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Cross-border Discounted Cash Flow Valuation

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Abstract In cross-border discounted cash flow (DCF) valuation, theorists and practitioners face more problems than in national valuation. Although it is indicated in the literature that the value of a firm is independent of the used currency, it is ambiguous whether spot or forward exchange rates should be used. Furthermore, the derivation of the corresponding cost of capital considering the correlation between exchange rate risk and a firm's business risk in a consistent global capital market model is unclear, and the implementation of cross-border valuation in a two-phase model with a coherent growth rate is largely neglected. This paper develops a consistent framework for cross-border DCF valuation in a two-phase model focusing on an unlevered firm. Based on a sound analysis of the multi-period global capital asset pricing model, we derive valuation approaches in foreign and home currency with the corresponding costs of capital, considering the correlation between exchange rate risk and a firm's business risk. Afterwards, we illustrate the transfer of the valuation approaches to a levered firm with active and passive debt management. Finally, we discuss the implications of the different valuation approaches from both theoretical and practical perspectives.

Keywords Valuation · Global capital asset pricing model · Exchange rate risk · Debt financing

JEL-Classification G12, G32

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1 Introduction

Over the last decades, the relevance of cross-border investments has steadily increased, and many international companies have emerged. Alongside this development, cross-border valuation has become an important tool for analyzing investments in foreign countries. However, the literature on discounted cash flow (DCF) valuation (Berk and DeMarzo 2020; Brealey et al. 2020; Koller et al. 2020; Kruschwitz and Löffler 2020) is often restricted to national valuation. If different currencies occur, the literature indicates that the market value should be independent of the currency used in valuation (see, for example, Koller et al. 2020, p. 507). However, although this is a recently much discussed topic, especially in Germany (Baumgartner and Schüler 2023; Breuer and de Vargas 2021; Breuer et al. 2021; Schultze and Frey 2023), and important developments have been made (Drukarczyk and Schüler 2021; Schüler 2021), some important details remain insufficiently explained in a multi-period setting.

In this study, we derive a consistent cross-border DCF valuation model. We examine the global capital asset pricing model (GCAPM) to deduce conditions for its application in a multi-period setting. Based on the GCAPM, we focus on an unlevered firm and derive valuation approaches in home currency (HC), distinguishing between the conversion of cash flows with spot and forward exchange rates. Thereby, we establish a framework in which the HC-valuation approaches yield the same result at the valuation date as the valuation approach in foreign currency (FC). Furthermore, we incorporate our findings into a two-phase model and transfer important relationships to the valuation of a levered firm. Additionally, we compare the different valuation approaches and assess their practicability.

Complications in valuation with different currencies arise when a company generates cash flows in FC and managers or shareholders are interested in the HC value of these cash flows (Berk and DeMarzo 2020, p. 1099). To determine this value, an FC- or HC-valuation approach can be conducted. In the former, cash flows are discounted at a cost of capital based on the foreign capital market, and the valuation result is converted to HC at the current spot exchange rate. In the latter, cash flows are converted to HC first, and then discounted at a cost of capital expressed in HC (Koller et al. 2020, p. 508). It is expected that in the case of correct application, the FC- and HC-valuation approaches are equivalent and yield the same result (Koller et al. 2020, p. 507; for further analysis, see Butler et al. 2013; Schüler 2021).

Based on the existing literature, HC-valuation approaches can be distinguished by whether spot or forward exchange rates are used to convert cash flows. The literature mostly focuses on the HC-valuation approach with forward exchange rates (Erasmus and Ernst 2014; Berk and DeMarzo 2020, Chapter 31; Brealey et al. 2020, Chapter 27; Holthausen and Zmijewski 2020, Chapter 17; Koller et al. 2020, Chapter 27, Appendix G), whereas the use of expected spot exchange rates for the conversion of cash flows (Bekaert and Hodrick 2018, Section 15–16, O'Brien 2022) is rarely discussed. The connection between these approaches, the derivation of consistent discount rates, and other conditions for their application are only discussed in Schüler (2021) and Drukarczyk and Schüler (2021, Chapter 17). They derive detailed valuation equations in the single-period case. Furthermore, valuation equations

are stated for a multi-period case, but derivations and corresponding limitations are missing. Therefore, the main goal of our study is the consistent implementation of DCF valuation with HC-valuation approaches in a multi-period setting. We identify the following five components to achieve this goal.

The first component is the derivation of consistent discount rates. For national valuation, the cost of capital can be derived from the capital asset pricing model (CAPM). For international valuation, the literature rarely elaborates on the derivation of the cost of capital. O'Brien (2022) derives a discount rate that is consistent with the international capital asset pricing model (ICAPM), which considers the integration of capital markets (for literature on the ICAPM, see Adler and Dumas 1983; Sercu 1980, 2009; Solnik 1974; Stulz 1981). A simplified model is the GCAPM (O'Brien 1999; Schramm and Wang 1999; Schüler 2021; Stulz 1999), where it is assumed that the purchasing power parity (PPP) holds (Schramm and Wang 1999, p. 65; Koller et al. 2020, pp. 512–516; Holthausen and Zmijewski 2020, p. 865). While Fama (1977) derived conditions for the application of the multi-period local CAPM (LCAPM), corresponding conditions for deriving a deterministic cost of capital based on the multi-period GCAPM are not explicitly analyzed in the literature.

The second component is analyzing the integration of a correlation between exchange rate risk and business risk of a firm. Beyond the classification based on spot or forward exchange rates, HC-valuation approaches can be further distinguished by whether a correlation between exchange rate risk and a firm's business risk is considered (see Schüler 2021, p. 643). Regarding the inclusion of the correlation, Bekaert and Hodrick (2018) point out that it may be difficult to derive information on covariances in corporate valuation practice such that "As a practical matter, no one does this." (Bekaert and Hodrick 2018, p. 723). Berk and DeMarzo (2020) critically assess their assumption of zero correlation and state, "Such an assumption often makes sense if the firm operates as a local firm in the foreign market [...]. However, many firms use imported inputs in their production processes or export some of their output to foreign countries." (Berk and DeMarzo 2020, p. 1110). Following this argument, for international companies, the correlation between exchange rate risk and a firm's business risk should be considered.

The third component is the incorporation of different valuation approaches into a two-phase model, including an explicit forecast phase and a steady state phase (see, for example, Brealey et al. 2020, pp. 95–99). Whereas in the explicit forecast phase detailed planning of cash flows is conducted, it is assumed that the firm has reached a steady state thereafter. In this steady state phase, the expected values of all input parameters increase at a constant rate, implying that the expected market value also grows at this rate (Koller et al. 2020, pp. 259–260). If quantities occur in FC and grow at a constant rate, this growth has to be transferred to HC.

The fourth component displays the integration of debt financing. Since interest on debt is deductible from corporate income, debt financing has an immediate impact on the market value of a firm (Kruschwitz and Löffler 2020, p. 73). In DCF valuation, active (Harris and Pringle 1985; Miles and Ezzell 1980, 1985) or passive debt management (Ashton and Atkins 1978; Modigliani and Miller 1958) is usually assumed. Although we focus on an unlevered firm, and most results for the valuation of an unlevered firm can be transferred to a levered firm, some relationships do not

follow directly. Furthermore, the cited literature mostly either assumes the firm to be equity-financed or does not explicitly consider debt financing. The concept of passive debt management is stated in Schüller (2021), but active debt management has not yet been theoretically analyzed in cross-border valuation. Therefore, we clarify the extension of the valuation approaches to a levered firm.

The fifth component is to deduce implications of our results for corporate valuation theory and practice. At first, the FC-valuation approach may seem more practical since it is similar to national valuation (for an overview on national valuation, see, for example, Berk and DeMarzo 2020; Koller et al. 2020; Kruschwitz and Löffler 2020). However, the FC-valuation approach requires knowledge about the foreign capital market. In particular, “One problem with this approach is that it is sometimes difficult to determine the appropriate foreign currency discount rate.” (Bekaert and Hodrick 2018, p. 722). Furthermore, the FC-valuation approach does not incorporate possible misvaluation of exchange rates (for a debate of the FC- versus the HC-valuation approach regarding exchange rate misvaluation, see O’Brien 2017, pp. 86–87). Koller et al. (2020) recommend, “Use a domestic-capital WACC if the cross-border business is financed and taxed at domestic interest and tax rates. As international companies tend to borrow in their parent country at parent company currencies, this is the most common approach.” (Koller et al. 2020, p. 516). This highlights the importance of HC-valuation approaches.

Other general problems of cross-border valuation include the forecasting of cash flows, debt and interdependencies between subsidiaries in several countries, as well as the integration of corresponding country risks, legal requirements, differences in international accounting standards, and international taxation (for an overview of problems in international valuation, see, for example, Erasmus and Ernst 2014, pp. 16–25). An overview of existing approaches to integrate political risk can be found in Bekaert et al. (2016). For elaborations on special characteristics of international taxation, we refer to Berk and DeMarzo (2020, pp. 1103–1106) and Holthausen and Zmijewski (2020, pp. 871–881). Overall, an analysis of all aspects of cross-border valuation is beyond the scope of this study. Therefore, we focus on the DCF valuation of a foreign firm from the perspective of a domestic investor. In this setting, the FC-valuation approach is usually suitable; however, the described gaps in the literature on HC-valuation approaches already apply to this case of cross-border DCF valuation. Thus, we analyze this basic setting to derive a sound framework that can then be transferred to a more general setting.

In this study, we derive conditions and valuation equations for a consistent framework for cross-border DCF valuation of a foreign firm. We analyze the multi-period GCAPM and elaborate on necessary conditions for its application (Section 2). Based on this, we develop valuation equations for different HC-valuation approaches using the FC-valuation approach as a starting point. In particular, we analyze HC-valuation approaches with spot and forward exchange rates, with and without explicitly considering the correlation between exchange rate risk and business risk of a firm, respectively. We thereby clarify the effects of such a correlation and provide a justification for not considering the correlation in the approaches discussed in the literature. In particular, we show conditions in which all approaches yield the same result at the valuation date. Furthermore, we clarify that correct future mar-

ket values can only be derived if covariances are explicitly considered (Section 3). Moreover, we transfer important relationships to active and passive debt management (Section 4). All results are embedded into a two-phase model and analyzed for practicability (Section 5).

2 Capital Market Model

To build a sound valuation model, we need a foundation based on a capital market model. We want to establish fundamental assumptions upon which we develop valuation equations. In corporate valuation practice, the application of the LCAPM is a popular model to estimate the cost of equity. However, the LCAPM is based on segmented markets (Sercu 2009, p. 679). In a cross-border valuation with integrated markets, the cost of equity should be estimated based on the GCAPM (Krapl and O'Brien 2016; Koller et al. 2020, p. 512). Several studies have examined the GCAPM (O'Brien 1999; Schramm and Wang 1999; Stulz 1999) and the influence of the choice of model on the cost of capital (Ejara et al. 2020; Koedijk et al. 2002; Koedijk and van Dijk 2004a, b), but conditions for the application in a multi-period context have not yet been analyzed. Fama (1977) conducted a study on the multi-period LCAPM. In particular, Fama excludes diversification options across periods; that is, the portfolio decision of an investor at some time t cannot be used to hedge against uncertainty at time $t + 1$. To do so, it is assumed that all input variables are deterministic at the valuation date. It follows that expectations and covariances do not depend on available information (Fama 1977). To provide a sound basis for cross-border DCF valuation, we transfer the results of Fama (1977) and examine the application of the GCAPM in a multi-period context.

We examine two countries with HC and FC, respectively. We assume that the capital market participants of both countries are risk-averse and have homogeneous expectations. Furthermore, the capital markets of both countries are assumed to be perfect, complete, and fully integrated. This implies that there are no restrictions or barriers and that the markets are free of arbitrage opportunities (Sercu 2009, p. 667). The global capital market results from the two perfectly correlated local capital markets.¹ The derived relationships also hold if more countries are considered. The global market then comprises more countries, but the relationships obtained in this section remain valid.

