits of the relationship, sR.

The best response is

$$h_i^{\mathrm{BR}}(h_j;\lambda) = \min\left\{1, \frac{\lambda ab/2 + (\gamma+s)(1+\rho h_j)}{(1-\lambda/2)ac + (\lambda/2)ab}\right\}. \tag{24}$$

The equilibrium remains unique and symmetric, including at the boundaries (Appendix A.4).

We write $h^*(\lambda, s)$ for the common equilibrium helping level, displaying the reward weight and the private value of the relationship while suppressing the other parameter arguments. If $(\gamma+s)(1+\rho) < (1-\lambda/2)ac$, solving the two first-order conditions gives

$$h^*(\lambda, s) = \frac{\lambda ab/2 + \gamma + s}{(1-\lambda/2)ac + (\lambda/2)ab - \rho(\gamma+s)}. \tag{25}$$

If $(\gamma+s)(1+\rho) \geq (1-\lambda/2)ac$, the equilibrium instead has $h^*(\lambda, s) = 1$. The lower boundary $h^*(\lambda, s) = 0$ occurs only when $\lambda = \gamma = s = 0$.

At a nonsaturated equilibrium,

$$\frac{\partial h^*}{\partial \lambda} = \frac{a^2bc + [ac - ab(1+\rho)](\gamma+s)}{2[(1-\lambda/2)ac + (\lambda/2)ab - \rho(\gamma+s)]^2} > 0.$$

The numerator equals $ab[ac - (1+\rho)(\gamma+s)] + ac(\gamma+s) > 0$ under nonsaturation. A higher team reward weight therefore increases helping until the time constraint binds.

Complete team rewards maximize aggregate utility, which need not maximize output. At $\lambda = 1$, each member's utility is $W/2$, so every maximizer of W is a Nash equilibrium. Uniqueness therefore

[3] The restriction $\lambda \geq 0$ is not needed for budget balance. When $\lambda < 0$, a member's pay rises with own output relative to the colleague's. Such schemes may invite uncooperative behavior (Lazear, 1989), which we do not model.

implies that complete team rewards implement it. When $s = 0$, $h^*(1, 0) = h^Y$; for $s > 0$, the private relationship benefits instead induce $h^*(1, s) > h^Y$.

4.2 The Output-Maximizing Reward Weight

Write $Y^*(\lambda, s) = Y(h^*(\lambda, s), h^*(\lambda, s))$ for equilibrium team output and $\lambda^Y(s)$ for an output-maximizing weight, suppressing the other parameters. Anticipating members' responses, the organization solves

$$\lambda^Y(s) \in \arg\max_{\lambda \in [0,1]} Y^*(\lambda, s).$$

At a nonsaturated equilibrium, the marginal effect of the reward weight is

$$\frac{\partial Y^*}{\partial \lambda} = 2(a(b + c) - 2\gamma\rho)\big[h^Y - h^*(\lambda, s)\big]\frac{\partial h^*}{\partial \lambda}.$$

Thus, increasing λ raises output when helping is insufficient and lowers output when helping is excessive. Because the technology is unchanged, the output-maximizing allocation h^Y remains the benchmark.

To find a reward weight that implements this allocation, substitute $h_i = h_j = h^Y$ into (23). Rearranging gives

$$\lambda = \frac{2\{ach^Y - (\gamma + s)(1 + \rho h^Y)\}}{ach^Y + ab(1 - h^Y)} = 1 - \frac{s}{\bar{s}}. \tag{26}$$

The last equality uses (11) and (15). The weight is feasible if and only if $s \leq \bar{s}$; larger private values would require a negative team reward weight. The threshold s_{sat} identifies when even individual rewards assign all time to helping.

Proposition 4. *For $0 \leq s < s_{\text{sat}}$, the unique output-maximizing reward weight is*

$$\lambda^Y(s) = \max\left\{0, 1 - \frac{s}{\bar{s}}\right\}. \tag{27}$$

It implements h^Y if and only if $s \leq \bar{s}$. When $0 < s < \bar{s}$, the optimal weight strictly increases with common ability:

$$\frac{\partial \lambda^Y}{\partial a} = \frac{sbc}{[c + b(1 + \rho)]\bar{s}^2} > 0. \tag{28}$$

For $s \geq s_{\text{sat}}$, every $\lambda \in [0, 1]$ is optimal and implements $h^*(\lambda, s) = 1$.

Proof. For $s \leq \bar{s}$, equation (26) uniquely implements h^Y. Differentiating $1 - s/\bar{s}$ with respect to a and using (20) yields (28). For $\bar{s} < s < s_{\text{sat}}$, helping is already excessive at $\lambda = 0$, and raising λ increases it until saturation. Hence $\lambda = 0$ uniquely maximizes output. For $s \geq s_{\text{sat}}$, all reward weights induce saturation. □

The private value of the relationship and team rewards substitute for each other as helping incentives. For $0 < s < \bar{s}$, the optimal weight falls at rate $d\lambda^Y/ds = -1/\bar{s}$, reaching zero at $s = \bar{s}$. This parallels Wang and He (2008), who find that stronger preferences for social rewards reduce the monetary incentives needed to encourage team activities. Appendix C extends (27) to general concave production and supermodular relationship technologies under the stated regularity conditions.

For $\bar{s} < s < s_{\text{sat}}$, individual rewards maximize output within the permitted class, but they leave helping excessive. The organization can remove the additional incentive from team rewards; it cannot remove the incentive arising from the private value of the relationship. Complementarity reinforces the substitution between the two incentives. Where $0 < s < \bar{s}$,

$$\frac{\partial \lambda^Y}{\partial \rho} = \frac{s}{\bar{s}^2} \frac{\partial \bar{s}}{\partial \rho} < 0.$$

Stronger complementarity requires less weight on team rewards to implement the output-maximizing allocation, even though that allocation itself changes with ρ.

Common ability and the private value of the relationship therefore have opposite implications for reward design. Higher ability calls for a larger team reward weight where the optimal mix is interior. This rewards the direct assistance that more able members would otherwise reduce as its opportunity cost rises. The comparison holds the private value of the relationship and the remaining technology parameters fixed. At $s = 0$, complete team rewards already implement maximum output; when helping is excessive under individual rewards, the optimal weight remains zero until the increase in ability makes team rewards useful.

4.3 Output under Optimal Rewards

Let $\widehat{Y}(s) = \max_{\lambda \in [0,1]} Y^*(\lambda, s)$ denote team output after the reward weight has been optimized. As before, parameters other than the displayed argument are suppressed. Proposition 4 gives

$$\widehat{Y}(s) = \begin{cases} Y^Y, & 0 \leq s \leq \bar{s}, \\ Y^*(0, s), & s > \bar{s}. \end{cases}$$

The inverted-U pattern in Proposition 1 concerns fixed individual rewards. With optimal rewards, the increasing part becomes a plateau: for $s \leq \bar{s}$, the organization offsets changes in the private value of the relationship by adjusting the team reward weight. Output then falls for $\bar{s} < s < s_{\text{sat}}$ and remains constant after saturation. Thus, optimal rewards support maximum output over an interval of private values of the relationship, rather than at a single value.

First consider an increase in common ability, holding s fixed. Where $s < \bar{s}$, reward adjustment implements h^Y, and the envelope condition gives

$$\frac{d\widehat{Y}}{da} = \frac{dY^Y}{da} = c[1 - (h^Y)^2] + b[2h^Y - (h^Y)^2] > 0. \tag{29}$$

Thus, in the insufficient-helping region, adjusting the team reward weight prevents the output reversal in Proposition 3. The positive output effect of higher ability also extends to the regions in which optimal rewards leave helping excessive or saturated.

We can now revisit an increase in relationship productivity γ, holding s fixed and allowing the organization to reoptimize rewards. Where $s < \bar{s}$, the output-maximizing allocation remains implementable

for small changes in γ. The envelope condition for (10) yields

$$\frac{d\widehat{Y}}{d\gamma} = \frac{dY^Y}{d\gamma} = 2\left[2h^Y + \rho(h^Y)^2\right] > 0.$$

In this region, reward adjustment prevents the allocation loss. Where $\bar{s} < s < s_{\text{sat}}$, however, the optimal reward weight is already zero, so optimized output coincides locally with baseline output. The negative effect established in Proposition 2 can therefore survive reward optimization.

Corollary 1. *Suppose Assumption 1 holds and rewards are chosen to maximize team output over* $\lambda \in [0, 1]$.

(i) Holding b, c, γ, ρ, s *fixed, optimized team output* $\widehat{Y}$ *is strictly increasing in common ability* a *throughout the region satisfying Assumption 1.*

(ii) For $\bar{s} < s < s_{\text{sat}}$, $d\widehat{Y}/d\gamma < 0$ *if and only if* (17) *holds. In particular, under* (18)*, the derivative is negative for all* s *sufficiently close to* s_{sat} *from below.*

Proof. For part (i), when $s < \bar{s}$, equation (29) gives $d\widehat{Y}/da > 0$. When $\bar{s} < s < s_{\text{sat}}$, the optimal weight is locally zero and $h^* > h^Y$, so Proposition 3(ii) gives the same sign. When $s > s_{\text{sat}}$, all reward weights induce $h^* = 1$, and $\widehat{Y} = ab + 2\gamma(2 + \rho)$ gives $d\widehat{Y}/da = b > 0$.

At $s = \bar{s}$, the two expressions for optimized output agree because $h^* = h^Y$. At $s = s_{\text{sat}}$, the nonsaturated equilibrium converges to $h^* = 1$. Thus optimized output is continuous at both transitions. Both $\bar{s}$ and s_{sat} are strictly increasing in a, so there are at most two transition points as ability changes. Strict increase within each region and continuity at these points imply strict increase throughout the admissible range. This conclusion includes the saturation boundary, where the two one-sided derivatives need not coincide.

For part (ii), when $\bar{s} < s < s_{\text{sat}}$, the optimal reward weight is locally zero, so $\widehat{Y} = Y^*(0, s)$ and the exact condition follows from Proposition 2(ii). Since $\bar{s} < s_{\text{sat}}$, the same proposition yields the final claim. □

The contrast in Corollary 1 comes from the direction of the change in helping. Team rewards can offset the reduction in helping caused by higher ability when individual rewards induce insufficient helping. Greater relationship productivity instead raises helping. Once individual rewards already induce excessive helping, the lower bound $\lambda = 0$ prevents the organization from countering this increase through a further reduction in the team reward weight.

For the parameters in Figure 1, $\bar{s} = (3 - 5\gamma)/7$. Hence the fixed private value of the relationship $s = 0.42$ exceeds $\bar{s}$ whenever $\gamma > 0.012$. Throughout the declining portion of panel (b), the optimal reward weight is therefore $\lambda^Y = 0$, and reward optimization leaves the output decline unchanged. For $\gamma < 0.012$, in contrast, optimal rewards implement h^Y, so optimized output differs from the fixed-individual-reward curve shown in the figure.

