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Resource Allocation and Firm Scope

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ABSTRACT

Resource Allocation and Firm Scope*

We develop a theory of firm scope in which integrating two firms into one facilitates the allocation of resources, but leads to weaker incentives for effort, compared with non-integration. Our theory makes minimal assumptions about the underlying agency problem. Moreover, the benefits and costs of integration originate from the same problem – to allocate resources efficiently, the integrated firm's top management must obtain information about the possible use of resources from division managers. The division managers' job is to create profitable investment projects. Giving the managers incentives to do so biases them endogenously towards their own divisions, and gives them a motive to overstate the quality of their projects in order to receive more resources. We show that paying managers based on firm performance in addition to individual performance can establish truthful upward communication, but creates a free-rider problem and raises the cost of inducing effort. This effect exists even though with perfect information, centralized resource allocation would improve the managers' incentives. The resulting tradeoff between a better use of resources and diminished incentives for effort determines whether integration or non-integration is optimal. Our theory thus provides a simple answer to Williamson's "selective-intervention" puzzle concerning the limits of firm size and scope. In addition, we provide an incentive-based argument for the prevalence of hierarchically structured firms in which higher-level managers coordinate the actions of lower-level managers.

JEL Classification: D23, D82, L22, M52

Keywords: theory of the firm, coordination, authority, incentives, strategic information transmission

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1 Introduction

Economic resources are often allocated within firms by the “visible hand” of managers, rather than by the invisible hand of the market. The coordination and resource allocation functions of an emerging professional top management were crucial for the growth of U.S. firms into large multi-divisional corporations in the early 20th century: “...[the] producing and distributing units within a modern business enterprise are monitored and coordinated by middle managers. Top managers, in addition to evaluating and coordinating the work of middle managers, [take] the place of the market in allocating resources for future production and distribution” (Chandler 1977, p. 7). Clearly, however, there must be costs associated with this solution; otherwise all economic activity could be carried out in firms.

We develop a theory of firm scope in which the benefit of integration relates to the resource allocation function of top management, and the cost to the strategic use of information by middle managers, and its consequences for effort incentives. Our theory has two salient features. First, both the benefits and costs of horizontal integration originate from the same economic problem: that of aggregating and using information that is dispersed among the members of an organization, a problem first emphasized by Hayek (1945). Second, we make minimal assumptions about the underlying agency problem. In particular, our theory does not rely on incongruent preferences over decisions among agents, rent-seeking or influence activities, or preferences for empire-building.

Our basic story is the following. Production in firms typically involves some resources that are costly to trade across firm boundaries. When two firms integrate, they can pool their resources and potentially allocate them more efficiently within the new firm. Suppose that upon integration, the two firms become divisions of one firm in which a common top management, or “CEO”, has the authority to allocate resources between the divisions. To do so efficiently, the CEO needs to know about the production possibilities of the divisions. This information, however, typically resides with the division managers.

The managers’ role in our model is to create investment opportunities, or “projects”. Project quality (profitability) depends on the effort of the responsible manager. Realizing a project requires resources, and the more resources a project receives, the higher its payoff (the division’s performance). A non-integrated firm operates with given resources, and can ensure its manager’s effort by providing appropriate performance-based incentives. In the integrated firm, it is efficient to shift resources to the best projects. But here, individual incentives may give a

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1 More precisely, project quality and resources are complements in the divisions’ production functions, but division performance is increasing in resources invested for any project quality.
manager a motive to overstate the quality of his project, since even if his project is not good, he stands to gain from receiving more resources as they increase expected division performance. Thus, the need to provide incentives for effort endogenously creates a motive similar to “empire-building” even though the managers have no intrinsic preferences for resources.

For integration to serve its purpose, the firm must establish truthful upward communication. We show that this is possible by paying managers not only for their own division’s performance, but also for that of the other division, or equivalently, firm performance. Doing so, however, creates a free-rider problem and reduces the managers’ incentives to provide effort relative to those they face under non-integration. This effect, highlighted by Dessein et al. (2005) in a different setting, appears solely because of the managers’ private information; with perfect information, resource allocation under the authority of the CEO would improve the managers’ incentives. Whether integration is optimal therefore depends on how the benefit of a better use of resources compares to the higher costs of inducing effort.

