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Improving inflation forecasting by modeling expectation de-anchoring in low-inflation periods

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Institute for Monetary and Financial Stability
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Improving Inflation Forecasting by Modeling Expectation De-Anchoring in Low-Inflation Periods*

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Abstract

Inflation is often overpredicted during persistent low-inflation periods. We evaluate a model in which inflation expectations can de-anchor, allowing transitions to a low-inflation equilibrium at the effective lower bound. We assess relative and absolute forecast performance using conventional and state-dependent tests. Relative to a model with expectations anchored at the inflation target, the transition model yields substantial accuracy gains during Japan's prolonged low-inflation episode and for euro area core inflation in the 2010s. These gains do not extend to the United States, where inflation remained closer to target. The absolute forecast evaluation also provides comparatively favorable evidence for the transition model in Japan and for euro area core inflation. Central bank projections are particularly strong at short horizons, reflecting their richer real-time information sets. For euro area core inflation during the low-inflation period, however, the transition model's forecast errors were roughly 50 percent lower than the ECB's for the following year and two years ahead.

Keywords: Inflation Forecasting, Inflation Expectation De-anchoring, State-dependent Forecast Evaluation, Effective Lower Bound, Multiple Equilibria, Central Bank Projections

JEL Codes: C52, C53, E31, E37

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1 Introduction

In standard macroeconomic models, long-run inflation expectations are anchored to the steady-state inflation rate, typically the central bank's inflation target. Forecasts from such models often imply a relatively rapid return to the steady state after shocks (see, e.g., Wickens, 2014) and can therefore be systematically biased when inflation remains persistently away from target. Although professional forecasters can in principle depart from this model-implied mean reversion, central-bank and survey forecasts often exhibit similar patterns in practice, tending to overpredict inflation in low-inflation periods and underpredict it when inflation is high.[1]

Macroeconomic models that can explain periods of persistent deviations from the inflation target could therefore be useful for improving the accuracy of inflation forecasts at times when accurate forecasts are most important for appropriate monetary policy. In this paper, we examine whether a model that allows for downward de-anchoring of inflation expectations can improve inflation forecasting during periods of persistently below-target inflation. We use the model to study low-inflation episodes in Japan, the euro area, and the United States. During Japan's prolonged low-inflation period, long-term inflation expectations were weakly anchored from the late 1990s through the early 2010s and remained incompletely anchored after the Bank of Japan introduced its 2 percent inflation target in 2013 (Hogen and Okuma, 2025). For the euro area, there is evidence of de-anchoring of inflation expectations during the low-inflation period preceding the 2021/2022 inflation surge (see, e. g., Nautz, Pagenhardt, and Strohsal, 2017; Natoli and Sigalotti, 2018; Corsello, Neri, and Tagliabracci, 2021; Christoffel and Farkas, 2025). For the United States, in contrast, the evidence for de-anchoring of inflation expectations during this period is less conclusive (e. g., Strohsal, Melnick, and Nautz, 2016; Grishchenko, Mouabbi, and Renne, 2019; Fischer, 2023).

The model we use, hereafter referred to as the transition model, builds on Lansing (2021) and the literature on target and low-inflation equilibria in New Keynesian models with an ELB (Benhabib et al., 2001; Aruoba et al., 2018).[2] It allows the economy to move between a target equilibrium, in which inflation fluctuates around the target and the nominal interest rate is typically positive, and a low-inflation equilibrium, in which inflation remains below target and the ELB typically binds. Persistently below-target inflation raises agents' perceived probability of the low-inflation equilibrium and lowers their inflation expectations. A decline in the natural interest rate reinforces this mechanism by increasing the frequency with which the ELB binds and making it more difficult for monetary policy to return inflation to target. The model is therefore designed to capture low-inflation, low-interest-rate environments, including the euro area in the 2010s and Japan's more persistent episode since the late 1990s.

[1] Evidence of related patterns, in different forms, is provided for the Eurosystem/ECB staff projections by Kontogeorgos and Lambrias (2022) and Granziera et al. (2025); for Federal Reserve staff projections by Joutz and Stekler (2000), Sinclair et al. (2010), and Rossi and Sekhposyan (2016); for the Bank of Japan by Tsuchiya (2021); and for the Bank of England, the ECB, and the Fed by Goodhart and Pradhan (2025). Similar over- and underpredictions have also been documented for survey-based forecasts such as the Survey of Professional Forecasters or the Blue Chip forecasts (see, e.g., Rossi and Sekhposyan, 2016).

[2] Hecker and Wolters (2025) estimate a related model with endogenous equilibrium transitions.

Forecast evaluation based on full-sample averages may conceal economically important heterogeneity in forecast performance, including changes in relative predictive accuracy over time (Rossi and Sekhposyan, 2010) and systematic forecast errors that offset across inflation environments. We therefore use conventional tests of relative predictive accuracy, together with Mincer–Zarnowitz regressions, for the full sample and for low-inflation episodes and other distinct inflation environments. Because these subsamples rely on predefined calendar periods, we complement them with the state-dependent forecast evaluation framework of Odendahl et al. (2023). Using inflation as the state variable and a logistic smooth-transition specification, we apply the framework to relative loss differentials when studying relative forecast accuracy and to moment conditions assessing the state-dependent weak efficiency of forecasts.

Using a calibration largely based on Lansing (2021), we find substantial cross-country differences in the subjective probabilities agents assign to the two equilibria. In Japan, the model assigns substantial weight to the low-inflation equilibrium from the late 1990s onward. For the euro area, the perceived probability of the low-inflation equilibrium rises substantially from 2015 onward and returns to zero only with the 2021 inflation surge. In the United States, by contrast, it generally remains low, reflecting that inflation stayed closer to target.

During Japan's pronounced low-inflation period from 1998 to 2011, the transition model yields significantly more accurate inflation forecasts than a counterpart model in which inflation expectations are assumed to remain anchored at the inflation target at all times (hereafter: the target model). For the euro area, the transition model correctly predicts for core inflation in 2015, when inflation is particularly low, that inflation will remain around 1%, while the target model anticipates a rapid return to the inflation target instead. Because headline inflation is volatile and exhibits occasional sharp spikes, the difference in forecast accuracy between the two model versions is not statistically significant over the low-interest-rate, low-inflation period from 2010 to 2019. For core inflation, however, the transition model significantly improves forecast accuracy, reducing forecast errors by up to about 50 percent. In the United States, transition and target forecasts are generally very similar, with a slight overall advantage for the target model. These patterns are consistent with the model's design: it delivers substantial gains when low inflation is sufficiently persistent to shift expectations away from the target, while performing similarly to the target model when the target equilibrium dominates. Its disadvantages arise mainly around abrupt changes in the inflation environment, when the equilibrium probabilities adjust only gradually.

We also analyze whether inflation forecasts based on the transition model would have improved on actual central bank projections. To align the model's information set as closely as possible with that available to economists at central banks, we use real-time data to generate the model's forecasts. For Japan, the transition model's current-fiscal-year forecasts are less accurate than those of the Bank of Japan, while the difference is smaller for next-fiscal-year forecasts. Much of the Bank of Japan's advantage in 2012–2019 reflects its ability to account for the 2014 consumption-tax increase: when the model forecasts are compared with Bank of Japan projections excluding this tax effect, the differences are no longer statistically significant. The Bank of Japan nevertheless frequently overpredicts inflation during the prolonged low-inflation period,

whereas the transition model does not exhibit an upward bias. For the euro area, the ECB's projections are particularly accurate at short horizons. During the particularly low-inflation period from 2014 to 2018, however, the ECB systematically overpredicted core inflation, and the transition model would have reduced forecast errors by around 50 percent at the following-year horizon and by around 60 percent at the two-year-ahead horizon. For the United States, where the low-inflation period was less pronounced, the transition model does not improve on the Fed's Greenbook/Tealbook forecasts, although the differences are generally small except at the one-quarter-ahead horizon.

The state-dependent tests add an important dimension to the conventional forecast evaluation by showing that full-sample results can conceal economically relevant variation across inflation environments. This is most apparent for Japanese core inflation and also evident for euro area core inflation: the transition model performs substantially better during persistent low-inflation periods, whereas the target model gains after abrupt increases in inflation as the transition weights adjust only gradually. Because forecast errors are particularly large during the recent inflation surge, this relatively short episode can offset much of the transition model's earlier gains in full-sample measures even though both models perform poorly during the surge.

The absolute forecast evaluation points to a broad cross-country pattern similar to that found in the relative forecast evaluation. The transition model exhibits comparatively favorable calibration and weak-efficiency properties in Japan and for euro area core inflation, whereas no comparable advantage emerges in the United States. The low-inflation model performs poorly in most absolute tests. The results for central bank projections are not uniformly favorable either.

Our paper contributes most directly to the literature on inflation forecasting with structural macroeconomic models. Such models often imply a rapid return of inflation to its steady state, which may partly reflect the assumption that inflation expectations remain anchored at the target. For the euro area, Christoffel et al. (2011) show that forecasts from the ECB's New Area-Wide Model (NAWM) return inflation rapidly to target following shocks and generate persistent forecast errors. Iversen et al. (2016) provide related evidence for Sweden, where inflation forecasts from the Riksbank's DSGE model revert more rapidly to the target than forecasts from its BVAR model, which approach the target more gradually and can remain below it during low-inflation periods. The BVAR forecasts are more accurate, yet the Riksbank's published forecasts are much closer to those from the DSGE model, possibly reflecting a desire to see inflation return to target within two to three years.

A related literature examines state-dependent biases in central bank inflation projections and their tendency to return rapidly to target. Granziera et al. (2025) show that standard time-series models exhibit state-dependent biases of the opposite sign to those of central bank projections, tending to overpredict inflation when it is above target and underpredict it when it is low, and therefore cannot explain central banks' overprediction of inflation during low-inflation periods. They also find that errors in external assumptions about interest rates, exchange rates, and oil prices cannot fully account for the ECB's forecast errors. They therefore consider whether the rapid return of published projections to the inflation target reflects an attempt to steer

inflation expectations toward the target. Goodhart and Pradhan (2025) offer a complementary explanation. They show that two-year-ahead inflation projections from the Bank of England, the ECB, and the Fed tend to remain close to the inflation target and argue that persistent errors at this horizon may reflect an overestimation of monetary-policy effects and of the degree of expectation anchoring. Similarly, Conrad et al. (2025) document that ECB inflation projections cluster tightly around the inflation target at a two-year horizon. Evidence consistent with target-oriented projection behavior is provided by Paloviita et al. (2021), who document stronger and faster mean reversion in ECB projections than in realized inflation, and by Charemza and Ladley (2016), who find a target bias in inflation forecasts from ten inflation-targeting central banks but no clear evidence of such bias among non-central-bank professional forecasters.

The model's expectation-formation mechanism is motivated by evidence that inflation expectations respond to experienced inflation rather than remaining mechanically tied to the central bank's target. D'Acunto et al. (2023) emphasize the role of personally experienced prices in household expectation formation, while Candia et al. (2023) provide related evidence for firms and question whether their expectations are firmly anchored even in stable, low-inflation environments. Bernanke (2024) likewise recommends empirically grounded models that do not impose permanently anchored inflation expectations. Accordingly, agents in the model update the probabilities assigned to the target and low-inflation equilibria based on recent forecast performance, allowing inflation expectations to de-anchor endogenously.

Although inflation rose globally after 2021 and even the Bank of Japan subsequently raised its policy rate, the possibility of renewed low-inflation, low-interest-rate episodes remains relevant. The increase in natural interest rates during the inflation surge may have been temporary, and some recent estimates of r^* have returned close to the low levels of the previous decade.[3] Cho et al. (2025) estimate, based on interest rate derivatives, that the risk of the U.S. economy returning to the ELB is 9% at the seven-year horizon and remains at similar levels at longer horizons. Switzerland has—after a mild increase in inflation between 2021 and 2023—already returned to interest and inflation rates close to zero.

2 Model

In the following, we describe the model, its solution, and its calibration. The model builds on Lansing (2021).

2.1 Core model equations

The demand side of the economy is described by an IS curve,

$$y_t = \hat{\mathbb{E}}_t y_{t+1} - \alpha \left[i_t - \hat{\mathbb{E}}_t \pi_{t+1} - r_t \right] + v_t, \tag{1}$$

[3]The U.S. estimates based on Laubach and Williams (2003) are declining but remain somewhat above post–global financial crisis levels, whereas those of Holston et al. (2017) are close to the record lows of the previous decade for both the United States and the euro area (see Federal Reserve Bank of New York, 2026).

and the supply side by a Phillips curve:

$$\pi_t = \beta \hat{\mathbb{E}}_t \pi_{t+1} + \kappa y_t + u_t. \tag{2}$$

Monetary policy is described by a smoothed rule for the unconstrained shadow policy rate i_t^*, while the actual nominal policy rate i_t is subject to the ELB:

$$i_t^* = \rho i_{t-1}^* + (1-\rho)\left[r_t^* + \pi^* + g_\pi(\bar{\pi}_t - \pi^*) + g_y(y_t - y^T)\right], \tag{3a}$$

$$i_t = \max\{\underline{i}, i_t^*\}. \tag{3b}$$

Here, y_t denotes the output gap, π_t the annualized quarter-on-quarter inflation rate, π^* the central bank's inflation target, i_t^* the unconstrained shadow policy rate, and i_t the actual nominal policy rate, which is subject to the ELB $\underline{i} = 0$. The variables r_t and r_t^* denote the short-run and long-run natural real interest rates, respectively, while v_t and u_t denote demand and supply shocks. The variable $\bar{\pi}_t$ approximates annual inflation and y^T denotes the output gap at the target endpoint; both are defined in what follows. The operator $\hat{\mathbb{E}}_t$ denotes agents' subjective expectations, whose formation differs from standard rational expectations and is described in Section 2.3.

The demand and supply shocks follow AR(1) processes,

$$v_t = \rho_v v_{t-1} + \epsilon_{v,t}, \qquad \epsilon_{v,t} \sim \mathcal{N}\left[0, \sigma_v^2(1-\rho_v^2)\right], \tag{4}$$

$$u_t = \rho_u u_{t-1} + \epsilon_{u,t}, \qquad \epsilon_{u,t} \sim \mathcal{N}\left[0, \sigma_u^2(1-\rho_u^2)\right]. \tag{5}$$

The processes for the long- and short-run natural real interest rates are

$$r_t^* = r_{t-1}^* + \eta_t, \qquad \eta_t \sim \mathcal{N}(0, \sigma_\eta^2), \tag{6}$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r) r_t^* + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2), \tag{7}$$

respectively, and $\bar{\pi}_t$ is an exponentially weighted moving average of current and past quarterly inflation rates,

$$\bar{\pi}_t = \omega \pi_t + (1-\omega)\bar{\pi}_{t-1}, \tag{8}$$

with the smoothing parameter ω chosen such that $\bar{\pi}_t$ approximates annual inflation.

2.2 Long-run endpoints and local forecast rules

In the absence of shocks, the long-run IS curve implies the Fisher relation between the nominal interest rate, inflation, and the natural real interest rate. Combined with a monetary policy rule satisfying the Taylor principle and the ELB, this gives rise to two long-run endpoints,[4] corresponding to the two steady states of New Keynesian models with an ELB constraint first shown by Benhabib et al. (2001). The first is the target endpoint, at which inflation equals the

[4]We use the term "endpoints" rather than "steady states" because the long-run natural interest rate process $\{r_t^*\}$ has a unit root and is therefore non-stationary.

inflation target and the ELB is slack: $\pi^T = \pi^*$, $i_t^T = r_t^* + \pi^*$, and $y^T = (1-\beta)\pi^*/\kappa$. The second is the low-inflation endpoint, at which the ELB binds: $\pi_t^L = \underline{i} - r_t^* = -r_t^*$, $i_t^L = \underline{i} = 0$, and $y_t^L = (1-\beta)(\underline{i} - r_t^*)/\kappa = (1-\beta)(-r_t^*)/\kappa$.

We consider local approximations around the two endpoints. Following Lansing (2021), we assume that, in the local approximation around the target endpoint, agents expect the ELB not to bind. The resulting system satisfies the determinacy conditions of Blanchard and Kahn (1980) and can be solved using standard linear rational-expectations methods. We refer to this unique local solution as the target equilibrium. It yields linear one-step-ahead forecast rules for the output gap and inflation. We denote the forecasts generated by these rules by $\mathbb{E}_t^T y_{t+1}$ and $\mathbb{E}_t^T \pi_{t+1}$.

In the local approximation around the low-inflation endpoint, by contrast, agents expect the ELB to bind permanently. The resulting system has one fewer unstable eigenvalue than required for determinacy and therefore admits a continuum of bounded solutions with one degree of indeterminacy. As our baseline, we use the Minimum State Variable (MSV) solution of McCallum (1983), which selects from this set a solution characterized by a minimal set of state variables.[5] We refer to the selected MSV solution as the low-inflation equilibrium. It yields corresponding linear one-step-ahead forecast rules for the output gap and inflation. We denote the forecasts generated by these rules by $\mathbb{E}_t^L y_{t+1}$ and $\mathbb{E}_t^L \pi_{t+1}$. Appendix B discusses indeterminacy and solution selection and derives the MSV solution.

Because indeterminacy permits alternative selections from the set of bounded equilibria, we assess the robustness of our results in Appendix E using the method of Farmer et al. (2015).[6] In our model, which has one degree of indeterminacy, this approach augments the system by reclassifying the non-fundamental inflation-expectation error as an additional fundamental shock. Appendix B derives the alternative solution, and Appendix E provides a comprehensive robustness analysis of the forecasting results.

2.3 Endogenous forecast switching, model solution, and benchmark models

In the transition model, agents form subjective one-step-ahead expectations by combining the forecasts from the target and low-inflation equilibria,

$$\hat{\mathbb{E}}_t x_{t+1} = \mu_t \mathbb{E}_t^T x_{t+1} + (1-\mu_t)\mathbb{E}_t^L x_{t+1}, \qquad x \in \{y,\pi\}, \tag{9}$$

where μ_t and $1-\mu_t$ denote the weights on the target and low-inflation forecasts, respectively. We interpret these weights as the subjective probabilities that agents attach to convergence toward the corresponding long-run endpoints.

[5] A set of state variables is minimal if no state variable can be omitted while preserving a valid solution. Different minimal sets may in principle exist. Under McCallum's additional requirement that the solution remain valid across admissible values of the model's structural parameters, the MSV solution is unique in our application.

[6] Other approaches to characterizing indeterminate equilibria include those of Lubik and Schorfheide (2003) and Bianchi and Nicolò (2021). Together with the approach of Farmer et al. (2015), these methods provide equivalent representations of the same set of indeterminate equilibria for a broad class of linear models, although they need not select the same particular equilibrium without additional restrictions.

At each date t, agents choose the weight that minimizes the RMSFE of the combined one-step-ahead forecasts for the output gap and inflation over the preceding $T_w = 8$ quarters,

$$\mu_t = \arg\min_{\mu\in[0,1]} \left\{ \frac{1}{T_w} \sum_{k=1}^{T_w} \sum_{x\in\{y,\pi\}} \left[x_{t-k} - \mu \mathbb{E}^T_{t-k-1} x_{t-k} - (1-\mu)\mathbb{E}^L_{t-k-1} x_{t-k} \right]^2 \right\}^{1/2}. \tag{10}$$

We implement the constrained minimization as a grid search. Because all forecast errors entering equation (10) are realized by period $t-1$, μ_t is predetermined when the period-t model is solved. We set $\mu_t = 1$ until the T_w one-step-ahead forecast errors required to compute the weight are available and update it recursively thereafter. The resulting subjective expectations influence current inflation and the output gap, rendering the model self-referential across periods: if a series of adverse shocks shifts weight toward the low-inflation forecast, expected future inflation and the expected future output gap decline, depressing their current values. These realizations subsequently enter the forecast errors used to update the weight, potentially reinforcing agents' belief in convergence toward the low-inflation endpoint.[7]

At the start of period t, the weight μ_t and the predetermined state variables are known. Together, the local forecast rules described in Section 2.2, their weighted combination in equation (9), and equations (1)–(3) form a simultaneous system that determines agents' subjective expectations and the current endogenous variables y_t, π_t, and i^*_t. A joint solution is necessary because the target forecast rules depend on contemporaneous smoothed inflation $\bar{\pi}_t$, and hence on π_t, as well as on the current shadow rate i^*_t. By contrast, the low-inflation forecast rules depend only on predetermined state variables and can therefore be evaluated before solving for the current endogenous variables.

We solve the resulting system regime by regime. First, we impose the candidate non-binding regime, $i_t = i^*_t$, and retain the solution if $i^*_t \geq \underline{i}$. If this condition is violated, we solve the system again imposing $i_t = \underline{i}$ and retain the solution if $i^*_t \leq \underline{i}$. This procedure implements the piecewise policy rule $i_t = \max\{\underline{i}, i^*_t\}$.

For comparison, we use the two local models as benchmarks. The target model assumes that the ELB never binds, so that $i_t = i^*_t$ at all dates, whereas the low-inflation model assumes that the ELB always binds, so that $i_t = \underline{i}$ at all dates. The transition model instead combines forecasts from the two local models using the time-varying weight μ_t and determines endogenously in each period whether the ELB binds.

2.4 Calibration

We largely follow the calibration of Lansing (2021) and use common parameter values across the three economies. These are reported in Table 1. The inflation target is country-specific, while the smoothing parameter ω is country- and inflation-measure-specific and is chosen such that $\bar{\pi}_t$ closely approximates annual inflation. The model is calibrated to capture the low sensitivity

[7] Alternatively, one could use the approach of Aruoba et al. (2018), who employ a sunspot shock to switch between the two local equilibria. The approach of Lansing (2021) is more suitable, in our view, for cases in which a transition toward the low-inflation endpoint has begun but has not yet been completed, as is plausible for the low-inflation periods in the euro area and the United States.

of consumption to interest rate changes (see, e.g., Campbell and Mankiw, 1989) and a relatively flat Phillips curve. The coefficients g_π, g_y, and ρ follow the calibration of Lansing (2021), which draws on Taylor (1999). Following Lansing (2021), we set $T_w = 8$. In Lansing's simulations, this value produces an ELB frequency close to that observed in U.S. data. The shock-process parameters are also taken from Lansing (2021).

Table 1: Calibration

Parameter	Value	Description
α	0.15	Interest rate coefficient in IS equation
β	0.995	Discount factor
κ	0.025	Slope of the Phillips curve
σ_v	0.003	Unconditional standard deviation of the demand shock
σ_u	0.010	Unconditional standard deviation of the supply shock
ρ_v	0.85	Persistence of the demand shock
ρ_u	0.32	Persistence of the supply shock
g_π	1.5	Systematic monetary policy response to inflation
g_y	1.0	Systematic monetary policy response to the output gap
ρ	0.8	Interest rate smoothing parameter
$\underline{i}$	0	Effective lower bound for the policy rate
ρ_r	0.875	Persistence of the short-run natural rate
σ_ε	0.0098	Standard deviation of the short-run natural rate shock
σ_η	0.0012	Standard deviation of the long-run natural rate shock
T_w	8	Window length for computing the forecast weights

We calibrate $\pi^* = 0.02$ for the United States and the euro area. The Bank of Japan introduced its 2 percent inflation target in January 2013. Before this, the literature assumes an implicit inflation target of 1% (see, e.g., Cuba-Borda and Singh, 2024). Hence, we use $\pi^* = 0.01$ until 2012Q4 and $\pi^* = 0.02$ thereafter.

3 Relative Forecast Accuracy across Models

We use quarterly data on inflation, the output gap, a policy rate, and the long-run natural interest rate. Unlike in many DSGE models, where variables are defined as percentage deviations from steady state, variables are measured in the data as they are defined in the model, allowing the data to be directly mapped to π_t, y_t, i_t, and r_t^* without adding steady state values in the measurement equations. The sample periods differ across Japan, the euro area and the United States due to data availability.

For Japan, we use CPI inflation, the Bank of Japan's output gap estimates, the Bank of Japan's basic discount rate as the policy rate, and the natural interest rate as estimated using the Holston et al. (2023) methodology and reported by Nakano et al. (2024). The sample covers 1985Q1–2025Q1 and includes data used for model initialization and forecast construction at the forecast origins, as well as subsequent inflation realizations used to evaluate forecasts. The same convention applies to the sample spans reported below. For the euro area, inflation is based on the Harmonized Index of Consumer Prices (HICP), the output gap is derived from the European Commission's potential output estimate, and we use the ECB's main refinancing operations rate as the policy rate, and the natural interest rate estimates of Holston et al. (2017). The sample spans 1999Q1–2025Q3. For the United States, we use CPI inflation, the output

gap based on the potential output estimates of the Congressional Budget Office, the effective federal funds rate as the policy rate, and the natural interest rate estimates of Holston et al. (2017). The sample spans 1988Q1–2025Q3. We repeat all analyses for all three economies using core inflation instead of headline inflation to avoid temporary fluctuations in the equilibrium probabilities caused by volatile energy and food prices.[8]

The model-based forecasts evaluated in the subsequent sections are generated recursively. At each forecast origin t, we compute the forecast weight μ_t from past one-step-ahead forecast errors using equation (10). For each horizon $j = 1, \ldots, H$, the predetermined variables are advanced to period $t + j$, future shock innovations are set to their conditional means of zero, and the natural interest rates evolve according to equations (6) and (7). We then apply the period-by-period solution described in Section 2.3 to obtain $y_{t+j|t}$ and $\pi_{t+j|t}$, while the remaining solution variables are carried forward to the next forecast horizon. Because the future realizations required to update equation (10) are unavailable at the forecast origin, μ_t is held fixed along each transition-model forecast path and recomputed at the next forecast origin using the newly available information.

We compare the forecasts of the transition model to those of the target and low-inflation models. This allows us to assess whether accounting for potential equilibrium transitions improves forecasting accuracy, whether a standard target-equilibrium model suffices, or whether, in environments with a persistently binding ELB such as Japan, a low-inflation-equilibrium model is preferable.

3.1 Equilibrium probabilities

To understand when differences in forecasting accuracy among the three models may arise, we first examine the equilibrium probabilities for the three economies. When the probability of the target equilibrium, μ_t, is close to one, the transition and target models produce similar forecasts. As μ_t declines, the transition model places greater weight on the low-inflation equilibrium and its forecasts diverge from those of the target model. The transition model should therefore be most useful when low inflation is sufficiently persistent to shift the equilibrium probabilities, whereas short-lived movements in inflation need not generate forecasting gains.

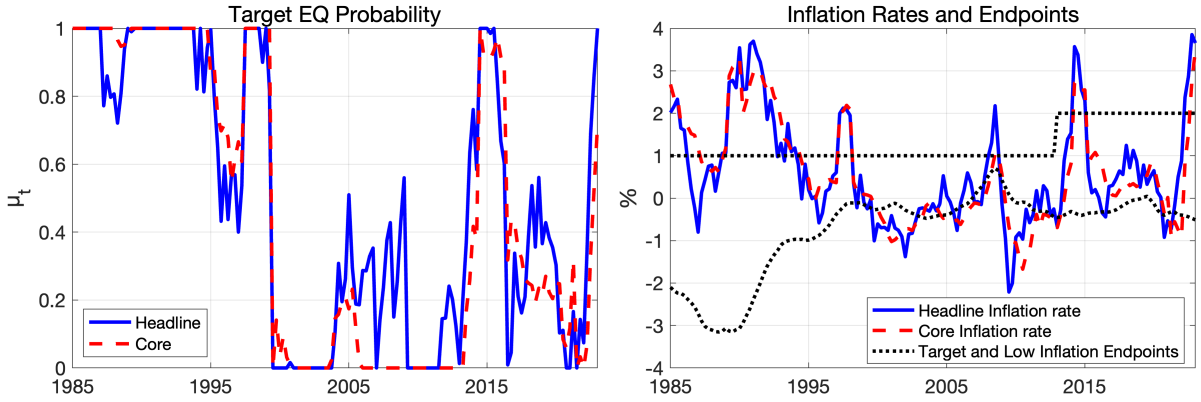
Figure 1 shows the model-implied probability of the target equilibrium in the left panels, computed separately using headline and core inflation. For ease of interpretation, the right panels display year-on-year inflation rather than the annualized quarter-on-quarter inflation measure used in the model, together with the target endpoint, $\pi_t^T = \pi^*$, and the low-inflation endpoint, $\pi_t^L = -r_t^*$. As the economy moves closer to one endpoint, the associated local model tends to deliver more accurate one-step-ahead forecasts. However, because μ_t is based on forecast errors over a rolling eight-quarter window, the corresponding weight adjusts only gradually.[9]

[8] Data sources and transformations, including those for the real-time data and central bank projections used in Section 4, are reported in the Appendix.

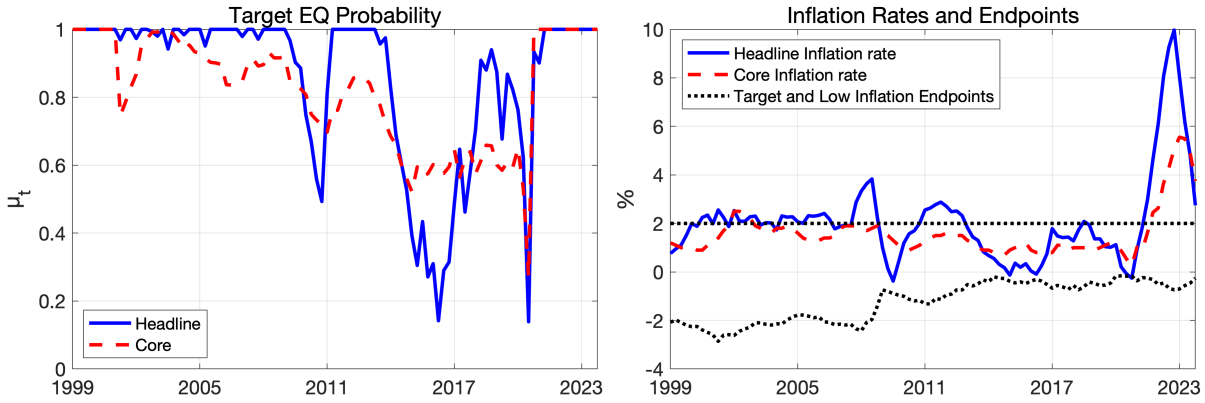
[9] The output gap also enters the computation of μ_t, but inflation is the main driver because the two output-gap endpoints are relatively close, particularly when r_t^* is low.

Figure 1: Equilibrium Probabilities and Inflation Endpoints

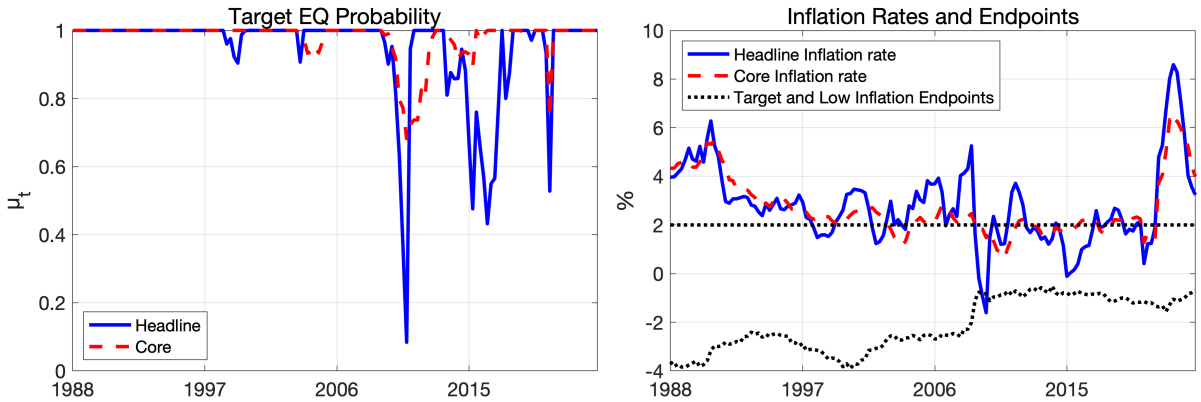
Japan



Euro area



United States



Notes: Left panels show the model-implied probability of the target equilibrium, computed separately using headline and core inflation. Solid blue lines correspond to headline inflation and dashed red lines to core inflation. Right panels show headline inflation (solid blue lines) and core inflation (dashed red lines) in year-on-year percentage terms. The upper black dotted lines mark the target inflation endpoint π^*; the lower black dotted lines depict the low-inflation endpoint $-r_t^*$.

