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ABSTRACT

Employment Protection, Firm Selection, and Growth

This paper analyzes the effect of firing costs on aggregate productivity growth. For this purpose, a model of endogenous growth through selection and imitation is developed. It is consistent with recent evidence on firm dynamics and on the importance of reallocation for productivity growth. In the model, growth is driven by selection among heterogeneous incumbent firms, and is sustained as entrants imitate the best incumbents. In this framework, firing costs not only induce misallocation of labor, but also affect growth by affecting firms’ exit decisions. Importantly, charging firing costs only to continuing firms raises growth by promoting selection. Also charging them to exiting firms is akin to an exit tax, hampers selection, and reduces growth – by 0.1 percentage points in a calibrated version of the model. With job turnover very similar in the two settings, this implies that the treatment of exiting firms matters for welfare and growth. In addition, the impact on growth rates is larger in sectors where firms face larger idiosyncratic shocks, as in services. This fits evidence that recent EU-U.S. growth rate differences are largest in these sectors and implies that firing costs can play a role here. A brief empirical analysis of the impact of firing costs on the size of exiting firms supports the model’s conclusions.

JEL Classification: E24, J63, J65, L11, L16, O40

Keywords: endogenous growth theory, firm dynamics, labor market regulation, firing costs, entry and exit, firm selection

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1 Introduction

This paper analyzes the effect of labor market regulation on productivity growth, a topic that is much less researched than their impact on the level of productivity or on employment. For this purpose, a heterogeneous-firm model with endogenous growth is developed. Besides being consistent with recent evidence on firm dynamics and on the importance of reallocation for productivity growth, the model can also account for the fact that recent productivity growth differences between the U.S. and the EU were particularly strong in the IT-using service sector. Employment protection legislation (EPL) here does not only affect the efficiency of the allocation of labor across plants or the incentive to work or to search as in most of the existing literature, but also affects the endogenous growth of aggregate productivity through its impact on the market selection process through the entry and exit margins.

Recent empirical research on firm dynamics has highlighted the importance of entry and exit and the heterogeneity of firms and plants. For example, Dwyer (1998) finds that productivity differs by a factor 3 between establishments in the 9th and the 2nd decile of the productivity distribution in the U.S. textile sector. Foster, Haltiwanger and Krizan (2001) (FHK) find that in the U.S. Census of Manufactures, more than a quarter of the increase in aggregate productivity between 1977 and 1987 was due to entry and exit. This is even more pronounced in the retail sector, as they find in their (2006) paper. The contribution of exit to aggregate productivity is positive in almost all of the 24 industrial and developing countries analyzed by Bartelsman, Haltiwanger and Scarpetta (2004) (BHS). Gabler and Licandro (2005) find in a calibration exercise that around half of U.S. post-war productivity growth can be traced to the process of market selection, entry, and exit.\(^1\)

The importance of entry and exit varies across industries. Generally, they contribute more to growth in sectors with high turbulence and with high TFP growth (BHS). These were precisely the sectors where Europe lagged U.S. productivity growth in recent years (van Ark, Inklaar and McGuckin 2002, Blanchard 2004). To be precise, labor productivity in IT-using services,\(^2\) a sector making up 26% of U.S. GDP in 2000, grew by 5.4% yearly in the U.S. and by only 1.4% in the EU in the period 1995-2000, and thereby made the largest contribution to the aggregate productivity growth.


\(^2\)These are defined as those service industries with an above-median ratio of IT capital services to capital services in U.S. data for the year 2000.
productivity growth difference of 2.5% vs 1.4% in that period. Differences in other sectors of comparable importance were much smaller.

Theory suggests that EPL imposes tighter constraints on firms in these more turbulent sectors (this idea is present in the literature at least since Bentolila and Bertola (1990)). Indeed, Pierre and Scarpetta (2004) provide empirical evidence that innovative firms feel particularly constrained by EPL. Going beyond this, Scarpetta, Hemmings, Tressel and Woo (2002) provide tentative evidence that EPL is negatively related to productivity growth both directly, and indirectly through reduced firm entry. Similarly, Gust and Marquez (2004) establish an empirical link between EPL and lower growth that passes through lower use of information technology.³

These pieces of evidence suggest the following account: productivity growth is higher in high-turbulence industries. In these industries, EPL constrains firms more strongly. With stricter EPL in continental Europe compared to the U.S., this fits the pattern of recent productivity growth differences showing up particularly in the IT-using service sector.

This paper takes this evidence as a point of departure. The mechanism of growth through selection and experimentation developed here fits many facts on firm dynamics and introduces a relationship between turbulence and growth. Most importantly, it allows quantifying the effect of firing costs along several margins, including their growth effect via entry, exit, and selection. The basic model is similar to the one developed in Gabler and Licandro (2005). In its treatment of firing costs, the analysis is related to the seminal paper by Hopenhayn and Rogerson (1993), and the more recent ones by Alvarez and Veracierto (2001), Veracierto (2001), and Samaniego (2006a). These four all analyze the effect of firing costs on the level of aggregate productivity. They employ a setting of exogenous growth and concentrate on the static efficiency of the allocation of labor. Bertola (1994), conversely, analyzes the effect of hiring and firing costs on growth, using a model of endogenous growth through variety expansion. In such a setting, firing costs affect entry but not exit, so that the selection effect that is crucial here cannot arise.⁴

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³Additional effects can arise through specialization, as argued by Saint-Paul (2002). Scarpetta et al. (2002) find that industries with wider productivity dispersion have higher average productivity. If the high level of productivity dispersion is due to large variance of idiosyncratic shocks, employment protection legislation is more binding in these industries. Indeed, Cunat and Melitz (2005) provide some evidence that high-EPL countries tend to specialize in low-dispersion industries, avoiding the industries where EPL has more bite. In a similar vein, Samaniego (2006b) analyzes how EPL can constrain technology adoption and shape specialization patterns in the presence of exogenous embodied technical progress. EPL might also induce firms to experiment less upon entry, and to choose more predictable business models. This fits with evidence from Haltiwanger, Jarmin and Schank (2003) that there is less productivity dispersion among entering firms in Germany compared to the U.S.. As a result, high-EPL countries can have lower growth because they specialize in lower-growth industries.

⁴A paper that analyzes efficient scrapping, albeit in the context of business cycles, is Caballero and Hammour (1996).
In the model developed here, firms receive idiosyncratic productivity shocks and therefore differ in their productivity and employment. Growth arises and is sustained endogenously through the interaction of selection (among incumbents) and imitation (by entrants). Each period, the least productive incumbents are eliminated, implying that the average productivity of remaining firms grows. Entry sustains growth: Entrants try to imitate firms close to the technological frontier. They do not succeed fully, but on average enter a constant fraction below it. Hence, there is a spillover from incumbents to entrants through the location of the frontier. How much the economy benefits from it depends on how much entry and exit, and thus selection, there is, so growth is driven by both selection and imitation. In addition, growth depends on the variance of productivity shocks. A higher variance, as observed in the service sector, makes high productivity draws more likely. While it also makes very low draws more likely, these are cut off by subsequent exit. As a result, selection is stricter, and growth is faster; however, there is a tradeoff between the dynamic gain of faster growth and the static loss due to the cost of higher, possibly excessive, entry and exit.

In this context, labor market regulation affects the entry and exit incentives of firms, and thereby the engines of growth in this model. It is well-known that firing costs, as one-sided adjustment costs, induce an inaction region in firms’ employment policy when productivity is stochastic. As a consequence, labor is not optimally allocated across firms, implying lower aggregate productivity. Firm value is also lower, which is the mechanism leading to less entry and lower growth in Bertola (1994).

In the present paper, there is an additional effect through exit and selection. To analyze it, it is crucial to distinguish if exiting firms have to pay firing costs, or are exempt (or can default on them). This distinction is also made by Samaniego (2006a) in an environment of exogenous growth. The crucial observation is that firing costs have two distinct effects: they are not only an adjustment cost but also a tax on exit. The latter discourages exit of low-productivity firms, thereby weakens the selection process, and reduces productivity growth through selection. When exiting firms are exempt, firing costs lower a firm’s continuation value relative to the value of exit, thereby promoting exit of low-productivity firms, strengthening selection, and increasing growth relative to the frictionless economy. The net welfare effect depends on the relative size of the static and the dynamic effects, i.e. if faster growth outweighs consumption losses due to misallocation of labor. Both effects are stronger when the variance of idiosyncratic shocks is larger – so EPL has a stronger effect on growth in the service sector.

To quantitatively evaluate the impact of labor market regulation on observed differences
in productivity growth and in the behavior of entrants, the model is calibrated to the U.S. business sector. Then the effects of introducing firing costs of one year’s wages, close to the level observed in many continental European countries, is evaluated. Results show that charging firing costs only to continuing firms promotes selection and thereby growth. However, this is outweighed by the effect of the misallocation of labor, so welfare falls by an equivalent of 6% of permanent consumption. When firing costs are also charged to exiting firms they act as an exit tax and slow down growth by 0.1 percentage point, so there are two sources of welfare loss. These losses are higher in the service sector, with growth declining by 0.3 points when charging firing costs to exiting firms. Job turnover always drops significantly, but only marginally more when charging exiting firms. This suggests that even when there are technological costs of job turnover (which are not modeled here), e.g. when there are search frictions, achieving this small additional reduction in job turnover is probably not worth the cost it imposes in terms of lower growth.

