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Working Paper
The technology of skill formation

IZA Discussion Papers, No. 2550

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IZA DP No. 2550

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January 2007
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Discussion Paper No. 2550  
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ABSTRACT

The Technology of Skill Formation*

This paper develops a model of skill formation that explains a variety of findings established in the child development and child intervention literatures. At its core is a technology that is stage-specific and that features self productivity, dynamic complementarity and skill multipliers. Lessons are drawn for the design of new policies to alleviate the consequences of the accident of birth that is a major source of human inequality.

JEL Classification: J13, J24, I38

Keywords: self productivity, skill multiplier, dynamic complementarity, accident of birth

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* This research was supported by NIH R01-HD043411, NSF SES-024158, the Committee for Economic Development with a grant from The Pew Charitable Trusts and the Partnership for America's Economic Success, and the J.B. Pritzker Consortium on Early Childhood Development at the Harris School of Public Policy, University of Chicago. Flavio Cunha also acknowledges support from the Claudio Haddad dissertation fund at the University of Chicago. The views expressed in this paper are those of the authors and not necessarily those of the funders listed here. The first draft of this paper, written with Pedro Carneiro, was presented at a conference at the Minneapolis Federal Reserve, October 2003. We thank Gary S. Becker, Janet Currie and Greg J. Duncan for helpful comments. A website at http://jenni.uchicago.edu/tech-skill/ contains supporting material.
The Technology of Skill Formation
by Flavio Cunha and James Heckman*

It is well documented that people are diverse on a vast array of abilities, that these abilities account for a substantial portion of the variation found across people in their socioeconomic success, and that persistent ability gaps across children from various socioeconomic groups open up at early ages before children enter school. The family plays a powerful role in shaping these abilities through genetics, parental investments and through choice of child environments. From a variety of intervention studies, it is known that ability gaps in children from different socioeconomic groups can be reduced if remediation is attempted at early enough ages. The remediation efforts that appear to be most effective are those that supplement family environments for disadvantaged children. Cunha et al. (2006a), henceforth CHLM, present a comprehensive survey and discussion of this literature.

This paper uses a simple economic model of skill formation to organize this and other evidence summarized below and the findings of related literatures in psychology, education and neuroscience. The existing theoretical literature on child development in economics treats childhood as a single period (see, e.g., Becker and Nigel Tomes, 1986; S. Rao Aiyagari et al., 2002; Roland Benabou, 2002). The implicit assumption in this approach is that inputs into the production of skills at different stages of childhood are perfect substitutes. We argue that to account for a large body of evidence, it is important to build a model of skill formation with multiple stages of childhood, where inputs at different stages are complements and where there is self-productivity of investment. In addition, in order to rationalize the evidence, it is important to recognize three distinct credit constraints operating on the family and its children. (i) The inability of a child to choose its parents. This is the fundamental constraint imposed by the accident of birth. (ii) The inability of parents to borrow against their children’s future income to finance investments in them. (iii) The inability of parents to borrow against their own income to finance investments in their children.
This paper summarizes findings from the recent literature on child development and presents a model that explains them. A model that is faithful to the evidence must recognize that (a) parental influences are key factors governing child development; (b) early childhood investments must be distinguished from late childhood investments; (c) an equity-efficiency trade-off exists for late investments, but not for early investments; (d) abilities are created, not solely inherited, and are multiple in variety; (e) the traditional ability-skills dichotomy is misleading. Both skills and abilities are created; and (f) the “nature versus nurture” distinction is obsolete. These insights change the way we interpret evidence and design policy about investing in children. Point (a) is emphasized in many papers. Point (b) is ignored in models that consider only one period of childhood investment. Points (c), (d) and (e) have received scant attention in the formal literature on child investment. Point (f) is ignored in the literature that partitions the variance of child outcomes into components due to nature and components due to nurture.

I Observations About Human Diversity and Human Development and Some Facts Our Model Explains

Any analysis of human development must reckon with three empirically well established observations about ability. The first observation is that ability matters. A large number of empirical studies document that cognitive ability is a powerful determinant of wages, schooling, participation in crime and success in many aspects of social and economic life. The frenzy generated by Richard J. Herrnstein and Charles A. Murray’s book, *The Bell Curve*, because of its claims of genetic determinism, obscured its real message, which is that cognitive ability is an important predictor of socioeconomic success. (See, e.g., James J. Heckman, 1995, and Richard J. Murnane et al., 1995.)

A second observation, more recently established, is that abilities are multiple in nature. Noncognitive abilities (perseverance, motivation, time preference, risk aversion, self-esteem, self-control, preference for leisure) have direct effects on wages (controlling for schooling), schooling, teenage pregnancy, smoking, crime, performance on achievement tests and many other aspects of social and economic life. (Samuel Bowles et al., 2001; Lex Borghans et al., 2006; Heckman
The third observation is that the nature versus nurture distinction is obsolete. The modern literature on epigenetic expression teaches us that the sharp distinction between acquired skills and ability featured in the early human capital literature is not tenable (see, e.g., Leslie Pray, 2004). Additive “nature” and “nurture” models, while traditional and still used in many studies of heritability and family influence, mischaracterize how ability is manifested. Abilities are produced, and gene expression is governed by environmental conditions (Eric Turkheimer et al., 2003). Measured abilities are susceptible to environmental influences, including in utero experiences, and also have genetic components. These factors interact to produce behaviors and abilities that have both a genetic and an acquired character. Genes and environment cannot be meaningfully parsed by traditional linear models that assign variance to each component.

Taking these observations as established, we develop a simple economic model to explain the following six facts from the recent empirical literature. First, ability gaps between individuals and across socioeconomic groups open up at early ages, for both cognitive and noncognitive skills. See Figure 1 for a prototypical figure which graphs a cognitive test score by age of child by socioeconomic status of the family. CHLM present many graphs of child cognitive and noncognitive skills by age showing early divergence and then parallelism during school-going years across children with parents of different socioeconomic status. Levels of child skills are highly correlated with family background factors like parental education and maternal ability, which, when statistically controlled for, largely eliminate these gaps (see Carneiro and Heckman, 2003, CHLM and our website). Experimental interventions with long term followup confirm that changing the resources available to disadvantaged children improves their adult outcomes. See the studies surveyed in CHLM or David Blau and Currie (2006). Schooling quality and school resources have relatively small effects on ability deficits and have little effect on test scores by age across children from different socioeconomic groups as displayed in Figure 1 and related graphs (see Heckman et al., 2004; Stephen W. Raudenbush, 2006).

