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ABSTRACT

Regulating Damage Clauses in (Labor) Contracts *

We analyze the role of damage clauses in labor contracts using a model in which a worker may want to terminate his current employment relationship and work for another firm. We show that the initial parties to a contract have an incentive to stipulate excessive damage clauses, which leads to ex post inefficiencies. This result is due to rent seeking motives a) between the contracting parties vis-à-vis third parties and b) among the contracting parties themselves. We then show that, by imposing an upper bound on the amount of enforceable damages, a regulator can induce a Pareto improvement; in some cases even the first best can be achieved.

JEL Classification: K12, K31, M12

Keywords: damage clauses, penalty doctrine, breach of contract, asymmetric information, labor contracts

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1 Introduction

Motivation In many circumstances, courts are reluctant to enforce damage clauses which are deemed excessive. The most prominent example is the penalty doctrine, as for example formulated in the US Uniform Commercial Code § 2-718 (1978): "Damages for breach by either party may be liquidated in the agreement but only at an amount which is reasonable in the light of the anticipated or actual harm caused by the breach.... A term fixing unreasonably large liquidated damages is void as a penalty".1 In a similar vein, labor market legislation often voids contracts which cover a considerable period of time and which cannot be unilaterally terminated.2 In Germany for example, it is possible to sign temporary employment contracts which can be terminated before expiration only if both parties agree, i.e. temporary employment contracts are in principle exempt from the contractual notice of termination in German Labor Law (§620 Ab.1 BGB). However, this only holds if the temporary contract lasts no longer than 5 years, in which case the worker can always unilaterally terminate the relationship after a period of notice of 6 month (§624 BGB). As another example, according to a new legislation in European professional soccer, the maximum duration of contracts between players and their clubs is also five years.3

Obviously, such rules disable parties to a contract from agreeing on certain damage clauses or contract durations and thus constitute a restriction on the freedom of contract. Consequently, by arguing that rational parties would never sign inefficient contracts, many scholars have criticized such restrictions on the grounds of being potentially detrimental from a social point of view.4

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1 Similar formulations also exist in other legal contexts, see e.g. Restatement (Second) of Contracts § 356(1).
2 Of course, any contract can be terminated if all parties to a contract agree to do so.
3 It might not seem obvious why restrictions on contract durations are intimately related to those on damage clauses and vice versa. However, it seems natural to assume that damage payments, either as specified in the contract or agreed on in renegotiations, are (at least weakly) increasing in the remaining length of the initial contract. As such this simply reflects the notion that “waiting is costly” for the worker and/or the new firm; this notion is used in Aghion and Bolton (1987) who, in fact, use the terms interchangeably. Moreover, it drops out as an equilibrium feature of a renegotiation game in which the worker simultaneously bargains with both, his current and a potential new employer in the form Nash-Bargaining, together with the reasonable assumption that the initial employer gets a higher share of the surplus, when it can prevent the worker from switching employers in the threat point, see Feess and Muehlheusser (2003). Since there exists a one-to-one relationship between contract duration and damage payment, restrictions on contract durations can be interpreted as restrictions on maximum damage clauses and vice versa.
4 See e.g. Brightman (1925), Stigler (1975, Ch.7), and Epstein (1989).
Proponents of such rules often argue that excessive damage clauses or contract durations may lead to inefficient breach decisions and thus to allocative distortions (see e.g. Goetz and Scott (1977)). In the labor market context for example, it is often argued that, when a worker and an employer are tied together by a long term contract, this could prevent the worker from always working for the firm in which his productivity is maximum; the reason being either the firm not always agreeing to a separation whenever this is efficient or the worker refusing to switch employers. It is clear that the validity of this claim crucially depends on the informational environment and whether or not the initial contract can be renegotiated. Moreover, even if valid, this argument does not explain why parties to a contract should stipulate socially excessive damage clauses or contract durations in the first place. However, as recent contract-theoretic work (which is discussed below) has shown, they might do so because it is jointly beneficial as a rent seeking device vis-à-vis third parties.

Framework and results  We analyze a model in which a wealth-constrained worker and his employer bargain over an initial contract stipulating a wage and a damage clause in case the worker should not honor the contract. After the initial contract has been signed, a potential new employer emerges, and the worker receives additional private information about which of the two firms he prefers to work for. In equilibrium, there will be ex post inefficiencies in the sense that the worker will not always switch employers whenever it is efficient to do so. This result is driven by the interplay of two rent seeking motives: First and reminiscent from the literature, when negotiating the initial contract, the worker and his employer have a joint incentive to stipulate an excessive damage clause in order to reduce the expected profit of the new firm. Second, since the worker is wealth-constrained, when distributing the joint surplus through the terms of initial contract, the initial employer is also concerned about rents accruing to the worker. This tends to create a further distortion which leads to even higher damage clauses and thus to a larger allocative inefficiency. It is shown that the extent of these inefficiencies crucially

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5 Arguing along the lines of Coase (1960), absent frictions like asymmetric information, costly bargaining or wealth constraints, bargaining will always lead to ex post efficiency since all parties can be made better off by agreeing on a switch of the worker to that employer where the social surplus is maximum, independent whether he has a binding contract with the old employer or not. Thus, there does not seem to be a need for restrictions. In reality, however, situations of frictionless bargaining seem to be rare. In this paper, the worker’s private information and wealth constraint will make the Coase Theorem inapplicable.
depends on the level of the worker’s outside option, which we use as a proxy for the relative bargaining power of the worker at the initial contracting stage.

Given this result the question then arises, whether a social planner can improve upon the outcome under freedom of contract by setting an upper bound on the level of the damage clause which is enforceable in court and which can therefore contractually agreed on. We show that by setting this upper bound appropriately, the regulator can always induce a Pareto improvement; in some cases even the first best can be achieved. Again, the optimal upper bound is shown to depend on the worker’s productivity and the distribution of bargaining power between the worker and his initial employer. This result hints at a potential inefficiency induced by common penalty doctrine practice, where simply the expectation damage measure is used as an upper bound.6

**Relation to the Literature** Earlier contributions on breach remedies such as Shavell (1980) and Rogerson (1984) have confined attention to situations where third parties cannot reap a positive share of the surplus. In this case, the joint surplus of the contracting parties coincides with the social surplus so that there is no incentive to stipulate contract terms which are socially inefficient. However, a large body of literature has shown that this is no longer true in situations where third parties do have market power: Diamond and Maskin (1979) analyze a search model where parties contract with each other but continue to search for better matches. They show that there is an incentive to stipulate high damages in the initial contract because this will increase the payoff in the new partnership. As they note, ”the rationale for these contracts is solely to ‘milk’ future partners for damage payments” (Diamond and Maskin (1979, p. 294)). In a different context, Aghion and Bolton (1987) analyze the role of contracts in the context of entry prevention. Again, they show that it is optimal for parties to stipulate a damage clause which prevents entry inefficiently often. Chung (1992) extends this model by introducing specific investment and compares a *penalty doctrine rule*, where damages are only enforced up to expectation damages, with a *freedom of contract rule* where no upper bound is in place. He shows that, under the latter rule, the initial contracting parties choose a damage clause which is inefficiently high from a social point of view and which also leads

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6 See the discussion in Chung (1992, pp.290).
to ex post inefficiencies.\footnote{Apart from the rent-extraction motives considered here, the literature has also analyzed the penalty doctrine from alternative viewpoints: Hermalin and Katz (1993) inquire whether a court might be more efficient by mandating contract terms than sophisticated parties. Also in the context of damage clauses they show that, by not enforcing damage clauses at all, the court can increase social welfare. Interestingly however, they also show that it is unclear that putting an upper bound on enforceable damages clauses is always the right thing to do. In particular, they show that there also exist parameter constellation in which it is optimal for the court to impose a minimum level of damage clauses (see also Stole (1992)). Matouschek and Ramezzana (2006) argue that contract-imposed frictions (such as damage or exclusive dealing clauses) can induce negative search externalities for other potential trading partners in the market.}

Contrary to these contributions and apart from identifying a second rent-seeking motive due to the wealth-constraints, in the present paper the optimal upper bound on enforceable damages is determined endogenously, whereas in Chung (1992) it is exogenous, while Aghion and Bolton (1987) do not consider such restrictions at all.