Finally, we also provide some indicative empirical evidence, confirming that firing costs do indeed affect the selection process in the way modeled here. To do this, we investigate the impact of different types of firing cost on the average size of exiting firms in industry-level panel data for 10 OECD countries.

To summarize, the paper makes two main contributions. Firstly, it provides a growth model that is consistent with facts on firm dynamics, and that can account for the fact that recent productivity growth differences between the U.S. and the EU were particularly strong in the IT-using service sector. Secondly, it provides a theoretical analysis of the effect of firing costs on productivity growth that highlights the importance of the treatment of exiting firms. In particular, charging firing costs to exiting firms reduces growth by hampering selection, with only a small additional reduction in job turnover. This shows that inhibiting the market selection mechanism comes at a cost.

The paper is organized as follows. In the next section, a simple heterogeneous firm model with growth by selection and experimentation is set up. In Section 3, it is solved for optimal behavior of all agents, equilibrium is defined, an algorithm for calculating it is given, and the determination of the growth rate is discussed. In the following section, the model is calibrated, and in Section 5, the quantitative effects of firing costs are explored. Section 6 provides some supporting empirical evidence, and section 7 concludes.
2 The Model Economy

Time is discrete and the horizon infinite. The economy is populated by a continuum of infinitely-lived consumers of measure one, a continuum of active firms of endogenous measure, a large pool of potential entrants, and a sector of perfectly competitive mutual funds.

Consumers value consumption and dislike working; this is summarized in the period utility function $u(c_t, n_t) = \ln c_t - \theta n_t$. They discount the future using a discount factor $\beta < 1$. They can consume or invest in shares $a_t$ of the mutual funds that pay a net return $r_t$; wages and the return to the portfolio provide them with income.

The mutual funds finance investment in firms and transfer profits as dividends to shareholders. Since the sector is competitive, they do not make any profits and return the entire net profits of the production sector to consumers as dividends. Given perfect competition and assuming symmetry, they all hold the market portfolio and pay the same return $r_t$ on assets. Hence, they can be summarized into one representative mutual fund. Effectively, their existence eliminates liquidity constraints on firms, which are not in focus here.

Firms: Firms produce a homogeneous good using labor as their only, variable input, with a positive and diminishing marginal product. This good serves as the numéraire of the economy. To remain active, firms also incur a fixed operating cost $c^f_t$ each period; this grows over time at the growth rate of output, $g$.

Firms differ in productivity. This arises because each firm receives idiosyncratic productivity shocks; more precisely, its log productivity follows a random walk. This is a very simple way of capturing the role of idiosyncratic shocks established by the empirical literature.\textsuperscript{5} It also renders the persistence of firm level productivity found in the data. Recent evidence in favor of a random walk as the main driver of firms’ productivity is provided by Franco and Philippon (2007). The production technology can then be summarized in Assumption 1 and in the production function

$$y_{it} = \exp(s_{it}) \, n_{it}^{\alpha}, \quad 0 < \alpha < 1,$$

where $y_{it}$ denotes output of firm $i$ in period $t$, $\exp(s_{it})$ is its productivity level, and $n_{it}$ employment.\textsuperscript{5}

\textsuperscript{5}Empirical work on firm dynamics agrees on the importance of idiosyncratic shocks to firm-level productivity. Without going to a detailed dynamic analysis of firm-level data, this can be inferred from the high correlation of contemporaneous entry and exit rates for most industries (this does not fit well with aggregate or industry-level shocks as main driver of firms’ fate), from the fact that productivity differences among firms are larger within than between industries (FHK), and from the fact that there are frequent changes in the identity of industry leaders.
**Assumption 1** Log productivity evolves according to

\[ s_{it} = s_{i,t-1} + \epsilon_{it}, \]  

where the innovation \( \epsilon \) is distributed normally with mean zero and variance \( \sigma^2 \).

What is crucial about Assumption 1 is that a firm’s productivity is not stationary. While the random walk could be tweaked to also fit deviations from Gibrat’s law (the independence of firm growth rates and size) that have been found in the data, this would not substantially alter the growth mechanism.

**Firing costs:** Adjusting employment is costless in the benchmark case. This will be compared to the case with employment protection legislation (EPL) in the form of firing costs of \( c^n \) times a period’s wages for each worker fired. This policy can take two forms, one where firing costs always have to be paid when firing a worker, including upon exit (denoted by \( F_x = 1 \)), and another one where firing costs only have to be paid if the firm also remains active in the subsequent period; i.e. exiting firms are exempted from firing costs (denoted by \( F_x = 0 \)). An active firm’s profit function can then be written as

\[ \pi_{it} = \pi(s_{it}, n_{it}, n_{i,t-1}, w_t) = \exp(s_{it}) n_{it}^\alpha - w_t n_{it} - c_f - h(n_{it}, n_{i,t-1}), \]  

where \( w_t \) denotes the period-\( t \) wage and the function \( h(n_{it}, n_{i,t-1}) \) summarizes firing costs. It is given by

\[ h(n_{it}, n_{i,t-1}) = c^n w_t \cdot \begin{cases} \max(0, n_{i,t-1} - n_{it}) & \text{if } F_x = 1, \\ \max(0, n_{i,t-1} - n_{it}) & \text{if } F_x = 0 \land n_{it} > 0, \\ 0 & \text{if } F_x = 0 \land n_{it} = 0. \end{cases} \]

The dependence of \( h(\cdot) \) on previous period’s employment makes the employment choice a dynamic decision when there are firing costs, and implies that a firm’s individual state variables are \((s_{it}, n_{i,t-1})\).

At the beginning of any period, firms can decide whether to exit at the end of that period. This is costless in the benchmark case and when exiting firms are exempt from firing costs \((F_x = 0)\); otherwise \((F_x = 1)\), the exiting firm has to cover the firing cost for reducing its workforce from \( n_{i,t-1} \) to 0.
Entry: Entering firms have to pay a sunk entry cost \( c_e^t \) that grows at the same rate as output. This can be interpreted as an irreversible investment into setting up production facilities.\(^6\) Entrants try to imitate the best firms in the economy; for the sake of concreteness, assume that they identify the best 1% of firms with the frontier of the economy. Denote average productivity of the target group with \( s^\text{max} \). In practice, entrants are on average less productive than incumbents; for instance, FHK report that active firms that entered within the last 10 years are on average 99% as productive as incumbents. One possible explanation is that they cannot copy incumbents perfectly due to tacitness of knowledge embodied in these firms. Assumption 2 formalizes the imitation process. (See Figure 1 for an illustration.)

Assumption 2 Entrants draw their initial log productivity \( s_0 \) from a normal distribution with mean \( s^\text{max} - \kappa \) (\( \kappa > 0 \)) and variance \( \sigma_e^2 \). Denote its pdf by \( \eta_t(s^0) \).

The assumption implies that, as the distribution of incumbents moves rightward, the distribution of entrants’ log productivity tracks it at a constant distance \( \kappa \). Because \( \kappa > 0 \), entrants are on average less productive than the best incumbents. In the configuration depicted in Figure 1, entrants are on average less productive than the average incumbent while entering into all deciles of the productivity distribution. This is the pattern suggested by the data; more on this in Section 4.

Assumption 2 describes an externality; incumbents’ productivity spills over to entrants. Together with the selection process, this externality drives growth. It can be interpreted in other ways besides imitation. For instance, incumbents’ productivity is an indicator of knowledge in the economy. If entrants can draw on that, either as a spillover or because it is embodied in the production facilities they acquire upon entry, they benefit from incumbents’ productivity.

The intensity of experimentation, parametrized by \( \sigma_e^2 \), influences the growth process. A higher \( \sigma_e^2 \) implies that the probability of drawing an extreme, including very high, productivity rises. On the other hand, the higher probability of bad draws means that the entry process consumes more resources, making the net effect ambiguous. While entrants could be thought to optimally choose the degree to which they experiment, take \( \sigma_e^2 \) as fixed by technology for the purpose of this paper.

Let \( \tilde{\mu}(s, n-1) \equiv M \mu(s, n-1) \) be the measure of firms with states \( (s, n-1) \), where \( M \) is the number of firms in the economy, and \( \mu(s, n-1) \) is a density function. The assumption of a con-

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\(^6\)Empirical evidence shows that in practice, a large part of investment is irreversible in the sense that the resale value of assets is very low. This is more pronounced the more specific and the less tangible the asset, and the thinner the resale market. For evidence, see e.g. Ramey and Shapiro (2001).
tinum of firms that are all independently affected by the same stochastic process, together with
the absence of aggregate uncertainty, implies that the aggregate distribution evolves determin-
istically. As a consequence, although the identity of firms with any \((s, n-1)\) is not determined,
their measure is deterministic. Moreover, the underlying probability distributions can be used
to describe the evolution of the cross-sectional distribution.\footnote{Formally, this follows from the Glivenko-Cantelli Theorem (see e.g. Billingsley 1986). For a more thorough
discussion, see Feldman and Gilles (1985) and Judd (1985).}

**Timing:** The structure of the economy implies the following timing. At the beginning of any
period, firms decide if they stay or exit, and potential entrants decide whether to enter. All firms
that stay or enter pay the fixed operating cost \(c_f\), and entrants in addition pay the entry cost
\(c_e\). Then incumbent firms receive their productivity innovations and entrants draw their initial
productivity. Firms demand labor, workers supply it, and the wage adjusts to clear the labor
market. Production occurs, agents consume, and profits are realized. Firms that reduced labor
or exited pay the firing cost. After this, the whole process resumes. Hence, the dynamic choices
of entry, exit, and employment are all made based on firms’ expectations of future productivity.