Figure 2 illustrates the common-ability comparison. Under individual rewards, helping changes from excessive to insufficient at $a = 0.78$. Panel (a) shows that output initially rises on both sides of this boundary. At $h^* = h^Y$, the marginal reallocation effect is zero, so the direct productivity gain dominates

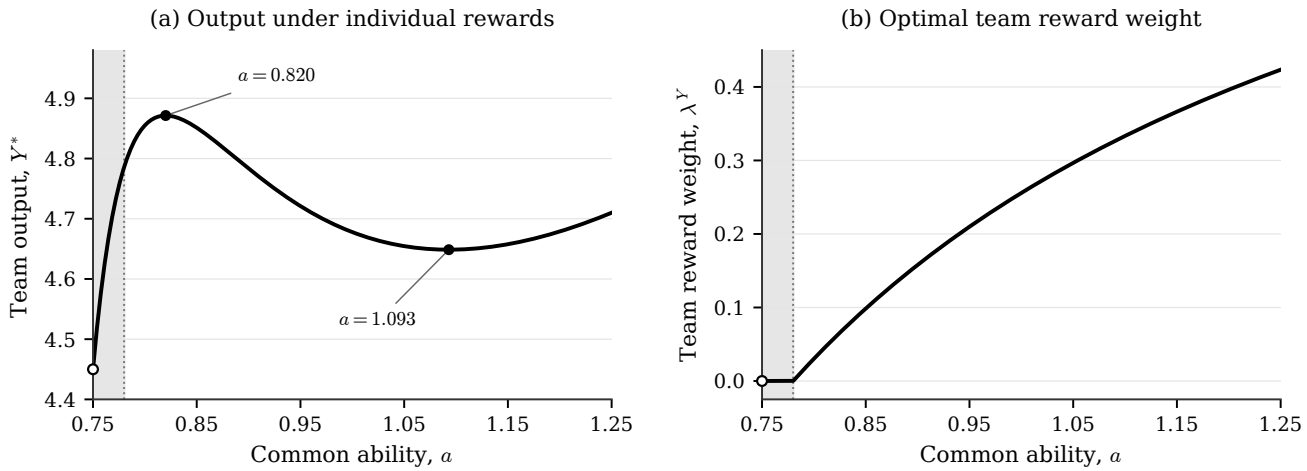


Figure 2: Common ability, team output, and optimal rewards

Notes: $c = 2$, $b = 5$, $\gamma = 0.05$, $\rho = 5$, and $s = 0.2$. Panel (a) holds individual rewards fixed ($\lambda = 0$); panel (b) plots the output-maximizing team reward weight. In both panels, gray and white regions denote excessive and insufficient helping under individual rewards, respectively, with equality at the dotted line $a = 0.78$. Assumption 1 holds throughout. All displayed allocations for $0.75 < a \leq 1.25$ are interior; open endpoints denote limits as a approaches 0.75 from above.

nearby. Output falls for approximately $0.820 < a < 1.093$, where the loss from reduced helping exceeds this gain. At higher ability levels in the figure, the helping response weakens and output rises again.

Panel (b) shows the reward response. The optimal weight is zero while individual rewards induce excessive helping and rises with ability once they induce insufficient helping. These positive weights implement h^Y and make optimized output increase with a.

5 Team Organization

Team size, leadership roles, and member composition also affect relationship formation. We examine each change separately, starting from the baseline technology and individual rewards. The team-size analysis extends the number of members; the leadership analysis changes the timing of choices in a two-member team; the composition analysis returns to simultaneous choice and allows private relationship values to differ.

5.1 Team Size and Optimal Team Formation

We now let an organization divide a fixed workforce into teams. The organization chooses team size, and members choose their helping time. We first retain individual rewards and then consider whether reward adjustment eliminates the role of team size.

5.1.1 Team Size and Helping Incentives

Consider a team of $n \geq 2$ identical members. Member i divides one unit of time between own-task effort e_i and total helping h_i, so $e_i = 1 - h_i$. The team operates a rule that distributes each member's

helping equally among the other $n-1$ members. Thus, the helping that i gives to j is $h_{ij} = h_i/(n-1)$. Members choose their total helping time but not its distribution among recipients.[4]

We retain the pairwise relationship technology in (2). For each pair i, j, its relationship quality is $R_{ij} = h_{ij} + h_{ji} + \rho h_{ij} h_{ji}$. Let H_i denote the total helping that i receives, and R_i the sum of the qualities of i's relationships. Substitution of the equal-distribution rule gives

$$H_i = \frac{1}{n-1} \sum_{j \neq i} h_j, \qquad R_i = h_i + H_i + \frac{\rho}{n-1} h_i H_i. \tag{30}$$

The first term of R_i records help given, the second help received, and the third the complementarity of these inputs in each pair. Both H_i and R_i reduce to the corresponding expressions in Section 3.1 when $n = 2$. For a symmetric allocation with total helping h per member, the complementarity term in R_i is $\rho h^2/(n-1)$. Its decline with team size follows from dividing a fixed helping budget equally among more colleagues. The team-size results below therefore depend on this scaling of pairwise relationship formation; a different normalization need not yield the same size effects.

A member's task output and utility under individual rewards are

$$y_i = \frac{ac}{2}(1 - h_i^2) + ab\left(H_i - \frac{H_i^2}{2}\right) + \gamma R_i, \qquad u_i = y_i + sR_i. \tag{31}$$

We continue to evaluate the sum of task output, $Y = \sum_{i=1}^{n} y_i$, excluding the private benefits of relationships. Common ability, the production parameters, and private value s do not depend on n. Assumption 1 continues to hold.

Given the other members' helping choices, i solves the same type of strictly concave problem as in the two-person model. Its interior first-order condition is

$$ach_i = (\gamma + s)\left(1 + \frac{\rho H_i}{n-1}\right).$$

An additional unit of i's helping is distributed across $n-1$ relationships. Its marginal contribution to relationship formation depends on the help received from those colleagues, but that complementarity is diluted as n increases. We focus on the region

$$0 < \gamma + s < \frac{ac}{1+\rho}, \tag{32}$$

which ensures that each member allocates positive time to both activities for every $n \geq 2$. It also gives a unique equilibrium under the equal-distribution rule. The equilibrium is symmetric, with

$$h^*(n) = \frac{\gamma + s}{ac - \dfrac{\rho(\gamma + s)}{n-1}}. \tag{33}$$

[4]If members were free to choose recipients, unequal allocations could also arise. In particular, the complementarity in relationship formation could encourage assistance to concentrate within particular pairs. We take equal distribution as a team operating rule to isolate how team size affects total helping incentives; we do not claim that members would always choose this allocation rule themselves.

For the output benchmark, let h denote a common helping choice. Equations (30) and (31) imply that output per member is

$$\frac{Y(h;n)}{n} = \frac{ac}{2} + (ab + 2\gamma)h - \frac{1}{2}\left(a(b+c) - \frac{2\gamma\rho}{n-1}\right)h^2. \tag{34}$$

Under Assumption 1, total output is strictly concave in the n helping choices. Its unique maximizer is symmetric and interior. The output-maximizing helping per member is therefore

$$h^Y(n) = \frac{ab + 2\gamma}{a(b+c) - \dfrac{2\gamma\rho}{n-1}}. \tag{35}$$

As in the baseline, we call helping excessive or insufficient according to whether $h^*(n)$ exceeds or falls short of $h^Y(n)$. The private value of the relationship at which these levels coincide is

$$\bar{s}_n = \frac{c(ab + 2\gamma)}{b + c + \dfrac{b\rho}{n-1}} - \gamma. \tag{36}$$

Proposition 5. *Suppose Assumption 1 and* (32) *hold, with* $\rho > 0$.

(i) Equilibrium helping $h^*(n)$ *is strictly decreasing in team size* n.

(ii) The threshold $\bar{s}_n$ *is strictly increasing in* n. *Helping is excessive if* $s > \bar{s}_n$, *equal to the output-maximizing level if* $s = \bar{s}_n$, *and insufficient if* $s < \bar{s}_n$.

Proof. See Appendix A.5 for the equilibrium uniqueness argument and the comparisons. □

A small team gives each member a strong incentive to reinforce the help received from colleagues. As the same amount of helping is spread across more relationships, this mutual reinforcement weakens. Both equilibrium and output-maximizing helping can fall with team size, but the threshold $\bar{s}_n$ rises. Thus, at a fixed private value s, helping can be excessive in a small team and insufficient in a larger one. This change in the helping distortion is the basis for the team-formation problem.

5.1.2 Optimal Team Formation

Suppose the organization has a fixed workforce of N identical employees and divides them into equally sized teams, without spillovers across teams or costs of creating a team. If each team has n members, there are N/n teams and total organization output is $(N/n)Y^*(n)$, where $Y^*(n) = Y(h^*(n); n)$. Hence the organization chooses n to maximize output per member:

$$n^{\text{opt}} \in \arg \max_{\substack{n \in \{2,\ldots,N\} \\ n \mid N}} \frac{Y^*(n)}{n}. \tag{37}$$

The divisibility condition $n \mid N$ means that all teams have the same integer size. To characterize the choice analytically, we first relax n to a continuous variable on $[2, \infty)$, retaining the symmetric equilibrium in (33). We denote a finite interior maximizer of this relaxation by n°.

An increase in team size reduces equilibrium helping. When helping is initially excessive, this can improve time allocation. But a larger team also weakens the productive complementarity in relationship formation, and sufficiently low equilibrium helping can become insufficient. The two effects determine whether output per member reaches an interior maximum.

Evaluating (34) at $h = h^*(n)$ and differentiating gives

$$\frac{d[Y^*(n)/n]}{dn} = \frac{\rho(\gamma+s)^2}{[ac(n-1)-\rho(\gamma+s)]^3}\Big[\rho(\gamma+s)(ab+\gamma) - (n-1)\{ac(ab+3\gamma)-(\gamma+s)a(b+c)\}\Big]. \tag{38}$$

The denominator is positive under (32). The derivative's sign is therefore determined by the bracketed term, which is affine in n and changes sign at most once. The continuous relaxation has the following solution.

Proposition 6. *Suppose Assumption 1 and* (32) *hold, with* $\rho > 0$.

(i) If

$$\frac{ac(ab+3\gamma)}{a(b+c)+\rho(ab+\gamma)} - \gamma < s < \frac{c(ab+3\gamma)}{b+c} - \gamma,$$

output per member has a unique interior maximizer,

$$n^\circ = 1 + \frac{\rho(\gamma+s)(ab+\gamma)}{ac(ab+3\gamma)-(\gamma+s)a(b+c)} > 2. \tag{39}$$

At this solution, $\partial n^\circ/\partial s > 0$.