We further show that the hierarchical structure of the integrated firm described is not just intuitive but in a certain sense optimal. An alternative structure would be to do without a CEO, and simply give one of the division managers the authority over the firm’s resources. We show, however, that providing a manager with incentives to allocate resources efficiently is unambiguously more costly than getting him to communicate truthfully to a CEO.

We obtain these results in a model (see Section 3) that consists of two functionally independent production units, which can be run either as self-standing firms or as divisions of an integrated firm. Ownership and control are separated; that is, the managers are agents of the firms’ (or firm’s) owners.

The resources in our model are not commodities — it would be pointless to establish a firm to allocate resources that can easily be traded in the marketplace. Instead, we envision firms endowed with resources — such as teams of engineers with special capabilities — that are to some extent firm-specific but can be allocated to different business activities. We also assume that the resources are costly to trade across firm boundaries, but relatively easy to allocate to different uses inside a firm under someone’s authority. To capture these ideas formally in a simple but plausible way, we follow the recent literature on authority in organizations in assuming that resources are not (spot-)contractible, but that the authority over resources is contractible. Each unit, run as an individual firm, must rely on its own resources. In the

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integrated firm, the units’ resources can be pooled and placed under the authority one person, which for most of our analysis we assume is the CEO.

The effect of integration on managerial incentives is driven by two countervailing effects, which we examine in benchmark cases in Sections 4.2 and 4.3. First, since the firm’s resources are shifted to the division with the better project, good projects receive more resources than under non-integration, and bad projects less. Competition between managers for the firm’s resources thus leads to a greater spread in expected performance between having a good and a bad project, and thus increases the marginal benefit of effort. As a result of this competition effect — which appears in its pure form if we assume that the CEO has perfect information about the divisions’ projects —, inducing managers to exert effort is less costly than in a non-integrated firm (see Stein, 2002, and Inderst and Laux, 2005, for similar results).

Second, however, there is also an information rent effect, because the project types are the managers’ private information: division managers with a bad project must be rewarded for revealing the quality of their project truthfully. The resulting decrease in the wedge between the payoffs for a good and a bad project makes it more expensive to induce effort, as in Levitt and Snyder (1997). We show that when contracts can be made contingent not only on division performance but on messages about project quality as well, the information rent effect precisely cancels the competition effect; that is, expected wages under integration and non-integration are the same. Under these contracting assumptions, integration dominates non-integration because it leads to a better resource allocation.

In practice, however, message-contingent contracts are often not feasible. In Section 4.4, we assume that contracts can be based only on division performance. Messages are not contractible; that is, we have a setting of strategic communication, or “cheap talk”. Each manager communicates the quality of his unit’s project to the CEO, who then allocates resources based on the managers’ messages. Now, the firm can no longer directly compensate a manager who reports a bad project. We show that truthful communication can be established by rewarding each manager for the other division’s performance in addition to his own. Doing so, however, creates a free-rider problem and leads to strictly higher costs of inducing effort. In other words, with cheap talk, the information rent effect dominates the competition effect.

This result always holds when the firm can only pay separate bonuses for each division’s performance, which in our model with binary division payoffs means that wage contracts are linear. In the bulk of the paper, we focus on this case for simplicity of exposition. We also show, however, that almost all of our main results carry over to the more general case in which wages can be conditioned on joint realizations of the divisions’ performances. The only exception occurs
when the probability of creating a good project is very low; truthtelling can then be ensured by paying a large bonus if both divisions have a high payoff. The resulting wage costs are then the same as under non-integration, just like in the case with message-contingent contracts. We summarize the results for the “non-linear” case throughout the main text, and present the full analysis in Appendix B.