3 Equilibrium

This section starts with the derivation of optimal behavior for all agents. Then, equilibrium is
defined, and an algorithm for calculating it is given. A discussion of the selection mechanism
and of the determination of the growth rate follows.

3.1 Optimal Behavior

Consumers maximize utility by choosing asset holdings and labor supply. Firms maximize the
expected discounted sum of profits by choosing employment, entry, and exit. These decisions
shape the law of motion of the firm productivity distribution, and thereby determine the growth
rate.

**Consumers:** The consumer problem is completely standard. Utility maximization yields the
Euler equation

\[
\frac{c_{t+1}}{c_t} = \beta(1 + r_t).
\]

(4)

Defining \(g^c\) as the growth rate of consumption, this implies that the prevailing gross interest
rate in the economy is \(1 + r_t = (1 + g^c)/\beta\). Moreover, consumers supply labor in accordance
with the first order condition $c_t = w_t/ \theta$.

**Employment:** Active firms face a standard dynamic optimization problem. This is particularly simple in the case with no firing costs, since then it is a sequence of static problems, and a firm’s productivity $s$ is the only firm-level state variable. Call labor demand for this case $n_0(s, w)$. With firing costs, last period’s employment $n_{-1}$ also becomes a state variable for the firm. The aggregate state variable is the firm productivity distribution $\mu$. Together with firms’ employment policies, it determines the labor-market clearing wage $w$. To underline this dependence, in the following, both are used as arguments of the firm’s value and policy functions, although for the firm, the aggregate state matters only because it drives the wage. So denote the firm’s employment policy for the more general problem (with firing costs) by $n(s, n_{-1}, w; \mu)$.

The associated Bellman equation is

$$V(s, n_{-1}, w; \mu) = \max_n \left\{ \pi(s, n, n_{-1}, w) + \frac{1}{1 + r} \max \left( E[V(s', n, w'; \mu')|s], V^x \right) \right\},$$

(5)

where the profit function as defined in equation (3) includes the fixed cost and the adjustment cost of labor, the inner max operator indicates the option to exit, $V^x$ denotes the value of exit as detailed in (6) below, and primes denote next-period values. Note that, since aggregates are deterministic, the firm faces uncertainty only about its own future productivity $s'$, not about future wages and firm distributions.

This is a standard problem, existence and uniqueness of the value function follow from standard arguments. In addition, two properties carry over from the profit function: The value function is increasing and convex in $s$ given $n_{-1}$, and weakly decreasing in $n_{-1}$ given $s$ if there are firing costs. Whereas the employment policy $n(s, n_{-1}, w; \mu)$ increases monotonically in $s$ in the frictionless economy, it features a constant part around $n_{-1}$ when $c^a > 0$. This is a standard effect of non-convex adjustment costs. It is illustrated in Figures 2 ($F_x = 1$) and 3 ($F_x = 0$). Intuitively, when a firm’s productivity increases a little, it will not immediately raise employment because productivity might fall again, and reducing employment again then would be costly. Analogously, when a firm’s productivity falls slightly, it will not immediately fire workers because productivity might recover and it would have paid the firing cost prematurely.

When firms are exempted from paying the firing cost upon exit ($F_x = 0$), firms that suffer a negative productivity shock so large that they are forced to exit will not adjust employment downward immediately, but keep it constant and fire all workers upon exit. So given an $n_{-1}$, the employment policy is constant for $s$ very low or around $n_{-1}$, and strictly increasing elsewhere.
This translates into the “canyon” in Figure 3. Denoting the domains of \( s \) and \( n_{-1} \) with \( S \) and \( N \) respectively, the employment policy function and the law of motion for \( s \) then jointly define a transition function \( Q : S \times N \rightarrow S \times N \) that moves firms over productivity and employment states. They also define a transition probability function \( q : (S \times N) \times (S \times N) \rightarrow [0,1] \) that gives the probability of going from state \( (s, n_{-1}) \) to state \( (s', n) \). Clearly, \( Q \) and \( q \) depend on \( w \) and \( \mu \); these arguments are omitted for simplicity.

The value of incumbent firms and their employment also decrease in the wage \( w \). Moreover, better (in the first-order stochastic dominance sense) future firm distributions, by implying higher future wages, reduce firm value and employment.

**Exit:** Firms exit if the expected value of continuing conditional on current states is less than that of exiting. The latter is equal to firing costs due if these have to be paid upon exit, and zero otherwise:

\[
V^x = -F_x c^n w n = \begin{cases} 
0 & \text{if } c^n = 0 \lor F_x = 0, \\
-c^n w n & \text{if } F_x = 1.
\end{cases}
\] (6)

Exit and re-entry also yields zero net value due to free entry – see equation (8) below. The value of exit is thus constant in \( s \). Since the value function is strictly increasing in \( s \) for any \( n_{-1} \), there is a unique threshold \( s_x \) where the expected value of continuing equals the value of exit. Firms exit when they draw an \( s \) below this. The exit threshold then is a function of \( n_{-1}, w', \) and \( \mu' \), defined by

\[
s_x(n_{-1}, w'; \mu') = \{ s | E[V(s', n, w'; \mu')|s] = V^x \}.
\] (7)

Taking into account the exit decision leads to a modification of the transition function to become \( Q_x : \tilde{S} \times N \rightarrow (\tilde{S} \cup \overline{S}) \times N \), where now the support of the productivity state \( s \) is partitioned into \( \tilde{S} = \{ s | s \geq s_x(n_{-1}, w'; \mu') \} \) (continue) and \( \overline{S} = \{ s | s < s_x(n_{-1}, w; \mu') \} \) (exit). The latter is an absorbing state. Note that the partition may differ across different elements of \( N \). The probability of going from \( (s, n_{-1}) \in \tilde{S} \times N \) to \( (s', n) \in (\tilde{S}(N) \cup \overline{S}(N)) \times N \) then is given by a function \( q_x(\cdot) \).

The dependence of the exit threshold on the other variables is crucial for the selection effect. Clearly, \( s_x \) increases in \( w' \). It also increases in the future productivity of other firms, \( \mu' \). As the value function is weakly decreasing in \( n_{-1} \), the exit threshold is weakly increasing in it. Finally, with firing costs upon exit, the value of exit is lower, and so is the exit threshold. In this sense, firing costs on exiting firms act as a tax on exit and discourage exit, particularly of
low-productivity firms. By worsening the distribution of surviving firms, they can slow down growth, as shown below.

**Entry:** Potential entrants enter until the expected net value of doing so is driven to zero. So in equilibrium, the free entry condition

\[ E[V^e(s^0, w_t; \mu_t)] = c_t^e \]

holds. (Alternatively, if \( E[V^e(s^0, w_t; \mu_t)] < c_t^e \), no entry takes place.) Since \( c_t^e \) and the distribution of \( s^0 \) are exogenous features of technology, this equation pins down the wage, given a firm distribution. A wage below (above) its equilibrium value would trigger additional (reduced) entry, driving up (down) the wage.

All firms’ decisions combined and the process for idiosyncratic shocks yield the law of motion for the firm productivity distribution

\[ \mu'(s', n) = \begin{cases} \int_N \int_{\bar{S}(N)} \mu(s, n-1) q(s', n|s, n-1) \, ds \, dn-1 & \text{if } n > 0, \\ \eta(s^0 = s')/M & \text{if } n = 0 \end{cases} \]

with firing costs, and simply \( \mu'(s') = \int_{\bar{S}} \mu(s) q(s'|s) \, ds + \eta(s^0 = s')/M \) otherwise. In both cases, the integral describes the motion of incumbents. Exit is captured by the restriction of the domain of the integral to surviving firms, and entry is given by \( \eta(\cdot) \). All elements for analyzing equilibrium of this economy have been assembled now. The next steps now are to define a competitive equilibrium, describe briefly how to compute its balanced growth path, and analyze the determination of the growth rate.

### 3.2 Equilibrium Definition

Define a *competitive equilibrium* of this economy as sequences of real numbers \( \{w_t\}_{t=0}^{\infty} \) and \( \{M_t\}_{t=0}^{\infty} \), functions \( n(s, n-1, w; \mu) \), \( V(s, n-1, w; \mu) \), and \( s_x(n-1, w; \mu) \), and a sequence of probability density functions \( \{\mu_t(s, n-1)\}_{t=0}^{\infty} \) such that:

(i) Consumers choose consumption, asset holdings, and labor supply optimally, so the interest rate is given by equation (4);

(ii) all active firms choose employment optimally according to the employment policy \( n(\cdot) \), yielding value \( V(\cdot) \) as described by equation (5) for all \( (s, n-1, w; \mu) \);

(iii) exit is optimal: \( s_x(\cdot) \) is given by equation (7) and firms exit if they draw an \( s < s_x(\cdot) \), given \( n-1, w, \mu, \) and \( V^x \);
entry is optimal and free: given a distribution $\eta(s^0)$ over entrants’ productivities $s^0$, an entry cost $c^e$, $w$ and $\mu$, firms enter until the net value of entry equals its cost (equation (8));

the labor market clears: given $M$, $w$ and $\mu$, aggregate labor demand equals supply as chosen by households; and

the firm distribution is defined recursively by equation (9) given $\mu_0$, $M_t$ and $s_{xt}$.

