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Coordinating Bank Dividend and Capital Regulation*

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Abstract

This paper examines how state-dependent dividend restrictions (taxes and bans) and capital requirements influence a bank's optimal capital buffers accumulation and risk-taking decisions. In the model, the bank distributes dividends and issues costly equity to maximise shareholder value, while its loans generate stochastic income under time-varying macroeconomic conditions. We solve the bank's stochastic control problem and derive the distribution of its capital buffers in closed form. We find that imposing dividend restrictions in bad macroeconomic states generates an intertemporal trade-off, as it encourages capital buffers accumulation in those states but promotes

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dividend payouts in the good ones. Furthermore, we show that the policy can undermine financial stability by reducing the bank's value and weakening its incentives to recapitalise in all states. Coordinating dividend taxes with counter-cyclical capital requirements can mitigate value losses and ease the trade-off, but it also exacerbates disincentives for recapitalisation. Finally, we show that when the bank can (optimally) reduce its risky loans in bad states, capital buffers generated through dividend restrictions mitigate the contraction by dampening its precautionary motive against costly recapitalisation. (JEL: C61; G21; G32; G35; G38)

Keywords: Capital requirements; dividend bans; dividend taxes; policy coordination; stochastic optimal control.

1 Introduction

At the height of the COVID-19 crisis, motivated by the observation that banks did not adjust their dividends during the Great Financial Crisis (Acharya et al., 2011; Cziraki et al., 2024; Belloni et al., 2024), banking regulators worldwide recommended, and in some cases enforced, dividend payout and share buyback restrictions.[1] These unprecedented measures aimed to preserve banks' credit capacity by ensuring adequate capital buffers, sustaining loans and, more critically, preventing systemic defaults.

Empirical evidence suggests that the short-term outcome of these policies has been two-sided. On the one hand, Andreeva et al. (2023) and Sanders et al. (2024) find that dividend restrictions and their announcements have negatively impacted bank equity valuations, consistent with the notion that shareholders demand higher returns in response to lower and delayed future payouts. On the other hand, Li et al. (2020) and Hardy (2021) argue that they were effective in improving the balance sheets of banks and, ultimately, in avoiding a credit crunch. Couaillier et al. (2025) find that dividend policies played a pivotal role in

[1] The ECB advised suspensions to start in March 2020; regular payments resumed in the fourth quarter of 2021. The FED imposed restrictions in June 2020, partially easing them in December 2020. The restrictions ended between June and July 2021, allowing banks to revert to pre-pandemic dividend policies.

sustaining bank lending, highlighting that capital buffer releases alone would not have been sufficient to achieve the same outcome.

From a theoretical standpoint, the problem of evaluating dividend regulation – such as bans and, more broadly, taxation – during adverse macroeconomic conditions has only recently received attention (see Vadasz, 2022 and Kroen, 2022) and remains poorly understood. This is because such policies often involve complex interactions with banks' decision-making and other forms of regulation. This paper develops a theory to evaluate how dividend taxes (or bans) contingent on the aggregate state of the economy affect a bank's optimal recapitalisation, dividend payout, and risk-taking decisions in both the short and long term, as well as their interaction with traditional capital requirements.

We model the optimal control problem of a bank that holds a fixed amount of loans and insured deposits.[2,3] Loans generate stochastic cash flows, whose expected returns and volatility depend on the aggregate state of the economy, as in Hackbarth et al. (2006) and Bolton et al. (2013).[4] The regulator imposes dividend restrictions in the form of taxes or bans and capital requirements, depending on the aggregate state of the economy. Similarly to Décamps et al. (2011) and Moreno-Bromberg and Rochet (2014), the bank decides whether to default or, at a cost, to issue equity (and in what amount) when capital requirements become binding; it retains cash flows as capital buffers or pays them out as dividends to avoid costly recapitalisation, aiming to maximise shareholder value.

In the first part of the paper, we solve the bank's stochastic control problem in closed form, demonstrating that optimal payouts follow a threshold strategy. In particular, we show that dividends are paid only when their marginal value, proportional to the state-contingent dividend tax, exceeds that of accumulating additional capital buffers. Otherwise, the bank takes no action. Notably, adopting state-contingent dividend taxes is equivalent to imposing

[2]In principle, the firm we model can represent either a financial or a non-financial company. However, our model is particularly well-suited to financial firms, which are typically subject to capital requirements.

[3]We relax the fixed loans assumption in the last part of the paper.

[4]Chay and Suh (2009) provide empirical evidence that cash flow uncertainty is a key determinant of firms' dividend payout decisions.

a dividend ban when the tax rate is high enough. Next, we show that recapitalising the bank is optimal if its costs are sufficiently small ("incentive compatible"). If that is the case, when the capital constraint binds and there are no dividend taxes, the bank injects equity until the reserves reach the dividend payout threshold. In the presence of dividend restrictions, the optimal recapitalisation target falls below the payout threshold.

Equipped with the bank's optimal controls, we analytically derive the stationary distribution of its capital buffers. We interpret this distribution either as a measure of the bank's "ex-ante" credit capacity or as the cross-sectional distribution of buffers that a regulator should consider in an economy populated by an infinite number of identical banks. We use this distribution to evaluate the long-term outcomes of different policies.

The second part of the paper employs numerical analysis to investigate how dividend and capital regulation affect the bank's optimal decisions and the distribution of capital buffers, and to derive policy implications. Motivated by the COVID-19 policy case, we examine a scenario in which dividend taxes are higher during a bad economic state, characterised by high cash flow volatility and low expected returns. To isolate the effects of dividend taxes, we first consider a-cyclical capital requirements.[5] Consistent with its scope, the policy encourages the accumulation of reserves in the targeted state by increasing the corresponding dividend payout threshold. This happens because higher taxes lower the marginal value of dividends. At the same time, however, the policy reduces the bank's value not only in the bad state ("ex-post") but also in the good state ("ex-ante") as shareholders internalise the prospect of lower future returns. Consequently, the bank finds it optimal to increase dividend payouts (i.e., reduce capital buffers) in the good state to compensate for these losses partially. These predictions warn that regulatory uncertainty regarding dividend restrictions can backfire by generating lower capital buffers in the long run (on this point, see also Attig et al., 2021) and encourage counter-cyclical equity issuance strategies, as suggested by Baron

[5]For simplicity, our model abstracts from the possibility of share buybacks. We believe this assumption does not significantly affect the scope of our results, since the policy motivating our analysis applied equally to both dividends and buybacks.

(2020). The second finding of the numerical analysis is that imposing tighter restrictions (i.e., higher taxes or outright bans) during bad macroeconomic conditions reduces the bank's optimal recapitalisation targets across all states. Additionally, the resulting shareholder value losses lower the incentive-compatible cost threshold beyond which shareholders are unwilling to inject equity when capital constraints are binding. This result suggests that state-contingent tax policies may increase default risk, potentially amplifying concerns about financial stability.

In the third part of the paper, we examine whether coordinating dividend taxes with cyclical capital requirements can help mitigate the adverse effects of the dividend policy. This analysis is particularly relevant in light of the findings of Dursun-de Neef et al. (2023), which demonstrate that the combination of recommendations to suspend dividends and relax capital requirements was crucial in sustaining lending during the COVID-19 pandemic. According to our simulations, coordinating countercyclical capital requirements with dividend taxes can mitigate the adverse effects of the latter by redistributing value losses across states and reducing the dispersion of the capital buffer distribution in the long run. However, these benefits come at the cost of creating additional disincentives for recapitalisation.

The fourth and final part of the paper extends our baseline framework, allowing the bank to optimally adjust loans in the bad state and examines how this decision is affected by the dividend policy. In line with the empirical evidence in Couaillier et al. (2025), our numerical simulations confirm that dividend restrictions can significantly reduce loan contractions. The reason is that extra capital buffers from retaining dividends mitigate the bank's precautionary motive against costly recapitalisation, encouraging more risk-taking. Higher recapitalisation costs in the bad state of the economy lead banks to cut loans more, partially offsetting the effect of the policy.

The paper is organised as follows. Section 2 situates the paper within the existing literature. Section 3 introduces the model, and Section 4 provides its analytical solution. Section 5 analyses the model numerically and discusses its policy implications. Section 6 concludes.

2 Related literature

Our work is related to recent empirical papers that examine the (primarily short-term) effects of dividend restrictions during the COVID-19 pandemic. Andreeva et al. (2023) and Sanders et al. (2024) find that banks subject to dividend suspension policies experienced a temporary drop in equity valuation but were able to increase their lending to the economy. Hardy (2021) show that banks' CDS spreads declined after this measure, suggesting an improvement in their safety. Dautovic et al. (2023) find that dividend restrictions were effective in limiting banks' pro-cyclical behaviour, thereby improving their capital buffers. Mücke (2023) show that mutual funds permanently reduced their ownership stakes in banks under payout restrictions. We complement this literature by providing a theoretical framework that analyses and characterises the joint effects of dividends and capital regulation on a bank's endogenous decisions in both the short and long run. Furthermore, our analysis of the joint effects of dividend and capital regulation provides a new theoretical perspective on state-dependent capital requirements. In particular, it complements evidence that relaxing capital requirements during crises without real capital or liquidity support may fail to address long-term capitalisation shortfalls (Bischof et al., 2023), and that the effect of capital requirement changes on lending is state-dependent, supporting a counter-cyclical capital buffer in normal times (Lang and Menno, 2025).

To the best of our knowledge, only a few papers have theoretically examined the effects of banks' dividend regulation. Goodhart et al. (2010) study the impact of dividend restrictions on an interbank market, showing that it may reduce defaults and improve welfare. Lindensjö and Lindskog (2020) solve the optimal control problem of a financial company facing dividend restrictions, finding that they may increase default risk. Vadasz (2022) explores the ex-post intervention problem between a regulator and a bank in a two-period game. None of these papers considers that dividend regulation may change with the aggregate state of the economy.

A closely related part of the literature investigates the effects of dividend regulation in equilibrium macroeconomic models with financial frictions, in the spirit of Gertler and Kiyotaki (2010) and Brunnermeier and Sannikov (2014). Schroth (2021), for example, shows that dividend restrictions can be beneficial both during normal times and financial crises, as they encourage banks to hold equity buffers. On the other hand, however, those restrictions may exacerbate the crisis severity. Using DSGE models with a banking sector, Munoz (2021) and Ampudia et al. (2023) show that dividend restrictions can stimulate bank lending while simultaneously reducing bank valuations. Our paper delivers similar predictions but starts from a dynamic corporate finance and banking perspective. This choice provides greater analytical tractability and allows us to examine not only the direct (and joint) effects of dividend restrictions and capital regulation on banks' capital buffer accumulation, but also their indirect effects through endogenous recapitalisation decisions, a channel that the existing literature has largely abstracted from.

In this respect, our paper falls within the extensive literature that uses continuous-time models to study firms' optimal cash management (Décamps et al., 2011; Gryglewicz, 2011) and its dividend policy implications (DeMarzo and Sannikov, 2006, 2016).[6] We differentiate our work from these studies by considering state-dependent dividend restrictions, *jointly with* macroeconomic uncertainty, following the approach of Hackbarth et al. (2006). In contrast to Bolton et al. (2013), we solve our baseline model in closed form, as in Hackbarth et al. (2006), while simultaneously incorporating dividends *and* capital requirements, as well as endogenous equity issuance decisions.

Methodologically, we build on the continuous-time stochastic control literature on dividends pioneered by Jeanblanc-Picqué and Shiryaev (1995). In particular, we tackle a problem that features Markovian regime switching, as in Jiang and Pistorius (2012) and Ferrari et al. (2022), and consider endogenous equity issuance, as in Løkka and Zervos (2008). Unlike in Radner and Shepp (1996), cash flows are exogenous. Our solution employs a guess-and-verify approach, similar to that described in Sotomayor and Cadenillas (2011).

[6] We refer the reader to Bolton et al. (2024) for a comprehensive review of this large literature.

We distinguish ourselves from these previous studies in four dimensions. First, we consider macroeconomic uncertainty not only in expected cash flows, as in Reppen et al., 2020, but also in their volatility. Second, we are the first to incorporate both dividend taxes (or bans) and capital regulation into an optimal dividend framework with state-dependent cash flows. Third, we analytically derive the stationary distribution of banks' capital buffers under the optimal control. Fourth, we develop and solve numerically an extended version of the model that considers reserve remuneration and the possibility of adjusting loan exposure depending on the state of the economy.

3 Model

Time is continuous and indexed by $t \in [0, \infty)$. As in Guo et al. (2005) and Hackbarth et al. (2006), we consider a bank subject to aggregate uncertainty on the state of the economy, modelled via a bi-variate Markov chain $I_t \in \{1, 2\}$, with transition intensity λ_{I_t}. A risk-neutral manager runs the bank in the best interest of its shareholders.

3.1 Balance sheet and cash flows

The bank's balance sheet comprises insured deposits D and two types of assets: a fixed portfolio of illiquid loans L, and liquid reserves X_t (cash or central bank deposits). The book value of equity E_t satisfies the identity:

$$E_t + D = X_t + L. \tag{1}$$

Deposits earn the risk-free rate $\rho \geq 0$, whereas reserves earn no return, for simplicity.[7] The loan portfolio generates stochastic operating cash flows that follow an arithmetic Brownian motion, with drift and diffusion coefficients $\bar{\mu}_{I_t}$ and σ_{I_t} that depend on the state

[7]We extend the model and relax this assumption in Section 5.4.

of the economy I_t. After servicing deposits, the net cash flow accruing to reserves is

$$\mu_{I_t}\, dt + \sigma_{I_t}\, dW_t, \qquad \mu_{I_t} \coloneqq \bar{\mu}_{I_t} - \rho D, \tag{2}$$

where W_t is a standard Brownian motion defined on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$. We assume $\bar{\mu}_1 \geq \bar{\mu}_2 > 0$ and $\sigma_2 \geq \sigma_1 > 0$. Accordingly, state $I_t = 1$ (expansion) offers higher expected returns and lower volatility, while state $I_t = 2$ (recession) exhibits lower returns and higher risk. Throughout the paper, we will refer to $I_t = 1$ as the "good" state and $I_t = 2$ as the "bad" state.

3.2 Dividend and capital regulation

The bank's net cash flows can be retained as reserves or distributed to shareholders as dividends. The manager chooses a cumulative dividend process Z_t, which is non-decreasing, right-continuous, and adapted to $\mathbb{F}$. Dividends are taxed asymmetrically across states: a dividend of \$1 paid in state I_t yields an after-tax value $\beta_{I_t} \in [0, 1]$ to shareholders. Consistent with the COVID-19 policy case, we assume $\beta_1 = 1$ and $\beta_1 \geq \beta_2$, so that dividends are taxed more heavily (or even prohibited) during recessions.

The regulator requires the bank to hold sufficient equity to repay depositors in full if loans are liquidated at a fire-sale price $\alpha \in [0, 1]$, as in Décamps et al. (2011). Provided $D > \alpha L$, the capital requirement is:

$$E_{R,I_t} \geq L(1 - \alpha) + \Gamma_{I_t}, \tag{3}$$

where $\Gamma_{I_t} \in \mathbb{R}$ captures additional regulatory buffers ($\Gamma_{I_t} > 0$) or subsidies/guarantees ($\Gamma_{I_t} < 0$). Substituting (1) into (3) yields the minimum reserve level:

$$X_t \geq D - \alpha L + \Gamma_{I_t} =: x_{R,I_t}. \tag{4}$$

Since X_t is stochastic, the reserves constraint (4) becomes occasionally binding at random times $(\tau_n)_{n\geq 1}$. When this occurs, the manager must either liquidate the bank or recapitalize it by issuing new equity at a cost. The choice is captured by a variable $b_n \in \{0,1\}$, where $b_n = 0$ denotes liquidation and $b_n = 1$ recapitalization. In the event of liquidation, shareholders receive the residual buffer $\Gamma^+_{I_t}$ but forfeit all future dividends. If the bank is recapitalized, shareholders inject an amount $G_n > 0$ and pay a fixed cost $\kappa > 0$.

3.3 Reserves process and admissible strategy

In this section, we model the dynamics of the bank's reserve process and the admissible strategy, based on the setting described informally above.

As a first step, let $\tau_0 = 0$, $I_0 = i$, and $X_0 = x \geq \min\{x_{R,1}, x_{R,2}\}$. Let $Z^{(0)}_t$ denote the initial dividend process chosen by the bank manager, defined for $t \geq 0$, which is non-decreasing, right-continuous, and adapted to the filtration $\mathbb{F}$. Next, given $Z^{(0)}$, we define the auxiliary process

$$\tilde{X}^{(0)}_t = X_0 + \int_0^t (\mu_{I_s}\, ds + \sigma_{I_s}\, dW_s) - Z^{(0)}_t, \quad t \geq 0, \tag{5}$$

and define the first intervention time as

$$\tau_1 = \inf\left\{t \geq 0 : \tilde{X}^{(0)}_t \leq x_{R,I_t}\right\}, \tag{6}$$

with the convention $\inf \emptyset = \infty$. We then proceed recursively as follows for $n = 1, 2, \ldots$:

1. At time $\tau_n < \infty$, the manager chooses one of the following two actions:

   (i) $b_n = 1$ (recapitalisation): select an amount $G_n > 0$ and set

$$\tilde{X}^{(n)}_{\tau_n} = \tilde{X}^{(n-1)}_{\tau_n^-} + G_n \geq x_{R,I_{\tau_n}}. \tag{7}$$

(ii) $b_n = 0$: (liquidation): the process is stopped. To simplify the formal construction, we *fictitiously* assume that the manager still selects a recapitalisation amount $G_n > 0$ and set $\tilde{X}^{(n)}_{\tau_n}$ as above. The actual liquidation will be explicitly accounted for when formulating the bank's optimisation problem in Section 3.4.