Second, in both animal and human species, there is compelling evidence of critical and
sensitive periods in the development of the child. Some skills or traits are more readily acquired at certain stages of childhood than other traits (see the evidence summarized in Eric I. Knudsen et al., 2006). For example, on average, if a second language is learned before age 12, the child speaks it without an accent (Elissa L. Newport, 1990). If syntax and grammar are not acquired early on, they appear to be very difficult to learn later on in life (Steven Pinker, 1994). A child born with a cataract on the eye will be blind if the cataract is not removed within the first year of life.

Different types of abilities appear to be manipulable at different ages. IQ scores become stable by age 10 or so, suggesting a sensitive period for their formation below age 10. (See Kenneth Hopkins and Glenn Brecht, 1975.) There is evidence that adolescent interventions can affect noncognitive skills (see CHLM). This evidence is supported in the neuroscience that establishes the malleability of the prefrontal cortex into the early 20s (Ronald E. Dahl, 2004). This is the region of the brain that governs emotion and self-regulation.

On average, the later remediation is given to a disadvantaged child, the less effective it is. A study by Thomas G. O’Connor et al. (2000) of adopted Romanian infants reared in severely deprived orphanage environments before their adoption supports this claim. The later the Romanian orphan was rescued from the social and emotional isolation of the orphanage, the lower was his or her cognitive performance at age 6. Classroom remediation programs designed to combat early cognitive deficits have a poor track record.

At historically funded levels, public job training programs and adult literacy and educational programs, like the GED, that attempt to remediate years of educational and emotional neglect among disadvantaged individuals have a low economic return and produce meager effects for most persons. A substantial body of evidence suggests that returns to adolescent education for the most disadvantaged and less able are lower than the returns for the more advantaged (Costas Meghir and Mårten Palme, 2001; Carneiro and Heckman, 2003, and the evidence they cite; Carneiro et al., 2006).

The available evidence suggests that for many skills and abilities, later remediation for
disadvantage may be possible but is much more costly than early remediation to achieve a
given level of adult performance (Cunha and Heckman, 2006b). The economic returns to job
training, high school graduation and college attendance are lower for less able persons. (See
Carneiro and Heckman, 2003, and the studies they cite.)

Third, despite the low returns to interventions targeted toward disadvantaged adolescents,
the empirical literature shows high economic returns for remedial investments in young disad-
vantaged children. See W. Steven Barnett (2004), the evidence in CHLM and the papers they
cite. This finding is a consequence of dynamic complementarity and self-productivity captured
by the technology developed in the next section.

Fourth, if early investment in disadvantaged children is not followed up by later investment,
its effect at later ages is lessened. Investments appear to be complementary and require follow
up to be effective. Currie and Duncan Thomas (1995) document a decline in the performance
of Head Start minority participants after they leave the program, return to disadvantaged
environments, and receive the low levels of investment experienced by many disadvantaged
children.\^7

Fifth, the effects of credit constraints on a child’s adult outcomes depend on the age at which
they bind for the child’s family. Recent research summarized in Carneiro and Heckman (2002,
2003) and in CHLM demonstrates the quantitative insignificance of family credit constraints
in the child’s college-going years in explaining a child’s enrollment in college. Controlling
for cognitive ability, under meritocratic policies currently in place in American society, family
income during the child’s college-going years plays only a minor role in determining child college
participation, although much public policy is predicated on precisely the opposite point of view.
Holding ability fixed, minorities are more likely to attend college than others despite their lower
family incomes (see Stephen V. Cameron and Heckman, 2001, and the references they cite).
Augmenting family income or reducing college tuition at the stage of the life cycle when a child
goes to college does not go far in compensating for low levels of previous investment.

Carneiro and Heckman present evidence for the United States that only a small fraction (at
most 8%) of the families of adolescents are credit constrained in making college participation decisions. This evidence is supported in research by Cameron and Christopher Taber (2004) and Ralph Stinebrickner and Todd R. Stinebrickner (2006). Permanent family income plays an important role in explaining educational choices, insofar as it is a proxy for the high level of investment in abilities and skills that wealthier families provide, but it is not synonymous with family income in the adolescent years, nor with tuition and fees.

However, there is some evidence that credit constraints operating in the early years have effects on adult ability and schooling outcomes (Duncan and Jeanne Brooks-Gunn, 1997; Gordon B. Dahl and Lance J. Lochner, 2004; Pamela Morris et al., 2005; and Duncan and Ariel Kalil, 2006). Carneiro and Heckman (2003) show that controlling for family permanent income reduces the estimated effect of early income on child outcomes. Permanent income has a strong effect on child outcomes. The strongest evidence for an effect of the timing of parental income for disadvantaged children is in their early years. The best documented market failure in the life cycle of skill formation in contemporary American society is the inability of children to buy their parents or the lifetime resources that parents provide and not the inability of families to secure loans for a child’s education when the child is an adolescent.

Sixth, socioemotional (noncognitive) skills foster cognitive skills and are an important product of successful families and successful interventions in disadvantaged families. Emotionally nurturing environments produce more capable learners. The Perry Preschool Program, which was evaluated by random assignment, did not boost participant adult IQ but enhanced performance of participants on a number of dimensions, including scores on achievement tests, employment and reduced participation in a variety of social pathologies. See Lawrence J. Schweinhart et al. (2005) and the figures and tables on the Perry program posted at our website.8

II A Model of Skill Formation

We now develop a model of skill formation that can explain the six facts just presented as well as additional findings from the literature on child development. We use the terms “skill” and
“ability” interchangeably. Both are produced by environments, investment, and genes.

Agents possess a vector of abilities at each age. These abilities (or skills) are multiple in nature and range from pure cognitive abilities (e.g. IQ) to noncognitive abilities (patience, self control, temperament, risk aversion, time preference). These abilities are used with different weights in different tasks in the labor market and in social life more generally. Achievement test scores, sometimes confused with IQ scores (e.g., Herrnstein and Murray, 1994), are not pure measures of ability and are affected by cognitive, noncognitive and environmental inputs (See, e.g., Karsten T. Hansen et al., 2004, and Cunha and Heckman, 2006a).

The human skill formation process is governed by a multistage technology. Each stage corresponds to a period in the life cycle of a child. While the child development literature recognizes stages of development (see, e.g., Erik H. Erikson, 1950), the economics of child development does not. Inputs or investments at each stage produce outputs at the next stage. Like Yoram Ben-Porath (1967), we use a production function to determine the relationship between inputs and the output of skill. Unlike Ben-Porath, in our model qualitatively different inputs can be used at different stages and the technologies can be different at different stages of child development.