In a further extension, Spier and Whinston (1995) also allow for renegotiation in addition to specific investments. In their framework of complete information, while renegotiation induces efficient breach decisions and thus eliminates any ex post inefficiency, the initial contract still exhibits inefficiencies in the form of inducing socially excessive investment incentives, the reason again being a rent-seeking motive vis-à-vis third parties. Contrary to Spier and Whinston (1995) (but in line with Aghion and Bolton (1987) and Chung (1992)), ex post inefficiencies do play an important role in our framework, and it should be noted that this is not an artefact of precluding renegotiation since it would have to occur under asymmetric information.\footnote{It is a well-known result in mechnism design under asymmetric information that, generically, any renegotiation procedure will induce ex post inefficiencies, see e.g. Myerson and Satterthwaite (1983) and Schweizer (2005). Moreover, the extent of the inefficiency will generally depend on the damage clause stipulated in the initial contract, and it will typically be the larger, the higher the damage clause.}

In a labor market context, Posner and Triantis (2004) analyze covenants not to compete which disallow workers to work for certain alternative employers other than their current one. When renegotiation is possible, a covenant becomes akin to a damage clause and, consequently, a worker and his initial employer have a joint incentive ex ante to excessively restrict the mobility of the worker.

This paper is organized as follows: The basic model is set up in section 2. The scenario in which any damage clause is assumed to be enforceable is analyzed in section 3, while section 4 considers the case in which a regulator imposes an upper bound on the enforceable damage payments. In section 5, the results form both scenarios are compared while section 6 discusses the main assumptions and concludes.
2 The basic model

We consider the contracting problem between a firm and a worker (both risk-neutral) in which the worker can either work for the firm with productivity $\beta \in \mathbb{R}^+$ or choose his outside option which yields reservation utility $U \in [0, \infty)$. The firm, which has a reservation payoff of zero, is able to make a take-it-or-leave-it offer for a contract $\Omega = (w_I, r)$ which specifies a wage rate $w_I \in \mathbb{R}_0^+$ and a damage clause $r \in \mathbb{R}$ which the worker has to pay if he chooses not to work for the firm after the contract has been signed. Thus, although any level of $r$ is assumed to be feasible, the worker is wealth-constrained so that negative wages are excluded.\footnote{Assuming the worker to be risk-neutral but wealth-constrained is also consistent with the modern contract-theoretic formulation of the Shapiro and Stiglitz (1984) efficiency wage model, see e.g. Tirole (1999, p. 745) or Laffont and Martimort (2002, p. 174). This assumption will also be discussed in more detail in section 6.}

For instance, this might be due to the fact that the worker cannot commit ex ante to work for a negative wage. The values of the outside options of the worker and the firm, respectively are commonly known.

Denote the acceptance decision by the worker concerning the initial contract by $D_I \in \{0, 1\}$. If he rejects the initial offer ($D_I = 0$), the game ends and the worker and the firm earn payoffs of $U$ and 0, respectively. After accepting it ($D_I = 1$), a new firm enters the scene. In order to distinguish both firms, we refer to the initial firm as ”firm $I$” and to the new firm as ”firm $E$”. The worker’s productivity in firm $E$ is also $\beta$. From the worker’s point of view, however, both firms differ with respect to a private benefit $b_k$ he receives when working for firm $k \in \{I, E\}$.\footnote{The case where the worker incurs effort costs when working for either firm is completely analogous.} The private benefit when working for firm $I$ is normalized to zero, i.e. $b_I \equiv 0$. The private benefit when working for firm $E$, $b_E$, is a random variable which is distributed in the interval $[-a, a]$ with $a > 0$, according to a distribution function $F(b_E)$ with positive continuous density $f(b_E)$ and zero mean. The preference parameter $b_E$ is private information to the worker, which he learns after the initial contract with firm $I$ has been signed.\footnote{We could equivalently assume that there are many potential new employers with different levels of $b_E$ (e.g. due to commuting distance or the “public image” of the employer), and that there is uncertainty ex ante which of these firms will have a job opening in the future.}

The distribution $F(b_E)$ is common knowledge.

After the worker has learned his type, firm $E$ may want to hire the worker by offering a wage $w_E \in \mathbb{R}_0^+$ in form of a take-it-or-leave-it offer. The acceptance decision of the worker
in this case is denoted by \( D_E(w_E) \in \{0, 1\} \). If the worker accepts the offer \((D_E = 1)\), he pays damages \( r \) to firm \( I \) and then starts to work for firm \( E \) and receives wage \( w_E \). If the worker rejects the offer \((D_E = 0)\), he works for firm \( I \) and gets wage \( w_I \) while firm \( E \) receives a reservation payoff of zero. As discussed in the introduction, renegotiation of the initial contract is precluded. Finally, the worker accepts an offer when indifferent.

Depending on the worker’s acceptance decisions, \( D_I \) and \( D_E \), payoffs and social welfare are summarized in Table 1:

<table>
<thead>
<tr>
<th></th>
<th>Firm I</th>
<th>Firm E</th>
<th>Worker</th>
<th>Social Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_I = 0 )</td>
<td>0</td>
<td>0</td>
<td>( U )</td>
<td>( U )</td>
</tr>
<tr>
<td>( D_I = 1, D_E = 0 )</td>
<td>( \beta - w_I )</td>
<td>0</td>
<td>( w_I )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>( D_I = 1, D_E = 1 )</td>
<td>( r )</td>
<td>( \beta - w_E )</td>
<td>( b_E + w_E - r )</td>
<td>( \beta + b_E )</td>
</tr>
</tbody>
</table>

Table 1: Payoffs

Denoting the efficient acceptance decisions of the worker by \( D_k^F \) for \( k = I, E \), he should work for firm \( E \) whenever \( b_E \) is non-negative, i.e. \( D_E^F = 1 \iff b_E \geq 0 \). Moreover, it is efficient for the worker to accept firm \( I \)’s offer whenever the expected surplus from doing so is higher than his outside option \( U \), i.e. \( D_I^F = 1 \iff \beta + EV(b_E \mid b_E \geq 0) \geq U \) where \( EV \) denotes “expected value”. We assume \( \beta > \Upsilon \) which is sufficient to ensure that it is never efficient for the worker to pursue his outside option.