2. For $t \geq \tau_n$, the manager chooses a new dividend process $Z^{(n)}_t$ (non-decreasing, right-continuous, $\mathbb{F}$-adapted, and such that $Z^{(n)}_{\tau_n} = 0$). Given $Z^{(n)}$, we define the auxiliary process

$$\tilde{X}^{(n)}_t = \tilde{X}^{(n)}_{\tau_n} + \int_{\tau_n}^t (\mu_{I_s}\, ds + \sigma_{I_s}\, dW_s) - Z^{(n)}_t, \quad t \geq \tau_n, \tag{8}$$

as well as the next intervention time

$$\tau_{n+1} = \inf \left\{ t \geq \tau_n : \tilde{X}^{(n)}_t \leq x_{R,I_t} \right\}. \tag{9}$$

Using this recursive construction, we define the overall reserve process as

$$X_t = \begin{cases} \tilde{X}^{(0)}_t, & \text{for } t \in [0, \tau_1), \\ \\ \tilde{X}^{(n)}_t, & \text{for } t \in [\tau_n, \tau_{n+1}), \quad n \geq 1, \end{cases} \tag{10}$$

and introduce the following class of admissible strategies.

Definition 1 (**Admissible strategy**)**.** *An admissible strategy is a tuple of sequences*

$$A = \{(Z^{(n)}, b_{n+1}, G_{n+1})\}_{n \geq 0}$$

constructed recursively as described above and satisfying the following conditions:

(i) For each $n \geq 0$, $(Z^{(n)}_t)_{t \in [\tau_n, \tau_{n+1})}$ is $\mathbb{F}$-adapted, right-continuous, and non-decreasing.

(ii) Dividend payments never cause an immediate violation of the capital requirement:

$$\Delta Z^{(n)}_t := Z^{(n)}_t - Z^{(n)}_{t^-} < \tilde{X}^{(n)}_{t^-} - x_{R,I_t} \quad \forall t \in (\tau_n, \tau_{n+1}). \tag{11}$$

(iii) For $n \geq 1$, $b_n \in \{0,1\}$ is $\mathcal{F}_{\tau_n}$-measurable and $G_n \geq 0$ is $\mathcal{F}_{\tau_n}$-measurable and strictly positive if $b_n = 1$.

We denote the set of admissible strategies by $\mathcal{A}$. Given an admissible strategy $A \in \mathcal{A}$, the cumulative dividend process Z_t is defined recursively as

$$Z_t = \begin{cases} Z_t^{(0)}, & t \in [0, t_1), \\ Z_{\tau_n^-} + Z_t^{(n)}, & t \in [\tau_n, \tau_{n+1}),\ n \geq 1. \end{cases} \tag{12}$$

3.4 Optimisation problem

Given $A \in \mathcal{A}$, we define $\ell_A := \inf\{n : \ b_n = 0\}$, with the usual convention $\inf \emptyset = \infty$. The bank's gain function is

$$J(x, i; A) = \mathbb{E}\left[\int_0^{\tau_{\ell_A}} e^{-\delta t} \beta_{I_t} dZ_t - \sum_{n=1}^{\ell_A - 1} e^{-\delta \tau_n}(G_n + \kappa)\right], \qquad \forall A \in \mathcal{A}, \tag{13}$$

where $\delta > 0$ is a discount rate, with the convention $\sum_{n=1}^{0} = 0$. The bank's value function (i.e., shareholder value) is then given by

$$V(x, i) \coloneqq \sup_{A \in \mathcal{A}} J(x, i; A). \tag{14}$$

The well-posedness of the problem is guaranteed by $\delta > 0$. The condition $\bar{\mu}_i - \rho D > 0$ for $i = 1, 2$ ensures that the net cash flow remains positive, allowing the reserve process to grow meaningfully and preventing trivial solutions.

4 Model solution

The first part of this section derives the solution to the bank's optimisation problem analytically. For this purpose, we focus on the "baseline" case in which capital requirements are a-cyclical, i.e. not contingent on the state of the economy ($x_{R,1} = x_{R,2} = x_R$). We will consider cyclical capital requirements in Section 5. The second part of the section derives and analyses the (stationary) distribution of the bank's reserves under the optimal strategy.

4.1 Shareholder value and optimal strategy

To tackle the problem (14), we follow a *guess-and-verify* approach.[8] More specifically, we formulate a set of optimality conditions for a candidate value function v based on heuristic considerations. Then, we use verification arguments to prove that $v = V$.

We expect the value function to solve (in a suitable sense) the following system of Hamilton-Jacobi-Bellman Variational Inequalities (HJBVI):

$$\max \Big\{ \mathcal{L}_i v(x,i) - \lambda_i \left[v(x,i) - v(x, 3-i) \right], \beta_i - v'(x,i) \Big\} = 0, \qquad i = 1,2, \quad x > x_R, \tag{15}$$

where $v : [x_R, \infty) \times \{1,2\} \to \mathbb{R}$, and $\mathcal{L}_i$ are differential operators acting on functions $\phi \in C^2(x_R, \infty)$ as follows:

$$\mathcal{L}_i \phi = \frac{1}{2}\sigma_i^2 \phi'' + \mu_i \phi' - \delta\phi, \qquad i = 1,2.$$

Associated with a smooth enough solution to (15), there are the following *continuation* and *intervention* regions for $i = 1,2$:

$$\mathcal{C}_i \coloneqq \{ x > x_R : \quad v'(x,i) > \beta_i \}, \tag{16}$$

$$\mathcal{S}_i \coloneqq \{ x > x_R : \quad v'(x,i) = \beta_i \}. \tag{17}$$

Equipped with these objects, we conjecture that the optimal control has a threshold structure, as in Sotomayor and Cadenillas (2011), meaning that for each $i = 1,2$, there is a reserve level $\tilde{x}_i$ below which the bank does not pay dividends. Indeed, we expect that it is optimal to accumulate reserves as long as their marginal value ($v'(\cdot, i)$) exceeds that of

[8] This approach is the most used in the classical corporate finance literature starting from Leland (1994). For a *direct* approach to a similar problem, building on the theory of viscosity solutions and stating the optimality conditions as *necessary*, we refer to Akyildirim et al. (2014).

dividends (β_i). Formally, we guess that the above regions are $\mathcal{C}_i = (x_R, \tilde{x}_i)$ and $\mathcal{S}_i = [\tilde{x}_i, \infty)$ for $i = 1, 2$, implying that

$$\tilde{x}_i = \inf\left\{x \geq x_R : v'(x, i) \leq \beta_i\right\}. \tag{18}$$

Next, we make some conjectures to construct a sufficiently smooth solution to (15). First, we assume that $v(\cdot, i) \in C([x_R, \infty)) \cap C^2((x_R, \infty))$. Second, we postulate $x_R < \tilde{x}_1 < \tilde{x}_2$, based on the intuition that it is optimal to pay more dividends when assets yield higher returns and carry less uncertainty. Third, we impose appropriate boundary conditions at the regulatory threshold x_R. The boundary conditions determine whether recapitalisation is optimal at x_R and, if so, the amount of equity to issue.

If the manager liquidates the firm, the shareholders receive the maximum between the capital buffer and zero (Γ_i^+). Otherwise, the optimal recapitalisation policy ($\hat{G}$) must be feasible and "incentive-compatible", i.e. its benefits must be larger than or equal to its costs. Therefore, we require that $\hat{G} = \operatorname{argmax}\left\{v(x_R + G; i) - G - \kappa\right\}$, with $v(x_R + \hat{G}, i) \geq \hat{G} + \kappa$ and $\hat{G} \in \mathcal{G} := [0, \tilde{x}_i - x_R]$. As either liquidation or recapitalisation must be optimal, we impose

$$v(x_R, i) = \max\left\{\max_{G \in \mathcal{G}}\left\{v(x_R + G, i) - G - \kappa\right\}, \Gamma_i^+\right\} \qquad \text{for } i = 1, 2. \tag{19}$$

We now construct a function that meets all these conditions.

In the intervals $[\tilde{x}_i, \infty)$, the function must satisfy $v'(\cdot, i) = \beta_i$. In the interval $(\tilde{x}_1, \tilde{x}_2)$, we define $v'(\cdot, 2)$ as the unique solution to

$$\frac{1}{2}\sigma_2^2 v'''(x, 2) + \mu_2 v''(x, 2) - (\delta + \lambda_2)\, v'(x, 2) + \lambda_2 = 0, \tag{20}$$

with boundary conditions $v'(\tilde{x}_2, 2) = \beta_2 > 0$ (optimality condition), and $v''(\tilde{x}_2, 2) = 0$ (super contact condition, see Dumas, 1991). In the interval $(x_R, \tilde{x}_1)$, we find the functions $v'(\cdot, i)$

as unique solutions to the following system:

$$\begin{cases} \frac{1}{2}\sigma_1^2 v'''(x,1) + \mu_1 v''(x,1) - (\delta+\lambda_1)\, v'(x,1) + \lambda_1 v'(x,2) = 0, \\ \\ \frac{1}{2}\sigma_2^2 v'''(x,2) + \mu_2 v''(x,2) - (\delta+\lambda_2)\, v'(x,2) + \lambda_2 v'(x,1) = 0. \end{cases} \tag{21}$$

with boundary conditions $v'(\tilde{x}_1, 1) = 1$ (optimality condition), $v''(\tilde{x}_1, 1) = 0$ (super-contact condition), $v'(\tilde{x}_1^-, 2) = v'(\tilde{x}_1^+, 2)$, and $v''(\tilde{x}_1^-, 2) = v''(\tilde{x}_1^+, 2)$ (continuity conditions).

Solving (20) and (21) for $v'(\cdot, i)$ and integrating over the corresponding intervals yields the following proposition.

Proposition 1 (**Solution to the HJBVI system**). *Recall that $\delta > 0$, $\lambda_i > 0$, $\mu_1 \geq \mu_2 > 0$, $\sigma_1 \leq \sigma_2$ and assume that $\beta_2 > \lambda_2/(\lambda_2+\delta)$. Then, the following holds.*

1. *Fix $\tilde{x}_1, \tilde{x}_2$ such that $x_R < \tilde{x}_1 < \tilde{x}_2$ and define*

$$v(x,1) = K_1 + \begin{cases} (x-\tilde{x}_1), \quad x \in [\tilde{x}_1, \infty), \\ \sum_{j=1}^4 A_j \left(e^{\alpha_j(x-\tilde{x}_1)} - 1\right), \quad x \in [x_R, \tilde{x}_1), \end{cases} \tag{22}$$

$$v(x,2) = K_2 + \begin{cases} \beta_2(x-\tilde{x}_2), \quad x \in [\tilde{x}_2, \infty), \\ \frac{\lambda_2(x-\tilde{x}_2)}{\delta+\lambda_2} + \sum_{j=1}^2 \tilde{A}_j \left(e^{\tilde{\alpha}_j(x-\tilde{x}_1)} - e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}\right), \quad x \in [\tilde{x}_1, \tilde{x}_2), \\ \frac{\lambda_2(\tilde{x}_1-\tilde{x}_2)}{\delta+\lambda_2} + \sum_{j=1}^2 \tilde{A}_j (1 - e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}) + \sum_{j=1}^4 B_j \left(e^{\alpha_j(x-\tilde{x}_1)} - 1\right), \quad x \in [x_R, \tilde{x}_1), \end{cases} \tag{23}$$

where $\alpha_j < \alpha_2 < 0 < \alpha_3 < \alpha_4$ and $\tilde{\alpha}_1 < 0 < \tilde{\alpha}_2$ are the real roots of

$$\underbrace{\left(\delta + \lambda_1 - \alpha\mu_1 - \frac{\sigma_1^2}{2}\alpha^2\right)}_{:=G_1(\alpha)} \underbrace{\left(\delta + \lambda_2 - \alpha\mu_2 - \frac{\sigma_2^2}{2}\alpha^2\right)}_{:=G_2(\alpha)} = \lambda_1\lambda_2, \tag{24}$$

$$\frac{1}{2}\sigma_2^2\tilde{\alpha}^2 + \mu_2\tilde{\alpha} = \delta + \lambda_2, \tag{25}$$

and $(A_1, A_2, A_3, A_4, \tilde{A}_1, \tilde{A}_2) \in \mathbb{R}^6$ *and* $(K_1, K_2) \in \mathbb{R}^2_+$ *solve the following linear system:*[9]

$$
\begin{cases}
\sum_{j=1}^4 A_j\alpha_j - 1 = 0,\\
\sum_{j=1}^4 A_j\alpha_j^2 = 0,\\
\sum_{j=1}^4 B_j\alpha_j - \frac{\lambda_2}{\delta+\lambda_2} - \sum_{h=1}^2 \tilde{A}_h\tilde{\alpha}_h = 0,\\
\sum_{j=1}^4 B_j\alpha_j^2 - \sum_{h=1}^2 \tilde{A}_h\tilde{\alpha}_h^2 = 0,\\
\sum_{h=1}^2 \tilde{\alpha}_h\tilde{A}_h e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)} - \beta_2 + \frac{\lambda_2}{\delta+\lambda_2} = 0\\
\sum_{h=1}^2 \tilde{\alpha}_h^2\tilde{A}_h e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)} = 0.\\
\mu_2\beta_2 - (\delta+\lambda_2)K_2 + \lambda_2(K_1 + \tilde{x}_2 - \tilde{x}_1) = 0,\\
\mu_1 - (\delta+\lambda_1)K_1 + \lambda_1\left(K_2 + \frac{\lambda_2}{\delta+\lambda_2}(\tilde{x}_1 - \tilde{x}_2)\right) = 0,
\end{cases}
\tag{26}
$$

with $B_j = A_j\lambda_1^{-1}G_1(\alpha_j)$. *Moreover, assume that*

$$
\begin{cases}
\sum_{j=1}^4 A_j\alpha_j^3 > 0,\\
\sum_{j=1}^4 B_j\alpha_j^3 > 0,\\
\sum_{j=1}^4 A_j\alpha_j^3 e^{\alpha_j(x_R-\tilde{x}_1)} > 0,\\
\sum_{j=1}^4 B_j\alpha_j^3 e^{\alpha_j(x_R-\tilde{x}_1)} > 0.
\end{cases}
\tag{27}
$$

Then, $v''(\cdot, i) < 0$ *in* $(x_R, \tilde{x}_i)$ *and* (22) *and* (23) *solve* (15) *in a classical sense.*

2. *Consider the framework in Point 1 and assume that*

$$
\sum_{j=1}^4 B_j\alpha_j e^{\alpha_j(x_R-\tilde{x}_1)} - 1 > 0. \tag{28}
$$

[9] Assuming that a solution to this system exists.

Then, $x_2^* \in (x_R, \tilde{x}_2]$ *is the unique solution of*

$$\mathbf{1}_{[x_R,\tilde{x}_1]}(x_2^*)\sum_{j=1}^{4} B_j\alpha_j e^{\alpha_j(x_2^*-\tilde{x}_1)} + \mathbf{1}_{(\tilde{x}_1,\tilde{x}_2]}(x_2^*)\left(\frac{\lambda_2}{\delta+\lambda_2}+\sum_{h=1}^{2}\tilde{\alpha}_h\tilde{A}_h e^{\tilde{\alpha}_h(x_2^*-\tilde{x}_1)}\right) - 1 = 0. \tag{29}$$

3. *Consider the framework in Points 1 and 2 and set* $\Gamma_i^+ = 0$*. Moreover, assume that* $\tilde{x}_1, \tilde{x}_2$ *solve the following algebraic system:*

$$\begin{cases} \sum_{j=1}^{4} A_j\left(e^{\alpha_j(x_R-\tilde{x}_1)}-1\right) + (\tilde{x}_1 - x_R) + \kappa = 0, \\ \frac{\lambda_2\tilde{x}_1}{\delta+\lambda_2} - \sum_{j=1}^{2}\tilde{A}_j(1-e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}) - \sum_{j=1}^{4} B_j\left(e^{\alpha_j(x_R-\tilde{x}_1)}-1\right) + \\ -\left(\frac{\lambda_2\tilde{x}_1}{\delta+\lambda_2} + \sum_{j=1}^{2}\tilde{A}_j(1-e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}) + \sum_{j=1}^{4} B_j\left(e^{\alpha_j(x_2^*-\tilde{x}_1)}-1\right)\right)\mathbf{1}_{(x_R,\tilde{x}_1)} + \\ -\left(\frac{\lambda_2 x_2^*}{\delta+\lambda_2} + \sum_{j=1}^{2}\tilde{A}_j\left(e^{\tilde{\alpha}_j(x_2^*-\tilde{x}_1)} - e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}\right)\right)\mathbf{1}_{(\tilde{x}_1,\tilde{x}_2)} - (x_2^* - x_R) + \kappa = 0. \end{cases} \tag{30}$$

Then, (22) *and* (23) *satisfy the boundary conditions* (19).

Proof. See Appendix A.1. □

If we suppose that all the above assumptions hold, then v equals the value function. In that case, we can use this proposition to determine the bank's optimal strategy by solving a system of algebraic equations. The procedure is as follows.

The bank only pays dividends in the state i when its reserves reach $\tilde{x}_i$. For a given $x_2^* \in (x_R, \tilde{x}_2]$, the payout thresholds solve the system in (30). When reserves reach x_R, liquidation is never optimal ($\hat{\tau}_l = \infty$ or, equivalently, $\hat{b}_n = 1$), provided that κ is sufficiently small to ensure that (19) is positive. If that is the case, the optimal recapitalisation in state i injects equity until the reserves reach the target level x_i^*. Therefore, $\hat{G}(i) = x_i^* - x_R$.