Ben-Porath focuses on adult investments where time and its opportunity cost play important roles. For child investments, parents make decisions and child opportunity costs are less relevant. The outputs at each stage in our technology are the levels of each skill achieved at that stage. Some stages of the technology may be more productive in producing some skills than other stages, and some inputs may be more productive at some stages than at other stages. The stages that are more effective in producing certain skills are called “sensitive periods” for the acquisition of those skills. If one stage alone is effective in producing a skill (or ability), it is called a “critical period” for that skill.

An important feature of our technology is that the skills produced at one stage augment the skills attained at later stages. This effect is termed self-productivity. It embodies the idea that skills acquired in one period persist into future periods. It also embodies the idea that skills are
self-reinforcing and cross-fertilizing. For example, emotional security fosters child exploration and more vigorous learning of cognitive skills. This has been found in animal species (Stephen J. Suomi, 1999; Michael J. Meany, 2001; Judy Cameron, 2004) and in humans (Duncan et al., 2006; C. Cybele Raver et al., 2006, interpreting the ability of a child to pay attention as a socioemotional skill). A higher stock of cognitive skill in one period raises the stock of next period cognitive skills. A second key feature of skill formation is *dynamic complementarity*. Skills produced at one stage raise the productivity of investment at subsequent stages. In a multistage technology, complementarity implies that levels of skill investments at different ages bolster each other. They are synergistic. Complementarity also implies that early investment should be followed up by later investment in order for the early investment to be productive. Together, dynamic complementarity and self-productivity produce multiplier effects which are the mechanisms through which skills beget skills and abilities beget abilities.

Dynamic complementarity, self-productivity of human capital, and multiplier effects imply an equity-efficiency trade-off for late child investments but not for early investments. These concepts, embedded in alternative market settings, explain the six facts from the recent literature summarized in the previous section. These features of the technology of skill formation have consequences for the design and evaluation of public policies toward families. In particular, they show why the returns to late childhood investment and remediation for young adolescents from disadvantaged backgrounds are so low, while the returns to early investment in children from disadvantaged environments are so high.

We now formalize these concepts in an overlapping generations model. An individual lives for $2T$ years. The first $T$ years the individual is a child of an adult parent. From age $T + 1$ to $2T$ the individual lives as an adult and is the parent of a child. The individual dies at the end of the period in which he is $2T$ years-old, just before his child’s child is born. At every calendar year there are an equal and large number of individuals of every age $t \in \{1, 2, \ldots, 2T\}$. To simplify the notation, we do not explicitly subscript generations.

A household consists of an adult parent and his child. Parents invest in their children
because of altruism. They have common preferences and supply labor inelastically. Let $I_t$ denote parental investments in child skill when the child is $t$ years-old, where $t = 1, 2, \ldots, T$.

The output of the investment process is a skill vector. The parent is assumed to fully control the investments in the skills of the child, whereas in reality, as a child matures, he gains much more control over the investment process.\(^{12}\) We ignore investments in the child’s adult years to focus on new ideas in this paper. We also keep government inputs (e.g., schooling) implicit. They can be modeled as a component of $I_t$.

We now describe how skills evolve over time. Assume that each agent is born with initial conditions $\theta_1$. Let $h$ denote parental characteristics (e.g., IQ, education, etc.). At each stage $t$, let $\theta_t$ denote the vector of skill stocks. The technology of production of skill when the child is $t$ years old is

\[
\theta_{t+1} = f_t(h, \theta_t, I_t),
\]

for $t = 1, 2, \ldots, T$. We assume that $f_t$ is strictly increasing and strictly concave in $I_t$, and twice continuously differentiable in all of its arguments.\(^{13}\)

Technology (1) is written in recursive form. Substituting in (1) for $\theta_t, \theta_{t-1}, \ldots$, repeatedly, one can rewrite the stock of skills at stage $t + 1, \theta_{t+1}$, as a function of all past investments.\(^{14}\)

\[
\theta_{t+1} = m_t(h, \theta_1, I_1, \ldots, I_t), \ t = 1, \ldots, T.
\]

Dynamic complementarity arises when $\partial^2 f_t (h, \theta_t, I_t) / \partial \theta_t \partial I_t' > 0$, i.e., when stocks of skills acquired by period $t - 1 (\theta_t)$ make investment in period $t (I_t)$ more productive. Such complementarity explains why returns to educational investments are higher at later stages of the child’s life cycle for more able children (those with higher $\theta_t$). Students with greater early skills (cognitive and noncognitive) are more efficient in later learning of both cognitive and noncognitive skills. The evidence from the early intervention literature suggests that the enriched early preschool environments provided by the Abecedarian, Perry and CPC interventions promote
greater efficiency in learning in school and reduce problem behaviors. See Currie and Blau (2006) and CHLM.

Self-productivity arises when $\partial f_t(h, \theta_t, I_t) / \partial \theta_t > 0$, i.e., when higher stocks of skills in one period create higher stocks of skills in the next period. For the case of skill vectors, this includes own and cross effects. The joint effects of self-productivity and dynamic complementarity help to explain the high productivity of investment in disadvantaged young children but the lower return to investment in disadvantaged adolescent children for whom the stock of skills is low and hence the complementarity effect is lower. These are facts two and three presented in Section I.

This technology is sufficiently rich to describe learning in rodents and macaque monkeys. More emotionally secure young animals explore their environments more actively and learn more quickly. This technology also explains the evidence that the ability of the child to pay attention affects subsequent academic achievement. Cross-complementarity serves to explain fact six. This technology also captures the critical and sensitive periods in humans and animals documented by Knudsen et al. (2006). We now define these concepts precisely.