For exchange rates, we use the direct quotation, which means that the cost of one unit of FC is given in units of HC (Sercu 2009, p. 72). Furthermore, we assume the Fisher hypothesis to hold (Bekaert and Hodrick 2018, pp. 395–396) and the nominal risk-free interest rates to be deterministic for all periods.² We require the covered

¹ Alternatively, one could start with two capital markets that are not perfectly correlated. An example of such a setting can be found in Sercu (2009, pp. 668–669). However, the assumption of fully integrated markets is widely accepted and is a condition for the application of the GCAPM (Stulz 1999; Koedijk et al. 2002; Sercu 2009, Chapter 19; Krapl and O'Brien 2016; Holthausen and Zmijewski 2020, Chapter 17; Koller et al. 2020, Chapter 27), such that we use it for the analysis in our study.

² The implications of Siegel's paradox (Siegel 1972; Solnik 1993) are not discussed.

interest parity (CIP) (Sercu 2009, pp. 123–125) but not the uncovered interest parity (Sercu 2009, p. 430; Holthausen and Zmijewski 2020, pp. 865–867) to hold.³ The CIP is given by (Koller et al. 2020, p. 511)

$$F_t = S_0 \cdot \frac{\prod_{k=1}^t (1 + i_k^{\text{HC}})}{\prod_{k=1}^t (1 + i_k^{\text{FC}})}, \quad (1)$$

where S_0 is the current spot exchange rate and F_t is the forward exchange rate of period t , which is the rate used in a contract made today to exchange a fixed amount in period t . Furthermore, i_k^{FC} and i_k^{HC} depict the risk-free interest rates of period k in FC and HC, respectively. Superscript FC (HC) always refers to a quantity expressed in FC (HC). Equation 1 states that it is irrelevant whether an amount is invested at the risk-free interest rate in FC for t periods and converted at the forward exchange rate, or if the same amount is exchanged at the current spot exchange rate and invested at the risk-free interest rate in HC for t periods.

To rule out arbitrage, we require further for $r \leq t$ (Sercu 2009, pp. 123–125)

$$\tilde{F}_{r,t} = \tilde{S}_r \cdot \frac{\prod_{k=r+1}^t (1 + i_k^{\text{HC}})}{\prod_{k=r+1}^t (1 + i_k^{\text{FC}})}, \quad (2)$$

where $\tilde{F}_{r,t}$ is the forward exchange rate that is used in contracts made in period r to exchange a fixed amount in period t , and $\tilde{S}_r$ is the spot exchange rate of period r . These quantities are uncertain at the valuation date but certain from the view of period r . Note that we always label uncertain variables by a tilde. Thus, the condition of Eq. 2 states the CIP from the view of period r . These assumptions imply that the forward exchange rate of period t does not have to be equal to the expected spot exchange rate of this period, but the forward exchange rate is the certainty equivalent of the spot exchange rate (Sercu 2009, p. 135).

We also assume the purchasing power parity (PPP) to hold (Schramm and Wang 1999, p. 65; Koller et al. 2020, Appendix G; Brealey et al. 2020, pp. 725–726; Holthausen and Zmijewski 2020, p. 865). We do not consider personal taxes. These assumptions are in line with the assumptions of the GCAPM (O'Brien 1999, p. 74, Koller et al. 2020, p. 827–829) and coincide with those of Schüller (2021, pp. 619–621).

In the GCAPM, the global market portfolio replaces the local market portfolio of the LCAPM (O'Brien 1999, p. 74) and contains securities from the capital markets of both countries (Koller et al. 2020, p. 513). Since we assume PPP to hold, all investors hold the same global market portfolio (Koller et al. 2020, p. 828). However, the rate of return of the global market portfolio depends on the currency it is expressed in. We start by analyzing the properties of the GCAPM in HC before we further examine the relations of future spot exchange rates, and thereby, derive the security market line of the GCAPM in FC.

To transfer the results of Fama (1977), we require the expectation of the return of the global market portfolio expressed in HC of some period t to be independent of

³ Implications for the case that the uncovered interest parity holds are discussed in Schüller (2021).

the available information before time t . Therefore, the return $\tilde{r}_t^{M,HC}$ is independent of the σ -algebra $\mathcal{F}_{t-1}$, where $(\mathcal{F}_t)_{t \in \mathbb{N}}$ is the natural filtration of the process $(\tilde{r}_t^{M,HC})_{t \in \mathbb{N}}$. This implies $\mathbb{E}_\theta[\tilde{r}_t^{M,HC}] = \mathbb{E}[\tilde{r}_t^{M,HC}]$ for $\theta < t$, where $\mathbb{E}_\theta[\cdot]$ denotes the expectation conditioned on $\mathcal{F}_\theta$. This assumption is the first requirement for deterministic input parameters.

The same assumption is required for the return of a security k . The expected return of the security in some period t must be independent of the information available before time t , which implies $\mathbb{E}_\theta[\tilde{r}_t^{k,HC}] = \mathbb{E}[\tilde{r}_t^{k,HC}]$ for $\theta < t$. It follows that the covariance of the return r_t^k and the return of the global market portfolio is also independent of the available information. Consequently, this also holds for the global beta $\beta_t^{k,HC}$. It follows that all input parameters of the security market line are deterministic, which yields

$$\mathbb{E}[\tilde{r}_t^{k,HC}] = i_t^{HC} + (\mathbb{E}[\tilde{r}_t^{M,HC}] - i_t^{HC}) \cdot \beta_t^{k,HC} = i_t^{HC} + MRP_t^{HC} \cdot \beta_t^{k,HC}, \quad (3)$$

where MRP denotes the global market risk premium.

To convert the quantities from HC to FC, we analyze the characteristics of the exchange rates. The change of future spot exchange rates is defined as

$$\tilde{\delta}_t^S = \frac{\tilde{S}_t}{\tilde{S}_{t-1}} - 1. \quad (4)$$

To obtain deterministic input parameters, we require further the expected change in spot exchange rates of period t to be independent of the occurred states before this time. It implies that, although the spot exchange rates of period $t-1$ and t are random variables, the ratio $\tilde{\delta}_t^S$ is independent of the information available before time t and thus,⁴

$$\mathbb{E}_\theta[\tilde{\delta}_t^S] = \mathbb{E}[\tilde{\delta}_t^S] \quad \text{for } \theta < t. \quad (5)$$

Since the two capital markets are fully integrated, all properties of the capital market in HC transfer to that in FC. Again, the expected return of some period t is assumed to be independent of the available information before time t . Furthermore, the expected return of some security k expressed in FC does not depend on available information and follows the security market line

$$\mathbb{E}[\tilde{r}_t^{k,FC}] = i_t^{FC} + (\mathbb{E}[\tilde{r}_t^{M,FC}] - i_t^{FC}) \cdot \beta_t^{k,FC} = i_t^{FC} + MRP_t^{FC} \cdot \beta_t^{k,FC}. \quad (6)$$

⁴ Eq. 5 is equivalent to the process $(\tilde{S}_t / \mathbb{E}[\tilde{S}_t])_{t \in \mathbb{N}}$ being a martingale, see the Appendix (Sect. 6.1) for further elaborations.

We can convert the expected return from FC to HC by (O'Brien 2017, pp. 58, 124)

$$\begin{aligned}\mathbb{E}[\tilde{r}_t^{M,HC}] &= \left(1 + \mathbb{E}[\tilde{r}_t^{M,FC}]\right) \cdot \left(1 + \mathbb{E}[\tilde{\delta}_t^S]\right) + \text{Cov}\left[\tilde{r}_t^{M,FC}, \tilde{\delta}_t^S\right] - 1 \\ &= \mathbb{E}[\tilde{r}_t^{M,FC}] + \mathbb{E}[\tilde{\delta}_t^S] + \mathbb{E}\left[\tilde{r}_t^{M,FC} \cdot \tilde{\delta}_t^S\right].\end{aligned}\quad (7)$$

The return in FC is multiplied by the change in spot exchange rates, and a covariance is added since these quantities can interact. Alternatively, in the second line of Eq. 7, the return of the global market portfolio expressed in HC can be computed by adding the expected return of the market portfolio expressed in FC to the expected change of spot exchange rates. Moreover, the expectation of the product has to be added to consider interaction between those quantities.

Overall, we have derived conditions for deterministic input parameters at the valuation date by transferring the results of Fama (1977). Furthermore, we have derived a link between the capital markets in HC and FC via spot exchange rates.

3 Valuation of an Unlevered Firm

3.1 Assumptions

We follow the line of Schöler (2021) and consider an unlevered foreign company that generates cash flows in FC. This company is to be valued from the perspective of a domestic investor such that the market value of the firm needs to be expressed in HC. It follows that one has to decide on the currency of the valuation: Either the cash flow is first converted into HC and then discounted in HC (HC-valuation approach), or the cash flow is discounted in FC and then converted to HC (FC-valuation approach) (Schramm and Wang 1999, p. 64; Koller et al. 2020, p. 508).

If the HC-valuation approach is used, another distinction can be made. The conversion of cash flows can either be done using spot or forward exchange rates. We distinguish the HC-valuation approaches further into whether a correlation between exchange rate risk and business risk of the firm is explicitly considered, which yields four possible HC-valuation approaches. If we do not explicitly consider the correlation in the conversion of cash flows, we label the approach as a *modified* approach. Note that we do not assume the correlation to be zero but analyze the effects of neglecting it in the conversion of cash flows. Since it may be difficult to derive information on covariances in corporate valuation practice, this is a useful concept (Bekaert and Hodrick 2018, p. 723). We derive a consistent framework in which the FC-valuation approach and all HC-valuation approaches yield the same market value at the valuation date.

In this section, we derive valuation equations for an unlevered firm. We assume that future free cash flows (FCF) expressed in FC of the foreign company are given, where the FCF always describes the cash flow of an unlevered firm. The projection of the cash flows is based on a two-phase model: For the explicit forecast phase, which comprises T periods, a detailed business plan is developed. Based on that, period-

specific cash flows are computed. After the explicit forecast phase, it is assumed that the firm has reached a steady state, in which all input parameters grow at the state-specific rate $\tilde{g}^{\text{FC}}$. It follows that the expectation of all input parameters grows at the constant rate $\mathbb{E}[\tilde{g}^{\text{FC}}]$, which does not depend on the information available at time t . In particular, for the steady state, it is assumed that the expected FCF grows at this rate. We analyze the effects of this steady state on the HC-valuation approaches by deriving corresponding growth rates in HC. In the following, we always start with the FC-valuation approach and then derive valuation equations for the HC-valuation approaches.

3.2 FC-valuation Approach

We use the FC-approach as a starting point since it is well-known from national valuation. Only the multiplication of the market value at the valuation date by the current spot exchange rate constitutes an additional step. Using a recursive valuation approach, the expected market value in FC of some period t is

$$\mathbb{E}[\tilde{V}_t^{u,\text{FC}}] = \frac{\mathbb{E}[\tilde{CF}_{t+1}^{\text{FC}}] + \mathbb{E}[\tilde{V}_{t+1}^{u,\text{FC}}]}{1 + \rho_{t+1}^{u,\text{FC}}}, \quad (8)$$

where $\tilde{CF}_t^{\text{FC}}$ is the FCF and $\tilde{V}_t^{u,\text{FC}}$ is the market value of the unlevered firm of period t . If the unlevered firm is considered, we denote it by a superscript u . The discount rate $\rho^{u,\text{FC}}$ displays the unlevered cost of equity, which coincides with the expected return and follows from applying the security market line of the GCAPM in FC, see Eq. 6.⁵

The expected market value at the beginning of the steady state phase can be derived with the Gordon-Shapiro formula (Gordon and Shapiro 1956).⁶ Note that we assume the business risk, and therefore the unlevered cost of equity, of the foreign company to be constant in the steady state phase. The resulting market value in FC at the valuation date is multiplied by the current spot exchange rate, which yields the market value in HC: $V_0^{u,\text{HC}} = V_0^{u,\text{FC}} \cdot S_0$.