(ii) If

$$s \le \frac{ac(ab+3\gamma)}{a(b+c)+\rho(ab+\gamma)} - \gamma,$$

output per member is decreasing on $n > 2$ *and the continuous optimum is* $n = 2$.

(iii) If

$$s \ge \frac{c(ab+3\gamma)}{b+c} - \gamma,$$

output per member is strictly increasing in n *and has no finite maximizer over* $n \ge 2$.

Proof. In part (i), the bracketed term in (38) decreases with n and is positive at $n = 2$. It vanishes at n° in (39), so output per member increases below n° and decreases above it. In part (ii), the bracketed term is nonpositive at $n = 2$ and strictly decreases thereafter. In part (iii), $ac(ab+3\gamma)-(\gamma+s)a(b+c) \le 0$, so the derivative is positive for every finite n. Differentiating the interior solution with respect to s gives

$$\frac{\partial n^\circ}{\partial s} = \frac{\rho ac(ab+\gamma)(ab+3\gamma)}{[ac(ab+3\gamma)-(\gamma+s)a(b+c)]^2} > 0.$$

□

At an interior optimum, the gain from reducing excessive helping offsets the loss of productive complementarity. Using $dh^*(n)/dn = -\rho[h^*(n)]^2/(n-1)^2$ in the derivative of (34), the first-order

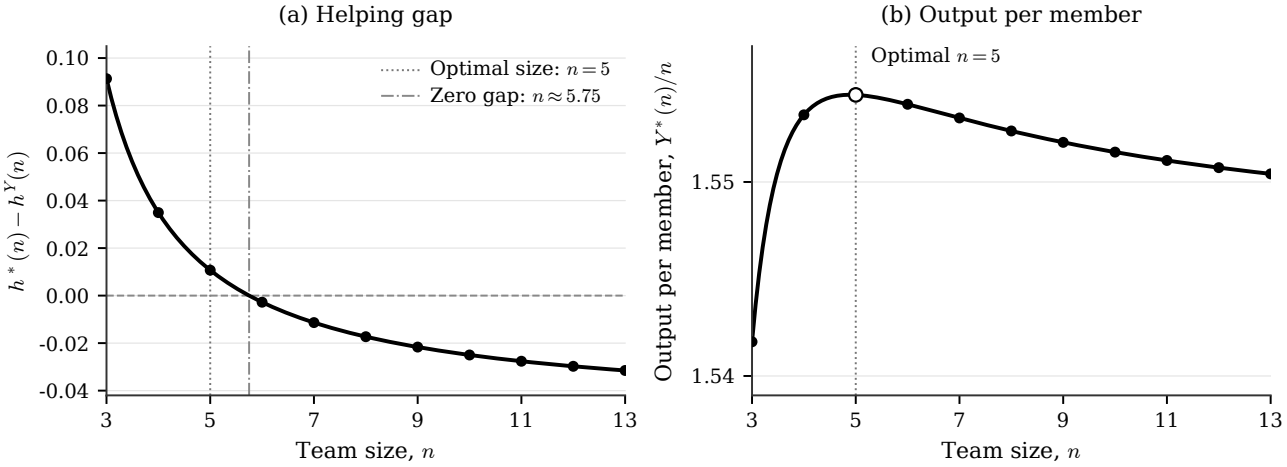


Figure 3: Team size, helping distortions, and output per member

Notes: $a = 1$, $c = 2$, $b = 2$, $\gamma = 0.05$, $\rho = 1$, and $s = 0.9$. Panel (a) plots $h^*(n) - h^Y(n)$; panel (b) plots output per member. The dotted vertical lines and circled dot mark the best integer size, $n = 5$. The horizontal dashed and vertical dash-dotted lines mark a zero helping gap. Both panels omit $n = 2$ for readability, where output per member is approximately 1.304. All displayed equilibria are interior.

condition gives

$$h^*(n^\circ) - h^Y(n^\circ) = \frac{\gamma}{a(b+c) - \dfrac{2\gamma\rho}{n^\circ - 1}}.$$

Thus, when $\gamma > 0$, helping remains strictly excessive at the continuous optimum. When $\gamma = 0$, the continuous optimum instead implements output-maximizing helping.

When the interior solution exists, a higher private value of the relationship increases equilibrium helping at a given team size. The organization can counter this increase by allocating employees to larger teams, where the mutual reinforcement of helping choices is weaker. For integer team sizes, the optimal choice in (37) lies among the feasible sizes immediately below and above n°, which must be compared directly. If the continuous objective instead decreases throughout the feasible range, the smallest feasible size is optimal; if it increases throughout, the largest feasible size is optimal.

Figure 3 illustrates the trade-off. Output per member peaks at $n^\circ \simeq 4.895$, while the helping gap vanishes only at $n \simeq 5.750$. Thus, further enlargement can reduce output even while helping remains excessive. The best integer size is $n = 5$, which is feasible in (37) when $5 \mid N$.

The organization can also adjust rewards. For n members, extend (22) to $u_i = (1-\lambda)y_i + \lambda Y/n + sR_i$, giving each member an equal share of team output in the team component. Equilibrium helping increases with λ until saturation (Appendix A.6). At the output-maximizing allocation, the marginal effect of each member's helping on team output is zero. The individual first-order condition then reduces to $s = (1-\lambda)\bar{s}_n$. Thus, when helping is not already excessive under individual rewards, the implementing weight is $\lambda_n^Y = 1 - s/\bar{s}_n$; otherwise, no nonnegative weight corrects excessive helping. For Figure 3, individual rewards induce excessive helping at $n = 2, \ldots, 5$, while six-member teams implement $h^Y(6)$ with a small team reward. Optimizing rewards still favors $n = 5$ when this size is feasible (Appendix A.6). Reward design and team formation therefore remain distinct instruments.

5.2 Leadership and Relationship Formation

We return to two-member teams with common ability and a common private value of the relationship. The baseline team is horizontal: both members choose their effort allocations simultaneously. We now let one member, the leader (L), commit to an allocation of effort between own-task work and helping. The follower (F) observes this allocation and then chooses their own. Since $e_i = 1 - h_i$, each allocation is fully described by h_i: the leader commits to h_L, and the follower then chooses h_F. Each retains utility $u_i = y_i + sR$, with $i \in \{L, F\}$ and $R = h_L + h_F + \rho h_L h_F$. Rewards remain individual. The leader cannot direct the follower or allocate rewards; the roles differ only in the timing of commitment. All parameters are common knowledge. Unlike the leading-by-example model of Hermalin (1998), where effort conveys private information, commitment here operates through complementarity in relationship formation.

Maintain Assumption 1, and suppose $\rho > 0$ and $0 < \gamma + s < ac/(1+\rho)$. The follower's best response is interior for every $h_L \in [0,1]$:

$$h_F(h_L) = \frac{(\gamma + s)(1 + \rho h_L)}{ac}. \tag{40}$$

An additional unit of the leader's helping raises the follower's helping by $\rho(\gamma + s)/(ac) \in (0,1)$. Anticipating this response, the leader solves

$$\max_{h_L \in [0,1]} u_L\big(h_L, h_F(h_L)\big).$$

At an interior choice, the first-order condition is

$$\begin{aligned} ach_L = {} & (\gamma + s)(1 + \rho h_F) \\ & + \frac{\rho(\gamma + s)}{ac}\big[ab(1 - h_F) + (\gamma + s)(1 + \rho h_L)\big]. \end{aligned}$$

The final term is absent under simultaneous choice. It is the benefit to the leader from the additional help and relationship benefits induced through the follower's response. This term is strictly positive, so commitment gives the leader an additional incentive to help.

Use the superscript S for subgame-perfect equilibrium outcomes under leadership. Thus $R^S = h_L^S + h_F^S + \rho h_L^S h_F^S$, while simultaneous choice yields $R^* = 2h^* + \rho(h^*)^2$. Let $u_i^* = u_i(h^*, h^*)$ denote each member's utility under simultaneous choice.

Proposition 7. *Under the maintained assumptions, the game with prior helping commitment has a unique subgame-perfect equilibrium.*

(i) Both members help more than under simultaneous choice, with

$$h_L^S > h_F^S > h^*, \qquad R^S > R^*.$$

(ii) Both members gain in utility: $u_L^S > u_L^*$ and $u_F^S > u_F^*$.

(iii) If $h^* \geq h^Y$, team output strictly falls: $Y^S < Y^*$. If $h^* < h^Y$, the output difference can have either sign.

Proof. See Appendix A.7 for the equilibrium solution, including boundary choices, and the output comparison. □

Anticipating the follower's response, the leader allocates more effort to helping and less to own-task work than under simultaneous choice. This induces the follower to help more and strengthens their social relationship. In the symmetric simultaneous-choice team, $h^* < h^Y$ also means that the relationship is less developed than at the output-maximizing allocation. Leadership can improve output in such a team. Yet it can also draw too much time away from own-task work. If relationship formation is already at or beyond the output-maximizing level, the same leadership structure strictly lowers output. The loss comes from the additional time devoted to building the relationship, which displaces own-task effort.

Both members nevertheless gain in utility, including the private benefits of their stronger relationship. Members can therefore prefer a leadership structure that lowers team output.

5.3 Differences in the Private Value of the Relationship

We now return to simultaneous choice in a two-member team and ask whether matching members with equal private relationship values necessarily maximizes output. Members retain common ability a but can differ in how much they value the relationship. Rewards remain individual ($\lambda = 0$), and the common production technology and Assumption 1 are unchanged.

5.3.1 Equilibrium Helping

Let $s_i \geq 0$ denote member i's private value of the relationship. Member i's private benefits of the relationship are then $s_i R$, and utility becomes

$$u_i = y_i + s_i R. \tag{41}$$

The production and relationship formation functions are unchanged. In particular, relationship productivity γ remains common to both members. Since s_i does not enter team output, the output-maximizing allocation remains symmetric, with helping h^Y given by (12).

Given h_j, member i maximizes (41) over $h_i \in [0, 1]$. Strict concavity is unchanged. Replacing s by s_i in the baseline best response gives

$$h_i^{\mathrm{BR}}(h_j) = \min\left\{1, \frac{(\gamma + s_i)(1 + \rho h_j)}{ac}\right\}. \tag{42}$$

Solving the two interior first-order conditions gives each member's equilibrium helping:

$$h_i^* = \frac{(\gamma + s_i)\{ac + \rho(\gamma + s_j)\}}{a^2c^2 - \rho^2(\gamma + s_1)(\gamma + s_2)}.$$

We restrict attention to parameters for which this solution has a positive denominator and both members allocate positive time to each activity:

$$a^2c^2 - \rho^2(\gamma + s_1)(\gamma + s_2) > 0, \qquad 0 < h_i^* < 1 \quad \text{for } i = 1, 2. \tag{43}$$

The best responses in (42) are capped affine functions. Their composition has Lipschitz constant at most $\rho^2(\gamma+s_1)(\gamma+s_2)/(a^2c^2) < 1$, so the equilibrium is unique even allowing boundary choices. Comparisons with a symmetric team also maintain interiority.