Period $t^*$ is a critical period for $\theta_{t+1}$ if

$$\frac{\partial \theta_{t+1}}{\partial I_s} = \frac{\partial m_t(h, \theta_1, I_1, \ldots, I_t)}{\partial I_s} \equiv 0 \text{ for all } h, \theta_1, I_1, \ldots, I_t, s \neq t^*,$$

but

$$\frac{\partial \theta_{t+1}}{\partial I_{t^*}} = \frac{\partial m_t(h, \theta_1, I_1, \ldots, I_t)}{\partial I_{t^*}} > 0 \text{ for some } h, \theta_1, I_1, \ldots, I_t.$$ 

This condition says that investments in $\theta_{t+1}$ are productive in period $t^*$ but not in any other period $s \neq t^*$. Period $t^*$ is a sensitive period for $\theta_{t+1}$ if

$$\left. \frac{\partial \theta_{t+1}}{\partial I_s} \right|_{h=\bar{h}, \theta_1=\bar{\theta}, I_1=\bar{i}_1, \ldots, I_t=\bar{i}_t} < \left. \frac{\partial \theta_{t+1}}{\partial I_{t^*}} \right|_{h=\bar{h}, \theta_1=\bar{\theta}, I_1=\bar{i}_1, \ldots, I_t=\bar{i}_t}.$$

In words, period $t^*$ is a sensitive period relative to period $s$ if, at the same level of inputs, investment is more productive in stage $t^*$ than in another stage $s \neq t^*.^{15}
Suppose for simplicity that $T = 2$. In reality, there are many stages in childhood, including in utero experiences. Assume that $\theta_1, I_1, I_2$ are scalars. The adult stock of skills, $h' (= \theta_3)$, is a function of parental characteristics, initial conditions and investments during childhood $I_1$ and $I_2$:

\begin{equation}
(3) \quad h' = m_2(h, \theta_1, I_1, I_2).
\end{equation}

The literature in economics assumes only one period of childhood. It does not distinguish between early investment and late investment. This produces the conventional specification which is a special case of the technology (3), where

\begin{equation}
(4) \quad h' = m_2(h, \theta_1, \gamma I_1 + (1 - \gamma) I_2)
\end{equation}

and $\gamma = 1/2$. In this case, adult stocks of skills do not depend on how investments are distributed over different periods of childhood. For example, take two children, $A$ and $B$, who have identical parents and the same initial condition $\theta_1$, but have different investment profiles: child $A$ receives no investment in period one and receives $I$ units of investment in period two, $I_1^A = 0, I_2^A = I$, while child $B$ receives $I$ units of investment in period one and zero units of investment in period two, $I_1^B = I, I_2^B = 0$. According to (4), when $\gamma = 1/2$, children $A$ and $B$ will have the same stocks of skills as adults. The timing of investment is irrelevant. Neither period one nor period two is critical.

The polar opposite of perfect substitution is perfect complementarity:

\begin{equation}
(5) \quad h' = m_2(h, \theta_1, \min \{I_1, I_2\}).
\end{equation}

Technology (5) has the feature that adult stocks of skills critically depend on how investments are distributed over time. For example, if investment in period one is zero, $I_1 = 0$, then it does not pay to invest in period two. If late investment is zero, $I_2 = 0$, it does not pay to invest
early. For the technology of skill formation defined by (5), the best strategy is to distribute investments evenly, so that $I_1 = I_2$. Complementarity has a dual face. It is essential to invest early to get satisfactory adult outcomes. But it is also essential to invest late to harvest the fruits of the early investment. Such dynamic complementarity helps to explain the evidence by Currie and Thomas (1995) that for disadvantaged minority students, early investments through Head Start have weak effects in later years if not followed up by later investments. This is fact four on our list. Our explanation is in sharp contrast to the one offered by Becker (1991) who explains weak Head Start effects by crowding out of parental investment by public investment. That is a story of substitution against the invested child in a one-period model of childhood. Ours is a story of dynamic complementarity.

A more general technology that captures technologies (4) and (5) as special cases is a standard CES:

$$h' = m_2 \left( h, \theta_1, \left[ \gamma (I_1)^\phi + (1 - \gamma) (I_2)^\phi \right]^{\frac{1}{\phi}} \right)$$

for $\phi \leq 1$ and $0 \leq \gamma \leq 1$. The CES share parameter $\gamma$ is a skill multiplier. It reveals the productivity of early investment not only in directly boosting $h'$ (through self-productivity) but also in raising the productivity of $I_2$ by increasing $\theta_2$ through first-period investments. Thus $I_1$ directly increases $\theta_2$ which in turn affects the productivity of $I_2$ in forming $h'$. $\gamma$ captures the net effect of $I_1$ on $h'$ through both self-productivity and direct complementarity.

The elasticity of substitution $1 / (1 - \phi)$ is a measure of how easy it is to substitute between $I_1$ and $I_2$. For a CES technology, $\phi$ represents the degree of complementarity (or substitutability) between early and late investment in producing skills. The parameter $\phi$ governs how easy it is to compensate for low levels of stage 1 skills in producing later skills.

When $\phi$ is small, low levels of early investment $I_1$ are not easily remediated by later investment $I_2$ in producing human capital. The other face of CES complementarity is that when $\phi$ is small, high early investment should be followed with high late investment if the early investment is to be harvested. In the extreme case when $\phi \rightarrow -\infty$, (6) converges to (5). This technology
explains facts two and three — why returns to education are low in the adolescent years for disadvantaged (low $h$, low $I_1$, low $\theta_2$) adolescents but are high in the early years. Without the proper foundation for learning (high levels of $\theta_2$) in technology (1), adolescent interventions have low returns.

In a one-period model of childhood, inputs at any stage of childhood are perfect substitutes. Application of the one-period model supports the widely held but empirically unsupported intuition that diminishing returns make investment in less advantaged adolescents more productive. As noted in fact two of Section I, the evidence suggests that just the opposite is true. We next embed the technology in a market environment with parental choice of inputs.

III The Optimal Lifecycle Profile of Investments

Using technology (6), we now show how the ratio of early to late investments varies as a function of $\phi$ and $\gamma$ as a consequence of parental choices in different market settings. Let $w$ and $r$ denote the wage and interest rates, respectively, in a stationary environment. At the beginning of adulthood, the parent draws the initial level of skill of the child, $\theta_1$, from the distribution $J(\theta_1)$. Upon reaching adulthood, the parent receives bequest $b$. The state variables for the parent are the parental skills, $h$, the parental financial resources, $b$, and the initial skill level of the child, $\theta_1$. Let $c_1$ and $c_2$ denote the consumption of the household in the first and second period of the lifecycle of the child. The parent decides how to allocate the resources among consumption and investments at different periods as well as bequests $b'$ which may be positive or negative. Assuming that human capital (parental and child) is scalar, the budget constraint is:

\begin{equation}
(7) \quad c_1 + I_1 + \frac{c_2 + I_2}{(1 + r)} + \frac{b'}{(1 + r)^2} = wh + \frac{wh}{(1 + r)} + b.
\end{equation}