3.3 HC-valuation Approach with Spot Exchange Rates

In this approach, we analyze the conversion of cash flows from FC to HC using spot exchange rates. For the expectation of the FCF of period t follows (Sercu 2009, p. 668)

$$\begin{aligned} \mathbb{E}[\tilde{CF}_t^{\text{HC},S}] &= \mathbb{E}[\tilde{CF}_t^{\text{FC}} \cdot \tilde{S}_t] \\ &= \mathbb{E}[\tilde{CF}_t^{\text{FC}}] \cdot \mathbb{E}[\tilde{S}_t] + \text{Cov}[\tilde{CF}_t^{\text{FC}}, \tilde{S}_t]. \end{aligned} \quad (9)$$

⁵ According to the theoretical foundation presented in Sect. 2, we obtain *deterministic* costs of capital. For a further discussion of this, see Kruschwitz and Löffler (2020, p. 33).

⁶ The growth rates are assumed to be iid, see Kruschwitz and Löffler (2020, p. 50).

The conversion of the cash flow results in an expectation of the product of the FCF and the spot exchange rate. When using a product of expectations, the covariance of these quantities must be added, see the second line of Eq. 9. Superscript S indicates that we exchange the FCF at the respective spot exchange rate using this pattern.

We can convert the expected market value from FC to HC following the same pattern, that is,

$$\mathbb{E}[\tilde{V}_t^{u,HC,S}] = \mathbb{E}[\tilde{V}_t^{u,FC}] \cdot \mathbb{E}[\tilde{S}_t] + \text{Cov}[\tilde{V}_t^{u,FC}, \tilde{S}_t]. \quad (10)$$

To derive a recursive valuation formula, the converted cash flows need to be discounted at a domestic cost of capital. Thus, we use the cost of equity $\rho_{t+1}^{u,HC,S}$ that is estimated with the security market line of the GCAPM in HC, see Eq. 3. The expected market value of some period t can be derived as

$$\mathbb{E}[\tilde{V}_t^{u,HC,S}] = \frac{\mathbb{E}[\tilde{CF}_{t+1}^{HC,S}] + \mathbb{E}[\tilde{V}_{t+1}^{u,HC,S}]}{1 + \rho_{t+1}^{u,HC,S}}. \quad (11)$$

Thus, the sum of the FCF and the market value of period $t + 1$ is discounted at the unlevered cost of equity. All quantities are expressed in HC.

To establish a connection between the discount rates in FC and HC, we compare this approach to the FC-valuation approach and use our results on the multi-period GCAPM to obtain (Schüler 2021, Eq. 31 and O'Brien 2022, Eq. 10)

$$\rho_t^{u,HC,S} = (1 + \rho_t^{u,FC}) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) + \text{Cov}[\tilde{r}_t^{u,FC}, \tilde{\delta}_t^S] - 1, \quad (12)$$

see the Appendix (Sect. 6.2.1). Thus, to deduce the unlevered cost of equity in HC, the cost of equity in FC is converted to HC by multiplying it by the expected change in spot exchange rates, and the covariance of the return of the unlevered firm and the change in spot exchange rates is added. The derived cost of equity is deterministic since the distribution of the return of the unlevered firm and the change in spot exchange rates does not depend on future information, see Section 2. This assumption and thus our analysis of the GCAPM is essential. Naturally, we have obtained the analogous link for the costs of equity in HC and FC as we did for the return of the global market portfolio, see Eq. 7.

To obtain a growth rate in HC, we compute the change of the expected market values from period t to $t + 1$ conditioned on all available information at time t , see the Appendix (Sect. 6.2.2), and conclude (Schüler 2021, Eq. 39)

$$\mathbb{E}[\tilde{g}^{HC,S}] = (1 + \mathbb{E}[\tilde{g}^{FC}]) \cdot (1 + \mathbb{E}[\tilde{\delta}^S]) + \text{Cov}[\tilde{g}^{FC}, \tilde{\delta}^S] - 1. \quad (13)$$

To obtain this growth rate, the growth rate in FC is multiplied by the change in spot exchange rates and a covariance is added. The computation follows the same structure as the computation of the unlevered cost of equity in HC, see Eq. 12. Again, the derived conditions for the application of the GCAPM are essential. Only with this, we can show that the expected growth rate in HC is deterministic at the

valuation date.⁷ As for the FC-valuation approach, the Gordon-Shapiro formula can be used to calculate the terminal value.

As the current spot exchange rate is deterministic, for $t = 0$, the covariance disappears in Eq. 10 and this market value coincides with the market value derived by the FC-valuation approach. We conclude that if the correlation between exchange rate risk and business risk is explicitly considered in the conversion of cash flows, it also has to be considered in the cost of capital. This happens automatically if the cost of equity based on the domestic capital market is applied. Furthermore, the correlation also has to be explicitly considered for the conversion of the growth rate.⁸

3.4 Modified HC-valuation Approach with Spot Exchange Rates

In this approach, we analyze consequences of omitting the covariance in the conversion of the FCF. We specify

$$\mathbb{E}[\widetilde{CF}_t^{\text{HC}, S^*}] = \mathbb{E}[\widetilde{CF}_t^{\text{FC}}] \cdot \mathbb{E}[\widetilde{S}_t]. \quad (14)$$

This expectation is based on the constructed cash flow

$$\widetilde{CF}_t^{\text{HC}, S^*} = \widetilde{CF}_t^{\text{FC}} \cdot \widetilde{S}_t - \text{Cov}[\widetilde{CF}_t^{\text{FC}}, \widetilde{S}_t].$$

Since the covariance is deterministic, it disappears when taking expectations. Thus, the expectation of the FCF in HC of Eq. 14 does not coincide with the expectation from Eq. 9. The fact that the underlying definition of the FCF does not depict a real but a modified cash flow on the capital market in HC is denoted by the superscript star S^* .

The conversion of the expected market value applying this pattern results in

$$\mathbb{E}[\widetilde{V}_t^{u, \text{HC}, S^*}] = \mathbb{E}[\widetilde{V}_t^{u, \text{FC}}] \cdot \mathbb{E}[\widetilde{S}_t]. \quad (15)$$

This is a modified market value, in analogy to the modified cash flow: Compared to the HC-valuation approach with spot exchange rates, the covariance is omitted. To obtain the correct market value at the valuation date with a recursive valuation equation using the modified cash flow and the modified market value, we adjust the discount rate accordingly. The approach is based on the FC-valuation approach, in

⁷ In the steady state phase, we assume a constant change of exchange rates. It follows that the cost of equity and the growth rate in HC are constant. If this was not assumed, both quantities would be time-dependent.

⁸ The valuation equation and the formula for the growth rate coincide with Schüler (2021, Eqs. 39, 42). However, the author does not elaborate on underlying conditions. Furthermore, in Schüler (2021, Eq. 42), it is not labeled how the expected market value of period one is converted to HC but it is left up to the reader to differentiate. In particular, the expectation “ $\mathbb{E}[\widetilde{V}_{t, \text{HC}, 1}]$ ” appears in both parts of Eq. 42. However, these terms do not coincide since it makes a difference whether forward or spot exchange rates are used for the conversion of future quantities to HC (see also Section 3.7). We clarify this by distinguishing between different expectations and by labeling how a certain value is converted.

which the valuation equation is expanded with future expected spot exchange rates, see the Appendix (Sect. 6.2.3). For the modified expected market value at period t follows

$$\mathbb{E}[\tilde{V}_t^{u,HC,S^*}] = \frac{\mathbb{E}[\tilde{C}F_{t+1}^{HC,S^*}] + \mathbb{E}[\tilde{V}_{t+1}^{u,HC,S^*}]}{1 + \rho_{t+1}^{u,HC,S^*}}, \quad (16)$$

where

$$\rho_t^{u,HC,S^*} = (1 + \rho_t^{u,FC}) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) - 1. \quad (17)$$

The corresponding discount rate for this approach is the product of the cost of equity in FC and the change in spot exchange rates. Thus, if covariances are neglected in the numerator, they also have to be neglected in the denominator. At the valuation date, the covariance of the current market value and current spot exchange rate amounts to zero, such that Eqs. 10 and 15 coincide and we obtain the correct valuation result. This is because covariances mathematically cancel out of the valuation equation at the valuation date (see also Schüller 2021, Eq. 11, who showed this in a single-period setting). However, covariances do not cancel out in the computation of future market values, such that future market values differ compared to the HC-valuation approach with spot exchange rates. Therefore, we refer to future market values as modified market values. Due to our assumptions from Section 2, the discount rate from Eq. 17 is deterministic. However, it cannot be derived by applying the GCAPM in HC. Instead, the discount rate is based on the cost of equity in FC. It follows that it cannot be referred to as a cost of capital. Therefore, we refer to it as a modified discount rate. Overall, we call this a *modified* approach.

To apply the approach with expected spot exchange rates for the steady state phase, we need to convert the growth rate in FC to a modified growth rate in HC. To do so, we compute the change in market values of some period t to $t + 1$ and obtain

$$\mathbb{E}[\tilde{g}^{HC,S^*}] = (1 + E[\tilde{g}^{FC}]) \cdot (1 + \mathbb{E}[\tilde{\delta}^S]) - 1. \quad (18)$$

As for the conversion of other quantities in this approach, the computation of the growth rate is conducted without an explicit consideration of covariances.

3.5 HC-valuation Approach with Forward Exchange Rates

When using forward exchange rates for the conversion of the FCF to HC, we agree today on a contract to exchange the expected FCF of period t at the forward exchange rate F_t (Bekaert and Hodrick 2018, p. 106). At that time, the FCF usually does not coincide with this expectation. Depending on the development of the FCF, we have either exchanged too much or too little at the forward exchange rate F_t . Consequently, we have to exchange the outstanding amount at the uncertain spot

exchange rate $\widetilde{S}_t$ (Berk and DeMarzo 2020, p. 1098). For the converted FCF in HC follows

$$\widetilde{CF}_t^{\text{HC},F} = \underbrace{\mathbb{E}[\widetilde{CF}_t^{\text{FC}}] \cdot F_t}_{\text{forward transaction}} + \underbrace{(\widetilde{CF}_t^{\text{FC}} - \mathbb{E}[\widetilde{CF}_t^{\text{FC}}]) \cdot \widetilde{S}_t}_{\text{spot transaction}}.$$

Taking expectations yields

$$\mathbb{E}[\widetilde{CF}_t^{\text{HC},F}] = \mathbb{E}[\widetilde{CF}_t^{\text{FC}}] \cdot F_t + \text{Cov}[\widetilde{CF}_t^{\text{FC}}, \widetilde{S}_t]. \quad (19)$$

Since this approach combines spot and forward exchange rates in the conversion of cash flows, we also need a combination of discount rates, using both spot and forward exchange rates, to obtain the correct result at the valuation date (for more details consider the Appendix, Sect. 6.3). This is very complex and therefore, not practical. Consequently, we will not consider the HC-valuation approach with forward exchange rates further.

3.6 Modified HC-valuation Approach with Forward Exchange Rates

By considering the spot transaction in the HC-valuation approach with forward exchange rates, the conversion of the FCF results in a complex combination of spot and forward exchange rates. Consequently, it may be useful to leave out this spot transaction and thus, only consider the forward transaction. We define this adjusted expectation regarding forward exchange rates as

$$\mathbb{E}[\widetilde{CF}_t^{\text{HC},F*}] = \mathbb{E}[\widetilde{CF}_t^{\text{FC}}] \cdot F_t. \quad (20)$$

Compared to Eq. 19, the covariance is neglected. Thus, it is a modified cash flow, which we denote by superscript F^* .