Subtracting the equilibrium helping levels yields

$$h_1^* - h_2^* = \frac{ac(s_1 - s_2)}{a^2c^2 - \rho^2(\gamma + s_1)(\gamma + s_2)}. \tag{44}$$

Thus, the member with the higher private value of the relationship helps more and devotes less time to their own task. The difference arises from preferences, even though the members have the same productivity in both activities.

We vary s_1 and s_2 while holding their mean s fixed:

$$s_1 + s_2 = 2s.$$

The symmetric comparison is the pair (s, s), with equilibrium helping h^* as in Lemma 1. Order the members so that $s_1 \geq s_2$. A larger difference is then represented by increasing s_1 and setting $s_2 = 2s - s_1$, while maintaining $s_i \geq 0$ and the interiority conditions.

Adding the two equilibrium helping levels and differentiating along this change gives

$$\frac{d(h_1^* + h_2^*)}{ds_1} = \frac{\rho ac\{2ac + \rho(2\gamma + s_1 + s_2)\}(s_2 - s_1)}{[a^2c^2 - \rho^2(\gamma + s_1)(\gamma + s_2)]^2}. \tag{45}$$

When $\rho > 0$, the derivative is strictly negative for $s_1 > s_2$ and zero at equality. Hence a nonzero difference lowers total helping relative to the symmetric pair. When $\rho = 0$, total helping is unchanged.

The reduction in helping comes from weaker mutual reinforcement. The product of the two interior best-response slopes is $\rho^2(\gamma + s_1)(\gamma + s_2)/(a^2c^2)$. For a given mean, the product $(\gamma + s_1)(\gamma + s_2)$ falls as the private values of the relationship become more unequal. Increasing one member's private value of the relationship while reducing the other's therefore reduces the amplification between their helping choices. This comparison changes both private values of the relationship in opposite directions. Raising only one member's private value of the relationship would instead increase that member's helping and, when $\rho > 0$, the other member's helping as well.

5.3.2 Team Output

Less total helping can improve output when a symmetric team helps excessively. However, unequal helping also creates a loss because the members share the same production technology. To separate these effects, we extend the output-loss identity in (14) to arbitrary helping choices:

$$\begin{aligned} Y^Y - Y(h_1, h_2) = (a(b + c) - 2\gamma\rho) \left(\frac{h_1 + h_2}{2} - h^Y \right)^2 \\ + \frac{a(b + c) + 2\gamma\rho}{4}(h_1 - h_2)^2. \end{aligned} \tag{46}$$

Both coefficients are positive under Assumption 1. The first term measures the loss from average helping departing from the output-maximizing level. The second measures the loss from unequal helping, holding its average fixed.

When the symmetric pair helps excessively, lower average helping can reduce the first loss, while unequal helping raises the second. Output improves only if the former effect dominates. If helping is initially insufficient, both changes instead move the allocation farther from the optimum.

Near the symmetric pair, a fixed-mean difference changes average helping only at second order, but the difference in helping at first order. Consequently, both terms in (46) can affect output at second order; Appendix A.8 derives their coefficients. An explicit sufficient condition for such a difference to raise output is

$$\begin{aligned} 2\rho\{ac + \rho(\gamma + s)\}(a(b + c) - 2\gamma\rho)\left[h^* - h^Y\right] \\ > ac(a(b + c) + 2\gamma\rho). \end{aligned} \tag{47}$$

This condition compares the gain from reducing excessive helping with the loss from unequal time allocation, evaluated at the symmetric pair. It requires $\rho > 0$ and $h^* > h^Y$, but neither requirement alone ensures that it holds. A sufficiently small difference means choosing $s_1 \neq s_2$ close to s, while maintaining their mean at s.

Proposition 8. *For comparisons satisfying* (43) *in both teams and holding* $s_1 + s_2 = 2s$ *fixed:*

(i) If $h^* \leq h^Y$*, every nonzero difference in private values of the relationship strictly lowers equilibrium team output relative to the pair* (s, s)*.*

(ii) Under (47)*, every sufficiently small nonzero difference in private values of the relationship strictly raises equilibrium team output relative to the pair* (s, s)*.*

(iii) If $\rho = 0$*, every nonzero difference in private values of the relationship strictly lowers equilibrium team output relative to the pair* (s, s)*.*

Proof. See Appendix A.8 for the loss comparison and the local output-improvement condition. □

A difference in preferences can therefore improve output by correcting excessive helping. The output-maximizing allocation remains symmetric, so the gain reflects a smaller allocation distortion, not a technological advantage from specialization. Part (ii) does not imply that large differences improve output.

Without complementarity, fixed-mean differences leave average helping unchanged. Only the loss from unequal helping remains:

$$Y(h^*, h^*) - Y(h_1^*, h_2^*) = \frac{(b + c)(s_1 - s_2)^2}{4ac^2} > 0 \tag{48}$$

for $s_1 \neq s_2$. Complementarity is therefore necessary for an output gain in this specification.

Figure 4 shows how a larger difference reduces average helping while making individual choices less equal. With $\rho = 3$, the symmetric pair helps excessively, and output initially rises, reaching a gain of approximately 1.39% at $s_1 - s_2 \simeq 0.302$. Larger differences eventually lower output below the symmetric-team level. With $\rho = 0$, every positive difference lowers output. Each curve compares output with its own symmetric-team benchmark.

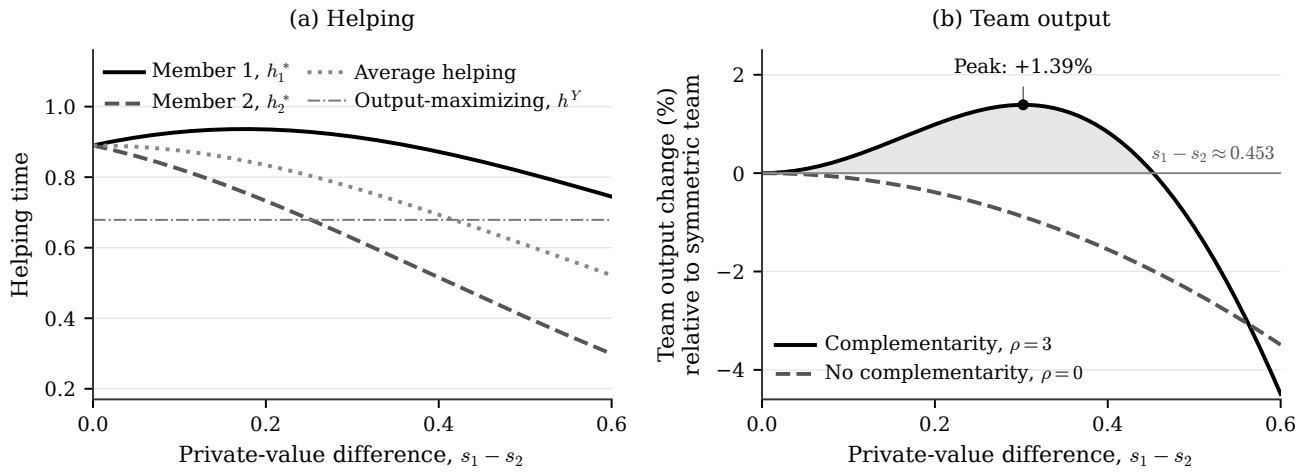


Figure 4: Differences in the private value of the relationship and team output

Notes: $a = 1$, $c = 2$, $b = 3$, $\gamma = 0.065$, $s_1 + s_2 = 0.84$, and $s_1 \geq s_2$, with $s_1 - s_2 \in [0, 0.6]$. Rewards depend on individual task output. Panel (a) uses $\rho = 3$. Panel (b) compares $\rho = 3$ with $\rho = 0$ and normalizes output by the corresponding equilibrium at $s_1 = s_2 = 0.42$. Assumption 1 and the interiority conditions in (43) hold throughout.

5.3.3 Helping and Individual Task Output

We next compare members within a team, allowing any $\rho \geq 0$ without varying their private values. Even with identical technologies, greater helping need not imply higher individual task output.

The productive benefits of the relationship γR enter both task outputs equally and cancel when their difference is taken. Using (3) and the time constraints, we obtain

$$y_1 - y_2 = -(h_1 - h_2)\left\{ab + (ac - ab)\frac{h_1 + h_2}{2}\right\}. \tag{49}$$

The expression in braces is a weighted average of the positive productivities ab and ac. We write y_i^* for member i's task output at equilibrium and retain $e_i^* = 1 - h_i^*$ for own-task effort.

Proposition 9. *At any equilibrium satisfying* (43)*,*

$$s_1 > s_2 \quad \Longrightarrow \quad h_1^* > h_2^*, \qquad e_1^* < e_2^*, \qquad y_1^* < y_2^*.$$

Proof. Equation (44) gives $h_1^* > h_2^*$, and the time constraints imply $e_1^* < e_2^*$. The positive coefficient in braces in (49) then gives $y_1^* < y_2^*$. □

The member who has the higher private value of the relationship devotes less time to their own task and receives less helping from the other member. Both effects lower that member's task output.

Individual task output therefore reflects the allocation of helping as well as productivity. A lower task output need not indicate lower ability. At the same time, because helping can be excessive for team output, a greater helping input is not by itself evidence of a more valuable contribution to the team.

6 Discussion

6.1 Managerial Implications

Reward design should account for both relationship preferences and common ability. Where the optimal mix is interior, stronger private relationship values call for less weight on team output, while higher common ability calls for more. Skill development and rewards should therefore be considered together: training that improves own-task work and direct assistance can lower output under individual rewards, but raises output when rewards are reoptimized. This comparison holds relationship productivity and preferences fixed. Once individual rewards themselves induce excessive helping, reward adjustment within the available class cannot restore the output-maximizing allocation.

Team formation and composition provide further ways to influence helping. With evenly distributed assistance, larger teams weaken mutual reinforcement and can alleviate excessive helping, although further expansion can lower output. The output-maximizing team size increases with the private value of the relationship when the optimum is interior. Moderate differences in members' private values can also improve output when a symmetric team helps excessively, holding their mean value and common ability fixed.