The time structure of the basic game is as follows (see also Figure 1): At date 1, the initial contract \( \Omega = (w_I, r) \) is offered. At date 2, the worker decides whether or not to accept the offer. If \( D_I = 1 \), then at date 3, the worker learns \( b_E \). At date 4, firm \( E \) offers \( w_E \) which, at date 5, the worker again either accepts or rejects. Afterwards, the worker works either for firm \( I \) or for firm \( E \).
Denote by $q$ and $B$, respectively the probability that the worker accepts the offer by firm $E$ and the expected private benefit conditional on having accepted firm $E$’s offer, i.e. $q \equiv \Pr(D_E = 1)$ and $B \equiv EV(b_E \mid D_E = 1)$. For $D_I = 1$, expected payoffs for firm $I$, firm $E$ and the worker, respectively, are:

\[
\begin{align*}
\pi_I &= (1 - q) \cdot (\beta - w_I) + q \cdot r, \\
\pi_E &= q \cdot (\beta - w_E), \\
\pi_W &= (1 - q) \cdot w_I + q \cdot (w_E + B - r),
\end{align*}
\]

For the worker, for instance, with probability $(1 - q)$ there is no transfer in which case he works for firm $I$ and gets the initial wage $w_I$, while with probability $q$ he works for firm $E$ and gets wage $w_E$ plus expected private benefit $B$, and has to pay $r$ to firm $I$. The payoffs for firms $I$ and $E$ can be interpreted analogously. Finally, expected social welfare in case that $D_I = 1$ is simply given by adding up all payoffs, i.e.

\[ SW = \beta + q \cdot B. \]

### 3 No Regulation of Damage Clauses

In this section we analyze the case in which there is no legal restriction on the damage clause $r$, i.e. any $r \in \mathbb{R}$ is assumed to be enforceable in court. For further reference, it is useful to define $R$ as the sum of the wage in firm $I$ and the liquidated damage clause: $R \equiv w_I + r$. We will refer to $R$ as the worker’s “total switching cost” when leaving firm $I$ consisting of his opportunity costs $w_I$ and the damage payment $r$.

**Date 5** Solving the game backwards, at date 5 the worker will accept firm $E$’s offer, whenever his net benefit from doing so is non-negative, i.e. $D_E = 1$ iff $b_E + w_E - r \geq w_I \iff b_E + w_E \geq R$. The borderline type which is just indifferent between accepting and rejecting is then implicitly defined by

\[ \tilde{b}_E - R + w_E = 0. \]
Given $\tilde{b}_E$, the probability of a transfer is

$$q = 1 - F(R - w_E)$$  \hfill (6)

**Date 4** For date 4, the wage offered by firm $E$ solves the following maximization problem

$$\max_{w_E} (1 - F(R - w_E)) \cdot (\beta - w_E)$$  \hfill (7)

Assuming an interior solution, the optimal value $w^*_E(R)$ satisfies the following first order condition:

$$-1 + F(R - w^*_E) + f(R - w^*_E)(\beta - w^*_E) = 0.$$  \hfill (8)

The optimal wage offered by firm $E$ trades off the higher probability of acceptance when choosing a high $w_E$ vs. the direct beneficial effect of a lower wage. Also denoting other equilibrium variables with a "*", we have the following result for the continuation game at date 4:

**Lemma 1** i) For the continuation game at date 4, we have

$$\tilde{b}^*_E(R) = R - w^*_E(R), \quad q^*(R) = 1 - F(R - w^*_E(R)), \quad \text{and} \quad B^*(R) = \frac{\int_{a}^{b} f(b)db}{\int_{b}^{\tilde{b}^*_E(R)} f(b)db}.$$  

ii) For the comparative statics with respect to $R$, we have $0 < \frac{dw^*_E}{dR} < 1$, $\frac{d\tilde{b}^*_E}{dR} > 0$, $\frac{dw^*_E}{dR} < 0$ and $\frac{dB^*}{dR} > 0$ as long as $-f(R - w^*_E) - f'(R - w^*_E)(\beta - w^*_E) < 0$ holds.

**Proof.** See Appendix A.  \hfill ■

A higher level of $R$ induces firm $E$ to offer a higher wage, the borderline type which agrees to a transfer is higher and therefore, the probability of a transfer is lower and the expected private benefit, given that the offer has been accepted is higher. The condition in part ii) is satisfied as long as $F(\cdot)$ is not too concave and thus when it is uniform as considered below.

**Dates 3 and 2** At date 3, nature draws $b_E$, so the next stage in which one of the parties chooses an action is at date 2, where the worker decides whether or not to accept
firm $I$’s offer. Clearly, taking into account the continuation of the game, the worker will do so only if his expected payoff from doing so is (weakly) higher than his outside option, i.e. $D^*_I = 1$ iff

$$
(1 - q^*(R)) \cdot w_I + q^*(R) \cdot [w^*_E(R) + B^*(R) - r] = w_I + q^*(R) \cdot [w^*_E(R) + B^*(R) - R] \geq U
$$

(9)

**Date 1** At date 1, firm $I$’s payoff is given by

$$
\pi_I = \begin{cases} 
(1 - q^*(R)) \cdot (\beta - w_I) + q^*(R) \cdot r & \text{if } D_I = 1 \\
0 & \text{if } D_I = 0 
\end{cases}
$$

(10)

The following lemma proves useful for the further analysis:

**Lemma 2** For any $R \equiv w_I + r$ given and $w_I > 0$, firm $I$ can always be made strictly better off by increasing $r$ and decreasing $w_I$ by the same amount, thereby keeping $R$ constant.

**Proof.** Simply note that $\pi_I = (1 - q^*(R)) \cdot (\beta - w_I) + q^*(R) \cdot r$ can also be written as

$$
\pi_I = (1 - q^*(R)) \cdot \beta + q^*(R) \cdot R - w_I
$$

which is strictly decreasing in $w_I$. ■

The lemma implies that, for $R$ given, firm $I$ prefers to offer a wage rate $w_I$ as low as possible to the worker. Although $w_I$ and $r$ are prefect substitutes vis a vis firm $E$, firm $I$ prefers to obtain a higher damage payment $r$ rather than paying a higher wage $w_I$. Clearly, there is a limit in lowering $w_I$ due to the non-negativity constraint.

Using Lemma 2 firm $I$’s maximization problem for the case $D_I = 1$ can be stated as

$$
\max_{w_I, R} \pi_I = (1 - q^*(R)) \cdot \beta + q^*(R) \cdot R - w_I
$$

(11)

subject to the worker’s participation constraint and the non-negativity constraint for $w_I$:

$$
w_I + q^*(R) \cdot [w^*_E(R) + B^*(R) - R] \geq U
$$

(12)

$$
w_I \geq 0.
$$

(13)

where the respective damage clause is then just the residual, i.e. $r = R - w_I$. 9
Before analyzing the contract offered by firm $I$ in more detail, we will for later purpose first derive two benchmark results concerning the switching cost $R$. It is assumed that these two solutions are interior and therefore implicitly given by the respective first order condition.