The conversion of the market value from FC to HC following this pattern results in a modified expected market value, that is,

$$\mathbb{E}[\widetilde{V}_t^{u,\text{HC},F*}] = \mathbb{E}[\widetilde{V}_t^{u,\text{FC}}] \cdot F_t. \quad (21)$$

The derivation of a recursive valuation equation is analogous to the modified HC-valuation approach with spot exchange rates (see the Appendix, Sect. 6.2.4 for a derivation). For the modified expected market value of period t follows

$$\mathbb{E}[\widetilde{V}_t^{u,\text{HC},F*}] = \frac{\mathbb{E}[\widetilde{CF}_{t+1}^{\text{HC},F*}] + \mathbb{E}[\widetilde{V}_{t+1}^{u,\text{HC},F*}]}{1 + \rho_{t+1}^{u,\text{HC},F*}}, \quad (22)$$

where

$$\rho_t^{u,\text{HC},F*} = (1 + \rho_t^{u,\text{FC}}) \cdot (1 + \delta_t^F) - 1, \quad (23)$$

and

$$\delta_t^F = \frac{F_t}{F_{t-1}} - 1 = \frac{1 + i_t^{\text{HC}}}{1 + i_t^{\text{FC}}} - 1. \quad (24)$$

The corresponding discount rate for this approach is computed by multiplying the cost of equity in FC by the change in forward exchange rates. This discount rate cannot be derived with the GCAPM in HC. Thus, we do not refer to it as a cost of capital but use the term modified discount rate.

The modified expected growth rate expressed in HC using forward exchange rates follows as

$$\mathbb{E}[\tilde{g}^{\text{HC}, F^*}] = (1 + \mathbb{E}[\tilde{g}^{\text{FC}}]) \cdot (1 + \delta^F) - 1. \quad (25)$$

At the valuation date, the forward exchange rate is equal to the current spot exchange rate, such that Eq. 21 coincides with Eq. 10. In particular, at the valuation date, the market value derived with this approach equals the market values derived with the other approaches. We conclude that to obtain the correct valuation result at the valuation date, the covariance is neglected in the conversion of cash flows, the discount rate and the growth rate.⁹ Consequently, as outlaid above, the discount rate cannot be referred to as a cost of capital but constitutes a modified discount rate.

3.7 Interim Summary of the Valuation of an Unlevered Firm

We started with the FC-valuation approach for the valuation of an unlevered foreign firm, which is well-known from national valuation. Afterwards, we have presented HC-valuation approaches with spot and forward exchange rates. We have distinguished the HC-valuation approaches further whether the correlation between exchange rate risk and business risk of the firm is explicitly considered in the conversion of cash flows. If we do not consider covariances, we call the approach a *modified* approach. For each approach, we derive corresponding discount rates. For the HC-valuation approach with spot exchange rates, the discount rate is based on the domestic capital market. In contrast, the modified HC-valuation approaches require modified discount rates, which use the foreign cost of capital. They are converted to HC by multiplying them by the change in exchange rates. Since the resulting discount rates are not directly based on the GCAPM, we do not refer to them as a “cost of capital”. Furthermore, to apply the HC-valuation approaches in a two-phase model, we convert the growth rate from FC to HC. Depending on the valuation approach, the conversion of the growth rate differs. Overall, if the correlation is omitted in the conversion of cash flows, it also has to be neglected in the computation of discount rates and growth rates.

We highlight that future market values derived by different HC-valuation approaches do not coincide: The correct future market value can only be derived with

⁹ The valuation equation and the formula for the growth rate coincide with Schüler (2021, Eqs. 39, 42). For a further comparison, see Footnote 8.

the HC-valuation approach with spot exchange rates. The application of a modified HC-valuation approach results in modified market values, see Eqs. 15 and 21. Furthermore, with the FC-valuation approach, it is not intended to derive future market values in HC. However, at the valuation date, the current spot exchange rate is deterministic such that it is equal to the current forward exchange rate, and the covariance of the current market value and current spot exchange rate amounts to zero. Thus, the market value computed with the FC-valuation approach coincides for $t = 0$ with the market value of the HC-valuation approaches, see Eqs. 10, 15 and 21. Consequently, at the valuation date, all valuation approaches yield the same result.

We conclude that for the valuation of an unlevered firm, all approaches can be applied to obtain the correct market value at the valuation date, but it is essential to use consistent assumptions. Since the HC-valuation approach with forward exchange rates was impractical, we did not consider it further. In the following, we transfer our results on the HC-valuation approach with spot exchange rates and the modified HC-valuation approaches to the valuation of a levered firm.

4 Valuation of a Levered Firm

4.1 Assumptions and Overview of Financing Policies

Since interest on debt is deductible from corporate income, debt financing has a direct impact on the market value of a firm. However, it also adds another risk component. In addition to business risk and exchange rate risk that we considered in the valuation of an unlevered firm, financial risk has to be taken into account (Copeland et al. 2014, p. 535, Schüler 2021, Section 4). To derive valuation equations on a clear theoretical basis and focus on the effects of foreign currency debt, we do not include the costs of financial distress and possibility of default, which implies that debt is risk-free. This assumption is common in other fundamental analyses (see, for example, Miles and Ezzell 1985) and can be relaxed by using the cost of debt instead of the risk-free interest rate (Arnold et al. 2018; Brealey et al. 2020; Clubb and Doran 1995; Inselbag and Kaufold 1997).

We continue to value a foreign company from the view of a domestic investor. This foreign company issues debt in FC. When passive debt management is used, the amount of debt D_t^{FC} is defined deterministically at the valuation date for all periods $t \in \mathbb{N}$ (Modigliani and Miller 1958, 1963). When active debt management is assumed, the debt-to-market value ratio θ_t is defined deterministically at the valuation date.¹⁰ It follows that the amount of debt $\tilde{D}_t^{\text{FC}} = \theta_t \cdot \tilde{V}_t^{\ell, \text{FC}}$ is a random variable (Harris and Pringle 1985; Miles and Ezzell 1980, 1985), where $\tilde{V}_t^{\ell, \text{FC}}$ denotes the market value of the levered company. Note that if the levered firm is considered,

¹⁰ The expressions “active” and “passive” debt management are for example also used in Appleyard and Strong (1989) and Clubb and Doran (1991). Miles and Ezzell (1980) also use the term “active rebalancing by the firm”. However, active and passive debt management can also be referred to as “financing based on market values” and “autonomous financing”, respectively (see, for example, Kruschwitz and Löffler 2020).

we denote it by superscript ℓ . As before, we also label in the exponent whether a quantity is expressed in FC or HC and how it is converted. The conversions follow the same pattern as in Section 3. We consider these two financing strategies and give an overview of important relationships, starting with active debt management.

4.2 Valuation Approaches for Active Debt Management

In this section, we assume active debt management according to Miles and Ezzell (Miles and Ezzell 1980, 1985). Thus, tax shields in FC are certain in the period of their occurrence but uncertain from the view of all previous periods. For national valuation, Miles and Ezzell (1980) have shown that the unlevered cost of equity is the appropriate discount rate for the uncertain tax shields (for DCF methods with active debt management in national valuation, see, for example, Kruschwitz and Löffler 2020, pp. 96–114). We transfer their results to the valuation of a foreign company. We do not explicitly state formulas for the application of a two-phase model since the results from Section 3 still hold. In particular, the derived formulas for growth rates, see Eqs. 13, 18 and 25, in conjunction with the Gordon-Shapiro formula, can also be applied for a levered firm with active debt management.

For active debt management the FCF method can be applied circularity-free. It uses the weighted average cost of capital (WACC). For the FC-valuation approach, the WACC can be computed as

$$\rho_t^{\tau, \text{FC}} = \rho_t^{\ell, \text{FC}} \cdot (1 - \theta_{t-1}) + i_t^{\text{FC}} \cdot (1 - \tau^{\text{FC}}) \cdot \theta_{t-1}, \quad (26)$$

where τ^{FC} is the foreign corporate tax rate and $\rho^{\ell, \text{FC}}$ denotes the levered cost of equity for active debt management in FC. The levered cost of equity can be derived by applying the GCAPM in FC with the levered beta. Alternatively, the levered cost of equity can be derived by applying the adjustment formula for active debt management to the unlevered cost of equity (Miles and Ezzell 1985) as

$$\rho_t^{\ell, \text{FC}} = \rho_t^{u, \text{FC}} + (\rho_t^{u, \text{FC}} - i_t^{\text{FC}}) \cdot \left(1 - \frac{\tau^{\text{FC}} \cdot i_t^{\text{FC}}}{1 + i_t^{\text{FC}}}\right) \cdot L_{t-1}, \quad (27)$$

where $L_{t-1} = \frac{\theta_{t-1}}{1 - \theta_{t-1}}$ denotes the leverage at time $t - 1$. The adjustment formula considers that tax shields are certain in the period of their occurrence and uncertain from the view of all previous periods. Using the levered cost of equity, the equity value can be alternatively derived with the FtE method. The FtE depicts the payments to the shareholders of a levered firm. Since the computation of the FtE requires information about future debt levels, a circularity problem arises, which can be solved by using a spreadsheet software. This is well-known from national valuation.

The conversion of quantities to HC follows mostly the same pattern as before. In particular, the conversion of the market value of the levered firm for the HC-valuation approach with spot exchange rates and the modified HC-valuation approaches follows the same pattern as the conversion of the market value of the unlevered firm. For a detailed derivation of the conversion of the levered cost of equity, see the Appendix (Sect. 6.4.1). For the FCF method, the corresponding WACC for the

three HC-valuation approaches needs to be specified. For the modified HC-valuation approaches, the relation between the WACC in FC and in HC is the same as for the unlevered cost of equity. For the HC-valuation approach with spot exchange rates the relation slightly differs. We derive

$$\rho_t^{\tau, \text{HC}, S} = (1 + \rho_t^{\tau, \text{FC}}) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) + (1 - \theta_{t-1}) \cdot \text{Cov}[\tilde{r}_t^{\ell, \text{FC}}, \tilde{\delta}_t^S] - 1. \quad (28)$$

For a detailed computation of Eq. 28 see the Appendix (Sect. 6.4.2). While the first term remains the same as before, the covariance enters only with the factor $1 - \theta_{t-1}$. Since the risk-free interest rate is deterministic, the corresponding covariance amounts to zero.

Inserting the formula for the WACC in FC into the WACCs in HC, respectively, yields

$$\rho_t^{\tau, \text{HC}, j} = \rho_t^{\ell, \text{HC}, j} \cdot (1 - \theta_{t-1}) + \eta_t^{\tau, \text{HC}, j} \cdot \theta_{t-1}, \quad (29)$$

where $j \in \{S, S^*, F^*\}$ and

$$\eta_t^{\tau, \text{HC}, S} = \eta_t^{\tau, \text{HC}, S^*} = (1 + i_t^{\text{FC}} \cdot (1 - \tau^{\text{FC}})) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) - 1 \quad (30)$$

$$\eta_t^{\tau, \text{HC}, F^*} = (1 + i_t^{\text{FC}} \cdot (1 - \tau^{\text{FC}})) \cdot (1 + \delta_t^F) - 1, \quad (31)$$

see the Appendix (Sect. 6.4.3). The structure of the WACC, see Eq. 29, does not change. It is the sum of the levered cost of equity, which is weighted by the equity-to-market value ratio, and the cost of capital $\eta_t^{\tau, \text{HC}}$, which is weighted by the debt-to-market value ratio. The deductibility of interest on debt is considered in $\eta_t^{\tau, \text{HC}}$: Since the tax shield appears in FC, the risk-free interest rate in FC is multiplied by the factor $1 - \tau$ and is converted to HC only afterwards. The risk-free interest rate in FC is deterministic such that the computation of $\eta_t^{\tau, \text{HC}}$ does not involve a covariance. In particular, $\eta_t^{\tau, \text{HC}, S}$ and $\eta_t^{\tau, \text{HC}, S^*}$ coincide, see Eq. 30. Note, that the foreign debt-to-market value ratio θ is the same from an HC-perspective such that it is used in the computation of the WACC in HC, see the Appendix (Sect. 6.4.3).