Since $v(\cdot, i)$ is differentiable, we can use (19) and the boundary condition $v(\tilde{x}_1, 1) = 1$ to obtain that $x_1^* = \tilde{x}_1$ and $x_2^* \in (x_R, \tilde{x}_2]$ as the unique solution of (29). In other words, the bank finds it optimal to recapitalise exactly to its dividend payout threshold in the good state and *below* the payout threshold in the bad state ($x_2^* \leq \tilde{x}_2$) because the marginal

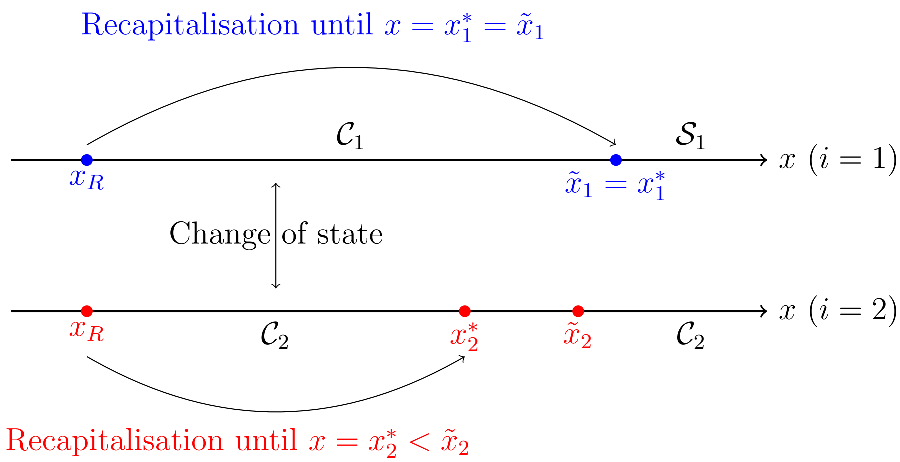

Figure 1: The figure displays the model's solution structure and the optimal strategy in the good state ($i = 1$) and the bad state ($i = 2$). The thresholds $\tilde{x}_i$ for $i = 1, 2$ indicate the optimal dividend payout. The thresholds x_2^* and $x_1^* = \tilde{x}_1$ represent the optimal recapitalisation targets relative to the regulatory threshold x_R in States 2 and 1, respectively. $\mathcal{C}_i$ and $\mathcal{S}_i$ denote the associated continuation and intervention regions.

value of dividends (β_2) is less than one. Since v is concave, $v(\tilde{x}_2, 2) = \beta_2 \leq v(x_2^*, 2) = 1$, and each state has a unique payout threshold and recapitalisation target. For a given $\hat{G}(i)$, the following condition defines the maximum cost level that ensures that recapitalisation is incentive-compatible:

$$\bar{\kappa} = \min \left\{ v(x_1^*, 1) - (x_1^* - x_R), v(x_2^*, 2) - (x_2^* - x_R) \right\}. \tag{31}$$

Figure 1 summarises the model's solution structure and the corresponding optimal strategy.

The next theorem verifies that v is indeed the value function and formally expresses the optimal strategy we described above.

Theorem 1 (**Verification**). *Let all the assumptions of Proposition 1 hold and let $v(\cdot, i)$ be the functions constructed therein. Moreover, let $(x, i) \in (x_R, \infty) \times \{1, 2\}$. Then, $v(x, i) =$*

$V(x,i)$ and the control $\hat{A} = (\hat{Z}, (\hat{b}, \hat{G})) \in \mathcal{A}$ such that

$$\begin{cases} \hat{b}_n = 1, \\ \hat{G}_n = x^*_{I_{\hat{\tau}_n^-}} - x_R, \\ \hat{Z}_t = \hat{Z}_{\tau_n} + \sup_{s\in[\hat{\tau}_n,t)} \left[x^*_{I_{\hat{\tau}_n^-}} + \int_{\hat{\tau}_n}^s (\mu_{I_r} dr + \sigma_{I_r} dW_r) - \tilde{x}_{I_{\hat{\tau}_n^-}} \right]^+, \qquad t \in [\hat{\tau}_n, \hat{\tau}_{n+1}), \end{cases} \tag{32}$$

where $\hat{\tau}_0 := 0$ and $\hat{\tau}_n$ is defined recursively as $\hat{\tau}_{n+1} = \inf\{t \geq \hat{\tau}_n : \ \hat{X}_{t^-} = x_R\}$ being $\hat{X}_t$ the associated state process, is optimal.

Proof. See Appendix A.2. □

Remark 1. ***(Dividend tax threshold)*** *The solution structure described in Proposition 1 and Theorem 1 holds under the parametric restriction that dividend taxes are not too high, i.e. $\beta_2 > \lambda_2/(\lambda_2 + \delta)$. In contrast, when $\beta_2 \leq \lambda_2/(\lambda_2 + \delta)$, an inspection of the conditions set shows that the solution structure used to obtain them breaks down. The same conditions suggest that the guess should feature an alternative structure, with $\tilde{x}_2 = \infty$. The correct structure has $\mathcal{C}_2 = (x_R, \infty)$, regardless of the value of $\beta_2 \in [0, \lambda_2/(\lambda_2 + \delta))$. A more detailed discussion of this case appears in Appendix A.4.*

Remark 1 clarifies that, when taxes are excessively high in the bad state, the bank finds it optimal to postpone dividend payments until the good state materialises. Therefore, setting $\beta_2 \leq \lambda_2/(\delta + \lambda_2)$ is equivalent to imposing a dividend ban in the bad state. The level of β_2 that triggers the "ban-like" behaviour increases with the probability of transitioning from the bad state to the good state (λ_2) and decreases with the manager's impatience (δ).

4.2 Capital buffers distribution

This section employs heuristic arguments to derive an analytical characterisation of the long-run (i.e., stationary) probability density function (pdf) of the bank's reserves.[10] The

[10] Throughout this section, we consider the case $x_2^* \leq \tilde{x}_1 \leq \tilde{x}_2$. Analogous formulas can be derived when $x_2^* > \tilde{x}_1$.

result is corroborated by numerical simulations consistent with the underlying stochastic dynamics. Section 5 will use the pdf to evaluate the effects of dividend taxes and capital regulation on the bank's capital buffers.

Let $\hat{X}_t$ be the optimal controlled state process described in Theorem 1, and let $\pi(x,i)$ denote its *stationary probability density* at point x in state i. We assume that this density exists and satisfies the following regularity conditions: $\pi(\cdot,1)$ is continuous on $[x_R,\tilde{x}_1]$ and smooth on $(x_R,\tilde{x}_1)$; $\pi(\cdot,2)$ is continuous on $[x_R,\tilde{x}_2]$ and smooth on $(x_R,x_2^*)\cup(x_2^*,\tilde{x}_2)$, except possibly at x_2^*. We then define the "flux" operator over the interior regions $(x_R,\tilde{x}_1)$ and $(x_R,x_2^*)\cup(x_2^*,\tilde{x}_2)$:[11]

$$F(x,1) := \mu_1\pi(x,1) - \frac{\sigma_1^2}{2}\pi'(x,1), \quad x \in (x_R,\tilde{x}_1), \tag{33}$$

$$F(x,2) := \mu_2\pi(x,2) - \frac{\sigma_2^2}{2}\pi'(x,2), \quad x \in (x_R,x_2^*)\cup(x_2^*,\tilde{x}_2). \tag{34}$$

Over these regions, the pair of functions $\pi(x,i)$, $i=1,2$, is expected to satisfy the following system of Kolmogorov forward equations (KFEs) (see, e.g. Achdou et al., 2022):

$$\begin{cases} -F'(x,1) + \lambda_2\pi(x,2) - \lambda_1\pi(x,1) = 0, & x \in (x_R,\tilde{x}_1), \\ -F'(x,2) + \lambda_1\pi(x,1)\mathbf{1}_{\{x<\tilde{x}_1\}} - \lambda_2\pi(x,2) = 0, & x \in (x_R,x_2^*)\cup(x_2^*,\tilde{x}_2). \end{cases} \tag{35}$$

We look for an analytical solution of (35) that satisfies a set of natural conditions which formalise the transport of probability mass from the regulatory threshold to the recapitalisation targets in states $i=1,2$ and the reflection at the dividend payments boundaries. The first condition ensures that the reserves process does not accumulate probability mass at x_R since, upon reaching this threshold, it is immediately and irreversibly transferred to

[11] The flux measures the rate of probability flow across a point; a positive (negative) value signifies net rightward (leftward) transport driven by drift and diffusion.

x_i^*. Hence, we set[12]

$$\pi(x_R, i) = 0, \quad i = 1, 2. \tag{36}$$

The second condition ensures that the probability mass removed at x_R is re-injected at x_i^*. In the bad state $i = 2$, where re-injection occurs at the interior point $x_2^* \in (x_R, \tilde{x}_2)$, we require the probability density function to be continuous,

$$\pi(x_2^{*-}, 2) = \pi(x_2^{*+}, 2). \tag{37}$$

The balance of mass at this point, considering the flow coming from recapitalisation, leads to the equality

$$F(x_2^{*+}, 2) - F(x_2^{*-}, 2) + F(x_R^+, 2) = 0, \tag{38}$$

where $F(x_R^+, 2) = \lim_{x \to x_R^+} F(x, 2)$.[13] In the good state ($i = 1$), re-injection occurs exactly at the reflecting barrier $\tilde{x}_1$. At this point, the flux must account for two contributions: first, the probability mass removed at x_R in the same state, and second, the mass accumulated in the bad state ($i = 2$) between $\tilde{x}_1$ and $\tilde{x}_2$, which is instantaneously transferred to $\tilde{x}_1$ when the economy re-enters into State 1. Thus, considering the reflection, the mass balance leads to the equality

$$F(\tilde{x}_1^-, 1) - F(x_R^+, 1) + \lambda_2 \int_{\tilde{x}_1}^{\tilde{x}_2} \pi(x, 2) dx = 0. \tag{39}$$

The third condition requires that the flux in the bad state vanishes at the dividend payout threshold $\tilde{x}_2$ (reflecting barrier):

$$F(\tilde{x}_2^-, 2) = 0. \tag{40}$$

In the limiting case where $\beta_2 = \beta_1 = 1$, the recapitalisation target coincides with the dividend payout threshold in both states; that is, $x_2^* = \tilde{x}_2$ too. Hence, conditions (37), (38), and (40)

[12] See Yaegashi et al. (2019) for a discussion of similar conditions in a uni-variate setting.

[13] The negative sign on the right-hand side of (38) ensures that the probability mass *leaving* the domain through the left boundary x_R ($F(x_R^+, 2) < 0$), is re-injected into the system at the same *positive* rate at x_2^*.

reduce to

$$F(\tilde{x}_2^-, 2) - F(x_R^+, 2) = 0. \tag{41}$$

Following a procedure analogous to that used to construct the candidate value function in Section 4.1, we conjecture that $\pi(x, i)$ can be represented as a linear combination of exponential basis functions. We then verify the candidate solution by substituting these expressions into (35) on the intervals (x_R, x_2^*), $(x_2^*, \tilde{x}_1)$, and $(\tilde{x}_1, \tilde{x}_2)$. The coefficients of the basis functions are uniquely determined by enforcing conditions (36)-(40) at x_R and x_i^*, as well as the usual value-matching and smooth-pasting conditions at x_2^* and $\tilde{x}_1$. The outcome of this procedure when $\beta_2 < 1$ is summarised in the following proposition. A detailed derivation is provided in Appendix A.3. Details on the limiting case $\beta_2 = \beta_1 = 1$ are provided in the supplementary material.

Proposition 2. ***(Capital buffers stationary density)*** *Assume that $x_R < x_2^* < \tilde{x}_1 < \tilde{x}_2$.*[14] *Then, the stationary density solving* (35) *with boundary conditions* (36)-(40) *has the following structure:*

$$\pi(x,1) = \begin{cases} P_1 e^{r_1 x} + P_2 e^{r_2 x} + P_3 e^{r_3 x} + P_4 e^{r_4 x}, & x \in (x_R, x_2^*), \\ \tilde{P}_1 e^{r_1 x} + \tilde{P}_2 e^{r_2 x} + \tilde{P}_3 e^{r_3 x} + \tilde{P}_4 e^{r_4 x}, & x \in (x_2^*, \tilde{x}_1), \\ 0, & x \in (\tilde{x}_1, \infty), \end{cases} \tag{42}$$

$$\pi(x,2) = \begin{cases} Q_1 e^{r_1 x} + Q_2 e^{r_2 x} + Q_3 e^{r_3 x} + Q_4 e^{r_4 x}, & x \in (x_R, x_2^*), \\ \tilde{Q}_1 e^{r_1 x} + \tilde{Q}_2 e^{r_2 x} + \tilde{Q}_3 e^{r_3 x} + \tilde{Q}_4 e^{r_4 x}, & x \in (x_2^*, \tilde{x}_1), \\ H_1 e^{s_1 x} + H_2 e^{s_2 x}, & x \in (\tilde{x}_1, \tilde{x}_2), \\ 0, & x \in (\tilde{x}_2, \infty), \end{cases} \tag{43}$$

[14] A similar statement can be obtained in case $x_R < \tilde{x}_1 < x_2^* < \tilde{x}_2$.

in which $r_1 < r_2 < 0 < r_3 < r_4$ *and* $s_1 < 0 < s_2$ *are the real roots of*

$$\underbrace{\left(\lambda_1 + r\mu_1 - \frac{\sigma_1^2}{2}r^2\right)}_{:=F_1(r)}\underbrace{\left(\lambda_2 + r\mu_2 - \frac{\sigma_2^2}{2}r^2\right)}_{:=F_2(r)} = \lambda_1\lambda_2, \tag{44}$$

$$\frac{\sigma_2^2}{2}s^2 - \mu_2 s = \lambda_2, \tag{45}$$

and the vector $(P_1, P_2, P_3, P_4, \tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \tilde{P}_4, H_1, H_2) \in \mathbb{R}^{10}$ *is the unique solution of the following 9-rank homogeneous linear system:*

$$\begin{cases} \sum_{j=1}^4 \left(P_j - \tilde{P}_j\right) e^{r_j x_2^*} = 0, \\ \sum_{j=1}^4 \left(P_j - \tilde{P}_j\right) r_j e^{r_j x_2^*} = 0, \\ \sum_{j=1}^4 P_j e^{r_j x_R} = 0, \\ \sum_{j=1}^4 \left(\tilde{P}_j e^{r_j \tilde{x}_1}\left(\frac{\sigma_1^2}{2} r_j - \mu_1\right) - \frac{\sigma_1^2}{2} P_j r_j e^{r_j x_R}\right) - \lambda_2 \sum_{h=1}^2 \frac{H_h}{s_h}\left(e^{s_h \tilde{x}_2} - e^{s_h \tilde{x}_1}\right) = 0, \\ \sum_{j=1}^4 \left(Q_j - \tilde{Q}_j\right) e^{r_j x_2^*} = 0, \\ \sum_{j=1}^4 \tilde{Q}_j e^{r_j \tilde{x}_1} - \sum_{h=1}^2 H_h e^{s_h \tilde{x}_1} = 0, \\ \sum_{j=1}^4 \tilde{Q}_j r_j e^{r_j \tilde{x}_1} - \sum_{h=1}^2 H_h s_h e^{s_h \tilde{x}_1} = 0, \\ \sum_{j=1}^4 Q_j e^{r_j x_R} = 0, \\ \sum_{h=1}^2 H_h \left(\mu_2 e^{s_h \tilde{x}_2} - \frac{\sigma_2^2}{2} s_h e^{s_h \tilde{x}_2}\right) = 0, \\ \sum_{j=1}^4 r_j \left(e^{r_j x_2^*}\left(Q_j - \tilde{Q}_j\right) - Q_j e^{r_j x_R}\right) = 0, \end{cases} \tag{46}$$

where $Q_j = \lambda_2^{-1} F_1(r_j) P_j$ *and* $\tilde{Q}_j = \lambda_2^{-1} F_1(r_j) \tilde{P}_j$*, subject to the additional normalization condition*

$$\sum_{i=1}^2 \int_{x_R}^{\tilde{x}_i} \pi(x, i)\, dx = 1. \tag{47}$$

5 Policy implications

In this section, we parameterise the model and numerically analyse its policy implications by examining the effects of dividend taxes on optimal controls, shareholder value, and the resulting capital buffer distribution. We then extend the baseline model to incorporate counter-cyclical capital requirements and examine their interaction with dividend regulation.

5.1 Parameters

We normalise the stock of deposits $D = 1$ and choose L to match the US bank deposit-to-loan ratio in Q4 2022 (about 1.5385), according to S&P Global. According to FRED, we set the rate of return on deposits at $\rho = 0.0043$. Consistently, we obtain $x_R = 1 - 0.6 \times 1.5385 = 0.0769$. We calibrate the fixed recapitalisation cost $\kappa = 0.087$ to yield a price-to-book ratio at the dividend payout threshold in the good state $(v(\tilde{x}_1, 1)/(L + \tilde{x}_1 - D))$ of about 1.04. This is the value observed across US commercial banks, according to the NYU Stern database. The drift and diffusion parameters, $\bar{\mu}_i$ and σ_i, and the discount rate δ are similar to Guo et al. (2005). The intensities of regime changes λ_i, and the haircut α come from Hackbarth et al. (2006). Lastly, we set $\beta_2 = 0.87$, corresponding to a 10 p.p. increment in the dividend tax rate from 0.3 in the good regime to 0.4 in the bad one.[15]

5.2 The effects of dividend taxes

5.2.1 Bank's optimal control and shareholder value

To evaluate the effect of state-contingent dividend taxes, we compare the bank's optimal control and shareholder value when $\beta_2 < \beta_1 = 1$ with the "no-policy" scenario where $\beta_2 = \beta_1 = 1$. The red dotted and solid blue lines in Figure 2 display $v(x, i)$ in the good (Panel (a)) and bad (Panel (b)) states as a function of the reserve level in these two cases. The black crosses and the blue (red) diamonds on the x axis show the regulatory threshold (x_R) and

[15] After normalizing the value of $\beta_1 = 1$, the parameter β_2 can be obtained by solving: $1 - 0.3 = \frac{1-0.4}{\beta_2}$.