The integration decision hence depends on the relative importance of allocating resources efficiently and inducing effort. In particular, in our model in which effort is either high or low, the optimal solution may be to integrate, but to offer flat wages that lead to truthtelling but no effort. (With continuous effort, the solution would be to provide lower-powered incentives.3)

In Section 5, we show in what way the symmetric hierarchy considered thus far is the optimal structure for an integrated firm. Suppose that instead of hiring a CEO, the owner gives the authority over resources to division manager 1, to whom manager 2 must report his project quality. Now, only one truthtelling constraint needs to be satisfied — manager 2’s. On the other hand, incentives must be structured to ensure that manager 1 allocates resources efficiently, instead of favoring his own division. While these constraints are very similar to the truthtelling constraint they replace, there is an important difference. In the hierarchy with CEO, manager 1 reports his project quality without knowing division 2’s project, whereas if manager 1 has authority over resources, he allocates them after learning about division 2’s project. The manager’s better information in the latter case exacerbates the agency problem for the firm, and translates into unambiguously more restrictive incentive constraints.

In the remainder of this Introduction, we explain what we consider the paper’s contribution. A discussion of related literature not mentioned here follows in Section 2.

The main contribution of our paper is to the theory of the firm. From Coase (1937) and Williamson (1975, 1985) to the property-rights and incentive-system theories of the firm4, most of the literature focuses on the “make-or-buy” decision, that is, on vertical integration. In contrast, we study horizontal integration, which has received much less attention in the literature despite its importance — after all, the emergence of the modern multidivisional firm was primary a

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3 See Footnote 15 on a variation of our model with continuous effort. More generally, with binary effort, a higher cost of inducing effort means that the principal must pay a higher bonus to induce high effort; she may then prefer to induce low effort instead. With continuous effort, the principal usually responds to a higher cost of inducing effort by offering lower-powered incentives, which lead to lower effort in equilibrium. Thus, the results may appear different depending on whether effort is binary or continuous, but are economically the same.

result of horizontal expansion.

In particular, we offer a simple answer to Williamson’s (1985, Ch. 6) “selective intervention puzzle”, which asks why is it not possible for two firms to integrate, to let a CEO coordinate their actions where appropriate, and otherwise to be run as before. Our answer is that coordination makes it necessary to aggregate dispersed information, but establishing truthful upward communication comes at a cost of muted incentives for effort.

This answer has two important virtues. One is that it relies on minimal, and natural, assumptions about the underlying agency problem: division managers need to be given incentives to create value in their divisions, and have private information about their investment opportunities that require corporate resources. Another virtue is that both the benefits and costs of integration derive from the same primitive, the aggregation of dispersed information (see Hart 1995, and Gibbons, 2005, on this methodological postulate).

Several other answers have been advanced in the literature. Williamson himself argued informally that integration leads to weaker incentives, because with high-powered incentives, managers would not take care of the firm’s assets or would engage in “accounting contrivances”. Milgrom and Roberts (1988, 1990) and Meyer, Milgrom and Roberts (1992) argue that firms use bureaucratic practices or lower-powered incentives in order to curb unproductive “influence activities” that agents engage in to promote their own interests rather than those of the firm. In the incentive-system theory of Holmström, Milgrom and Tirole (see Footnote 4), lower-powered incentives can emerge as optimal solution when some of an agent’s tasks can be less well measured than others (see also Baker et al., 2001). In contrast, we do not rely on multiple tasks — productive or unproductive — as a primitive in our model. Rather, managers communicate to the CEO, in addition to providing effort, because their information is needed to allocate resources; their influence originates from their ability to misrepresent information.

Other answers to Williamson’s puzzle are based on the idea that top management’s decisions aimed at realizing the benefits of integration may have adverse consequences for others in the firm. In Hart and Holmström (2002), the top management of an integrated firm maximizes the firm’s profit, but neglects the private benefits of workers and managers who upon integration lose decision rights, which reduces the total surplus created. In Mailath et al. (2005), the cost of integration is that it may reduce the value of managers’ human capital and hence make it more costly to induce their effort. These papers belong to the literature on authority in organizations already mentioned, see Footnote 2 above and Section 2.

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5 Note that communication is not simply another task, because the effort-substitution effect that is key in the incentive-system approach is absent in our model, since communication is costless.
Although our contracting assumptions follow this literature, authority plays a different role in our model. Rather than appealing to agency problems that arise when top management makes decisions that adversely affect those at lower ranks, the puzzle we seek to solve is, why are there limits to integration even if top management intervenes selectively and beneficially? We therefore refrain from assuming any divergence of preferences between division managers, CEO, and owner, except that the managers’ effort is privately costly. In this setting, the cost of giving an unbiased CEO authority over resources is not per se that it reduces the managers’ incentives — on the contrary, with perfect information, their incentives improve. Instead, the problem is that coordination through centralized decision-making requires aggregating the private information of managers who are endogenously biased because they want their projects to succeed. Establishing truth-telling then comes at a higher cost of providing incentives for effort.