Leadership should be assessed by its effect on effort allocation as well as relationship formation. By committing to an effort allocation first, the leader induces more helping from the follower and strengthens social ties. This can improve output where these ties are insufficiently developed relative to the output-maximizing allocation. When relationship formation already displaces too much own-task work, leadership lowers output despite making both members better off.

Performance assessment should distinguish individual task output from contributions made through helping. Among equally able members, the person who values the relationship more helps more but records lower task output. Assessments should therefore consider assistance provided and received alongside own-task output. Helping time alone is also insufficient: when helping is excessive, a greater input need not imply a more valuable contribution.

6.2 Interpreting the Relationship–Performance Link

Relationship quality, its private value, and its productivity must be distinguished when interpreting workplace performance. Quality R is formed through helping; s measures private value per unit of quality, and γ measures its contribution to production. The inverted-U pattern under individual rewards compares equilibria as s varies, holding other conditions fixed. It does not imply an unconditional inverted-U association between observed closeness and performance across workplaces with different technologies or rewards. Such comparisons require attention to the preferences and incentives generating the observed relationships.

Examining the productivity reversal requires observing time allocation as well as output. A laboratory experiment could give pairs fixed time budgets for own tasks and helping, with an explicit production technology defining the output-maximizing allocation. A treatment could raise relationship productivity while holding other production parameters fixed. Comparing output under the two technologies at the same time allocation would measure the direct gain. Under the new technology, comparing output at that allocation with output after participants adjust their choices would measure

the reallocation effect. A negative overall response could then be distinguished from an absence of productive relationship benefits.

Proposition 9 also predicts that, under individual rewards and with equal abilities, the member who values the relationship more helps more but has lower task output. Researchers could combine records of helping from work logs or code reviews with surveys of private relationship values. Among similarly skilled pairs working under individual rewards, researchers could examine whether values, helping time, and task output align with this prediction, accounting for task characteristics and workload.

6.3 Scope and Future Research

Appendix D adds a share of private relationship benefits to the organization's objective. For a positive range of these shares, the optimized objective can still fall as relationship productivity rises. When all private benefits are included, the objective equals aggregate member utility, whose optimized value increases with relationship productivity.

Appendix B allows receiving help to consume time. These recipient time costs reduce symmetric equilibrium helping and can reverse strategic complementarity; they can also raise output by reducing excessive helping. The analysis retains individual rewards and the two-member technology.

Two further extensions concern relationship formation. Allowing members to choose recipients could produce concentrated helping networks and alter the team-size results, which rely on even distribution. Modeling relationship quality as a stock would distinguish time spent building relationships from time spent maintaining them, and would permit an analysis of team duration and rotation alongside learning and handover costs.

7 Conclusion

Workplace relationships can raise output at fixed effort allocations yet reduce it once members adjust their helping choices. Helping advances colleagues' tasks and builds a relationship with both productive and private benefits. Those private benefits can correct insufficient helping but also induce excessive helping. We characterize when an increase in relationship productivity reduces equilibrium output despite its direct productive benefit.

The same incentives also affect how a fixed workforce should be organized into teams. When members distribute helping equally among colleagues, expanding a team weakens the mutual reinforcement of helping choices. Excessive helping in small teams may then give way to insufficient helping in larger ones. An interior optimal team size can arise, and it increases with the private value of the relationship under the conditions identified in the model. Leadership through prior commitment to an effort allocation strengthens the social relationship and benefits both members, but lowers output when relationship formation already takes too much time from own-task work.

The private benefits of the relationship and team rewards provide substitute incentives for helping. Differences in private relationship values can also mitigate excessive helping. Higher common ability calls for more weight on team rewards when the optimal mix is interior. With rewards optimized, improvements in common ability strictly raise team output. Yet reward optimization need not eliminate excessive helping or the output decline caused by greater relationship productivity.

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Appendix

A Proofs

A.1 Proof of Lemma 1

Proof.
Expected utility against any distribution of the other member's helping is strictly concave in one's own helping. A best response is therefore unique, so any Nash equilibrium is in pure strategies. If $\gamma + s = 0$, equation (5) immediately gives $h_1 = h_2 = 0$. Suppose $\gamma + s > 0$. Both best responses are then strictly positive.

If both members choose less than one, subtracting their first-order conditions gives

$$[ac + \rho(\gamma + s)](h_1 - h_2) = 0.$$

Hence their helping choices coincide. An equilibrium in which only one member chooses one cannot exist. To see this, suppose $h_1 = 1$ and $h_2 < 1$. Since the two best-response functions are identical and nondecreasing, member 1's best response to $h_2 < 1$ is no greater than member 2's best response to $h_1 = 1$, which is $h_2 < 1$. This contradicts $h_1 = 1$. Thus every equilibrium is symmetric.

For common helping h, the equilibrium condition is

$$h = \min\left\{1, \frac{(\gamma + s)(1 + \rho h)}{ac}\right\}.$$

If $(\gamma + s)(1 + \rho) < ac$, the slope of the linear expression on the right is less than one. Its unique fixed point is $h = (\gamma+s)/[ac-\rho(\gamma+s)] < 1$. If $(\gamma+s)(1+\rho) \geq ac$, consider the difference between that linear expression and h. This difference is positive at $h = 0$ and nonnegative at $h = 1$. Because it is affine, it is strictly positive at every $h < 1$. Consequently, the only fixed point of the capped best response is $h = 1$. These cases give (7) and the stated condition for an interior equilibrium. □

A.2 Proof of Proposition 2

Proof.
For part (i), both terms in (16) are nonnegative when $h^* \leq h^Y$. If $h^* > 0$, the first is strictly positive. If $h^* = 0$, then $\gamma = s = 0$, and the second equals $2b/c > 0$.

For part (ii), fix a, c, b, γ, ρ and let s approach the saturation threshold at this value of γ. At each (s, γ), the derivative with respect to γ holds s fixed. Then $h^* \to 1$, $R^* \to 2 + \rho$, and $\partial h^*/\partial\gamma \to (1 + \rho)^2/(ac)$. Equation (16) gives

$$\begin{aligned} \lim_{s\uparrow s_{\text{sat}}} \frac{dY^*}{d\gamma} &= 2(2 + \rho) + \frac{2(1 + \rho)^2}{ac}[2\gamma(1 + \rho) - ac] \\ &= 2 - 2\rho - 2\rho^2 + \frac{4\gamma(1 + \rho)^3}{ac}. \end{aligned} \tag{A.1}$$

The limit in (A.1) is negative under (18). Continuity then gives a nonempty interval below s_{sat} on

which $dY^*/d\gamma < 0$. The upper bound on γ in (18) is below $ac/[2(1+\rho)]$, so Assumption 1 and $s_{\text{sat}} > 0$ hold throughout this construction.

For part (iii), when $\rho = 0$ and $s < s_{\text{sat}}$, equilibrium helping is $h^* = (\gamma + s)/(ac)$. Substitution into the total derivative of (13) gives

$$\frac{dY^*}{d\gamma} = \frac{2}{ac}\big[ab(1-h^*) + ach^* + 2\gamma\big] > 0.$$

The inequality follows from $b, c > 0$ and $0 \le h^* < 1$. □

A.3 Proof of Proposition 3

Proof.
Part (i) follows from (19) and $dR^*/da = 2(1+\rho h^*)\,\partial h^*/\partial a$. For part (ii), the direct term in (21) is strictly positive. When $h^* \ge h^Y$, the reallocation term is nonnegative, so output strictly increases.

For a negative derivative, take $(a, c, b, \gamma, \rho, s) = (1, 2, 5, 1/20, 5, 1/5)$. Assumption 1 holds strictly, and $h^* = 1/3 < h^Y = 51/65$. At this point,

$$\frac{\partial h^*}{\partial a} = -\frac{8}{9}, \qquad \left.\frac{\partial Y}{\partial h_i}\right|_{h_1 = h_2 = h^*} = \frac{44}{15}, \qquad c[1-(h^*)^2] + b[2h^* - (h^*)^2] = \frac{41}{9}.$$

Equation (21) therefore gives

$$\frac{dY^*}{da} = \frac{41}{9} + 2\left(\frac{44}{15}\right)\left(-\frac{8}{9}\right) = -\frac{89}{135} < 0.$$

All maintained inequalities and the output decline are strict and persist in a neighborhood, establishing a nonempty open set.

For part (iii), with $\rho = 0$, $h^* = (\gamma + s)/(ac)$ and $\partial h^*/\partial a = -h^*/a$. Substitution into (21) yields

$$\frac{dY^*}{da} = c + (b+c)(h^*)^2 - \frac{4\gamma h^*}{a} \ge c(1-h^*)^2 + b(h^*)^2 > 0,$$

where the weak inequality follows from $ac > 2\gamma$ and $h^* \ge 0$. The final inequality holds for every $h^* \in [0,1]$, since $b, c > 0$. If $\gamma + s = 0$, helping remains zero and $dY^*/da = c > 0$ directly. □

A.4 Equilibrium under Mixed Rewards

Each member's utility has second derivative $-[(1-\lambda/2)ac + (\lambda/2)ab] < 0$. As in Appendix A.1, strict concavity rules out nondegenerate mixed strategies, and identical nondecreasing best responses rule out an equilibrium with only one member at the upper bound. Subtracting the interior first-order conditions gives

$$[(1-\lambda/2)ac + (\lambda/2)ab + \rho(\gamma+s)]\,(h_1 - h_2) = 0,$$

so every interior equilibrium is symmetric. Unless $\lambda = \gamma = s = 0$, both best responses are strictly positive. The capped affine fixed-point argument in Appendix A.1, applied to (24), then gives (25) when $(\gamma+s)(1+\rho) < (1-\lambda/2)ac$, and $h = 1$ otherwise. When $\lambda = \gamma = s = 0$, the unique equilibrium

is $h = 0$.

A.5 Proof of Proposition 5

Proof. With received helping H_i as defined in (30), the best response is

$$h_i^{\mathrm{BR}} = \min\left\{1, \frac{\gamma + s}{ac}\left(1 + \frac{\rho H_i}{n-1}\right)\right\}.$$

Under (32), the upper bound is slack for all $H_i \in [0,1]$, and each best response is strictly positive. Moreover, changing opponents' helping profiles changes a best response by at most $\rho(\gamma+s)/[ac(n-1)]$ times their maximum change. This coefficient is smaller than one. The best-response map is therefore a contraction, and the game has a unique equilibrium. Symmetry yields (33).

The Hessian of total output has diagonal entries $-ac - ab/(n-1)$ and off-diagonal entries $[2\gamma\rho - ab(n-2)]/(n-1)^2$. Its eigenvalues are

$$-a(b+c) + \frac{2\gamma\rho}{n-1}, \qquad -ac - \frac{ab + 2\gamma\rho}{(n-1)^2}.$$

The second has multiplicity $n-1$. Both are negative under Assumption 1. The symmetric interior stationary point in (35) is therefore the unique output-maximizing allocation, even when the organization is not restricted to symmetric time choices.