The levered cost of equity for the HC-valuation approach with spot exchange rates, that is $\rho_t^{\ell, \text{HC}, S}$, can be derived from the GCAPM in HC. Compared with the derivation of the unlevered cost of equity, the levered beta instead of the unlevered beta is used. Again, the modified HC-valuation approaches are not based on the domestic capital market such that the GCAPM in HC cannot be applied. Nevertheless, adjustment formulas can be used. In the Appendix (Sect. 6.4.4), for the (modified) HC-valuation approach with spot exchange rates, we obtain,

$$\rho_t^{\ell, \text{HC}, j} = \rho_t^{\mu, \text{HC}, j} + \left(\rho_t^{\mu, \text{HC}, j} - \eta_t^{\text{HC}, S} \right) \cdot \left(1 - \frac{\tau^{\text{FC}} \cdot i_t^{\text{FC}}}{1 + i_t^{\text{FC}}} \right) \cdot L_{t-1}, \quad (32)$$

where $j \in \{S, S^*\}$ and

$$\eta_t^{\text{HC}, S} = (1 + i_t^{\text{FC}}) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) - 1; \quad (33)$$

and for the modified HC-valuation approach with forward exchange rates

$$\rho_t^{\ell, \text{HC}, F^*} = \rho_t^{u, \text{HC}, F^*} + (\rho_t^{u, \text{HC}, F^*} - i_t^{\text{HC}}) \cdot \left(1 - \frac{\tau \cdot i_t^{\text{FC}}}{1 + i_t^{\text{FC}}}\right) \cdot L_{t-1} . \quad (34)$$

The overall structure of the adjustment formulas remains unchanged: To the unlevered cost of equity in HC, a risk premium is added. If spot exchange rates are used, see Eq. 32, the risk premium depends on the difference of the unlevered cost of equity and $\eta^{\text{HC}, S}$. In the computation of $\eta^{\text{HC}, S}$, the risk-free interest rate in FC is multiplied by the change in spot exchange rates. We subtract $\eta^{\text{HC}, S}$ instead of the risk-free interest rate in HC to consider the exchange rate risk of the tax shields. Compared with $\eta^{\tau, \text{HC}, S}$, which was used in the computation of the WACC, the tax shield is not considered. Future forward exchange rates are certain such that the corresponding tax shields in HC do not contain exchange rate risk and the risk-free interest rate is used in Eq. 34. The third term of the adjustment formulas coincides for all approaches and considers that tax shields are certain in the period of their occurrence such that they can be discounted at the risk-free interest rate for one period. The involved quantities are expressed in FC since the tax shield appears in FC. At last, the expression is multiplied by the leverage ratio. The formulas for unlevering and relevering beta follow accordingly. Using these adjustment formulas, also the FtE method can be applied. As for national valuation, all DCF methods are equivalent and at the valuation date, the FC-valuation approach and all HC-valuation approaches coincide.

4.3 Valuation Approaches for Passive Debt Management

In this section, we assume passive debt management (for DCF methods with passive debt management in national valuation, see, for example, Kruschwitz and Löffler 2020, pp. 90–95). Thus, all tax shields in FC are certain. We state the APV method since it can be applied without circularity problems. It uses the value-additivity principle by computing the value of the unlevered firm and the value of the tax shield separately. Valuation equations for the unlevered firm are derived in Section 3. It remains to analyze the value of the tax shield in FC and HC, respectively. Afterwards, we shortly discuss the FtE method for passive debt management.

Since future debt levels are certain under the assumption of passive debt management, the tax shield in FC of some period t , $TS_t^{\text{FC}} = \tau^{\text{FC}} \cdot i_t^{\text{FC}} \cdot D_{t-1}^{\text{FC}}$, is certain. Discounting future tax shields at the risk-free interest rate yields the value of the tax shield $V_t^{TS, \text{FC}}$.

To derive valuation equations for the HC-valuation approaches, the conversions follow the same pattern as before. Since the tax shield is a deterministic quantity under the assumption of passive debt management, covariances amount to zero. In particular, both approaches using spot exchange rates coincide. We obtain

$$\begin{aligned} \mathbb{E}[\widetilde{TS}_t^{\text{HC}, S}] &= \mathbb{E}[\widetilde{TS}_t^{\text{HC}, S^*}] = TS_t^{\text{FC}} \cdot \mathbb{E}[\widetilde{S}_t] \\ TS_t^{\text{HC}, F^*} &= TS_t^{\text{FC}} \cdot F_t . \end{aligned}$$

The tax shields converted with spot exchange rates are not risk-free anymore since they contain exchange rate risk. Thus, they are discounted by $\eta_t^{\text{HC},S}$, which is defined as in Section 4.2. Since forward exchange rates are deterministic, the tax shields converted with forward exchange rates are certain in HC such that the risk-free interest rate in HC is the correct discount rate. As before, the value of the tax shield of period $t \neq 0$ differs between the approaches but coincides at the valuation date.

For passive debt management, in the steady state phase, the debt levels grow at expected rates instead of growing state-specific. Thus, for the HC-valuation approach with spot exchange rates, the expected growth rate amounts to

$$\mathbb{E}[\tilde{g}^{\text{HC},S*}] = (1 + E[\tilde{g}^{\text{FC}}]) \cdot (1 + \mathbb{E}[\tilde{\delta}^S]) - 1, \quad (35)$$

where compared with before, the covariance disappears. In line with the calculation of the tax shield, the calculation of the growth rate coincides with the growth rate of the modified HC-valuation approach with spot exchange rates. Thus, the Gordon-Shapiro formula can be applied to calculate the value of the tax shield for passive debt management, but for the HC-valuation approach with spot exchange rates, the growth rate differs from the growth rate of the FCF.

To derive the equity value with the FtE method, the levered cost of equity is again necessary. However, the GCAPM cannot be applied to derive this discount rate. For passive debt management, the distribution of the return of the levered firm, $\tilde{r}_t^{\ell,\text{FC}}$, at some time t depends on the occurred states before this time, since financing risk differs between the states of each period. In particular, the expectation is dependent on the available information at some time $\theta < t$. This is not consistent with our assumptions of the multi-period GCAPM such that the levered discount rate does not constitute a cost of capital (see also Kruschwitz and Löffler 2020, pp. 107–109 on this characteristic in national valuation). Nevertheless, also for passive debt management, it is possible and widely used to derive a suitable discount rate $\rho^{\ell,\text{FC}}$ to obtain a consistent valuation equation for the FtE method. Similar to the procedure in the modified HC-valuation approaches, it can be derived by applying the adjustment formula for passive debt management to the unlevered cost of equity (Inselbag and Kaufold 1997, Eq. 7).

In the derivation of valuation equations for the HC-valuation approach with spot exchange rates, we always use that the covariance of the return of the firm and the change in spot exchange rates does not depend on future information. Since for passive debt management, financing risk differs between the states of some period, this assumption is not fulfilled. It follows that the shown relations no longer hold and the levered cost of equity cannot be derived. In particular, it is not possible to apply the adjustment formula in a similar form as for active debt management. Note also that in the steady state phase, the expected FtE does not increase at a constant growth rate. Since the value of the tax shield increases at a different growth rate than the FCF, the growth of the FtE is not constant (see Dierkes and Schäfer 2021, for how to incorporate different growth rates into terminal value calculation). Due to the dependency of covariances on future information, we do not further consider the FtE method for the HC-valuation approach with spot exchange rates. For the

modified HC-valuation approaches, the conversion of the Flow to Equity and the levered cost of equity follow the same pattern as before.

5 Implications for International Valuation in Theory and Practice

We have presented a consistent framework for valuing a foreign firm using the FC-valuation approach, the HC-valuation approach with spot exchange rates, and the modified HC-valuation approach with spot and forward exchange rates, respectively. A summary of our findings is presented in Fig. 1. We started with an unlevered firm and showed that the FC-valuation approach and the different HC-valuation approaches yield the same result at the valuation date (labeled as “y” for “yes” in Fig. 1). This also holds for the valuation of a levered firm with either active or passive debt management. Thereby, we confirm existing results on the irrelevance of the chosen valuation approach. However, it has not been pointed out before that future market values do not coincide: The correct future market values can only be obtained with the HC-valuation approach with spot exchange rates. The modified HC-valuation approaches yield modified future market values (labeled as “n” for “no” in Fig. 1). Furthermore, in the FC-valuation approach, it is not intended to determine future market values in HC (gray space in Fig. 1). If only the market value at the valuation date is of interest, all approaches are theoretically equivalent.

As the first component of our goal of deriving a consistent framework for DCF valuation with HC-valuation approaches, we identified the derivation of consistent discount rates. We show that the application of the GCAPM in HC is only possible for the HC-valuation approach with spot exchange rates, regardless of whether an unlevered or a levered firm is considered. In the FC-valuation approach, the corresponding discount rate is based on the foreign capital market. This also holds for the modified HC-valuation approaches: Since the modified discount rates are adjusted to the neglect of covariances in the conversion of cash flows, they are computed

Characteristic	unlevered firm				active debt man.				passive debt man.			
	FC	HC <i>S</i>	HC <i>S*</i>	HC <i>F*</i>	FC	HC <i>S</i>	HC <i>S*</i>	HC <i>F*</i>	FC	HC <i>S</i>	HC <i>S*</i>	HC <i>F*</i>
valuation at $t = 0$	y	y	y	y	y	y	y	y	y	y	y	y
valuation at $t > 0$		y	n	n		y	n	n		y	n	n
cost of capital based on domestic market		y	n	n		y	n	n		n	n	n
neglect of covariance	y	n	y	y	y	n	y	y	y	n	y	y
constant growth of the FCF	y	y	y	y	y	y	y	y	y	y	y	y
neglect of covariance in TS					y	n	y	y	y	y	y	y
constant growth of VTS					y	y	y	y	y	y	y	y
constant growth of the FTE					y	y	y	y	y	n	y	y

Fig. 1 Characteristics of different valuation approaches, where FC denotes the FC-valuation approach; *S* denotes the HC-valuation approach with spot exchange rates; *S** denotes the modified HC-valuation approach with spot exchange rates; and *F** denotes the modified HC-valuation approach with forward exchange rates; “y” (“n”) means that a certain characteristic is (not) met, and a gray space means that a certain characteristic does not apply

using the cost of equity in FC. In particular, they cannot be directly derived from the GCAPM, such that we do not refer to them as a “cost of capital”. Overall, the analysis of the foreign capital market can be avoided only by applying the HC-valuation approach with spot exchange rates.

The second component was the correct integration of a correlation between exchange rate risk and the firm’s business risk. We demonstrate that this correlation is explicitly considered in the HC-valuation approach with spot exchange rates by adding a covariance term in the conversion of cash flows, debt levels, costs of capital, and growth rates. It is possible to neglect the covariance in the conversion of cash flows, but then it also has to be neglected in the computation of all other quantities to obtain the correct valuation result at the valuation date. This results in a modified valuation approach.

The third component was the incorporation of the valuation approaches into a two-phase model. For each approach, we presented formulas for computing growth rates, such that for an unlevered firm, the Gordon-Shapiro formula can be applied to each HC-valuation approach.

The fourth component was the integration of debt financing, distinguishing between active and passive debt management. For active debt management, we demonstrate that the overall structure of adjustment formulas remains unchanged, but adaptations are required to consider exchange rate risk. Furthermore, the computed growth rates of the unlevered firm can be transferred to the valuation of a levered firm. The debt levels, and therefore the value of the tax shield and the FtE, grow at the same rate as the FCF. As for national valuation, only the FCF method can be applied circularity-free, such that its application is the most convenient choice, but it is also possible to apply another method since all DCF methods are equivalent.

For passive debt management, the multi-period GCAPM cannot be applied to derive the levered cost of equity, because the expected return depends on available information and is therefore not deterministic at the valuation date. As a consequence, the discount rate in the HC-valuation approach with spot exchange rates cannot be deduced in the same way as for active debt management. Moreover, in this approach, the growth rate of the tax shield value differs from that of the FCF, resulting in an inconstant growth of the FtE. The APV method can be applied circularity-free, making it the best choice for passive debt management.