For Japan, the probability of the target equilibrium remains high from the mid-1980s to the mid-1990s, when inflation generally fluctuates around its target. It declines sharply in the late 1990s as inflation and r_t^* fall toward zero. Before the inflation target is raised in 2013, the headline-based probability remains predominantly below 50% but fluctuates more than its core-based counterpart. During this period, the relatively small distance between the two inflation endpoints makes the estimated weight particularly sensitive to temporary movements in headline inflation. Core inflation provides a more persistent signal: the probability of the target equilibrium falls to zero in the late 1990s and remains low until 2014. The subsequent rise in the target-equilibrium probability coincides with the increase in the inflation target and the introduction of quantitative and qualitative easing in 2013, followed by the temporary increase in inflation associated with the 2014 consumption-tax hike. Thereafter, the model alternates between assigning greater weight to the two equilibria before the recent inflation surge raises both probabilities sharply, with the headline-based probability returning to the target equilibrium more decisively. Overall, the Japanese evidence points to a persistent role for the low-inflation equilibrium and shows that core inflation provides a more stable signal during Japan's prolonged low-inflation period.

For the euro area, the probabilities indicate a less complete transition than in Japan. They decline temporarily around the global financial crisis but recover thereafter. A more persistent shift toward the low-inflation equilibrium begins around 2014: the headline-based probability falls sharply and fluctuates considerably, whereas the core-based probability declines more gradually and remains around 60% for much of the low-inflation period. Taken together, the probabilities characterize the euro area as a mixture of the two equilibria: the low-inflation equilibrium becomes economically important but does not dominate as persistently as in Japan. As inflation rises sharply from 2021 onward, both probabilities return to one, indicating that the low-inflation episode has ended.

In the United States, by contrast, the target equilibrium remains dominant over most of the sample. Headline inflation produces several short-lived shifts toward the low-inflation equilibrium, but the probability repeatedly returns close to one. The core-based probability shows only brief and mild declines after the global financial crisis, in 2015, and during the COVID-19 crisis, while remaining well above 50% throughout. The equilibrium probabilities therefore provide little indication of a persistent de-anchoring episode comparable to those in Japan or the euro area.

These findings are broadly consistent with previous studies. Lansing (2021) and Aruoba et al. (2018) find that the United States remained predominantly near the target equilibrium, while Aruoba et al. (2018) conclude that the low-inflation equilibrium has prevailed in Japan since the late 1990s. This evidence supports our conclusion that the low-inflation equilibrium is substantially more important for Japan than for the United States.

These probability patterns have clear implications for relative forecast performance. The transition and target models should perform similarly when the target equilibrium dominates, as in the early Japanese sample and most of the U.S. sample. Larger differences should arise during Japan's prolonged low-inflation episode and for euro area core inflation in the 2010s, when

the transition model assigns substantial weight to the low-inflation equilibrium. However, the gradual adjustment of the equilibrium weights may temporarily put the transition model at a disadvantage after abrupt changes in the inflation environment. Section 3.2 examines whether the empirical results confirm these predictions.

3.2 Conventional Analysis of Relative Forecast Accuracy

For the evaluation of forecast performance, we consider inflation forecasts at horizons of up to eight quarters. For each economy, the full evaluation sample begins at the tenth observation, the first forecast origin at which the eight one-step-ahead forecast errors entering equation (10) are available. The first nine observations are therefore used for initialization. The reported sample periods refer to forecast origins rather than forecast realizations. We use the same forecast origins at all horizons. Consequently, realizations for longer-horizon forecasts may fall beyond the stated sample endpoint and are used only for forecast evaluation. We compute the relative root mean squared forecast errors (rRMSFEs) of the transition model relative to the target and low-inflation models and test whether the differences are statistically significant using the Diebold–Mariano test (Diebold and Mariano, 1995), with the small-sample adjustment proposed by Harvey et al. (1997). We evaluate the forecasts separately for each economy and for headline and core inflation. The analysis covers the full sample and several subsamples defined by changes in the equilibrium probabilities. Table 2 reports the results based on revised data.

3.2.1 Japan

For Japan, the transition and target models have very similar full-sample RMSFEs, with a slight advantage for the transition model, although most differences are statistically insignificant. Both significantly outperform the low-inflation model, reducing forecast errors by about 20–30%. These full-sample averages, however, conceal pronounced changes in relative forecast performance across inflation environments. The transition model gains relative to the target model when persistent declines in inflation and r_t^* shift its forecasts toward the low-inflation equilibrium, but may perform worse following an abrupt rise in inflation because the equilibrium weights adjust only gradually.

From 1987 to 1997, inflation is generally positive, estimates of r_t^* are around 2% or higher, and the probability of the target equilibrium is close to one. The transition model therefore produces forecasts very similar to those of the target model, and the differences in accuracy are not statistically significant. By contrast, the low-inflation model predicts convergence toward the negative inflation endpoint $-r_t^*$ and performs significantly worse at all horizons.

The ranking changes during the pronounced low-inflation period from 1998 to 2011. As inflation and r_t^* fall toward zero, the transition model shifts weight toward the low-inflation equilibrium, whereas the target model continues to predict a return to the 1% target. The transition model reduces forecast errors relative to the target model by up to 20% for headline inflation and up to 50% for core inflation, with statistically significant gains at all horizons. Its forecasting accuracy is similar to that of the low-inflation model, with a modest advantage for the latter that is statistically significant for core inflation at short horizons. Because the equilibrium

Table 2: Relative Forecast Accuracy across Models

	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	4	8	1	2	4	8
Japan								
Transition/Target								
1987Q2–2023Q1	0.97	0.97	1.02	1.04	0.92*	0.91	0.96	0.99
1987Q2–1997Q4	1.02	1.03	1.07	1.07	1.01	1.03	1.03	1.00
1998Q1–2011Q4	0.89***	0.83***	0.81***	0.85**	0.56***	0.53***	0.57***	0.69***
2012Q1–2019Q4	0.97	0.95	1.00	0.91	0.92	0.90	0.97	0.90
2020Q1–2023Q1	1.08	1.24	1.66	1.97	1.03	1.14	1.32	2.06*
Transition/Low-Inflation								
1987Q2–2023Q1	0.81***	0.73***	0.74**	0.74*	0.73***	0.66***	0.67***	0.68*
1987Q2–1997Q4	0.66***	0.55***	0.54***	0.55*	0.57***	0.48***	0.45***	0.44**
1998Q1–2011Q4	1.02	1.04	1.07	1.05	1.08**	1.11**	1.15	1.17
2012Q1–2019Q4	0.97	0.96	1.04	1.04	0.94	0.95	1.02	1.07
2020Q1–2023Q1	0.86**	0.83*	0.85	0.82	0.90	0.89	0.93	0.87
Target/Low-Inflation								
1987Q2–2023Q1	0.83***	0.75***	0.72**	0.71*	0.79***	0.73***	0.70**	0.69
1987Q2–1997Q4	0.65***	0.53***	0.50***	0.52*	0.56***	0.46***	0.43***	0.44**
1998Q1–2011Q4	1.15**	1.25***	1.32***	1.22**	1.94***	2.08***	2.01***	1.68***
2012Q1–2019Q4	1.01	1.01	1.04	1.14	1.03	1.05	1.06	1.18
2020Q1–2023Q1	0.80	0.67	0.52*	0.42***	0.88	0.78	0.71	0.42***
Euro Area								
Transition/Target								
2001Q2–2023Q3	1.00	1.02	1.03	1.08	0.89***	0.91*	0.88**	0.98
2001Q2–2009Q4	1.00	1.01	1.00	1.01	1.01	0.99	0.85	0.83
2010Q1–2019Q4	1.01	1.01	1.02	1.08	0.59***	0.48***	0.50***	0.69***
2020Q1–2023Q3	1.00	1.03	1.04	1.11	0.97	1.03	1.05	1.17
Transition/Low-Inflation								
2001Q2–2023Q3	0.63***	0.60***	0.59***	0.62***	0.37***	0.36***	0.35***	0.39***
2001Q2–2009Q4	0.44***	0.37***	0.33***	0.33***	0.19***	0.17***	0.15***	0.15***
2010Q1–2019Q4	0.63***	0.62***	0.65*	0.73	0.30***	0.24***	0.28***	0.41***
2020Q1–2023Q3	0.77***	0.75***	0.74**	0.79	0.58***	0.57***	0.54***	0.59***
Target/Low-Inflation								
2001Q2–2023Q3	0.63***	0.59***	0.57***	0.57***	0.42***	0.39***	0.39***	0.40***
2001Q2–2009Q4	0.44***	0.36***	0.33***	0.33***	0.19***	0.17***	0.17***	0.18***
2010Q1–2019Q4	0.63***	0.61**	0.63	0.68	0.52***	0.50***	0.56**	0.58***
2020Q1–2023Q3	0.77***	0.73***	0.71**	0.71	0.59***	0.55***	0.51***	0.51**
United States								
Transition/Target								
1990Q2–2023Q3	1.02**	1.04*	1.04*	1.04**	1.01**	1.01*	1.03*	1.03
1990Q2–2009Q4	1.00	1.00	1.01	1.01*	1.00	1.00	1.01	1.02
2010Q1–2019Q4	1.16*	1.17	1.13	1.06	1.19*	1.12*	1.13	1.02
2020Q1–2023Q3	1.01	1.02	1.04	1.05	1.00	1.00	1.02	1.04
Transition/Low-Inflation								
1990Q2–2023Q3	0.55***	0.48***	0.46***	0.45***	0.34***	0.30***	0.29***	0.28***
1990Q2–2009Q4	0.49***	0.40***	0.36***	0.37***	0.23***	0.19***	0.17***	0.15***
2010Q1–2019Q4	0.65***	0.66***	0.59***	0.63**	0.26***	0.32***	0.31***	0.42***
2020Q1–2023Q3	0.70***	0.61***	0.62***	0.57*	0.62***	0.53***	0.53***	0.48**
Target/Low-Inflation								
1990Q2–2023Q3	0.54***	0.47***	0.44***	0.44***	0.34***	0.29***	0.28***	0.27***
1990Q2–2009Q4	0.49***	0.40***	0.36***	0.36***	0.23***	0.19***	0.17***	0.15***
2010Q1–2019Q4	0.57***	0.57***	0.52***	0.59**	0.22***	0.29***	0.27***	0.41***
2020Q1–2023Q3	0.69***	0.60***	0.60***	0.54*	0.62***	0.53***	0.52***	0.46**

Notes: Entries are relative RMSFEs based on revised data; values below one favor the model in the numerator. The reported sample periods refer to forecast origins. Stars indicate rejection of the null hypothesis of equal predictive accuracy based on the Diebold–Mariano test, with *, **, and *** denoting significance at the 10%, 5%, and 1% levels, respectively.

probabilities depend on forecast performance over the preceding eight quarters, the transition model adjusts only gradually toward the low-inflation equilibrium.

During the "Abenomics" period from 2012 to 2019, inflation and the equilibrium probabilities fluctuate between the two equilibria. Although assigning positive weight to the low-inflation equilibrium can improve the transition forecast, frequent changes in inflation conditions limit this advantage. Its forecast accuracy therefore remains similar to that of both the target and low-inflation models, with a slight advantage over the target model but no statistically significant differences.

The ranking shifts once more during the inflation surge after 2020. The target model generally performs best and the low-inflation model worst, with the transition model in between, although the differences are statistically significant only in a few cases. The same gradual adjustment mechanism now works in reverse, causing the transition model to respond more slowly to the abrupt rise in inflation. Because the full-sample relative RMSFEs are based on squared forecast errors pooled across all observations rather than averages of the subsample relative RMSFEs, the relatively large errors during this short but pronounced inflation surge receive considerable weight. They offset much of the transition model's substantial gains during the longer low-inflation period, leaving only a slight full-sample advantage.

3.2.2 Euro area

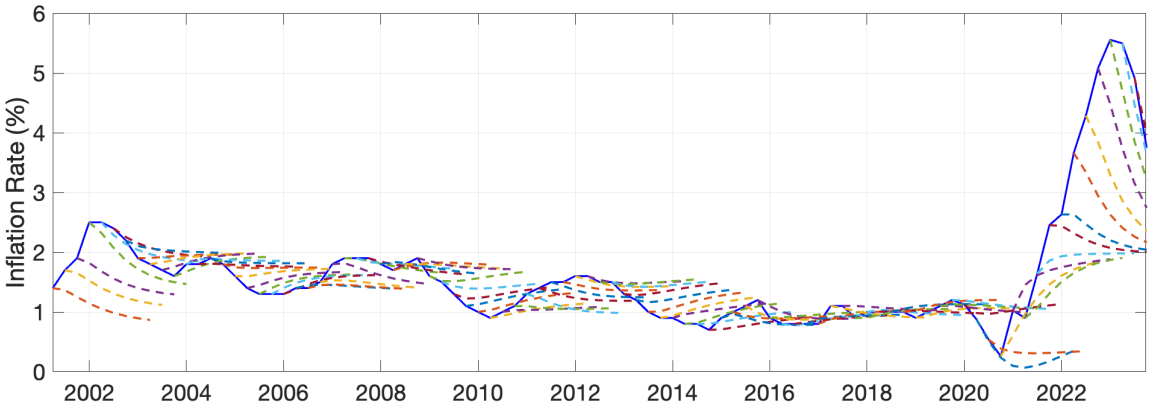
For the euro area, the transition and target models have very similar full-sample RMSFEs for headline inflation, with no statistically significant differences. For core inflation, by contrast, the transition model has a modest full-sample advantage, with statistically significant differences at most horizons. The low-inflation model is generally less accurate than both because it predicts convergence toward the low-inflation endpoint even when inflation remains substantially above it. The transition model's advantage is particularly pronounced for core inflation during the low-interest-rate, low-inflation period from 2010 to 2019.

During this period, the transition model reduces core-inflation forecast errors relative to the target model by approximately 30–50%, with statistically significant differences at all horizons. The probability of the low-inflation equilibrium remains non-negligible, while the target equilibrium continues to receive most of the weight. The transition model can therefore accommodate persistent below-target inflation without imposing either a rapid return to target or a permanent shift to the low-inflation equilibrium. It also substantially outperforms the low-inflation model. Its forecasting gains arise from combining the two equilibria rather than from treating either one as permanently dominant.

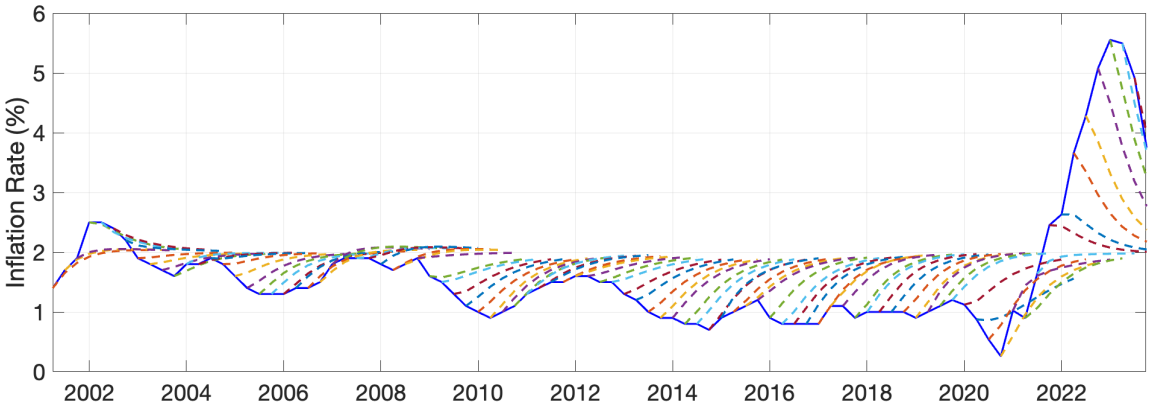
Figure 2 illustrates the source of these gains. During much of the 2010s, the target model predicts a relatively rapid return of core inflation to the 2% target and therefore repeatedly overpredicts its realization. The transition model allows for more persistent below-target inflation and does not exhibit the same systematic overprediction. For forecasts originating around 2015, for example, the transition model predicts that core inflation will remain near 1%, close to the subsequent realization, whereas the target model forecasts a rapid return toward 2%.

Figure 2: Core Inflation Forecasts, Euro Area, Year-on-Year

(a) Transition Model



(b) Target Model



Notes: The solid line shows realized year-on-year core inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

The transition model's advantage for core inflation does not extend to headline inflation during 2010–2019. Forecasts originating around 2015, shown in the Appendix, illustrate why. The transition model predicts greater persistence below target, whereas the target model predicts a faster return toward 2%. Unlike core inflation, however, realized headline inflation begins to rise in 2016 and subsequently approaches 2%. The target model's faster mean reversion is therefore closer to the realization in this episode. Over the full 2010–2019 period, the two models have similar headline-inflation RMSFEs, with no statistically significant differences.

During the inflation surge after 2020, the transition and target models have similar forecast accuracy, with a slight advantage for the target model, although none of the differences are statistically significant. As in Japan, the gradual adjustment of the equilibrium weights disadvantages the transition model after the abrupt change in the inflation environment. All three models nevertheless underpredict the inflation surge by such a wide margin that the differences between them are secondary relative to the magnitude of their forecast errors. The target model's somewhat smaller errors during this short but volatile period offset part of the transi-

tion model's earlier gains, reducing its full-sample advantage for core inflation. Overall, the euro area results show that combining the two equilibria is particularly valuable during a persistent but incomplete shift toward the low-inflation equilibrium, as observed for core inflation during the 2010s.

3.2.3 United States

For the United States, the transition and target models have very similar full-sample RMSFEs, with only a slight advantage for the target model that is statistically significant in several cases, though often only at the 10% level. The low-inflation model performs considerably worse throughout. Across subsamples, the advantage of the target model is concentrated mainly in the 2010–2019 period, while the transition and target models perform very similarly before 2010 and after 2019.

This pattern is consistent with the equilibrium probabilities in Figure 1. The target equilibrium remains dominant, with temporary movements toward the low-inflation equilibrium concentrated in 2010–2019. The differences in forecast performance are likewise concentrated in this period: declines in inflation temporarily shift some weight toward the low-inflation equilibrium, and the gradual adjustment of the equilibrium weight causes the transition model to carry low inflation forward even as inflation begins to move back toward target, whereas the target model mean-reverts more quickly. Consequently, assigning positive weight to the low-inflation forecast rule does not produce the gains observed during the pronounced low-inflation episode in Japan or for euro area core inflation during the 2010s, but instead slightly reduces forecast accuracy relative to the target model during these temporary shifts. The U.S. results thus provide a useful contrast: without a persistent shift toward the low-inflation equilibrium, the transition mechanism offers no systematic forecasting gain relative to the target model.

Across the three economies, the low-inflation model performs worse than both alternatives over the full samples. Japan's pronounced low-inflation period is an important exception, during which the low-inflation model remains competitive with the transition model. This illustrates that a single-equilibrium model can perform well when the economy remains close to the corresponding equilibrium for a sustained period, but may be less suitable across changing inflation environments.

3.3 State-Dependent Tests of Relative Forecast Accuracy

To assess whether relative predictive accuracy across the three models varies with the inflation environment, we apply the state-dependent forecast-evaluation framework of Odendahl, Rossi, and Sekhposyan (2023) (ORS hereafter). In the preceding analysis based on conventional DM tests, we examined such state dependence indirectly by comparing relative forecast performance across predefined subsamples. The ORS framework instead allows relative predictive accuracy to vary directly with the inflation environment, so we apply the test to the full evaluation samples. We use the corresponding inflation measure observed at the forecast origin as the state variable and apply the framework to the squared-error loss differential between competing

forecasts. Section 5.2 applies the state-dependent framework to the forecast-efficiency moment instead.

3.3.1 Test Framework

For the relative-accuracy application, we define the squared-error loss differential between forecasts i and j as

$$\Delta L_{t+h|t}^{(i,j)} = \left(\pi_{t+h} - \widehat{\pi}_{t+h|t}^{(i)}\right)^2 - \left(\pi_{t+h} - \widehat{\pi}_{t+h|t}^{(j)}\right)^2. \tag{11}$$

Negative values indicate that forecast i is more accurate, while positive values favor forecast j. We model the conditional mean of the loss differential using a logistic smooth-transition specification,

$$\Delta L_{t+h|t}^{(i,j)} = \delta_0 + \delta_1 G(S_t;\varphi) + \varepsilon_{t+h}, \tag{12}$$

where S_t denotes headline or core inflation observed at the forecast origin. Following Teräsvirta (2006), the transition function is

$$G(S_t;\varphi) = G(S_t;\gamma,\tau) = \{1 + \exp\left[-\tau(S_t - \gamma)\right]\}^{-1}, \qquad \varphi = (\gamma,\tau) \in \mathbb{R} \times \mathbb{R}_{>0}. \tag{13}$$

The parameter γ determines the inflation level around which the transition occurs, while τ governs the smoothness of the transition.

The joint null hypothesis is H_0: $\delta_0 = \delta_1 = 0$. Under the null, the conditional mean of the loss differential is zero in every inflation state. A rejection may therefore reflect unequal average predictive accuracy, variation in relative predictive accuracy across inflation states, or both. Comparing the ORS and conventional DM results helps distinguish different forms of forecast-performance heterogeneity. An ORS rejection in the absence of a DM rejection is particularly informative, as it may indicate that similar average forecast accuracy conceals offsetting differences across inflation states. When both tests reject, the evidence is consistent with differences in average predictive accuracy and potentially also with state dependence. Conversely, a DM rejection without an ORS rejection may arise when the average difference is small and largely state-invariant, since the broader ORS joint test may have less power against such an alternative than the DM test, which focuses directly on average forecast loss. When neither test rejects, there is no evidence that similar average accuracy masks systematic differences across inflation states.

The transition parameters φ are not identified under the null hypothesis. For each admissible value of φ, the null hypothesis can nevertheless be tested using a Wald statistic $W_P(\varphi)$, where P denotes the number of forecast origins in the evaluation sample. To eliminate the dependence on a particular value of φ, ORS construct the following functionals over the parameter space

Φ:

$$g_\Phi(W_P) = \begin{cases} \sup_{\varphi \in \Phi} W_P(\varphi), & \text{sup-}W \\ \int_\Phi W_P(\varphi)\, dw(\varphi), & \text{ave-}W \\ \log\left\{ \int_\Phi \exp\left[\frac{1}{2} W_P(\varphi) \right] dw(\varphi) \right\}, & \text{exp-}W. \end{cases} \tag{14}$$

Here, $w(\varphi)\colon \Phi \to \mathbb{R}$ denotes a weighting function over Φ. Following ORS, we use a uniform weighting scheme. The sup-W statistic takes the largest Wald statistic over the parameter space, the ave-W statistic averages the Wald statistics, and the exp-W statistic places relatively greater weight on large Wald statistics. Under the regularity conditions in ORS and for compact Φ, these statistics converge to the corresponding functionals of a χ^2 process. Their null distributions are not nuisance-parameter-free, however. ORS therefore propose a simulation-based procedure to obtain p-values, which we follow.

In our application, the model parameters are calibrated rather than recursively estimated, so parameter-estimation error does not enter the test. To account for potential serial correlation in the loss differential, particularly that induced by overlapping multi-step forecasts, we follow the adjusted simulation algorithm of ORS and use a Bartlett kernel with bandwidth

$$B = 4(P/100)^{2/9} + 1,$$

where P denotes the number of forecast origins in the evaluation sample. We conduct the test at horizons $h \in \{1,2,4,8\}$. Following ORS, the parameter space for γ is determined by the 15th and 85th percentiles of observed inflation, while τ ranges from 0.1 to 5. We inspect the transition functions implied by this parameter space over the observed range of S_t to ensure that the τ-grid spans gradual to relatively sharp transitions and provides broad coverage of distinct transition shapes. We approximate the null distributions of the sup-W, ave-W, and exp-W statistics using $J = 1{,}000$ simulation replications under H_0: $\delta_0 = \delta_1 = 0$.

3.3.2 Results

The three statistics yield very similar conclusions in our application, so we report results based on the exp-W statistic. Table 3 reports the same full-sample relative RMSFEs as Table 2, while its significance markers refer to the exp-W test of the joint state-dependent null rather than to the conventional DM test. The relative RMSFEs summarize average forecast performance and help interpret the results, whereas the markers indicate whether the conditional mean of the loss differential is zero across inflation states, not whether the reported RMSFEs differ significantly from one.

For Japan, the conventional full-sample results indicate similar forecast accuracy for headline inflation, and the ORS test likewise does not reject its joint null. Thus, there is no evidence that similar average accuracy masks systematic differences in relative forecast performance across inflation environments. For core inflation, the conventional DM tests do not reject equal pre-

Table 3: Full-Sample Relative RMSFEs and State-Dependent Test Results

	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	4	8	1	2	4	8
				Japan				
1987Q2–2023Q1								
Transition/Target	0.97	0.97	1.02	1.04	0.92**	0.91***	0.96*	0.99
Transition/Low-Inflation	0.81*	0.73**	0.74**	0.74**	0.73**	0.66***	0.67**	0.68**
Target/Low-Inflation	0.83	0.75**	0.72**	0.71*	0.79**	0.73***	0.70***	0.69**
				Euro Area				
2001Q2–2023Q3								
Transition/Target	1.00	1.02	1.03	1.08	0.89***	0.91	0.88**	0.98
Transition/Low-Inflation	0.63***	0.60***	0.59***	0.62***	0.37***	0.36***	0.35***	0.39***
Target/Low-Inflation	0.63***	0.59***	0.57***	0.57***	0.42***	0.39***	0.39***	0.40***
				United States				
1990Q2–2023Q3								
Transition/Target	1.02	1.04	1.04	1.04**	1.01	1.01	1.03	1.03*
Transition/Low-Inflation	0.55***	0.48***	0.46***	0.45***	0.34***	0.30***	0.29***	0.28***
Target/Low-Inflation	0.54***	0.47***	0.44***	0.44***	0.34***	0.29***	0.28***	0.27***

Notes: Entries are relative RMSFEs based on revised data; values below one favor the model in the numerator. The reported sample periods refer to forecast origins. Stars indicate rejection of the joint null hypothesis of equal average predictive accuracy and no state dependence in the loss differential, H_0: $\delta_0 = \delta_1 = 0$, based on the exp-W statistic. They do not indicate whether the reported relative RMSFE differs significantly from one. The state variable is headline or core inflation observed at the forecast origin, as indicated. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

dictive accuracy, despite relative RMSFEs favoring the transition model, particularly at short horizons. The ORS test, by contrast, rejects its joint null at most horizons. The ORS rejections indicate that the conventional full-sample DM results do not provide a complete characterization of relative predictive performance. This interpretation is supported by the conventional subsample results in Section 3.2.1, which reveal a pronounced reversal in relative forecast performance across inflation environments: the transition model performs substantially better during the prolonged low-inflation period, whereas the target model performs better during the post-2020 inflation surge. For Japanese core inflation, the results therefore provide a clear case in which conventional full-sample tests alone give an incomplete assessment because they do not capture substantial variation in relative forecast performance across inflation environments.

For the euro area, the ORS results broadly confirm the conventional evidence. For headline inflation, neither the conventional DM test nor the ORS test rejects at any horizon, providing no evidence that the similar full-sample accuracy of the transition and target models masks offsetting differences across inflation states. The conventional subsample results corroborate this interpretation, as relative forecast performance remains similar across inflation environments. For core inflation, by contrast, both the ORS and conventional DM tests reject at selected short and medium horizons. These joint rejections are consistent with differences in average predictive accuracy and potentially also with state dependence. The pronounced variation in relative forecast performance across inflation environments documented in the conventional subsample results in Section 3.2.2 supports the latter interpretation.

For the United States, the conventional full-sample results slightly favor the target model over the transition model, whereas the ORS test rejects its joint null only at the longest horizon. The more limited ORS evidence is consistent with the broader ORS joint test having less power against small differences in average accuracy. Overall, the evidence suggests that the transition model offers no systematic forecasting gain in the United States, but also that the target model's average advantage is relatively modest.

For comparisons involving the low-inflation model, the ORS test rejects its joint null in nearly all cases, while the full-sample relative RMSFEs show that the low-inflation model is less accurate on average than both alternatives across all three economies.

Overall, the results show that combining the ORS analysis with the conventional DM and subsample evidence yields a more complete picture of both average predictive accuracy and how it varies across inflation environments.

4 Model-Based Forecasts versus Central Bank Projections

Having established when the transition model improves on the two single-equilibrium models, we next examine whether these gains remain economically relevant when the model-based forecasts are compared with central bank projections that draw on richer information sets and judgment. We initialize the recursive forecasting procedure described in Section 3 using real-time data vintages wherever available and pseudo-real-time data constructed with the relevant publication lags otherwise. In the real-time exercises, ω is re-calibrated for each data vintage. The resulting evaluation samples are shorter than those in Section 3 because real-time data and central bank projections are available for more limited periods.

4.1 Japan

We use projections from the Bank of Japan's *Outlook for Economic Activity and Prices* for current- and next-fiscal-year CPI inflation excluding fresh food, available from October 2003 onward.[10] Initially, the projections were published once a year in the fourth quarter. From 2005 to 2007, they were published in the second and fourth quarters, and thereafter on a quarterly basis. The forecast horizon therefore varies across Bank of Japan publications, as does the number of projections available for each fiscal year. For each publication, we construct model-based forecasts for the same current- and next-fiscal-year outcomes.[11]

Real-time inflation vintages are available monthly. At each model forecast origin, the corresponding vintage generally contains data through the final month of the preceding quarter. We aggregate these data to quarterly frequency and use an autoregressive model, with lag length selected by the Akaike Information Criterion, to nowcast inflation at the model forecast origin.

[10] Since April 2022, the Bank of Japan has also published projections for CPI inflation excluding fresh food and energy. We do not evaluate these projections because the available sample is too short.

[11] We also experimented with evaluating forecasts for the different horizons arising from the varying distance between forecast origins and the end of the current and next fiscal years, but the number of observations per horizon is very small. In addition, we transformed the central bank's fixed-event projections into fixed-horizon forecasts using the optimal weighting approach of Knüppel and Vladu (2025). Forecast accuracy deteriorated substantially, making these transformed forecasts unsuitable for comparison with the model-based forecasts.

The structural model forecast then begins in the subsequent quarter, which is also the quarter in which the corresponding Bank of Japan projection is published. We combine the available quarterly observations and the autoregressive nowcast with the subsequent model forecasts to construct projections for the same current- and next-fiscal-year outcomes reported by the Bank of Japan. This construction avoids using information that was not yet available at the model forecast origin, while the Bank of Japan may draw on a richer and more up-to-date information set when producing its projections. Real-time output-gap vintages are available from April 2017 onward and contain data through two quarters before publication, so we use an autoregressive model to estimate the previous and current quarters. For earlier forecast origins, we use revised output-gap data while imposing the relevant publication lag. We use the first-month average of the unrevised policy rate as the current-quarter nowcast. For the slowly changing one-sided natural-rate estimate, we assume that the current-quarter value was available at the forecast origin.

We compare the model-based forecasts with both the Bank of Japan's baseline projections and alternative projections excluding direct consumption-tax effects, particularly those associated with the 2014 increase. Figure 3 shows forecasts for the current and next fiscal years. Because the horizontal axis shows the dates of the corresponding Bank of Japan projections, realized inflation in the lower panel is shifted one year to the left to align each realization with the corresponding projection. The upper panel of Table 4 reports the models' RMSFEs relative to the baseline projections, with values in parentheses referring to comparisons with the alternative projections excluding the tax effect.

For current-fiscal-year forecasts, all three models are significantly less accurate than the Bank of Japan over the full sample. Much of this advantage, however, reflects developments that are difficult for the model-based forecasts to capture. In particular, the models do not incorporate the 2014 consumption-tax increase. Excluding its effect substantially narrows the full-sample differences, which are no longer statistically significant. During the 2010s, the differences become particularly small, with the transition model roughly matching and the target model outperforming the Bank of Japan once the tax effect is excluded. By contrast, the Bank of Japan tracks the post-2020 inflation surge more accurately.

For next-fiscal-year forecasts, the differences are substantially smaller and vary with the inflation environment. None of these differences is statistically significant. During persistent low inflation, the transition and low-inflation models remain competitive, whereas the target model is less accurate. During the post-2020 inflation surge, the Bank of Japan is more accurate than the transition and low-inflation models, while the target model remains competitive and even slightly outperforms the Bank of Japan. These shifts in relative performance help explain the modest full-sample differences.