Parameter	Meaning	Value
μ_1	Cash flow drift, good state	0.05
μ_2	Cash flow drift, bad state	0.02
σ_1	Cash flow volatility, good state	0.25
σ_2	Cash flow volatility, bad state	0.3
κ	Recapitalisation cost	0.087
δ	Discount rate	0.03
x_R	Capital requirement	0.0769
ρ	Return on deposits	0.0043
α	Haircut	0.6
β_2	Dividend regulation	0.87
$\frac{L}{D}$	Loan-to-deposit ratio	1.5385
$1/\lambda_1$	Average duration, good state	10
$1/\lambda_2$	Average duration, bad state	6.7
Γ_1, Γ_2	Capital buffers	0

Table 1: The table reports the values of the parameters used in the numerical simulations.

the optimal dividend payout ($\tilde{x}_i$) in the good (bad) state. The red star in panel (b) indicates the optimal recapitalisation target. Four key patterns emerge from the comparison.

First, dividend taxes reduce shareholder value at all levels of x and economic states. This is expected because the policy introduces an additional distortion beyond the capital requirement. The value losses are more severe in percentage terms for any reserve value in the bad state than in the good state, as the policy affects the latter only indirectly. Furthermore, value losses are always more severe when capital buffers are lower, except in the bad state, where they increase slightly after reaching a tipping point around $x = 1$.

Second, we discuss the effects of dividend taxes on the bank's optimal payout strategy. On the one hand, consistent with its intended purpose, the policy encourages the accumulation of additional capital buffers (i.e., fewer dividend payments) in the bad state, shifting $\tilde{x}_2$ from approximately 0.95 to 1.7. The higher the tax, the more willing the bank is to delay dividend payments. This outcome is apparent in Table 2, which shows the optimal $\tilde{x}_i$ and x_2^* as functions of β_2.[16] On the other hand, the policy "backfires" in the good state, where

[16]Note that, consistent with Remark 1, the last rows of the table highlight that the bad-state payout threshold $\tilde{x}_2$ "explodes" when $\beta_2 \to \lambda_2/(\delta + \lambda_2)$ ($\approx 0.3\bar{3}$ with our parameters).

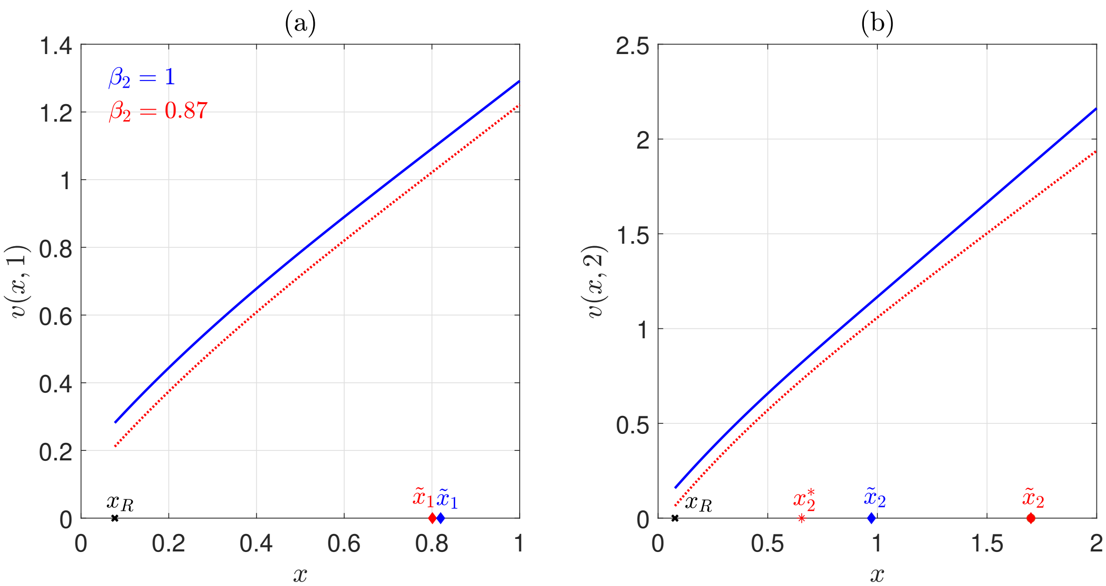

Figure 2: This figure displays shareholder value, $v(x, i)$, as a function of bank reserves x in the good state ($i = 1$, Panel (a)) and the bad state ($i = 2$, Panel (b)). The blue solid line and the red dotted line represent the value without dividend restrictions ($\beta_2 = 1$) and with dividend restrictions ($\beta_2 = 0.87$), respectively. The black cross on the x-axis marks the regulatory threshold x_R. Red and blue diamonds indicate the dividend payout thresholds $\tilde{x}_i$ with and without dividend restrictions, respectively. The red star denotes the recapitalisation target under dividend restrictions, x_2^*, in the bad state.

β_2	$\tilde{x}_1$	$\tilde{x}_2$	x_2^*
1.000	0.820	0.950	0.950
0.990	0.819	0.941	0.815
0.970	0.814	1.061	0.747
0.950	0.811	1.142	0.724
0.900	0.804	1.421	0.667
0.870	0.801	1.700	0.655
0.850	0.800	2.663	0.649
0.834	0.800	17.024	0.649

Table 2: This table reports the optimal dividend thresholds $\tilde{x}_i$ for $i = 1, 2$ and the recapitalisation target x_2^* in the bad state under the baseline parameterisation in Table 1, for different levels of the dividend restriction parameter β_2.

the bank reduces $\tilde{x}_1$ from 0.82 to 0.80. The reason is that the bank compensates for the tax in the bad state by anticipating dividends in the good one. However, notice that there is a significant asymmetry in the magnitude of the threshold shifts between the good and bad states. In our numerical analysis, enforcing dividend restrictions reduces the payout threshold in the good state by only 2.5%, while it increases it by more than 30% in the bad state.

Third, dividend taxes reduce the bank's optimal recapitalisation targets (x_i^*) in every state. In the good state, this result directly follows the change in the payout dividend threshold $\tilde{x}_1$ because it coincides with the recapitalisation target ($\tilde{x}_1 = x_1^*$). In contrast, $\tilde{x}_2 > x_2^*$ in the bad state because the marginal cost of paying dividends (β_2) is always lower than that of injecting liquid reserves and $v''(x, 2) < 0$ (see (29)). Notably, x_2^* decreases with β_2 (up to a limit value as $\beta_2 \to \frac{\delta+\lambda_2}{\lambda_2}$) and lies *below* the payout threshold in the good state $\tilde{x}_1$. Hence, dividend taxes reduce the bank's willingness to issue equity when reserves hit the regulatory threshold, x_R.

Fourth, the overall influence of the dividend tax on the bank's capital buffers ex-ante (i.e., independent of i) is ambiguous because the ways it affects its optimal control are conflicting: while the increase in $\tilde{x}_2$ yields higher capital buffers, the reduction in $\tilde{x}_1$ and x_2^* may curb them. To determine which effect prevails, the following section analyses the impact of the

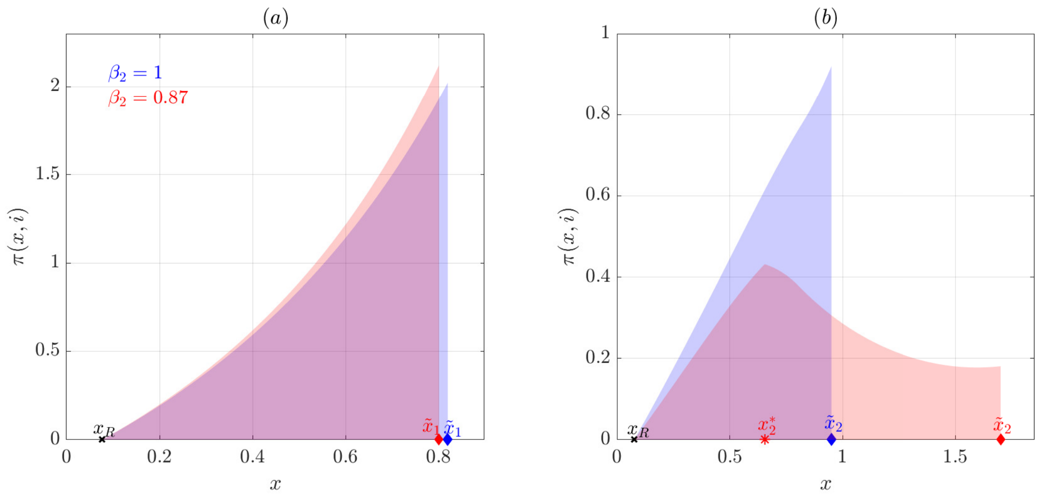

Figure 3: This figure displays the stationary pdf of bank reserves in the good state ($i = 1$, Panel (a)) and in the bad state ($i = 2$, Panel (b)). The blue and red shaded areas represent the pdfs without dividend restrictions ($\beta_2 = 1$) and with dividend restrictions ($\beta_2 = 0.87$), respectively. The black cross on the x-axis marks the regulatory threshold, x_R. Blue diamonds indicate the dividend payout thresholds ($\tilde{x}_i$), with $\tilde{x}_1$ shown in the good state (Panel (a)) and $\tilde{x}_2$ in the bad state (Panel (b)). The red star in the bad state (Panel (b)) denotes the recapitalisation target, x_2^*.

policy on the bank's stationary capital buffers distribution.

5.2.2 Long-run capital buffers and recapitalisation incentives

This section uses the density function derived in Proposition 2 to construct a proxy for the bank's long-run "credit capacity". We use such a proxy to evaluate how credit capacity is influenced by dividend policies, both conditionally and unconditionally on the aggregate state of the economy. Although the model features a fixed loan supply, the long-run capital buffers are the amount of available resources that could potentially support new lending. From this point onward, we use the terms "long-run capital buffers" and "credit capacity" interchangeably.

As a first step in the analysis, we examine the shape of the bank's capital buffer distribution, $\pi(x, i)$, across different reserve levels x and aggregate states i. For this purpose, Figure 3 displays $\pi(x, i)$ and the optimal control when $\beta_2 = \beta_1 = 1$ (blue) and $\beta_2 < \beta_1 = 1$ (red).

As a first observation, higher dividend taxes in the bad state do not substantially alter the shape of the pdf in the good state (panel (a)). However, they transfer some probability density to lower reserve levels because the reduction of β_2 shifts the optimal dividend payout threshold $\tilde{x}_1$ to the left, even though the probability of $i = 1$ remains unchanged. In contrast, the tax policy largely affects the shape of the pdf in the bad state (panel (b)). In particular, the dispersion of $\pi(x, 2)$ increases sharply because a lower β_2 shifts $\tilde{x}_2$ to the right, widening the support of the reserves' distribution. Additionally, dividend taxes generate a steep mass point at the recapitalisation target x_2^* because $x_2^* < \tilde{x}_1 < \tilde{x}_2$.[17]

To evaluate the long-run effect of the dividend tax policy, we use $\pi(x, i)$ to define the following measure of the bank's credit capacity, conditional on state i:

$$M_i[\pi] := \frac{\int_{x_R}^{\tilde{x}_i} x\, \pi(x, i)\, dx}{\int_{x_R}^{\tilde{x}_i} \pi(x, i)\, dx}, \tag{48}$$

and its variance

$$S_i[\pi] := \frac{\int_{x_R}^{\tilde{x}_i} x^2\, \pi(x, i)\, dx}{\int_{x_R}^{\tilde{x}_i} \pi(x, i)\, dx} - M_i[\pi]^2 \tag{49}$$

as a measure of dispersion. Consistently, we define the bank's unconditional (or ex-ante) credit capacity as

$$M\,[\pi] := \sum_{i=1}^{2} \int_{x_R}^{\tilde{x}_i} x\pi(x, i)dx, \tag{50}$$

with dispersion $S\,[\pi] := \sum_{i=1}^{2} \int_{x_R}^{\tilde{x}_i} x^2\pi(x, i)dx - M\,[\pi]^2$. Table 3 collects the value of these objects using the parameters in Table 1 and different levels of β_2. The analysis delivers the following implications.

First, state-contingent dividend taxes reduce the bank's credit capacity in the good state but enhance it in the bad state. However, the loss in the former is always more than offset by the gains in the latter (Columns 1 and 2). As a result, imposing dividend taxes increases overall credit capacity (Column 3). Second, higher dividend taxes slightly reduce the dispersion of reserves in the good state but significantly increase it in the bad state. Hence, bank credit capacity dispersion increases overall (Columns 5-7).

[17] Notice that, due to the regime-switching, the pdf of x in state 2 displays a small but interior mass point even when $\beta_2 = 1$, coinciding with the dividend payout threshold $\tilde{x}_1$ in State 1.

β_2	M_1	M_2	M	S_1	S_1	S	$\bar{\kappa}$
1.00	0.5986	0.6600	0.6232	0.0280	0.0418	0.0344	0.1351
0.95	0.5937	0.7342	0.6499	0.0272	0.0625	0.0461	0.1187
0.90	0.5893	0.8209	0.682	0.0266	0.1045	0.0706	0.1090
0.87	0.5874	0.8847	0.7063	0.0264	0.1529	0.0982	0.1057
0.85	0.5868	0.9831	0.7453	0.0263	0.2968	0.1722	0.1043

Table 3: The table reports the bank's long-run capital buffers, M_i (i.e., the expectation over the stationary pdf $\pi(x,i)$), and the dispersion of capital buffers, S_i, both conditional (Columns 2–3 and 5–6) and unconditional (Columns 4 and 7) on the state of the economy ($i = 1, 2$). It also reports the maximal incentive-compatible recapitalisation cost, $\bar{\kappa} := x_R + \min\{v(x_1^*, 1) - x_1^*,\, v(x_2^*, 2) - x_2^*\}$ (see Eq. (31)), for different levels of the dividend tax parameter, β_2.

The relatively small negative impact of dividend taxes on credit capacity in the good state, compared to their large positive effects in the bad state, and overall, suggests that the benefits of the policy outweigh its costs. Indeed, one can verify that dividend taxes make recapitalisation events less frequent in each state i and overall. To this aim, Table 4 reports a numerical approximation of the average waiting time between subsequent recapitalisation events, formally defined as $\hat{\tau}(i) := \inf\{t \geq \tau_n : \hat{X}_t = x_{R,i}, \hat{X}_{\tau_n} = x_i^*\}$ (state-contingent) and $\hat{\tau} := \sum_{i=1}^{2} \hat{\tau}(i) \cdot \lambda_{3-i}/(\lambda_1 + \lambda_2)$ (ex-ante) for different values of β_2.[18] Despite the tax, the average recapitalisation time is longer in the good state than in the bad state. However, the effect of adjusting the recapitalisation policy to a lower level on credit capacity is sizable. Specifically, if we were to ignore the bank's endogenous response in terms of the recapitalisation threshold x_2^* and instead keep it fixed at $\tilde{x}_2$, credit capacity would be nearly 20% higher.

The global evaluation of the dividend tax policy becomes even less straightforward when noticing that, as a result of value losses, higher dividend taxes reduce the maximal equity issuance cost that is incentive-compatible ($\bar{\kappa}$, see Eq. (31)). The last column of Table 3 displays this phenomenon, showing that, for example, a 10% increase in dividend taxes is associated with a 28% reduction in the level of $\bar{\kappa}$.

[18] The numbers are obtained by averaging 5,000 Monte Carlo simulations of the bank's reserves process under the optimal strategy (32).

β_2	$\hat{\tau}(1)$	$\hat{\tau}(2)$	$\hat{\tau}$
1.000	11.89	11.56	11.76
0.970	12.44	11.99	12.26
0.870	13.06	12.66	12.90
0.834	13.12	12.74	12.97

Table 4: The table reports a numerical approximation of the average waiting time (years) between subsequent recapitalisation events for different values of the dividend tax parameter β_2. The state-contingent waiting time is defined as $\hat{\tau}(i) := \inf\{t \geq \tau_n : \hat{X}_t = x_{R,i}, \hat{X}_{\tau_n} = x_i^*\}$, while the ex-ante (overall) waiting time is given by $\hat{\tau} := \sum_{i=1}^{2} \hat{\tau}(i) \cdot \lambda_{3-i}/(\lambda_1 + \lambda_2)$.

It is relevant to stress that all the figures reported in the table exceed the baseline cost level adopted in our parametrisation ($\kappa = 0.087$), ensuring that the bank always finds it optimal to recapitalise when $x = x_R$. We nevertheless interpret this tightening of the incentive-compatibility constraint as evidence that dividend restrictions may endogenously increase default risk.

5.2.3 Comparative statics

Table 5 presents a comparative static analysis which shows that our results hold qualitatively under substantial variations in the main parameters of the model. A higher drift in the good or bad states lowers the payout thresholds because it releases the bank's precautionary motive (Rows 3-4). Higher levels of cash flow volatility generate the opposite effect (see, e.g., Row 2). Decreasing the probability of visiting the bad state (λ_1) fosters dividend payouts without the tax while mitigating the additional payout incentive in the good state induced by the tax. Moreover, it increases the maximum incentive-compatible cost level while leaving the optimal recapitalisation target unaffected. The probability of transitioning from the bad to the good state (λ_2) has similar effects on $\tilde{x}_1$, x_2^*, and $\bar{\kappa}$, while also leading to a further postponement of dividends in the bad state (Row 6).