**Benchmark I: First Best** First we determine $R^F$, the level of $R$ which maximizes expected social welfare, i.e.

$$R^F \in \arg \max_R \ SW(R) = \beta + q^*(R) \cdot B^*(R) \tag{14}$$

where $R^F$ solves the following first order condition:

$$q''(R^F) \cdot B^*(R^F) + q^*(R^F) \cdot B''(R^F) = 0 \tag{15}$$

The first term is the marginal loss from a lower level of $q$ while the second term is the gain due to a higher expected private benefit $b_E$ if the transfer is realized (see Lemma 1 part ii)). Note that $q^*(R^F)$ will ensure that the worker will choose to accept firm $E$’s offer exactly when this is socially desirable, i.e. it induces $\bar{b}_E = 0.12$

**Benchmark II: Maximization of Joint Payoff** As a second benchmark case, we determine the level of switching costs $R^J$ which maximizes the joint payoff of firm $I$ and the worker, i.e.

$$R^J \in \arg \max_R \ J(R) \equiv \pi_I + \pi_W = (1 - q^*(R)) \cdot \beta + q^*(R) \cdot (w^*_E(R) + B^*(R))$$

$$= SW(R) - q^*(R) \cdot (\beta - w^*_E(R)) \tag{16}$$

\[12\] Checking that the first order condition (15) is satisfied at $\bar{b}_E = 0$ yields

$$\frac{-f(0) \cdot \int_0^a b \cdot f(b)db}{1 - F(0)} + \frac{-f(0) \cdot 0 \cdot F(0) + f(0) \cdot \int_0^a b \cdot f(b)db}{(1 - F(0))^2} = 0.$$
so that the respective following first order condition is

\[ q''(R_I) \cdot B^*(R_I) + q^*(R_I) \cdot B''^*(R_I) - \left[ q''(R_I)(\beta - w^*_E) - q^*(R_I) \left( \frac{dw^*_E}{dR} \right) \right] = 0. \] (17)

This immediately leads to the following result:

**Proposition 1** The maximization of the joint surplus of the worker and firm I induces excessive switching costs for the worker, i.e. \( R_I > R_F \) holds. This prevents inefficiently many transfers, i.e. \( q^*(R_I) < q^*(R_F) \).

**Proof.** See Appendix B. ■

Intuitively, \( R_I \) will not ensure that the worker will always be transferred whenever it is efficient. Rather, it leads to an inefficiency in a sense that the probability of a transfer is too small. The reason for this inefficiency is the extraction of rents from firm E which can be shared ex ante between the worker and firm I. To see this note that it follows from Lemma 1 that firm E’s equilibrium profit is strictly decreasing in \( R_I \):

\[
\left(1 - F(\tilde{\beta}_E^*(R)) \right) \cdot \left( \frac{dw^*_E(R)}{dR} \right) - f(\tilde{\beta}_E^*(R)) \cdot \frac{\tilde{\beta}_E^*(R)}{dR} \cdot (\beta - w^*_E(R)) < 0. \] (18)

It follows that there is the following trade-off when negotiating the initial contract: On the one hand does a high level of \( R_I \) lead to higher rent extraction from firm E, whereby both, \( w_I \) and \( r \) have the function of increasing the "threat point" of the worker and firm I vis-a-vis firm E. Moreover, they are perfect substitutes in performing this function. On the other hand, this also prevents efficient transfers for some realizations of \( b_E \). These two effects are balanced at the margin by \( R_I \). Note that this result is qualitatively robust against changes in how the surplus is shared ex ante between the worker and firm I:\(^{13}\)

We can now analyze the initial contract offered by firm I at date 1 in more detail: Since firm I has all the bargaining power at the initial contracting stage, it prefers to concede a stake of the joint surplus as small as possible to the worker. For the remainder of the analysis, it will be instructive to specify the model in more detail which allows for closed-form solutions as well as a more tractable comparison of the scenarios with and without regulation of damage clauses. To this end, we assume that \( b_E \) is

\(^{13}\)It can be shown that even if the player could make a take-it-or-leave-it offer to firm I, he would choose a contract which prevents efficient transfers for some realizations of \( b_E \).
uniformly distributed on $[-1, 1]$, and that $\beta > \frac{3}{2}$ which ensures that all relevant levels of $R$ are positive and that all equilibrium values at date 4 are interior.\(^{14}\) With the uniform distribution $F(b_E) = \frac{1 + b_E}{2}$, the equilibrium outcome at date 4 is easily calculated as $w^*_E(R) = \frac{R + \beta - 1}{2}$, $\tilde{w}_E(R) = \frac{R - \beta + 1}{2}$, $q^*(R) = \frac{1 - R + \beta}{4}$ and $B^*(R) = \frac{3 + R - \beta}{4}$. Furthermore, we get $R^F = \beta - 1$, and $R^J = \beta + \frac{1}{3} > R^F$. Finally, define $R_{\text{min}} := \beta - \frac{3}{2} > 0$ and $R_{\text{max}} := \beta + \frac{1}{2}$. We then have the following result:

**Proposition 2** In the scenario without regulation of damage clauses, the optimal initial contract offered by firm I stipulates:

i) $w^*_I(U) = -\frac{3}{36} + U \geq 0$ and $r^*(U) = R^J - w^*_I = \beta + \frac{36}{161} - U$ for all $U \in \left[\frac{1}{36}, U\right]$

ii) $w^*_I(U) \equiv 0$ and $r^*(U) = R^*(U) \in (R^J, R_{\text{max}})$ for all $U \in \left(\frac{1}{64}, \frac{1}{36}\right)$.

iii) $w^*_I(U) \equiv 0$ and $r^*(U) \equiv R_{\text{max}}$ for all $U \leq \frac{1}{64}$.

iv) The worker chooses to accept this contract independent of this outside option.

**Proof.** See Appendix C.  

The intuition behind this result (also illustrated in figure 2) can be explained as follows: When the reservation utility of the worker is high enough, constraint (12) is binding at a positive wage so that firm I maximizes $JS(R)$ minus a constant thus implements $R^J$ which explains part i) of the Proposition. As for part ii), when $U$ decreases, then the wage $w_I$ which satisfies the participation constraint of the worker with equality would be negative and therefore violates constraint (13). In this case, the worker gets $w^*_I = 0$ and would earn a rent if firm I would continue to choose $R^J$. In order to avoid conceding a rent to the worker, the damage clause $r^*$ offered by firm I leads to $R^* > R^J$ which even more distorts the equilibrium level of switching costs from its first best level $R^F$. In part ii), firm I continues to keep the worker at his reservation payoff at the expense of an even higher level of $R$. By how much $R^*$ exceeds $R^J$ depends on $U$. Finally, as for part iii), if $U$ is sufficiently low, then firm I prefers to concede a rent to the worker in order not to induce too high a level of switching costs. Thus, in addition to the rent seeking motive vis a vis firm $E$, there is an additional effect related to the rent for the worker, which both tend to lead to excessive damage clauses.\(^{15}\)

\(^{14}\)While significantly less tractable, a general solution to the optimal contracting problem does not generate any qualitatively different insights. It is available from the author upon request.

\(^{15}\)It is interesting to compare this to Aghion and Bolton (1987, pp. 395), where the overall inefficiency decreases as a result of introducing further (incentive) constraints (due to a second dimension of asymmetric information). That is, while additional constraints (partly) cancel each other in Aghion and Bolton (1987), they add up here which results in a larger degree of inefficiency.
Figure 2: The optimal contract without regulation of damage clauses.