The last condition implies that the sequence of firm distributions is consistent with the law of motion generated by the entry and exit rules. The rest of the paper will deal only with balanced growth equilibria.

3.3 Balanced growth

Define a balanced growth equilibrium or balanced growth path (BGP) as a competitive equilibrium in which output, consumption, wages, and aggregate productivity grow at a constant rate $g$, the firm log productivity distribution shifts up the productivity scale in steps of $g$ (see Figure 4), its shape is invariant, and the firm employment distribution, the interest rate, the number of firms, the firm turnover rate, and other dynamic characteristics of the firm distribution are constant. In this case, the economy can be made stationary by applying the transformation $\hat{z}_t = z_t e^{-gt} = z$ to all growing variables $z$, $\hat{x}_t = x_t = x$ to all constant variables $x$, and $\hat{s}_{it} = s_{it} - gt$ to the firm-level productivity state. (In the following, I will refer to the “stationarized” as opposed to the “growing” economy. To distinguish them, the stationarized variables carry hats. Note that the transformation of $s$ also affects the transition functions $Q$ and $q$.) So in the stationarized economy, firm productivity evolves according to

$$\hat{s}_{it} = \hat{s}_{i,t-1} - g + \epsilon_{it}.$$ 

The random walk gets a downward drift (for positive aggregate growth rates) because the whole firm productivity distribution shifts up at rate $g$, so in expectation, firms fall back by $g$ every period relative to the distribution.

Now it is easy to use the free entry condition and the law of motion of the productivity distribution to pin down the equilibrium wage and growth rate. The free entry condition becomes

$$E[V^e(s^0, \hat{w}; g)] = c^e.$$  \hspace{1cm} (E)

What matters for firms is not the entire firm productivity distribution, but just its growth rate and the (stationarized) wage. Firms are affected by other firms’ productivity only through the
wage. Knowing that the wage grows at a constant rate, they do not need to keep track of the productivity distribution. The growth rate determines how fast firms fall behind and how hard it is for them to keep up with the distribution. The value of entry hence decreases in $g$ and in $\hat{w}$. The line labeled $E$ in Figure 5 depicts the combinations of $\hat{w}$ and $g$ for which the free entry condition holds, given entrants’ productivity distribution $\eta$ and the entry investment $c^e$. It is downward-sloping because entry value decreases in both $\hat{w}$ and $g$, so to make the condition hold, a rise in $g$ has to be compensated by a fall in $\hat{w}$ and vice versa. As the wage goes to zero, the value of entry takes off to infinity, so the growth rate also has to go to infinity for $E$ to hold.

The law of motion of the productivity distribution (for simplicity, for the case without firing costs) becomes

$$\hat{\mu}(\hat{s}') = \int_{\bar{S}} \hat{\mu}(\hat{s}) \hat{q}(\hat{s}'|\hat{s}) d\hat{s} + \eta(\hat{s}^0 = \hat{s}')/\hat{M}. \quad (S)$$

Entry, exit, and growth interact in such a way as to keep the stationarized distribution $\mu$ constant. The combinations of $\hat{w}$ and $g$ that satisfy the law of motion trace out the upward-sloping line in Figure 5. A higher wage makes selection tougher, raises the exit threshold, thereby increases average productivity next period, and thus implies a higher growth rate. For this reason, refer to this relation as the selection line, labeled $S$. The intersection of the two curves gives the equilibrium growth rate and stationarized wage on the BGP. The shape of the curves ensures that this pair exists and is unique. Appendix A describes how to compute the stationarized balanced growth equilibrium.

One conceptual concern here is to ensure that starting with some initial productivity distribution, the variance of the distribution remains finite as time goes by, and is not blown up by the random walk in firms’ productivity. Growth ensures this by making the weight of each cohort’s contribution to aggregate variance decline faster than the within-cohort variance can rise due to the random walk. As a result, each cohort’s contribution to aggregate variance declines as the cohort ages, and the variance of the aggregate productivity distribution is bounded. For a more detailed argument, see Appendix B.

### 3.4 The growth rate

The growth rate $g$ is driven by the selection process and by the distance $\kappa$ between entrants’ and incumbents’ mean productivity. Intuitively, the process is as follows. In the growing economy, the productivity of incumbents follows a random walk. This implies that for a given set of firms, each firm’s productivity is constant in expectation, but the variance of those firms' productivity distribution grows over time. However, with exit, the exit threshold truncates the firms’ pro-
ductivity distribution from below. As a result, the distribution can only expand upwards, and average productivity of this set of firms grows. In this way, selection drives growth. Figure 6 depicts the productivity distribution of survivors from a cohort that entered some periods ago compared to that of all incumbent firms; the former has higher mean productivity. While this process bears some similarity to the one in Jovanovic (1982), there is also a crucial difference: Whereas in Jovanovic (1982), firms gradually discover their underlying, given productivity, in the present case firms’ productivity actually evolves over time. Selection hence is not just a process of passive discovery of the most productive, but is active, driven by changes at the firm level.

As time goes by and firms keep on exiting, the distribution thins out. This is why entry is needed to sustain growth: In a stationary equilibrium, the measure of firms is constant, and exiting firms are replaced by entering ones. Yet while exiting firms are at the bottom of the distribution, entering firms are more productive – otherwise they would not enter. As a result, the productivity distribution shifts to the right: the bottom firms are replaced by more productive entrants, while some firms in the upper part of the distribution are lucky, receive positive shocks, and move that part of the distribution to the right.

For this process, both non-stationarity of individual firms’ productivity and the dependence of entrants’ average productivity on that of incumbents, i.e. both Assumptions 1 and 2, matter. Without the latter, say with entrants always drawing from the same distribution, selection would still have some effect. However, there would not be a balanced growth path of the type analyzed here, as the productivity distribution would fan out, and thin out, over time. Without the former assumption, the growth engine is choked off. Exogenous increases in entrants’ productivity, resulting in a vintage-type model, could still yield a growth path similar to the one analyzed here, but with the crucial difference that all growth would result from entry and exit. This is at odds with the evidence.

For a more formal analysis of the sources of growth, the law of motion of the firm distribution can be decomposed and rewritten. While the following is for the frictionless case, it carries over to the case with firing costs, at the cost of significantly more complicated notation. Denoting aggregate output by $Y$ and a firm’s output by $y(s)$, the aggregate growth rate is simply

$$ g = \frac{Y' - Y}{Y} = \frac{1}{Y} \left[ \int S \mu'(s) y(s) \, ds - \int S \mu(s) y(s) \, ds \right]. \quad (10) $$
Substituting in for $\mu'$ using the law of motion from equation (9) and rearranging, this yields

$$g = \frac{1}{Y} \left[ \int_S [(Qx\mu)(s) - (1 - e)\mu(s)] y(s) \, ds + e \int_S [\eta(s) - \mu(s)] y(s) \, ds \right],$$

where $e = 1/M$ is the exit rate. Note that this cannot be used directly for calculating the equilibrium growth rate since $Q_x$, and thereby $\mu'$, depend on $g$. This is why finding $g$ is a fixed-point problem. The first integral in equation (11) gives the effect of selection, i.e. the difference between the output-weighted average productivity of surviving firms (with productivity distribution $Q_x\mu$), and that of firms active in the preceding period. The tougher market selection is, and the more firms at the low end of the productivity distribution exit, the larger this term. This also makes it clear that the growth rate and welfare do not have to behave the same way; in the extreme, eliminating all but the most efficient firms would imply a strong selection effect, but harm welfare due to decreasing returns to scale at the firm level and the direct cost of turnover (financing a lot of entry every period). The second term in (11) is the effect of entry, or imitation. It decreases in $\kappa$, the distance of entrants from the frontier. Splitting $\mu$ into the distributions of continuing and exiting firms, $\mu_{cont}$ and $\mu_{exit}$, and using $\mu'(s) = Q\mu_{cont}(s) + e\eta'(s)$, the growth rate can be further rewritten as

$$g = \frac{1}{Y} \left[ \int_S [(Q\mu_{cont})(s) - \mu_{cont}(s)] y(s) \, ds + \int_S \eta(s) - \mu_{exit}(s) y(s) \, ds \right].$$

The second integral captures the part of growth due to entry and exit. As long as entrants are more productive than exiting firms – and this must be the case for there to be positive entry in equilibrium – it is positive, and the combined processes of selection and imitation lead to growth in the aggregate. The first integral captures growth due to experimentation in surviving firms: different firms experience positive or negative innovations to their productivity. Because productivity follows a geometric random walk, the unweighted average of their growth rates is zero. The average level still rises because a firm’s optimal choice of output is a convex function of productivity, so that, by Jensen’s inequality, a mean-preserving spread of the productivity distribution induces a rise in total output. Calibrating the share of growth that is due to entry and exit allows assigning the right relative importance to these two sources of growth.