Another aspect worth highlighting concerns the effects of a change in the regulatory parameter defined in (3), Γ_i, for $i = 1, 2$. The last two rows of the Table 5 examine the

	$\tilde{x}_1$		$\tilde{x}_2$		x_2^*		$\bar{\kappa}$	
β_2	1.00	0.87	1.00	0.87	1.00	0.87	1.00	0.87
Baseline	0.821	0.801	0.954	1.700	0.954	0.655	0.135	0.106
$\sigma_2 = 0.35$	0.827	0.811	1.051	1.938	1.051	0.710	0.118	0.087
$\mu_1 = 0.07$	0.794	0.776	0.952	1.698	0.952	0.653	0.288	0.251
$\mu_2 = 0.03$	0.820	0.798	0.941	1.651	0.941	0.637	0.194	0.157
$\lambda_1 = 0.05$	0.816	0.806	0.954	1.700	0.954	0.655	0.186	0.160
$\lambda_2 = 0.2$	0.821	0.804	0.950	3.324	0.950	0.672	0.164	0.141
$\Gamma_1 = \Gamma_2 = 0.05$	0.871	0.851	1.007	1.750	1.007	0.705	0.133	0.106
$\Gamma_1 = \Gamma_2 = -0.05$	0.769	0.751	0.923	1.650	0.923	0.605	0.133	0.106

Table 5: The table compares the optimal dividend thresholds, $\tilde{x}_i$, and the recapitalisation target, x_2^*, in the bad state under the baseline parameterisation reported in Table 1, and for different combinations of the dividend restriction parameter β_2, the bad-state volatility σ_2, the good- and bad-state drift parameters μ_i, the regime-switching intensities λ_i, and the capital buffer parameters Γ_i, for $i = 1, 2$.

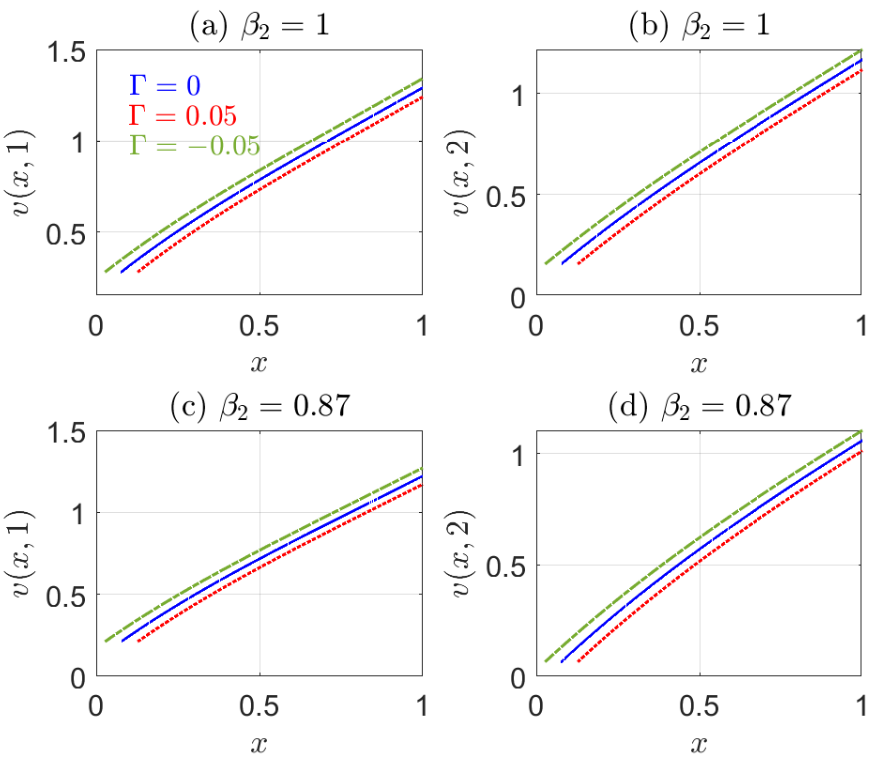

Figure 4: The figure displays the bank's value function $v(x, i)$ in the good state ($i = 1$, left panels) and the bad state ($i = 2$, right panels) for different values of the capital buffer parameters $\Gamma_1 = \Gamma_2 = \Gamma$, when the dividend tax parameter is $\beta_2 = 1$ (top panels) and $\beta_2 = 0.87$ (bottom panels).

cases where $\Gamma_1 = \Gamma_2 = \Gamma = \pm 0.05$.[19] Figure 4 displays the effect of changing Γ on the value function.

As intuition suggests, tighter (looser) capital requirements are associated with higher (lower) recapitalisation thresholds and lower (higher) bank valuations. Furthermore, changes in Γ do not affect the maximum incentive-compatible recapitalisation costs. This occurs because shareholder value shifts in the opposite direction of the change in capital requirements.

Although relatively straightforward, these effects are significant as they suggest regulators can mitigate shareholder value losses–one of the adverse impacts of the dividend tax policy–by adjusting capital requirements simultaneously. We explore this aspect further in the next section.

5.3 Coordinating dividend and capital regulation

This section extends the model to incorporate counter-cyclical capital requirements and discusses how this modification affects the structure of the solution. Then, it explores the policy challenge of coordinating dividend taxation and capital regulation numerically. This analysis is motivated by the fact that, while recommending dividend suspensions, the ECB temporarily eased capital requirements for banks during the COVID-19 crisis (Matyunina and Ongena, 2022).

While our framework can accommodate pro-cyclical capital requirements, we focus on the special case in which they are counter-cyclical. This choice is justified by the Basel III framework and by the extensive literature that shows how pro-cyclical buffers may destabilise financial institutions and thus exacerbate crises (see, for example Repullo and Suarez, 2013; Valencia and Bolanos, 2018, and the references therein). In the supplementary material, we demonstrate how to incorporate pro-cyclical capital requirements into our model. There, we also show that, consistent with the aforementioned literature, counter-cyclical capital requirements can mitigate the unintended consequences of dividend taxes, whereas pro-

[19] This analysis will also serve as a benchmark when we discuss cyclical capital regulation in Section 5.3.3.)

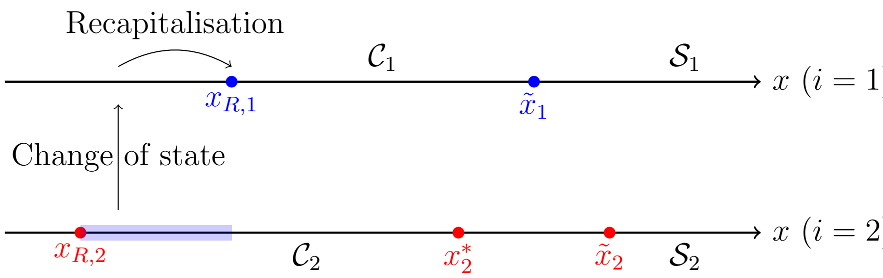

Figure 5: The figure visualises the model's solution structure and the optimal strategy in the good state ($i = 1$) and the bad state ($i = 2$) when capital requirements are counter-cyclical ($x_{R,1} > x_{R,2}$). The thresholds $\tilde{x}_i$, for $i = 1, 2$, indicate the optimal dividend payout levels. The thresholds x_2^* and $x_1^* = \tilde{x}_1$ represent the optimal recapitalisation targets relative to the regulatory thresholds $x_{R,i}$ when $i = 2$ and $i = 1$, respectively. $\mathcal{C}_i$ and $\mathcal{S}_i$ denote the associated continuation and intervention regions. The shaded blue area represents the buffer region in which the regulator requires the bank to recapitalise immediately (or liquidate) should the economy shift from State 2 to State 1.

cyclical requirements can only exacerbate them.

5.3.1 Solution structure with counter-cyclical capital requirements

To model counter-cyclical capital requirements, we consider $\Gamma_1 > \Gamma_2$ or, equivalently, $x_{R,1} > x_{R,2}$ (see Eq. (4)). This assumption requires adjusting the solution structure of the model by considering the additional "buffer" region $(x_{R,2}, x_{R,1})$. The regulator requires the bank to immediately recapitalise (or liquidate) in this region should the economy shift from State 2 to State 1, as shown in Figure 5. Accordingly, we set

$$v(x, 1) = \max\{v(\tilde{x}_1, 1) - (\tilde{x}_1 - x) - \kappa, 0\} \text{ for } x \in (x_{R,2}, x_{R,1}) \tag{51}$$

and find $v(x, 2)$ as the unique solution of

$$\frac{1}{2}\sigma_2^2 v''(x, 2) + \mu_2 v'(x, 2) - (\delta + \lambda_2)\, v(x, 2) + \lambda_2 v(x, 1) = 0, \tag{52}$$

with boundary conditions $v(x_{R,1}^+, 2) = v(x_{R,1}^-, 2)$ and $v'(x_{R,1}^+, 2) = v'(x_{R,1}^-, 2)$.

Since the bank always finds it optimal to recapitalize when κ is small enough (see Eq. (51)), in the remaining regions (i.e., when $x > x_{R,1}$), the value function equals the one described in Section 4.1 after setting $x_{R,1} = x_R$. Hence, we can still characterise the model's solution analytically by solving a system of algebraic equations. We provide details in the supplementary material.

5.3.2 Capital buffers distribution with counter-cyclical capital requirements

With countercyclical capital buffers, the KFE system (35) is modified as follows:

$$\begin{cases} -F'(x,1) + \lambda_2 \pi(x,2) - \lambda_1 \pi(x,1) = 0, & x \in (x_{R,1}, \tilde{x}_1), \\ -F'(x,2) + \lambda_1 \pi(x,1) \mathbf{1}_{\{x_{R,1} < x < \tilde{x}_1\}} - \lambda_2 \pi(x,2) = 0, & x \in (x_{R,2}, x_2^*) \cup (x_2^*, \tilde{x}_2). \end{cases} \tag{53}$$

Furthermore, we impose $\pi(x,1) = 0$ for $x \in (x_{R,2}, x_{R,1})$ and we have to account that, when the state suddenly changes from state 2 to 1, the mass accumulated across the buffer interval is transported at $x_{R,1}$. Indeed, if a transition from the bad to the good state occurs when $x \in (x_{R,2}, x_{R,1})$, the process is instantaneously shifted to $x_{R,1}$ and subsequently to the dividend payout threshold $\tilde{x}_1$. Hence, (39) is modified as follows:

$$F(\tilde{x}_1^-, 1) - F(x_{R,1}^+, 1) + \lambda_2 \int_{x_{R,2}}^{x_{R,1}} \pi(x,2)\, dx + \lambda_2 \int_{\tilde{x}_1}^{\tilde{x}_2} \pi(x,2)\, dx = 0. \tag{54}$$

Further details on how these modifications affect the analytical expression of the stationary pdf in Proposition 2 are provided in the supplementary material.

5.3.3 The effects of counter-cyclical capital requirements

We now numerically examine the effects of counter-cyclical capital requirements on the bank's optimal strategy and their interaction with dividend taxes. We adopt as a benchmark the a-cyclical case discussed in Section 4 ($\Gamma_1 = \Gamma_2 = \Gamma$), assuming that the regulator sets Γ_i to maintain a constant *mean* capital requirement across states. In particular, we choose

$x_{R,1}$	$x_{R,2}$	$\tilde{x}_1$	$\tilde{x}_2$	x_2^*	M_1	M_2	M	S_1	S_2	S	$\bar{\kappa}$
0.077	0.077	0.801	1.700	0.655	0.588	0.885	0.706	0.026	0.153	0.098	0.106
0.087	0.062	0.806	1.688	0.643	0.593	0.877	0.707	0.026	0.153	0.096	0.102
0.097	0.047	0.812	1.677	0.631	0.600	0.870	0.708	0.026	0.153	0.094	0.098
0.117	0.017	0.824	1.656	0.609	0.609	0.864	0.711	0.027	0.149	0.092	0.087

Table 6: Columns 3–9 of this table report the bank's optimal dividend thresholds ($\tilde{x}_i$), the recapitalization target in the bad state of the economy (x_2^*), the long-run capital buffers M_i (i.e., the expectation with respect to the stationary pdf $\pi(x, i)$), and the dispersion of capital buffers S_i, both conditional and unconditional on the state of the economy ($i = 1, 2$), for different levels of state-dependent capital requirements ($x_{R,i}$, Columns 1 and 2). The last column reports the associated maximal incentive-compatible recapitalization cost: $\bar{\kappa} := \min\{v(x_1^*, 1) - (x_1^* - x_{R,1}), v(x_2^*, 2) - (x_2^* - x_{R,2})\}$.

$\Gamma_1 = \Gamma > 0$ and $\Gamma_2 = -\Gamma_1$, such that[20]

$$x_R = \underbrace{(D - \alpha L + \Gamma)}_{=x_{R,1}} \underbrace{\frac{\lambda_2}{\lambda_1 + \lambda_2}}_{=\mathbb{P}\{i=1\}} + \underbrace{(D - \alpha L - \Gamma)}_{=x_{R,2}} \underbrace{\frac{\lambda_1}{\lambda_1 + \lambda_2}}_{=\mathbb{P}\{i=2\}}. \tag{55}$$

Table 6 reports the bank's optimal strategy, credit capacity and dispersion, and the maximal incentive-compatible recapitalisation cost for different levels of Γ. Row 1 displays the benchmark case where $\Gamma = 0$. We obtain the following predictions.

First, adopting counter-cyclical capital requirements increases the bank's credit capacity in the good state (Columns 3 and 6), while reducing it in the bad state (Columns 4 and 6) and overall (Column 8), relative to the benchmark. When both dividend taxes *and* capital regulation are used, the bank's unconditional credit capacity remains higher than when dividend regulation is not applied (i.e., $\beta_2 = \beta_1 = 1$). Therefore, coordinating the two policies enables the imposition of dividend taxes without curtailing credit capacity in the good state. Table 6 supports this claim.

The second effect of the policy is to reduce the dispersion of capital buffers, even after

[20] The appendix examines a simpler case where the regulator relaxes capital requirements only in the bad state (i.e., $x_{R,1} = x_R$ and $x_{R,2} = x_{R,1} - \Gamma_2$ with $\Gamma_2 > 0$). As expected, the policy lowers the dividend payout threshold, diminishes recapitalisation incentives, and reduces average credit capacity and dispersion across states. Furthermore, it enhances the bank's value in both states.

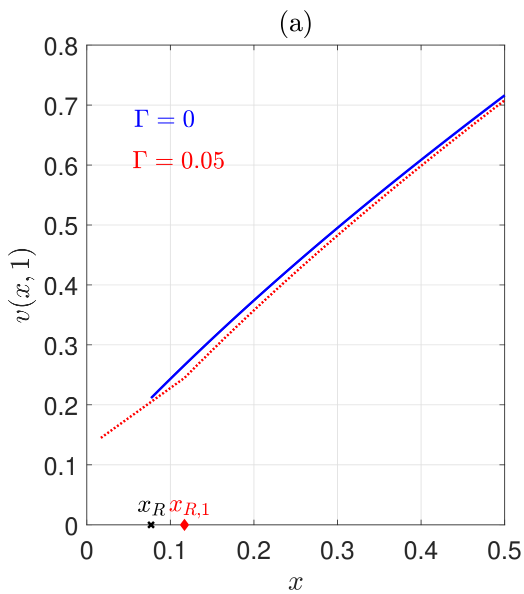

Figure 6: This figure compares the bank's shareholder value, $v(x, i)$, as a function of bank reserves x in the good state ($i = 1$, Panel (a)) and the bad state ($i = 2$, Panel (b)), for different levels of the counter-cyclical capital requirement parameter Γ. The black cross on the x-axis marks the a-cyclical regulatory threshold x_R, while the red diamonds indicate the counter-cyclical regulatory thresholds $x_{R,i}$ for $i = 1, 2$.

accounting for its (lower) expected value. For example, our simulations show that the coefficient of variation of the reserves distribution ($\sqrt{S[\pi]}/M[\pi]$) decreases by approximately 3.2% when $x_{R,1} = 0.117$ and $x_{R,2} = 0.017$, compared to when $x_{R,1} = x_{R,2} = 0.077$.

Figure 6 displays the model's third prediction by showing the bank's value function for $\Gamma = 0$ ($x_{R,1} = x_{R,2} = 0.077$) and $\Gamma = 0.05$ ($x_{R,1} = 0.117$, $x_{R,2} = 0.017$).[21] The plot reveals that relaxing the capital requirement in the bad state helps mitigate shareholder value losses, which can be substantial when dividends are taxed (Panel (b)), as discussed in the previous sections. However, the gain comes at the cost of reducing bank value in the good state (Panel (a)). These outcomes arise because tighter (looser) capital requirements curb (encourage) banks' precautionary motives in the good (bad) state, leading them to anticipate (delay) their dividend payments (Columns 3 and 4).

These results suggest that coordinating dividend taxes (or bans) with counter-cyclical capital regulation can mitigate some of the adverse effects of the former policy from the point

[21] Computing plots for different values of Γ produces qualitatively similar results.

of view of the bank (namely a reduction in its value in the good state) and of the regulator (the reduction in the bank's capital buffers and the larger dispersion of its reserves (credit capacity) across states. From a policy standpoint, this provides a theoretical foundation for the regulator's decision to pair dividend restrictions with looser capital requirements during the COVID-19 crisis. However, the positive outcomes of policy coordination come with a cautionary note. Specifically, the model's fourth policy prediction shows that adopting counter-cyclical capital requirements lowers the optimal recapitalisation target in the bad state (x_2^*). Moreover, the maximal incentive-compatible recapitalisation cost, $\bar{\kappa}$ (see Columns 5 and 12), decreases. These effects occur because, although the policy increases the bank's value for all states above x_R, it reduces the value *at* $x_{R,2} < x_R$, which plays a critical role in determining the optimal recapitalisation (see (19)).