Taken together, the results show that the Bank of Japan's richer information set is most valuable for current-fiscal-year forecasts and event-specific developments, while next-fiscal-year performance depends more strongly on the inflation environment. This mirrors the results in Section 3: models placing greater weight on the low-inflation equilibrium remain competitive during persistent low inflation but can perform less well when inflation rises abruptly. Figure 3 also shows

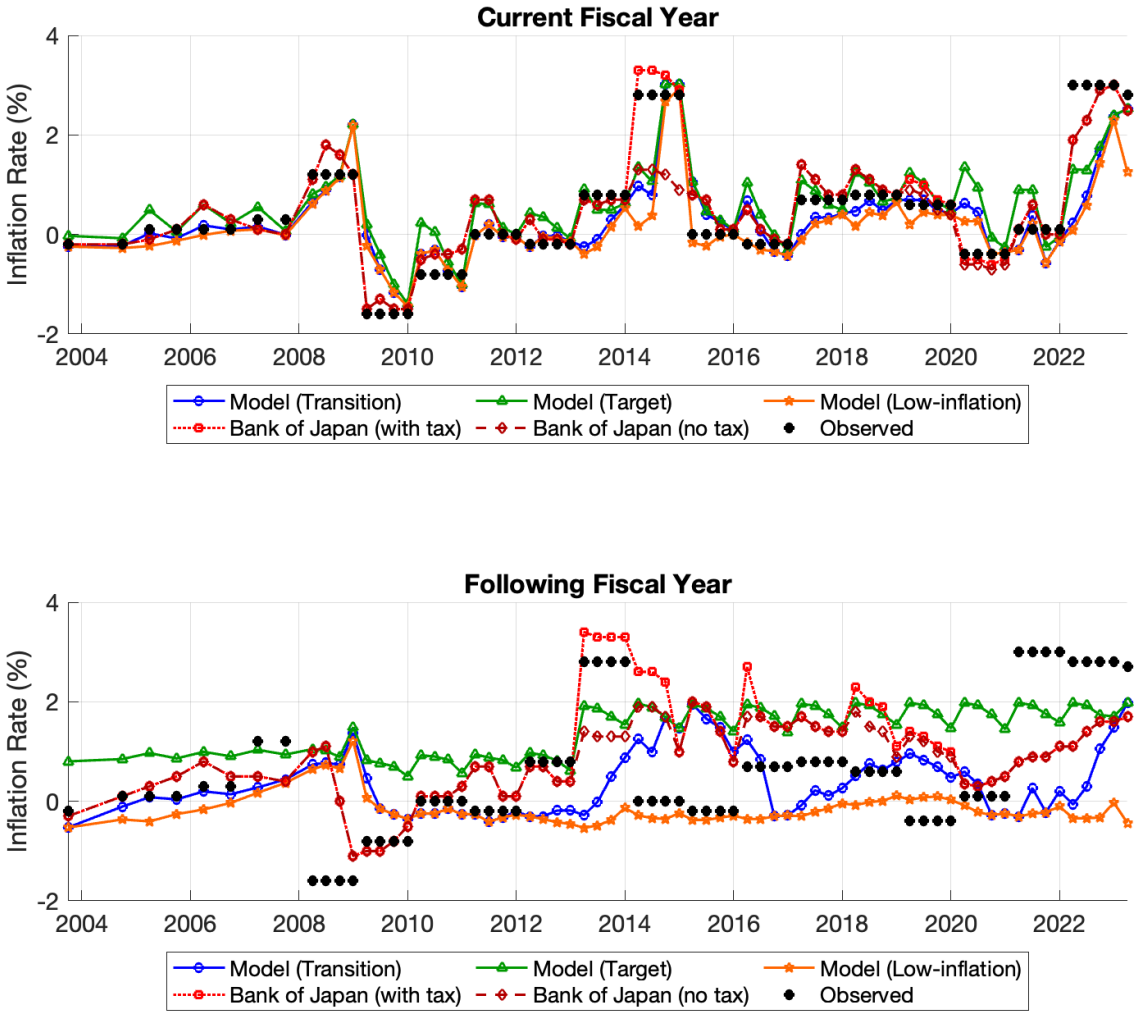


Figure 3: Model-Based and Bank of Japan Inflation Forecasts

Notes: The upper and lower panels show forecasts for the current and next fiscal years, respectively. The horizontal axis shows the dates of the corresponding Bank of Japan projections. In the lower panel, realized inflation is shifted one year to the left to align each realization with the corresponding projection. The alternative Bank of Japan projections exclude the effects of the 2014 consumption-tax increase. Inflation is measured by the CPI excluding fresh food.

that the Bank of Japan frequently overpredicts inflation during the prolonged low-inflation period, whereas the transition model does not exhibit an upward bias. The transition model may therefore serve as a useful complement to the Bank of Japan's projections.

4.2 Euro area

For the euro area, the evaluation sample begins in 2001 for headline inflation, when real-time inflation vintages first become available, and in 2009 for core inflation, when ECB projections for core inflation first become available. We use quarterly ECB staff projections from the Macroeconomic Projection Database (MPD). For headline inflation, the MPD provides quarterly projections of year-on-year HICP inflation. We combine these projections with the relevant observed price-index levels to recover the implied quarter-on-quarter inflation rates. We compare these rates with the model-based forecasts at horizons of one, two, four, and eight quarters. For core inflation, the MPD provides year-on-year projections for the current year, the following year, and two years ahead. These are fixed-event forecasts, as in the case of Japan, and we construct the corresponding model-based forecasts for the same horizons.

Table 4: Accuracy of Model-Based Forecasts Relative to Central Bank Projections

Japan

Forecast Horizon:	Current Fiscal Year			Next Fiscal Year		
Transition/Central Bank	Headline Inflation Excluding Fresh Food					
2003Q3–2023Q1	1.99**	(1.38	excl. tax)	1.17	(1.23	excl. tax)
2003Q3–2011Q4	1.43			1.20		
2012Q1–2019Q4	1.77	(0.97	excl. tax)	1.00	(1.09	excl. tax)
2020Q1–2023Q1	2.84			1.47		
Target/Central Bank						
2003Q3–2023Q1	1.95***	(1.35	excl. tax)	1.13	(1.18	excl. tax)
2003Q3–2011Q4	1.86			1.55		
2012Q1–2019Q4	1.60	(0.88	excl. tax)	1.04	(1.14	excl. tax)
2020Q1–2023Q1	2.57**			0.90		
Low-Infl./Central Bank						
2003Q3–2023Q1	2.21*	(1.53	excl. tax)	1.28	(1.34	excl. tax)
2003Q3–2011Q4	1.37			1.17		
2012Q1–2019Q4	2.08	(1.14	excl. tax)	0.98	(1.06	excl. tax)
2020Q1–2023Q1	3.15			1.84		

Euro Area

Forecast Horizon:		1	2	4	8		Current Year	Next Year	Two Years Ahead
Transition/Central Bank		Headline Inflation					Core Inflation		
2001Q2	AR-nowcast	1.07	1.04	1.11	1.04	2009Q2	2.86*	1.37	1.10
-2023Q3	CB-nowcast	1.00	1.02	1.11	1.04	-2023Q3			
2001Q2	AR-nowcast	1.10	0.94	0.95	0.97	2009Q2	1.67***	1.04	0.84
-2011Q1	CB-nowcast	0.99	0.93	0.95	0.97	-2019Q4			
2011Q2	AR-nowcast	0.96	1.02	1.07	0.99	2014Q1	1.51**	0.53***	0.40**
-2019Q4	CB-nowcast	0.99	1.01	1.08	0.99	-2018Q4			
2020Q1	AR-nowcast	1.08	1.08	1.17	1.07	2020Q1	3.14**	1.42	1.17
-2023Q3	CB-nowcast	1.01	1.05	1.16	1.07	-2023Q3			
Target/Central Bank									
2001Q2	AR-nowcast	1.02	0.97	0.99	0.93*	2009Q2	2.69**	1.28**	0.98
-2023Q3	CB-nowcast	0.96	0.96	0.99	0.93*	-2023Q3			
2001Q2	AR-nowcast	1.03	0.85**	0.86***	0.91*	2009Q2	2.18***	1.99***	1.35***
-2011Q1	CB-nowcast	0.93	0.85*	0.86***	0.91*	-2019Q4			
2011Q2	AR-nowcast	0.98	1.09	1.15*	0.94	2014Q1	2.05***	2.15***	1.45***
-2019Q4	CB-nowcast	1.05	1.10	1.15*	0.94	-2018Q4			
2020Q1	AR-nowcast	1.02	0.98	1.00	0.92	2020Q1	2.83**	1.12	0.85
-2023Q3	CB-nowcast	0.95	0.96	1.00	0.92	-2023Q3			

United States

Forecast Horizon:		1	2	3	4	1	2	3	4
Transition/Central Bank		Headline Inflation				Core Inflation			
1996Q2	AR-nowcast	1.45***	0.99	1.01	1.03	1.08	1.05	1.07	1.06
-2019Q4	CB-nowcast	1.34**	0.99	1.01	1.03	1.06	1.03	1.07	1.06
1996Q2	AR-nowcast	1.42**	0.92	0.96	0.99	0.91	1.01	1.04	1.04
-2009Q4	CB-nowcast	1.32	0.93	0.96*	0.99	0.94	0.98	1.04	1.04
2010Q1	AR-nowcast	1.57***	1.18	1.12	1.10	1.37**	1.07	1.09	1.07
-2019Q4	CB-nowcast	1.43**	1.18	1.12	1.10	1.27*	1.06	1.09	1.06
Target/Central Bank									
1996Q2	AR-nowcast	1.39***	0.93	0.96**	0.98	0.95	0.93	0.94	0.93
-2019Q4	CB-nowcast	1.29*	0.94	0.96**	0.98	0.94	0.92	0.94	0.93
1996Q2	AR-nowcast	1.41**	0.92	0.96*	0.99	0.89	1.00	1.03	1.01
-2009Q4	CB-nowcast	1.31	0.92	0.95**	0.99	0.93	0.97	1.02	1.01
2010Q1	AR-nowcast	1.32**	0.98	0.96	0.95	1.04	0.89*	0.90	0.88
-2019Q4	CB-nowcast	1.22*	0.99	0.97	0.95	0.96	0.88*	0.90	0.88

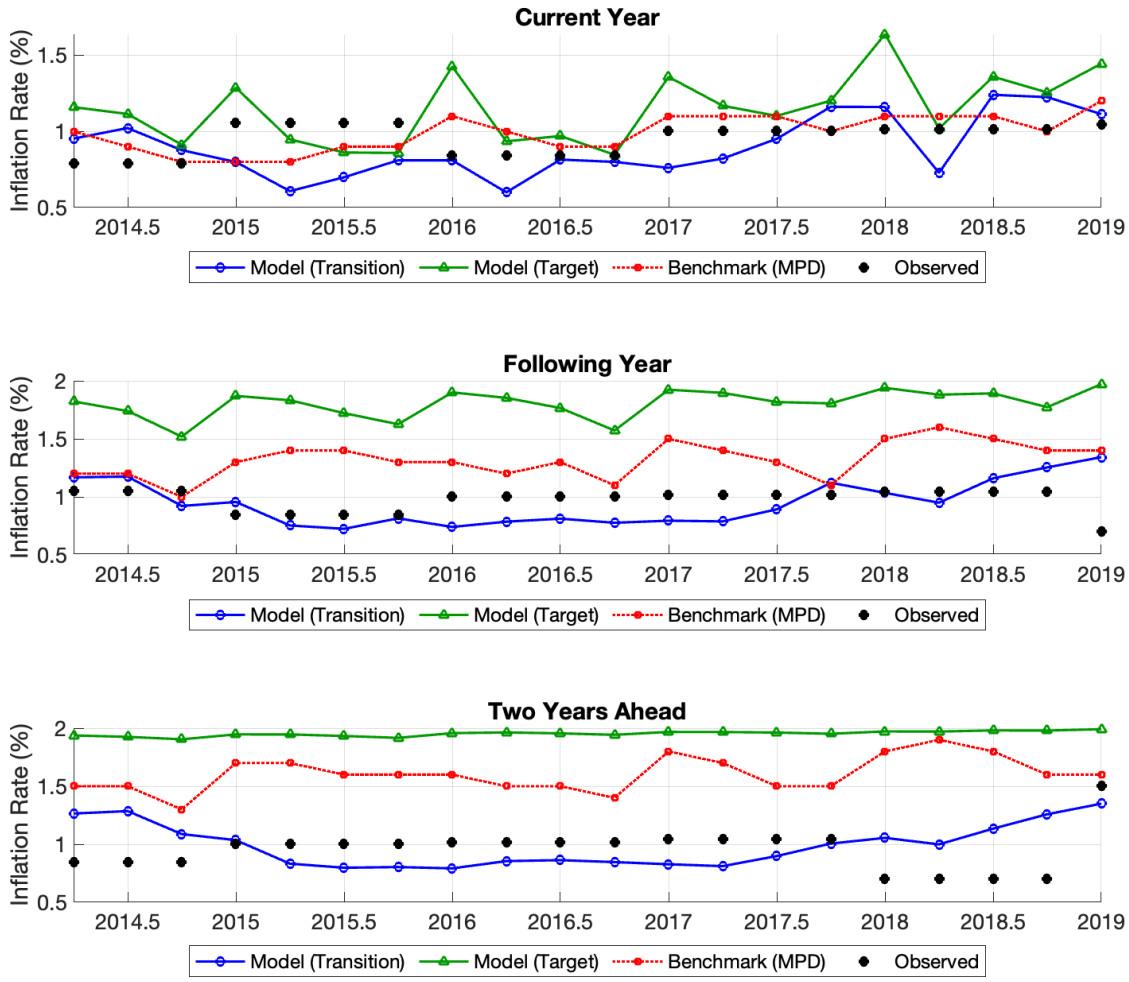
Notes: Entries are relative RMSFEs; values below one favor the model-based forecast. Sample periods refer to model forecast origins. AR-nowcast and CB-nowcast denote model forecasts initialized with autoregressive and central bank nowcasts, respectively. For Japan, values in parentheses use Bank of Japan projections excluding the 2014 tax increase. Stars indicate rejection of equal predictive accuracy based on the Diebold–Mariano test, with *, **, and *** denoting significance at the 10%, 5%, and 1% levels, respectively.

Real-time inflation vintages contain observations through the quarter preceding the forecast origin. For headline inflation, we initialize the model forecasts in two ways. In the CB-nowcast specification, we append the ECB's current-quarter inflation nowcast to the real-time vintage, thereby giving the model the same information about current-quarter inflation as the ECB projection. In the AR-nowcast specification, current-quarter inflation is instead estimated using an autoregressive model. Core inflation requires a different construction because the MPD reports annual fixed-event projections rather than a quarterly forecast path. As for Japan, we set the model forecast origin one quarter before the corresponding ECB projection, use an autoregressive model to nowcast inflation at that origin, and let the structural model forecast begin in the ECB publication quarter. We then combine the available observations, the autoregressive nowcast, and the subsequent model forecasts to construct projections for the same current-, following-, and two-year-ahead outcomes reported by the ECB. Because the MPD does not provide a corresponding quarterly core-inflation nowcast, we report only this AR-nowcast specification. Real-time output-gap vintages from the European Commission's AMECO database are available from 2011Q2 onward and provide annual estimates, which we assign to all quarters of the corresponding year. For earlier forecast origins, we use the revised output-gap data employed in Section 3, imposing availability only through the forecast origin. For headline inflation, we therefore report separate subsamples ending in 2011Q1 and beginning in 2011Q2. This distinguishes forecasts based entirely on real-time data from those combining real-time inflation vintages with revised output-gap data. As for Japan, we use the average policy rate in the first month of the current quarter as the nowcast for that quarter and assume that the slowly changing natural-rate estimate for the current quarter was available at the forecast origin.

Results for the euro area are reported in the middle panel of Table 4. In what follows, we restrict the comparison with central bank projections to the transition and target models, since the low-inflation model performs substantially worse than both models throughout for the euro area and the United States. For headline inflation, initializing the models with the ECB nowcast rather than the AR nowcast noticeably reduces their one-quarter-ahead relative RMSFEs, while making little difference at longer horizons. Overall, the model-based forecasts and ECB projections have similar forecast accuracy. The ECB has a slight advantage over the transition model at most horizons, whereas the target model tends to be slightly more accurate than the ECB projections. These differences are generally small and mostly statistically insignificant.

For core inflation, the ECB's projections are significantly more accurate than both model-based forecasts for the current year over the full sample. At the following-year horizon, the ECB significantly outperforms the target model. The difference relative to the transition model is somewhat larger but statistically insignificant. At the two-year-ahead horizon, the differences are small and statistically insignificant. The full-sample results, however, mask a pronounced change in relative performance for forecasts originating during the period when core inflation was particularly low, from 2014Q1 to 2018Q4. For current-year forecasts, the ECB remains significantly more accurate than both models. At the following-year and two-year-ahead horizons, by contrast, the transition model significantly outperforms the ECB, reducing RMSFEs by about 50% and 60%, respectively. This horizon pattern is consistent with the ECB's richer informa-

Figure 4: Model-Based and ECB Core Inflation Forecasts



Notes: The horizontal axis shows the dates of the corresponding ECB projections. Realized year-on-year core inflation is aligned with the corresponding projection. In the following-year and two-year-ahead panels, the realizations are therefore shifted one and two years to the left, respectively.

tion set being most valuable for current-year forecasts, while the transition model's allowance for persistent below-target inflation becomes more valuable at longer horizons. Figure 4 shows that the ECB systematically overpredicts core inflation during this period, whereas the transition model captures its persistence more accurately. The target model, by contrast, remains significantly less accurate than the ECB at all three horizons, as its forecasts return relatively quickly toward the inflation target. This mirrors the large differences between the transition- and target-model forecasts for core inflation during the 2010s documented in Section 3.

4.3 United States

For the United States, we use Greenbook/Tealbook staff projections for quarter-on-quarter headline and core CPI inflation. The evaluation sample begins in 1996Q2, when real-time output-gap estimates first become available. The Greenbook/Tealbook data include real-time quarterly inflation and output-gap estimates as well as current-quarter nowcasts. In the CB-nowcast specification, we use the Greenbook/Tealbook current-quarter nowcast to initialize the model. When two Greenbook/Tealbook rounds fall within a quarter, we use the second

publication for both the projections and the corresponding current-quarter nowcast. Real-time core CPI vintages begin in 1998Q4. For earlier forecast origins, we use the revised core CPI series because historical revisions are small. We report both CB-nowcast and AR-nowcast specifications and apply the same timing conventions for the policy rate and natural interest rate as for Japan and the euro area.

The results for the United States are reported in the lower panel of Table 4. For both headline and core inflation, the transition model generally has somewhat higher RMSFEs than the Greenbook/Tealbook, whereas the target model generally has somewhat lower RMSFEs, with statistically significant advantages at some medium horizons. Overall, the differences in forecast accuracy are generally modest. A notable exception is headline inflation at the one-quarter-ahead horizon, where the Greenbook/Tealbook is substantially more accurate than the transition and target models.

The one-quarter-ahead headline results illustrate the importance of the central bank's richer and more timely information set. Initializing the models with the Greenbook/Tealbook current-quarter nowcast narrows the gap, but does not eliminate it. As a timing sensitivity check, replacing the second within-quarter vintage used in our baseline with the earlier vintage substantially reduces the central bank's advantage, both economically and, in several specifications, statistically, while relative forecast accuracy at longer horizons changes little. This suggests that the informational advantage of the Greenbook/Tealbook is concentrated at very short horizons and that providing the model with the central bank nowcast captures only part of this advantage.

The transition model also does not improve on the Greenbook/Tealbook projections during the 2010–2019 low-inflation period. During this period, the Greenbook/Tealbook projections tended to carry low inflation forward more persistently and were therefore closer in this respect to the transition model than to the target model. The target model generally performs somewhat better than the Greenbook/Tealbook projections, although these differences are mostly statistically insignificant. This ranking mirrors the results in Section 3: U.S. low-inflation episodes were comparatively short-lived, the target-equilibrium probability repeatedly returned close to one, and inflation remained closer to target than in the euro area or Japan.

4.4 State-Dependent Comparisons with Central Bank Projections

As in Section 3.3, we apply the ORS framework to the squared-error loss differentials between the model-based forecasts and central bank projections, allowing their relative predictive accuracy to vary with the inflation environment. Because the sup-W, ave-W, and exp-W statistics again yield very similar conclusions, we report the exp-W statistic. Since the ORS results largely confirm the conventional forecast comparisons, we report the full-sample relative RMSFEs together with the results of the ORS test of the joint null of equal average predictive accuracy and no state dependence in the loss differential in Appendix Table A.9.

One exception involving the transition model occurs for euro area core inflation at the next-year horizon, where the ORS test provides weak evidence of differences in predictive performance not detected by the conventional DM test. This pattern is consistent with relative predic-

tive performance varying across inflation environments. The conventional subsample results in Section 4.2 support this interpretation: although the full-sample relative RMSFE favors the ECB, the transition model achieves substantial gains during the low-inflation period. Other differences between the ORS and conventional tests are isolated and do not change the overall conclusions.

Compared with the model-to-model analysis in Section 3.3, the state-dependent tests therefore add less beyond the conventional full-sample evidence in the central-bank comparisons. This closer correspondence may partly reflect the shorter evaluation samples in several of these comparisons, which provide fewer observations spanning distinct inflation environments, as well as the fact that differences relative to information-rich central bank projections are often already visible in average forecast accuracy, particularly at short horizons.

Taken together, Sections 3 and 4 show that the transition model's forecasting gains are concentrated in persistent low-inflation environments rather than arising systematically across countries and inflation environments. Central bank projections are particularly accurate at short horizons, while the transition model can match or outperform them at longer horizons in persistent low-inflation environments. The state-dependent analysis provides additional information beyond conventional full-sample tests in selected model-to-model comparisons, while the ORS and DM results are much more closely aligned in the central-bank comparisons. Together with the conventional subsample evidence, these results show that full-sample averages can in some cases conceal economically relevant variation in relative forecast performance across inflation environments. Having established the relative performance of the competing forecasts, Section 5 turns to their calibration and state-dependent forecast efficiency.

5 Forecast Calibration and State-Dependent Forecast Efficiency

To complement the analysis of relative forecast accuracy, we examine whether the forecasts satisfy standard calibration and efficiency conditions. A forecasting approach may outperform an alternative without satisfying these conditions. We proceed in two closely related steps. First, Mincer–Zarnowitz regressions assess whether forecast errors have zero mean and are orthogonal to the forecasts. Second, the ORS analysis focuses on the latter orthogonality condition and examines the corresponding forecast-efficiency moment while allowing its conditional mean to depend on the inflation environment.

5.1 Linear Forecast Calibration

We assess linear forecast calibration using Mincer–Zarnowitz regressions. Specifically, we regress realized inflation on a constant and the corresponding forecast,

$$\pi_{t+h} = \beta_0 + \beta_1 \hat{\pi}_{t+h|t} + \varepsilon_{t+h}, \tag{15}$$

and test the joint null hypothesis that $\beta_0 = 0$ and $\beta_1 = 1$. Under the null, forecast errors have zero mean and are uncorrelated with the forecasts. The reported p-values are based on Wald tests using Newey–West heteroskedasticity- and autocorrelation-consistent standard errors. For

Table 5: Full-Sample p-Values for the Mincer–Zarnowitz Joint Test (Revised Data)

	Headline Inflation				Core Inflation			
Horizon:	1	2	4	8	1	2	4	8
				Japan (1987Q2–2023Q1)				
Transition	0.11	0.18	0.01	0.01	0.55	0.55	0.31	0.08
Target	0.02	0.00	0.02	0.15	0.00	0.00	0.01	0.07
Low-Inflation	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
				Euro Area (2001Q2–2023Q3)				
Transition	0.06	0.14	0.03	0.05	0.02	0.17	0.41	0.50
Target	0.01	0.12	0.74	0.01	0.00	0.00	0.31	0.00
Low-Inflation	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
				United States (1990Q2–2023Q3)				
Transition	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03
Target	0.02	0.01	0.05	0.11	0.00	0.00	0.00	0.05
Low-Inflation	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Entries are p-values for the joint Wald test of $\beta_0 = 0$ and $\beta_1 = 1$, based on Newey–West heteroskedasticity- and autocorrelation-consistent standard errors with $h - 1$ lags. Detailed coefficient estimates and subsample results are reported in Appendix Tables A.1–A.3.

fixed-horizon forecasts, we use $h - 1$ lags, with h measured in quarters. For annual fixed-event forecasts, we use $h_{\max} - 1$ lags, where $h_{\max}$ denotes the maximum forecast horizon in quarters of the respective fixed event. We estimate the regressions separately for the revised-data and real-time forecast exercises discussed above. Tables 5 and 6 report the full-sample p-values for the joint test. To keep these tables compact, the underlying coefficient estimates and the subsample results are reported in Appendix Tables A.1–A.8. We use these additional results below to interpret non-rejections and to assess whether calibration differs across inflation environments.

For the revised-data forecasts, the joint calibration null is rejected in most specifications. The main exception is the transition model, which yields a systematic pattern of non-rejections in Japan and the euro area, particularly for core inflation. In Japan, the null is not rejected across the core-inflation horizons and also for headline inflation at short horizons. In the euro area, the favorable results are concentrated mainly at medium horizons for core inflation, with some additional non-rejections for headline inflation at short horizons. The target model yields only isolated non-rejections without a comparably clear pattern, while the low-inflation model is rejected throughout the full sample.

The subsample results in Appendix Tables A.1–A.3 show that calibration varies substantially across inflation environments. For the transition model, the comparatively favorable full-sample evidence for Japan and the euro area does not extend uniformly across subperiods. In the United States, rejections remain predominant across subsamples. At the same time, some specifications rejected over the full sample are not rejected in particular periods. The clearest example is the low-inflation model in Japan, for which the calibration restrictions are rejected over the full sample but not in several specifications during the pronounced low-inflation period. These differences between full-sample and subsample results motivate the state-dependent analysis below.

The coefficient estimates in the same tables help assess how informative these non-rejections are. A non-rejection is less informative when the point estimates are far from the joint restrictions, since it may then primarily reflect imprecise estimation rather than estimates consistent with calibration. This concern is generally less pronounced for the full-sample non-rejections of the transition model, although it applies to some subsample results. A few target-model non-rejections are more clearly associated with extreme coefficient estimates and substantial estimation uncertainty.

The cross-country pattern is consistent with the equilibrium probabilities discussed above. In Japan and the euro area, changes in the weight on the low-inflation equilibrium allow the transition model to generate forecasts that differ materially from those of the target model, and the transition model exhibits comparatively favorable calibration results, particularly for core inflation. In the United States, by contrast, the target equilibrium receives most of the weight, so transition-model forecasts remain close to those of the target model. The transition mechanism therefore has limited scope to alter the calibration properties of the target model, consistent with the largely unfavorable calibration results for both models.

We report the real-time results primarily as a robustness check and to compare the model-based forecasts with central bank projections over common real-time evaluation samples. Overall, the model-based results in Table 6 broadly preserve the cross-country patterns found in the revised-data analysis, although differences in forecast horizons and evaluation samples preclude a direct one-to-one comparison. In Japan, the calibration restrictions for the transition model are not rejected for current-fiscal-year forecasts but are rejected for next-fiscal-year forecasts, broadly consistent with the more favorable short-horizon than medium-horizon results for headline inflation using revised data. The target model is rejected at both fiscal-year horizons, while the low-inflation model yields only an isolated non-rejection for the current fiscal year. In the euro area, transition-model headline forecasts are predominantly rejected, whereas the restrictions are not rejected for core inflation beyond the current year, broadly echoing the more favorable medium-horizon core-inflation results using revised data. The target model yields more non-rejections, particularly for headline inflation, but several are accompanied by extreme coefficient estimates and substantial estimation uncertainty. In the United States, the transition and low-inflation models are rejected throughout. The target model yields considerably more non-rejections than in the revised-data exercise, particularly beyond the shortest horizon, although several corresponding coefficient estimates remain some distance from the calibration restrictions.

Central bank projections satisfy the calibration restrictions only selectively. The full-sample results in Table 6 are favorable for the ECB projections, particularly for core inflation, but the subsample results in the Appendix show that their calibration deteriorates during the low-inflation period. This is consistent with the forecast-accuracy evidence in Section 4: at medium horizons, the ECB systematically overpredicted core inflation during this period, whereas the transition model captured its persistence more accurately. For the Bank of Japan, the calibration restrictions are rejected for both fiscal-year forecasts when the 2014 consumption-tax effect is included, but are not rejected for either horizon once this effect is excluded. Green-

Table 6: Full-Sample p-Values for the Mincer–Zarnowitz Joint Test (Real-Time Data)

Japan (2003Q3–2023Q1)								
Forecast Horizon:	Current Fiscal Year				Next Fiscal Year			
	Headline Inflation Excluding Fresh Food							
Transition	0.59				0.00			
Target	0.01				0.00			
Low-Inflation	0.09				0.00			
Bank of Japan	0.03 (0.17 excl. tax)				0.02 (0.52 excl. tax)			
Euro Area (headline: 2001Q2–2023Q3; core: 2009Q2–2023Q3)								
Forecast Horizon:	1	2	4	8	Current Year	Next Year	Two Years Ahead	
	Headline Inflation				Core Inflation			
Transition	0.04	0.03	0.06	0.04	0.00	0.38	0.28	
Target	0.18	0.58	0.33	0.20	0.05	0.86	0.00	
Low-Inflation	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
ECB	0.08	0.13	0.12	0.00	0.80	0.26	0.53	
United States (1996Q2–2019Q4)								
Forecast Horizon:	1	2	3	4	1	2	3	4
	Headline Inflation				Core Inflation			
Transition	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Target	0.00	0.24	0.63	0.69	0.03	0.40	0.12	0.20
Low-Inflation	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Greenbook/Tealbook	0.39	0.00	0.00	0.08	0.00	0.00	0.00	0.00

Notes: Entries are p-values for the joint Wald test of $\beta_0 = 0$ and $\beta_1 = 1$, based on Newey–West heteroskedasticity- and autocorrelation-consistent standard errors. For fixed-horizon forecasts, the lag length is $h-1$, with h measured in quarters; for annual fixed-event forecasts, it is $h_{\max}-1$, where $h_{\max}$ denotes the maximum forecast horizon in quarters of the respective fixed event. The model-based forecasts use autoregressive inflation nowcasts. For the Bank of Japan, values in parentheses refer to projections excluding the effects of the 2014 consumption-tax increase. Detailed coefficient estimates and subsample results are reported in Appendix Tables A.4–A.8.

book/Tealbook projections yield mixed results for headline inflation and are rejected throughout for core inflation. Rejections of the calibration restrictions are therefore not confined to the model-based forecasts, and central bank projections do not exhibit uniformly better calibration properties.

5.2 State-Dependent Forecast Efficiency

The full-sample Mincer–Zarnowitz tests do not reveal whether departures from their joint restrictions are stable over time or concentrated in particular inflation environments. This distinction matters because forecast-optimality violations may arise only during particular episodes and may be obscured by conventional full-sample tests in the presence of instability (Rossi and Sekhposyan, 2016). Motivated by this possibility, we apply the ORS framework to the forecast-efficiency moment and allow its conditional mean to depend on the inflation environment. For forecasting approach i, the moment is defined as

$$FE^{(i)}_{t+h|t} = \left(\pi_{t+h} - \hat{\pi}^{(i)}_{t+h|t}\right)\hat{\pi}^{(i)}_{t+h|t}. \tag{16}$$

Under weak forecast efficiency, its conditional mean is zero, corresponding to orthogonality between the forecast error and the corresponding forecast. Relative to the Mincer–Zarnowitz test, the ORS analysis focuses on this condition and does not separately test whether forecast errors have zero mean.[12] To allow the conditional mean to vary smoothly with the inflation environment, we use the logistic smooth-transition specification

$$FE^{(i)}_{t+h|t} = \delta_0 + \delta_1 G(S_t; \varphi) + \varepsilon_{t+h}, \tag{17}$$

where S_t denotes headline or core inflation observed at the forecast origin and $G(S_t; \varphi)$ is the logistic transition function defined above. The joint null hypothesis is H_0: $\delta_0 = \delta_1 = 0$. Under the null, the conditional mean of $FE^{(i)}_{t+h|t}$ is zero across inflation states. The null therefore combines a zero average forecast-efficiency moment with the absence of state dependence. A rejection may result from a nonzero average moment, variation in the moment across inflation states, or both.

We use the same nuisance-parameter space, weighting scheme, and simulation procedure as in the ORS analysis in Section 3.3. As the sup-W, ave-W, and exp-W statistics produce broadly similar results, we report simulated p-values for the exp-W statistic based on $J = 1{,}000$ simulations.