With selection and experimentation driving growth, firm-level productivity innovations take on large importance. Their effect can easily be seen in the framework of Figure 5. Firm value being convex in productivity, an increase in $\sigma^2$ or $\sigma_e^2$ raises the expected value of entry given $w$ and $g$. This shifts the $E$ curve up. It also leads to stricter selection, as a larger variance of shocks means that more firms go “over the cliff” each period. This in turn shifts the $S$ curve up.
As a result, more variable firm-level productivity unambiguously raises growth by reinforcing selection.

### 3.5 Optimality

Growth in this economy is driven by selection among surviving firms and a spillover to entering firms. In the decentralized equilibrium, firms do not take this into account. In particular, in their exit decision, firms do not take into account how their decision to exit or to remain active influences the productivity of entering firms. Likewise, entrants only consider private benefits. Therefore, a social planner could improve upon the decentralized equilibrium by taking this into account. The following paragraphs provide a brief discussion about where the decentralized equilibrium deviates from the optimal outcome, and with which instruments a planner could implement the optimal outcome as a competitive equilibrium.

Suppose that there is a benevolent social planner that maximizes the representative consumer’s utility. Further suppose that the planner faces the same technological constraints as firms in the decentralized equilibrium, but can directly impose an exit threshold \( \hat{s}_x \) for all firms in the economy. This then determines the firm distribution and the growth rate. The planner also faces the decision of how much output to allocate to consumption, and how much to the construction and operation of firms. This intertemporal decision yields the condition

\[
\epsilon^e = EV^e. \tag{13}
\]

This is analogous to the free entry condition in the competitive equilibrium. Marginal utility weights on social costs and benefits just cancel as they appear on both sides.

In addition, the planner can influence the expected value of entering firms \( EV^e \) by the choice of exit threshold. That choice aims at obtaining the best tradeoff between a high-productivity firm distribution and a high growth rate on the one hand, and the associated cost of firm turnover on the other hand. (More exit implies more selection and faster growth as discussed above, but also higher costs of financing entry investment.) This implies choosing \( \hat{s}_x \) to maximize the value of a portfolio of firms minus the social cost of turnover. The stationary formulation of the problem thus is

\[
\max_{\hat{s}_x} \int \hat{\mu}(\hat{s})V(\hat{s}) \, d\hat{s} - \epsilon c^e = \max_{\hat{s}_x} \int_{\tilde{S}} (\hat{Q}\hat{\mu})(\hat{s})V(\hat{s}) \, d\hat{s} + \epsilon \int \eta(\hat{s})V(\hat{s}) \, d\hat{s} - \epsilon \epsilon^e, \tag{14}
\]

subject to the law of motion of \( \hat{\mu} \). (Remember that \( \tilde{S} \) denotes the set of continuing firms, i.e. \( \tilde{S} = \{ \hat{s} | \hat{s} \geq \hat{s}_x(\hat{n}_{-1}, \hat{\nu}, g) \} \).) In this objective, \( \tilde{S}, \hat{\mu}, \epsilon, \) and \( V \) all depend on \( \hat{s}_x \). Using
\[ \dot{Q}V(\dot{s}, \cdot) = E[V|\dot{s}] \] and differentiating with respect to \( \dot{s} \) yields the first order condition

\[ -\dot{\mu}(\dot{s})E[V|\dot{s}] + \int_{\bar{S}} \frac{\partial \dot{\mu}(\dot{s})}{\partial \dot{s}} E[V|\dot{s}] d\dot{s} + \int_{\bar{S}} \dot{\mu}(\dot{s}) \frac{\partial E[V|\dot{s}]}{\partial \dot{s}} d\dot{s} + \frac{\partial e}{\partial \dot{s}} (EV^e - c^e) = 0. \] (15)

Note that the third term combines changes in expected value for both continuing and entering firms. Applying (13) yields the simplified condition

\[ \dot{\mu}(\dot{s})E[V|\dot{s}] = \int_{\bar{S}} \frac{\partial \dot{\mu}(\dot{s})}{\partial \dot{s}} E[V|\dot{s}] d\dot{s} + \int_{\bar{S}} \dot{\mu}(\dot{s}) \frac{\partial E[V|\dot{s}]}{\partial \dot{s}} d\dot{s}. \] (16)

Compare this to the exit condition (7) in the decentralized equilibrium. There, firms exit if their expected value is smaller than the value of exit, \( V^x \). The planner, in contrast, takes into account the impact of the exit decision on the remaining distribution (first term on the right-hand side (RHS)) and on the value of other firms (second term). The former is positive, as a higher exit threshold translates into a better long-run distribution. The latter is positive for low \( \dot{s} \) and negative for large ones, for the same reason as there is a unique \( \dot{s} \) in the competitive equilibrium: Firm value is negative for low \( \dot{\dot{s}} \) due to the fixed cost; and the option to exit is valuable in that situation. Hence, there is a range of \( \dot{s} \) for which the RHS is positive, and it may become negative for high \( \dot{s} \).

To see how this relates to the competitive equilibrium, insert two equilibrium conditions from there into equation (16). First, privately optimal choice of \( \dot{s} \) implies the first order condition \( \frac{\partial E[V|\dot{s}]}{\partial \dot{s}} = 0 \). Hence, the second term on the RHS of (16) is zero. Second, the left-hand side (LHS) of equation (16) equals \( \dot{\mu}(\dot{s})V^x \) by the exit condition (7). The fact that the first term on the RHS of (16) is positive (the selection effect) now implies that the LHS should also be positive to achieve a social optimum, i.e. \( V^x > 0 \). Since \( V^x = 0 \) in the competitive equilibrium without firing cost or with \( F^x = 0 \), and \( V^x < 0 \) when \( F^x = 1 \), the decentralized equilibrium is not optimal. Charging firing costs to exiting firms – an exit tax – is even worse, as detailed in Section 5.

This reasoning also shows how the optimal allocation can be implemented as a competitive equilibrium: by an exit subsidy (so \( V^x > 0 \)) that makes (16) hold with equality. Since \( V^x \) is continuous and monotonic in \( s \), every \( s_x \) can be achieved in (7) by setting the right \( V^x \).

Other instruments that affect firms’ continuation value (such as a lump-sum tax, or EPL) or otherwise affect firm turnover (such as an entry subsidy) could also be used to control the exit threshold. Compared to them, the exit subsidy has the smallest impact on other choices. It hence constitutes the cleanest instrument.

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8To see this, write the firm’s exit problem as \( \max_{\dot{s}_x} E[V^*(\dot{s}, \dot{s}_x)] \), where \( V^*(\dot{s}, \dot{s}_x) \) is the value of a firm with productivity \( \dot{s} \) that applies the rule to exit for productivity below \( \dot{s}_x \).
4 Benchmark Economy

To derive quantitative conclusions, we calibrate the model to the U.S. non-farm business sector. This is a good no-firing cost benchmark since both procedural inconveniences and severance pay due upon an individual no-fault dismissal are zero in the U.S. according to the OECD’s indicators of employment protection published in Nicoletti, Scarpetta and Boylaud (2000). Other measures of employment protection are among the lowest worldwide, too (see also Gwartney and Lawson 2006).

To calibrate the model, commonly used values from the literature are used for some baseline parameters, while the remaining ones are chosen jointly such that the distance between a set of informative model moments and corresponding data moments is minimized. Distance here is measured as the mean squared relative deviation. This objective may have many local minima. To find the global minimum, a genetic algorithm as laid out in Dorsey and Mayer (1995) is used.

An exception is $\kappa$, the productivity of entrants relative to incumbents. Because of its importance for growth, it is set to exactly match its empirical counterpart, measured in precisely the same way. The closest available empirical measure is the productivity of firms that entered within the last ten years. Foster et al. (2001) report it to be 99% of average productivity.

The parameter values adopted from the literature are 0.64 for the labor share $\alpha$ and 0.95 for the discount factor $\beta$. The disutility of labor $\theta$ is set such that labor force participation fits the value of 66% reported by the BLS and the ILO. The upper bound of the grid for $s$ is chosen such that the largest plant has 1500 employees. According to U.S. Census Statistics of U.S. Businesses data reported by Rossi-Hansberg and Wright (forthcoming), less than 0.05% of plants are larger than this.

The four parameters that remain to be assigned are the variance of the log productivity distribution of entrants, $\sigma^2_e$, the variance of the the idiosyncratic productivity shock hitting incumbents, $\sigma^2$, the fixed operating cost $c^f$, and the entry cost $c^e$. They are chosen to jointly match two moments referring to the entire economy, and two moments related to entry and post-entry behavior: the job turnover rate, average plant size, the four-year survival rate of entrants, and the share of aggregate productivity growth due to entry and exit. These moments are chosen because they capture different aspects of the firm distribution and its dynamics and therefore allow a relatively full description. Average plant size helps to scale the economy as it pins down the level of the fixed operating cost and the entry cost, given their ratio. The job turnover rate is driven to a large extent by the variance of productivity shocks. Entrants’ survival rate
and the share of growth due to entry and exit are informative about the variance of entrants’ productivity distribution and about the entry cost. They also allow fixing the importance of the entry and selection process at a realistic value. The next few paragraphs briefly discuss definition, values, and sources for the data moments.