The final step of the analysis assesses the effect of counter-cyclical capital regulation on the average waiting time (in years) after each recapitalisation. Table 7 presents a numerical approximation of these quantities conditional on each state i and overall for various levels of Γ when $\beta_2 = 0.87$. According to these simulations, coordinating counter-cyclical dividend taxes and capital requirements can effectively reduce the frequency of recapitalisation (see columns 3-5). However, when the policy becomes excessively cyclical, its effectiveness during downturns diminishes (see column 4). This occurs because when $x_{R,2}$ becomes too low, the positive effect on capital buffers from a higher $\tilde{x}_1$ in the good state is partially offset by the increasingly negative impact of having lower $\tilde{x}_2$ and x_2^* in the bad state (see Table 6).

5.4 Model extensions

One important limitation of our model is that loan exposure is fixed, preventing us from examining how dividend restrictions affect bank risk-taking. In addition, we assume that liquid reserves are non-remunerated and that equity issuance costs (κ) are constant. This section relaxes these assumptions by developing and solving an extended version of the model in which reserves (X_t) and deposits (D) are remunerated at the rate ρ, and the bank can

$x_{R,1}$	$x_{R,2}$	$\hat{\tau}(1)$	$\hat{\tau}(2)$	$\hat{\tau}$
0.077	0.077	13.06	12.66	12.90
0.087	0.062	13.11	12.97	13.05
0.097	0.047	13.22	12.94	13.10
0.117	0.017	13.38	12.84	13.16

Table 7: The table reports the simulated average waiting time (years) between successive recapitalisation events for different values of the counter-cyclical capital requirements $x_{R,1}$ and $x_{R,2}$, when the dividend tax parameter is $\beta_2 = 0.87$. The state-contingent waiting time is defined as $\hat{\tau}(i) := \inf\{t \geq \tau_n : \hat{X}_t = x_{R,i}, \hat{X}_{\tau_n} = x_i^*\}$, while the ex-ante (overall) waiting time is given by $\hat{\tau} := \sum_{i=1}^{2} \hat{\tau}(i) \cdot \lambda_{3-i}/(\lambda_1 + \lambda_2)$.

choose to reduce its loan exposure by a constant $\theta \in [0, 1]$ in the bad state (the remainder of the bank's assets earns the reserves/deposits rate ρ and carries no risk). Furthermore, we allow recapitalisation costs to vary across states, being higher in the bad state ($\kappa_2 \geq \kappa_1$).[22]

In this more general framework, the dynamics of the bank's net cash flow accruing to reserves in (2) is modified as follows:

$$\mu_{I_t}^{\theta} dt + \sigma_{I_t}^{\theta} dW_t, \tag{56}$$

where

$$\mu_{I_t}^{\theta} := \bar{\mu}_{I_t} - \theta \mathbf{1}_{\{I_t=2\}} \left(\bar{\mu}_{I_t} - \rho\right) - \rho D, \qquad \sigma_{I_t}^{\theta} := \left(1 - \theta \mathbf{1}_{\{I_t=2\}}\right) \sigma_{I_t}. \tag{57}$$

Consequently, the auxiliary process in (8) becomes

$$\tilde{X}_t^{(n)} = \tilde{X}_{\tau_n}^{(n)} + \int_{\tau_n}^{t} \left(\left(\rho \tilde{X}_s^{(n)} + \mu_{I_s}^{\theta} \right) ds + \sigma_{I_s}^{\theta} \, dW_s \right) - Z_t^{(n)}, \quad t \geq \tau_n. \tag{58}$$

In the same spirit of Klimenko and Moreno-Bromberg (2016), the bank chooses the loan adjustment θ to maximise the *ex-ante* value of its equity, net of the initial investment in

[22] A standard condition that ensures that the dividend control problem is well-defined and that the value function is finite is that the shareholders' discount rate exceeds the return on retained reserves (see e.g. Rochet and Villeneuve, 2005). We thus assume that the discount rate $\delta > \rho$. The condition is trivially satisfied in the baseline model as reserves are not remunerated.

liquid reserves, minus a quadratic loan-adjustment cost:

$$W(\theta) := (V(\tilde{x}_1, 1; \theta) - \tilde{x}_1)\,\mathbb{P}\{i = 1\} + (V(x_2^*, 2; \theta) - x_2^*)\,\mathbb{P}\{i = 2\} - \xi(L\theta)^2, \tag{59}$$

where $\xi \in \mathbb{R}^+$.[23] The rest of the model remains as described in Section 3.

5.4.1 Extended model solution

To solve the extended model, we set $x_{R,1} = x_{R,2} = x_R$ and conjecture that the value function is concave as in the baseline case. Then, we follow the same steps as in Section 4.1, postulating that $x_R < \tilde{x}_2^* < \tilde{x}_1 < \tilde{x}_2$. We thus look for a candidate value function $v = V$ that satisfies the following ODE system:

$$\begin{cases} \dfrac{(\sigma_1^\theta)^2}{2} v''(x,1) + \left(x\rho + \mu_1^\theta\right) v'(x,1) - \delta v(x,1) - \lambda_1\left[v(x,1) - v(x,2)\right] = 0, & x \in (x_R, \tilde{x}_1), \\ \dfrac{(\sigma_2^\theta)^2}{2} v''(x,2) + \left(x\rho + \mu_2^\theta\right) v'(x,2) - \delta v(x,2) - \lambda_2\left[v(x,2) - v(x,1)\right] = 0, & x \in (x_R, \tilde{x}_2), \end{cases} \tag{60}$$

with boundary conditions

$$v(x_R, i) = \max\left\{v(x_i^*, i) - (x_i^* - x_R) - \kappa_i,\, \Gamma_i^+\right\}, \qquad i = 1, 2, \tag{61}$$

$$v'(\tilde{x}_1, 1) = 1, \tag{62}$$

$$v'(\tilde{x}_2, 2) = \beta_2, \tag{63}$$

where $x_1^* = \tilde{x}_1$. The recapitalisation target and dividend payout thresholds (x_2^*, $\tilde{x}_1$, and $\tilde{x}_2$) are pinned down by the optimality condition $v(x_2^*, 2) = 1$ and the super-contact condition $v''(\tilde{x}_i, i) = 0$ for $i = 1, 2$.

Unfortunately, the presence of the term $x\rho$ in (60) makes an analytical solution of the ODE system infeasible. We therefore resort to numerical methods to approximate $v(x, i)$.

[23] We note that, if $V(\cdot, i)$ is concave, $\tilde{x}_1$ and x_2^* correspond to the optimal initial reserve levels in the good and bad states, respectively.

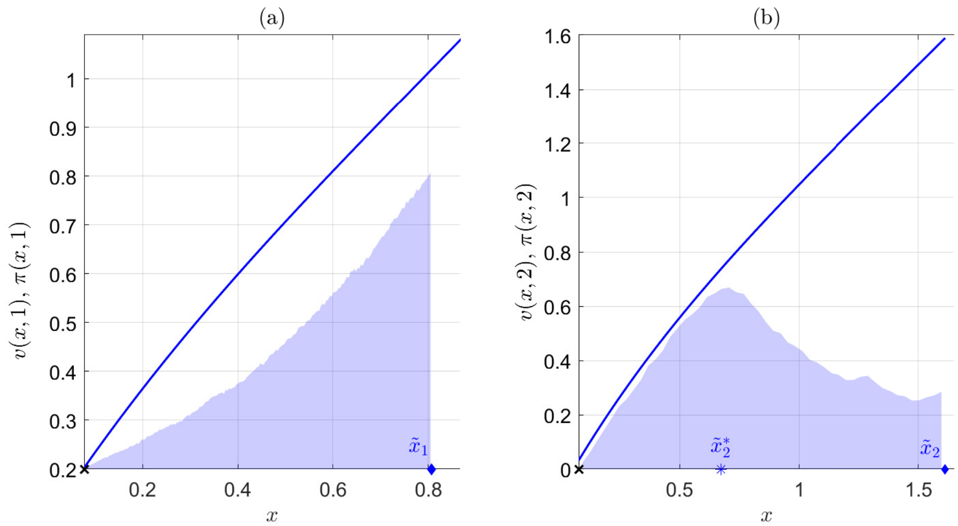

Figure 7: This figure displays numerical approximations of the bank's shareholder value ($v(x, i)$, solid blue line) and the stationary density of reserves ($\pi(x, i)$, shaded blue area) in the extended model as functions of the reserve level (x). Panel (a) refers to the good state ($i = 1$), while Panel (b) refers to the bad state ($i = 2$). The black cross on the x-axis marks the regulatory threshold (x_R). Blue diamonds indicate the optimal dividend payout thresholds ($\tilde{x}_i$) for $i = 1, 2$. The blue star denotes the optimal recapitalisation target in the bad state, (x_2^*). Simulations adopt the baseline parameters summarised in Table 1 and the loan adjustment $\theta = 0.1$.

Similarly, we approximate the stationary pdf $\pi(x, i)$ via Monte Carlo simulations. Figure 7 displays the outcome of our numerical approximation using the parameters in Table 1 and $\theta = 0.1$. Details of the numerical method and the MATLAB code for its implementation can be found in the supplementary material.

5.4.2 The effect of dividend taxes when reserves are remunerated

As a first step in the analysis, we set $\theta = 0$ and investigate how state-dependent recapitalisation costs and reserve remuneration affect our main results. Our simulations use the parametrisation of Table 1. Following Arnold et al. (2018), we set $\kappa_2 = 0.11$, which is roughly two percentage points higher than $\kappa_1 = 0.087$ in the good state of the baseline model.

Table 8 compares the optimal dividend payout thresholds ($\tilde{x}_1$ and $\tilde{x}_2$) and the optimal recapitalisation target (x_2^*) in the baseline model ($x\rho = 0$, $\kappa_2 = \kappa_1$) and the following three extensions: (i) reserves remuneration, constant costs ($x\rho > 0$, $\kappa_2 = \kappa_1$); (ii) time-varying

	$\tilde{x}_1$		$\tilde{x}_2$		x_2^*	
β_2	1.00	0.87	1.00	0.87	1.00	0.87
Baseline: $x\rho = 0$, $\kappa_1 = \kappa_2$	0.821	0.801	0.954	1.700	0.954	0.655
Extension (i): $x\rho > 0$, $\kappa_1 = \kappa_2$	0.854	0.832	0.994	2.032	0.994	0.681
Extension (ii): $x\rho = 0$, $\kappa_2 > \kappa_1$	0.832	0.813	1.012	1.752	1.012	0.707
Extension (iii): $x\rho > 0$, $\kappa_2 > \kappa_1$	0.866	0.846	1.053	2.084	1.053	0.735

Table 8: The table compares the optimal dividend thresholds, $\tilde{x}_i$, and the recapitalisation target, x_2^*, in the bad state under the baseline model with Extensions (i) (remunerated reserves), Extension (ii) (state-dependent recapitalisation costs), and Extension (iii) (remunerated reserves *and* state-dependent recapitalisation costs) for different values of the dividend restriction parameter β_2. Simulations adopt the baseline parameters in Table 1, with $\kappa_1 = \kappa_2 = \kappa$ in Extension (i) and $\kappa_2 = 0.11$ in Extensions (ii) and (iii).

costs ($x\rho = 0$, $\kappa_2 > \kappa_1$); (iii) reserves remuneration, time-varying costs ($x\rho > 0$, $\kappa_2 > \kappa_1$), with and without dividend taxes ($\beta = 1$ vs $\beta = 0.87$). As intuition suggests, remuneration of the bank's reserves makes capital buffer accumulation more attractive, thereby shifting all dividend payout thresholds and the recapitalisation target to the right (Row 2). Higher recapitalisation costs in the bad state generate a similar effect, due to the precautionary motive to avoid costlier equity issuance (Row 3). Notably, neither the isolated nor the joint effect of these two forces affects the qualitative outcome of the dividend restriction policy inferred from the baseline model analysis (Row 4).

5.4.3 The effects of dividend taxes on bank risk taking

Next, we investigate the effect of dividend taxes on the bank's risk-taking (i.e., loans adjustment) decisions. To this end, we follow steps similar to those used to compute the optimal (static) capital structure in Leland (1994). First, we fix θ and solve the bank's optimal control problem to compute its value function $V(x, i; \theta)$. Second, we numerically compute the time-zero loan adjustment θ^* that maximises (59) for two scenarios: $\beta_2 = 1$ (benchmark) and $\beta_2 < 1$ (tax policy), and compare the results. For this exercise, we use the same parameters of Section 5.4.2 and set ξ to target the average observed annual drop in the stock of commercial loans during the Great Financial Crisis (around 5% according to

	θ^*		$\tilde{x}_1(\theta^*)$		$\tilde{x}_2(\theta^*)$		$x_2^*(\theta^*)$	
β_2	1.00	0.87	1.00	0.87	1.00	0.87	1.00	0.87
Baseline: $x\rho = 0$, $\kappa_1 = \kappa_2$	5%	2%	0.818	0.799	0.896	1.673	0.896	0.648
Extension (i): $x\rho > 0$, $\kappa_1 = \kappa_2$	5%	2%	0.850	0.830	0.933	1.998	0.933	0.674
Extension (ii): $x\rho = 0$, $\kappa_2 > \kappa_1$	6%	3%	0.826	0.812	0.937	1.710	0.937	0.697
Extension (iii): $x\rho > 0$, $\kappa_2 > \kappa_1$	6%	3%	0.860	0.843	0.974	2.032	0.974	0.723

Table 9: The table compares the optimal dividend thresholds, $\tilde{x}_i$, the optimal recapitalisation target, x_2^*, and the optimal loan adjustment, θ^*, under the baseline model with Extensions (i) (remunerated reserves), Extension (ii) (state-dependent recapitalisation costs), and Extension (iii) (remunerated reserves *and* state-dependent recapitalisation costs) for different values of the dividend restriction parameter β_2. Simulations adopt the baseline parameters in Table 1, with $\kappa_1 = \kappa_2 = \kappa$ in Extension (i) and $\kappa_2 = 0.11$ in Extensions (ii) and (iii).

FRED) in the benchmark scenario.

Table 9 reports the optimal controls $(\tilde{x}_1, \tilde{x}_2, x_2^*)$ and the optimal loan adjustment (θ^*) in the baseline model and extended models (i)-(iii). In line with the empirical evidence motivating this paper (Hardy, 2021; Couaillier et al., 2025), the simulations show that dividend taxes reduce the bank's optimal loan adjustments (i.e., contraction) relative to the baseline case by more than half (Row 1). Intuitively, this occurs because higher capital buffers dampen the bank's precautionary motive against costly default. Consistently, higher recapitalisation costs in the bad state prompt banks to further reduce loans in that state, while moderating the relative impact of the dividend tax (Rows 2 and 4). Notably, remunerating the bank's reserves does not qualitatively affect this outcome (Row 3).

6 Conclusion

We have modelled and solved the optimal control problem of a bank which takes dividends and re-capitalisation decisions under macroeconomic uncertainty, cyclical capital requirements, and dividend restrictions (taxes and bans). Our framework provides several testable policy implications, complementing recent empirical literature on the (short-term) effects of bank dividend suspension policies in the EU and the US.

First, the model predicts that state-contingent dividend taxes (or bans) negatively affect shareholder value not only during crises (ex post) but also in good times (ex ante). Second, taxing dividends in bad macroeconomic states incentivises the bank to pay out more in good times, reducing its corresponding capital buffers (credit capacity). This creates a trade-off between maintaining capital buffers (credit capacity) in good versus bad macroeconomic conditions. Third, dividend taxes may lead to dispersion in the bank's capital buffers over the long term. Furthermore, they may undermine financial stability by diminishing the bank's recapitalisation incentives. Policymakers can coordinate dividend restrictions with counter-cyclical capital requirements to reallocate value losses and credit capacity between good and bad states. However, lower capital requirements could potentially exacerbate disincentives for recapitalisation.

The tractability assumptions of our baseline model carry a few limitations. For example, our bank's optimisation problem does not include investments and assumes fixed loans and deposits. As a result, even though our analysis of credit capacity proxies the potential support the bank may provide to the real economy under different policies, it does not fully capture how that interacts with its risk-taking incentives. To partially address this issue, the paper develops an extended version of the model that allows the bank to adjust its loans in the bad state. The optimal (static) loan adjustment is chosen to maximise shareholders' ex-ante value, net of their initial investment. We have used numerical simulations to show that, by encouraging the accumulation of additional capital buffers and thereby reducing the bank's precautionary motive, dividend restrictions can mitigate the bank's optimal loan contraction. A more comprehensive analysis of the relationship between bank risk-taking and dividend restrictions would require a model in which the risky assets explored are adjusted dynamically, as in Rochet and Villeneuve (2005). Another limitation of our model is that the bank's optimal responses perfectly anticipate the policy the regulator will enact in each possible state. In reality, regulatory uncertainty poses interesting nuances to our problem for two reasons. First, the regulator may refrain from acting when the economy deteriorates.

This can be accounted for by allowing for the possibility that, in the bad state, the policy is enacted only with a certain likelihood. Solving such an extension for our model would require a numerical implementation, in a set up similar to Kakhbod et al. (2024). Second, the regulator may imperfectly observe the state of the economy. The arising incomplete information framework, where the macroeconomic state is unobservable (see e.g. Huang et al., 2019; De Angelis, 2020) could be used to study the optimal policy from the regulator's point of view, which we believe is an interesting avenue for future research.