Table 7 reports the ORS results for the revised-data model forecasts. In the corresponding Mincer–Zarnowitz analysis, the transition model was the main exception to otherwise frequent calibration rejections, with comparatively favorable results in Japan and the euro area, particularly for core inflation. This broad cross-country assessment is preserved under the ORS test, although the rejection patterns differ in several respects. For headline inflation, the ORS test results are more favorable than the Mincer–Zarnowitz results for the transition model in both Japan and the euro area, with fewer rejections of weak forecast efficiency. The Mincer–Zarnowitz coefficient estimates help explain this difference: positive intercepts are accompanied by slopes substantially below one, indicating that the forecasts are miscalibrated with respect to the level and variation of realized inflation. These deviations can partly offset in the single forecast-efficiency moment assessed by the ORS test. Non-rejections by the ORS test therefore do not overturn the evidence of miscalibration, but show that the corresponding weaker orthogonality condition is not rejected. For core inflation, the transition model continues to yield comparatively favorable results in Japan and the euro area, although the ORS and Mincer–Zarnowitz rejection patterns do not coincide in all cases. In the United States, by contrast, both tests yield predominantly unfavorable results for the transition model. The target model yields a more mixed ORS pattern, with non-rejections concentrated mainly in euro area headline inflation, while the low-inflation model is rejected in almost all specifications. Overall, the ORS evidence therefore refines rather than overturns the Mincer–Zarnowitz assessment.

Table 8 reports the state-dependent weak-efficiency results for the real-time model forecasts and central bank projections on common evaluation samples. Overall, the central bank projections

[12] ORS refer to this as the forecast-efficiency moment. We use the qualifier "weak" to emphasize that orthogonality is assessed with respect to the forecast itself rather than the full information set available at the forecast origin.

Table 7: Simulated p-Values for the State-Dependent Test of Weak Forecast Efficiency: Model-Based Forecasts

	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	4	8	1	2	4	8
				Japan				
1987Q2–2023Q1								
Transition model	0.698	0.125	0.126	0.136	0.609	0.006	0.127	0.173
Target model	0.126	0.000	0.013	0.114	0.000	0.000	0.000	0.021
Low-Inflation model	0.032	0.015	0.016	0.022	0.102	0.015	0.010	0.018
				Euro Area				
2001Q2–2023Q3								
Transition model	0.164	0.253	0.442	0.609	0.104	0.052	0.137	0.416
Target model	0.134	0.204	0.439	0.518	0.000	0.000	0.001	0.172
Low-Inflation model	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000
				United States				
1990Q2–2023Q3								
Transition model	0.032	0.012	0.011	0.021	0.000	0.000	0.000	0.000
Target model	0.053	0.030	0.029	0.037	0.000	0.000	0.000	0.000
Low-Inflation model	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Notes: Entries are simulated p-values for the exp-W statistic testing H_0: $\delta_0 = \delta_1 = 0$ in equation (17), under which the conditional mean of the weak forecast-efficiency moment is zero across inflation states. The results are based on $J = 1{,}000$ simulations. The state variable S_t is headline or core inflation observed at the forecast origin, as indicated. The tests use revised data.

are rejected less frequently than the model-based forecasts. Weak forecast efficiency is not rejected for any of the ECB projections or for the Bank of Japan projections once the 2014 consumption-tax effect is excluded. The Greenbook/Tealbook projections are also rejected only in a few cases. Compared with the Mincer–Zarnowitz results in Table 6, the central bank evidence is therefore substantially more favorable in the ORS analysis. Many departures from the stronger calibration restrictions do not carry over to the weaker orthogonality condition tested by ORS.

The real-time model results help assess whether the relatively favorable ORS evidence for central bank projections mainly reflects the shorter evaluation samples. The model-based forecasts also receive a more favorable assessment on some real-time samples, most clearly in the United States, indicating that the evaluation sample matters. Using the change in model-based results as a benchmark for assessing the role of the evaluation sample, we find that the ECB and, after the tax adjustment, the Bank of Japan remain comparatively favorable on the common real-time samples. The U.S. results provide less evidence of a systematic central-bank advantage, while the low-inflation model remains the least favorable of the model-based forecasts across both exercises.

Comparing the calibration and weak-efficiency results with the relative forecast-accuracy evidence reveals the clearest overlap for euro area core inflation, where the transition model combines substantial accuracy gains during persistent low-inflation periods with comparatively favorable full-sample calibration and weak-efficiency results. Together, the two evaluations sup-

Table 8: Simulated p-Values for the State-Dependent Test of Weak Forecast Efficiency (Real-Time Data)

Japan (2003Q3–2023Q1)								
Forecast Horizon:	Current Fiscal Year				Next Fiscal Year			
	Headline Inflation Excluding Fresh Food							
Transition	0.002				0.091			
Target	0.000				0.000			
Low-Inflation	0.036				0.000			
Bank of Japan	0.151 (0.375 excl. tax)				0.039 (0.162 excl. tax)			
Euro Area (headline: 2001Q2–2023Q3; core: 2009Q2–2023Q3)								
Forecast Horizon:	1	2	4	8	Current Year	Next Year	Two Years Ahead	
	Headline Inflation				Core Inflation			
Transition	0.034	0.219	0.621	0.648	0.219	0.128	0.001	
Target	0.022	0.140	0.433	0.526	0.000	0.000	0.066	
Low-Inflation	0.001	0.000	0.000	0.000	0.513	0.000	0.000	
ECB	0.286	0.755	0.770	0.312	0.414	0.133	0.389	
United States (1996Q2–2019Q4)								
Forecast Horizon:	1	2	3	4	1	2	3	4
	Headline Inflation				Core Inflation			
Transition	0.493	0.272	0.218	0.094	0.000	0.136	0.078	0.141
Target	0.413	0.289	0.580	0.255	0.000	0.452	0.180	0.307
Low-Inflation	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Greenbook/Tealbook	0.456	0.605	0.947	0.508	0.075	0.130	0.129	0.033

Notes: Entries are simulated p-values for the exp-W statistic testing H_0: $\delta_0 = \delta_1 = 0$ in equation (17), under which the conditional mean of the weak forecast-efficiency moment is zero across inflation states. The results are based on $J = 1{,}000$ simulations. The state variable S_t is headline or core inflation observed at the forecast origin, as indicated. The tests use the corresponding real-time forecast samples. The model-based forecasts use autoregressive inflation nowcasts.

port the usefulness of the transition model in persistent low-inflation environments, without establishing that its forecasts are uniformly calibrated or efficient.

6 Conclusion

We investigate whether allowing inflation expectations to de-anchor from the inflation target improves inflation forecasts. The transition model can generate substantial gains when both equilibria are relevant, as when a shift toward the low-inflation equilibrium has begun but remains incomplete. It can also adapt as the relative importance of the two equilibria changes over time, allowing it to accommodate samples that span periods of inflation fluctuating around target and persistent low-inflation episodes. When one equilibrium clearly dominates, its forecast accuracy is similar to that of the corresponding single-equilibrium model. The gradual adjustment of the equilibrium probabilities can, however, delay the transition forecast's response to abrupt changes in the inflation environment.

These properties also define the model's role in practical forecasting. Central bank projections exploit much richer information sets and remain particularly strong at short horizons. At medium horizons, however, persistent changes in underlying inflation become more important, and the transition model can provide a useful complementary perspective. This is particularly evident for euro area core inflation, where it avoids the persistent overprediction observed in

ECB projections during the low-inflation period. A similar pattern appears in Japan, where Bank of Japan projections frequently overpredict inflation during the prolonged low-inflation period, whereas the transition model does not exhibit an upward bias. Taken together, the results suggest that the transition framework can complement standard model-based and information-rich institutional forecasts, offering particular gains during persistent low-inflation periods.

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Appendix to "Improving Inflation Forecasting by Modeling Expectation De-Anchoring in Low-Inflation Periods"

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A Data and Central Bank Projections

A.1 Macroeconomic Data

A.1.1 United States

Headline Inflation We use the Consumer Price Index for All Urban Consumers: All Items in U.S. City Average (`CPIAUCSL`), seasonally adjusted, with quarterly observations constructed as averages of the monthly index. Revised data are obtained from FRED: `https://fred.stlouisfed.org/series/CPIAUCSL`. Real-time vintages are obtained from the Federal Reserve Bank of Philadelphia Real-Time Data Research database: `https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/cpi`.

Core Inflation We use the Consumer Price Index for All Urban Consumers: All Items Less Food and Energy in U.S. City Average (`CPILFESL`), seasonally adjusted, with quarterly observations constructed as averages of the monthly index. Revised data are obtained from FRED: `https://fred.stlouisfed.org/series/CPILFESL`. Real-time vintages are obtained from the Federal Reserve Bank of Philadelphia Real-Time Data Research database: `https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/pcpix`. Real-time core CPI vintages are available from 1998Q4 onward. For earlier forecast origins, we use revised core CPI data, for which historical revisions are small.

Natural Rate of Interest The natural rate of interest is taken from the one-sided estimates provided by the Federal Reserve Bank of New York; see Federal Reserve Bank of New York (2026). Data are obtained from `https://www.newyorkfed.org/research/policy/rstar`.

Policy Rate We use quarterly averages of the Effective Federal Funds Rate (`FEDFUNDS`) from FRED. Data are obtained from `https://fred.stlouisfed.org/series/FEDFUNDS`.

Output Gap The revised output gap is the natural logarithm of the ratio of real GDP to potential GDP. Real GDP is measured by `GDPC1` and potential GDP by `GDPPOT`, both from FRED, and obtained from `https://fred.stlouisfed.org/series/GDPC1` and `https://fred.stlouisfed.org/series/GDPPOT` respectively. For the real-time output gap, real GDP vintages are obtained from the Federal Reserve Bank of Philadelphia: `https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/routput`, and potential GDP vintages from ALFRED: `https://alfred.stlouisfed.org/series/downloaddata?seid=GDPPOT`. The real-time evaluation sample begins in 1996Q2, when real-time output-gap estimates first become available.

A.1.2 Euro Area

Headline Inflation Revised headline inflation is based on the Harmonised Index of Consumer Prices, all items, euro area changing composition (`ICP.M.U2.Y.000000.3.INX`), working-day and seasonally adjusted, index 2015=100. Quarterly observations are averages of the monthly index. Data are obtained from the ECB: `https://data.ecb.europa.eu/data/datasets/ICP/ICP.M.U2.Y.000000.3.INX`. Real-time data are obtained from the ECB Real-Time Database, series `P_C_OV.X`: `https://data-api.ecb.europa.eu/service/data/RTD/M.S0.N.P_C_OV.X?includeHistory=true&format=csvdata`. The real-time index is seasonally adjusted using X-13ARIMA-SEATS. When more than one vintage is available within a quarter, we use the latest release in the reference quarter.

Core Inflation Revised core inflation is based on the Harmonised Index of Consumer Prices excluding energy and food, euro area changing composition (`ICP.M.U2.Y.XEF000.3.INX`), working-day and seasonally adjusted, index 2015=100. Quarterly observations are averages of the monthly index. Data are obtained from the ECB: `https://data.ecb.europa.eu/data/datasets/ICP/ICP.M.U2.Y.XEF000.3.INX`. Real-time data are obtained from the ECB Real-Time Database. Since an exactly corresponding real-time series excluding both energy and all food is not available over the required vintage sample, we use HICP excluding energy and unprocessed food (`P_C_XEFUN.X`): `https://data-api.ecb.europa.eu/service/data/RTD/M.S0.N.P_C_XEFUN.X?includeHistory=true&format=csvdata`. The real-time index is seasonally adjusted using X-13ARIMA-SEATS. When more than one vintage is available within a quarter, we use the latest release in the reference quarter.

Natural Rate of Interest The natural rate of interest is taken from the one-sided estimates provided by the Federal Reserve Bank of New York; see Federal Reserve Bank of New York (2026). Data are obtained from `https://www.newyorkfed.org/research/policy/rstar`.

Policy Rate We use the ECB interest rate for main refinancing operations, end of month (series `BBIN1.M.D0.ECB.ECBMIN.EUR.ME`), from the Deutsche Bundesbank. Data are obtained from `https://api.statistiken.bundesbank.de/rest/data/BBIN1/M.D0.ECB.ECBMIN.EUR.ME?format=csv&lang=en`.

Output Gap The revised output gap is the natural logarithm of real GDP divided by potential GDP. Potential GDP is taken from the AMECO database and interpolated to quarterly frequency using the method of Chow and Lin (1971). Real GDP is measured in chain-linked volumes at constant prices. Real-time output-gap data are constructed from AMECO archive vintages: `https://economy-finance.ec.europa.eu/economic-research-and-databases/economic-databases/ameco-database/ameco-archive_en`. Real-time AMECO vintages are available from 2011Q2 onward. For earlier forecast origins, we use revised output-gap data while restricting the information set to observations available at the respective forecast origin.

A.1.3 Japan

Headline Inflation For revised-data analysis, headline inflation is based on the national Consumer Price Index, all items, from the long-term monthly e-Stat series, index 2020=100. We seasonally adjust the long historical series using X-13ARIMA-SEATS and construct quarterly indices as averages of monthly observations. Data are obtained from `https://www.e-stat.go.jp/stat-search/files?page=1&layout=datalist&toukei=00200573&tstat=000001150147&cycle=0&tclass1=000001150151&tclass2=000001150152&tclass3=000001150153&tclass4=000001150156&stat_infid=000032103842&tclass5val=0`.

Core Inflation For revised-data analysis, core inflation is measured by CPI excluding fresh food and energy. For the real-time analysis and the comparison with Bank of Japan projections, core inflation is measured by CPI excluding fresh food, reflecting the definition available in the historical real-time vintages and used by the Bank of Japan projections. The revised core series is constructed from the long historical e-Stat CPI series and seasonally adjusted using X-13ARIMA-SEATS. Data are obtained from `https://www.e-stat.go.jp/stat-search/files?page=1&layout=datalist&toukei=00200573&tstat=000001150147&cycle=0&tclass1=000001150151&tclass2=000001150152&tclass3=000001150153&tclass4=000001150156&stat_infid=000032103842&tclass5val=0`.

Real-time CPI vintages are taken from the seasonally adjusted Consumer Price Index for Japan and Ku-area of Tokyo monthly reports. Vintages begin in July 2001 and contain data from January 2000 onward. Quarterly indices are calculated as averages of the monthly indices. Because CPI vintages with different base years cover different sample periods, the series are spliced across base years. When a series with the current base year does not extend sufficiently far back, it is extended using the growth rates of the corresponding series with the previous base year. Data are obtained from `https://www.e-stat.go.jp/en/stat-search?page=1&layout=normal&toukei=00200573&survey=price%20index&metadata=1&data=1`.

Natural Rate of Interest The natural rate of interest is based on the estimates of Nakano et al. (2024) that use the methodology of Holston et al. (2023). Their estimates begin in 1992Q1. We extend the series back to 1985 using the pre-1992 growth rates of an earlier Laubach–Williams-type estimate reported by Okazaki and Sudo (2018). The data associated with Nakano et al. (2024) are available directly from the Bank of Japan, `https://www.boj.or.jp/en/research/wps_rev/wps_2024/data/wp24e12.xlsx`; the earlier series is available in the replication package from Davis et al. (2024): `https://data.mendeley.com/datasets/vb8ptgynpp/1`.

Policy Rate We use the Bank of Japan's monthly historical policy-rate series, reported as the Basic Discount Rate and Basic Loan Rate (series `IR01'MADR1M`). Data are obtained from `https://www.stat-search.boj.or.jp/ssi/mtshtml/ir01_m_1_en.html`.

Output Gap We use quarterly output-gap estimates from the Bank of Japan: `https://www.boj.or.jp/en/research/research_data/gap/index.htm`. Real-time output-gap vintages are provided by the Bank of Japan and are available from April 2017 onward. For earlier forecast origins, we use revised output-gap data while imposing the relevant publication lag.

A.2 Central Bank Projections

A.2.1 United States

Headline and Core Inflation We use Greenbook/Tealbook projections for headline CPI inflation (`gPCPI`) and core CPI inflation (`gPCPIX`) from the Federal Reserve Bank of Philadelphia Greenbook Data Set. The available vintages span 1980Q1–2019Q4. When two Greenbook/Tealbook rounds fall within a quarter, we use the second publication for both the Greenbook/Tealbook projections and the current-quarter nowcast used to initialize the model. Data are obtained from `https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/philadelphia-data-set`.

A.2.2 Euro Area

Headline Inflation We use quarterly HICP headline inflation for the euro area as annual growth rates from the ECB Macroeconomic Projection Database (`MPD.Q.U2.HIC.A`). Vintages span 1999Q1–2024Q1. The projections are published following Governing Council monetary policy meetings in March, June, September, and December. `https://data.ecb.europa.eu/data/datasets/MPD`. The ECB projections are transformed into quarter-on-quarter inflation rates for comparison with the model-based forecasts.

Core Inflation We use HICP inflation excluding energy and food from the ECB Macroeconomic Projection Database (`MPD.A.U2.HEF.A`). Vintages span 2009Q2–2024Q1. The projections are published quarterly and provide forecasts for the current year, the following year, and the year after that. `https://data.ecb.europa.eu/data/datasets/MPD`

A.2.3 Japan

Core Inflation We use Bank of Japan projections for CPI inflation excluding fresh food for the current and next fiscal year, available from October 2003 onward. The publication frequency increased over time and has been quarterly since 2008. Data are obtained from `https://www.boj.or.jp/en/mopo/outlook/index.htm`.

B Solutions of the Low-Inflation Model

B.1 Indeterminacy and Solution Selection

Near the low-inflation endpoint, agents expect the ELB to bind permanently. Under this assumption, the linearized model has one fewer unstable eigenvalue than required by the determinacy conditions of Blanchard and Kahn (1980). It therefore admits a continuum of bounded rational-expectations solutions with one degree of indeterminacy. Standard solution algorithms do not select a unique equilibrium without an additional restriction.

Several approaches have been proposed to characterize or select among indeterminate equilibria. These include the Minimum State Variable (MSV) solution of McCallum (1983) and the methods of Lubik and Schorfheide (2003), Farmer, Khramov, and Nicolò (2015), and Bianchi and Nicolò (2021).

We use the MSV solution as our baseline. The MSV criterion selects a solution whose law of motion depends on a minimal set of state variables, in the sense that no state variable can be omitted while preserving a valid solution. Different minimal sets may in principle exist. McCallum's additional requirement that the solution remain valid across admissible values of the structural parameters selects a unique solution in our application.

As a robustness check, we also solve the model using the method of Farmer, Khramov, and Nicolò (2015). For a linear model with m degrees of indeterminacy, their approach constructs an augmented determinate system by treating m non-fundamental expectational errors as additional fundamental shocks. Under their regularity condition, alternative choices of expectational errors can yield observationally equivalent representations provided that the covariance matrix of the augmented shocks is adjusted accordingly. In our calibrated application, the covariance structure is restricted rather than estimated freely, so the choice of expectational error is not innocuous. Since our model has one degree of indeterminacy, we treat the inflation-expectation error as the additional fundamental shock, consistent with our focus on inflation expectations.

B.2 MSV Solution

To solve the model around the low-inflation endpoint, rewrite the IS and PC equations in terms of deviations of y_t and π_t from their endpoint values $\pi_t^L = -r_t^*$ and $y_t^L = -(1-\beta)/\kappa r_t^*$. Defining $\tilde{y}_t = y_t - y_t^L$ and $\tilde{\pi}_t = \pi_t - \pi_t^L$ and using $i_t \equiv 0$ (as the ELB is assumed to permanently bind near the low-inflation endpoint), the IS and PC equations become

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} + \alpha \mathbb{E}_t \tilde{\pi}_{t+1} + \alpha \tilde{r}_t + v_t \tag{1}$$

$$\tilde{\pi}_t = \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \kappa \tilde{y}_t + u_t \tag{2}$$

where $\tilde{r}_t = r_t - r_t^*$ denotes the deviation of the short natural real interest rate from its long-run counterpart; note that $\tilde{r}_t$ follows the process

$$\tilde{r}_t = \rho_r \tilde{r}_{t-1} - \rho_r \eta_t + \epsilon_t \tag{3}$$

Defining $x_t = \begin{pmatrix} \tilde{y}_t & \tilde{\pi}_t \end{pmatrix}'$ and $\xi_t = \begin{pmatrix} \tilde{r}_t & v_t & u_t \end{pmatrix}'$, we can write the model in matrix form as

$$\underbrace{\begin{pmatrix} 1 & 0 \\ -\kappa & 1 \end{pmatrix}}_{=A_1} x_t = \underbrace{\begin{pmatrix} 1 & \alpha \\ 0 & \beta \end{pmatrix}}_{=A_2} \mathbb{E}_t x_{t+1} + \underbrace{\begin{pmatrix} 1 & 0 & \alpha \\ 0 & 1 & 0 \end{pmatrix}}_{=B} \xi_t \tag{4}$$

Since

$$\xi_t = \underbrace{\begin{pmatrix} \rho_r & 0 & 0 \\ 0 & \rho_v & 0 \\ 0 & 0 & \rho_u \end{pmatrix}}_{=\Sigma} \xi_{t-1} + \begin{pmatrix} -\rho_r \eta_t + \epsilon_t \\ \epsilon_{v,t} \\ \epsilon_{u,t} \end{pmatrix} \tag{5}$$

the MSV solution is given by

$$x_t = \Gamma \xi_t \tag{6}$$

where Γ satisfies

$$\operatorname{vec} \Gamma = [I_6 - \Sigma \otimes A]^{-1} \operatorname{vec} B \tag{7}$$

and where $A = A_1^{-1} A_2$.

B.3 FKN Solution

We next derive the alternative solution based on Farmer, Khramov, and Nicolò (2015). Because the model has one degree of indeterminacy, one expectational error must be introduced as an additional fundamental shock. We use the inflation-expectation error for this purpose. Define

$$\tilde{\pi}_t^{sun} \equiv \mathbb{E}_t \tilde{\pi}_{t+1} \tag{8}$$

and

$$\tilde{s}_t \equiv \tilde{\pi}_t - \tilde{\pi}_{t-1}^{sun} \tag{9}$$

Then $\tilde{s}_t$ is a martingale difference, and we interpret $\tilde{s}_t$ as a new structural shock. Let $x_t = \begin{pmatrix} \tilde{y}_t & \tilde{\pi}_t & \tilde{\pi}_t^{sun} \end{pmatrix}'$ and $\xi_t = \begin{pmatrix} \tilde{r}_t & v_t & u_t & \tilde{s}_t \end{pmatrix}'$, and write the augmented system consisting of the IS and PC equations and the definition of $\tilde{s}_t$ as

$$\underbrace{\begin{pmatrix} 1 & 0 & -\alpha \\ -\kappa & 1 & -\beta \\ 0 & 1 & 0 \end{pmatrix}}_{=A_1} x_t = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{=A_2} \mathbb{E}_t x_{t+1} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{=A_3} x_{t-1} + \underbrace{\begin{pmatrix} \alpha & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{=B} \xi_t \tag{10}$$

where ξ_t follows the process

$$\xi_t = \underbrace{\begin{pmatrix} \rho_r & 0 & 0 & 0 \\ 0 & \rho_v & 0 & 0 \\ 0 & 0 & \rho_u & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{=\Sigma} \xi_{t-1} + \begin{pmatrix} -\rho_r \eta_t + \epsilon_t \\ \epsilon_{v,t} \\ \epsilon_{u,t} \\ \tilde{s}_t \end{pmatrix} \tag{11}$$

Guess that the solution takes the form

$$x_t = Cx_{t-1} + D\xi_t \tag{12}$$

Then

$$\mathbb{E}_t x_{t+1} = C^2 x_{t-1} + (CD + D\Sigma)\xi_t \tag{13}$$

Plugging this into the above matrix equation and comparing coefficients,

$$A_1 C = A_2 C^2 + A_3 \tag{14}$$
$$A_1 D = A_2(CD + D\Sigma) + B \tag{15}$$

A valid solution for C must have two zero eigenvalues and one nonzero stable eigenvalue; it can be shown that the unique solution satisfying this is given by

$$C = \begin{pmatrix} 0 & 0 & (2-\nu)/(2\kappa) \\ 0 & 0 & 1 \\ 0 & 0 & \nu/(2\beta) \end{pmatrix} \tag{16}$$

where

$$\delta = 1 + \beta + \alpha\kappa \tag{17}$$
$$\nu = \delta - \sqrt{\delta^2 - 4\beta} \tag{18}$$

(and where $\delta^2 > 4\beta$ such that ν is well-defined). D is then recovered as

$$\operatorname{vec} D = [I_4 \otimes A_1 - I_4 \otimes (A_2 C) - \Sigma \otimes A_2]^{-1} \operatorname{vec} B \tag{19}$$

and is explicitly given by

$$D = \begin{pmatrix} 2\alpha\beta/\gamma_r & 2\beta/\gamma_v & -\gamma/(\gamma_u\kappa) & \gamma/(\gamma_s\kappa) \\ 0 & 0 & 0 & 1 \\ -2\alpha\kappa/\gamma_r & -2\kappa/\gamma_v & -2(1-\rho_u)/\gamma_u & 2/\gamma_s \end{pmatrix} \tag{20}$$

where

$$\gamma = 2 + 2\alpha\kappa - \nu \tag{21}$$

$$\gamma_r = \gamma + 2\beta(1 - \rho_r) \tag{22}$$

$$\gamma_v = \gamma + 2\beta(1 - \rho_v) \tag{23}$$

$$\gamma_u = \gamma + 2\beta(1 - \rho_u) \tag{24}$$

$$\gamma_s = \gamma + 2\beta \tag{25}$$

Unlike the MSV solution, the FKN solution includes lagged inflation expectations as an additional endogenous state and can therefore generate different local dynamics around the low-inflation endpoint. For the robustness exercise, we replace the MSV low-inflation forecast rule by the FKN rule while leaving the target-equilibrium solution and all other model components unchanged. Appendix E reports the resulting equilibrium probabilities and forecasting results.

B.3.1 State and Shock Recovery

Taking the FKN forecast rule to the data requires the shocks (v_t, u_t) and the expectation state $\tilde{\pi}_t^{sun}$ at each date. Under the MSV solution these can be recovered period by period by inverting the observation equations. Under the FKN solution this inversion is no longer unique, because the two observables $(\tilde{y}_t, \tilde{\pi}_t)$ must be attributed to the disturbances $(v_t, u_t, \tilde{s}_t)$ and the unobserved state $\tilde{\pi}_{t-1}^{sun}$. We therefore cast the FKN solution in state-space form, with state vector $(\tilde{\pi}_t^{sun}, v_t, u_t)'$, the natural-rate gap $\tilde{r}_t$ as an exogenous regressor, and $(\tilde{y}_t, \tilde{\pi}_t)$ observed without error, and recover the states and shocks recursively with a Kalman filter.

The state-space specification requires a value for the variance of the sunspot shock $\tilde{s}_t$, on which the model itself is silent. We normalize σ_s so that one-standard-deviation innovations to the sunspot and supply shocks have equal absolute impact effects on inflation expectations $\tilde{\pi}_t^{sun}$, that is, $|D_{3,4}|\,\sigma_s = |D_{3,3}|\,\sigma_u$, where $D_{i,j}$ denotes the entries of the matrix D above and σ_s, σ_u are the standard deviations of the innovations $\tilde{s}_t$ and $\epsilon_{u,t}$.

For the forecasting exercises, we use the filtered states. The shocks and expectations attributed to each date t are therefore conditioned only on information available through that date and do not use subsequent observations.

C Mincer–Zarnowitz Results: MSV Solution

Table A.1: Mincer–Zarnowitz Regression Results: MSV Solution (Revised Data)

		Headline Inflation				Core Inflation			
Forecast Horizon:		1	2	4	8	1	2	4	8
		Japan							
Transition									
1987Q2–2023Q1	β_0	0.25	0.29	0.63	0.75	0.09	0.15	0.38	0.56
	β_1	0.78	0.80	0.10	-0.02	1.09	1.03	0.53	0.20
	p-value	0.11	0.18	0.01	0.01	0.55	0.55	0.31	0.08
1987Q2–1997Q4	β_0	0.87	0.48	1.55	1.87	0.58	0.04	0.46	0.08
	β_1	0.56	1.13	-0.13	-0.85	0.86	1.49	1.13	1.46
	p-value	0.13	0.15	0.08	0.23	0.22	0.04	0.19	0.51
1998Q1–2011Q4	β_0	-0.21	-0.26	-0.31	-0.12	-0.26	-0.37	-0.44	-0.39
	β_1	0.32	0.15	-0.38	-0.50	0.88	0.60	0.18	-0.72
	p-value	0.28	0.28	0.00	0.00	0.00	0.01	0.00	0.00
2012Q1–2019Q4	β_0	0.54	0.75	1.42	1.04	0.40	0.59	1.02	0.94
	β_1	0.23	-0.05	-0.97	-0.60	0.42	0.11	-0.55	-0.66
	p-value	0.05	0.05	0.00	0.00	0.01	0.00	0.00	0.00
2020Q1–2023Q1	β_0	0.93	1.47	2.29	3.21	0.86	1.41	1.97	2.97
	β_1	1.07	1.05	0.08	0.40	1.45	1.93	-0.23	0.39
	p-value	0.17	0.03	0.02	0.00	0.31	0.07	0.20	0.00
Target									
1987Q2–2023Q1	β_0	-0.33	-0.89	-0.36	-0.16	-0.77	-1.17	-0.60	-0.21
	β_1	0.91	1.28	0.81	0.71	1.30	1.46	0.95	0.67
	p-value	0.02	0.00	0.02	0.15	0.00	0.00	0.01	0.07
1987Q2–1997Q4	β_0	0.75	-1.32	-7.11	2.70	0.46	-1.94	-8.13	0.12
	β_1	0.57	2.51	8.24	-1.41	0.90	3.14	9.27	1.24
	p-value	0.37	0.24	0.10	0.89	0.37	0.03	0.00	0.64
1998Q1–2011Q4	β_0	-0.45	-1.32	-0.31	6.56	-1.35	-2.80	-5.49	1.63
	β_1	0.38	1.28	-0.01	-6.83	1.86	2.92	5.28	-2.06
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2012Q1–2019Q4	β_0	-0.04	-0.31	2.38	4.91	-0.34	-1.09	0.56	4.95
	β_1	0.50	0.58	-0.91	-2.32	0.68	1.01	0.06	-2.38
	p-value	0.03	0.01	0.00	0.00	0.01	0.00	0.00	0.00
2020Q1–2023Q1	β_0	-1.24	-10.59	-36.46	25.54	-1.23	-9.91	-70.49	18.78
	β_1	1.63	6.79	20.27	-11.33	1.58	6.42	37.87	-8.04
	p-value	0.66	0.00	0.01	0.00	0.56	0.01	0.00	0.00
Low-Inflation									
1987Q2–2023Q1	β_0	0.65	0.28	0.08	0.17	0.63	0.22	0.02	0.01
	β_1	0.08	-0.69	-0.92	-0.87	0.24	-0.67	-0.90	-0.96
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1987Q2–1997Q4	β_0	1.31	0.61	0.11	-0.65	1.51	0.85	0.20	-0.37
	β_1	-0.21	-0.64	-0.85	-1.14	-0.08	-0.53	-0.82	-1.05
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1998Q1–2011Q4	β_0	-0.16	-0.21	-0.43	-0.30	-0.11	-0.24	-0.40	-0.54
	β_1	0.35	0.23	-0.57	-0.97	1.06	0.76	0.23	-0.93
	p-value	0.38	0.67	0.10	0.00	0.24	0.21	0.17	0.00
2012Q1–2019Q4	β_0	0.69	0.60	-0.12	0.07	0.65	0.67	0.11	-0.28
	β_1	0.31	-0.73	-3.25	-1.93	0.50	0.09	-2.27	-3.05
	p-value	0.01	0.00	0.01	0.01	0.02	0.02	0.01	0.00
2020Q1–2023Q1	β_0	1.44	3.57	4.13	2.52	1.36	3.43	5.67	1.89
	β_1	1.55	5.16	3.85	-1.84	1.49	4.86	7.83	-2.57
	p-value	0.04	0.00	0.00	0.00	0.22	0.05	0.03	0.00

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h - 1$ lags.