4 Regulation of damage clauses

In the last section we saw that freedom of contract leads to a distortion of the damage clause from its efficient level in order to reduce rents of firm $E$ and, depending on $U$, also of the worker. Consequently in this section, we explore whether a regulator can improve upon the outcome under freedom of contract. In particular, we inquire how an upper bound on the enforceable damage clause might improve matters. From a practical point of view, an upper bound $\tau$ means that all $r > \tau$ will not be enforced by the court and are therefore not contractible. Moreover, any $\tau < 0$ would have the unrealistic implication that it was impossible for parties to write a contract without specifying a damage clause at all (which is equivalent to stipulating $r = 0$). Therefore, only $\tau \geq 0$ are assumed to be feasible. The timing of the game is unchanged except that at date 0, the regulator sets $\tau$. The continuation of the game for date 4 as established by Lemma 1 as well as the acceptance decision of the worker at date 2 remain unchanged. Therefore in a first step, we have to determine the contract offered by firm $I$ at date 1 for $U$ and $\tau$ given, and then the regulator’s optimal choice of $\tau$ for date 0.
Firm $I$’s choice at date 1  At date 1, for the case $D_I = 1$, firm $I$’s maximization problem is given by:

$$\max_{w_I, R} \pi_I = (1 - q^*(R)) \cdot \beta + q^*(R) \cdot R - w_I$$  \hspace{1cm} (19)$$

subject to the participation constraints of the worker, the non-negativity constraint of the worker and the constraint that the damage clause must not exceed $\tau$:

$$w_I + q^*(R) \cdot [w^*_E(R) + B^*(R) - R] \geq U$$  \hspace{1cm} (20)$$

$$w_I \geq 0$$  \hspace{1cm} (21)$$

$$R - w_I \leq \tau$$  \hspace{1cm} (22)$$

Compared to the scenario without regulation, because of the additional constraint $R - w_I = r \leq \tau$, the solution to the problem is more complex, since the relevant parameter space enlarges to $[0, U] \times \mathbb{R}_0^+$. From an economic point of view, this means that when $\tau$ is sufficiently low, firm $I$ is constrained in using a high damage clause as a substitute for the worker’s wage to implement a given level of switching costs $R$ vis a vis firm $E$. As stated below, this will affect the optimal contract in several ways:

Clearly, by how much a given $\tau$ will change the optimal contract offered by firm $I$ (if at all) will also depend $U$. It is therefore instructive to define the following three threshold combinations of $U$ and $\tau$: i) $U^H(\tau) := \beta + \frac{13}{36} - \tau$, ii) $U^L(\tau) := \beta - \frac{7}{36} - \tau < U^H(\tau)$ \forall $\tau \geq 0$ and iii) $U^{PC}(\tau) := \frac{\beta + 1 - \tau}{16}$ which gives the level of $U$ which satisfies participation constraint (20) with equality when $w_I = 0$ and $r = \tau$. Then, we have the following result (which is also illustrated in figure 3):

**Proposition 3** With regulation of damage clauses, the optimal initial contract offered by firm $I$ stipulates:
Again, the worker accepts this contract independent of this outside option.

**Proof.** See Appendix D. □

The intuition is as follows: As seen in Proposition 2, absent any restriction on \( r \), whenever \( w_i^* > 0 \), firm \( I \) keeps the worker at his reservation payoff \( U \), in which case the optimal choice would be \( R_J \). However, when additionally \( r \leq \tau \) has to hold, this
may no longer be optimal: When $\tau$ and/or $U$ are sufficiently low, then firm $I$ is willing to pay a positive wage not because of the participation constraint of the worker, but to distort $R^*$ not too much from $R^J$ (region A1), thereby leaving a rent to the worker. For intermediate values of $\tau$ and/or $U$, implementing $R^J$ would require too high a wage so that firm $I$ still optimally chooses some $R^* < R^J$ (region A2). In this case, the worker is kept at his reservation utility and no longer earns a rent. As for part A3), when $\tau$ and/or $U$ is sufficiently high, firm $I$ implements $R^J$ with the minimum wage necessary to satisfy the worker’s participation constraint. As for parts B1-B3, the worker does not get a positive wage and thus earns a rent, as long as $U$ is sufficiently low (region B1). In region B2, by stipulating high damages, firm $I$ is able to compensate for this and thus is able to eliminate the rent of the worker. Finally, $\tau$ becomes neutral when it is sufficiently high so that optimal contract is the same as in the scenario without regulation. Region B3 captures the case where $U$ is very low, while the other two cases are captured by parts B2 and A3. Note finally that, whenever $w^*_I > 0$, the switching cost induced in equilibrium does not exceed $R^J$, since the only reason to do so is a binding wealth constraint.

The Regulator’s optimal choice of $\tau$ at date 0 Using the previous results, we can now determine the regulator’s optimal choice of $\tau$ at date 0 depending on $U$. Since we have seen that an inefficiently high level of $R$ is implemented for all $U$ in the unregulated case, it can never be optimal for the regulator to set $\tau > \tau^\text{max}$, because it can always replicate the outcome of the unregulated case by simply setting $\tau = \tau^\text{max}$ while the expected social surplus is strictly decreasing in $R$ for $R > \tau_F$. For the regulator’s optimal choice, the following result holds:

**Proposition 4** At date 0, the regulator can induce $\tau_F$ by setting

$$
\tau(U) = \begin{cases} 
R^F & \text{for } U \in [0, \frac{1}{4}] \\
R^F + \frac{1}{4} - U & \text{for } U \in (\frac{1}{4}, R^F + \frac{1}{4}) 
\end{cases}
$$

When $U \in (R^F + \frac{1}{4}, U)$, then it is optimal for the regulator to set $\tau(U) \equiv 0$ thereby implementing some $R \in (R^F, R^J)$.  

16
Proof. See Appendix E. ■

Intuitively, as long as $U$ is sufficiently low, the regulator can simply implement $R^F$ by choosing $\tau = R^F$ since $w_I^* = 0$ holds in the continuation game (i.e. as long as we are in region $B1$). When $U$ increases and $w_I^* > 0$ holds in the continuation game (i.e. as we move to region $A2$), the regulator can still implement $R^F$ by choosing $\tau$ appropriately. However, there is a limit due to $\tau \geq 0$. Thus for all $U > R^F + \frac{1}{4}$, $R^F$ can no longer be reached and the best choice of the regulator is $\tau^* = 0$ so that $R > R^F$ results. Thus, when $U$ is sufficiently large, the best choice for the regulator is to impose a ban of damage clauses. Note that our result that the upper bound on enforceable damage clauses is not a constant, but depends, among others, on $R^F = \beta - 1$ and $U$ (and thus on the worker’s productivity $\beta$ and on the distribution of bargaining power between the worker and the firm) hints at an inefficiency associated with current penalty doctrine practice of setting the upper bound equal to expectation damages.

Figure 4: The optimal maximum damage clause in $U - \tau$-space.
5 Comparison of both scenarios

In this section we compare the results from the regulated and the unregulated case. Using the result from Proposition 2, we know that in equilibrium, the minimum level of switching costs is $R^J$. From Proposition 1 we know that expected social welfare is strictly decreasing in $R$ in this range. Therefore the maximum level of expected social welfare attainable in the unregulated case is $SW(R^J)$. However, this is less than what can be achieved in the scenario with regulation, no matter in which region as it was shown that either $R^F$ or some $R^* < R^J$ is implemented and thus the minimum level of expected social welfare is strictly higher than $SW(R^J)$. This is summarized in the following proposition:

**Proposition 5** For all $U \in [0, U']$, the equilibrium switching costs are strictly higher in the unregulated case. As a result, expected social welfare is strictly higher in the regulated case.

Thus, this proposition provides a strong case for restricting the freedom of contract: For any $U$ does an upper bound on the enforceable damage clauses increase expected social welfare.

6 Conclusion

The previous analysis provides a strong case why restricting the freedom of contract by putting an upper bound on contractually stipulated damage clauses is welfare improving: Due to a rent seeking motive, the initial parties to a contract have an incentive to stipulate an excessive damage clause in the initial contract.\(^\text{16}\) This clause prevents efficient transfers of the worker for some realizations of $b_E$. Moreover, when the non-negativity constraint for the wage rate is binding, firm $I$ has an incentive to increase $R$ even more to avoid/reduce the rent of the worker. The regulator can counterbalance these incentives by setting $\tau$ appropriately.