Average plant size (employment) is 26.4 for the U.S. business sector according to Bartelsman et al. (2003, Table 2). The job turnover rate is the sum of job creation and job destruction at continuing, entering and exiting plants in a year, divided by total employment in that year; it is a crucial dynamic feature of the plant distribution. According to the Bureau of Labor Statistics, it is 28% yearly in the U.S.. Cross-country differences in this variable are significant, as documented by Davis, Haltiwanger and Schuh (1996). The U.S. value is on the high side among developed economies.

Important statistics for understanding entry and its implications are entrants’ survival rates and the share of growth due to entry and exit. Matching them well is important for obtaining a good estimate of $\kappa$ since it is calculated using the relative productivity of surviving entrants. In the U.S., the four-year survival rate, i.e. the proportion of entrants of a given cohort still active four years later, is 63% according to Bartelsman et al. (2004) (BHS, using the U.S. Census Longitudinal Business Database). The share of productivity growth due to entry and exit is 26% for the U.S. manufacturing sector and higher in retailing according to Foster et al. (2001, 2006). Other studies find similar estimates, BHS give an overview. For the economy as a whole, the figure of 26% used here is hence a lower bound, implying that results related to selection obtained here are rather conservative.

Calibration targets and model values are given in Table 1. Adopted parameter values are given in Table 2. Model statistics fit all targets closely. The calibration also fits reasonably well in dimensions that were not targeted. In particular, the productivity dispersion falls comfortably within the range of 2 to 4 reported by Dwyer (1998) from U.S. data, and the seven-year growth rate of surviving entrants is close to the data moment of 40%. Hence, the shape of the distribution, its dynamic behavior, and entrants’ performance match the data well. Remarkably, the implied growth rate, without being targeted, is also of a reasonable magnitude. It thus seems safe to conclude that the model of growth through selection and experimentation presented here provides a good description of the way selection and reallocation promote growth in the U.S..
Table 1: Calibration: Model statistics, calibration targets (U.S., all data for 1990s)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>model</th>
<th>U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average employment at plant</td>
<td>26.4</td>
<td>26.4</td>
</tr>
<tr>
<td>Labor force participation</td>
<td>66%</td>
<td>66%</td>
</tr>
<tr>
<td>Relative productivity of entrants</td>
<td>99%</td>
<td>99%</td>
</tr>
<tr>
<td>Job turnover rate</td>
<td>26.6%</td>
<td>28%</td>
</tr>
<tr>
<td>Four-year survival rate of entrants</td>
<td>62.0%</td>
<td>63%</td>
</tr>
<tr>
<td>Share of aggregate productivity growth due to entry and exit</td>
<td>26.7%</td>
<td>26%</td>
</tr>
<tr>
<td>Productivity dispersion</td>
<td>3.5</td>
<td>2 to 4</td>
</tr>
<tr>
<td>Seven-year growth rate of entrants</td>
<td>45.0%</td>
<td>40%</td>
</tr>
<tr>
<td>Output per capita growth</td>
<td>2.44%</td>
<td></td>
</tr>
</tbody>
</table>


5 Firing costs and productivity growth

The objective of the paper is the analysis of the impact of firing costs on aggregate productivity growth. Since growth is endogenous in the model developed above, frictions can affect not only the level (as in previous literature), but also the growth rate of output and productivity. This section explores their effect first theoretically, then empirically.

5.1 Theoretical discussion

Firing costs affect firms in two ways: they constitute a friction to the adjustment of labor, and they are a tax on exit, if charged to exiting firms. Their effects can most easily be seen via their impact on the entry and selection curves in Figure 5, and in the light of the discussion of optimality in Section 3.5.

Firstly, as an adjustment friction, firing costs cause firms’ employment to deviate from what is optimal in the frictionless economy. This lowers firm value and the incentive to enter or to continue in operation for any \( w \) and \( g \), shifting the entry \( (E) \) curve towards the origin. (See Figure 7.) This shift is similar no matter how exiting firms are treated. Although firm value drops a bit less when exiting firms are exempt from firing costs, the difference is quantitatively small.

Secondly, firing costs affect firms’ exit decision. Consider first the case where firing costs
Table 2: Calibration: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.64</td>
<td>Labor share</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.95</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1.09</td>
<td>Disutility of working</td>
</tr>
<tr>
<td>$\sigma^2_e$</td>
<td>0.829</td>
<td>Variance of log productivity distribution of entrants</td>
</tr>
<tr>
<td>$\sigma^2_\gamma$</td>
<td>0.113</td>
<td>Variance of idiosyncratic productivity shock</td>
</tr>
<tr>
<td>$c_f$</td>
<td>1.3%</td>
<td>Fixed operating cost, % of avg firm output</td>
</tr>
<tr>
<td>$c_e$</td>
<td>260%</td>
<td>Cost of entry, % of avg firm output</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>2.1</td>
<td>Log prodty difference best incumbents/avg entrant</td>
</tr>
</tbody>
</table>

are charged only to continuing firms and not to exiting firms ($F_x = 0$). The value of exit does not change compared to the benchmark economy. Because of the fall in the continuation value, however, the exit threshold $\hat{s}_x$ rises for any $\hat{w}$ and $g$. For the selection equation ($S$) to hold, a higher growth rate is required, shifting the selection curve in Figure 7 up (dashed line). Together with the shift in the entry curve, this can translate into a rise in $g$ if the selection curve is not too steep, or the entry curve not too flat. In terms of optimality, the higher exit threshold comes closer to the socially optimal one that solves equation (16), but at the cost of distorting firms’ labor demand policy. Hence, while firing costs achieve stricter selection, they are a very “dirty” instrument for this.

If also charged upon exit ($F_x = 1$), firing costs act as a tax on exit. They then provide an incentive towards continuing and shift the exit threshold away from its optimal value, and the selection curve down (dotted line). This is because compared to the benchmark economy, both the expected value of continuing and the value of exit fall. The latter drops slightly more because it implies bearing firing costs immediately, whereas continuing implies bearing them only later, so they get discounted when entering the value of continuing.\(^9\) Only the difference between the changes to the expected value of continuing and the value of exit affects the exit threshold and selection, so they react less than with the exemption, and the selection curve shifts little. Combined with the shift in the entry curve, however, growth unambiguously falls.

To summarize, firing costs intensify selection and can raise growth when only charged to continuing firms, while they reduce both when also charged to exiting firms. Besides the effect on the growth rate, there is a double level effect. First, because of the adjustment cost, firms’

\(^9\)A similar point is already present in Bentolila and Bertola’s (1990) discussion of optimal firing of a marginal worker and its consequences for average employment.
employment will not always be optimal, reducing allocative efficiency exactly as in Hopenhayn and Rogerson (1993). Moreover, due to the lower wage, average firm size rises. With less and larger firms, the production structure is less efficient than in the benchmark economy because of decreasing returns. As a result, welfare is unambiguously lower in the case where firing costs are always due, whereas the relative size of changes in the growth rate and levels matters when exiting firms are exempt.

5.2 Quantitative evaluation

This section reports quantitative results on the effect of altering the benchmark economy by introducing firing costs of $c^n$ times the equilibrium wage for each worker fired. $c^n$ is set to one, i.e. a year’s wages. This is close to the average over continental European countries according to the OECD’s employment protection indicators.

Results are reported in Table 3 and fit the qualitative patterns described above. The most salient result are the changes in growth rates. Introducing firing costs decreases the growth rate by around 1 tenth of a percentage point when firing costs are always charged. When exiting firms are exempt, the growth rate rises by almost a tenth of a percentage point. For the rest, results fit the qualitative results outlined above. Job turnover is reduced, while average plant size rises due to lower wages and labor hoarding as firms aim to avoid firing costs.

The increase in growth when exiting firms are exempt from firing costs begs a question about welfare: what is the net welfare effect of the change in growth and the levels effect? (Without the exemption, welfare clearly drops.) The easiest way of evaluating this is for an instant jump to the new balanced growth path, thus neglecting the transition. The two stationarized BGPs are

Table 3: Results: Introducing firing costs (always: $F_x = 1$, exit exemption: $F_x = 0$)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Benchmark</th>
<th>$F_x = 0$</th>
<th>$F_x = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average employment at plant</td>
<td>26.4</td>
<td>31.9</td>
<td>26.5</td>
</tr>
<tr>
<td>Labor force participation</td>
<td>66.0%</td>
<td>65.7%</td>
<td>65.1%</td>
</tr>
<tr>
<td>Relative productivity of entrants</td>
<td>99%</td>
<td>100.6%</td>
<td>99.5%</td>
</tr>
<tr>
<td>Job turnover rate</td>
<td>26.6%</td>
<td>9.4%</td>
<td>9.0%</td>
</tr>
<tr>
<td>Four-year survival rate of entrants</td>
<td>62.0%</td>
<td>61.7%</td>
<td>62.4%</td>
</tr>
<tr>
<td>Productivity dispersion</td>
<td>3.5</td>
<td>3.3</td>
<td>3.4</td>
</tr>
<tr>
<td>Output per capita growth</td>
<td>2.44%</td>
<td>2.52%</td>
<td>2.35%</td>
</tr>
<tr>
<td>Consumption and wage (bm = 100)</td>
<td>100</td>
<td>92.5</td>
<td>96.6</td>
</tr>
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</table>
linked by the fixed average productivity of entrants, a measure of the technology state unaffected by the policy change, permitting the levels comparison. Such a jump is what the consumption and wage numbers in Table 3 refer to. For this exercise, welfare drops substantially despite the rise in growth, by an equivalent of 6% of consumption in the benchmark economy. The levels effect hence substantially outweighs the growth rate effect.