Table A.2: Mincer–Zarnowitz Regression Results: MSV Solution (Revised Data)

		Headline Inflation				Core Inflation			
Forecast Horizon:		1	2	4	8	1	2	4	8
		Euro Area							
Transition									
2001Q2–2023Q3	β_0	-0.53	0.36	1.49	2.85	-0.57	-0.39	-0.24	0.92
	β_1	1.53	1.11	0.41	-0.46	1.51	1.44	1.35	0.53
	p-value	0.06	0.14	0.03	0.05	0.02	0.17	0.41	0.50
2001Q2–2009Q4	β_0	0.05	1.80	2.96	5.40	0.45	1.68	1.89	2.03
	β_1	0.97	0.11	-0.50	-1.72	0.73	-0.02	-0.20	-0.32
	p-value	1.00	0.64	0.23	0.03	0.62	0.05	0.00	0.00
2010Q1–2019Q4	β_0	0.60	1.15	1.54	1.96	0.55	0.63	1.07	1.55
	β_1	0.60	0.06	-0.33	-0.53	0.49	0.42	-0.05	-0.40
	p-value	0.34	0.05	0.00	0.00	0.38	0.26	0.00	0.00
2020Q1–2023Q3	β_0	0.00	1.50	4.25	12.14	-0.04	0.59	1.44	5.45
	β_1	1.84	1.80	0.51	-4.53	1.54	1.51	1.30	-1.09
	p-value	0.02	0.04	0.00	0.00	0.01	0.03	0.00	0.00
Target									
2001Q2–2023Q3	β_0	-2.10	-5.09	4.35	36.42	-2.39	-5.10	-1.68	15.91
	β_1	2.11	3.68	-1.13	-17.31	2.18	3.52	1.70	-7.18
	p-value	0.01	0.12	0.74	0.01	0.00	0.00	0.31	0.00
2001Q2–2009Q4	β_0	-0.02	1.57	4.64	41.97	-1.37	-3.33	-2.99	7.95
	β_1	0.99	0.22	-1.32	-19.90	1.57	2.48	2.25	-3.20
	p-value	0.97	0.85	0.62	0.03	0.00	0.00	0.00	0.00
2010Q1–2019Q4	β_0	-1.06	-3.72	0.31	-49.09	0.14	0.50	3.69	-11.93
	β_1	1.38	2.67	0.43	25.61	0.57	0.32	-1.39	6.62
	p-value	0.06	0.01	0.04	0.01	0.00	0.00	0.00	0.00
2020Q1–2023Q3	β_0	-1.19	-2.86	4.48	66.54	-1.33	-4.25	-5.71	23.00
	β_1	2.16	3.66	0.32	-31.76	2.01	3.80	4.86	-9.91
	p-value	0.02	0.05	0.09	0.00	0.01	0.02	0.03	0.00
Low-Inflation									
2001Q2–2023Q3	β_0	2.28	2.89	2.44	2.61	1.92	1.96	1.89	2.02
	β_1	1.69	0.94	0.28	0.43	1.08	0.36	0.21	0.30
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2001Q2–2009Q4	β_0	2.44	0.29	2.01	3.63	0.66	-0.28	0.56	1.53
	β_1	0.76	-1.19	0.01	0.82	-1.48	-1.30	-0.54	0.00
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2010Q1–2019Q4	β_0	1.37	0.22	-0.28	1.87	0.98	0.76	0.51	1.21
	β_1	0.57	-1.92	-2.06	0.80	-0.58	-0.58	-0.72	0.16
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2020Q1–2023Q3	β_0	2.38	5.11	7.04	6.04	1.97	3.91	5.47	4.52
	β_1	2.15	3.72	3.98	3.42	2.06	3.14	3.75	2.01
	p-value	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h - 1$ lags.

Table A.3: Mincer–Zarnowitz Regression Results: MSV Solution (Revised Data)

		Headline Inflation				Core Inflation			
Forecast Horizon:		1	2	4	8	1	2	4	8
		United States							
Transition									
1990Q2–2023Q3	β_0	0.52	1.68	2.50	2.40	-0.81	-1.26	1.84	2.00
	β_1	1.06	0.52	0.05	0.10	1.64	1.99	0.35	0.25
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03
1990Q2–2009Q4	β_0	1.16	2.90	3.28	10.41	-2.11	-4.61	-1.81	4.42
	β_1	0.70	-0.15	-0.39	-4.02	2.18	3.56	2.15	-1.09
	p-value	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00
2010Q1–2019Q4	β_0	1.59	3.03	2.30	1.79	0.88	2.11	2.22	0.22
	β_1	0.13	-0.97	-0.39	0.21	0.61	-0.15	-0.14	1.16
	p-value	0.10	0.00	0.00	0.00	0.02	0.00	0.00	0.00
2020Q1–2023Q3	β_0	0.97	2.12	8.54	7.51	1.55	-0.04	8.80	10.37
	β_1	1.43	1.55	-1.88	-1.86	1.06	2.29	-2.19	-3.27
	p-value	0.02	0.00	0.00	0.00	0.01	0.00	0.00	0.00
Target									
1990Q2–2023Q3	β_0	-0.10	-0.74	4.42	10.36	-1.36	-4.68	0.66	5.86
	β_1	1.29	1.70	-0.95	-3.94	1.86	3.66	0.95	-1.71
	p-value	0.02	0.01	0.05	0.11	0.00	0.00	0.00	0.05
1990Q2–2009Q4	β_0	1.15	3.08	1.73	24.91	-2.32	-6.13	-4.07	5.61
	β_1	0.70	-0.24	0.40	-11.27	2.27	4.28	3.27	-1.68
	p-value	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.18
2010Q1–2019Q4	β_0	-0.02	5.21	4.82	-86.02	-0.29	2.13	-0.29	-45.29
	β_1	0.99	-1.92	-1.62	45.13	1.21	-0.14	1.19	24.35
	p-value	0.99	0.24	0.64	0.07	0.37	0.72	0.65	0.25
2020Q1–2023Q3	β_0	0.67	0.45	28.19	74.15	1.63	0.72	19.90	41.83
	β_1	1.49	2.24	-11.86	-35.62	1.02	1.87	-7.83	-19.15
	p-value	0.02	0.00	0.00	0.00	0.01	0.00	0.00	0.00
Low-Inflation									
1990Q2–2023Q3	β_0	3.03	2.55	2.28	2.50	2.88	2.61	2.35	2.65
	β_1	0.65	-0.05	-0.15	-0.04	0.53	0.04	-0.08	0.08
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1990Q2–2009Q4	β_0	3.01	0.84	1.74	2.82	3.12	1.13	0.55	1.57
	β_1	0.32	-0.79	-0.29	0.10	0.55	-0.61	-0.67	-0.26
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2010Q1–2019Q4	β_0	1.95	0.38	-3.04	-3.14	2.09	2.49	1.47	-0.19
	β_1	1.03	-1.68	-4.90	-5.58	1.20	0.83	-0.51	-2.61
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2020Q1–2023Q3	β_0	4.00	5.66	-1.86	-1.14	3.89	5.25	0.30	0.24
	β_1	1.25	0.54	-5.95	-4.34	0.80	0.79	-3.71	-3.24
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h - 1$ lags.

Table A.4: Mincer–Zarnowitz Regression Results: MSV Solution (Real-Time Data)

Japan			
Forecast Horizon:		Current Fiscal Year	Next Fiscal Year
Transition		Headline Inflation Excluding Fresh Food	
2003Q3–2023Q1	β_0	0.11	0.61
	β_1	1.06	-0.11
	p-value	0.59	0.00
2003Q3–2011Q4	β_0	-0.14	-0.25
	β_1	0.98	-0.73
	p-value	0.64	0.00
2012Q1–2019Q4	β_0	0.24	1.05
	β_1	0.89	-0.68
	p-value	0.11	0.00
2020Q1–2023Q1	β_0	0.42	1.85
	β_1	1.12	0.43
	p-value	0.20	0.04
Target			
2003Q3–2023Q1	β_0	-0.24	-1.09
	β_1	1.11	1.17
	p-value	0.01	0.00
2003Q3–2011Q4	β_0	-0.35	0.28
	β_1	0.94	-0.66
	p-value	0.08	0.00
2012Q1–2019Q4	β_0	-0.11	0.88
	β_1	1.00	-0.14
	p-value	0.57	0.00
2020Q1–2023Q1	β_0	-0.19	-0.42
	β_1	1.26	1.34
	p-value	0.17	0.91
Low-Inflation			
2003Q3–2023Q1	β_0	0.27	0.29
	β_1	1.13	-1.71
	p-value	0.09	0.00
2003Q3–2011Q4	β_0	-0.10	-0.34
	β_1	1.00	-0.90
	p-value	0.80	0.00
2012Q1–2019Q4	β_0	0.44	0.18
	β_1	0.90	-1.98
	p-value	0.03	0.00
2020Q1–2023Q1	β_0	0.55	1.47
	β_1	1.40	-2.27
	p-value	0.00	0.00
Central Bank			
2003Q3–2023Q1	β_0	-0.13 (-0.05 excl. tax)	-0.06 (-0.03 excl. tax)
	β_1	0.98 (1.10 excl. tax)	0.62 (0.72 excl. tax)
	p-value	0.03 (0.17 excl. tax)	0.02 (0.52 excl. tax)
2003Q3–2011Q4	β_0	-0.18	-0.34
	β_1	0.92	0.36
	p-value	0.04	0.01
2012Q1–2019Q4	β_0	-0.15 (-0.19 excl. tax)	-0.49 (0.83 excl. tax)
	β_1	0.90 (1.36 excl. tax)	0.65 (-0.15 excl. tax)
	p-value	0.00 (0.20 excl. tax)	0.00 (0.00 excl. tax)
2020Q1–2023Q1	β_0	0.11	-0.02
	β_1	1.09	2.10
	p-value	0.18	0.00

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For annual fixed-event forecasts, we use $h_{\max} - 1$ lags, where $h_{\max}$ denotes the maximum forecast horizon in quarters of the respective fixed event. For Japan, "incl. tax" and "excl. tax" refer to Bank of Japan projections including and excluding the effects of the 2014 consumption-tax increase, respectively.

Table A.5: Mincer–Zarnowitz Regression Results: MSV Solution (Real-Time Data)

Euro Area										
Forecast Horizon:			1	2	4	8		Current Year	Next Year	Two Years Ahead
Transition			Headline Inflation					Core Inflation		
2001Q2	AR Nowcast	β_0	0.58	1.65	3.14	3.21	2009Q2	0.43	1.05	2.16
–2023Q3		β_1	0.97	0.32	-0.67	-0.71	–2023Q3	0.74	0.48	-0.33
		p-value	0.04	0.03	0.06	0.04		0.00	0.38	0.28
2001Q2	CB Nowcast	β_0	-0.02	1.43	3.13	3.20				
–2023Q3		β_1	1.27	0.46	-0.67	-0.71				
		p-value	0.11	0.04	0.07	0.04				
2001Q2	AR Nowcast	β_0	2.43	2.80	3.18	2.47	2009Q2	0.53	1.12	1.07
–2011Q1		β_1	-0.19	-0.40	-0.58	-0.22	–2019Q4	0.49	-0.05	0.06
		p-value	0.01	0.00	0.00	0.00		0.00	0.00	0.00
2001Q2	CB Nowcast	β_0	1.15	2.91	3.18	2.47				
–2011Q1		β_1	0.46	-0.44	-0.58	-0.22				
		p-value	0.46	0.00	0.00	0.00				
2011Q2	AR Nowcast	β_0	0.59	0.78	1.16	1.76	2014Q1	0.90	0.95	1.10
–2019Q4		β_1	0.49	0.23	-0.23	-0.46	–2018Q4	0.06	0.02	-0.15
		p-value	0.17	0.07	0.00	0.00		0.00	0.00	0.00
2011Q2	CB Nowcast	β_0	0.65	0.76	1.16	1.76				
–2019Q4		β_1	0.42	0.25	-0.24	-0.45				
		p-value	0.12	0.07	0.00	0.00				
2020Q1	AR Nowcast	β_0	2.48	4.40	8.09	7.96	2020Q1	1.30	3.44	5.58
–2023Q3		β_1	1.03	0.31	-2.36	-2.61	–2023Q3	0.61	-0.02	-1.51
		p-value	0.06	0.02	0.00	0.00		0.00	0.00	0.00
2020Q1	CB Nowcast	β_0	1.12	3.80	8.09	7.92				
–2023Q3		β_1	1.62	0.71	-2.35	-2.59				
		p-value	0.03	0.02	0.00	0.00				
Target			Headline Inflation					Core Inflation		
2001Q2	AR Nowcast	β_0	-1.20	-1.87	13.96	30.12	2009Q2	0.21	-0.34	22.25
–2023Q3		β_1	1.74	2.09	-6.09	-14.19	–2023Q3	0.79	1.07	-10.55
		p-value	0.18	0.58	0.33	0.20		0.05	0.86	0.00
2001Q2	CB Nowcast	β_0	-2.09	-3.89	13.41	29.24				
–2023Q3		β_1	2.11	3.09	-5.80	-13.75				
		p-value	0.05	0.34	0.35	0.20				
2001Q2	AR Nowcast	β_0	2.02	1.50	8.45	34.43	2009Q2	0.35	0.85	-10.11
–2011Q1		β_1	0.03	0.29	-3.16	-16.18	–2019Q4	0.56	0.12	5.77
		p-value	0.17	0.72	0.07	0.02		0.00	0.00	0.00
2001Q2	CB Nowcast	β_0	-0.01	1.86	8.32	34.58				
–2011Q1		β_1	0.99	0.10	-3.09	-16.26				
		p-value	0.99	0.74	0.09	0.02				
2011Q2	AR Nowcast	β_0	-1.04	-4.21	-2.57	-85.54	2014Q1	0.83	1.13	-0.80
–2019Q4		β_1	1.35	2.93	1.81	44.29	–2018Q4	0.11	-0.09	0.90
		p-value	0.04	0.00	0.00	0.00		0.00	0.00	0.00
2011Q2	CB Nowcast	β_0	0.15	-2.78	-1.32	-91.16				
–2019Q4		β_1	0.59	2.11	1.15	47.17				
		p-value	0.02	0.00	0.00	0.00				
2020Q1	AR Nowcast	β_0	0.87	2.96	27.14	62.09	2020Q1	1.08	3.66	22.26
–2023Q3		β_1	1.52	0.95	-11.80	-29.77	–2023Q3	0.65	-0.12	-9.74
		p-value	0.08	0.11	0.03	0.00		0.01	0.03	0.00
2020Q1	CB Nowcast	β_0	-1.27	-1.58	26.42	57.15				
–2023Q3		β_1	2.30	3.18	-11.40	-27.23				
		p-value	0.05	0.08	0.04	0.00				

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h-1$ lags. For annual fixed-event forecasts, we use $h_{\max} - 1$ lags, where $h_{\max}$ denotes the maximum forecast horizon in quarters of the respective fixed event.

Table A.6: Mincer–Zarnowitz Regression Results: MSV Solution (Real-Time Data)

Euro Area										
Forecast Horizon:			1	2	4	8		Current Year	Next Year	Two Years Ahead
Low-Inflation			Headline Inflation					Core Inflation		
2001Q2	AR Nowcast	β_0	2.41	2.50	2.23	2.68	2009Q2	0.85	1.91	2.59
–2023Q3		β_1	1.36	0.41	0.08	0.49	–2023Q3	0.67	0.54	1.23
		p-value	0.00	0.00	0.00	0.00		0.00	0.00	0.00
2001Q2	CB Nowcast	β_0	2.33	2.68	2.24	2.69				
–2023Q3		β_1	1.82	0.64	0.09	0.49				
		p-value	0.00	0.00	0.00	0.00				
2001Q2	AR Nowcast	β_0	1.57	1.32	2.91	3.33	2009Q2	0.96	0.84	1.17
–2011Q1		β_1	-0.84	-0.53	0.45	0.68	–2019Q4	0.19	-0.40	0.03
		p-value	0.00	0.00	0.00	0.00		0.00	0.00	0.00
2001Q2	CB Nowcast	β_0	2.57	1.21	2.92	3.33				
–2011Q1		β_1	1.04	-0.63	0.46	0.68				
		p-value	0.00	0.00	0.00	0.00				
2011Q2	AR Nowcast	β_0	1.19	-0.08	0.09	2.59	2014Q1	0.97	0.97	1.24
–2019Q4		β_1	0.29	-2.06	-1.16	2.19	–2018Q4	-0.03	-0.01	0.54
		p-value	0.00	0.00	0.00	0.00		0.00	0.00	0.00
2011Q2	CB Nowcast	β_0	1.13	0.05	0.09	2.60				
–2019Q4		β_1	-0.12	-1.87	-1.17	2.21				
		p-value	0.00	0.00	0.00	0.00				
2020Q1	AR Nowcast	β_0	3.46	5.07	4.06	10.50	2020Q1	1.67	3.36	4.62
–2023Q3		β_1	1.48	0.91	-1.76	11.58	–2023Q3	0.56	-0.29	2.11
		p-value	0.00	0.00	0.00	0.00		0.00	0.00	0.00
2020Q1	CB Nowcast	β_0	2.59	5.53	4.38	10.53				
–2023Q3		β_1	2.32	3.19	-1.21	11.61				
		p-value	0.00	0.00	0.00	0.00				
Central Bank			Headline Inflation					Core Inflation		
2001Q2	AR Nowcast	β_0	0.51	1.24	1.43	3.01	2009Q2	-0.02	-0.31	1.87
–2023Q3		β_1	0.94	0.53	0.40	-0.49	–2023Q3	1.03	1.28	-0.07
		p-value	0.08	0.13	0.12	0.00		0.80	0.26	0.53
2001Q2	CB Nowcast	β_0	0.51	1.24	1.43	3.01				
–2023Q3		β_1	0.94	0.53	0.40	-0.49				
		p-value	0.08	0.13	0.12	0.00				
2001Q2	AR Nowcast	β_0	1.09	2.46	2.47	2.12	2009Q2	0.12	0.76	2.21
–2011Q1		β_1	0.57	-0.23	-0.19	-0.03	–2019Q4	0.86	0.22	-0.68
		p-value	0.02	0.00	0.00	0.00		0.17	0.00	0.00
2001Q2	CB Nowcast	β_0	1.09	2.46	2.47	2.12				
–2011Q1		β_1	0.57	-0.23	-0.19	-0.03				
		p-value	0.02	0.00	0.00	0.00				
2011Q2	AR Nowcast	β_0	0.68	0.60	0.79	2.46	2014Q1	0.77	1.09	1.43
–2019Q4		β_1	0.36	0.32	0.06	-0.78	–2018Q4	0.19	-0.09	-0.29
		p-value	0.04	0.09	0.00	0.00		0.00	0.00	0.00
2011Q2	CB Nowcast	β_0	0.68	0.60	0.79	2.46				
–2019Q4		β_1	0.36	0.32	0.06	-0.78				
		p-value	0.04	0.09	0.00	0.00				
2020Q1	AR Nowcast	β_0	2.56	4.86	6.32	7.47	2020Q1	0.30	2.07	5.85
–2023Q3		β_1	0.74	-0.01	-0.58	-1.68	–2023Q3	0.96	0.64	-1.30
		p-value	0.15	0.12	0.03	0.10		0.24	0.01	0.00
2020Q1	CB Nowcast	β_0	2.56	4.86	6.32	7.47				
–2023Q3		β_1	0.74	-0.01	-0.58	-1.68				
		p-value	0.15	0.12	0.03	0.10				

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h-1$ lags. For annual fixed-event forecasts, we use $h_{\max} - 1$ lags, where $h_{\max}$ denotes the maximum forecast horizon in quarters of the respective fixed event.

Table A.7: Mincer–Zarnowitz Regression Results: MSV Solution (Real-Time Data)

United States										
Forecast Horizon:			1	2	3	4	1	2	3	4
Transition			Headline Inflation				Core Inflation			
1996Q2	AR Nowcast	β_0	2.49	2.41	2.18	2.16	1.03	1.34	1.49	1.68
–2019Q4		β_1	-0.19	-0.19	-0.04	-0.03	0.55	0.37	0.29	0.18
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1996Q2	CB Nowcast	β_0	1.52	2.58	2.18	2.17	1.00	1.24	1.50	1.68
–2019Q4		β_1	0.36	-0.28	-0.04	-0.03	0.56	0.42	0.29	0.19
		p-value	0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.00
1996Q2	AR Nowcast	β_0	3.40	2.10	1.91	3.13	-0.08	-0.94	-1.20	-1.06
–2009Q4		β_1	-0.48	0.16	0.23	-0.39	1.12	1.57	1.70	1.62
		p-value	0.00	0.08	0.31	0.12	0.03	0.01	0.03	0.15
1996Q2	CB Nowcast	β_0	1.59	3.03	1.60	3.27	0.36	-1.28	-1.26	-1.05
–2009Q4		β_1	0.42	-0.32	0.39	-0.47	0.89	1.75	1.73	1.61
		p-value	0.15	0.01	0.30	0.11	0.18	0.00	0.03	0.15
2010Q1	AR Nowcast	β_0	2.36	2.79	2.41	2.32	1.42	1.90	1.99	2.11
–2019Q4		β_1	-0.45	-0.84	-0.48	-0.41	0.31	-0.03	-0.03	-0.09
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2010Q1	CB Nowcast	β_0	1.84	2.93	2.45	2.33	1.26	1.86	2.01	2.11
–2019Q4		β_1	-0.05	-0.92	-0.50	-0.41	0.40	0.00	-0.04	-0.08
		p-value	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Target			Headline Inflation				Core Inflation			
1996Q2	AR Nowcast	β_0	2.71	1.95	1.21	0.87	0.05	-0.37	-1.24	-1.57
–2019Q4		β_1	-0.29	0.08	0.47	0.65	1.04	1.24	1.70	1.86
		p-value	0.00	0.24	0.63	0.69	0.03	0.40	0.12	0.20
1996Q2	CB Nowcast	β_0	1.09	2.74	1.19	1.11	0.25	-0.64	-1.14	-1.59
–2019Q4		β_1	0.55	-0.33	0.48	0.52	0.93	1.38	1.65	1.87
		p-value	0.25	0.05	0.62	0.68	0.16	0.33	0.16	0.19
1996Q2	AR Nowcast	β_0	3.48	2.16	0.80	0.52	-0.62	-2.32	-3.97	-5.68
–2009Q4		β_1	-0.51	0.12	0.80	0.95	1.38	2.25	3.09	3.94
		p-value	0.00	0.11	0.47	0.42	0.02	0.00	0.00	0.00
1996Q2	CB Nowcast	β_0	1.58	3.29	0.29	0.85	0.17	-2.61	-3.95	-5.61
–2009Q4		β_1	0.42	-0.45	1.06	0.78	0.97	2.39	3.07	3.90
		p-value	0.19	0.01	0.47	0.41	0.31	0.00	0.00	0.00
2010Q1	AR Nowcast	β_0	2.63	5.14	4.35	4.11	0.67	1.57	1.09	1.88
–2019Q4		β_1	-0.52	-1.94	-1.40	-1.25	0.70	0.16	0.46	0.05
		p-value	0.00	0.14	0.40	0.44	0.30	0.36	0.39	0.15
2010Q1	CB Nowcast	β_0	0.74	4.45	4.57	4.28	0.39	1.29	1.28	1.77
–2019Q4		β_1	0.57	-1.52	-1.50	-1.33	0.83	0.31	0.35	0.11
		p-value	0.65	0.08	0.29	0.42	0.45	0.46	0.34	0.16

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h - 1$ lags.

Table A.8: Mincer–Zarnowitz Regression Results: MSV Solution (Real-Time Data)

United States										
Forecast Horizon:			1	2	3	4	1	2	3	4
Low-Inflation			Headline Inflation				Core Inflation			
1996Q2	AR Nowcast	β_0	1.47	1.16	1.29	1.34	1.95	1.67	1.69	1.69
–2019Q4		β_1	-0.85	-0.57	-0.43	-0.39	-0.14	-0.20	-0.17	-0.16
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1996Q2	CB Nowcast	β_0	2.09	1.13	1.29	1.33	1.96	1.69	1.69	1.69
–2019Q4		β_1	-0.10	-0.59	-0.43	-0.39	-0.12	-0.19	-0.17	-0.16
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1996Q2	AR Nowcast	β_0	1.29	0.91	0.91	1.46	1.73	0.88	0.49	0.42
–2009Q4		β_1	-0.97	-0.66	-0.56	-0.34	-0.34	-0.54	-0.61	-0.60
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1996Q2	CB Nowcast	β_0	2.59	0.55	0.96	1.45	1.64	0.95	0.49	0.42
–2009Q4		β_1	0.13	-0.83	-0.54	-0.34	-0.43	-0.51	-0.61	-0.60
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2010Q1	AR Nowcast	β_0	1.58	0.69	0.45	-0.23	2.03	2.16	2.30	1.69
–2019Q4		β_1	-0.58	-1.20	-1.35	-1.99	0.58	0.38	0.39	-0.28
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2010Q1	CB Nowcast	β_0	1.86	0.85	0.43	-0.23	2.03	2.21	2.27	1.71
–2019Q4		β_1	0.49	-1.06	-1.40	-2.00	0.71	0.45	0.35	-0.26
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Central Bank			Headline Inflation				Core Inflation			
1996Q2	AR Nowcast	β_0	0.31	2.95	2.69	2.01	0.83	1.06	1.05	1.17
–2019Q4		β_1	0.95	-0.46	-0.30	0.05	0.57	0.45	0.47	0.41
		p-value	0.39	0.00	0.00	0.08	0.00	0.00	0.00	0.00
1996Q2	CB Nowcast	β_0	0.31	2.95	2.69	2.01	0.83	1.06	1.05	1.17
–2019Q4		β_1	0.95	-0.46	-0.30	0.05	0.57	0.45	0.47	0.41
		p-value	0.39	0.00	0.00	0.08	0.00	0.00	0.00	0.00
1996Q2	AR Nowcast	β_0	0.30	3.67	3.25	2.13	0.92	0.55	0.51	0.61
–2009Q4		β_1	0.95	-0.64	-0.45	0.12	0.54	0.70	0.71	0.66
		p-value	0.70	0.00	0.00	0.27	0.01	0.05	0.06	0.03
1996Q2	CB Nowcast	β_0	0.30	3.67	3.25	2.13	0.92	0.55	0.51	0.61
–2009Q4		β_1	0.95	-0.64	-0.45	0.12	0.54	0.70	0.71	0.66
		p-value	0.70	0.00	0.00	0.27	0.01	0.05	0.06	0.03
2010Q1	AR Nowcast	β_0	0.32	2.31	2.48	2.55	0.77	1.86	1.80	2.05
–2019Q4		β_1	0.94	-0.37	-0.40	-0.45	0.59	0.00	0.07	-0.04
		p-value	0.44	0.05	0.06	0.05	0.08	0.01	0.00	0.00
2010Q1	CB Nowcast	β_0	0.32	2.31	2.48	2.55	0.77	1.86	1.80	2.05
–2019Q4		β_1	0.94	-0.37	-0.40	-0.45	0.59	0.00	0.07	-0.04
		p-value	0.44	0.05	0.06	0.05	0.08	0.01	0.00	0.00

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h - 1$ lags.

D Additional State-Dependent Results: MSV Solution

D.1 Comparisons with Central Bank Projections

Table A.9 reports the state-dependent forecast-accuracy tests comparing the baseline MSV model-based forecasts with the central bank projections on the real-time evaluation samples.

Table A.9: Full-Sample Relative RMSFEs and State-Dependent Test Results: Model-Based Forecasts versus Central Bank Projections

Japan								
2003Q3–2023Q1	Headline Inflation Excluding Fresh Food							
Forecast Horizon:	Current Fiscal Year				Next Fiscal Year			
	incl. tax		excl. tax		incl. tax		excl. tax	
Transition/CB	1.99^{*}		1.38		1.17		1.23	
Target/CB	1.95^{***}		1.35		1.13^{**}		1.18^{**}	
Low-Infl./CB	2.21^{*}		1.53		1.28		1.34^{*}	
Euro Area (AR nowcast)								
2001Q2–2023Q3	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	4	8	Current Year	Next Year	Two Years Ahead	
Transition/CB	1.07	1.04	1.11	1.04	2.86^{**}	1.37^{*}	1.10	
Target/CB	1.02	0.97	0.99	0.93	2.69^{***}	1.28^{**}	0.98	
Low-Infl./CB	1.56^{***}	1.65^{***}	1.74^{***}	1.63^{***}	3.99^{***}	2.60^{***}	2.10^{***}	
Euro Area (CB nowcast)								
2001Q2–2023Q3	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	4	8	Current Year	Next Year	Two Years Ahead	
Transition/CB	1.00	1.02	1.11	1.04	—	—	—	
Target/CB	0.96	0.96	0.99	0.93	—	—	—	
Low-Infl./CB	1.49^{***}	1.63^{***}	1.74^{***}	1.63^{***}	—	—	—	
United States (AR nowcast)								
1996Q2–2019Q4	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	3	4	1	2	3	4
Transition/CB	1.45^{**}	0.99	1.01	1.03	1.08	1.05	1.07	1.06
Target/CB	1.39^{*}	0.93	0.96^{**}	0.98	0.95	0.93	0.94	0.93
Low-Infl./CB	2.45^{***}	1.98^{***}	2.15^{***}	2.23^{***}	5.33^{***}	5.28^{***}	5.54^{***}	5.50^{***}
United States (CB nowcast)								
1996Q2–2019Q4	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	3	4	1	2	3	4
Transition/CB	1.34^{**}	0.99	1.01	1.03	1.06	1.03	1.07	1.06
Target/CB	1.29^{*}	0.94	0.96^{**}	0.98	0.94	0.92	0.94	0.93
Low-Infl./CB	2.35^{***}	1.97^{***}	2.15^{***}	2.23^{***}	5.28^{***}	5.26^{***}	5.53^{***}	5.50^{***}

Notes: Entries are relative RMSFEs; values below one favor the model-based forecast. Sample periods refer to model forecast origins. AR-nowcast and CB-nowcast denote initialization with autoregressive and central bank nowcasts, respectively. For Japan, "incl. tax" and "excl. tax" refer to Bank of Japan projections including and excluding the effects of the 2014 consumption-tax increase. Stars indicate rejection of the joint null hypothesis of equal average predictive accuracy and no state dependence in the loss differential, $H_0 : \delta_0 = \delta_1 = 0$, based on the exp-$W$ statistic. They do not indicate whether the reported relative RMSFE differs significantly from one. The state variable is headline or core inflation observed at the forecast origin, as indicated. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

E Robustness to the FKN Solution

This appendix assesses the robustness of the forecasting results to replacing the MSV solution for the low-inflation equilibrium with the alternative FKN solution. The FKN solution of Farmer, Khramov, and Nicolò (2015) introduces lagged beliefs as an additional endogenous state and therefore changes the dynamics of the low-inflation model. The target-equilibrium solution and all other model components remain unchanged. Overall, the main empirical patterns are robust to the solution method. The FKN dynamics substantially improve the forecasting performance of the low-inflation model in several settings, while the transition forecast generally remains close to the MSV baseline. The discussion below focuses on these differences rather than repeating results that are essentially unchanged across the two solutions.

E.1 Equilibrium Probabilities and Forecast Dynamics

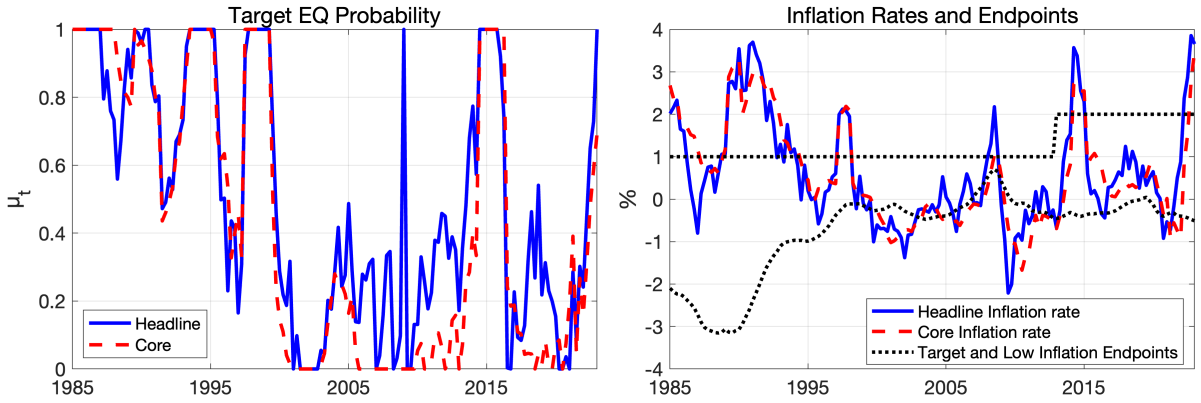
Figure A.1 shows that the broad cross-country ordering of equilibrium probabilities remains similar under FKN, although the FKN probabilities are generally more volatile and their levels can differ materially from the MSV baseline. In Japan, the low-inflation equilibrium continues to play a substantial role during the prolonged low-inflation period. In the euro area, FKN likewise assigns substantial probability to the low-inflation equilibrium during the 2010s. In the United States, the target equilibrium continues to dominate over most of the sample. The differences between the two solutions become particularly visible around the recent inflation surge and subsequent disinflation. In the euro area and the United States, the FKN probability of the target equilibrium declines again as inflation recedes from its peak, whereas the corresponding MSV probabilities return to or remain close to one. Japan provides a contrast, with the target-equilibrium probability rising at the end of the sample as inflation continues to increase.