Since many assumptions underlie the analysis, some of them shall now be discussed in more detail:

\(^{16}\)In a different context, Hua (2005) shows that, even with a rent-seeking motive, contracts might still be socially desirable as they can be used to mitigate rent seeking motives ex post, thereby increasing allocative efficiency.
Non-negativity constraint for \( w_I \). Concerning the supports for \( w_I \) and \( r \), the alternative interpretation of \( w_I \geq 0 \) as a wealth constraint appears somewhat critical, since this would imply that although the worker cannot afford to pay a wage \( w_I < 0 \), he is be able to pay any damage payment \( r \). Note however, that he has to pay \( r \) only if he works for firm \( E \), in which he gets \( w_E \geq 0 \) in return. Of course, since he also receives the private benefit \( b_E \), it is not assured that \( w_E > r \) holds in which case he might still need external funds to carry out the transfer. However at least, this suggests that a possible wealth constraint of the worker seems less strict with respect to \( r \) than with respect to \( w_I \). Therefore, we believe that assuming this extreme case to be justified. Alternatively, one could assume that firm \( E \) has to pay \( r \) in which case the problem would disappear and the results would be qualitatively unchanged. However, this seems to contradict reality. In fact, one of the very few segments of the labor market in which damages are to be paid by the new employer is the market for professional athletes. Finally, the case \( w_I \in \mathbb{R} \) is not an interesting one, because then the worker’s participation constraint can always be made binding by a sufficiently low \( w_I \) which implies that firm \( I \)’s offer will maximizes \( JS(R) \) minus a constant so that it will always offer \( R^I \).

Bargaining between the worker and firm \( I \). Note that contrary to many applications in the literature, the size of the expected surplus to be shared ex ante among the worker and firm \( I \) is not fixed in our model but endogenously determined by the nature of the initial contract. Concerning the modeling assumption how the surplus is split in the initial contract, it is clear that one could alternatively assume that the worker and firm \( I \) engage in Nash bargaining by maximizing the generalized Nash-Product \( \pi_I^\alpha \pi_W^{1-\alpha} \) where \( \alpha \in (0, 1) \) denotes the bargaining power of firm \( I \).\(^{17}\) However, apart from making the model less tractable, any split of the joint surplus which can be achieved when using the Nash bargaining approach by variation of \( \alpha \), can also be achieved by varying \( U \) accordingly in the present model.\(^{18}\) Therefore, it seems a reasonable approach to use the outside option of the worker as a proxy for his bargaining power while allowing firm \( I \) to make a take-it-or-leave-it offer.

\(^{17}\) For example, the paper by Diamond and Maskin (1979) discussed above analyzes a model with Nash bargaining to illustrate the existence of the rent seeking motive vis-à-vis third parties.\(^{18}\) See also Demougin and Helm (2005).
Bargaining between the worker and firm $E$ The assumption that the new firm is able to make a take-it-or-leave-it offer to the worker is strong but not crucial in order to derive the results qualitatively. What is important is that firm $E$ reaps a positive share of the surplus. Otherwise, we would be back in the case in which there would be no externalities from contracting so that no rent seeking motive vis a vis firm $E$ arises.\footnote{See Shavell (1980), Rogerson (1984) or Chung (1992) (for $\alpha = 0$).} In this case, firm $I$ and the worker would have the right incentives in case the worker’s outside option is binding, while there would still be a rent reduction incentive for firm $I$ when the non-negativity constraint $w_I \geq 0$ is binding.

Signaling and Screening Devices That workers are not able to signal their private information seems a reasonable assumption for some segments of the labor market while it is not for others where it might well be the case that signaling is either not possible or excessively costly. Since firm $E$ is assumed to have only the possibility of offering a wage rate $w_E$ to the worker, it is clear that there is no way in inducing different types to act differently since there would have to be at least a second choice variable which could be used in order to ensure incentive compatibility. Clearly, one might want to enrich the model in that direction.

Another interesting extension would be to include investment choices. For example, in case that the productivity of the worker in firms $I$ and $E$ is a function of firm $I$’s (general) investment, reducing the initial parties’ ability to stipulate liquidated damages would lead to lower investment incentives, which presumably would make the results concerning the desirability of such kinds of restrictions less unambiguous. This issue awaits further research.
Appendix

A Proof of Lemma 1

Part i): Follows immediately from substituting the equilibrium values in (5), (6) and the definition of $B$.

Part ii): From (8), applying the implicit function theorem we have

$$\frac{dw_E^*}{dR} = \frac{(-1) \cdot [f(R - w_E^*) + f'(R - w_E^*)(\beta - w_E^*)]}{(-1) \cdot [2f(R - w_E^*) + f'(R - w_E^*)(\beta - w_E^*)]}.$$ 

At an interior solution, the second order condition $(\beta - w_E^*) > 0$ must hold. As for the numerator, as long as $F(\cdot)$ is not too concave such that also $-f(R - w_E^*) - f'(R - w_E^*)(\beta - w_E^*) < 0$, we have

$$-f(R - w_E^*) - f'(R - w_E^*)(\beta - w_E^*) > -2f(R - w_E^*) - f'(R - w_E^*)(\beta - w_E^*)$$ 

since $f(R - w_E^*) > 0$ and thus $0 < \frac{dw_E^*}{dR} < 1$ holds. Note that at an interior solution, $(\beta - w_E^*)$ must be non-negative since otherwise, firm $E$ would obtain a negative payoff and thus prefer its reservation payoff of zero to making an offer to the worker.

From this result, it follows that \( \frac{dB^*}{dR} = 1 - \frac{dw_E^*}{dR} > 0 \) and \( \frac{dB^*}{dR} = -f(R - w_E^*)(1 - \frac{dw_E^*}{dR}) < 0 \).

For the comparative statics result for $B^*$ we have

$$\frac{dB^*}{dR} = \frac{\int\limits_{\tilde{b}_E^*(R)}^a f(b)db \cdot (-\tilde{b}_E^* \cdot f(\tilde{b}_E^*) \cdot \frac{\tilde{b}_E^*}{\tilde{b}_E^*(R)}) - \int\limits_{\tilde{b}_E^*(R)}^a b \cdot f(b)db \cdot (-f(\tilde{b}_E^*) \cdot \frac{\tilde{b}_E^*}{\tilde{b}_E^*(R)})}{\left(\int\limits_{\tilde{b}_E^*(R)}^a f(b)db\right)^2}$$

$$= \frac{-f(\tilde{b}_E^*) \cdot \frac{\tilde{b}_E^*}{\tilde{b}_E^*(R)}}{\left(\int\limits_{\tilde{b}_E^*(R)}^a f(b)db\right)^2} \left(\int\limits_{\tilde{b}_E^*(R)}^a f(b)db - \int\limits_{\tilde{b}_E^*(R)}^a b \cdot f(b)db \right) \left(\int\limits_{\tilde{b}_E^*(R)}^a f(b)db\right)^2 > 0.$$
Note that the second term in the numerator is negative, i.e.

\[
\tilde{b}_E^* < \frac{\int_a^b b \cdot f(b) db}{\int_a^b f(b) db} = B
\]

holds, because the expected value cannot be smaller than the lower bound of the integral.