While taking into account the transition would affect this number, the direction of the net effect is still clear. To see this, consider scrapping firing costs where before continuing firms had to pay them. This would eventually reduce yearly growth by 0.08 percentage points. The welfare consequence of an immediate reduction in growth by this amount corresponds to 1.5% lower consumption. Because the growth rate takes time to adjust, this is an upper bound on the growth rate effect. At the same time, firms instantly move to optimal employment (keeping the number and distribution of firms constant). This reallocation effect alone, without changing the firm distribution, raises consumption by 1.5%. More gains follow along the transition as more firms enter. Hence, the net effect of scrapping the firing cost cannot be negative.

The largest growth difference of course does not arise between the frictionless benchmark and the economy with firing costs, but between the two firing cost regimes. Imposing firing costs on exiting firms in addition to continuing firms reduces the growth rate by 1.7 percentage points. This result is particularly relevant since even if employment protection legislation is well-entrenched, there may be more flexibility in the treatment of exiting firms, which differs substantially across countries.\textsuperscript{10}

Moreover, as the contribution of exiting firms to job turnover is small, the exit exemption barely affects job turnover. Exempting exiting firms from firing costs can thus boost growth without affecting job turnover very much. As a consequence, even non-modeled costs of job turnover should not matter much for the comparison of the two regimes.

It is also clear that the presence of firing costs alone does not allow conclusions on growth. In effect, an empirical analysis that does not take into account that countries differ in their treatment of exiting firms is likely to find a much reduced, if any, effect of firing costs.

It is well-established that the variance of idiosyncratic shocks is larger in the service sector. For instance, the coefficient of variation of firm size is up to three times as high as in manufacturing

\textsuperscript{10}In many countries, laid-off workers may obtain claims on assets of the firm even if the firm is liquidated. The seniority of these claims varies widely across countries, cf. Johnson (2003), and determines whether any firing costs in the sense of the model are due. Note also that in cases where a firm’s creditors can determine the firm’s fate, their incentives concerning exit vs continuation are influenced by firing costs in the same way as they are for the firm’s owners.
Table 4: Results: Service sector (always: $F_x = 1$, exit exemption: $F_x = 0$)

<table>
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<th>Statistic</th>
<th>Benchmark</th>
<th>$F_x = 0$</th>
<th>$F_x = 1$</th>
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<td>Average employment at plant</td>
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<td>42.2</td>
<td>33.3</td>
</tr>
<tr>
<td>Labor force participation</td>
<td>67.1%</td>
<td>66.4%</td>
<td>65.3%</td>
</tr>
<tr>
<td>Relative productivity of entrants</td>
<td>98.3%</td>
<td>99.8%</td>
<td>98.5%</td>
</tr>
<tr>
<td>Job turnover rate</td>
<td>32.9%</td>
<td>12.5%</td>
<td>11.7%</td>
</tr>
<tr>
<td>Four-year survival rate of entrants</td>
<td>63.2%</td>
<td>62.4%</td>
<td>63.6%</td>
</tr>
<tr>
<td>Productivity dispersion</td>
<td>3.6</td>
<td>3.5</td>
<td>3.8</td>
</tr>
<tr>
<td>Output per capita growth</td>
<td>3.6%</td>
<td>3.8%</td>
<td>3.4%</td>
</tr>
<tr>
<td>Consumption (bm = 100)</td>
<td>100</td>
<td>90.7</td>
<td>96.1</td>
</tr>
</tbody>
</table>

Sectors (see BHS), job turnover is higher (see e.g. Davis, Faberman and Haltiwanger 2006), and firm turnover is higher (Bartelsman et al. 2003). In such a setting, employment protection legislation constrains firms more strongly. And indeed, recent growth differences between Europe and the U.S. were largest in a large subsector of the service sector, namely in IT-using services (van Ark, Melka, Mulder, Timmer and Ypma 2003, Blanchard 2004). Table 4 shows the effect of firing costs of a year’s wages in an economy where $\sigma^2$, the variance of the idiosyncratic shock, is raised from 0.113 to 0.14 to mimic the service sector. First note that this sector has a higher growth rate than the benchmark economy – this is the positive effect of $\sigma^2$ on the growth rate established in Section 3.4. Job turnover and productivity dispersion are also higher. Now firing costs have a stronger effect on the growth rate and on welfare for both settings of $F_x$. If firing costs are only charged to continuing firms ($F_x = 0$), there is stricter selection, and the growth rate rises almost by 0.4%. If they are also charged to exiting firms ($F_x = 1$), it drops by 0.13%. Welfare falls in both cases.

Hence, firing costs can have potentially large growth rate effects, particularly in sectors where firms face a very volatile environment, such as in services. This fits very well with the pattern of recent growth rate differences between the U.S. and Continental Europe. It also results that details of EPL regimes matter, and that dealing with exit efficiently is an important policy concern in its own right. In fact, charging firing costs to exiting firms does not reduce job turnover by much, but has potentially large costs in terms of growth compared to charging them only to continuing firms. Conclusions comparing the two firing cost regimes are hence robust to unmodeled costs of job turnover.
6 Some indicative empirical evidence

Selection is a crucial driver of growth in the model presented here. This section provides some indicative empirical evidence on the effect of labor market regulation on the selection process.

In the model, selection is governed by equation (7) that states that firms exit when their productivity falls below a threshold at which the expected value of maintaining the firm active equals the value of exiting the market. Firing costs charged to continuing firms reduce firm value and thereby raise this threshold. Firing costs charged upon exit reduce the value of exiting and thereby lower the threshold. These two predictions can be tested empirically using industry-level data for different countries. (It would be interesting to perform it with firm-level data.)

Most of the existing empirical literature studies the impact of some general measure of employment protection legislation (EPL) on gross job flows. For instance, Micco and Pagès (2004) and Haltiwanger, Scarpetta and Schweiger (2006) use cross-sectoral differences to apply a difference-in-difference approach and find that more stringent regulation strongly reduces industry-level job flows, particularly in sectors requiring higher flexibility. Autor, Kerr and Kugler (2007) use variation in the adoption of EPL across U.S. states and find lower employment flows, less firm entry, capital deepening, and a decline in the level of TFP following introduction of wrongful-discharge protections. These results fit with the level effects of EPL as analyzed already in Hopenhayn and Rogerson (1993) and also present here. Bassanini and Venn (2007) survey the (meagre) literature on EPL and growth and find it to be inconclusive. In their own detailed analysis using a difference-in-difference approach, they find a weak negative effect of EPL on growth. However, they do not take into account how exiting firms are treated. If this varies across countries, their small effect can just as well be an average of larger but opposite effects associated with the two regimes. The effect of EPL could thus be larger than they find.

Industry-level data used is from the OECD Firm Level Data Project.11 International comparability is ensured because the data is gathered using a common analytical framework and harmonized definitions of key concepts such as entry, exit, and the unit of measurement. The data set contains annual observations on total employment and the number of continuing, entering, and exiting firms for 34 private sector industries in 10 OECD economies. It does not allow inference about productivity. However, it gives the average size of exiting firms in an industry (total employment of exiting firms divided by their number). Since a firm’s employment is a

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11 This data is available on the OECD’s web site at http://www.oecd.org/document/4/0,2340,en_2649_34117_1962948_1_1_1_1,00.html. It covers Canada, Denmark, Finland, France, Germany, Italy, the Netherlands, Portugal, the United Kingdom, and the United States. Industries are mainly 2-digit ISIC (Rev. 3) industries; in a few cases, such as "Textiles, textile products, leather and footwear", some of these are aggregated.
non-decreasing function of its productivity, the model predictions carry over to the exit size (instead of productivity) threshold. This yields an unbalanced panel with 2178 observations over the years 1977 to 2000 and varying country coverage; or a balanced panel for all countries for 1989 to 1993 with 1021 observations. Results reported below are for the unbalanced panel; they are both qualitatively and quantitatively similar for the balanced panel.

The model predicts that the reaction of the exit size threshold to firing costs depends on whether they are charged only to continuing firms or also to exiting ones. OECD EPL indices published in Nicoletti et al. (2000) allow to distinguish this roughly. They report separately indicators on the “procedural inconveniences” and on the “direct cost” of dismissals. Both are indices ranging from 0 to 6 (least to most restrictive). For ease of reference, call them “firing difficulty” (FD) and “firing cost” (FC), respectively. The firing cost index mainly consists of the cost of severance payments due when dismissing a redundant worker. (Requirements regarding collective dismissals are usually at least as onerous.) The firing difficulty index measures the administrative and legal burden of firing, in particular whether a written statement to a third party is required, whether its approval is needed, and how long the delay up to the start of the notice period is.