These differences in equilibrium probabilities reflect the interaction between the richer FKN low-inflation dynamics and the forecast-performance-based weighting mechanism. The forecast plots in Appendix F show that the FKN low-inflation model can generate substantially different inflation dynamics from its MSV counterpart, particularly during the post-2020 inflation surge and subsequent disinflation in the euro area and the United States. Rather than rapidly moving inflation toward the low-inflation endpoint, the FKN low-inflation model can generate disinflation from elevated inflation levels. This can improve the relative one-step-ahead forecast performance of the low-inflation model over the rolling window used to construct μ_t, thereby reducing the probability assigned to the target equilibrium even while inflation remains above target. Because μ_t is determined by forecast errors over a rolling window rather than by the distance of current inflation from the two endpoints, the resulting probabilities need not move monotonically with the inflation level. This mechanism is particularly visible at the end of the euro area and U.S. samples, where the probability of the target equilibrium declines substantially during the disinflation. The FKN probabilities therefore have a less direct structural interpretation than under MSV. A decline in μ_t may reflect stronger predictive performance of the low-inflation model over the rolling window rather than a corresponding movement of the economy toward the low-inflation endpoint. As a result, the more flexible FKN low-inflation

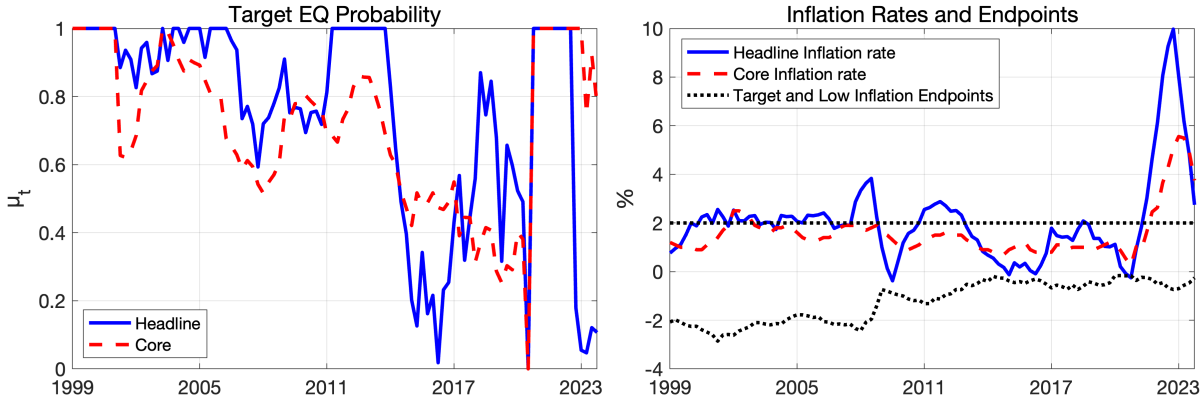
dynamics can shift the transition forecast away from its MSV counterpart without necessarily improving multi-horizon forecast accuracy.

Figure A.1: Equilibrium Probabilities and Inflation Endpoints (FKN Solution)

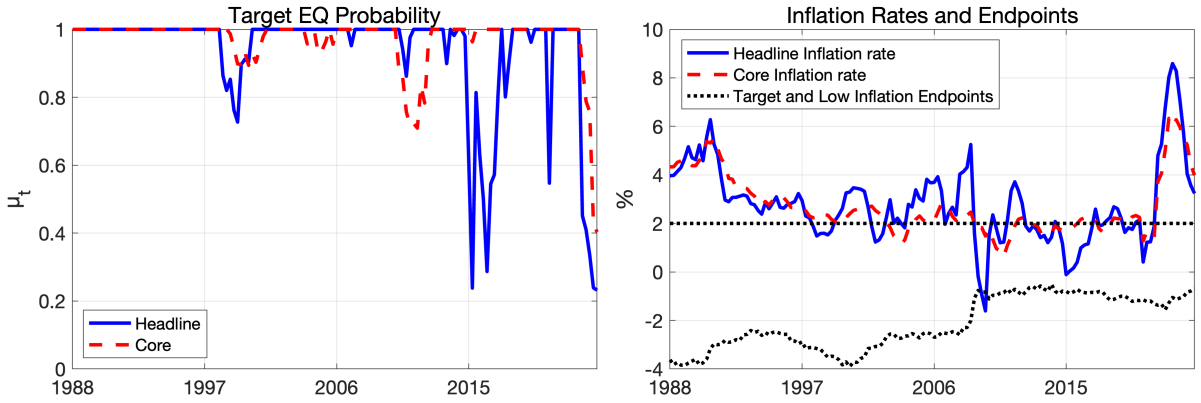
Japan



Euro Area



United States



Notes: Left panels show the model-implied probability of the target equilibrium. Solid lines use headline inflation, while dashed lines use core inflation. Right panels show headline (solid) and core (dashed) inflation in year-on-year percentage terms. The upper dotted lines mark the target inflation endpoint π^*, and the lower dotted lines depict the low-inflation endpoint $-r_t^*$.

E.2 Forecast Accuracy with Revised Data

Table A.10 shows that the main effect of the FKN solution is to make the low-inflation forecast substantially more competitive. Across the three economies, the FKN low-inflation forecast is markedly more accurate than its MSV counterpart relative to the common target benchmark. Despite this improvement, it remains less accurate than the transition and target forecasts over the full samples of the euro area and the United States. Japan is an important exception: during the prolonged low-inflation period, both the transition and low-inflation forecasts substantially outperform the target model, with the FKN low-inflation forecast performing similarly to, and in some cases better than, the transition forecast. The improvement in the low-inflation model does not, however, translate into systematic gains for the transition model, whose relative forecast accuracy generally remains close to the MSV baseline. In the euro area, the transition model continues to perform particularly well for core inflation during the prolonged low-inflation period, with results that are very similar to the MSV baseline. In the United States, the transition and target models remain approximately equally accurate, and the transition forecast performs very similarly under FKN and MSV.

These results are consistent with the forecast dynamics discussed above. The richer FKN dynamics allow the low-inflation model to accommodate persistent inflation movements and, in particular, disinflation from elevated inflation levels without rapidly pulling forecasts toward the low-inflation endpoint. This makes the low-inflation model useful over a broader range of inflation environments than under MSV. The improvement in the low-inflation forecast does not, however, translate one-for-one into gains for the transition model because FKN also changes the equilibrium probabilities that determine the relative influence of the forecasts from the two models. This interaction is particularly visible during the post-2020 disinflation in the euro area and the United States. As the probability of the target equilibrium declines, the low-inflation forecast receives more weight in the transition forecast, even though the target forecast remains more accurate over most horizons in this episode. The performance of the transition forecast therefore reflects the joint effect of the more flexible low-inflation dynamics and the associated changes in equilibrium probabilities.

E.3 Forecast Accuracy with Real-Time Data

The real-time comparisons in Table A.11 show that the broad central-bank comparisons remain similar under FKN, with some country-specific differences. In Japan, FKN improves both the transition and low-inflation forecasts relative to the Bank of Japan, bringing next-fiscal-year performance close to parity, while the Bank of Japan remains more accurate for current-fiscal-year forecasts. In the euro area, the FKN transition forecast is somewhat more accurate than its MSV counterpart relative to the ECB for core inflation, while differences for headline inflation are very small. This improvement does not materially change the main low-inflation-period result: the large transition-model gains for core inflation at the following-year and two-year horizons are essentially the same as under MSV. In the United States, the FKN transition forecast performs very similarly to its MSV counterpart relative to the Greenbook/Tealbook

Table A.10: Forecast Accuracy: FKN Solution (Relative RMSFEs, Revised Data)

	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	4	8	1	2	4	8
Japan								
Transition/Target								
1987Q2–2023Q1	0.97	0.96	1.01	1.04	0.91**	0.88**	0.92	0.99
1987Q2–1997Q4	1.03	1.04*	1.07	1.05	1.03*	1.05*	1.05**	1.01
1998Q1–2011Q4	0.90**	0.87***	0.85***	0.88**	0.54***	0.52***	0.55***	0.70***
2012Q1–2019Q4	0.98	0.95	0.97	0.92	0.93	0.89	0.95	0.91
2020Q1–2023Q1	0.98	1.05	1.49	1.99	0.97	1.01	1.13	2.00
Transition/Low-Inflation								
1987Q2–2023Q1	0.95*	0.94*	0.93	0.90	0.93***	0.91**	0.90*	0.87
1987Q2–1997Q4	0.91**	0.87***	0.83**	0.76	0.88***	0.83***	0.77**	0.68*
1998Q1–2011Q4	1.00	1.06	1.04	1.05	1.10**	1.14**	1.22*	1.17
2012Q1–2019Q4	1.00	0.99	1.03	1.05	0.99	1.02	1.07	1.09
2020Q1–2023Q1	0.90**	0.89**	0.89*	0.91	0.93*	0.91*	0.89	0.88**
Target/Low-Inflation								
1987Q2–2023Q1	0.98	0.98	0.93	0.86	1.02	1.04	0.98	0.88
1987Q2–1997Q4	0.89**	0.84***	0.78**	0.72	0.85***	0.79***	0.73**	0.67
1998Q1–2011Q4	1.11	1.22***	1.23***	1.19*	2.03***	2.20***	2.24***	1.67***
2012Q1–2019Q4	1.02	1.05	1.07	1.15	1.07	1.15	1.13	1.20
2020Q1–2023Q1	0.93	0.85	0.60	0.46	0.96	0.91	0.79	0.44
Euro Area								
Transition/Target								
2001Q2–2023Q3	1.00	1.01	1.03	1.13	0.91***	0.92	0.91	1.06
2001Q2–2009Q4	1.00	1.01	1.00	0.99	0.98	0.92	0.73	0.71
2010Q1–2019Q4	0.94	0.92	0.93	1.09	0.59***	0.48***	0.48***	0.72**
2020Q1–2023Q3	1.01	1.03	1.06	1.18	1.00	1.06	1.11	1.30
Transition/Low-Inflation								
2001Q2–2023Q3	0.91*	0.88**	0.80***	0.79**	0.66***	0.64***	0.60***	0.61***
2001Q2–2009Q4	0.72***	0.63***	0.56***	0.51***	0.38***	0.33***	0.26***	0.23***
2010Q1–2019Q4	0.84**	0.85	0.88	0.94	0.51***	0.42***	0.47***	0.65***
2020Q1–2023Q3	1.03	0.99	0.87	0.85	0.84	0.85*	0.75*	0.75
Target/Low-Inflation								
2001Q2–2023Q3	0.92	0.87**	0.78**	0.70*	0.72***	0.69***	0.65***	0.58**
2001Q2–2009Q4	0.72***	0.62***	0.56***	0.52***	0.39***	0.36***	0.35***	0.32***
2010Q1–2019Q4	0.89	0.93	0.95	0.86	0.87	0.87	0.98	0.89
2020Q1–2023Q3	1.03	0.96	0.82	0.72	0.84	0.80	0.68*	0.57
United States								
Transition/Target								
1990Q2–2023Q3	1.01*	1.01*	1.02**	1.02*	1.01**	1.01*	1.01**	1.01**
1990Q2–2009Q4	1.00*	1.01	1.01	1.01	1.01***	1.01**	1.03*	1.03
2010Q1–2019Q4	1.05	1.03	1.03	1.03	1.10*	1.06	1.05	1.00
2020Q1–2023Q3	1.00	1.01	1.01	1.03	1.00	1.00	1.00	1.00
Transition/Low-Inflation								
1990Q2–2023Q3	0.80***	0.74***	0.70***	0.65***	0.61***	0.56***	0.51***	0.46***
1990Q2–2009Q4	0.75***	0.67***	0.63***	0.58***	0.46***	0.40***	0.35***	0.28***
2010Q1–2019Q4	0.80**	0.78**	0.76**	0.78*	0.41***	0.51***	0.46***	0.58***
2020Q1–2023Q3	0.93	0.89	0.80	0.70	0.88	0.83	0.75	0.65
Target/Low-Inflation								
1990Q2–2023Q3	0.80***	0.73***	0.69***	0.64***	0.61***	0.55***	0.51***	0.46***
1990Q2–2009Q4	0.75***	0.67***	0.62***	0.57***	0.45***	0.39***	0.34***	0.27***
2010Q1–2019Q4	0.77**	0.76**	0.73**	0.76*	0.37***	0.48***	0.44***	0.58***
2020Q1–2023Q3	0.93	0.89	0.79	0.68	0.88	0.84	0.75	0.65

Notes: Entries are relative RMSFEs. Values below one favor the model in the numerator. Stars indicate rejection of the null hypothesis of equal predictive accuracy based on the Diebold–Mariano test, with *, **, and *** denoting significance at the 10%, 5%, and 1% levels, respectively. Sample periods refer to forecast origins.

projections. Despite the substantially improved FKN low-inflation dynamics documented above, the low-inflation forecast remains considerably less accurate than the central bank projections over the full samples of the euro area and the United States.

Overall, the FKN robustness exercise leaves the broad central-bank comparison from the main analysis largely unchanged. Central bank projections remain particularly strong at short and current-year horizons, while the transition model is relatively more competitive at longer horizons in persistent low-inflation environments.

Table A.11: Forecast Accuracy Relative to Central Bank Projections: FKN Solution (Real-Time Data)

Japan									
Forecast Horizon:		Current Fiscal Year				Next Fiscal Year			
Transition/Central Bank		Headline Inflation Excluding Fresh Food							
2003Q3–2023Q1		1.88**	(1.30	excl. tax)		1.05	(1.10	excl. tax)	
2003Q3–2011Q4		1.47				1.19			
2012Q1–2019Q4		1.80	(0.98	excl. tax)		0.98	(1.06	excl. tax)	
2020Q1–2023Q1		2.44				1.09			
Low-Inflation/Central Bank									
2003Q3–2023Q1		1.93*	(1.34	excl. tax)		1.09	(1.15	excl. tax)	
2003Q3–2011Q4		1.06				1.14			
2012Q1–2019Q4		1.90	(1.04	excl. tax)		0.90	(0.98	excl. tax)	
2020Q1–2023Q1		2.70				1.42			

Euro Area									
Forecast Horizon:		1	2	4	8		Current Year	Next Year	Two Years Ahead
Transition/Central Bank		Headline Inflation					Core Inflation		
2001Q2	AR Nowcast	1.06	1.02	1.10	1.06	2009Q2	2.73**	1.25	1.00
–2023Q3	CB Nowcast	0.98	0.99	1.08	1.06	-2023Q3			
2001Q2	AR Nowcast	1.08	0.91	0.90	0.94	2009Q2	1.69**	1.11	0.90
–2011Q1	CB Nowcast	0.96	0.89	0.90	0.93	-2019Q4			
2011Q2	AR Nowcast	0.96	1.01	1.06	1.02	2014Q1	1.50**	0.53***	0.42**
–2019Q4	CB Nowcast	1.00	1.00	1.06	1.02	-2018Q4			
2020Q1	AR Nowcast	1.07	1.06	1.15	1.11	2020Q1	2.98**	1.27	1.03
–2023Q3	CB Nowcast	0.99	1.02	1.13	1.10	-2023Q3			

United States									
Forecast Horizon:		1	2	3	4	1	2	3	4
Transition/Central Bank		Headline Inflation				Core Inflation			
1996Q2	AR Nowcast	1.43***	0.96	0.98	1.00	1.06	1.04	1.06	1.03
–2019Q4	CB Nowcast	1.31*	0.96	0.97	0.99	1.04	1.02	1.06	1.02
1996Q2	AR Nowcast	1.42**	0.93	0.97	1.01	0.93	1.06	1.11	1.11
–2009Q4	CB Nowcast	1.32	0.93	0.97	1.01	0.95	1.03	1.10	1.10
2010Q1	AR Nowcast	1.47***	1.05	1.00	0.99	1.30*	1.03	1.03	0.99
–2019Q4	CB Nowcast	1.30**	1.03	0.98	0.97	1.20*	1.01	1.03	0.98

Notes: Entries are relative RMSFEs. Values below one favor the model in the numerator. Stars indicate rejection of the null hypothesis of equal predictive accuracy based on the Diebold–Mariano test, with *, **, and *** denoting significance at the 10%, 5%, and 1% levels, respectively. For Japan, "incl. tax" and "excl. tax" refer to Bank of Japan projections including and excluding the effects of the 2014 consumption-tax increase, respectively. Sample periods refer to model forecast origins.

E.4 State-Dependent Tests of Relative Forecast Accuracy

Table A.12 reports full-sample relative RMSFEs across the model variants under the FKN solution, together with the state-dependent test of equal predictive accuracy.

Table A.12: Full-Sample Relative RMSFEs and State-Dependent Test Results: FKN Solution

	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	4	8	1	2	4	8
				Japan				
1987Q2–2023Q1								
Transition/Target	0.97	0.96	1.01	1.04	0.91^{**}	0.88^{***}	0.92^{*}	0.99
Transition/Low-Inflation	0.95	0.94^{*}	0.93	0.90	0.93	0.91^{*}	0.90^{**}	0.87^{*}
Target/Low-Inflation	0.98	0.98^{**}	0.93	0.86	1.02	1.04^{***}	0.98^{**}	0.88
				Euro Area				
2001Q2–2023Q3								
Transition/Target	1.00	1.01	1.03	1.13	0.91^{**}	0.92	0.91^{*}	1.06
Transition/Low-Inflation	0.91^{***}	0.88^{***}	0.80^{***}	0.79^{***}	0.66^{***}	0.64^{***}	0.60^{***}	0.61^{***}
Target/Low-Inflation	0.92^{**}	0.87^{**}	0.78^{**}	0.70^{**}	0.72^{***}	0.69^{***}	0.65^{***}	0.58^{***}
				United States				
1990Q2–2023Q3								
Transition/Target	1.01	1.01	1.02	1.02	1.01	1.01	1.01^{*}	1.01^{*}
Transition/Low-Inflation	0.80^{***}	0.74^{***}	0.70^{***}	0.65^{***}	0.61^{***}	0.56^{***}	0.51^{***}	0.46^{***}
Target/Low-Inflation	0.80^{***}	0.73^{***}	0.69^{***}	0.64^{***}	0.61^{***}	0.55^{***}	0.51^{***}	0.46^{***}

Notes: Entries are relative RMSFEs based on revised data; values below one favor the model in the numerator. The reported sample periods refer to forecast origins. Stars indicate rejection of the joint null hypothesis of equal average predictive accuracy and no state dependence in the loss differential, H_0: $\delta_0 = \delta_1 = 0$, based on the exp-W statistic. They do not indicate whether the reported relative RMSFE differs significantly from one. The state variable is headline or core inflation observed at the forecast origin, as indicated. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

E.5 State-Dependent Comparisons with Central Bank Projections

Table A.13 compares the FKN model-based forecasts with the central bank projections on the real-time samples.

Table A.13: Full-Sample Relative RMSFEs and State-Dependent Test Results: Model-Based Forecasts versus Central Bank Projections, FKN Solution

	Japan							
2003Q3–2023Q1	Headline Inflation Excluding Fresh Food							
Forecast Horizon:	Current Fiscal Year				Next Fiscal Year			
	incl. tax		excl. tax		incl. tax		excl. tax	
Transition/CB	1.88**		1.30		1.05		1.10	
Target/CB	1.95***		1.35		1.13**		1.18**	
Low-Infl./CB	1.93*		1.34		1.09		1.15	
	Euro Area (AR nowcast)							
2001Q2–2023Q3	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	4	8	Current Year	Next Year	Two Years Ahead	
Transition/CB	1.06	1.02	1.10	1.06	2.73**	1.25**	1.00	
Target/CB	1.02	0.97	0.99	0.93	2.69***	1.28**	0.98	
Low-Infl./CB	1.19**	1.20**	1.34**	1.32**	3.49***	1.97***	1.66**	
	Euro Area (CB nowcast)							
2001Q2–2023Q3	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	4	8	Current Year	Next Year	Two Years Ahead	
Transition/CB	0.98	0.99	1.08	1.06	—	—	—	
Target/CB	0.96	0.96	0.99	0.93	—	—	—	
Low-Infl./CB	1.07**	1.14**	1.29**	1.31**	—	—	—	
	United States (AR nowcast)							
1996Q2–2019Q4	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	3	4	1	2	3	4
Transition/CB	1.43**	0.96*	0.98	1.00	1.06	1.04	1.06	1.03
Target/CB	1.39*	0.93	0.96**	0.98	0.95	0.93	0.94	0.93
Low-Infl./CB	1.85***	1.34***	1.42***	1.45***	2.84***	2.79***	2.94***	2.96***
	United States (CB nowcast)							
1996Q2–2019Q4	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	3	4	1	2	3	4
Transition/CB	1.31**	0.96*	0.97	0.99	1.04	1.02	1.06	1.02
Target/CB	1.29*	0.94	0.96**	0.98	0.94	0.92	0.94	0.93
Low-Infl./CB	1.68***	1.33***	1.39***	1.44***	2.78***	2.75***	2.92***	2.93***

Notes: Entries are relative RMSFEs; values below one favor the model-based forecast. Sample periods refer to model forecast origins. AR-nowcast and CB-nowcast denote initialization with autoregressive and central bank nowcasts, respectively. For Japan, "incl. tax" and "excl. tax" refer to Bank of Japan projections including and excluding the effects of the 2014 consumption-tax increase. Stars indicate rejection of H_0: $\delta_0 = \delta_1 = 0$, which combines equal average predictive accuracy and no state dependence in the loss differential, based on the exp-W statistic. They do not indicate whether the reported relative RMSFE differs significantly from one. The state variable is headline or core inflation at the forecast origin. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

E.6 Mincer–Zarnowitz Results

The revised-data Mincer–Zarnowitz results show a calibration pattern for the FKN transition forecasts that is generally close to the MSV baseline. In Japan, the restrictions are not rejected for headline inflation at short horizons and more broadly for core inflation, closely matching the MSV pattern. The corresponding coefficient estimates are also similar in magnitude, with the core-inflation estimates at short horizons lying particularly close to the joint restrictions $\beta_0 = 0$

and $\beta_1 = 1$. In the euro area, the restrictions are not rejected in somewhat more headline-inflation specifications under FKN, while the core-inflation results remain broadly similar to MSV, with the more favorable evidence concentrated at medium horizons. The additional headline non-rejections do not uniformly provide stronger evidence of calibration because some of the corresponding coefficient estimates remain relatively far from the joint restrictions. In the United States, the joint calibration restrictions for the transition forecasts continue to be rejected throughout the full sample. For the low-inflation forecasts, FKN moves several coefficient estimates closer to the joint restrictions than MSV, but the restrictions remain rejected at all full-sample horizons in all three economies. Overall, changing from the MSV to the FKN solution leaves the main cross-country calibration pattern largely unchanged.

The real-time Mincer–Zarnowitz results are likewise broadly similar under FKN and MSV. In Japan, the current-fiscal-year restrictions for the transition model are not rejected under either solution. For the next fiscal year, FKN yields a non-rejection where MSV does not, but the coefficient estimates remain relatively far from the joint restrictions, providing only limited additional evidence of calibration. In the euro area, the rejection pattern changes little, with the comparatively favorable evidence remaining concentrated in core inflation beyond the current year. In the United States, FKN yields a few headline-inflation non-rejections not observed under MSV, but the corresponding coefficients remain far from the joint restrictions, while core inflation continues to be rejected under both solutions. For the low-inflation forecasts, the full-sample rejection pattern is unchanged from MSV: the current-fiscal-year restrictions are not rejected in Japan, while the restrictions are rejected for the next fiscal year in Japan and throughout the euro area and the United States.

Table A.14: Mincer–Zarnowitz Regression Results: FKN Solution (Revised Data)

		Headline Inflation				Core Inflation			
Forecast Horizon:		1	2	4	8	1	2	4	8
		Japan							
Transition									
1987Q2–2023Q1	β_0	0.23	0.24	0.53	0.75	0.14	0.17	0.31	0.53
	β_1	0.72	0.74	0.27	-0.01	0.97	1.00	0.74	0.28
	p-value	0.07	0.13	0.00	0.02	0.35	0.39	0.26	0.05
1987Q2–1997Q4	β_0	0.95	0.69	2.09	1.82	0.78	0.44	1.41	0.03
	β_1	0.46	0.82	-0.78	-0.76	0.69	1.09	0.10	1.63
	p-value	0.11	0.22	0.05	0.36	0.20	0.13	0.09	0.43
1998Q1–2011Q4	β_0	-0.22	-0.26	-0.31	-0.12	-0.22	-0.31	-0.41	-0.44
	β_1	0.34	0.14	-0.19	-0.30	0.72	0.52	0.28	-0.48
	p-value	0.13	0.04	0.00	0.00	0.01	0.00	0.00	0.00
2012Q1–2019Q4	β_0	0.51	0.69	1.43	1.23	0.38	0.48	0.91	0.89
	β_1	0.22	0.02	-0.73	-0.66	0.40	0.25	-0.33	-0.52
	p-value	0.02	0.14	0.00	0.00	0.01	0.00	0.00	0.00
2020Q1–2023Q1	β_0	0.84	1.23	2.07	3.33	0.74	1.10	1.59	3.02
	β_1	1.04	1.30	0.65	-0.03	1.19	1.65	1.45	-0.04
	p-value	0.18	0.03	0.02	0.00	0.48	0.03	0.09	0.00
Target									
1987Q2–2023Q1	β_0	-0.33	-0.89	-0.36	-0.16	-0.77	-1.17	-0.60	-0.21
	β_1	0.91	1.28	0.81	0.71	1.30	1.46	0.95	0.67
	p-value	0.02	0.00	0.02	0.15	0.00	0.00	0.01	0.07
1987Q2–1997Q4	β_0	0.75	-1.32	-7.11	2.70	0.46	-1.94	-8.13	0.12
	β_1	0.57	2.51	8.24	-1.41	0.90	3.14	9.27	1.24
	p-value	0.37	0.24	0.10	0.89	0.37	0.03	0.00	0.64
1998Q1–2011Q4	β_0	-0.45	-1.32	-0.31	6.56	-1.35	-2.80	-5.49	1.63
	β_1	0.38	1.28	-0.01	-6.83	1.86	2.92	5.28	-2.06
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2012Q1–2019Q4	β_0	-0.04	-0.31	2.38	4.91	-0.34	-1.09	0.56	4.95
	β_1	0.50	0.58	-0.91	-2.32	0.68	1.01	0.06	-2.38
	p-value	0.03	0.01	0.00	0.00	0.01	0.00	0.00	0.00
2020Q1–2023Q1	β_0	-1.24	-10.59	-36.46	25.54	-1.23	-9.91	-70.49	18.78
	β_1	1.63	6.79	20.27	-11.33	1.58	6.42	37.87	-8.04
	p-value	0.66	0.00	0.01	0.00	0.56	0.01	0.00	0.00
Low-Inflation									
1987Q2–2023Q1	β_0	0.51	0.62	0.68	0.65	0.45	0.59	0.65	0.51
	β_1	0.65	0.73	0.10	-0.74	0.97	1.10	0.62	-0.87
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
1987Q2–1997Q4	β_0	1.21	1.31	1.45	0.81	1.22	1.35	1.51	1.05
	β_1	0.41	0.54	-0.13	-1.78	0.55	0.57	-0.12	-1.68
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1998Q1–2011Q4	β_0	-0.14	-0.16	-0.40	-0.22	-0.07	-0.17	-0.28	-0.57
	β_1	0.33	0.31	-0.28	-0.28	0.85	0.66	0.49	-0.59
	p-value	0.05	0.06	0.00	0.00	0.56	0.34	0.38	0.00
2012Q1–2019Q4	β_0	0.61	0.75	0.97	0.84	0.52	0.56	0.81	0.74
	β_1	0.24	-0.11	-1.04	-1.27	0.41	0.37	-0.67	-1.57
	p-value	0.00	0.01	0.03	0.00	0.04	0.03	0.07	0.01
2020Q1–2023Q1	β_0	1.10	1.62	2.30	3.32	1.09	1.72	2.19	2.95
	β_1	1.00	1.44	0.69	-0.10	1.07	1.66	1.37	-0.32
	p-value	0.12	0.01	0.00	0.00	0.28	0.02	0.03	0.00

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h - 1$ lags.

Table A.15: Mincer–Zarnowitz Regression Results: FKN Solution (Revised Data)

		Headline Inflation				Core Inflation			
Forecast Horizon:		1	2	4	8	1	2	4	8
		Euro Area							
Transition									
2001Q2–2023Q3	β_0	-0.25	0.48	1.41	3.64	-0.31	0.06	0.33	1.58
	β_1	1.37	1.03	0.46	-0.99	1.34	1.14	0.97	0.08
	p-value	0.11	0.14	0.06	0.06	0.04	0.15	0.31	0.21
2001Q2–2009Q4	β_0	0.28	1.55	0.99	-0.12	0.28	1.03	1.10	1.13
	β_1	0.90	0.25	0.55	1.24	0.87	0.39	0.30	0.28
	p-value	0.86	0.36	0.69	0.23	0.55	0.45	0.25	0.11
2010Q1–2019Q4	β_0	0.32	0.58	1.01	2.36	0.44	0.35	0.31	1.64
	β_1	0.77	0.50	0.11	-0.80	0.55	0.61	0.58	-0.45
	p-value	0.69	0.29	0.08	0.00	0.08	0.17	0.12	0.00
2020Q1–2023Q3	β_0	1.18	3.21	4.94	8.69	0.84	1.65	2.40	4.33
	β_1	1.33	0.85	0.09	-2.57	1.18	1.00	0.82	-0.49
	p-value	0.06	0.02	0.01	0.00	0.02	0.01	0.00	0.00
Target									
2001Q2–2023Q3	β_0	-2.10	-5.09	4.35	36.42	-2.39	-5.10	-1.68	15.91
	β_1	2.11	3.68	-1.13	-17.31	2.18	3.52	1.70	-7.18
	p-value	0.01	0.12	0.74	0.01	0.00	0.00	0.31	0.00
2001Q2–2009Q4	β_0	-0.02	1.57	4.64	41.97	-1.37	-3.33	-2.99	7.95
	β_1	0.99	0.22	-1.32	-19.90	1.57	2.48	2.25	-3.20
	p-value	0.97	0.85	0.62	0.03	0.00	0.00	0.00	0.00
2010Q1–2019Q4	β_0	-1.06	-3.72	0.31	-49.09	0.14	0.50	3.69	-11.93
	β_1	1.38	2.67	0.43	25.61	0.57	0.32	-1.39	6.62
	p-value	0.06	0.01	0.04	0.01	0.00	0.00	0.00	0.00
2020Q1–2023Q3	β_0	-1.19	-2.86	4.48	66.54	-1.33	-4.25	-5.71	23.00
	β_1	2.16	3.66	0.32	-31.76	2.01	3.80	4.86	-9.91
	p-value	0.02	0.05	0.09	0.00	0.01	0.02	0.03	0.00
Low-Inflation									
2001Q2–2023Q3	β_0	1.05	1.58	2.00	2.24	0.98	1.34	1.54	1.68
	β_1	1.07	0.89	0.31	-0.49	1.00	0.90	0.67	0.10
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2001Q2–2009Q4	β_0	1.52	2.00	2.03	1.96	1.26	1.52	1.59	1.38
	β_1	0.58	0.03	-0.33	-0.53	0.73	0.62	0.42	-0.41
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2010Q1–2019Q4	β_0	0.80	0.93	1.08	1.43	1.06	1.11	1.09	1.08
	β_1	0.84	0.75	0.22	-0.59	0.03	-0.13	-0.50	0.36
	p-value	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
2020Q1–2023Q3	β_0	2.16	3.66	5.45	6.35	1.79	2.45	3.27	4.01
	β_1	0.98	0.66	-0.23	-1.48	0.87	0.77	0.41	-0.62
	p-value	0.04	0.00	0.01	0.00	0.00	0.00	0.00	0.00

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h - 1$ lags.