The fifth component was deducing practical implications. For this end, we need to distinguish between different occasions of cross-border valuation. We chose to analyze the valuation of a foreign company without debt in domestic currency and other interdependencies. This was necessary since even for this simple setting, the DCF framework of the HC-valuation approaches has not yet been clear. Although, the FC-valuation approach is usually the simplest choice for this occasion, our consistent implementation of HC-valuation approaches serves as an important basis.

If a company generates cash flows in both FC and HC and also issues debt in both FC and HC, the valuation becomes more extensive, but our results can be transferred (for characteristics on combining foreign and domestic debt, see Bekaert and Hodrick 2018, p 742; Schüler 2021, Section 5). If the HC-valuation approach with spot exchange rates is applied, all quantities and possible interdependencies can be expressed in HC. Furthermore, the required cost of capital can be determined on

the basis of the domestic capital market. If more countries are considered, the benefits of this approach increase further: It is still only necessary to analyze the domestic capital market. However, this approach has the disadvantage that converting the FCF from FC to HC requires information about the covariances $\text{Cov}[\widehat{CF}_t^{\text{FC}}, \widehat{S}_t]$. Thus far, no standardized approach exists to estimate these covariances. Nevertheless, depending on the extent of foreign operations and the sensitivity to exchange rate fluctuations, it is conceivable to start by estimating the correlation as a standardized measure in the interval $[-1; 1]$. Multiplying the correlation by the estimated standard deviations of the FCF and the spot exchange rate yields the covariance.

Other covariances are only theoretical quantities, so their estimation is not necessary for practical use: The covariance of the market value of the firm and the spot exchange rate, $\text{Cov}[\widehat{V}_t^{\text{FC}}, \widehat{S}_t]$, is redundant since the market value in HC is directly calculated via a recursive procedure. The growth rate of the FCF can be derived implicitly by estimating the growth rate of the converted FCF in HC, so that the estimation of the covariance of the growth rate in FC and the change in spot exchange rates, $\text{Cov}[\tilde{g}_t^{\text{FC}}, \tilde{\delta}_t^S]$, is also dispensable. Furthermore, in the case of determining the cost of capital with the GCAPM in HC, the estimation of the covariance of the return of the firm and the change in spot exchange rates, $\text{Cov}[\tilde{r}_t^{\text{FC}}, \tilde{\delta}_t^S]$ is not required. Consequently, only the estimation of the covariance of the foreign FCF and the spot exchange rate is necessary. This holds for an unlevered firm as well as for a levered firm with active or passive debt management.

In the literature of cross-border valuation, covariances are often neglected for practical reasons when converting cash flows. This suggests the application of a modified HC-valuation approach, see Fig. 1. If a modified HC-valuation approach is applied, one has to choose between forward and spot exchange rates. Whereas spot exchange rates have to be forecasted (for an overview of exchange rate forecasting, see, for example, Bekaert and Hodrick 2018, Chapter 10; the discussion is also ongoing in Germany, see, for example, Breuer et al. 2021, p. 1963–1964), forward exchange rates are known and therefore, easier to implement. Indeed, the literature often focuses on the approaches with forward exchange rates. Consequently, when choosing among the modified HC-valuation approaches, the modified HC-valuation approach with forward exchange rates can be more convenient. If either of the modified HC-valuation approaches is used for a more extensive cross-border valuation, all quantities are converted to HC. However, all involved foreign capital markets have to be analyzed to obtain the corresponding modified discount rates for discounting the converted cash flows. Furthermore, for quantities that already occur in HC, a discount rate based on the domestic capital market has to be used, thereby requiring an analysis of the domestic capital market. If the uncovered interest parity holds true (for a discussion of the uncovered interest parity, see, for example, Sercu 2009, pp. 430–431), expected spot exchange rates equal forward exchange rates such that both modified valuation approaches are equivalent (see also Schüller 2021). Alternatively, it is also possible to use the FC-valuation approach for each country individually and add up the resulting values; however, even with this approach, one must analyze all foreign capital markets. Additionally, for both the modified HC-valuation approaches and the FC-valuation approach, it is unclear how interdependencies are considered.

The use of a cost of capital derived from the domestic capital market in connection with a modified cash flow leads to deviating valuation results. O'Brien (2022) used a constant perpetual growth model with a cost of capital based on the ICAPM for his analysis. The author computed the deviation of market values that were deduced under different assumptions. In particular, the author compared a consistent application of an approach with spot exchange rates to an inconsistent approach in which covariances are neglected in the conversion of cash flows, but a domestic discount rate is used. O'Brien shows that the deviation can be substantial (O'Brien 2022). Similar results can be expected for the application of the GCPAM. If forward exchange rates are used in connection with a domestic discount rate, we expect similar results. Both cases can be subject for future research. Thereby, it is of particular interest whether modified cash flows computed with spot or forward exchange rates in combination with a domestic discount rate yield smaller deviations, and which input parameters have the greatest impact. It is conceivable that the deviations are small for companies with only few cross-border relations. This would make such a mixed approach legitimate and useful. For big multinational companies, more substantial deviations might occur. Therefore, we recommend to use the HC-valuation approach with spot exchange rates for more extensive cross-border valuations. The decisive advantage is that only the domestic capital market has to be analyzed to derive costs of capital. This is in line with Koller et al. (2020), who recommend a domestic cost of capital in most cases of a valuation of an international company. However, further research is needed to develop a standardized method for estimating covariances and expected spot exchange rates.

Overall, to obtain the correct results at the valuation date, one needs to maintain consistent assumptions throughout the valuation process. Depending on the business model of the firm, the appropriate approach should be selected. Other problems, such as the integration of country risks, legal requirements, differences in international accounting standards, international taxation, and interdependencies between subsidiaries regarding services, supplies, and financing, are not considered in this study. However, for these problems, we recommend applying the HC-valuation approach with spot exchange rates. All quantities are then available in HC, and adequate adjustments can be made. A more detailed elaboration is subject to further research. Overall, we have developed a consistent theoretical framework for international valuation that can be used as a basis for further research.

6 Appendix

6.1 Elaborations on the Change in Exchange Rates

We define

$$\widetilde{M}_t := \frac{\widetilde{S}_t}{\mathbb{E}[\widetilde{S}_t]}.$$

In the following, we show that Eq. 5 is equivalent to the process $(\widetilde{M}_t)_{t \in \mathbb{N}}$ being a martingale: Let $(\mathcal{G}_t)_{t \in \mathbb{N}}$ be the natural filtration of $(\widetilde{S}_t)_{t \in \mathbb{N}}$. Thus, $(\widetilde{M}_t)_{t \in \mathbb{N}}$ is also adapted to this filtration. First, we show that Eq. 5 implies that the expected change in spot exchange rates is equal to the change of expected spot exchange rates: For $t \in \mathbb{N}$, we obtain

$$\widetilde{S}_{t-1} = \widetilde{S}_{t-1} \cdot \frac{\mathbb{E}_{t-1}[\widetilde{S}_t]}{\mathbb{E}_{t-1}[\widetilde{S}_t]} = \frac{\mathbb{E}_{t-1}[\widetilde{S}_t]}{\mathbb{E}_{t-1}\left[\frac{\widetilde{S}_t}{\widetilde{S}_{t-1}}\right]} = \frac{\mathbb{E}_{t-1}[\widetilde{S}_t]}{\mathbb{E}\left[\frac{\widetilde{S}_t}{\widetilde{S}_{t-1}}\right]},$$

where we used Eq. 5 in the last step. Taking expectations yields

$$\mathbb{E}[\widetilde{S}_{t-1}] = \frac{\mathbb{E}[\widetilde{S}_t]}{\mathbb{E}\left[\frac{\widetilde{S}_t}{\widetilde{S}_{t-1}}\right]} \Leftrightarrow \mathbb{E}\left[\frac{\widetilde{S}_t}{\widetilde{S}_{t-1}}\right] = \frac{\mathbb{E}[\widetilde{S}_t]}{\mathbb{E}[\widetilde{S}_{t-1}]}. \quad (36)$$

Now, we can show that Eq. 5 implies that $(\widetilde{M}_t)_{t \in \mathbb{N}}$ is a martingale. We have

$$\mathbb{E}_{t-1}[\widetilde{M}_t] = \mathbb{E}_{t-1}\left[\frac{\widetilde{S}_t}{\mathbb{E}[\widetilde{S}_t]}\right] = \frac{\widetilde{S}_{t-1}}{\mathbb{E}[\widetilde{S}_t]} \cdot \mathbb{E}_{t-1}\left[\frac{\widetilde{S}_t}{\widetilde{S}_{t-1}}\right] = \frac{\widetilde{S}_{t-1}}{\mathbb{E}[\widetilde{S}_t]} \cdot \frac{\mathbb{E}[\widetilde{S}_t]}{\mathbb{E}[\widetilde{S}_{t-1}]} = \widetilde{M}_{t-1}, \quad (37)$$

where we used Eqs. 5 and 36 in the third equation. It remains to show that $(\widetilde{M}_t)_{t \in \mathbb{N}}$ being a martingale implies Eq. 5. By dividing Eq. 37 on both sides by $\widetilde{S}_{t-1}$ and taking expectations, Eq. 36 follows. We use this to show

$$\mathbb{E}_{t-1}[\widetilde{\delta}_t] = \frac{\mathbb{E}[\widetilde{S}_t]}{\widetilde{S}_{t-1}} \cdot \mathbb{E}_{t-1}[\widetilde{M}_t] = \frac{\mathbb{E}[\widetilde{S}_t]}{\widetilde{S}_{t-1}} \cdot \widetilde{M}_{t-1} = \frac{\mathbb{E}[\widetilde{S}_t]}{\mathbb{E}[\widetilde{S}_{t-1}]} = \mathbb{E}[\widetilde{\delta}_t].$$

With the law of total expectations, Eq. 5, and therefore the claim, follows.

As we assume PPP to hold, the expected change in spot exchange rates equals the expected change in inflation rates (Brealey et al. 2020, pp. 722–723; Koller et al. 2020, p. 828). It follows that by defining the process $(\widetilde{M}_t)_{t \in \mathbb{N}}$, we adjust the spot exchange rate for inflation effects. Since the resulting process is a martingale, Eq. 5 can be understood as the assumption that the process of the inflation-adjusted spot exchange rates $(\widetilde{M}_t)_{t \in \mathbb{N}}$ represents a fair betting game in the sense that for any given value $\widetilde{M}_{t-1}$, the inflation adjusted exchange rate on average does not change from period $t - 1$ to t . In other words, the development of the conditional expected values of the process $(\widetilde{M}_t)_{t \in \mathbb{N}}$ follows the rules of rational expectation formation. This means that all the information available in some period t is correctly evaluated when forming these conditional expected values.