Let $\beta$ denote the utility discount factor and $\delta$ denote the parental altruism toward the child. Let $u(\cdot)$ denote the utility function. The recursive formulation of the problem of the parent is:

\begin{equation}
(8) \quad V(h, b, \theta_1) = \max \left\{ u(c_1) + \beta u(c_2) + \beta^2 \delta E[V(h', b', \theta_1')] \right\}.
\end{equation}
The problem of the parent is to maximize (8) subject to (7), and technology (6).\textsuperscript{21}

When $\phi = 1$, so early and late investment are perfect CES substitutes, the optimal investment strategy is straightforward. The price of early investment is $1$. The price of the late investment is $1/(1 + r)$. Thus the parent can purchase $(1 + r)$ units of $I_2$ for every unit of $I_1$. The amount of human capital produced from one unit of $I_1$ is $\gamma$, while $(1 + r)$ of $I_2$ produces $(1 + r)(1 - \gamma)$ units of human capital. Thus, two forces act in opposite directions. High productivity of initial investment (the skill multiplier $\gamma$) drives the parent toward making early investments. The interest rate drives the parent to invest late. It is optimal to invest early if $\gamma > (1 - \gamma)(1 + r)$.

As $\phi \to -\infty$, the CES production function converges to the Leontief case and the optimal investment strategy is to set $I_1 = I_2$. In this case, investment in the young is essential. At the same time, later investment is needed to harvest early investment. On efficiency grounds, early disadvantages should be perpetuated, and compensatory investments at later ages are economically inefficient.

For $-\infty < \phi < 1$, the first-order conditions are necessary and sufficient given concavity of the technology in terms of $I_1$ and $I_2$. For an interior solution, we can derive the optimal ratio of early to late investment:

\begin{equation}
\frac{I_1}{I_2} = \left[\frac{\gamma}{(1 - \gamma)(1 + r)}\right]^{\frac{1}{1 - \phi}}.
\end{equation}

Figure 2 plots the ratio of early to late investment as a function of the skill multiplier $\gamma$ under different values of the complementarity parameter $\phi$, assuming $r = 0$. When $\phi \to -\infty$, the ratio is not sensitive to variations in $\gamma$. When $\phi = 0$, the function (6) is

\begin{equation}
h' = m_2 (h, \theta_1, I_1, I_2) = m_2 (h, \theta_1, I_1 I_2^{1-\gamma}).
\end{equation}

In this case, from equation (9), the optimal $I_1/I_2$ is close to zero for low values of $\gamma$, but explodes to infinity as $\gamma$ approaches one.\textsuperscript{22}
When CES complementarity is high, the skill multiplier $\gamma$ plays a limited role in shaping the ratio of early to late investment. High early investment should be followed by high late investment. As the degree of CES complementarity decreases, the role of the skill multiplier increases, and the higher the multiplier, the more investment should be concentrated in the early ages.

In a perfect credit market model, optimal investment levels are not affected by parental wages or endowments, or the parameters that characterize the utility function $u(\cdot)$.\textsuperscript{23} Note, however, that even in this “perfect” credit market setting, parental investments depend on parental skills, $h$, because these characteristics affect the returns to investment. From the point of view of the child, this is a market failure due to the accident of birth. Children would like to choose the optimal amount of parental characteristics $h$ to complement their initial endowment, $\theta_1$.\textsuperscript{24}

Consider the second credit constraint mentioned in the introduction: parental bequests must be non-negative, i.e., parents cannot leave debts to their children. The problem of the parent is to maximize (8) subject to (7), technology (6), and the liquidity constraint:

\begin{equation}
\text{(10)} \quad b' \geq 0.
\end{equation}

If constraint (10) binds, then early investment under lifetime liquidity constraints, $\hat{I}_1$, is lower than the early investment under the perfect credit market model, denoted $I_1^*$. The same is true for late investment: $\hat{I}_2 < I_2^*$. Under this type of market imperfection, underinvestment in skills starts at early ages and continues throughout the life cycle of the child. This explains fact one — that skill gaps open up early and are perpetuated.\textsuperscript{25}

In this second case, both early and late investment depend on parental initial wealth $b$ for the families for whom the constraint (10) binds. Children who come from constrained families with lower $b$ will have lower early and late investment. Interventions that occur at early stages would exhibit high returns, especially if they are followed up with resources to supplement late investment. However, once the early stage investment is realized, late remediation for
disadvantaged children would produce lower returns if early and late investment are not perfect substitutes and late investment is more productive the higher the level of early investment. This helps to explain fact five in Section I.

The effects of government policies on promoting the accumulation of human capital depend on the complementarity between early and late investment as well as on whether the policies were anticipated by parents or not. For example, the short-run effects of an unanticipated policy that subsidizes late investment will have weaker effects the greater the complementarity between early and late investment. If the technology is Leontief, there is no short-run impact of the policy on adolescent investment. At the time the policy is announced, poor parents have already made their early investment decisions and, in the Leontief case, it is not possible to compensate by increasing late investment as a response to the subsidy.

There is, however, a long-run effect of the policy. If the policy is a permanent change announced before the child is born, new parents will adjust both early and late investment in response to the subsidy to late investment. Note that the same is true for an exogenous increase in the return to education. If there is strong complementarity between early and late investment, in the short run we would expect weak reactions to the increase in returns to education as gauged by adolescent investment decisions for the children from very poor family backgrounds, but stronger reactions in the long run. This analysis provides an explanation for why the college enrollment response to unanticipated increases in the returns to college were initially so strong for adolescents from advantaged families and initially so weak for adolescents from less advantaged families. Adolescents from less advantaged families are more likely to lack the foundational skills that make college going productive, compared to adolescents from more advantaged families. See Figure D10 on the website.

There is no trade-off between equity and efficiency in early childhood investment. Government policies to promote early accumulation of human capital should be targeted to the children of poor families. However, the optimal second-period intervention for a child from a disadvantaged environment depends critically on the nature of technology (6). If $I_1$ and $I_2$
are perfect CES complements, then a low level of $I_1$ cannot be compensated at any level of investment by a high $I_2$. On the other hand, suppose that $\phi = 1$, so the technology $m_2$ can be written with inputs as perfect CES substitutes. In this case, a second-period intervention can, in principle, eliminate initial skill deficits (low values of $I_1$). At a sufficiently high level of second-period investment, it is technically possible to offset low first-period investment, but it may not be cost effective to do so. If $\gamma$ is sufficiently low relative to $r$, it is more efficient to postpone investment.