Table A.16: Mincer–Zarnowitz Regression Results: FKN Solution (Revised Data)

		Headline Inflation				Core Inflation			
Forecast Horizon:		1	2	4	8	1	2	4	8
		United States							
Transition									
1990Q2–2023Q3	β_0	0.17	0.44	2.69	2.32	-1.09	-2.67	1.02	0.40
	β_1	1.19	1.15	-0.06	0.14	1.75	2.69	0.78	1.08
	p-value	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.01
1990Q2–2009Q4	β_0	1.20	3.21	6.40	3.86	-2.25	-6.06	-1.05	4.19
	β_1	0.69	-0.31	-2.01	-0.68	2.25	4.30	1.77	-0.98
	p-value	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00
2010Q1–2019Q4	β_0	0.87	2.50	2.23	1.76	0.55	2.41	2.09	-0.66
	β_1	0.54	-0.50	-0.29	0.20	0.77	-0.31	-0.07	1.57
	p-value	0.67	0.07	0.00	0.00	0.17	0.02	0.00	0.09
2020Q1–2023Q3	β_0	0.71	1.83	12.49	7.26	1.66	1.13	19.02	19.26
	β_1	1.48	1.61	-3.86	-1.71	1.00	1.67	-7.35	-7.77
	p-value	0.02	0.00	0.00	0.00	0.01	0.00	0.00	0.01
Target									
1990Q2–2023Q3	β_0	-0.10	-0.74	4.42	10.36	-1.36	-4.68	0.66	5.86
	β_1	1.29	1.70	-0.95	-3.94	1.86	3.66	0.95	-1.71
	p-value	0.02	0.01	0.05	0.11	0.00	0.00	0.00	0.05
1990Q2–2009Q4	β_0	1.15	3.08	1.73	24.91	-2.32	-6.13	-4.07	5.61
	β_1	0.70	-0.24	0.40	-11.27	2.27	4.28	3.27	-1.68
	p-value	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.18
2010Q1–2019Q4	β_0	-0.02	5.21	4.82	-86.02	-0.29	2.13	-0.29	-45.29
	β_1	0.99	-1.92	-1.62	45.13	1.21	-0.14	1.19	24.35
	p-value	0.99	0.24	0.64	0.07	0.37	0.72	0.65	0.25
2020Q1–2023Q3	β_0	0.67	0.45	28.19	74.15	1.63	0.72	19.90	41.83
	β_1	1.49	2.24	-11.86	-35.62	1.02	1.87	-7.83	-19.15
	p-value	0.02	0.00	0.00	0.00	0.01	0.00	0.00	0.00
Low-Inflation									
1990Q2–2023Q3	β_0	1.92	2.40	2.56	2.57	1.68	2.15	2.42	2.50
	β_1	0.72	0.46	0.08	-0.27	0.91	0.89	0.46	0.31
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1990Q2–2009Q4	β_0	2.32	2.64	2.52	2.43	1.48	2.17	2.50	2.55
	β_1	0.41	-0.17	-0.07	-0.32	1.32	1.41	1.17	0.62
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2010Q1–2019Q4	β_0	1.36	1.98	1.91	2.10	1.42	1.88	1.93	1.97
	β_1	0.58	-0.79	-0.60	-0.01	0.64	-0.05	0.14	1.35
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2020Q1–2023Q3	β_0	3.13	4.77	6.30	5.71	3.20	4.09	5.34	5.00
	β_1	0.65	0.29	-0.79	-1.36	0.49	0.38	-0.53	-0.87
	p-value	0.05	0.00	0.00	0.00	0.01	0.00	0.00	0.00

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h - 1$ lags.

Table A.17: Mincer–Zarnowitz Regression Results: FKN Solution (Real-Time Data)

Japan			
Forecast Horizon:		Current Fiscal Year	Next Fiscal Year
Transition		Headline ex. fresh food Inflation	
2003Q3–2023Q1	β_0	0.06	0.39
	β_1	1.09	0.37
	p-value	0.69	0.14
2003Q3–2011Q4	β_0	-0.15	-0.27
	β_1	0.94	-0.29
	p-value	0.57	0.00
2012Q1–2019Q4	β_0	0.22	0.96
	β_1	0.88	-0.49
	p-value	0.08	0.00
2020Q1–2023Q1	β_0	0.16	0.81
	β_1	1.30	1.47
	p-value	0.14	0.00
Target			
2003Q3–2023Q1	β_0	-0.24	-1.09
	β_1	1.11	1.17
	p-value	0.01	0.00
2003Q3–2011Q4	β_0	-0.35	0.28
	β_1	0.94	-0.66
	p-value	0.08	0.00
2012Q1–2019Q4	β_0	-0.11	0.88
	β_1	1.00	-0.14
	p-value	0.57	0.00
2020Q1–2023Q1	β_0	-0.19	-0.42
	β_1	1.26	1.34
	p-value	0.17	0.91
Low-Inflation			
2003Q3–2023Q1	β_0	0.20	0.55
	β_1	1.13	0.51
	p-value	0.13	0.02
2003Q3–2011Q4	β_0	-0.06	-0.33
	β_1	1.01	-0.24
	p-value	0.80	0.00
2012Q1–2019Q4	β_0	0.36	0.69
	β_1	0.93	-1.68
	p-value	0.08	0.00
2020Q1–2023Q1	β_0	0.31	1.44
	β_1	1.31	1.58
	p-value	0.05	0.00
Central Bank			
2003Q3–2023Q1	β_0	-0.13 (-0.05 excl. tax)	-0.06 (-0.03 excl. tax)
	β_1	0.98 (1.10 excl. tax)	0.62 (0.72 excl. tax)
	p-value	0.03 (0.17 excl. tax)	0.02 (0.52 excl. tax)
2003Q3–2011Q4	β_0	-0.18	-0.34
	β_1	0.92	0.36
	p-value	0.04	0.01
2012Q1–2019Q4	β_0	-0.15 (-0.19 excl. tax)	-0.49 (0.83 excl. tax)
	β_1	0.90 (1.36 excl. tax)	0.65 (-0.15 excl. tax)
	p-value	0.00 (0.20 excl. tax)	0.00 (0.00 excl. tax)
2020Q1–2023Q1	β_0	0.11	-0.02
	β_1	1.09	2.10
	p-value	0.18	0.00

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For annual fixed-event forecasts, we use $h_{\max} - 1$ lags, where $h_{\max}$ denotes the maximum forecast horizon in quarters of the respective fixed event. For Japan, "incl. tax" and "excl. tax" refer to Bank of Japan projections including and excluding the effects of the 2014 consumption-tax increase, respectively.

Table A.18: Mincer–Zarnowitz Regression Results: FKN Solution (Real-Time Data)

Euro Area										
Forecast Horizon:			1	2	4	8		Current Year	Next Year	Two Years Ahead
Transition			Headline Inflation					Core Inflation		
2001Q2	AR Nowcast	β_0	0.59	1.39	2.67	3.53	2009Q2	0.37	0.56	1.21
–2023Q3		β_1	0.96	0.49	-0.36	-0.95	-2023Q3	0.76	0.78	0.38
		p-value	0.04	0.03	0.08	0.05		0.01	0.57	0.48
2001Q2	CB Nowcast	β_0	-0.06	0.98	2.52	3.73				
–2023Q3		β_1	1.28	0.75	-0.26	-1.08				
		p-value	0.11	0.07	0.09	0.07				
2001Q2	AR Nowcast	β_0	2.16	2.28	2.08	1.25	2009Q2	0.50	0.99	1.16
–2011Q1		β_1	-0.05	-0.12	0.02	0.48	-2019Q4	0.50	0.06	-0.01
		p-value	0.01	0.06	0.17	0.27		0.00	0.00	0.00
2001Q2	CB Nowcast	β_0	0.70	2.20	2.19	1.14				
–2011Q1		β_1	0.71	-0.07	-0.04	0.54				
		p-value	0.58	0.11	0.12	0.28				
2011Q2	AR Nowcast	β_0	0.58	0.72	1.01	1.93	2014Q1	0.91	0.90	1.17
–2019Q4		β_1	0.50	0.28	-0.11	-0.59	-2018Q4	0.05	0.07	-0.21
		p-value	0.13	0.06	0.00	0.00		0.00	0.00	0.00
2011Q2	CB Nowcast	β_0	0.66	0.67	1.04	2.04				
–2019Q4		β_1	0.40	0.31	-0.13	-0.66				
		p-value	0.09	0.06	0.00	0.00				
2020Q1	AR Nowcast	β_0	2.64	4.31	6.74	7.94	2020Q1	1.20	3.37	6.37
–2023Q3		β_1	0.91	0.34	-1.18	-2.48	-2023Q3	0.62	0.02	-1.71
		p-value	0.06	0.01	0.00	0.00		0.00	0.00	0.00
2020Q1	CB Nowcast	β_0	1.30	3.53	6.58	8.88				
–2023Q3		β_1	1.46	0.82	-1.06	-3.12				
		p-value	0.05	0.01	0.00	0.00				
Target			Headline Inflation					Core Inflation		
2001Q2	AR Nowcast	β_0	-1.20	-1.87	13.96	30.12	2009Q2	0.21	-0.34	22.25
–2023Q3		β_1	1.74	2.09	-6.09	-14.19	-2023Q3	0.79	1.07	-10.55
		p-value	0.18	0.58	0.33	0.20		0.05	0.86	0.00
2001Q2	CB Nowcast	β_0	-2.09	-3.89	13.41	29.24				
–2023Q3		β_1	2.11	3.09	-5.80	-13.75				
		p-value	0.05	0.34	0.35	0.20				
2001Q2	AR Nowcast	β_0	2.02	1.50	8.45	34.43	2009Q2	0.35	0.85	-10.11
–2011Q1		β_1	0.03	0.29	-3.16	-16.18	-2019Q4	0.56	0.12	5.77
		p-value	0.17	0.72	0.07	0.02		0.00	0.00	0.00
2001Q2	CB Nowcast	β_0	-0.01	1.86	8.32	34.58				
–2011Q1		β_1	0.99	0.10	-3.09	-16.26				
		p-value	0.99	0.74	0.09	0.02				
2011Q2	AR Nowcast	β_0	-1.04	-4.21	-2.57	-85.54	2014Q1	0.83	1.13	-0.80
–2019Q4		β_1	1.35	2.93	1.81	44.29	-2018Q4	0.11	-0.09	0.90
		p-value	0.04	0.00	0.00	0.00		0.00	0.00	0.00
2011Q2	CB Nowcast	β_0	0.15	-2.78	-1.32	-91.16				
–2019Q4		β_1	0.59	2.11	1.15	47.17				
		p-value	0.02	0.00	0.00	0.00				
2020Q1	AR Nowcast	β_0	0.87	2.96	27.14	62.09	2020Q1	1.08	3.66	22.26
–2023Q3		β_1	1.52	0.95	-11.80	-29.77	-2023Q3	0.65	-0.12	-9.74
		p-value	0.08	0.11	0.03	0.00		0.01	0.03	0.00
2020Q1	CB Nowcast	β_0	-1.27	-1.58	26.42	57.15				
–2023Q3		β_1	2.30	3.18	-11.40	-27.23				
		p-value	0.05	0.08	0.04	0.00				

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h-1$ lags. For annual fixed-event forecasts, we use $h_{\max} - 1$ lags, where $h_{\max}$ denotes the maximum forecast horizon in quarters of the respective fixed event.

Table A.19: Mincer–Zarnowitz Regression Results: FKN Solution (Real-Time Data)

Euro Area										
Forecast Horizon:			1	2	4	8		Current Year	Next Year	Two Years Ahead
Low-Inflation			Headline Inflation					Core Inflation		
2001Q2	AR Nowcast	β_0	1.42	1.85	2.15	2.19	2009Q2	0.71	1.53	1.74
–2023Q3		β_1	0.85	0.60	-0.07	-0.47	-2023Q3	0.66	0.36	0.05
		p-value	0.00	0.00	0.00	0.00		0.00	0.00	0.00
2001Q2	CB Nowcast	β_0	1.05	1.67	2.08	2.23				
–2023Q3		β_1	1.11	0.83	0.14	-0.57				
		p-value	0.00	0.00	0.00	0.00				
2001Q2	AR Nowcast	β_0	1.99	2.04	2.16	1.94	2009Q2	0.79	1.10	1.14
–2011Q1		β_1	0.11	0.08	-0.42	-0.74	-2019Q4	0.35	-0.23	-0.17
		p-value	0.00	0.00	0.00	0.00		0.00	0.00	0.00
2001Q2	CB Nowcast	β_0	1.52	2.05	2.18	2.01				
–2011Q1		β_1	0.60	0.05	-0.35	-0.58				
		p-value	0.00	0.00	0.00	0.00				
2011Q2	AR Nowcast	β_0	0.81	0.87	0.81	1.23	2014Q1	0.96	0.96	0.96
–2019Q4		β_1	0.77	0.74	0.44	-0.24	-2018Q4	-0.01	0.08	-0.09
		p-value	0.00	0.00	0.02	0.00		0.00	0.00	0.00
2011Q2	CB Nowcast	β_0	0.92	0.86	0.83	1.37				
–2019Q4		β_1	0.43	0.68	0.27	-1.32				
		p-value	0.00	0.01	0.02	0.00				
2020Q1	AR Nowcast	β_0	3.24	4.60	6.08	5.98	2020Q1	1.55	3.44	3.89
–2023Q3		β_1	0.66	0.17	-0.82	-1.46	-2023Q3	0.54	-0.03	-0.58
		p-value	0.03	0.01	0.00	0.00		0.00	0.00	0.00
2020Q1	CB Nowcast	β_0	2.35	4.17	5.97	6.45				
–2023Q3		β_1	1.02	0.45	-0.69	-1.80				
		p-value	0.04	0.01	0.01	0.00				
Central Bank			Headline Inflation					Core Inflation		
2001Q2	AR Nowcast	β_0	0.51	1.24	1.43	3.01	2009Q2	-0.02	-0.31	1.87
–2023Q3		β_1	0.94	0.53	0.40	-0.49	-2023Q3	1.03	1.28	-0.07
		p-value	0.08	0.13	0.12	0.00		0.80	0.26	0.53
2001Q2	CB Nowcast	β_0	0.51	1.24	1.43	3.01				
–2023Q3		β_1	0.94	0.53	0.40	-0.49				
		p-value	0.08	0.13	0.12	0.00				
2001Q2	AR Nowcast	β_0	1.09	2.46	2.47	2.12	2009Q2	0.12	0.76	2.21
–2011Q1		β_1	0.57	-0.23	-0.19	-0.03	-2019Q4	0.86	0.22	-0.68
		p-value	0.02	0.00	0.00	0.00		0.17	0.00	0.00
2001Q2	CB Nowcast	β_0	1.09	2.46	2.47	2.12				
–2011Q1		β_1	0.57	-0.23	-0.19	-0.03				
		p-value	0.02	0.00	0.00	0.00				
2011Q2	AR Nowcast	β_0	0.68	0.60	0.79	2.46	2014Q1	0.77	1.09	1.43
–2019Q4		β_1	0.36	0.32	0.06	-0.78	-2018Q4	0.19	-0.09	-0.29
		p-value	0.04	0.09	0.00	0.00		0.00	0.00	0.00
2011Q2	CB Nowcast	β_0	0.68	0.60	0.79	2.46				
–2019Q4		β_1	0.36	0.32	0.06	-0.78				
		p-value	0.04	0.09	0.00	0.00				
2020Q1	AR Nowcast	β_0	2.56	4.86	6.32	7.47	2020Q1	0.30	2.07	5.85
–2023Q3		β_1	0.74	-0.01	-0.58	-1.68	-2023Q3	0.96	0.64	-1.30
		p-value	0.15	0.12	0.03	0.10		0.24	0.01	0.00
2020Q1	CB Nowcast	β_0	2.56	4.86	6.32	7.47				
–2023Q3		β_1	0.74	-0.01	-0.58	-1.68				
		p-value	0.15	0.12	0.03	0.10				

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h-1$ lags. For annual fixed-event forecasts, we use $h_{\max} - 1$ lags, where $h_{\max}$ denotes the maximum forecast horizon in quarters of the respective fixed event.

Table A.20: Mincer–Zarnowitz Regression Results: FKN Solution (Real-Time Data)

United States										
Forecast Horizon:			1	2	3	4	1	2	3	4
Transition			Headline Inflation				Core Inflation			
1996Q2	AR Nowcast	β_0	2.59	2.15	1.87	2.09	0.94	1.42	1.61	1.72
–2019Q4		β_1	-0.25	-0.03	0.14	0.02	0.60	0.32	0.22	0.17
		p-value	0.00	0.01	0.07	0.01	0.00	0.00	0.00	0.00
1996Q2	CB Nowcast	β_0	1.32	2.70	1.90	2.14	0.89	1.23	1.62	1.66
–2019Q4		β_1	0.45	-0.34	0.12	-0.01	0.62	0.43	0.22	0.20
		p-value	0.07	0.01	0.12	0.02	0.00	0.01	0.00	0.00
1996Q2	AR Nowcast	β_0	3.46	2.89	4.39	6.01	-0.46	-1.71	-1.53	0.13
–2009Q4		β_1	-0.52	-0.25	-1.08	-1.93	1.32	1.99	1.90	1.01
		p-value	0.00	0.00	0.03	0.00	0.01	0.01	0.04	0.33
1996Q2	CB Nowcast	β_0	1.63	3.85	4.13	6.47	0.25	-2.31	-1.76	0.08
–2009Q4		β_1	0.40	-0.75	-0.93	-2.16	0.95	2.30	2.01	1.03
		p-value	0.14	0.00	0.02	0.00	0.15	0.00	0.03	0.33
2010Q1	AR Nowcast	β_0	2.41	2.58	1.88	1.93	1.44	2.08	2.09	1.95
–2019Q4		β_1	-0.44	-0.59	-0.07	-0.11	0.29	-0.13	-0.09	0.01
		p-value	0.00	0.10	0.30	0.04	0.00	0.00	0.00	0.00
2010Q1	CB Nowcast	β_0	1.48	3.50	2.17	1.98	1.18	2.00	2.14	1.88
–2019Q4		β_1	0.18	-1.14	-0.25	-0.14	0.43	-0.08	-0.12	0.05
		p-value	0.17	0.05	0.42	0.07	0.01	0.00	0.00	0.00
Target			Headline Inflation				Core Inflation			
1996Q2	AR Nowcast	β_0	2.71	1.95	1.21	0.87	0.05	-0.37	-1.24	-1.57
–2019Q4		β_1	-0.29	0.08	0.47	0.65	1.04	1.24	1.70	1.86
		p-value	0.00	0.24	0.63	0.69	0.03	0.40	0.12	0.20
1996Q2	CB Nowcast	β_0	1.09	2.74	1.19	1.11	0.25	-0.64	-1.14	-1.59
–2019Q4		β_1	0.55	-0.33	0.48	0.52	0.93	1.38	1.65	1.87
		p-value	0.25	0.05	0.62	0.68	0.16	0.33	0.16	0.19
1996Q2	AR Nowcast	β_0	3.48	2.16	0.80	0.52	-0.62	-2.32	-3.97	-5.68
–2009Q4		β_1	-0.51	0.12	0.80	0.95	1.38	2.25	3.09	3.94
		p-value	0.00	0.11	0.47	0.42	0.02	0.00	0.00	0.00
1996Q2	CB Nowcast	β_0	1.58	3.29	0.29	0.85	0.17	-2.61	-3.95	-5.61
–2009Q4		β_1	0.42	-0.45	1.06	0.78	0.97	2.39	3.07	3.90
		p-value	0.19	0.01	0.47	0.41	0.31	0.00	0.00	0.00
2010Q1	AR Nowcast	β_0	2.63	5.14	4.35	4.11	0.67	1.57	1.09	1.88
–2019Q4		β_1	-0.52	-1.94	-1.40	-1.25	0.70	0.16	0.46	0.05
		p-value	0.00	0.14	0.40	0.44	0.30	0.36	0.39	0.15
2010Q1	CB Nowcast	β_0	0.74	4.45	4.57	4.28	0.39	1.29	1.28	1.77
–2019Q4		β_1	0.57	-1.52	-1.50	-1.33	0.83	0.31	0.35	0.11
		p-value	0.65	0.08	0.29	0.42	0.45	0.46	0.34	0.16

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h - 1$ lags.

Table A.21: Mincer–Zarnowitz Regression Results: FKN Solution (Real-Time Data)

United States										
Forecast Horizon:			1	2	3	4	1	2	3	4
Low-Inflation			Headline Inflation				Core Inflation			
1996Q2	AR Nowcast	β_0	2.34	2.14	2.11	2.11	1.81	1.98	2.01	2.02
–2019Q4		β_1	-0.32	-0.31	-0.33	-0.13	0.39	0.15	0.17	0.09
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1996Q2	CB Nowcast	β_0	2.02	2.21	2.14	2.11	1.80	1.96	2.01	2.02
–2019Q4		β_1	0.20	-0.49	-0.37	-0.32	0.38	0.21	0.15	0.12
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1996Q2	AR Nowcast	β_0	2.64	2.41	2.33	2.40	1.80	2.12	2.21	2.28
–2009Q4		β_1	-0.33	-0.11	-0.21	0.12	0.66	0.65	0.59	0.69
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1996Q2	CB Nowcast	β_0	2.32	2.45	2.35	2.36	1.85	2.11	2.21	2.25
–2009Q4		β_1	0.18	-0.32	-0.17	-0.07	0.53	0.77	0.63	0.65
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2010Q1	AR Nowcast	β_0	1.95	1.84	1.83	1.80	1.69	1.84	1.89	1.99
–2019Q4		β_1	-0.35	-0.65	-0.36	-0.28	0.32	0.06	0.13	-0.07
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2010Q1	CB Nowcast	β_0	1.61	1.92	1.90	1.85	1.62	1.81	1.91	1.96
–2019Q4		β_1	0.23	-0.64	-0.49	-0.40	0.39	0.11	0.08	0.02
		p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Central Bank			Headline Inflation				Core Inflation			
1996Q2	AR Nowcast	β_0	0.31	2.95	2.69	2.01	0.83	1.06	1.05	1.17
–2019Q4		β_1	0.95	-0.46	-0.30	0.05	0.57	0.45	0.47	0.41
		p-value	0.39	0.00	0.00	0.08	0.00	0.00	0.00	0.00
1996Q2	CB Nowcast	β_0	0.31	2.95	2.69	2.01	0.83	1.06	1.05	1.17
–2019Q4		β_1	0.95	-0.46	-0.30	0.05	0.57	0.45	0.47	0.41
		p-value	0.39	0.00	0.00	0.08	0.00	0.00	0.00	0.00
1996Q2	AR Nowcast	β_0	0.30	3.67	3.25	2.13	0.92	0.55	0.51	0.61
–2009Q4		β_1	0.95	-0.64	-0.45	0.12	0.54	0.70	0.71	0.66
		p-value	0.70	0.00	0.00	0.27	0.01	0.05	0.06	0.03
1996Q2	CB Nowcast	β_0	0.30	3.67	3.25	2.13	0.92	0.55	0.51	0.61
–2009Q4		β_1	0.95	-0.64	-0.45	0.12	0.54	0.70	0.71	0.66
		p-value	0.70	0.00	0.00	0.27	0.01	0.05	0.06	0.03
2010Q1	AR Nowcast	β_0	0.32	2.31	2.48	2.55	0.77	1.86	1.80	2.05
–2019Q4		β_1	0.94	-0.37	-0.40	-0.45	0.59	0.00	0.07	-0.04
		p-value	0.44	0.05	0.06	0.05	0.08	0.01	0.00	0.00
2010Q1	CB Nowcast	β_0	0.32	2.31	2.48	2.55	0.77	1.86	1.80	2.05
–2019Q4		β_1	0.94	-0.37	-0.40	-0.45	0.59	0.00	0.07	-0.04
		p-value	0.44	0.05	0.06	0.05	0.08	0.01	0.00	0.00

Notes: Reported p-values refer to the joint Wald test of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$. Standard errors are Newey–West heteroskedasticity- and autocorrelation-consistent. For fixed-horizon forecasts, we use $h - 1$ lags.

E.7 State-Dependent Forecast Efficiency

Table A.22 reports the state-dependent weak-efficiency tests for the FKN model-based forecasts using revised data. Table A.23 reports the corresponding real-time results for the FKN model-based forecasts and central bank projections on common evaluation samples. The central bank results are included for comparison and are identical across solution methods by construction.

Table A.22: Simulated p-Values for the State-Dependent Test of Weak Forecast Efficiency: Model-Based Forecasts, FKN Solution

	Headline Inflation				Core Inflation			
Forecast Horizon:	1	2	4	8	1	2	4	8
				Japan				
1987Q2–2023Q1								
Transition model	0.418	0.060	0.128	0.169	0.428	0.026	0.121	0.109
Target model	0.126	0.000	0.013	0.114	0.000	0.000	0.000	0.021
Low-Inflation model	0.011	0.057	0.006	0.012	0.352	0.071	0.126	0.036
				Euro Area				
2001Q2–2023Q3								
Transition model	0.222	0.312	0.423	0.899	0.055	0.026	0.071	0.151
Target model	0.134	0.204	0.439	0.518	0.000	0.000	0.001	0.172
Low-Inflation model	0.079	0.125	0.333	0.377	0.002	0.027	0.052	0.203
				United States				
1990Q2–2023Q3								
Transition model	0.033	0.012	0.018	0.019	0.000	0.000	0.000	0.000
Target model	0.053	0.030	0.029	0.037	0.000	0.000	0.000	0.000
Low-Inflation model	0.002	0.021	0.065	0.098	0.000	0.000	0.009	0.114

Notes: Entries are simulated p-values for the exp-W statistic testing H_0: $\delta_0 = \delta_1 = 0$, under which the conditional mean of the weak forecast-efficiency moment is zero across inflation states. The results are based on $J = 1{,}000$ simulations. The state variable S_t is headline or core inflation observed at the forecast origin, as indicated. The tests use revised data.

Table A.23: Simulated p-Values for the State-Dependent Test of Weak Forecast Efficiency (Real-Time Data): FKN Solution

Japan (2003Q3–2023Q1)								
Forecast Horizon:	Current Fiscal Year				Next Fiscal Year			
	Headline Inflation Excluding Fresh Food							
Transition	0.001				0.175			
Target	0.000				0.000			
Low-Inflation	0.009				0.362			
Bank of Japan	0.151 (0.375 excl. tax)				0.039 (0.162 excl. tax)			
Euro Area (headline: 2001Q2–2023Q3; core: 2009Q2–2023Q3)								
Forecast Horizon:	1	2	4	8	Current Year	Next Year	Two Years Ahead	
	Headline Inflation				Core Inflation			
Transition	0.038	0.199	0.368	0.822	0.099	0.189	0.108	
Target	0.022	0.140	0.433	0.526	0.000	0.000	0.066	
Low-Inflation	0.110	0.225	0.227	0.068	0.580	0.641	0.328	
ECB	0.286	0.755	0.770	0.312	0.414	0.133	0.389	
United States (1996Q2–2019Q4)								
Forecast Horizon:	1	2	3	4	1	2	3	4
	Headline Inflation				Core Inflation			
Transition	0.438	0.167	0.140	0.103	0.000	0.153	0.060	0.110
Target	0.413	0.289	0.580	0.255	0.000	0.452	0.180	0.307
Low-Inflation	0.462	0.377	0.100	0.031	0.000	0.376	0.296	0.140
Greenbook/Tealbook	0.456	0.605	0.947	0.508	0.075	0.130	0.129	0.033

Notes: Entries are simulated p-values for the exp-W statistic testing H_0: $\delta_0 = \delta_1 = 0$, under which the conditional mean of the weak forecast-efficiency moment is zero across inflation states. The results are based on $J = 1{,}000$ simulations. The state variable S_t is headline or core inflation observed at the forecast origin, as indicated. The tests use the corresponding real-time forecast samples. The model-based forecasts use autoregressive inflation nowcasts.

E.8 Overall Assessment

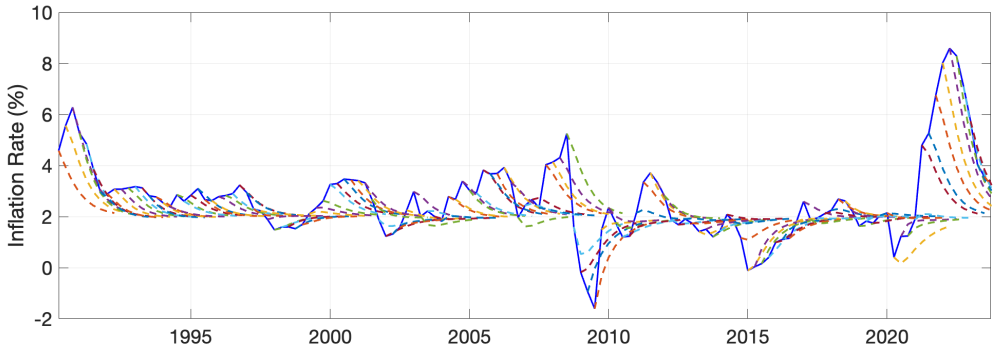
Taken together, the FKN robustness exercise confirms the broad cross-country forecasting patterns of the baseline analysis. The richer FKN dynamics substantially improve the relative forecast accuracy of the low-inflation model. Japan's prolonged low-inflation period remains the clearest case in which the FKN low-inflation model performs particularly well. This improvement does not, however, translate into systematic gains for the transition model, whose accuracy generally remains close to the MSV baseline. The Mincer–Zarnowitz calibration patterns also remain largely unchanged, while the weak-efficiency tests yield additional non-rejections for the low-inflation forecasts in some settings.

The main drawback of the richer FKN dynamics concerns the structural interpretation of the transition framework. Under FKN, the low-inflation model can generate disinflation from elevated inflation levels, improving its relative forecast performance even when inflation remains well above target. Since the equilibrium weights are determined by recent relative forecast performance, this can result in substantially more weight being assigned to the low-inflation forecast without a correspondingly clear movement of the economy toward the low-inflation equilibrium. The estimated weights are therefore less directly interpretable as equilibrium probabilities under FKN than under the MSV baseline. If forecast accuracy were the sole objective, still more flexible reduced-form forecasting approaches could in principle be considered. Our objective, however, is also to retain an economically interpretable link between forecast performance and movements between the target and low-inflation equilibria. We therefore retain MSV as the baseline solution while using FKN to show that the main forecasting patterns are robust to a more flexible treatment of the low-inflation dynamics.

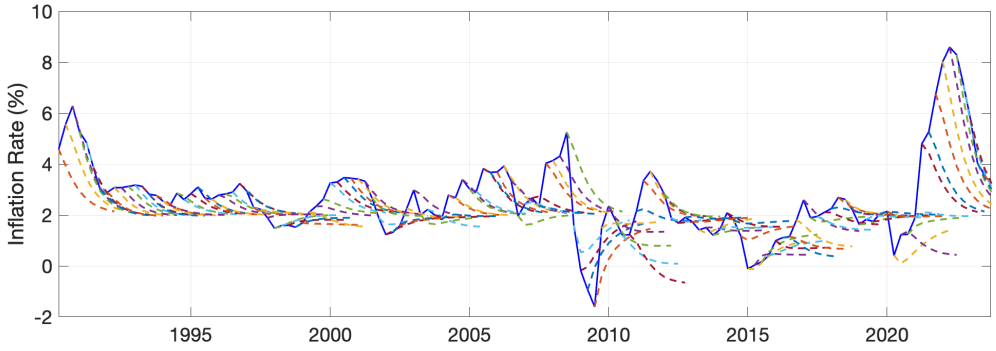
F Forecast Plots: MSV and FKN Solutions

This appendix section documents the forecast paths underlying the baseline MSV results and the FKN robustness exercise. Because the target-equilibrium solution is unchanged under FKN, the target forecast is shown only for the MSV solution.