6.2 Derivation of Equations for the Valuation of an Unlevered Firm

6.2.1 Derivation of Eq. 12

The return of the unlevered firm is defined as

$$\tilde{r}_{t+1}^{u,FC} = \frac{\widetilde{CF}_{t+1}^{FC} + \widetilde{V}_{t+1}^{u,FC}}{\widetilde{V}_t^u} - 1$$

Note that

$$\text{Corr}[\tilde{r}_{t+1}^{u,FC}, \widetilde{CF}_{t+1}^{FC}] = 1.$$

This correlation of 1 between the return of the unlevered firm and the FCF always holds true in our setting since it is a direct consequence of the characteristics of the GCAPM. It follows

$$\text{Cov}_t[\tilde{r}_{t+1}^{u,FC}, \tilde{\delta}_{t+1}^S] = \text{Cov}_t[\widetilde{CF}_{t+1}^{FC} + \widetilde{V}_{t+1}^{u,FC}, \tilde{S}_{t+1}] \cdot \frac{1}{\widetilde{V}_t^{u,FC} \cdot \tilde{S}_t}. \quad (38)$$

The market value that is computed using the HC-valuation approach with spot exchange rates needs to coincide with the market value that is computed using the FC-valuation approach and multiplied by the spot exchange rate. We obtain for the market value in HC of some period t

$$\widetilde{V}_t^{u,FC} \cdot \tilde{S}_t \stackrel{!}{=} \frac{\mathbb{E}_t[\widetilde{CF}_{t+1}^{FC}] \cdot \mathbb{E}_t[\tilde{S}_{t+1}] + \text{Cov}_t[\widetilde{CF}_{t+1}^{FC}, \tilde{S}_{t+1}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{u,HC,S}]}{1 + \rho_{t+1}^{u,HC,S}}.$$

Using

$$\mathbb{E}_t[\widetilde{V}_{t+1}^{u,HC,S}] = \mathbb{E}_t[\widetilde{V}_{t+1}^{u,FC}] \cdot \mathbb{E}_t[\tilde{S}_{t+1}] + \text{Cov}_t[\widetilde{V}_{t+1}^{u,FC}, \tilde{S}_{t+1}]$$

and solving for $1 + \rho_{t+1}^{u,HC,S}$ yields

$$\begin{aligned} 1 + \rho_{t+1}^{u,HC,S} &= \frac{(\mathbb{E}_t[\widetilde{CF}_{t+1}^{FC}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{u,FC}]) \cdot \mathbb{E}_t[\tilde{S}_{t+1}]}{\widetilde{V}_t^{u,FC} \cdot \tilde{S}_t} \\ &\quad + \frac{\text{Cov}_t[\widetilde{CF}_{t+1}^{FC}, \tilde{S}_{t+1}] + \text{Cov}_t[\widetilde{V}_{t+1}^{u,FC}, \tilde{S}_{t+1}]}{\widetilde{V}_t^{u,FC} \cdot \tilde{S}_t} \\ &= (1 + \rho_{t+1}^{u,FC}) \cdot \frac{\mathbb{E}_t[\tilde{S}_{t+1}]}{\tilde{S}_t} + \text{Cov}_t \left[\frac{\widetilde{CF}_{t+1}^{FC} + \widetilde{V}_{t+1}^{u,FC}}{\widetilde{V}_t^{u,FC}}, \frac{\tilde{S}_{t+1}}{\tilde{S}_t} \right] \\ &= (1 + \rho_{t+1}^{u,FC}) \cdot (1 + \mathbb{E}[\tilde{\delta}_{t+1}^S]) + \text{Cov}[\tilde{r}_{t+1}^{u,FC}, \tilde{\delta}_{t+1}^S]. \end{aligned}$$

In the second step we used Eq. 38. In the last step, we used that the return of the unlevered firm $\tilde{r}_{t+1}^{u,HC}$ and the change of spot exchange rates $\tilde{\delta}_{t+1}^S$ is independent of the information available at time t , see Section 2. In particular, the cost of capital in HC is deterministic.

6.2.2 Derivation of Eq. 13

The change of the expected market values from period t to $t + 1$ conditioned on information available at time t is computed as

$$\begin{aligned} 1 + \mathbb{E}_t[\tilde{g}_{t,t+1}^{HC,S}] &= \frac{\mathbb{E}_t[\tilde{V}_{t+1}^{u,HC,S}]}{\tilde{V}_t^{u,HC,S}} \\ &= \frac{\mathbb{E}_t[\tilde{V}_{t+1}^{u,FC}] \cdot \mathbb{E}_t[\tilde{S}_{t+1}] + \text{Cov}_t[\tilde{V}_{t+1}^{u,FC}, \tilde{S}_{t+1}]}{\tilde{V}_t^{u,FC} \cdot \tilde{S}_t} \\ &= (1 + E[\tilde{g}^{FC}]) \cdot (1 + \mathbb{E}_t[\tilde{\delta}_{t+1}^S]) + \text{Cov}_t[\tilde{g}^{FC}, \tilde{\delta}_{t+1}^S], \end{aligned}$$

where we used Eq. 38. Since the change in spot exchange rates and the growth rate in FC does not depend on the available information at time t , the expected growth rate is deterministic and Eq. 13 follows.

6.2.3 Derivation of Eq. 16

Converting the expected market value in FC of period t at the expected spot exchange rate of period t and expanding with $\mathbb{E}[\tilde{S}_{t+1}]$ yields

$$\mathbb{E}[\tilde{V}_t^{u,HC,S^*}] = \mathbb{E}[\tilde{V}_t^{u,FC}] \cdot \mathbb{E}[\tilde{S}_t] = \frac{(\mathbb{E}[\tilde{CF}_{t+1}^{FC}] + \mathbb{E}[\tilde{V}_{t+1}^{u,FC}]) \cdot \mathbb{E}[\tilde{S}_{t+1}]}{(1 + \rho_{t+1}^{u,FC}) \cdot (1 + \mathbb{E}[\tilde{\delta}_{t+1}^S])}.$$

Note, that we used Eq. 36 in the last step. By defining

$$\rho_t^{u,HC,S^*} = (1 + \rho_t^{u,FC}) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) - 1,$$

we obtain

$$\mathbb{E}[\tilde{V}_t^{u,HC,S^*}] = \frac{\mathbb{E}[\tilde{CF}_{t+1}^{HC,S^*}] + \mathbb{E}[\tilde{V}_{t+1}^{u,HC,S^*}]}{1 + \rho_{t+1}^{u,HC,S^*}}.$$

6.2.4 Derivation of Eq. 22

Converting the market value in FC of period t at the forward exchange rate F_t and expanding it with F_{t+1} yields the valuation equation at time t , that is,

$$\mathbb{E}[\widetilde{V}_t^{u,\text{HC},F*}] = \mathbb{E}[\widetilde{V}_t^{u,\text{FC}}] \cdot F_t = \frac{(\mathbb{E}[\widetilde{CF}_{t+1}^{\text{FC}}] + \mathbb{E}[\widetilde{V}_{t+1}^{u,\text{FC}}]) \cdot F_{t+1}}{(1 + \rho_{t+1}^{u,\text{FC}}) \cdot (1 + \delta_{t+1}^F)},$$

where

$$\delta_{t+1}^F = \frac{F_{t+1}}{F_t} - 1 = \frac{1 + i_{t+1}^{\text{HC}}}{1 + i_{t+1}^{\text{FC}}} - 1.$$

By defining

$$\rho_t^{u,\text{HC},F*} = (1 + \rho_t^{u,\text{FC}}) \cdot (1 + \delta_t^F) - 1 = (1 + \rho_t^{u,\text{FC}}) \cdot \frac{1 + i_t^{\text{HC}}}{1 + i_t^{\text{FC}}},$$

we obtain

$$\mathbb{E}[\widetilde{V}_t^{u,\text{HC},F*}] = \frac{\mathbb{E}[\widetilde{CF}_{t+1}^{\text{HC},F*}] + \mathbb{E}[\widetilde{V}_{t+1}^{u,\text{HC},F*}]}{1 + \rho_{t+1}^{u,\text{HC},F*}}.$$

6.3 Derivation of a Valuation Equation for the HC-valuation Approach with Forward Exchange Rates

To obtain a valuation equation for the HC-valuation approach with forward exchange rates, we need to derive the corresponding cost of capital to discount cash flows that are derived according to Eq. 19. To do so, we compare this approach to the market value derived by the FC-valuation approach and multiplied by the spot exchange rate. From the view of period t , we obtain

$$\widetilde{V}_t^{u,\text{FC}} \cdot \widetilde{S}_t \stackrel{!}{=} \frac{\mathbb{E}_t[\widetilde{CF}_{t+1}^{\text{FC}}] \cdot \widetilde{F}_{t,t+1} + \text{Cov}_t[\widetilde{CF}_{t+1}^{\text{FC}}, \widetilde{S}_{t+1}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{u,\text{HC},F}]}{1 + \rho_{t+1}^{u,\text{HC},F}}, \quad (39)$$

with

$$\mathbb{E}_t[\widetilde{V}_{t+1}^{u,\text{HC},F}] = \mathbb{E}_t[\widetilde{V}_{t+1}^{u,\text{FC}}] \cdot \widetilde{F}_{t,t+1} + \text{Cov}_t[\widetilde{V}_{t+1}^{u,\text{FC}}, \widetilde{S}_{t+1}].$$

Solving for $1 + \rho_{t+1}^{u,HC,F}$ yields

$$\begin{aligned} 1 + \rho_{t,t+1}^{u,HC,F} &= \frac{\left(\mathbb{E}_t[\widetilde{CF}_{t+1}^{FC}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{u,FC}]\right) \cdot \widetilde{F}_{t,t+1}}{\widetilde{V}_t^{u,FC} \cdot \widetilde{S}_t} \\ &\quad + \frac{\text{Cov}_t[\widetilde{CF}_{t+1}^{FC}, \widetilde{S}_{t+1}] + \text{Cov}_t[\widetilde{V}_{t+1}^{u,FC}, \widetilde{S}_{t+1}]}{\widetilde{V}_t^{u,FC} \cdot \widetilde{S}_t} \\ &= (1 + \rho_{t+1}^{u,FC}) \cdot \frac{\widetilde{F}_{t,t+1}}{\widetilde{S}_t} + \text{Cov}_t\left[\frac{\widetilde{CF}_{t+1}^{FC} + \widetilde{V}_{t+1}^{u,FC}}{\widetilde{V}_t^{u,FC}}, \frac{\widetilde{S}_{t+1}}{\widetilde{S}_t}\right] \\ &= (1 + \rho_{t+1}^{u,FC}) \cdot \frac{\widetilde{F}_{t,t+1}}{\widetilde{S}_t} + \text{Cov}[\widetilde{r}_{t+1}^{u,FC}, \widetilde{\delta}_{t+1}^S], \end{aligned}$$

where we used Eq. 38 in the second step. Due to our assumptions of the multiperiod GCAPM, the covariance does not depend on the available information at time t . By applying the arbitrage condition, see Eq. 2, we obtain

$$\frac{\widetilde{F}_{t,t+1}}{\widetilde{S}_t} = \frac{\widetilde{S}_t}{\widetilde{S}_t} \cdot \frac{1 + i_t^{HC}}{1 + i_t^{FC}} = \frac{F_{t+1}}{F_t},$$

and conclude

$$\rho_{t+1}^{u,HC,F} = (1 + \rho_{t+1}^{u,FC}) \cdot \frac{F_{t+1}}{F_t} + \text{Cov}\left[\widetilde{r}_{t+1}^{u,FC}, \widetilde{\delta}_{t+1}^S\right] - 1.$$

Note that we use the uncertain forward exchange rate $\widetilde{F}_{t,t+1}$ for the conversion of the cash flow. Note also that we equate the computation of the market value using this approach with forward exchange rates to $\widetilde{V}_t^{u,FC} \cdot \widetilde{S}_t$, see Eq. 39. Consequently, by using $\rho^{u,HC,F}$ as discount rate, we obtain a cash flow that is based on spot exchange rates. Thus, the resulting cash flow has to be discounted using $\rho^{u,HC,S}$.