The concepts of critical and sensitive periods are defined in terms of the technical possibilities of remediation. Many noneconomists frame the question of remediation for adverse environments in terms of what is technically possible — not what is economically efficient. Our analysis considers both technological possibilities and costs. From an economic point of view, critical and sensitive periods should be defined in terms of the costs and returns of remediation, and not solely in terms of technical possibilities.

Another source of market failure arises when parents are subject to lifetime liquidity constraints and constraints that prevent parents from borrowing against their own future labor income, which may affect their ability to finance investment in the child’s early years. This is the third constraint considered in the introduction. To analyze this case, assume that parental productivity grows exogenously at rate $\alpha$. Let $s$ denote parental savings. We write the constraints facing the parent at each stage of the life cycle of the child as:

(First stage) \[ c_1 + I_1 + \frac{s}{(1 + r)} = wh + b \]

(Second stage) \[ c_2 + I_2 + \frac{b'}{(1 + r)} = w (1 + \alpha) h + s, \]

where $s \geq 0$ and $b' \geq 0$. The restriction $s \geq 0$ says that the parent cannot borrow income from their old age to finance consumption and investment when the child is in the first stage of the life cycle. Some parents may be willing to do this, especially when $\alpha$ is high. In the case
when \( s \geq 0 \) and \( b' \geq 0 \) bind, and investments in different periods are not perfect substitutes, early income matters. To see this, note that if \( u(c) = \frac{(c^{\sigma} - 1)}{\sigma} \), the ratio of early to late investment is

\[
\frac{I_1}{I_2} = \left( \frac{\gamma}{(1 - \gamma)(1 + r)} \right)^{\frac{1}{1 - \phi}} \left[ \frac{(wh + b - I_1)}{\beta((1 + \alpha)wh - I_2)} \right]^{\frac{1 - \sigma}{1 - \phi}}.
\]

If early income is low with respect to late income, the ratio \( I_1/I_2 \) will be lower than the optimal ratio. The deviation from the optimal ratio will be larger the lower the elasticity of intertemporal substitution of consumption (captured by the parameter \( \sigma \)). Early income would not matter if \( \sigma = 1 \), which would be the case when consumption at stage one is a perfect substitute for consumption in stage two. Substitutability through parental preferences can undo lack of substitutability in the technology of skill formation.

Our analysis of credit constrained families joined with a low value of \( \phi \) interprets the evidence presented by Duncan and Brooks-Gunn, Morris et al., Duncan and Kalil, and Dahl and Lochner that the timing of family income in the early stages of childhood has some effect on the level of ability and achievement of the children. This is fact five of Section I. Our analysis also interprets the evidence of Carneiro and Heckman and Cameron and Taber that conditioning on child ability, family income in the adolescent years has only a minor effect on adolescent schooling choices.

## IV Cognitive and Noncognitive Skill Formation

A large body of research documents the socioemotional basis of reason (see Antonio R. Damasio, 1994; Joseph E. LeDoux, 1996). Our analysis goes beyond this literature to formalize a body of evidence that emotional skills promote learning. Mechanisms relating cortisol to stress and the effects of cortisol on brain development have been documented by Suomi (1999) and Meaney (2001) for animals. Duncan et al. (2006) and Raver et al. (2006) show that a child’s ability to pay attention facilitates later learning.

The framework developed in Section II readily accommodates skill vectors.\(^{27}\) The evidence summarized in Section I shows the importance of both cognitive and noncognitive skills in
determining adult outcomes. Child development is not just about cognitive skill formation although a lot of public policy analysis focuses solely on cognitive test scores. Let $\theta_t$ denote the vector of cognitive and noncognitive skills: $\theta_t = (\theta^C_t, \theta^N_t)$. Let $I_t$ denote the vector of investment in cognitive and noncognitive skills: $I_t = (I^C_t, I^N_t)$. We use $h = (h^C, h^N)$ to denote parental cognitive and noncognitive skills. At each stage $t$, we can define a recursive technology for cognitive skills ($k = C$), and noncognitive skills, ($k = N$):

$$
\theta_{t+1}^k = f_t^k (\theta^C_t, \theta^N_t, I^k_t, h^C, h^N), \; k \in \{C, N\}.
$$

Note that technology (11) allows for cross-productivity effects: cognitive skills may affect the accumulation of noncognitive skills and vice versa. They also allow for critical and sensitive periods to differ by skill, as required to account for fact two.

If cognitive and/or noncognitive skills determine costs of effort, time preference or risk aversion parameters, parental investments affect child and adult behavior. Our analysis of preference formation contrasts with the analyses of Hideo Akabayashi (1996) and Bruce A. Weinberg (2001). Those authors build principal-agent models where the parent (the principal) and the child (the agent) agree on contracts in which parents financial transfers are conditional on observable measures of effort (e.g., test scores in school). These contracts are designed so that the children are driven towards the level of effort desired by the parents. In our model, parents directly shape child preferences.

Accounting for preference formation enables us to interpret the success of many early childhood programs targeted to disadvantaged children which do not permanently raise IQ, but which permanently boost social performance. This is fact six of Section I. The controversy over Head Start fadeout may have been a consequence of relying only on cognitive measures to gauge performance. The Perry Preschool Program had an IQ fadeout but a lasting effect on a variety of participants through age 40. They work harder, are less likely to commit crime, and participate in many fewer social pathologies than do control group members.
V Estimates of the Technology

Cunha and Heckman (2006a) and Cunha et al. (2006b) estimate recursive multistage technology (6) with cognitive and noncognitive skills generating adult outcomes like schooling, earnings and occupational choice. They develop new econometric methods that extend factor analysis to a nonlinear setting. We refer the reader to those papers for econometric details and discussions of the rich panel data on child development that makes such estimation possible.

They find strong evidence of self-productivity and complementarity. Their evidence is consistent with the literature demonstrating malleability of the prefrontal cortex governing executive function and socioeconomic development as well as the stability of IQ measures after age 10 cited in Section I. They find higher substitutability of early and late investment in producing noncognitive skills and lower substitutability of early investment in producing cognitive skills. Higher stocks of noncognitive skills promote the self-productivity of cognitive skills; cognitive skill stocks promote the self-productivity of noncognitive skills. Higher levels of both cognitive and noncognitive skills raise the productivity of subsequent investment. There is evidence of sensitive periods for parental investment. The productivity of parental investment is higher in early stages for cognitive skills with a fall off in their productivity in later years. The productivity of parental investment is higher at later stages for noncognitive skills. This evidence is consistent with greater malleability of the prefrontal cortex governing socioemotional development into the early 20s, documented by Dahl (2004).