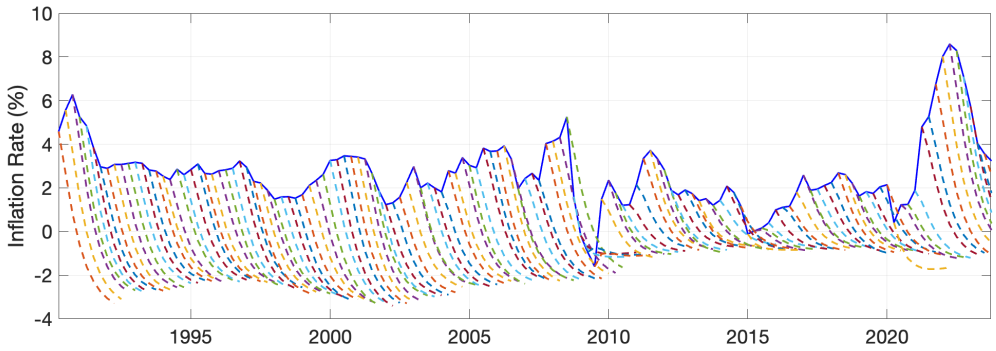
Figure A.2: Headline Inflation Forecasts for the United States, MSV Solution



(a) Target



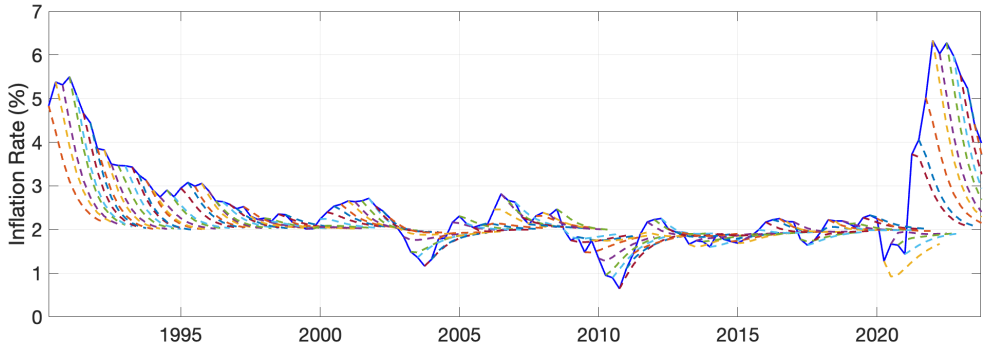
(b) Transition



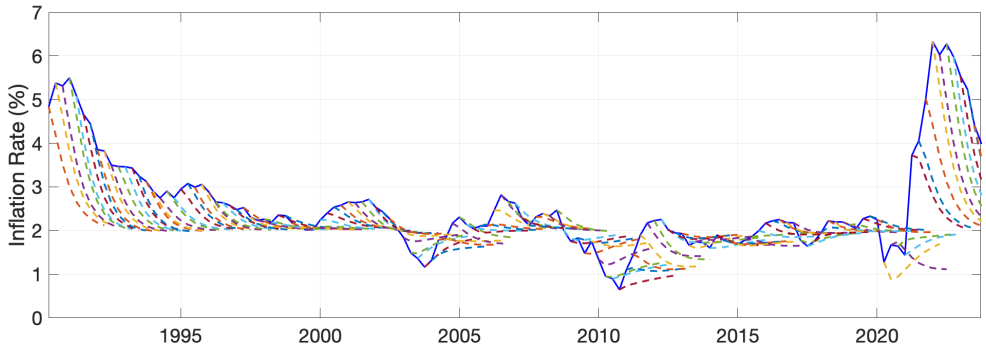
(c) Low-Inflation

Notes: The solid line shows realized year-on-year inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

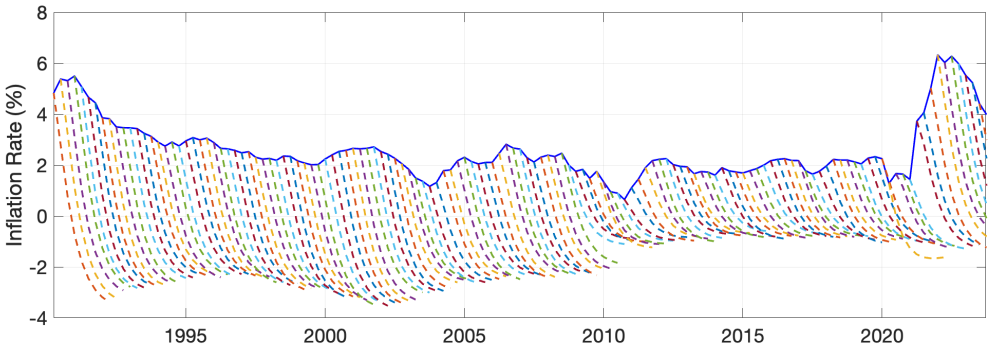
Figure A.3: Core Inflation Forecasts for the United States, MSV Solution



(a) Target



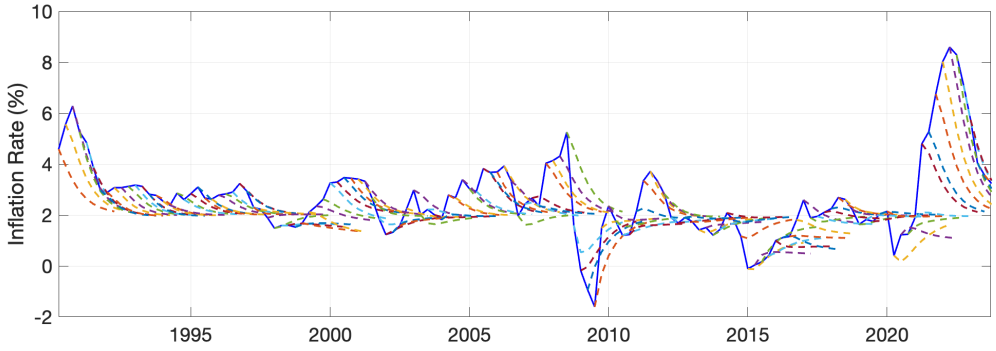
(b) Transition



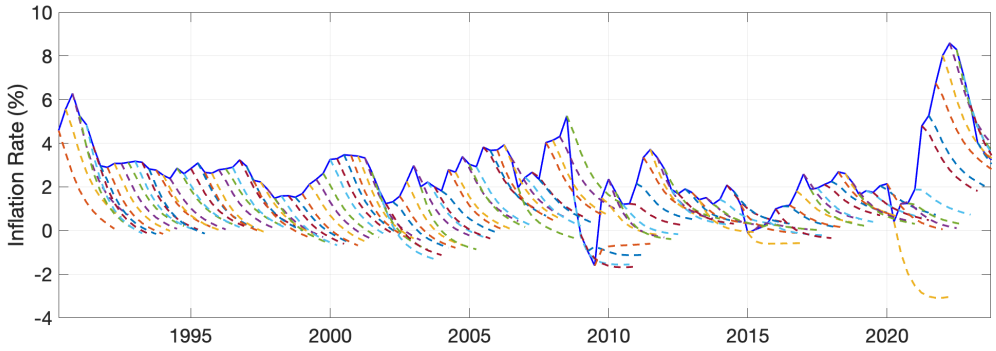
(c) Low-Inflation

Notes: The solid line shows realized year-on-year inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

Figure A.4: Headline Inflation Forecasts for the United States, FKN Solution



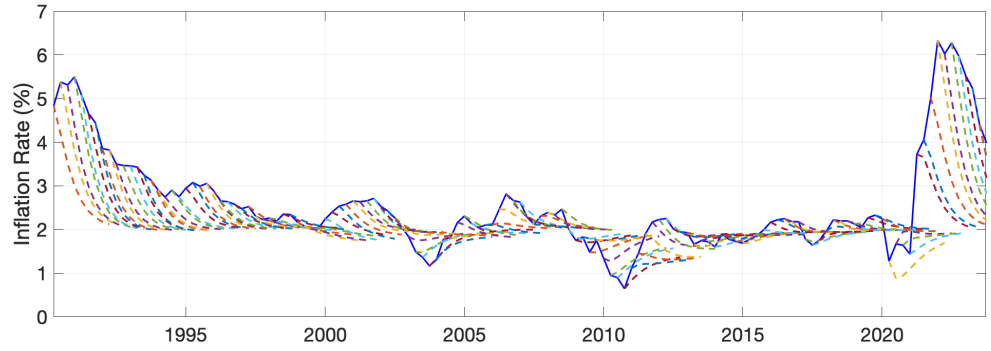
(a) Transition



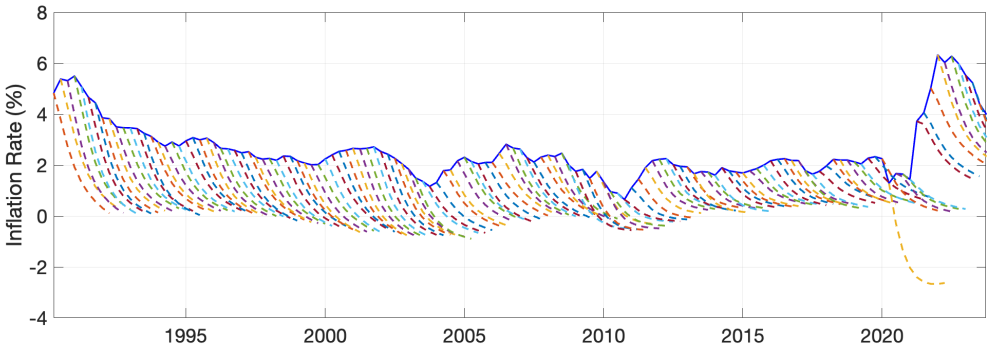
(b) Low-Inflation

Notes: The solid line shows realized year-on-year inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

Figure A.5: Core Inflation Forecasts for the United States, FKN Solution



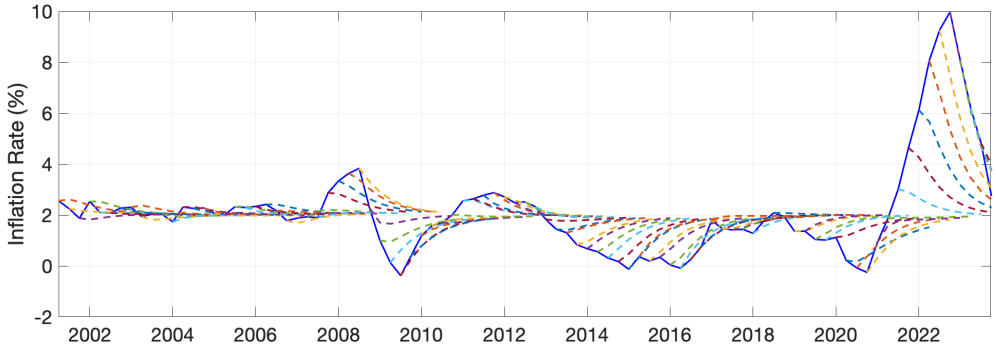
(a) Transition



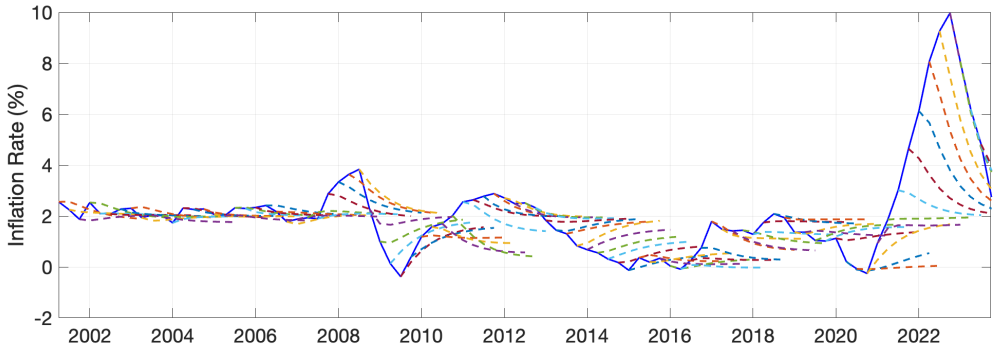
(b) Low-Inflation

Notes: The solid line shows realized year-on-year inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

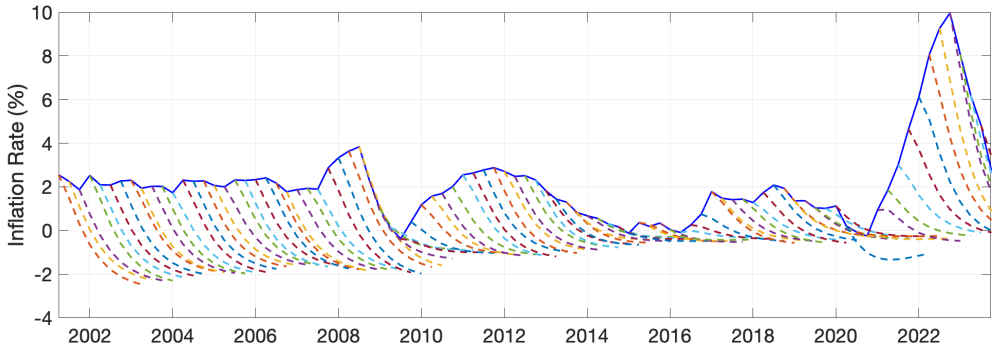
Figure A.6: Headline Inflation Forecasts for the Euro Area, MSV Solution



(a) Target



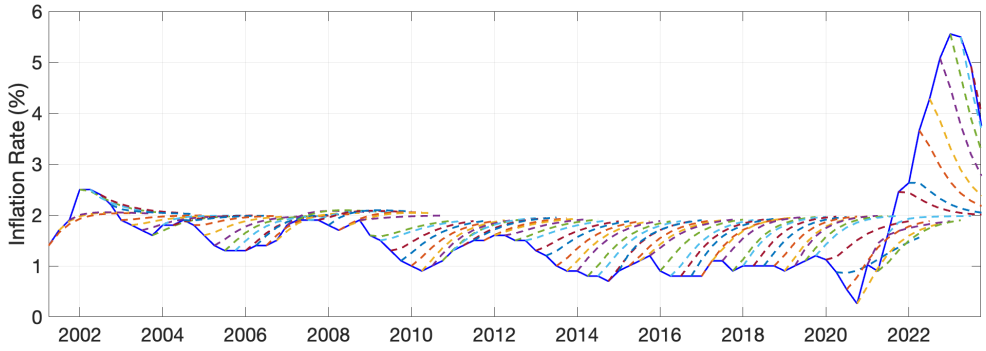
(b) Transition



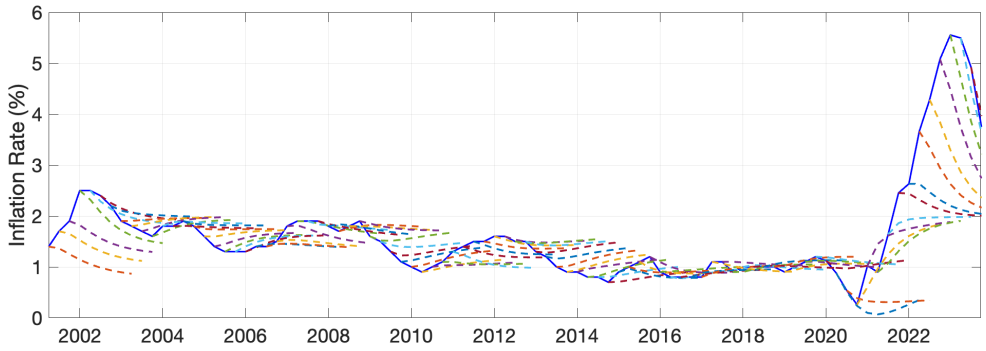
(c) Low-Inflation

Notes: The solid line shows realized year-on-year inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

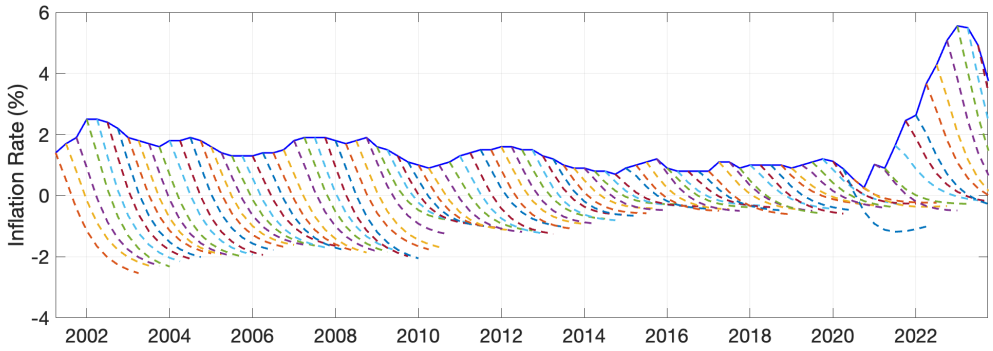
Figure A.7: Core Inflation Forecasts for the Euro Area, MSV Solution



(a) Target



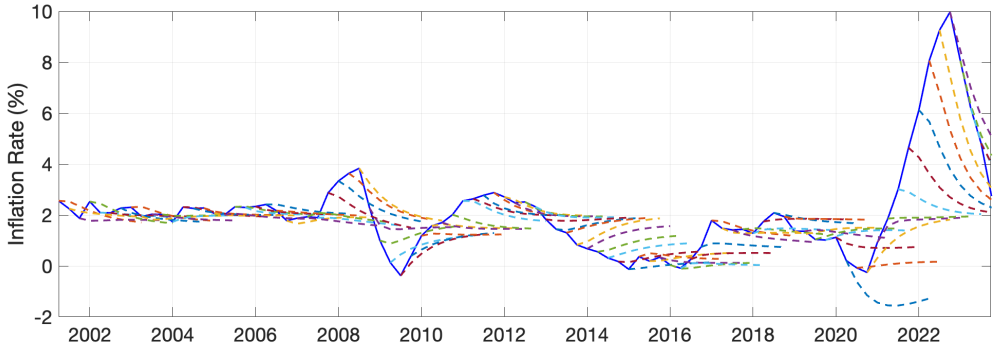
(b) Transition



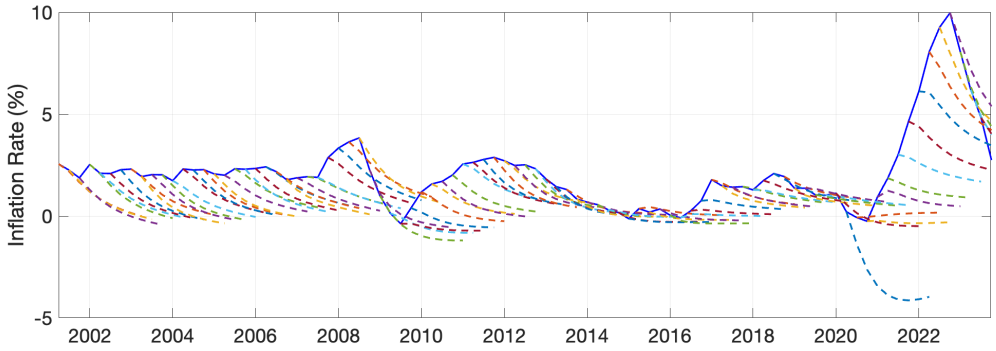
(c) Low-Inflation

Notes: The solid line shows realized year-on-year inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

Figure A.8: Headline Inflation Forecasts for the Euro Area, FKN Solution



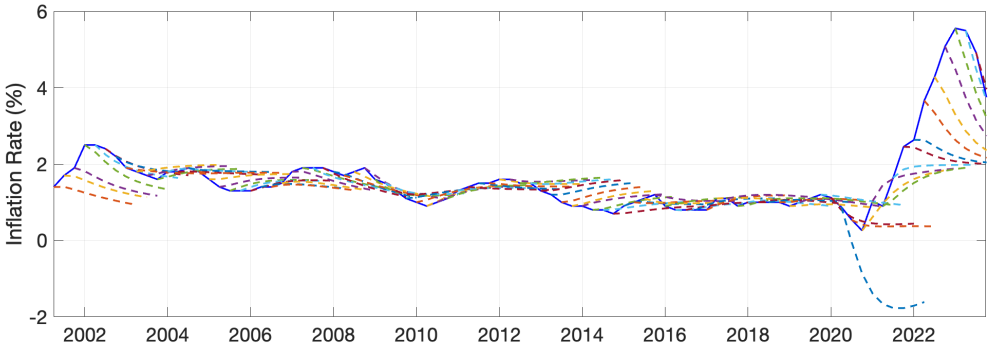
(a) Transition



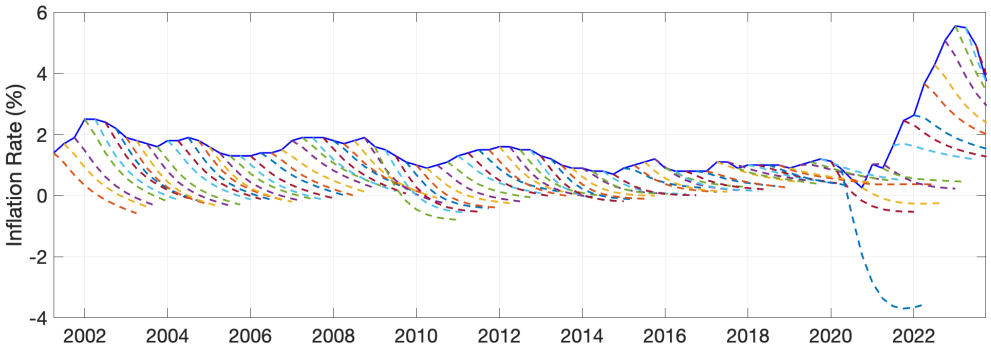
(b) Low-Inflation

Notes: The solid line shows realized year-on-year inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

Figure A.9: Core Inflation Forecasts for the Euro Area, FKN Solution



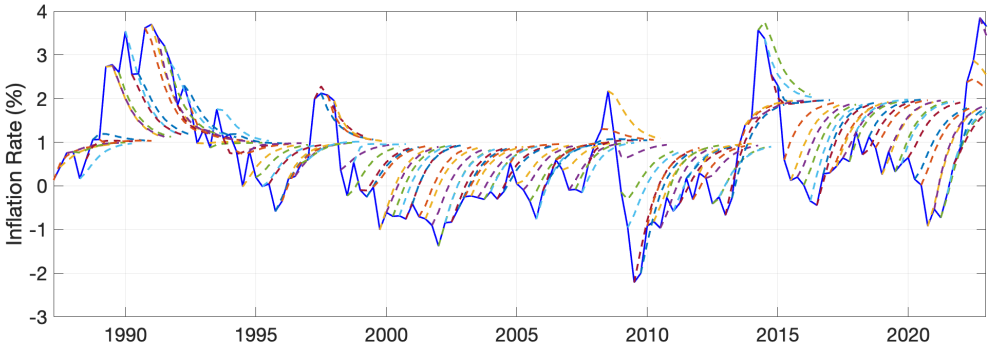
(a) Transition



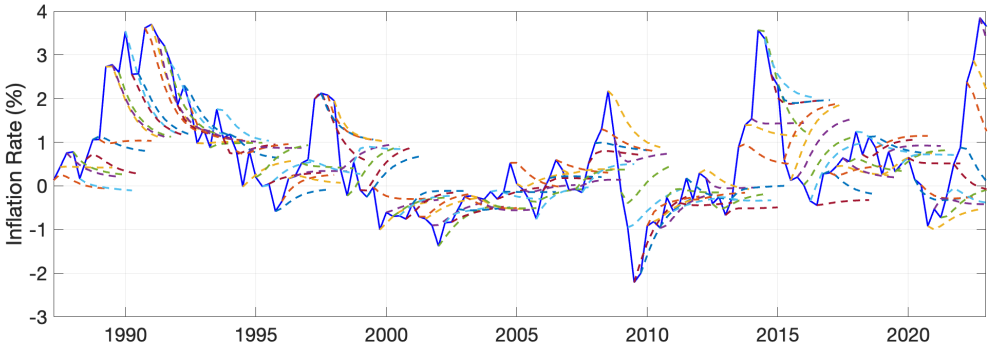
(b) Low-Inflation

Notes: The solid line shows realized year-on-year inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

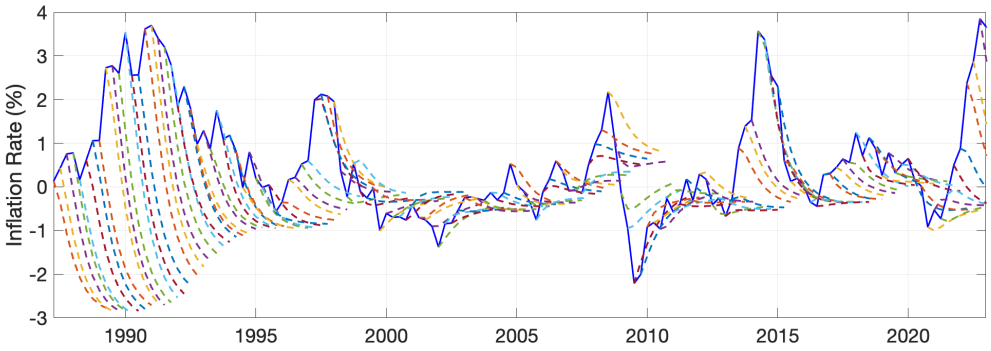
Figure A.10: Headline Inflation Forecasts for Japan, MSV Solution



(a) Target



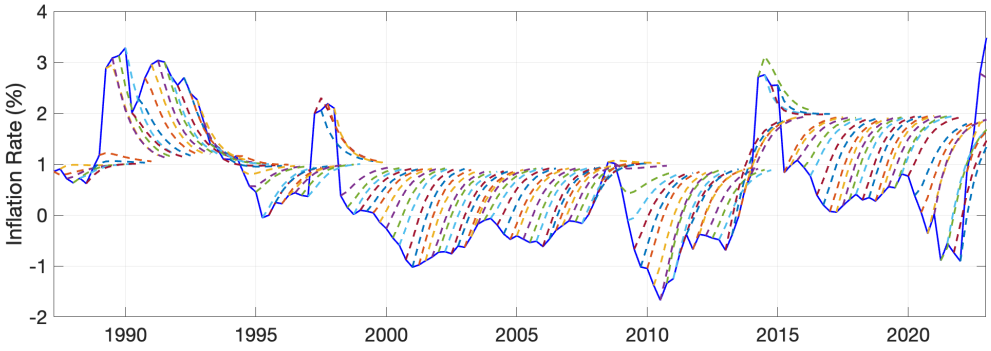
(b) Transition



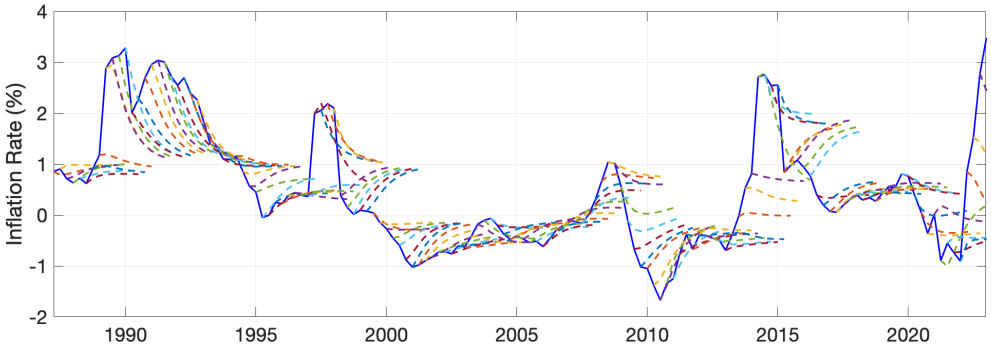
(c) Low-Inflation

Notes: The solid line shows realized year-on-year inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

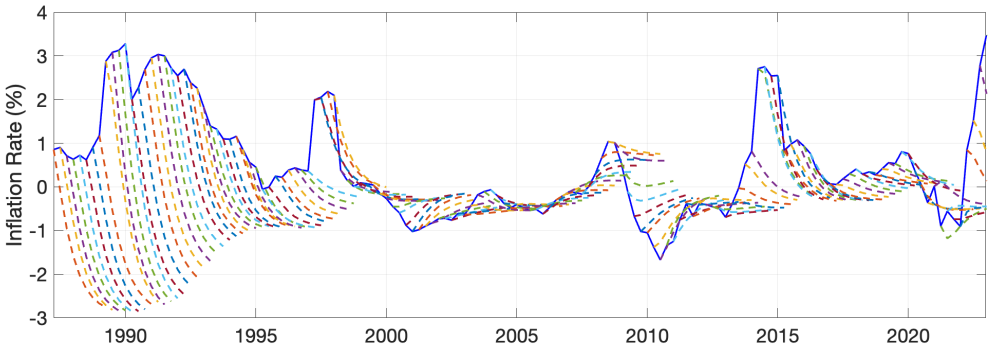
Figure A.11: Core Inflation Forecasts for Japan, MSV Solution



(a) Target



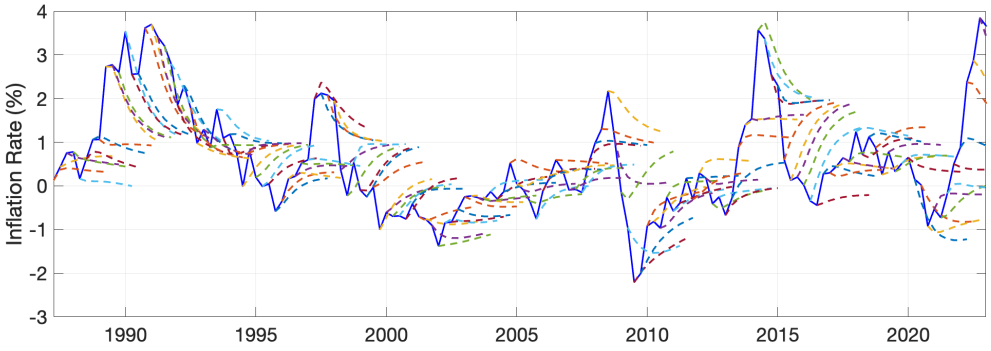
(b) Transition



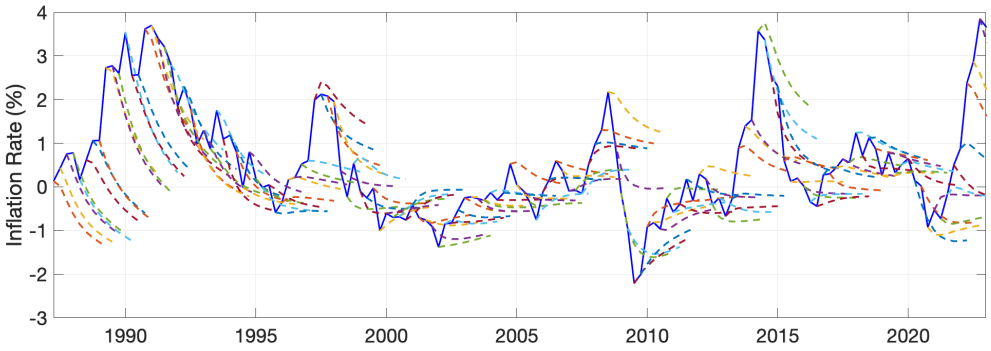
(c) Low-Inflation

Notes: The solid line shows realized year-on-year inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

Figure A.12: Headline Inflation Forecasts for Japan, FKN Solution



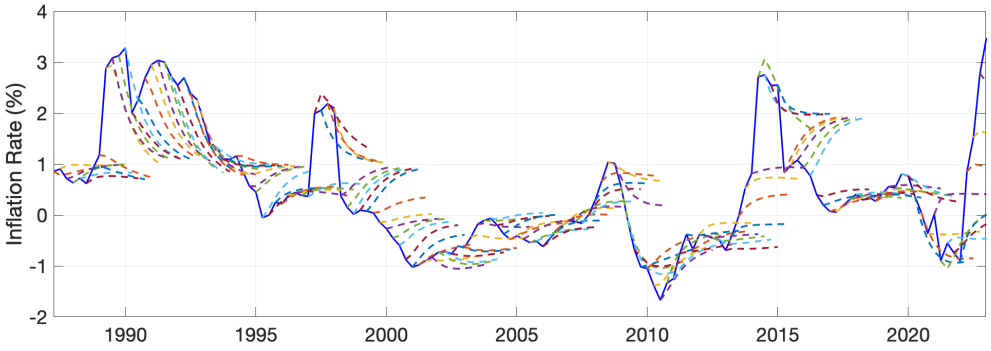
(a) Transition



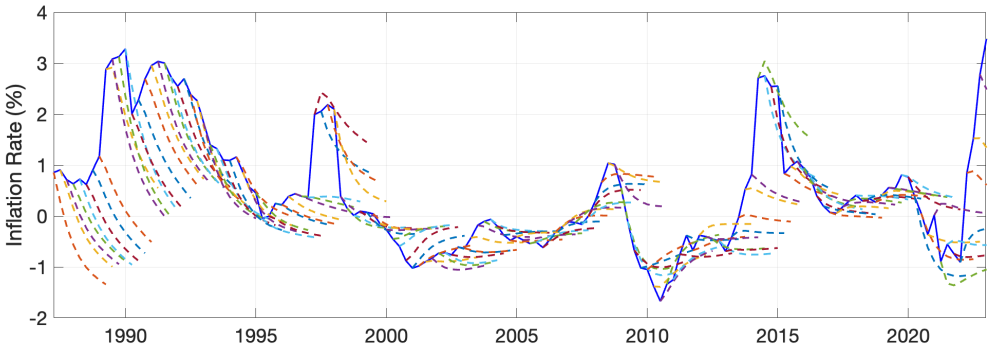
(b) Low-Inflation

Notes: The solid line shows realized year-on-year inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

Figure A.13: Core Inflation Forecasts for Japan, FKN Solution



(a) Transition



(b) Low-Inflation

Notes: The solid line shows realized year-on-year inflation. The dashed lines show forecast paths from successive forecast origins at horizons of up to eight quarters. For readability, the forecast paths show $\bar{\pi}_t$, the model's approximation to annual inflation, rather than the quarterly inflation rate π_t used in the empirical analysis.

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