6.4 Derivation of the Levered Cost of Equity and the WACC for Active Debt Management

6.4.1 Derivation of a Link Between the Levered Cost of Equity in HC and the Levered Cost of Equity in FC

To derive a link between the levered costs of equity in HC, $\rho^{\ell,HC,S}$, and the levered cost of equity in FC, $\rho^{\ell,FC}$, we compare the FC-valuation approach to the HC-valuation approach with spot exchange rates. From the view of period t , we obtain

$$\widetilde{E}_t^{\ell,FC} \cdot \widetilde{S}_t \stackrel{!}{=} \frac{\mathbb{E}_t[\widetilde{F}_t \widetilde{E}_{t+1}^{HC,S}] + \mathbb{E}_t[\widetilde{E}_{t+1}^{\ell,HC,S}]}{1 + \rho_{t+1}^{\ell,HC,S}}.$$

From rearranging follows

$$\begin{aligned}
 1 + \rho_{t+1}^{\ell, \text{HC}, S} &= \frac{\mathbb{E}_t[\widetilde{FtE}_{t+1}^{\text{HC}, S}] + \mathbb{E}_t[\widetilde{E}_{t+1}^{\ell, \text{HC}, S}]}{\widetilde{E}_t^{\ell, \text{FC}} \cdot \widetilde{S}_t} \\
 &= \frac{\left(\mathbb{E}_t[\widetilde{FtE}_{t+1}^{\text{FC}}] + \mathbb{E}_t[\widetilde{E}_{t+1}^{\ell, \text{FC}}]\right) \cdot \mathbb{E}_t[\widetilde{S}_{t+1}]}{\left(\mathbb{E}_t[\widetilde{FtE}_{t+1}^{\text{FC}}] + \mathbb{E}_t[\widetilde{E}_{t+1}^{\ell, \text{FC}}]\right) \cdot \widetilde{S}_t} \cdot (1 + \rho_{t+1}^{\ell, \text{FC}}) \\
 &\quad + \frac{\text{Cov}_t[\widetilde{FtE}_{t+1}^{\text{FC}} + \widetilde{E}_{t+1}^{\ell, \text{FC}}, \widetilde{S}_{t+1}]}{\widetilde{E}_t^{\ell, \text{FC}} \cdot \widetilde{S}_t} \\
 &= (1 + \rho_{t+1}^{\ell, \text{FC}}) \cdot (1 + \mathbb{E}_t[\widetilde{\delta}_{t+1}^S]) + \text{Cov}_t[\widetilde{r}_t^{\ell, \text{FC}}, \widetilde{\delta}_{t+1}^S].
 \end{aligned}$$

In Section 2, we describe that the expectation of the change in spot exchange rates does not depend on the information available at time t . Note that in the case of active debt management, the distribution of the return of the levered firm at some time t also does not depend on the occurred states before time t since financial risk is constant in every state of this period. It follows that the covariance does not depend on the available information at time t , such that the levered cost of equity $\rho_{t+1}^{\ell, \text{HC}, S}$ is deterministic. The relations for the modified HC-valuation approaches follow similarly.

6.4.2 Derivation of Eq. 28

To derive a link between the WACC in HC $\rho^{\tau, \text{HC}, S}$ and the WACC in FC $\rho^{\tau, \text{FC}}$, we apply the same pattern as in Sect. 6.4.1. From the view of period t , we obtain

$$\widetilde{V}_t^{\ell, \text{FC}} \cdot \widetilde{S}_t \stackrel{!}{=} \frac{\mathbb{E}_t[\widetilde{CF}_{t+1}^{\text{HC}, S}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{\ell, \text{HC}, S}]}{1 + \rho_{t+1}^{\tau, \text{HC}, S}}.$$

From rearranging follows

$$\begin{aligned}
 1 + \rho_{t+1}^{\tau, \text{HC}, S} &= \frac{\mathbb{E}_t[\widetilde{CF}_{t+1}^{\text{HC}, S}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{\ell, \text{HC}, S}]}{\widetilde{V}_t^{\ell, \text{FC}} \cdot \widetilde{S}_t} \\
 &= \frac{\left(\mathbb{E}_t[\widetilde{CF}_{t+1}^{\text{FC}}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{\ell, \text{FC}}]\right) \cdot \mathbb{E}_t[\widetilde{S}_{t+1}]}{\left(\mathbb{E}_t[\widetilde{CF}_{t+1}^{\text{FC}}] + \mathbb{E}_t[\widetilde{V}_{t+1}^{\ell, \text{FC}}]\right) \cdot \widetilde{S}_t} \cdot (1 + \rho_{t+1}^{\tau, \text{FC}}) \\
 &\quad + \frac{\text{Cov}_t[\widetilde{CF}_{t+1}^{\text{FC}} + \widetilde{V}_{t+1}^{\ell, \text{FC}}, \widetilde{S}_{t+1}]}{\widetilde{V}_t^{\ell, \text{FC}} \cdot \widetilde{S}_t} \\
 &= (1 + \rho_{t+1}^{\tau, \text{FC}}) \cdot (1 + \mathbb{E}_t[\widetilde{\delta}_{t+1}^S]) + (1 - \theta_t) \cdot \text{Cov}_t[\widetilde{r}_t^{\ell, \text{FC}}, \widetilde{\delta}_{t+1}^S].
 \end{aligned}$$

Note that in the last step we used, that the risk-free interest rate is deterministic, such that its covariance amounts to zero.

6.4.3 Derivation of Eq. 29

Inserting the formula for the WACC in FC, see Eq. 26 into Eq. 28 yields

$$\begin{aligned}\rho_t^{\tau, \text{HC}, S} &= (1 + \rho_t^{\ell, \text{FC}} \cdot (1 - \theta_{t-1}) + i_t^{\text{FC}} \cdot (1 - \tau^{\text{FC}}) \cdot \theta_{t-1}) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) \\ &\quad + (1 - \theta_{t-1}) \cdot \text{Cov}[\tilde{r}_t^{\ell, \text{FC}}, \tilde{\delta}_t^S] - 1 \\ &= \left((1 + \rho_t^{\ell, \text{FC}}) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) + \text{Cov}[\tilde{r}_t^{\ell, \text{FC}}, \tilde{\delta}_t^S] \right) \cdot (1 - \theta_{t-1}) \\ &\quad + \theta_{t-1} \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) + (i_t^{\text{FC}} \cdot (1 - \tau^{\text{FC}}) \cdot \theta_{t-1}) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) - 1.\end{aligned}$$

By inserting the relation between $\rho^{\ell, \text{FC}}$ and $\rho^{\ell, \text{HC}, S}$, we obtain

$$\begin{aligned}\rho^{\tau, \text{HC}, S} &= (1 + \rho^{\ell, \text{HC}, S}) \cdot (1 - \theta_{t-1}) + (1 + i_t^{\text{FC}} \cdot (1 - \tau^{\text{FC}}) \cdot \theta_{t-1}) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) - 1 \\ &= \rho^{\ell, \text{HC}, S} \cdot (1 - \theta_{t-1}) - \theta_{t-1} + (1 + i_t^{\text{FC}} \cdot (1 - \tau^{\text{FC}})) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) \cdot \theta_{t-1} \\ &= \rho^{\ell, \text{HC}, S} \cdot (1 - \theta_{t-1}) + \left((1 + i_t^{\text{FC}} \cdot (1 - \tau^{\text{FC}})) \cdot (1 + \mathbb{E}[\tilde{\delta}_t^S]) - 1 \right) \cdot \theta_{t-1}.\end{aligned}$$

Defining $\eta^{\tau, \text{HC}, S}$ as in Eq. 30 yields Eq. 29 for $j = S$. The modified WACCs $\rho^{\tau, \text{HC}, S^*}$ and $\rho^{\tau, \text{HC}, F^*}$ follow analogously. In the proof, we used that the debt ratios coincide in both currencies. This can be shown as follows. For the HC-valuation approach with spot exchange rates, we obtain

$$\tilde{\theta}_t^{\text{HC}, S} = \frac{\tilde{D}_t^{\text{HC}, S}}{\tilde{V}_t^{\text{HC}, S}} = \frac{\tilde{D}_t^{\text{FC}} \cdot \tilde{S}_t}{\tilde{V}_t^{\text{FC}} \cdot \tilde{S}_t} = \frac{\tilde{D}_t^{\text{FC}}}{\tilde{V}_t^{\text{FC}}} = \theta_t^{\text{FC}} = \theta_t.$$

We start with the debt ratio in HC on the left-hand side, for which we want to show that it is deterministic. In a first step, we insert the definition of the debt-to-market value ratio in HC. We obtain a ratio of stochastic quantities in HC. We do not use expected values here since we start with a stochastic quantity. The debt level and market value in HC are based on the respective quantities in FC, which are multiplied by the spot exchange rate. We do not use covariances here since we still consider stochastic quantities and do not take expectations. The spot exchange rates cancel out, yielding the fraction of the debt level in FC and the levered market value in FC. Both quantities are stochastic. However, this is equal to the debt-to-market value ratio, which is deterministic by definition of active debt management. Since the quantity on the right-hand side is deterministic, the quantity on the left-hand side is also deterministic. Thus, the debt-to-market value ratio in HC is deterministic and coincides with the debt-to-market value ratio in FC. The modified HC-valuation approach with spot exchange rates does not use state-dependent quantities but modified expectations, such that we obtain

$$\theta_t^{\text{HC}, S^*} = \frac{\mathbb{E}[\tilde{D}_t^{\text{HC}, S^*}]}{\mathbb{E}[\tilde{V}_t^{\text{HC}, S^*}]} = \frac{\mathbb{E}[\tilde{D}_t^{\text{FC}}] \cdot \mathbb{E}[\tilde{S}_t]}{\mathbb{E}[\tilde{V}_t^{\text{FC}}] \cdot \mathbb{E}[\tilde{S}_t]} = \frac{\mathbb{E}[\tilde{D}_t^{\text{FC}}]}{\mathbb{E}[\tilde{V}_t^{\text{FC}}]} = \theta_t^{\text{FC}} = \theta_t.$$

Again, the debt-to-market value ratio in HC coincides with the specified ratio θ . The modified HC-valuation approach with forward exchange rates follows analogously.

6.4.4 Derivation of Eq. 32

In the following, we show a detailed computation of Eq. 32 for $j = S$. Note that for the return of the levered firm in FC, the same adjustment formula holds as for the levered cost of equity, that is,

$$\tilde{r}_{t+1}^{\ell, \text{FC}} = \tilde{r}_{t+1}^{u, \text{FC}} + (\tilde{r}_{t+1}^{u, \text{FC}} - i_{t+1}^{\text{FC}}) \cdot \left(1 - \frac{\tau^{\text{FC}} \cdot i_{t+1}^{\text{FC}}}{1 + i_{t+1}^{\text{FC}}}\right) \cdot L_t^{\text{FC}}.$$

For the covariance of the return of the levered firm and the change in spot exchange rates follows

$$\text{Cov}[\tilde{r}_{t+1}^{\ell, \text{FC}}, \tilde{\delta}_{t+1}^S] = \text{Cov}[\tilde{r}_{t+1}^{u, \text{FC}}, \tilde{\delta}_{t+1}^S] + \text{Cov}[\tilde{r}_{t+1}^{u, \text{FC}}, \tilde{\delta}_{t+1}^S] \cdot \left(1 - \frac{\tau^{\text{FC}} \cdot i_{t+1}^{\text{FC}}}{1 + i_{t+1}^{\text{FC}}}\right) \cdot L_t^{\text{FC}}. \quad (40)$$

By using the adjustment formula for $\rho^{\ell, \text{FC}}$, see Eq. 27, and the relation between $\rho^{\ell, \text{FC}}$ and $\rho^{\ell, \text{HC}, S}$, we obtain

$$\begin{aligned} \rho_{t+1}^{\ell, \text{HC}, S} = & \left(1 + \rho_{t+1}^{u, \text{FC}} + (\rho_{t+1}^{u, \text{FC}} - i_{t+1}^{\text{FC}}) \cdot \left(1 - \frac{\tau^{\text{FC}} \cdot i_{t+1}^{\text{FC}}}{1 + i_{t+1}^{\text{FC}}}\right) \cdot L_t^{\text{FC}}\right) \cdot (1 + \mathbb{E}[\tilde{\delta}_{t+1}^S]) \\ & + \text{Cov}[\tilde{r}_t^{\ell, \text{FC}}, \tilde{\delta}_{t+1}^S] - 1. \end{aligned}$$

Inserting Eq. 40 and applying the formula for the link between $\rho^{u, \text{HC}, S}$ and $\rho^{u, \text{FC}}$, see Eq. 12, yields Eq. 32 for $j = S$, that is,

$$\rho_{t+1}^{\ell, \text{HC}, S} = \rho_{t+1}^{u, \text{HC}, S} + (\rho_{t+1}^{u, \text{HC}, S} - \eta_{t+1}^{\text{HC}, S}) \cdot \left(1 - \frac{\tau^{\text{FC}} \cdot i_{t+1}^{\text{FC}}}{1 + i_{t+1}^{\text{FC}}}\right) \cdot L_t^{\text{FC}}.$$

Equation 32 for $j = S^*$, and Eq. 34 can be obtained similarly.

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Declarations

Conflict of interest S. Dierkes and I. de Maeyer declare that they have no competing interests.

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