### B Proof of Proposition 1

For the comparison of \( R^F \) and \( R^I \), it suffices to compare the first order conditions (15) and (17): Since the term \( q^*(R^I)(\beta - w^*_E) + q^*(R^I)(-\frac{dw^*_E}{dR}) \) in (17) (measuring the effect of an increase of \( R \) on the expected profits of firm \( E \)) is negative, the marginal cost of increasing \( R \) for the worker and firm \( I \) is lower than the social marginal cost. Therefore, \( R^I > R^F \) holds which, from Lemma 1 also implies \( q^*(R^I) < q^*(R^F) \).

### C Proof of Proposition 2

After simplifying, firm \( I \)'s maximization can be re-written in the standard form for non-linear programming:

\[
\max_{w_I,R} \left( 1 - \frac{1 - R + \beta}{4} \right) \cdot \beta + \left( \frac{1 - R + \beta}{4} \right) \cdot R - w_I
\]

s.t. \(-U \geq -w_I + \frac{1}{16}(1 - R + \beta)(R - 1 - \beta) \)

\[0 \geq -w_I\]

The Lagrangian \( Z \) is then given by

\[
Z = \left( 1 - \frac{1 - R + \beta}{4} \right) \cdot \beta + \left( \frac{1 - R + \beta}{4} \right) \cdot R - w_I - y_1 \left( -w_I + \frac{1}{16}(1 - R + \beta)(R - 1 - \beta) \right) + y_2 w_I
\]

where \( y_1 \) and \( y_2 \) denote the respective multipliers. The objective function is quasi-concave in \( R \) and \( w_I \) and both constraints are quasi-convex in \( R \) and \( w_I \). Moreover, since neither the objective function is strictly concave in \( R, w_I \) nor are all constraints strictly convex in \( R, w_I \), to apply the result by Arrow and Enthoven (1961), two further conditions must
be met.\textsuperscript{20} Arrow and Enthoven (1961) prove that when this is the case, the Kuhn-Tucker
conditions are sufficient for a maximum. For your case, the respective Kuhn-Tucker
conditions are

\begin{equation}
\frac{\partial Z}{\partial R} = (2\beta - 2R + 1) + y_1\left(\frac{R - 1 - \beta}{8}\right) = 0 \tag{23}
\end{equation}

\begin{equation}
\frac{\partial Z}{\partial w_I} = -1 + y_1 + y_2 \neq 0 \tag{24}
\end{equation}

\begin{equation}
-U \geq -w_I + \frac{1}{16}(1 - R + \beta)(R - 1 - \beta) \tag{25}
\end{equation}

\begin{equation}
0 \geq -w_I \tag{26}
\end{equation}

\begin{equation}
y_1, y_2 \geq 0 \tag{27}
\end{equation}

\begin{equation}
0 = y_1 \left[w_I - \frac{1}{16}(1 - R + \beta)(R - 1 - \beta) - U\right] \tag{28}
\end{equation}

\begin{equation}
0 = y_2 w_I \tag{29}
\end{equation}

First, consider a solution with \(w_I^* > 0\), which implies \(y_2 = 0\) (from (29)) and \(y_1 = 1\). Then (23) yields \(R^* = \beta + \frac{1}{3} = R^f\) while (25) must be binding so that \(w_I = U - \frac{1}{36}\) which is positive for all \(U > \frac{1}{36}\) so that this is the solution to the problem for all \(U > \frac{1}{36}\). This explains part i) of the Proposition.

When \(w_I^* = 0\) then \(y_2 \geq 0\). Case 1: \(y_2 = 0\) which implies \(y_1 = 1\). Again (23) yields \(R^* = \beta + \frac{1}{3} = R^f\) while (25) yields \(w_I = U - \frac{1}{36}\) which is strictly increasing in \(U\) and equal to zero for \(U = \frac{1}{36}\). Case 2: \(y_2 > 0\). Again we have two sub-cases: Case 2a: \(0 < y_2 < 1\) and \(y_1 = 1 - y_2 > 0\). In this case, constraint (25) must also be binding so that we have a system with 3 equations ((23), (24) and (25)) and three endogenous variables \((R, y_1, y_2)\) which gives \(R^* = \tau\), \(y_1 = 2\frac{2\tau + 1 - 2\beta}{2\tau - 2\beta + 2\beta + 1 - \beta}\) and \(y_2 = \frac{\gamma + 2\tau - 2\beta + 1 - \beta - 2}{\tau - 2\tau + 2\beta + 1 - \beta}\), where \(\tau(U) = 1 + \beta - 4\sqrt{U}\). We then have \(y_1, y_1 > 0\) for \(U \in (\frac{1}{64}, \frac{1}{36})\) which implies that \(R^* \in (\beta + \frac{1}{3}, \beta + \frac{1}{2}) = (R^f, R^{\max})\).

Case 2b: \(y_2 = 1\) which implies \(y_1 = 0\). In this case, (23) yields \(R = \beta + \frac{1}{2} = R^{\max}\). Then, (25) is satisfied for \(U < \frac{1}{64}\).

As for part iv), the worker earns at least his outside option and therefore always chooses to accept firm \(I\)’s offer.

\textsuperscript{20}These conditions are that i) there must exist a point \((R^0, w_I^0)\) for which all constraints are satisfied as strict inequalities and ii), there must not exist a point \((R^1, w_I^1)\) at which the partial derivatives of all constraints with respect to \(R\), and \(w_I\) are zero. Both conditions are met in the present context.
D Proof of Proposition 3

The programming problem is identical to the one in the scenario without regulation, except that we have an additional constraint (22), which is also quasi-convex in $R$ and $w_I$. Denoting the respective multiplier by $y_3$, the Lagrangian is

$$Z = (1 - \frac{1}{4}(1 - R + \beta)) \cdot \beta + (1 - \frac{1}{4}(1 - R + \beta)) \cdot R - w_I - y_1 \left(-w_I + \frac{1}{16}(1 - R + \beta)(R - 1 - \beta)\right) + y_2 w_I - y_3 [R - w_I]$$

for which the respective Kuhn-Tucker conditions are

$$\frac{\partial Z}{\partial R} = (2\beta - 2R + 1) + y_1 \left(\frac{R - 1 - \beta}{8}\right) - y_3 \overset{!}{=} 0 \quad (30)$$

$$\frac{\partial Z}{\partial w_I} = -1 + y_1 + y_2 + y_3 \overset{!}{=} 0 \quad (31)$$

$$-U \geq -w_I + \frac{1}{16}(1 - R + \beta)(R - 1 - \beta) \quad (32)$$

$$0 \geq -w_I \quad (33)$$

$$\tau \geq R - w_I \quad (34)$$

$$y_1, y_2, y_3 \geq 0 \quad (35)$$

$$0 = y_1 \left[w_I - \frac{1}{16}(1 - R + \beta)(R - 1 - \beta) - U\right] \quad (36)$$

$$0 = y_2 w_I \quad (37)$$

$$0 = y_3 [\tau - R - w_I] \quad (38)$$

We start with parts A1-A3 where $w^*_I > 0$ so that it follows from (37) that $y_2 = 0$ must hold. Thus we have to consider 3 cases: Case 1 (which explains part A1 of the Proposition): $y_1 = 0$ and therefore $y_3 = 1$. In this case, from (30) we get $R^* = \frac{1}{2} = R^\text{min}$ and, from (38), $w^*_I = R^\text{min} - \tau > 0$ for all $\tau < R^\text{min}$ and thus, $r^* = \tau$ holds. From the participation constraint we have

$$-U + w^*_I - \frac{1}{16}(1 - R^* + \beta)(R^* - 1 - \beta) \geq 0 \Leftrightarrow U \leq \frac{71}{64} - \tau = U^L(\tau).$$