For $\rho > 0$, differentiation of (33) yields

$$\frac{dh^*(n)}{dn} = -\frac{\rho(\gamma+s)^2}{[ac(n-1) - \rho(\gamma+s)]^2} < 0.$$

The equality $h^*(n) = h^Y(n)$ is equivalent to $s = \bar{s}_n$, where $\bar{s}_n$ is given by (36). Further,

$$\frac{d\bar{s}_n}{dn} = \frac{a^2bc\rho(ab+2\gamma)}{(n-1)^2\left[a(b+c) + \dfrac{ab\rho}{n-1}\right]^2} > 0.$$

Since $h^*(n)$ is increasing in s and $h^Y(n)$ does not depend on s, the stated excessive and insufficient helping comparisons follow. □

A.6 Reward Adjustment in n-Member Teams

Under the equal-distribution rule, member i has utility $u_i = (1-\lambda)y_i + \lambda Y/n + sR_i$, where $\lambda \in [0,1]$. Material rewards sum to Y. The equilibrium is unique and symmetric.[5] When interior, the member's first-order condition is

$$\left(1 - \lambda + \frac{\lambda}{n}\right) ach = \frac{\lambda ab}{n}(1-h) + \left[s + \gamma\left(1 - \lambda + \frac{2\lambda}{n}\right)\right]\left(1 + \frac{\rho h}{n-1}\right). \tag{A.2}$$

[5] Strict concavity in own helping rules out mixed strategies. Marginal-payoff differences have the opposite sign to helping differences, ruling out asymmetry even at boundaries. The symmetric marginal payoff is affine with a positive intercept, giving a unique constrained solution.

Under this reward scheme, the symmetric helping level is nondecreasing in λ (strictly increasing before saturation). For an interior equilibrium, this follows by implicitly differentiating (A.2): the denominator is positive, and the sign of the helping response equals the sign of

$$(n-1)ach + ab(1-h) - \gamma(n-2)\left(1 + \frac{\rho h}{n-1}\right),$$

which is positive after substitution of the equilibrium condition. Team rewards thus cannot reduce excessive helping at a given n.

Substituting $h = h^Y(n)$ into (A.2) and using the output-maximizing condition

$$ach^Y(n) = ab[1 - h^Y(n)] + 2\gamma\left(1 + \frac{\rho h^Y(n)}{n-1}\right)$$

gives $\lambda = 1 - s/\bar{s}_n$. Hence, for $s \leq \bar{s}_n$, the output-maximizing allocation is implemented by a feasible reward weight; for $s > \bar{s}_n$, individual rewards ($\lambda = 0$) are output-maximizing within this reward class, but helping remains excessive.

For the parameters in Figure 3, $\bar{s}_5 \simeq 0.883$ and $\bar{s}_6 \simeq 0.905$. Thus $s = 0.9 > \bar{s}_5$ yields the optimal reward weight $\lambda = 0$ in five-member teams, with output per member approximately 1.5545. In six-member teams, $s < \bar{s}_6$, and $\lambda_6^Y = 1 - 0.9/0.9045 \simeq 0.005$ implements $h^Y(6)$, yielding output per member approximately 1.5540. For every $n \geq 6$, $\bar{s}_n \geq \bar{s}_6$, so the organization can implement the output-maximizing allocation. Its output per member is

$$\frac{Y^Y(n)}{n} = \frac{ac}{2} + \frac{(ab+2\gamma)^2}{2\left(a(b+c) - \dfrac{2\gamma\rho}{n-1}\right)},$$

which decreases strictly with n for $\gamma\rho > 0$. Hence, even after the reward weight is optimized for each integer team size, output per member is highest at $n = 5$. This is also the optimum in (37) whenever $5 \mid N$.

A.7 Proof of Proposition 7

Proof. Substituting (40) into the leader's utility gives

$$\frac{ac}{2}(1 - h_L^2) + ab\left[h_F(h_L) - \frac{h_F(h_L)^2}{2}\right] + (\gamma + s)\big[h_L + h_F(h_L) + \rho h_L h_F(h_L)\big].$$

The leader's objective has a strictly positive derivative at zero and second derivative

$$-\frac{a^2c^3 - \rho^2(2c-b)(\gamma+s)^2}{ac^2}.$$

If $a^2c^3 > \rho^2(2c-b)(\gamma+s)^2$, it is strictly concave. The equilibrium helping choices are then

$$h_L^S = \min\left\{1, \frac{(\gamma+s)\big[ac(c+\rho b)+\rho(\gamma+s)(2c-b)\big]}{a^2c^3-\rho^2(2c-b)(\gamma+s)^2}\right\},$$
$$h_F^S = \frac{(\gamma+s)(1+\rho h_L^S)}{ac}.$$

If $a^2c^3 \le \rho^2(2c-b)(\gamma+s)^2$, the objective is strictly increasing on $[0,1]$, so instead $h_L^S = 1$ and $h_F^S = (\gamma+s)(1+\rho)/(ac)$. At the simultaneous-choice allocation, the derivative of the leader's utility along the follower's best response is

$$\left.\frac{du_L(h_L, h_F(h_L))}{dh_L}\right|_{h_L=h^*} = \frac{\rho(\gamma+s)}{ac}\big[ab(1-h^*)+ach^*\big] > 0. \tag{A.3}$$

In the strictly concave case, the positive derivative at h^* implies that the unique constrained maximizer exceeds h^*. In the remaining cases the objective is strictly increasing and $h_L^S = 1 > h^*$. Since the follower's best response has slope in $(0,1)$ and crosses the diagonal at h^*, it follows that $h_L^S > h_F^S > h^*$. Relationship quality is strictly increasing in each member's helping, so $R^S > R^*$.

For part (ii), choosing $h_L = h^*$ remains feasible and induces $h_F = h^*$. The strictly positive derivative in (A.3) therefore gives $u_L^S > u_L^*$. For the follower, the envelope theorem gives

$$\frac{du_F(h_F(h_L), h_L)}{dh_L} = ab(1-h_L) + (\gamma+s)\big(1+\rho h_F(h_L)\big) > 0,$$

where the first argument of u_F is the follower's own helping. Hence $u_F^S > u_F^*$.

For part (iii), expanding (9) around (h^*, h^*) gives

$$\begin{aligned} Y^S - Y^* = {} & [a(b+c) - 2\gamma\rho](h^Y - h^*)\big[(h_L^S - h^*) + (h_F^S - h^*)\big] \\ & - \frac{a(b+c)}{2}\big[(h_L^S - h^*)^2 + (h_F^S - h^*)^2\big] \\ & + 2\gamma\rho(h_L^S - h^*)(h_F^S - h^*). \end{aligned}$$

The last two terms together are strictly negative by the strict concavity of output under Assumption 1. Since both members increase helping, $h^* \ge h^Y$ also makes the first term nonpositive. Leadership through helping commitment therefore strictly reduces team output in this case.

Both signs occur when $h^* < h^Y$. To establish this, set $\gamma = 0$ and fix $a, b, c, \rho > 0$. As $s \downarrow 0$, the equilibrium formulas imply

$$\lim_{s\downarrow 0} \frac{Y^S - Y^*}{s} = \frac{\rho b^2}{c^2} > 0.$$

Thus leadership raises output for sufficiently small positive s, when $h^* < h^Y$. At $s = \bar{s} = abc/[c+b(1+\rho)]$, helping is interior and $h^* = h^Y$, so output strictly falls. Equilibrium choices and output are continuous in s, including transitions to $h_L^S = 1$. Output therefore also falls for s just below $\bar{s}$, where $h^* < h^Y$. Both strict signs persist for sufficiently small positive γ. □

A.8 Proof of Proposition 8

Proof.
For part (i), equations (44) and (45) imply that any nonzero difference in private values of the relationship produces unequal helping and average helping no greater than h^*. If $h^* \leq h^Y$, the first loss in (46) cannot be smaller than for the symmetric pair. The second loss is strictly positive, whereas it is zero for the symmetric pair. Team output is therefore strictly lower. The loss identity follows by substituting (9) and (12) and completing the square in the average and difference of helping.

For part (ii), consider equilibrium output $Y(h_1^*, h_2^*)$ as s_1 varies with $s_2 = 2s - s_1$, holding the mean s and the production parameters fixed. At $s_1 = s$, output equals $Y(h^*, h^*)$. This symmetric-team benchmark is constant along the comparison. Symmetry implies

$$\left.\frac{dY(h_1^*, h_2^*)}{ds_1}\right|_{s_1=s} = 0.$$

The equilibrium formulas give the following derivatives at the symmetric pair:

$$\left.\frac{d^2[(h_1^* + h_2^*)/2]}{ds_1^2}\right|_{s_1=s} = -\frac{2\rho ac\{ac + \rho(\gamma + s)\}}{[a^2c^2 - \rho^2(\gamma + s)^2]^2},$$
$$\left.\frac{d(h_1^* - h_2^*)}{ds_1}\right|_{s_1=s} = \frac{2ac}{a^2c^2 - \rho^2(\gamma + s)^2}.$$

Average helping has zero first derivative at this pair, and the two helping levels coincide. Differentiating (46) twice therefore yields

$$\left.\frac{d^2Y(h_1^*, h_2^*)}{ds_1^2}\right|_{s_1=s} = \frac{4\rho ac\{ac + \rho(\gamma + s)\}(a(b + c) - 2\gamma\rho)\left[h^* - h^Y\right]}{[a^2c^2 - \rho^2(\gamma + s)^2]^2} - \frac{2(a(b + c) + 2\gamma\rho)a^2c^2}{[a^2c^2 - \rho^2(\gamma + s)^2]^2}.$$

This expression is strictly positive under (47). The symmetric pair is then a strict local minimum of equilibrium output along the fixed-mean comparison. All sufficiently small nonzero differences therefore raise output. The strict condition also implies $s > \bar{s} > 0$, so both private values of the relationship remain nonnegative for small changes. Interiority is preserved in a neighborhood because the maintained inequalities are strict.