A second contribution of our paper is to provide an incentive-based explanation for the prevalence of hierarchically structured firms, and for the division of labor between top and middle management documented by Chandler. Intuitively, allocating the firm’s resources is best done by an unbiased decision maker, whereas creating investment opportunities for a division is best done by someone whose incentives are strongly aligned with that division. These tasks turn out to be difficult to combine in the same job, a result reminiscent of Dewatripont and Tirole’s (1999). We discuss alternative explanations for hierarchies in Section 2.

2 Related Literature

Aside from the literature already discussed, our paper is related to several strands of the organizational economics and corporate finance literature.

Authority, communication and incentives: It is a pervasive theme in the organizational economics literature that a principal’s actions may undermine an agent’s incentive to exert effort or to communicate truthfully to the principal. For instance, in Aghion and Tirole (1997), a principal’s authority to overrule her agent reduces the agent’s incentive to generate information about different projects. In Dessein (2002), the agent communicates his private information about a project only partially (see also Marino and Matsusaka, 2004). In these and many related papers on authority in organizations (see Footnote 2), the underlying agency problem is a divergence of players’ preferences over possible actions. Several papers also rule out monetary incentives that might align players’ interests. Consequently, giving one player authority over a decision increases that player’s incentives to take a surplus-increasing action (e.g. to determine the profitability of available projects) but reduces the other players’ incentives. In contrast,
as discussed, there is no exogenous divergence of preferences over decisions in our model. The cause of agency problems is instead the managers' effort aversion and their private information. We also allow for incentive contracts as a way to align the players' incentives.

Closer to our paper is that of Levitt and Snyder (1997), who point out the conflict between providing effort incentives and establishing truthful communication. In their model, a principal can get an agent to communicate “bad news” about a project truthfully by paying him a reward for termination of the project. But if the project’s quality also depends on the agent’s up-front effort, then a reward for bad news undermines the agent’s effort incentives. Optimal incentives are therefore weaker than without communication. Our model exhibits the same tradeoff, except that with cheap talk and unverifiable resource allocations, managers cannot be rewarded directly for conveying bad news. Team-based performance incentives are then the only way to establish truthful communication. The multi-agent structure of our integrated firm thus plays an important role for the resulting solution.

**Team incentives:** Our paper offers a new rationale for the provision of “team” incentives, i.e. incentives based on the positive performance of others, to managers. Several reasons to use team incentives have been advanced in the literature. One is that for technological reasons, individual performance may not be measurable; team incentives may then be the only solution available (as in Alchian and Demsetz, 1972, and Holmström, 1982).

Partially team-based compensation also emerges when individual performance can be measured but when there are externalities between agents. In particular, there may be a choice between devoting effort to one’s own work and to helping others, as in the multitask models of Itoh (1991) and Auriol et al. (2002). Che and Yoo (2001) show that in a dynamic model with peer monitoring, team incentives can foster cooperative behavior among agents.

Our theory helps to explain the provision of team incentives to division managers of large firms, that is, in a context in which it is perhaps less natural to think of agents as directly helping or monitoring one another. Externalities (or “synergies”) between two production units exist because resources can be reallocated between the units. This requires centralized control but also relies on localized information. The purpose of team incentives, then, is to ensure truthful upward communication.

**Coordination:** Several recent papers study the costs and benefits of centralized decision-

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6 The same tradeoff was identified independently by Povel (1999) and Aghion, Bolton and Fries (1999). Povel argues that “soft” bankruptcy procedures (such as chapter 11 bankruptcy) make it easier for managers to inform creditors that their firm is in trouble. This facilitates early interventions, but also diminishes managers’ incentives ex ante. Aghion et al. investigate a similar tradeoff in the context of bank bailout policies.
making and coordination. In a closely related paper, Dessein, Garicano and Gertner (2005) study the interaction between an upstream (functional) manager and two downstream (product) managers in an organization that must choose between adapting products to market conditions and standardizing them to reduce costs. Each player has different information relevant to this choice. Dessein et al. show that the need to aggregate this information leads to a tradeoff between coordination and effort incentives.