Cunha et al. (2006b) estimate a strong interaction between initial endowments and parental investments that calls into question the conventional additive model of nature vs. nurture. This evidence is consistent with the modern literature on epigenetics. Nature and nurture interact to produce child outcomes and environmental effects that last across generations. Even $\theta_1$, endowment at birth, is affected by environmental factors as a large literature documents (see, e.g., Shonkoff and Phillips, 2000).
VI  Lessons for the Design of Policies

Cunha and Heckman (2006b) simulate the nonlinear model of skill formation estimated by Cunha et al. (2006b) to show the importance of self-productivity and complementarity for designing policies to reduce inequality. We focus our analysis on children from disadvantaged backgrounds because at current levels of social inequality they benefit the most from policies that supplement early environments.31 Disadvantaged children are at risk of being permanently poor and uneducated, and of participating in crime. In our simulation, disadvantaged children come from a background where mothers are in the first decile in the distribution of parental skills. If no intervention occurs, the children receive investments equivalent to the first decile of the distribution of parental investments.

Consider three different policies. The first policy is a Perry Preschool-like policy. It provides investment at early ages that moves children from the first decile of child cognitive skills at entry age to the fourth decile of child skills at the age of exit from the program. This gain can be achieved by moving parental investment from the bottom decile to around the seventh decile of the family investment distribution. In this policy, there is no follow-up investment. We also consider a second policy for the same target population that postpones remediation until adolescence. It compensates early shortfalls by investing larger amounts in adolescent stages of the life cycle to produce approximately the same high school graduation rates that are observed in the Perry program.

College tuition programs, adolescent literacy programs and mentoring programs are examples of such a policy. To achieve Perry-like outcomes for this population through adolescent investment, it is necessary to move adolescent investment to the top of the parental investment distribution. The present value of the costs of the investment in this adolescent remediation program is more than 35% larger than in the Perry Preschool program. Late remediation is possible but it is costly. The case for early childhood intervention is based more on the importance of sensitive periods in the life cycle of the child than on the importance of critical periods. We contrast early-only and late-only investment policies with a third policy that op-
timally distributes the resources spent in the second policy over the full life cycle of the child. A balanced investment strategy is the most efficient.

The first column of numbers in Table 1 reports high-school graduation, college enrollment, conviction, probation, and use of welfare if no intervention is made. Our model predicts a 41% high school graduation rate for this group, compared to 41.4% found in the Perry control group. Only 4.5% of the control group ever enroll in college. Around 22% of them will be convicted of a crime or be on probation at some point in their adult lives. About 18% will make use of welfare programs in their adult years.

The second column of numbers in Table 1 reports the performance of our Perry-like early intervention policy. This policy increases high school graduation and college enrollment rates to more than 65% and 12%, respectively. It reduces participation in crime. It makes the children more productive when they are adults. It cuts in half the probability that the child collects welfare benefits in his/her early adult years. These effects are comparable to those reported in the Perry preschool intervention (See, e.g., Schweinhart et al., 2005). Thus, with our technology, we can rationalize the results found in the Perry program as an intervention that boosts parental investments (but not parental characteristics) from the first decile of investment in children to the fourth decile.

The third column of the numbers in Table 1 displays the performance of a 35% more costly policy that produces comparable educational outcomes for those obtained in the Perry-like intervention. Adolescent interventions can be effective, but they are more costly than early interventions. The greater cost associated with later remediation arises from lost gains in self-productivity and dynamic complementarity from early investment that are a key feature of our model.

The empirical importance of dynamic complementarity — the fact that the marginal productivity of investment depends on the level of skills produced by previous investments — generates an important insight for the design of policies. For a fixed expenditure, policies that are balanced increase returns and are more productive than policies tailored to one segment of the
life cycle of the child. The returns to later investment are greater if higher early investment is made. Perry children made less use of remedial education than peers who did not receive treatment. The intervention made later schooling more effective. If early interventions are followed up with later interventions in an optimal fashion, outcomes can be considerably improved.

The fourth column of numbers in Table 1 presents the results from a balanced policy. It displays the outcomes that can be produced by an intervention that distributes the funds spent on the adolescent-only intervention in an optimal way. For a balanced program, high school graduation and college enrollment rates are, respectively, 91% and 37%. The reduction in conviction and probation rates is better than what is obtained from an adolescent-only policy, and welfare use is reduced to a low 2.6% rate. Complementarity implies that early investment is more productive if it is followed up with late investment. And late investment is more productive if it is preceded by early investment. The mechanism that makes the balanced intervention more effective has a very simple economic interpretation. When adolescent-only interventions are made, baseline skills are low and, consequently, so is the marginal productivity of later investment. A balanced investment program increases the stock of skills of the child coming into adolescence. Because the marginal productivity of later investment depends on the level of skills acquired prior to adolescence, the investment in the last period is more productive. Thus, the same amount of total investment distributed more evenly over the life cycle of the child produces more adult skills than a policy that concentrates attention on only one part of the child’s life cycle.

VII Summary and Extensions

A technology of cognitive and noncognitive skill formation that features self-productivity, dynamic complementarity and skill multipliers explains a variety of findings established in the child development and child intervention literatures. Although we have focused on cognitive and noncognitive skills, our analysis also applies to the formation of physical health capital. (See Cunha and Heckman, 2006c.) The evidence on the importance of early childhood environments on adult health (David J. P. Barker, 1998, and Anne Case et al., 2005) can be rationalized by
our technology. Stocks of cognitive and noncognitive skills facilitate the accumulation of health capital through self-regulation and choices. Stocks of health skills also raise the productivity of schooling (Alok Bhargava, 2008).
References


Cunha, Flavio and James J. Heckman. 2006b. “Investing in our Young People.” Unpublished manuscript, University of Chicago, Department of Economics.


Notes

*This research was supported by NIH R01-HD043411, NSF SES-024158, the Committee for Economic Development with a grant from The Pew Charitable Trusts and the Partnership for America’s Economic Success, and the J.B. Pritzker Consortium on Early Childhood Development at the Harris School of Public Policy, University of Chicago. Flavio Cunha also acknowledges support from the Claudio Haddad dissertation fund at the University of Chicago. The views expressed in this paper are those of the authors and not necessarily those of the funders listed here. The first draft of this paper, written with Pedro Carneiro, was presented at a conference at the Minneapolis Federal Reserve, October 2003. We thank Gary S. Becker, Janet Currie and Greg J. Duncan for helpful comments. A website at http://jenni.uchicago.edu/tech-skill/ contains supporting material.