For part (iii), $\rho = 0$ gives $h_i^* = (\gamma + s_i)/(ac)$. Average helping is unchanged, and substituting $h_1^* - h_2^* = (s_1 - s_2)/(ac)$ into (46) yields the strictly positive loss in (48). □

B Recipient Time Costs

The baseline assumes that receiving assistance does not consume the recipient's time. Suppose instead that each unit of helping supplied by member j requires μ units of member i's time, where $0 \leq \mu \leq 1$. We retain individual rewards and the two-member relationship technology. The time constraint becomes

$$e_i + h_i + \mu h_j = 1, \qquad e_i, h_i \geq 0.$$

The term μh_j represents time spent explaining one's task to the helper, discussing feedback, or clarifying the advice received. A similar bilateral time cost arises in the knowledge-transfer model of Hattori (2026), where transferring knowledge takes productive time from both the sender and the recipient. We take μ as given; a larger value need not mean that the help itself is less productive. The output functions remain $y_i = ac\left(e_i - \frac{e_i^2}{2}\right) + ab\left(h_j - \frac{h_j^2}{2}\right) + \gamma R$, with $R = h_1 + h_2 + \rho h_1 h_2$, and utility remains $u_i = y_i + sR$. The baseline is recovered at $\mu = 0$.

The time constraint now links the members' feasible allocations. Given h_j, member i can choose $h_i \in [0, 1 - \mu h_j]$. We analyze interior equilibria with $e_i, h_i > 0$. Member i's interior first-order condition is

$$ac(h_i + \mu h_j) = (\gamma + s)(1 + \rho h_j). \tag{B.1}$$

Its interior best-response slope is

$$\frac{dh_i^{\mathrm{BR}}}{dh_j} = \frac{\rho(\gamma + s)}{ac} - \mu.$$

Relationship formation still has complementarity $\rho \geq 0$. Nevertheless, helping choices become strategic substitutes when $\mu > \rho(\gamma + s)/(ac)$. Receiving more help then reduces the time available for own-task effort sufficiently to lower the recipient's desired helping.

Writing $h^*(\mu)$ for a symmetric interior equilibrium, equation (B.1) gives

$$h^*(\mu) = \frac{\gamma + s}{ac(1 + \mu) - \rho(\gamma + s)}, \qquad e^*(\mu) = 1 - (1 + \mu)h^*(\mu). \tag{B.2}$$

We restrict attention to $\gamma + s > 0$ and

$$(\gamma + s)(1 + \mu + \rho) < ac(1 + \mu), \tag{B.3}$$

so the displayed symmetric allocation is strictly interior.[6] Within this region,

$$\frac{\partial h^*}{\partial \mu} = -\frac{ac(\gamma + s)}{[ac(1 + \mu) - \rho(\gamma + s)]^2} < 0, \qquad \frac{\partial e^*}{\partial \mu} = \frac{\rho(\gamma + s)^2}{[ac(1 + \mu) - \rho(\gamma + s)]^2} \geq 0.$$

Thus the additional time required to receive help induces both members to supply less help. If $\rho > 0$, their adjustment is strong enough that the time devoted to their own tasks increases, despite the direct loss of time when help is received.

For a symmetric allocation $h_1 = h_2 = h$, the resulting output is

$$Y(h, h; \mu) = ac + 2(ab + 2\gamma)h - [ab + ac(1 + \mu)^2 - 2\gamma\rho]h^2. \tag{B.4}$$

For arbitrary feasible helping choices, the output function is

$$\begin{aligned} Y(h_1, h_2; \mu) = ac + (ab + 2\gamma)(h_1 + h_2) - \frac{ab + ac(1 + \mu^2)}{2}(h_1^2 + h_2^2) \\ + (2\gamma\rho - 2ac\mu)h_1 h_2. \end{aligned}$$

[6] At $\mu = 1$ and $\rho = 0$, any pair with $h_1, h_2 > 0$ and $h_1 + h_2 = (\gamma + s)/(ac)$ is an interior equilibrium under these restrictions. The comparisons below concern the symmetric equilibrium in (B.2).

Its Hessian has eigenvalues $-[ab+ac(1+\mu)^2-2\gamma\rho]$ and $-[ab+ac(1-\mu)^2+2\gamma\rho]$, both negative under Assumption 1. Consequently, any feasible interior output-maximizing allocation is uniquely determined and symmetric, with common helping

$$h^Y(\mu)=\frac{ab+2\gamma}{ab+ac(1+\mu)^2-2\gamma\rho}. \tag{B.5}$$

We use this expression only where it is feasible and strictly interior. Equating (B.2) and (B.5), the private value that implements output-maximizing helping is

$$\bar{s}(\mu)=\frac{c(1+\mu)(ab+2\gamma)}{c(1+\mu)^2+b(1+\rho)}-\gamma.$$

The threshold need not be monotone in the recipient's time cost. In particular,

$$\operatorname{sign}\left(\frac{d\bar{s}}{d\mu}\right)=\operatorname{sign}\left[ab(1+\rho)-ac(1+\mu)^2\right].$$

The decrease in equilibrium helping can therefore alleviate excessive helping, but the output-maximizing level also falls with μ.

To separate these effects, let $Y^*(\mu)=Y(h^*(\mu),h^*(\mu);\mu)$. Differentiating (B.4) gives

$$\frac{dY^*}{d\mu}=\underbrace{-2ac(1+\mu)(h^*)^2}_{\text{direct time cost}}+\underbrace{2[ab+ac(1+\mu)^2-2\gamma\rho](h^Y-h^*)\frac{\partial h^*}{\partial\mu}}_{\text{effect of the change in time allocation}}.$$

The first term is negative. The second is positive if helping is initially excessive. For example, take $a=1$, $c=2$, $b=3$, $\gamma=0.065$, $\rho=3$, and $s=0.42$. Equilibrium output is approximately 3.920 at $\mu=0$, 3.985 at $\mu=0.05$, and 3.944 at $\mu=0.10$. All these equilibria satisfy (B.3). The additional time cost initially raises output because it reduces excessive helping. This is a comparison of equilibrium allocations, not a technological benefit of recipient time costs: at fixed helping, output strictly decreases as μ increases.

C General Relationship and Production Technologies

This appendix considers distinct, concave production technologies for own-task work and helping. Members retain $e_i=1-h_i$, but now

$$R=R(h_1,h_2),\qquad y_i=f(1-h_i)+g(h_j)+\gamma R(h_1,h_2),\quad i\neq j.$$

Here f and g are own-task and helping production functions, respectively, incorporating a fixed common ability. Under individual rewards, $u_i=y_i+sR$, with $s,\gamma\geq 0$. Team output is $Y=y_1+y_2$ and excludes private relationship benefits. The baseline is the case $f(x)=ac(x-x^2/2)$ and $g(x)=ab(x-x^2/2)$.

Throughout this appendix, $\bar{s}$ and s_{sat} denote the corresponding thresholds under the general technologies. They reduce to (15) and (6), respectively, in the baseline specification.

For derivatives of the relationship function, write $\partial_i R=\partial R/\partial h_i$ and $\partial_{ij}R=\partial^2 R/(\partial h_i\partial h_j)$ for

$i, j \in \{1, 2\}$.

We impose the following regularity conditions. The functions f, g are twice continuously differentiable and concave on $[0, 1]$, with $f'(x) > 0$ for $x < 1$, $f'(1) = 0$, $f''(x) < 0$, $g'(x) > 0$ for $x < 1$, and $g''(x) \leq 0$. The condition $f'(1) = 0$ means that diverting a small amount of time from full own-task effort has no first-order output cost; it prevents a zero-helping region when $\gamma + s > 0$. The relationship function R is twice continuously differentiable, symmetric, increasing, and supermodular, with $R(0, 0) = 0$, $\partial_1 R, \partial_2 R > 0$, and $\partial_{12} R \geq 0$. Team output Y is strictly concave on $[0, 1]^2$ and has an interior maximizer. For the values of s and $\lambda \in [0, 1]$ considered, the game has a unique symmetric equilibrium, and each interior symmetric first-order condition crosses zero with strictly negative slope. Supermodularity alone does not ensure these properties.

For example, equilibrium uniqueness follows if, for all feasible helping pairs and reward weights in a bounded parameter region,

$$-\left(1 - \frac{\lambda}{2}\right) f''(1 - h_i) - \frac{\lambda}{2} g''(h_i) - (\gamma + s)\partial_{11} R(h_i, h_j) > (\gamma + s)\partial_{12} R(h_i, h_j). \tag{C.1}$$

Then each capped best response has a slope below one, and the joint best-response map is a contraction. Strict concavity of Y follows, for example, from diagonal dominance of its negative Hessian. We use the stated uniqueness and curvature requirements; (C.1) is sufficient, not necessary.

Symmetry of the relationship technology gives $\partial_1 R(h, h) = \partial_2 R(h, h) > 0$. The unique interior output-maximizing allocation is symmetric by strict concavity and satisfies

$$f'(1 - h^Y) = g'(h^Y) + 2\gamma \partial_1 R(h^Y, h^Y). \tag{C.2}$$

Assume $\gamma \partial_1 R(1, 1) < f'(0)$. Then the individual-reward saturation threshold is positive:

$$s_{\text{sat}} \equiv \frac{f'(0)}{\partial_1 R(1, 1)} - \gamma > 0. \tag{C.3}$$

C.1 The inverted-U relationship

Under individual rewards, the symmetric interior equilibrium solves

$$f'(1 - h^*) = (\gamma + s)\partial_1 R(h^*, h^*). \tag{C.4}$$

The implicit-function theorem gives

$$\frac{\partial h^*}{\partial s} = \frac{\partial_1 R(h^*, h^*)}{-f''(1 - h^*) - (\gamma + s)[\partial_{11} R(h^*, h^*) + \partial_{12} R(h^*, h^*)]} > 0. \tag{C.5}$$

The denominator is positive because the symmetric marginal payoff crosses zero with negative slope. Condition (C.1) at $\lambda = 0$ is sufficient for this property.

Proposition C.1. *Under the stated regularity conditions, let*

$$\bar{s} \equiv \gamma + \frac{g'(h^Y)}{\partial_1 R(h^Y, h^Y)}. \tag{C.6}$$

Then $0 < \bar{s} < s_{\text{sat}}$. For $0 \leq s < \bar{s}$, equilibrium helping is below h^Y and team output strictly increases with s; at $s = \bar{s}$, equilibrium helping equals h^Y and output is maximized; for $\bar{s} < s < s_{\text{sat}}$, helping is excessive and output strictly decreases with s. At and above saturation, equilibrium helping and output are constant in s.