These procedural difficulties apply mainly to continuing firms; exit directly creates facts. Firing costs, in contrast, should in principle affect all firms, including exiting ones. In practice, workers may not receive severance payments in full, depending on the rank of their entitlements relative to other creditors; however, some drain on the firm’s exit value remains. So while firing difficulty should only affect the expected value of continuing, and thereby raise the exit threshold, firing costs should also reduce the value of exit, and reduce the exit threshold. Hence, in a regression of the average size of exiting firms on both firing difficulty and firing costs, the coefficient on the former should be positive and the one on the latter negative.

Table 5 shows results of a regression of the (log) average size of exiting firms on measures of firing cost and firing difficulty. The coefficients on both firing costs and firing difficulty are significant, and their signs conform with the predictions of the model, i.e. higher firing difficulty is related to higher size of exiting firms, and higher firing costs to lower size (column A). Subsequent columns show that this result is robust to controlling for industry fixed effects (columns B and D to G), year effects (column G), and other variables that could affect the size of exiting firms, for instance the (log) average size of all firms (logn), product market regulation (PMR), the costs of bankruptcy procedures (BKC), and (log) costs of exporting (EXPC). Including logn is important because it allows identifying effects of EPL on exiting firms that go beyond those...
it has on all firms, e.g. due to labor hoarding. All controls have the signs suggested by the literature. Results are also robust to using just the time averages for each industry (columns C and H), to using the relative size of exiting firms as the dependent variable (effectively fixing the coefficient of logn at 1), or to excluding industries where selection processes are possibly weaker due to public intervention, such as agriculture, mining, utilities, telecoms, and construction, or to limiting the regression to the manufacturing or the service sector.

Due to lack of time variation in the policy variables, the difference-in-difference framework that Micco and Pagès (2004) and Autor et al. (2007) employ to reduce problems of omitted variable bias and endogeneity cannot be used here. However, at least the latter problem is likely to be less severe here due to the distinction of the two components of EPL and because of the nature of the dependent variable. For instance, in regressions of job turnover on EPL, it is imaginable that there is some unobserved factor contributing to high job turnover, and eliciting political pressure for stricter EPL, obscuring the effect of EPL on job turnover. In the present context, results are much more nuanced because we are considering the effects of two different policy variables. Moreover, it is hard to make up an additional channel linking the size of exiting firms to EPL.

7 Conclusion

This paper has analyzed the effect of firing costs on productivity growth, a topic that is currently receiving much attention in policy circles, notably in Europe, but has not been subject of much study in the theoretical literature. To perform the analysis, a model of growth through selection and experimentation has been developed, taking into account recent evidence on firm dynamics, particularly on the importance of job turnover, firm heterogeneity, and the contribution of entry

\[ \text{The product market regulation indicator is from Nicoletti et al. (2000). The negative sign is an indication that product market regulation protects incumbent firms: it drives down the exit threshold. The indicator of the cost of bankruptcy procedures is from the World Bank’s Doing Business database (http://www.doingbusiness.org), following a methodology developed by Djankov, Hart, McLiesh and Shleifer (2006). These are official costs going off the remaining asset value of exiting firms. This can explain the positive coefficient – higher costs prompt creditors to be more active, and cause firms to exit earlier. Costs of exporting come from the same source. Their negative impact on the exit threshold fits with predictions of recent heterogeneous-firm theories of trade in the spirit of Melitz (2003).}

\[ As a thought experiment, imagine that in some country/industry, for some reason, the average size of exiting firms was very high, and that this triggered demands for stricter EPL. This would raise the estimate of the EPL coefficient in question. Concerning the one on FC, this means biasing it towards zero; the true coefficient would then be even more negative than the estimate presented here. Concerning the other coefficient, the estimate presented here might then be too high; however the channel from large firm exit only to stricter procedural inconveniences seems far-fetched.\]
and exit to aggregate productivity growth. In the model, firms receive idiosyncratic productivity shocks and therefore differ in productivity and employment. Growth occurs endogenously due to selection among incumbents, and due to imitation by entrants. In a nutshell, selection eliminates the worst active firms, so that entrants direct their imitation efforts towards the remaining, better ones. Modeling mean productivity of entrants as a constant fraction of the productivity frontier, the model economy grows through rightward shifts of the firm productivity distribution. The more variable the fate of firms in the economy, the stronger the selection mechanism, and the faster growth.

In this setting, firing costs do not only induce a misallocation of labor, reduce firm value, and discourage entry, as in other models, but also discourage exit of low-productivity firms. This congests the selection process and slows down growth. Their effect is stronger the more variable firms’ productivity is. Through this mechanism, the model can match the fact that in recent years, productivity growth differences between the EU and the U.S. were largest in the high-turbulence IT-using service sector. Modeling aggregate productivity growth in close accordance with the evidence on firm dynamics and matching this fact is the first contribution of the paper.

The second one lies in the analysis of the treatment of exiting firms. Exempting exiting firms from firing cost speeds up the exit of inefficient firms and thereby growth. Since job turnover is not much higher than without the exemption, it is likely that the growth cost of charging firing costs to exiting firms exceeds any (here unmodeled) benefits of slightly reducing job turnover.

Calibrating the model to the U.S. economy allows for a quantitative evaluation. It turns out that firing costs always reduce welfare, even if only charged to continuing firms, as the misallocation of labor they induce outweighs the positive effect on growth. These results imply that EPL matters for productivity growth. Moreover, it is crucial how labor market policies affect efficient firm exit. Charging firing costs to exiting firms implies small reductions in job turnover, but large costs in terms of lower growth. Interfering with the market selection mechanism comes at a cost.
References


Appendix

A Algorithm for finding a stationary equilibrium

With the transformation to stationarity, an equilibrium can be calculated numerically as follows. (This assumes parameters are fixed as detailed in the Section 4.) The state space $S \times N$ is discretized into a grid of $200 \times 200$ points. Using more points does not significantly affect results. The $N$ grid is chosen such that it consists of the optimal employment quantities chosen by a firm in the frictionless economy for the points in $S$. Firm value can be obtained by value function iteration for each $(s, n_{-1})$ pair given $g$ and $w$. This also yields the exit thresholds $s_x(n_{-1}, w, g)$ as defined in equation (7), and the transition function $Q_x$, given $g$ and $w$. For any fixed $g$, the free entry condition (E) determines the equilibrium wage $w$, and thereby the exit threshold and transition function for that $g$. Using these, the ergodic firm productivity distribution can be obtained; in the frictionless case directly as $\tilde{\mu} = (I - Q'_x)^{-1} \eta$, and in the case with firing cost by iteration on the law of motion for $\mu$, equation (S). The mean of entrants’ initial log productivity distribution can be normalized to 0. Equilibrium $g$ then is the one that implies a $\mu$ and an $s_{\text{max}}$ consistent with Assumption 2.

B The stationary distribution

Despite the random walk in firms’ productivity, the variance of the distribution remains finite as time goes by. The variance of the aggregate productivity distribution is a weighted sum of the variance of individual cohorts’ productivity distributions. As productivity follows a random walk, each cohort’s variance would grow linearly in time if there was no exit. With exit predominantly hitting the lower part of the distribution, the variance for each cohort can grow linearly at most. To compensate this rise, each cohort’s weight in the overall variance has to decline more rapidly than this for the overall variance to be finite. This is where growth comes in. With exit decisions taken on the basis of expected value next period, it is known at $t - 1$ which firms will be around at $t$. A firm’s probability to still be around after that depends on its distance to the exit threshold, say $s_{\Delta t} \equiv s_t - s_x$, and on the aggregate growth rate. The probability of still being active in period $t + 1$ conditional on survival to $t$ and on productivity in $t - 1$ is $\mathcal{P}(\epsilon \geq g - s_{\Delta t-1})$, that of being active in period $t + h$ conditional on having made it to $t + h - 1$ is $\mathcal{P}(\epsilon \geq hg - s_{\Delta t-1})$, etc. With $\epsilon$ normal, or indeed for any cdf, these probabilities are decreasing functions of $h$; survival becomes less likely every period. With this “marginal”
probability of survival declining with time, the ex ante probability of surviving to $t + h$ also declines with time, and that faster than linearly. Hence, a cohort’s contribution to the variance of the aggregate productivity distribution shrinks with time and goes to zero as $h \to \infty$. 
Table 5: Regression of log average size of exiting firms on firing costs and firing difficulty, 10 OECD countries

<table>
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<th>A</th>
<th>B</th>
<th>C</th>
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<th>E</th>
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Standard errors in parentheses; stars indicate significance at 10 (*), 5 (**), and 1 (***) percent levels, respectively. Sources: OECD Firm Level Data Project, Nicoletti et al. (2000), World Bank Doing Business Database. FE: fixed effects estimation (industry effects), BE: between estimation (industry × country as observation). Regressors: FD: firing difficulty; FC: firing cost; logn: log average size of all firms in the country-industry; PMR: product market regulation; BKC: bankruptcy cost; EXPC: export cost.
Figure 1: Productivity distribution of entrants and incumbents, difference: $\kappa$

Figure 2: The employment policy function when firing costs are always charged ($F_x = 1$)
Figure 3: The employment policy function with firing cost exemption upon exit \((F_x = 0)\)

Figure 4: Growth through right-shifts of the firm productivity distribution
Figure 5: The free entry (E) and selection (S) conditions

Figure 6: The effect of selection on a cohort’s productivity distribution
Figure 7: Firing costs shift the free entry ($E$) and selection ($S$) conditions