Finally, we believe this paper can serve as a first step towards understanding the general equilibrium implications of the joint adoption of dividend and capital regulations by focusing on the endogenous response of a single bank to a system-wide restriction. Although we interpret the stationary distribution of reserves (capital buffers) as representing heterogeneity across similar banks, analysing how a pool of heterogeneous banks responds to this policy mix is a complex and interesting research question that naturally follows from our contribution.

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A Proofs and derivations

A.1 Proof of Proposition 1

We prove the three points of the proposition in turn.

Point 1. The proof is organised into two steps. As a first step, we construct the function $v(x,i)$ sequentially over the intervals $[\tilde{x}_i,\infty)$ for $i=1,2$, $(\tilde{x}_1,\tilde{x}_2)$, and $(x_R,\tilde{x}_1)$, and we verify that it is strictly concave in the intervals $(\tilde{x}_1,\tilde{x}_2)$ for $i=1,2$ and $(x_R,\tilde{x}_1)$.

(i) $x\in[\tilde{x}_i,\infty)$ for $i=1,2$. Integrating $v'(\cdot,i)=\beta_i$ in $[\tilde{x}_i,x]$ we obtain

$$\int_{\tilde{x}_i}^{x} v(s,i)ds = v(x,i)-v(\tilde{x}_i,i)=\beta_i\left(x-\tilde{x}_i\right). \tag{64}$$

Setting $K_i:=v(\tilde{x}_i,i)>0$ yields the first equations in (22) and (23) as they appear in the main text.

(ii) $x\in(\tilde{x}_1,\tilde{x}_2)$. In this interval, we require that $v'(\cdot,1)$ solves (20). We guess and verify (i.e., substitute the guess in the ODE and matching coefficients) that the solution has the following structure:

$$v'(x,2)=\frac{\lambda_2}{\delta+\lambda_2}+\sum_{h=1}^{2}\tilde{\alpha}_h\tilde{A}_h e^{\tilde{\alpha}_h(x-\tilde{x}_1)}. \tag{65}$$

with $\tilde{\alpha}_i$ as in (25). By imposing $v(\cdot,2)\in C^2$ at $x=\tilde{x}_2$, we obtain that $v'(\tilde{x}_2,2)=\beta_2$ and $v''(\tilde{x}_2,2)=0$. Using (65) to enforce these conditions reflect in the following equations:

$$\begin{cases}\sum_{h=1}^{2}\tilde{\alpha}_h\tilde{A}_h e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)}=\beta_2-\frac{\lambda_2}{\delta+\lambda_2},\\ \sum_{h=1}^{2}\tilde{\alpha}_h^2\tilde{A}_h e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)}=0,\end{cases} \tag{66}$$

as they appear in the system (26). Integrating (65) over $[x,\tilde{x}_2]$ we obtain

$$v(x,2)=v(\tilde{x}_2,2)+\frac{\lambda_2}{\delta+\lambda_2}\left(x-\tilde{x}_2\right)+\sum_{h=1}^{2}\tilde{A}_h\left(e^{\tilde{\alpha}_h(x-\tilde{x}_1)}-e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)}\right), \tag{67}$$

which yields the second equation of System (23) by evaluating (64) at $i=2$ and $\tilde{x}_2$, verifying $K_2:=v(\tilde{x}_2,2)>0$.

To show that $v(\cdot,2)<0$ is strictly concave in $(\tilde{x}_1,\tilde{x}_2)$, we set $u(s)=v'(\tilde{x}_2-s,2)$ and

$w(s) = v''(\tilde{x}_2 - s, 2)$. Then, by using Eq. (20) with boundary conditions $v'(\tilde{x}_2, 2) = \beta_2$ and $v''(\tilde{x}_2, 2) = 0$ we obtain the following dynamic system:

$$\begin{cases} u'(s) = -w(s), \qquad u(0) = \beta_2, \\ w'(s) = \frac{2}{\sigma_2^2}\left[\mu_2 w(s) - (\delta+\lambda_2)u(s) + \lambda_2\right], \qquad w(0) = 0. \end{cases}$$

Given that $(\delta+\lambda_2)\beta_2 > \lambda_2$, an analysis of the system shows that $w'(s) < 0$ for $s > 0$. This implies that $v'''(\cdot, 2) > 0$ in $(\tilde{x}_1, \tilde{x}_2)$.

(iii) $x \in (x_R, \tilde{x}_1)$. In this interval the functions $v'(\cdot, i)$ satisfy the system (21) with boundary conditions

$$v'(\tilde{x}_1, 1) = 1, \quad v''(\tilde{x}_1, 1) = 0, \quad v'(\tilde{x}_1^-, 2) = v'(\tilde{x}_1^+, 2), \quad v''(\tilde{x}_1^-, 2) = v''(\tilde{x}_1^+, 2). \tag{68}$$

By plugging the guesses

$$v'(x, 1) = \sum_{j=1}^{4} A_j \alpha_j e^{\alpha_j(x-\tilde{x}_1)} \text{ and } v'(x, 2) = \sum_{j=1}^{4} B_j \alpha_j e^{\alpha_j(x-\tilde{x}_1)} \tag{69}$$

into (21) and matching coefficients, we obtain the following characteristic equations:

$$A_j\left(\alpha_j\mu_1 + \frac{\sigma_1^2}{2}\alpha_j^2 - (\delta+\lambda_1)\right) + \lambda_1 B_j = 0,$$

$$B_j\left(\alpha_j\mu_2 + \frac{\sigma_2^2}{2}\alpha_j^2 - (\delta+\lambda_2)\right) + \lambda_2 A_j = 0,$$

for $j = 1, 2, 3, 4$. Solving the first equation for B_j, substituting it into the latter equation, and rearranging yields (24) as it appears in the Proposition. To verify that (24) has four real roots, let us define

$$f(\theta) := \underbrace{\left(\delta+\lambda_1 - \theta\mu_1 - \frac{\sigma_1^2}{2}\theta^2\right)}_{:=G_1(\theta)} \underbrace{\left(\delta+\lambda_2 - \theta\mu_2 - \frac{\sigma_2^2}{2}\theta^2\right)}_{:=G_2(\theta)} - \lambda_1\lambda_2,$$

and let θ_j^i be the roots of $G_i(\theta_j)$. It is straightforward to verify that $f(0) > 0$, $f(\infty) > 0$, $f(-\infty) > 0$, and $f(\theta_j^i) = -\lambda_1\lambda_2 < 0$ for $i = 1, 2$ and $j = 1, 2, 3, 4$. Then, by continuity and using that $\theta_1^i\theta_2^i = -2\left(\delta + \lambda_i\right)/\sigma_i^2 < 0$, (24) has four different four roots, two positive and two negative. Using (65) and (69) and their derivatives,

$$v''(x,1) = \sum_{j=1}^{4} A_j\alpha_j^2 e^{\alpha_j(x-\tilde{x}_1)}, \quad v''(x,2) = \sum_{j=1}^{4} B_j\alpha_j^2 e^{\alpha_j(x-\tilde{x}_1)}, \tag{70}$$

and

$$v''(x,2) = \sum_{h=1}^{2} \tilde{\alpha}_h^2 \tilde{A}_h e^{\tilde{\alpha}_h(x-\tilde{x}_1)}, \tag{71}$$

to enforce the boundary conditions (68) entail the following equations:

$$\begin{cases} \sum_{j=1}^4 A_j\alpha_j = 1, \\ \sum_{j=1}^4 A_j\alpha_j^2 = 0, \\ \sum_{j=1}^4 B_j\alpha_j = \frac{\lambda_2}{\delta+\lambda_2} + \sum_{h=1}^2 \tilde{A}_h\tilde{\alpha}_h, \\ \sum_{j=1}^4 B_j\alpha_j^2 = \sum_{h=1}^2 \tilde{A}_h\tilde{\alpha}_h^2, \end{cases} \tag{72}$$

as they appear in (26). Integrating (69) in $[x, \tilde{x}_1]$ we obtain

$$v(x,1) = v(\tilde{x}_1, 1) + \sum_{j=1}^{4} A_j\left(e^{\alpha_j(x-\tilde{x}_1)} - 1\right), \tag{73}$$

$$v(x,2) = v(\tilde{x}_1, 2) + \sum_{j=1}^{4} B_j\left(e^{\alpha_j(x-\tilde{x}_1)} - 1\right). \tag{74}$$

Using (73) and evaluating (64) with $i = 1$ at $\tilde{x}_1$ yields $K_1 := v(\tilde{x}_1, 1) > 0$ and, in turn, the second equation of System (22). Using (74) together with (67) to impose the continuity condition $v(\tilde{x}_1^-, 2) = v(\tilde{x}_1^+, 2)$ gives

$$v(\tilde{x}_1, 2) = K_2 + \frac{\lambda_2}{\delta+\lambda_2}\left(\tilde{x}_1 - \tilde{x}_2\right) + \sum_{h=1}^{2} \tilde{A}_h\left(1 - e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)}\right), \tag{75}$$

which can then be substituted into (67) to obtain the third equation of System (23).

To identify the free parameters K_1 and K_2, we impose the following conditions:

$$\mathcal{L}_i v(\tilde{x}_i, i) - \lambda_i \left[v(\tilde{x}_i, i) - v(\tilde{x}_i, 3-i)\right] = 0, \quad i = 1, 2,$$

which are obtained by passing the equality $\mathcal{L}_i v(x, i) - \lambda_i \left[v(x, i) - v(x, 3-i)\right] = 0$ to the limit as $x \to \tilde{x}_i^-$. Since $v'(\tilde{x}_i, i) = \beta_i$ and $v''(\tilde{x}_i, i) = 0$, they rewrite as

$$\mu_2\beta_2 - (\delta + \lambda_2)K_2 + \lambda_2(K_1 + \tilde{x}_2 - \tilde{x}_1) = 0, \tag{76}$$

and

$$\mu_1 - (\delta + \lambda_1)K_1 + \lambda_1 \left(K_2 + \frac{\lambda_2}{\delta + \lambda_2}(\tilde{x}_1 - \tilde{x}_2)\right) = 0, \tag{77}$$

which completes System (26).

To show that $v(\cdot, i)$ is strictly concave in $(x_R, \tilde{x}_1)$ under (27) we use that, in this interval, the functions $u(\cdot, i) := v'''(\cdot; i)$ solve the following ODE system:

$$\begin{cases} \frac{1}{2}\sigma_1^2 u''(x,1) + \mu_1 u'(x,1) - (\delta + \lambda_1)\, u(x,1) + \lambda_1 u(x,2) = 0, \\ \\ \frac{1}{2}\sigma_2^2 u''(x,2) + \mu_2 u'(x,2) - (\delta + \lambda_2)\, u(x,2) + \lambda_2 u(x,1) = 0. \end{cases} \tag{78}$$

The Feynman-Kac representation of u provides

$$u(x, i) = \mathbb{E}\left[e^{-\delta\tau} u(X_\tau^{x,i,\circ}, I_\tau^i)\right], \tag{79}$$

where $\tau = \inf\{t \geq 0 : \; X_t^{x,i,\circ} \notin (x_R, \tilde{x}_1)\}$, being $X_t^{x,i,\circ}$ the solution to

$$dX_t = \mu_{I_t} dt + \sigma_{I_t} dW_t, \qquad X_0^{x,i,\circ} = 0.$$

Condition (27) entails $u(x_R, i) > 0$ and $u(\tilde{x}_1, i) > 0$ for all $i = 1, 2$. Thus, from (79) we get $u(\cdot, i) > 0$. Hence, $v''(\cdot, i)$ is strictly increasing on $(x_R, \tilde{x}_1)$ for $i = 1, 2$. Since $v''(\tilde{x}_1^-, 1) = 0$ and $v''(\tilde{x}_1^-, 2) < 0$, we get that v'' is negative on $(x_R, \tilde{x}_1)$ and the claim follows. Next, we show that

$$v'(\tilde{x}_1, 2) < \frac{\lambda_1 + \delta}{\lambda_1}. \tag{80}$$

In the interval $(x_R, \tilde{x}_1)$, the function $v'(\cdot; 1)$ solve

$$\frac{1}{2}\sigma_1^2 v'''(x, 1) + \mu_1 v''(x, 1) - (\delta + \lambda_1)\, v'(x, 1) + \lambda_1 v'(x, 2) = 0. \tag{81}$$

By (27), we have $v'''(\tilde{x}_1^-, 1) > 0$. Recalling also that $v''(\tilde{x}_1, 1) = 0$ and $v'(\tilde{x}_1, 1) = 1$, plugging all these information into (81), and passing to the limit as $x \to \tilde{x}_1^-$, we get $\lambda_1 v'(\tilde{x}_1, 2) > (\lambda_1 + \delta)$, which verifies the claim.

As a second step of the proof, we show that the solution constructed in the first step solves the HJBVI (15). Most of the work has already been done. Indeed, looking at the previous steps, we see that we have constructed v such that all the following are met: (a) $v(\cdot, i) \in C^2((x_R, \infty); \mathbb{R})$; (b) $v'(x, i) = \beta_i$ in $[\tilde{x}_i, \infty)$; (c) $\mathcal{L}_i v(\cdot, i) - \lambda_i (v(\cdot, i) - v(\cdot, 3-i)) = 0$ in $(x_R, \tilde{x}_i)$; (d) $v(\cdot, i)$ are concave, which entails $v'(\cdot, i) \geq \beta_i$ for $i = 1, 2$. So, it remains to show that

$$H(x, i) := \mathcal{L}_i v(x, i) - \lambda_i (v(x, i) - v(x, 3 - i)) \leq 0, \qquad \forall x \in [\tilde{x}_i, \infty), \quad i = 1, 2.$$

First, we prove that $H(x, 2) \leq 0$ in $[\tilde{x}_2, \infty)$. Indeed, by (76), we have $H(\tilde{x}_2, 2) = 0$. Recalling that $v'(x, i) = \beta_i$ for $x \geq \tilde{x}_2$ and using that $\beta_2 > \lambda_2/(\lambda_2 + \delta)$, we have

$$H'(x, 2) = -(\lambda_2 + \delta)\beta_2 + \lambda_2 < 0, \quad \forall x \geq \tilde{x}_2.$$

Next, we prove that $H(x, 1) \leq 0$ in $[\tilde{x}_1, \infty)$. Indeed, by (77), we have $H(\tilde{x}_1, 1) = 0$.

Recalling that $v'(x,1) = 1$ for $x \in [\tilde{x}_1, \infty)$ and using concavity of $v(\cdot, 2)$ and (80), we get

$$H'(x,1) = -(\lambda_1 + \delta) + \lambda_1 v'(x,2) < 0, \quad \forall x \geq \tilde{x}_1.$$

Point 2. We obtain Eq. (28) by using (74) to impose

$$v'(x_R^+, 2) = \sum_{j=1}^{4} \alpha_j B_j e^{\alpha_j (x_R - \tilde{x}_1)} > 1. \tag{82}$$

Eq. (29) is obtained using (67) and (74) to enforce the optimality condition from (19):

$$v'(x_2^*, 2) = \sum_{j=1}^{4} \alpha_j B_j e^{\alpha_j (x_2 - \tilde{x}1)} = 1, \text{ if } x \in (x_R, \tilde{x}_1), \tag{83}$$

or

$$v'(x_2^*, 2) = \frac{\lambda_2}{\delta + \lambda_2} + \sum_{h=1}^{2} \tilde{A}_h \tilde{\alpha}_h e^{\tilde{\alpha}_h \left(x_2^* - \tilde{x}_1\right)} = 1, \text{ if } x \in (\tilde{x}_1, \tilde{x}_2). \tag{84}$$

The fact that (29) admits a unique solution follows from the strict concavity of $v(\cdot, 2)$, as established in Point 1, together with $v'(\tilde{x}_2, 2) = \beta_2 \leq 1$ and (82).

Point 3. This is straightforward, since (30) is simply a rewriting of the boundary conditions $v(x_R, i) = v(x_i^*, i) - (x_i^* - x_R) - \kappa$ for $i = 1, 2$ (see Eq. (19)) using Eqs. (64), (67), (73), and (74). Namely,

$$K_1 + \sum_{j=1}^{4} A_j \left(e^{\alpha_j (x_R - \tilde{x}_1)} - 1\right) = K_1 - (x_i^* - x_R) - \kappa \tag{85}$$

for $i = 1$, and

$$\sum_{j=1}^{4} B_j \left(e^{\alpha_j (x_R - \tilde{x}_1)} - 1\right) = \sum_{j=1}^{4} B_j \left(e^{\alpha_j (x_2^* - \tilde{x}_1)} - 1\right) - (x_2^* - x_R) - \kappa, \tag{86}$$

if $x_2^* \in [x_R, \tilde{x}_1)$, and

$$\frac{\lambda_2(\tilde{x}_1 - \tilde{x}_2)}{\delta + \lambda_2} + \sum_{j=1}^{2} \tilde{A}_j (1 - e^{\tilde{\alpha}_j(\tilde{x}_2 - \tilde{x}_1)}) + \sum_{j=1}^{4} B_j \left(e^{\alpha_j(x_R - \tilde{x}_1)} - 1 \right) =$$
$$= \frac{\lambda_2(x_2^* - \tilde{x}_2)}{\delta + \lambda_2} + \sum_{j=1}^{2} \tilde{A}_j \left(e^{\tilde{\alpha}_j(x_2^* - \tilde{x}_1)} - e^{\tilde{\alpha}_j(\tilde{x}_2 - \tilde{x}_1)} \right) - (x_2^* - x_R) - \kappa. \quad (87)$$

if $x_2^* \in [\tilde{x}_1, \tilde{x}_2)$ for $i = 2$.