1For example, Becker (1993, pp. 99–100) contrasts the implications for the earnings distribution of ability models of earnings and human capital models, claiming the latter are more consistent with the empirical evidence on earnings. The implicit assumption in his analysis and the literature it spawned is that ability is determined by “nature”, i.e., is genetic, and outside the influence of family investment strategies.

2There is some evidence that the modified genes are heritable (see Pray, 2004).

3Some recent evidence on gene-environment interactions resulting from child maltreatment is presented in Avshalom Caspi et al. (2002).

4Permanent income is the measure of socioeconomic status in this figure. See CHLM for the source of this figure and the precise definition of permanent income.

5These and other figures are posted at the website for this paper. See Figures D0–D8 on the website.

6See Figures D1–D3 on the website.

7Currie and Thomas (2000) present additional analyses of the Head Start Program.

8See the Figure D9 series.

9CHLM briefly discuss the evidence on this point and suggest a model of comparative ad-
vantage in occupational choice to supplement their model of skill formation.

10We discuss the comparison between our technology and that of Ben-Porath (1967) in Section B of the website.

11We develop our formal OLG model in Section C of our website.

12A sketch of such a model is discussed in Carneiro et al., 2003.

13These conditions are sufficient. There is no need for a differentiability requirement for \( h \), and the differentiability requirement with respect to \( \theta_t \) can be weakened.

14Examples are developed in Section A at our website.

15See CHLM for a definition of critical and sensitive periods in terms of technology (1). These definitions are developed further in Section B of the website.

16Our technology applies to in utero and post-natal investments as well. See Jack P. Shonkoff and Deborah Phillips (2000) for evidence on the importance of such investments.

17CHLM analyze the vector case. See also the supporting material on the website.

18Both periods are critical. Note that in this case the production function is not strictly differentiable as required in our definition. Our definition can be extended to deal with this limit case.

19We offer another explanation of the apparently weak Head Start effects below.

20Consider an example in which \( f_t (h, \theta_t, I_t) = [\eta_{1,t} (\theta_t)^{\phi_t} + \eta_{2,t} (I_t)^{\phi_t} + \eta_{3,t} (h)^{\phi_t}]^{\rho_t} \) for \( t = 1, 2; \rho_1 = 1, \rho_2 < 1; \) and \( \eta_{1,t} + \eta_{2,t} + \eta_{3,t} = 1 \). Then, substituting for \( \theta_2 \), we obtain \( m_2 (h, \theta_1, I_1, I_2) = [\eta_{1,2} (\eta_{1,1} (\theta_1)^{\phi_1} + \eta_{2,1} (I_1)^{\phi_1} + \eta_{3,1} (h)^{\phi_1})^{\rho_1} + \eta_{2,2} (I_2)^{\phi_2} + \eta_{3,2} (h)^{\phi_2}]^{\rho_2} \). The parameter \( \phi_2 \) describes how easy it is to compensate early neglect with second-period investment. The parameter \( \phi_1 \) describes the compensation possibilities for overcoming adverse initial conditions \( \theta_1 \) with first-period investment. The technology in the text is obtained by setting \( \phi_1 = \phi_2, \eta_{3,1} = \eta_{3,2} = \eta_{1,1} = 0, \) so that \( \gamma = \eta_{1,2} \eta_{2,1} \) and \( \eta_{2,2} = 1 - \gamma \). See Section A.1 at the website for further analysis of this and related cases.

21Section C of the website develops this analysis for an overlapping generations model.

22Table A1 on the website concisely summarizes this analysis.
We refer to parental resources specific to a given generation.

This thought experiment is whimsical. If parents create the child, through genes and environment, the child is not an independent actor. Under a homunculus theory, the child would have an identity independent of the parent.

Of course other reasons why skill gaps open up early and are perpetuated is variation in $h$ and $\theta_1$, the parental environmental and initial endowment variables, respectively.

This type of constraint is also analyzed by Elizabeth Caucutt and Lochner (2004).

This type of constraint is also analyzed by Elizabeth Caucutt and Lochner (2004).

See Section B at our website.

The Abecedarian early intervention program permanently boosted adult IQ. See CHLM.

See the D9 series on the website. The exact mechanism by which noncognitive skills are boosted is not yet established. It could be that noncognitive skills are created directly in the early years and persist. It could also be that the higher early cognitive skills that fade out foster noncognitive skills that persist. Both channels of influence could be in operation.

Anchoring test score outcomes in behavior avoids reliance on arbitrarily scaled test scores as a measure of output. See Cunha and Heckman (2006a) and Cunha et al. (2006b).

This is fact three of Section I.
Table 1. Comparison of different investment strategies

Disadvantaged children: first decile in the distribution of cognitive and noncognitive skills at age 6

Mothers are in first decile in the distribution of cognitive and noncognitive skills at ages 14–21

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Changing initial conditions: Moving children to the 4th decile of distribution of skills only through early investment</th>
<th>Adolescent intervention: Moving investments at last transition from 1st to 9th decile</th>
<th>Changing initial conditions and performing a balanced intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>High school graduation</td>
<td>0.4109</td>
<td>0.6579</td>
<td>0.6391</td>
<td>0.9135</td>
</tr>
<tr>
<td>Enrollment in college</td>
<td>0.0448</td>
<td>0.1264</td>
<td>0.1165</td>
<td>0.3755</td>
</tr>
<tr>
<td>Conviction</td>
<td>0.2276</td>
<td>0.1710</td>
<td>0.1773</td>
<td>0.1083</td>
</tr>
<tr>
<td>Probation</td>
<td>0.2152</td>
<td>0.1487</td>
<td>0.1562</td>
<td>0.0815</td>
</tr>
<tr>
<td>Welfare</td>
<td>0.1767</td>
<td>0.0905</td>
<td>0.0968</td>
<td>0.0259</td>
</tr>
</tbody>
</table>

Note: The adolescent-only and balanced intervention programs cost 35% more than the Perry program

Source: Cunha and Heckman (2006b)
Figure 1
Children of the NLSY
Average Standardized Score for PIAT Math by Permanent Income Quartile

Source: Full sample of the Children of the National Longitudinal Survey of Youth. See our website for a full explanation of this figure.

Figure 2
Ratio of Early to Late Investment in Human Capital
As a Function of the Skill Multiplier for Different Values of Complementarity