Proof. At h^Y, equation (C.2) gives the individual marginal payoff $-g'(h^Y) - \gamma\partial_1 R(h^Y, h^Y) < 0$ when $s = 0$. The unique stable individual-reward equilibrium at $s = 0$ therefore lies below h^Y. By (C.5), equilibrium helping is strictly increasing before saturation; by (C.3), it reaches one at s_{sat}. Substituting (C.2) into (C.4) at $h^* = h^Y$ gives (C.6), so the crossing occurs once and strictly before saturation. Along symmetric allocations,

$$\frac{dY(h,h)}{dh} = 2\left[-f'(1-h) + g'(h) + 2\gamma\partial_1 R(h,h)\right]. \tag{C.7}$$

Strict concavity of team output and its unique interior symmetric maximizer imply that this derivative is positive below h^Y and negative above it. Combining this fact with (C.5) proves the stated comparisons. Helping is fixed at one for $s \geq s_{\text{sat}}$, and hence output is constant in s. □

C.2 Optimal individual and team rewards

Consider the same reward mixture as in Section 4, $u_i = (1-\lambda)y_i + \lambda Y/2 + sR$, $\lambda \in [0,1]$. An interior symmetric equilibrium satisfies

$$\left(1 - \frac{\lambda}{2}\right) f'(1-h^*) = \frac{\lambda}{2} g'(h^*) + (\gamma + s)\partial_1 R(h^*, h^*). \tag{C.8}$$

The marginal utility of helping is the right-hand side minus the left-hand side of (C.8). Its derivative with respect to λ is $[f'(1-h) + g'(h)]/2 > 0$. Hence the unique equilibrium helping level increases with λ when interior and is nondecreasing otherwise.

Proposition C.2. *Under the stated regularity conditions, the output-maximizing reward weight satisfies*

$$\lambda^Y(s) = \max\left\{0, 1 - \frac{s}{\bar{s}}\right\} \qquad (0 \leq s < s_{\text{sat}}).$$

It implements the output-maximizing helping level if and only if $s \leq \bar{s}$. For $s \geq s_{\text{sat}}$, all reward weights induce saturation and hence the same output.

Proof. Insert the output-maximizing condition (C.2) into the member's condition (C.8), evaluated at h^Y. The expression simplifies to

$$0 = (\lambda - 1)g'(h^Y) + [s - \gamma + \lambda\gamma]\partial_1 R(h^Y, h^Y).$$

Solving yields $\lambda = 1 - s/[\gamma + g'(h^Y)/\partial_1 R(h^Y, h^Y)] = 1 - s/\bar{s}$. For $0 \leq s \leq \bar{s}$, this weight is feasible and implements the unique output-maximizing allocation. For $\bar{s} < s < s_{\text{sat}}$, equilibrium helping is already excessive at $\lambda = 0$ by Proposition C.1. Increasing λ only increases helping and cannot improve output. Thus the unique optimal feasible weight is zero. When $s \geq s_{\text{sat}}$, helping is saturated at $\lambda = 0$ and cannot rise further; all weights induce the same allocation. □

The linear adjustment rule is therefore not a special algebraic property of quadratic production. It follows from the equal division of team output under team rewards and the fact that productive and private benefits are generated by the same relationship R.

C.3 Relationship productivity and output

The productivity reversal requires a more specific comparison. At an interior equilibrium, γ and s enter (C.4) through their sum, so

$$\frac{\partial h^*}{\partial \gamma} = \frac{\partial h^*}{\partial s} > 0,$$

with the response given by (C.5). Combining this response with the output slope (C.7), the total effect is

$$\begin{aligned} \frac{dY^*}{d\gamma} &= 2R(h^*, h^*) \\ &\quad + 2\big[-f'(1-h^*) + g'(h^*) + 2\gamma \partial_1 R(h^*, h^*)\big] \frac{\partial h^*}{\partial \gamma}. \end{aligned} \tag{C.9}$$

Proposition C.3. *At a stable, interior equilibrium, an increase in relationship productivity reduces team output if and only if*

$$\big[f'(1-h^*) - g'(h^*) - 2\gamma \partial_1 R(h^*, h^*)\big] \frac{\partial h^*}{\partial \gamma} > R(h^*, h^*). \tag{C.10}$$

In particular, excessive helping is necessary, but not sufficient, for the productivity reversal.

Proof. Rearranging (C.9) gives (C.10). Since $\partial h^*/\partial \gamma > 0$ and $R(h^*, h^*) \geq 0$, the inequality requires the expression in brackets to be positive. By (C.7) and strict concavity of output, this places h^* strictly above h^Y. □

Multiplying both sides of (C.10) by two compares the output loss from the induced increase in helping with the direct productivity gain. Output falls when the former exceeds the latter.

Supermodularity does not by itself ensure that the inequality is satisfied. Nor is strict complementarity in relationship formation necessary in the general class. To see the latter point, let $R(h_1, h_2) = h_1 + h_2$, $f(x) = g(x) = 2(x - 0.05x^2 - 0.3x^3)$, and $\gamma = 0$. Both functions are increasing and strictly concave on $[0, 1]$, with $f'(1) = 0$, and $\partial_{12} R = 0$. At $s = 1.95$, the unique interior equilibrium is approximately $h^* = 0.880$, and (C.9) gives the right derivative $dY^*/d\gamma \simeq -1.286$ at $\gamma = 0$. Consequently, Proposition 2(iii) is specific to the baseline quadratic production function; complementarity is an amplification channel rather than a general necessary condition for reversal.

D Partial Internalization of Private Relationship Benefits

Suppose the organization's objective includes a share $\alpha \in [0, 1]$ of members' private relationship benefits:

$$V_\alpha(h_1, h_2) = Y(h_1, h_2) + 2\alpha s R(h_1, h_2). \tag{D.1}$$

At $\alpha = 0$, the objective is team output; at $\alpha = 1$, it is aggregate member utility W. One possible interpretation is partial capture of private relationship benefits through compensating wage adjustments.

We treat α as an exogenous weight and do not model wage determination or participation constraints. Only the organizational criterion changes; members' payoffs remain as in Sections 3 and 4.

D.1 Helping relative to the organizational objective

Consider a nonsaturated equilibrium under individual rewards ($\lambda = 0$), with helping h^* as in (7). At the symmetric allocation, the individual first-order condition gives $ach^* = (\gamma+s)(1+\rho h^*)$. Substitution into the marginal organizational objective yields

$$\left.\frac{\partial V_\alpha}{\partial h_i}\right|_{h_1=h_2=h^*} = ab(1-h^*) + [\gamma - (1-2\alpha)s](1+\rho h^*). \tag{D.2}$$

If $\alpha \geq 1/2$, the marginal value is strictly positive, so individual rewards cannot induce marginally excessive helping relative to V_α. Excessive helping requires both $\alpha < 1/2$ and a sufficiently large s. The restriction $\alpha < 1/2$ is also necessary, but not sufficient, for V_α to fall as γ rises.

D.2 Relationship productivity and reward adjustment

Write $V_\alpha^*(\lambda, s)$ for the value of (D.1) at the symmetric equilibrium under reward weight λ, and let $\widehat{V}_\alpha(s) = \max_{\lambda\in[0,1]} V_\alpha^*(\lambda, s)$. The other parameters, including γ, are suppressed in this notation. Holding s and α fixed, under individual rewards we have

$$\frac{dV_\alpha^*(0,s)}{d\gamma} = \frac{dY^*}{d\gamma} + 2\alpha s\frac{dR^*}{d\gamma}, \qquad \frac{dR^*}{d\gamma} = 2(1+\rho h^*)\frac{\partial h^*}{\partial \gamma} = \frac{2(1+\rho h^*)^3}{ac} > 0. \tag{D.3}$$

Here Y^* and R^* are evaluated under individual rewards. The additional term reflects the private benefits of the induced increase in helping.

At any nonsaturated parameter configuration with $dY^*/d\gamma < 0$, we must have $s > 0$. Define the critical fraction of private benefits that offsets this negative response by

$$\alpha^{\text{rev}} \equiv -\frac{dY^*/d\gamma}{2s\, dR^*/d\gamma}. \tag{D.4}$$

The derivatives in this expression are evaluated at the given (s, γ) under individual rewards.

Proposition D.1. *Fix a nonsaturated individual-reward equilibrium with $dY^*/d\gamma < 0$. Then $0 < \alpha^{\text{rev}} < 1/2$. For every $\alpha \in [0, \alpha^{\text{rev}}]$, the unique reward weight maximizing $V_\alpha^*(\lambda, s)$ is $\lambda = 0$. At this optimum,*

$$\frac{d\widehat{V}_\alpha}{d\gamma}\begin{cases} < 0, & 0 \leq \alpha < \alpha^{\text{rev}}, \\ = 0, & \alpha = \alpha^{\text{rev}}. \end{cases}$$

By contrast, at $\alpha = 1$, complete team rewards ($\lambda = 1$) implement an allocation maximizing $V_1 = W$, and the optimized objective is increasing in γ.

Proof. Because $dY^*/d\gamma < 0$, Proposition 2(i) implies $h^* > h^Y$ and hence $s > 0$. Equation (D.3) is strictly increasing in α and negative at $\alpha = 0$. At $\alpha = 1/2$, the marginal value in (D.2) is $ab(1-h^*) +$

$\gamma(1+\rho h^*) > 0$. The chain rule then gives $dV^*_{1/2}(0,s)/d\gamma = 2R^* + 2(\partial V_{1/2}/\partial h_i)\,(\partial h^*/\partial\gamma) > 0$. Hence the zero in (D.4) lies strictly between zero and 1/2.

For $\alpha \leq \alpha^{\text{rev}}$, equation (D.3) is nonpositive. As $R^* > 0$ and $\partial h^*/\partial\gamma > 0$, the chain rule implies that the derivative of $V_\alpha(h,h)$ with respect to h is strictly negative at $h = h^*$. This derivative is affine in h and strictly positive at $h = 0$, because

$$\frac{dV_\alpha(h,h)}{dh} = 2\{ab + 2(\gamma + \alpha s) - [a(b+c) - 2\rho(\gamma + \alpha s)]h\}.$$

It follows that $V_\alpha(h,h)$ is strictly concave and decreasing for all $h \geq h^*$. Since equilibrium helping weakly increases with λ and is strictly greater than h^* for every $\lambda > 0$, individual rewards uniquely maximize $V^*_\alpha(\lambda, s)$. Thus the derivative of the optimized objective equals (D.3), yielding the stated signs.

At $\alpha = 1$, each member's payoff under $\lambda = 1$ is $Y/2 + sR = V_1/2$. A global maximizer of V_1 is therefore a Nash equilibrium; uniqueness of the equilibrium under complete team rewards implies that $\lambda = 1$ implements it. Raising γ increases V_1 at every fixed allocation by $2R \geq 0$, strictly so at the optimum. Hence the optimized objective increases with γ. □

The critical fraction need not be small. Combining (6), (A.1), and (D.4) yields, for $\rho > (\sqrt{5}-1)/2$,

$$\lim_{\gamma \downarrow 0} \lim_{s \uparrow s_{\text{sat}}(\gamma)} \alpha^{\text{rev}} = \frac{\rho^2 + \rho - 1}{2(1+\rho)^2}.$$

This limit equals 11/32 when $\rho = 3$ and approaches 1/2 as complementarity increases.