A.2 Proof of Theorem 1

We only sketch the proof, as a rigorous argument would be unnecessarily technical. We refer the reader to two papers that deal with similar problems and provide complete proofs in similar settings: Løkka and Zervos (2008), in the case of no regime switching and no recapitalisation; and Ferrari et al. (2022), in the case of regime switching but with no recapitalisation or dividend taxes.

We divide the sketch of the proof in two steps.

Step 1 Let $A = (Z, (b, G)) \in \mathcal{A}$ be an arbitrary control and define $\tau_0 := 0$ and, recursively on $n \geq 0$, $\tau_{n+1} = \inf\{t \geq \tau_n : \ X_{t^-} = x_R\}$, being X_t the associated state process. Then, we have, by verification arguments in the interval $[0, \tau_1)$ (see Ferrari et al., 2022)

$$v(x, i) \geq \mathbb{E} \left[\int_0^{\tau_1^-} e^{-\delta t} \beta_{I_t} dZ_t + e^{-\delta \tau_1} v(x_R, I_{\tau_1}) \right]. \quad (88)$$

Using (88) and the boundary condition (19) we get

$$v(x, i) \geq \mathbb{E} \left[\int_0^{\tau_1^-} e^{-\delta t} \beta_{I_t} dZ_t + e^{-\delta \tau_1} \left(v(x_R + G_1; I_{\tau_1}) - G_1 - \kappa \right) \right].$$

Iterating the argument yields

$$v(x,i) \geq \mathbb{E}\left[\int_0^{\tau_n^-} e^{-\delta t}\beta_{I_t} dZ_t - \sum_{k=1}^{n} e^{-\delta\tau_k}\left(G_k + \kappa\right) + e^{-\delta\tau_n} v(x_R, I_{\tau_n})\right].$$

Letting $n \to \infty$ and observing that $\tau_n \to \infty$, we get $v(x,i) \geq J(x,i;A)$. Since A was arbitrarily chosen, we obtain that $v(x,i) \geq V(x,i)$ and conclude this part of the proof.

Step 2 Take $A = \hat{A}$ in Step 1. Then, by construction, all previous inequalities become equalities, which allows us to conclude that $v(x,i) = J(x,i;\hat{A})$. This last condition entails $J(x,i;\hat{A}) = v(x,i) = V(x,i)$, which completes the proof.

A.3 Proof of Proposition 2

In this appendix, we characterize the stationary density of bank reserves as a continuous, piecewise-defined function over the intervals (x_R, x_i^*) and $(x_i^*, \tilde{x}_i)$ for $i = 1, 2$. First, we adopt a guess-and-verify approach based on a sum of basis functions that satisfies KFEs system (35). Second, we impose appropriate boundary, value-matching, and smooth-pasting conditions at x_i^*, $\tilde{x}_i$, and x_R to determine the coefficients of the basis functions.

Step 1 In the region $(x_R, \tilde{x}_2^*)$, the stationary pdf $\pi(x,i)$ must satisfy the following coupled system of ODEs:

$$\begin{cases} -\mu_1\pi'(x,1) + \frac{\sigma_1^2}{2}\pi''(x,1) - \lambda_1\pi(x,1) + \lambda_2\pi(x,2) = 0, \\ -\mu_2\pi'(x,2) + \frac{\sigma_2^2}{2}\pi''(x,2) - \lambda_2\pi(x,2) + \lambda_1\pi(x,1) = 0. \end{cases} \tag{89}$$

Since this system is linear and homogeneous, we conjecture that its solution takes the following form:

$$\pi(x,1) = \sum_{j=1}^{4} P_j e^{r_j x}, \quad \pi(x,2) = \sum_{j=1}^{4} Q_j e^{r_j x}. \tag{90}$$

Substituting (90) into (89) and matching coefficients yields the characteristic equation (44), which has four real roots $r_1 < r_2 < 0 < r_4 < r_4$ (see Appendix A.1, Point (iv), for details on these roots), and the relationship $Q_j = \lambda_2^{-1} F_1(r_j) P_j$.

In the region $(x_2^*, \tilde{x}_1)$, the stationary pdf also satisfies (89), albeit with different boundary conditions. Thus, we guess that its solution takes the form

$$\pi(x,1) = \sum_{j=1}^{4} \tilde{P}_j e^{r_j x}, \quad \pi(x,2) = \sum_{j=1}^{4} \tilde{Q}_j e^{r_j x}, \tag{91}$$

and verify that $\tilde{Q}_j = \lambda_2^{-1} F_1(r_j) \tilde{P}_j$.

In the regions $x \in (\tilde{x}_i, \infty)$, we must have $\pi(x,i) = 0$ because $\tilde{x}_i$ acts as a reflecting barrier (see below for a more detailed discussion of this condition). Therefore, since $\tilde{x}_1 > \tilde{x}_2$, Eq. (35) implies that, on the interval $(\tilde{x}_2, \tilde{x}_1)$, $\pi(x,2)$ must solve

$$\lambda_2 \pi(x,2) + \mu_2 \pi'(x,2) = \frac{\sigma_2^2}{2} \pi''(x,2). \tag{92}$$

We guess that its solution takes the following form:

$$\pi(x,2) = \sum_{j=1}^{2} H_j e^{s_j x}. \tag{93}$$

Substituting (93) into (92) and matching coefficients yields the characteristic equation (45), whose real roots identify $s_2 < 0 < s_1$. Matching (90), (91), and (93) yields (42) and (43) as stated in the proposition.

Step 2 To determine the unknown coefficients in Step 1 (P_j, $\tilde{P}_j$, and H_h for $j = 1,2,3,4$ and $h = 1,2$), we use (91) and (93) to impose the following ten conditions, which define a linear system of ten equations in ten unknowns.

1. No mass at the regulatory threshold in State 1 (corresponding to (36) with $i = 1$ in

the main text):

$$\sum_{j=1}^{4} P_j e^{r_j x_R} = 0, \tag{94}$$

2. No mass at the regulatory threshold in State 2 (corresponding to (36) with $i = 2$ in the main text):

$$\sum_{j=1}^{4} Q_j e^{r_j x_R} = 0. \tag{95}$$

3. Preservation of mass after recapitalisation in State 1 (corresponding to (39) in the main text after imposing (36)):

$$-\sum_{j=1}^{4} \tilde{P}_j e^{r_j \tilde{x}_1} \left(\mu_1 - \frac{\sigma_1^2}{2} r_j \right) = \frac{\sigma_1^2}{2} \sum_{j=1}^{4} P_j r_j e^{r_j x_R} + \lambda_2 \sum_{h=1}^{2} \frac{H_h}{s_h} \left(e^{s_h \tilde{x}_2} - e^{s_h \tilde{x}_1} \right), \tag{96}$$

4. Preservation of mass after recapitalisation in State 2 (corresponding to (38) in the main text after imposing (36) and (37)):

$$\sum_{j=1}^{4} r_j e^{r_j x_2^*} \left(Q_j - \tilde{Q}_j \right) = \sum_{j=1}^{4} r_j e^{r_j x_R} Q_j. \tag{97}$$

5. Preservation of mass after dividends (corresponding to (40) in the main text):

$$\mu_2 \sum_{h=1}^{2} H_h e^{s_h \tilde{x}_2} - \frac{\sigma_2^2}{2} \sum_{h=1}^{2} H_h s_h e^{s_h \tilde{x}_2} = 0. \tag{98}$$

6. Continuity (but no differentiability) at $x = x_2^*$ in State 2 (corresponding to (37) in the main text):

$$\sum_{j=1}^{4} Q_j e^{r_j x_2^*} = \sum_{j=1}^{4} \tilde{Q}_j e^{r_j x_2^*}. \tag{99}$$

7. Continuity (value matching) at x_2^* in State 1:

$$\sum_{j=1}^{4} P_j e^{r_j x_2^*} = \sum_{j=1}^{4} \tilde{P}_j e^{r_j x_2^*}. \tag{100}$$

8. Differentiability (smooth pasting) at x_2^* in State 1:

$$\sum_{j=1}^{4} r_j P_j e^{r_j x_2^*} = \sum_{j=1}^{4} r_j \tilde{P}_j e^{r_j x_2^*}. \tag{101}$$

9. Continuity (value matching) at $\tilde{x}_1$ in State 2:

$$\sum_{j=1}^{4} \tilde{Q}_j e^{r_j \tilde{x}_1} = \sum_{h=1}^{2} H_h e^{s_h \tilde{x}_1}. \tag{102}$$

10. Differentiability (smooth pasting) at $\tilde{x}_1$ in State 2:

$$\sum_{j=1}^{4} \tilde{Q}_j r_j e^{r_j \tilde{x}_1} = \sum_{h=1}^{2} H_h s_h e^{s_h \tilde{x}_1}. \tag{103}$$

Rearranging (94)-(103) yields System (46) as stated in the proposition.

A.4 Solution structure with dividend bans

First, we show that when $\beta_2 \leq \lambda_2/(\lambda_2 + \delta)$, the solution structure in Proposition 1 does not hold. To this end, we follow the steps in Appendix (A.1), Point 1 (i), to derive the linear system (66), which has the following unique solution:

$$\begin{bmatrix} \tilde{A}_1 \\ \tilde{A}_2 \end{bmatrix} = \left(\beta_2 - \frac{\lambda_2}{\delta + \lambda_2}\right) \begin{bmatrix} \frac{e^{\tilde{\alpha}_1(\tilde{x}_1 - \tilde{x}_2)}}{\tilde{\alpha}_1(\tilde{\alpha}_2 - \tilde{\alpha}_1)} \\ -\frac{e^{\tilde{\alpha}_2(\tilde{x}_1 - \tilde{x}_2)}}{\tilde{\alpha}_2(\tilde{\alpha}_2 - \tilde{\alpha}_1)} \end{bmatrix}.$$

Plugging these coefficients in the solution structure (65) in Point 1 (ii) and rearranging, we get

$$v'(x,2) = \frac{\lambda_2}{\delta + \lambda_2} + \underbrace{\frac{e^{-\tilde{\alpha}_1(\tilde{x}_2 - x)} - e^{-\tilde{\alpha}_2(\tilde{x}_2 - x)}}{\tilde{\alpha}_2 - \tilde{\alpha}_1}}_{>0} \underbrace{\left(\beta_2 - \frac{\lambda_2}{\delta + \lambda_2}\right)}_{\leq 0} \leq \frac{\lambda_2}{\delta + \lambda_2}.$$

Since $\tilde{\alpha}_1 < 0 < \tilde{\alpha}_2$ (see Eq. (25)), the above equation contrasts with the optimality condition that requires $v'(x,2) > \beta_2$ on $(\tilde{x}_1, \tilde{x}_2)$ (see Eq. (20)).

Second, we build an alternative solution of (15) in which $\tilde{x}_2 = \infty$, and show that it is consistent with the parametric restriction $\beta_2 \leq \lambda_2/(\lambda_2 + \delta)$. Following the same steps of Section 4.1, we look for a function v such that, for $i = 1, 2$, the following hold:

a) $v(\cdot, i) \in C([x_R, \infty)) \cap C^1((x_R, \infty))$;

b) The associated continuation and intervention regions have the following structure:

$$\mathcal{C}_2 = (x_R, \infty), \qquad \mathcal{C}_1 = (x_R, \tilde{x}_1); \quad \mathcal{S}_1 = [\tilde{x}_1, \infty);$$

c) $v(\cdot, 1) \in C^2(\mathcal{C}_1)$ and $v(\cdot, 2) \in C^2(\mathcal{C}_2 \setminus \{\tilde{x}_1\})$.

Accordingly, we have $\tilde{x}_1 = \inf\{x > x_R : v'(x, 1) \leq 1\}$ and $\tilde{x}_2 = \infty$. The boundary conditions at x_R are given by (19) in the main text.

Following the approach in Appendix A.1, we now construct a pair of functions $v(x, i)$ for $i = 1, 2$ that fulfil conditions a), b), and c) over the intervals $[x_R, \tilde{x}_1)$ and $[\tilde{x}_1, \infty)$. We then impose the usual optimality and regularity conditions at $\tilde{x}_1$ to translate them into a set of algebraic requirements that determine the constant coefficients.

In the interval $[\tilde{x}_1, \infty)$, we set $v'(\cdot, 1) = 1$ and

$$v'(x, 2) = \frac{\lambda_2}{\delta + \lambda_2} + \tilde{\alpha}_1 \tilde{A}_1 e^{\tilde{\alpha}_1 (x - \tilde{x}_1)}, \tag{104}$$

where $\tilde{\alpha}_1 < 0$ is the negative root of (25) and $\tilde{A}_1 < 0$. In the interval $(x_R, \tilde{x}_1)$, we set the functions $v'(\cdot, i)$ as unique solutions to the system (21). This entails the same structure as in (69) and, in turn, (73) and (74). Integrating (73), (74), $v'(x, 1) = 1$, and (104) in $[x, \tilde{x}_1)$ we get

$$v(x, 1) = K_1 + \begin{cases} (x - \tilde{x}_1), & x \in [\tilde{x}_1, \infty), \\ \sum_{j=1}^{4} A_j \left(e^{\alpha_j (x - \tilde{x}_1)} - 1\right), & x \in [x_R, \tilde{x}_1), \end{cases} \tag{105}$$

$$v(x,2) = K_2 + \begin{cases} \frac{\lambda_2}{\delta+\lambda_2}(x-\tilde{x}_1) + \tilde{A}_1\left(e^{\tilde{\alpha}_1(x-\tilde{x}_1)} - 1\right), & x \in [\tilde{x}_1, \infty), \\ \sum_{j=1}^4 B_j\left(e^{\alpha_j(x-\tilde{x}_1)} - 1\right), & x \in [x_R, \tilde{x}_1), \end{cases} \tag{106}$$

where $B_j = \lambda_2^{-1}F_1(r_j)A_j$ for $j = 1,2,3,4$, and $K_i := v(\tilde{x}_1, i) > 0$. Using these equations to enforce the optimality condition $v'(\tilde{x}_1, 1) = 1$, the super-contact condition $v''(\tilde{x}_1, 1) = 0$, the value matching condition $v(\tilde{x}_1^-, 2) = v(\tilde{x}_1^+, 2)$ and the smooth-pasting condition $v'(\tilde{x}_1^-, 2) = v'(\tilde{x}_1^+, 2)$ yields the following algebraic system for $(A_1, A_2, A_3, A_4) \in \mathbb{R}^4$:

$$\begin{cases} \sum_{j=1}^4 A_j\alpha_j = 1, \\ \sum_{j=1}^4 A_j\alpha_j^2 = 0, \\ \sum_{j=1}^4 B_j\alpha_j = \frac{\lambda_2}{\delta+\lambda_2} + \tilde{A}_1\tilde{\alpha}_1, \\ \sum_{j=1}^4 B_j\alpha_j^2 = \tilde{\alpha}_1^2\tilde{A}_1. \end{cases}$$

Under similar assumptions as Proposition 1 and using that $v(\cdot, i)$ is differentiable and using (106) to enforce the optimality condition $v(x_2^*, 2) = 1$, one gets that $G(1)^* = \tilde{x}_1 - x_R$ and $G(2)^* = x_2^* - x_R$, where $x_2^* \in (x_R, \infty)$ is the unique solution of

$$\mathbf{1}_{[x_R,\tilde{x}_1]}\sum_{j=1}^4 B_j\alpha_j e^{\alpha_j(x_2^*-\tilde{x}_1)} + \mathbf{1}_{(\tilde{x}_1,\infty)}\left(\frac{\lambda_2}{\delta+\lambda_2} + \tilde{\alpha}_1\tilde{A}_1 e^{\tilde{\alpha}_1\left(x_2^*-\tilde{x}_1\right)}\right) - 1 = 0.$$

To pin down the remaining coefficients ($K_1 > 0$, $K_2 > 0$, and $\tilde{A}_1 < 0$) and the dividend threshold ($\tilde{x}_1 > x_R$) we use (105) and (106) to enforce $\mathcal{L}_i v(x,i) - \lambda_1\left[v(x,i) - v(x,i)\right] = 0$ for

$i = 1, 2$ as $x \to \tilde{x}_1^-$, and to impose the two boundary conditions in (19), which rewrite as

$$\begin{cases} \mu_2 \sum_{j=1}^4 B_j \alpha_j + \frac{\sigma_2^2}{2} \sum_{j=1}^4 B_j \alpha_j^2 - (\delta + \lambda_2) K_2 + \lambda_2 K_1 = 0, \\ \mu_1 - (\delta + \lambda_1) K_1 + \lambda_1 K_2 = 0, \\ \sum_{j=1}^4 A_j \left(e^{\alpha_j (x_R - \tilde{x}_1)} - 1 \right) + (\tilde{x}_1 - x_R) + \kappa = 0, \\ \sum_{j=1}^4 B_j \left(e^{\alpha_j (x_R - \tilde{x}_1)} - 1 \right) - \left(\frac{\lambda_2}{\delta + \lambda_2} (x_2^* - \tilde{x}_1) + \tilde{A}_1 \left(e^{\tilde{\alpha}_1 (x_2^* - \tilde{x}_1)} - 1 \right) \right) \mathbf{1}_{[x_R, \tilde{x}_1)} + \\ - \left(\sum_{j=1}^4 B_j \left(e^{\alpha_j (x_2^* - \tilde{x}_1)} - 1 \right) \right) \mathbf{1}_{[\tilde{x}_1, \infty)} + (x_2^* - x_R) + \kappa = 0. \end{cases} \tag{107}$$

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