Case 2 (A2): $y_1 > 0$ and $y_3 > 0$: For this case, (30), (31), (36) and (38) must hold simultaneously, so that we have 4 equations for 4 endogenous variables $(R, w_I, y_1, y_3)$ with solution $R^* = \rho$, $w^*_I = \rho - \tau$, $y^*_1 = \frac{2 - \rho - 2\beta - \beta^2}{2\rho - 2\rho^2 - 2\beta - 2\beta^2 - 2\beta - 8\rho - 8\beta + 8}$, $y^*_3 =$
\[
\frac{2\beta - 1 + \beta + 2 - 2\rho - \rho}{2\rho - \rho - 2\beta - 1 + \beta + 8}
\]

where
\[
\rho(U) = \beta - 7 + 4\sqrt{3 + \tau - \beta + U}.
\]

One calculates that \(y_1\) is strictly increasing in \(\rho\), so that \(y_1 > 0 \iff \rho > R_{\text{min}}\)
while \(y_3\) is strictly decreasing in \(\rho\) so that \(y_3 > 0 \iff \rho < \beta + \frac{1}{3} = R^J\). Using
the definition for \(\rho\), this is easily be transformed into a condition in \(U\) for
which one gets
\[
\rho(U) \in (R_{\text{min}}, R^J) \iff U \in (U^L(\tau), U^H(\tau))
\]

Finally, we have \(w_I > 0 \iff U > \frac{(\beta + 1 - \tau^2)}{16} = U^{PC}(\tau)\). These conditions have to
be compatible with each other. Therefore, one calculates that that \(U^L(\tau) >
(\langle \rangle) U^{PC}(\tau)\) for all \(\tau < (>) R_{\text{min}}\) and \(U^H(\tau) > (\langle \rangle) U^{PC}(\tau)\) for all \(\tau < (>) R^J\).
Therefore, in the relevant interval \((R_{\text{min}}, R^J)\), the relevant condition becomes
\(U \in [\max(U^L(\tau), U^{PC}(\tau)), U^H(\tau)]\) as stated in the proposition.

Case 3 (A3): \(y_1 = 1\) and thus \(y_3 = 0\): In this case, (30) and (36) must
hold with equality, which yields \(R^* = \beta + \frac{1}{3} = R^J\) and \(w_I^* = U - \frac{1}{36}\) which is
strictly increasing in \(U\) and therefore positive for \(U > \frac{1}{36}\) so that the condition
becomes again \(U > \frac{1}{36}\). Moreover, (34) is satisfied when
\[
w_I^* \geq R^* - \tau \iff U \geq \beta + \frac{13}{36} - \tau = U^H(\tau).
\]

Now consider the parts of the proposition where \(w_I^* = 0\) (B1-B3): Again, we have to
consider three cases:

Case 1 (B1): \(y_1 = 0, y_2 \geq 0\) and \(y_3 > 0\): In this case, one has to solve the equation
system with Eqns. (30), (31) and (38) for three endogenous variables \((R, y_2, y_3)\) which
yields \(R = \tau, y_2 = \frac{1}{4}(3 - 2\beta + 2\tau),\) and \(y_3 = \frac{1}{4}(2\beta + 1 - 2\tau)\). \(y_2\) is strictly increasing in \(\tau\)
and therefore non-negative for \(\tau \geq \beta - \frac{3}{2} = R_{\text{min}}\) while \(y_3\) is strictly decreasing in \(\tau\)
and positive for \(\tau < \beta + \frac{1}{3} = R_{\text{max}}\). To check for which levels of \(U\) this is consistent with \(w_I^* = 0\)
and condition (32), substitute \(R^*\) and \(w_I^* = 0\) in (32) to yield \(U \leq \frac{(\beta + 1 - \tau)^2}{16} = U^{PC}(\tau)\).

Case 2 (B2): \(y_1 > 0, y_2 \geq 0\), and \(y_3 = 0\): For this case, the equation system to solve consists of Eqns. (30), (31) and (36) for three endogenous variables \((R, y_1, y_2)\)
which leads to solution \( R^* = \phi, \ y_1 = 2 \frac{2\beta + 1 - 2\phi}{\phi + 2\beta + 1 - \beta - 2\phi}, \) and \( y_2 = \frac{2\phi + \beta - 2\beta + 1 - \beta - 2}{\phi + 2\beta + 1 - \beta - 2\phi} \) where \( \phi(U) = \beta + 1 - 4\sqrt{U}. \)

Substituting this into \( y_1 \) and \( y_2 \) reveals that \( y_1 > 0 \Leftrightarrow U > \frac{1}{64} \) and that \( y_2 \geq 0 \Leftrightarrow U \leq \frac{1}{36} \). Moreover, we have \( \phi(U = \frac{1}{64}) = R^{\max} \) and \( \phi(U = \frac{1}{36}) = R^J. \) Finally, we have to determine the levels of \( U \) for which (32) is also satisfied: Solving \( \tau - R^* \geq 0 \) for \( U \) yields \( U \geq \frac{(\beta + 1 - \tau)^2}{16} = U^{PC}(\tau). \) Since \( U^{PC}(\tau) \) is decreasing in \( \tau, \) we have \( U^{PC}(\tau) > (\leq) \frac{1}{64} \) for all \( \tau > (\leq) R^{\max} \) which leads to the relevant range \( U \in (\frac{1}{36}, \max(U^{PC}(\tau), \frac{1}{64})) \) as stated in the proposition.

Case 3 (B3): \( y_1 = 0, \ y_2 = 1, \) and \( y_3 = 0: \) For this last case, (30) yields \( R^* = \beta + \frac{1}{2} = R^{\max}. \) Since this case is only relevant for \( \tau \geq R^{\max}, \) condition (34) is trivially satisfied. For (32), substituting \( R^* \) and \( \w^*_I = 0 \) yields \( U \leq \frac{1}{64}. \)

### E Proof of Proposition 4

Given the continuation game as established by Proposition 3, the regulator maximizes expected social welfare with respect to \( \tau. \) As for part i), as long as \( \w^*_I = 0 \) holds on the equilibrium path, \( R^F = \beta - 1 \) can be achieved by simply setting \( \tau = R^F. \) The threshold value for \( U, \) for which this is no longer possible is determined by setting \( \tau = R^F \) in the equation according to which is \( \w^*_I = 0 \) holds, i.e.

\[
U^{PC}(\tau = R^F) = \frac{(\beta + 1 - R^F)^2}{16} = \frac{1}{4}
\]

For \( U > \frac{1}{4} \) and at \( \tau = R^F \) we are no longer in region B1 but in region A2. Also in this case, \( R^F \) can be implemented as long as \( U \) is not too high: Setting \( \rho \) as given above equal to \( R^F \) and solving for \( U \) yields \( \tau^* = R^F + \frac{1}{4} - U. \) Thus, the maximum level of \( U \) for which \( R^F \) can be implemented by a non-negative \( \tau \) is \( U = R^F + \frac{1}{4}. \) For all \( U > R^F + \frac{1}{4}, \) we are still in region A1 so that the worker’s participation constraint will be binding. This means that even for \( \tau = 0 \) we already have \( \w^*_I > R^F \) and thus the best the regulator can do is not increasing \( R \) even further by choosing \( \tau = 0. \) Since \( U^H(\tau = 0) > \beta, \) it follows from our initial assumption \( \beta > \underline{U} \) that the resulting level of \( R \) is strictly lower than \( R^J. \)Aghion and Bolton (1992)
References


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