While this tradeoff is also at work in our model, our paper differs from Dessein et al. in its purpose and contribution. In Dessein et al., production involves three players, each of whom provides an input. Dessein et al.’s interest is in understanding who should have the right to decide on adaptation vs. standardization, which depends on how the optimal sharing contracts in each case affect the players’ incentives to provide effort and to communicate. In our model, in contrast, the two production units can be run as separate firms. Our primary interest is in understanding when it is optimal for them to integrate; we are thus concerned with firm scope. Second, authority plays a different role in our paper. In Dessein et al., giving the functional manager authority over the firm’s choice reduces the product managers’ incentives even if information is not an issue. In our model, the CEO is an unbiased coordinator; as a result, the CEO’s authority itself improves the managers’ incentives (see the perfect-information case). As our stepwise analysis makes clear, a trade-off between resource allocation and incentives exists only because of the managers’ private information. Third, integration in our model requires only the two managers as members of the firm. We show that it is (in a certain sense) optimal to bring in a CEO as third party and to establish a firm with a pyramidal structure, in which the CEO’s job is to coordinate the divisions’ activities.

Alonso, Dessein and Matouschek (2006) study the optimal allocation of decision rights in a two-division firm. Similar to Dessein et al. (2005), the tension is between adaptation of each division’s action to its environment and coordination between the two divisions. There is either centralized decision-making with cheap talk between managers and headquarters, or decentralized decision-making with horizontal communication between the divisions. Like in our model, headquarters is unbiased.

In Alonso et al., a conflict between the divisions exists because each manager is exogenously biased towards his own division; monetary incentives are not considered. In contrast, we endogenize the division managers’ bias; it is caused by incentive contracts designed to ensure effort provision. This difference matters because optimal incentive contracts are different under integration and non-integration, and play a decisive role in the choice of firm scope (with which Alonso et al. are not concerned). Further, the nature of the coordination problem is quite dif-
Different in the two papers. In Alonso et al., coordinating the divisions’ actions means to choose similar actions, and as such does not require the managers’ private information about their environment. In our model, the coordination problem is about using the firm’s resources where they are most productive, which requires aggregating the managers’ information. As a result, centralization (integration) is more likely when coordination is more important, in contrast to a result of Alonso et al.

Coordination also plays a central role in the theories of Hart and Holmström (2002) (see the Introduction) and Hart and Moore (2005). The latter study the optimal allocation of authority over assets when agents have conflicting ideas about their best use; their theory helps to explain why in hierarchical firms, coordinators are typically senior to specialized agents. Our paper shares with Hart and Holmström (2002) the focus on firm scope, and with Hart and Moore (2005) the focus on resource allocation. Like in both papers, coordination requires integration under a central authority because (resource) decisions are non-contractible. Aside from these similarities, our theory is very different in that it emphasizes the interaction of effort incentives and communication, both of which are absent in the other two papers.

Hierarchical structure of organizations: Most theories that help to explain why firms are typically structured as hierarchies are based on team theory. That is, they focus on communication or information-processing costs rather than conflicting interests like agency models do. Early examples are the resource-allocation models of Crémer (1980), Aoki (1986) and Geanakoplos and Milgrom (1991). More recent theories include the communication-based theory of Bolton and Dewatripont (1994) and the knowledge-based theory of Garicano (2000).

In our theory, a hierarchy with a top manager as pure coordinator avoids the conflict of interest a division manager would have with the double task of running his division and of allocating resources. A hierarchy thus emerges as a response to an incentive problem. There are very few other explanations for the existence of hierarchies in the agency literature; instead, most agency-based models take a hierarchical structure as given. Classical agency-based explanations link the existence of a principal as residual claimant to her role as monitor of workers (Alchian and Demsetz, 1972) or as budget-breaker (Holmström, 1982). In Rayo (2006), a hierarchy can emerge as optimal solution in a model of team production with mutual monitoring that allows for both explicit profit shares and relational contracts. Alonso et al. (2006, which we already discussed above) and Athey and Roberts (2001) are closer to our paper; both emphasize the advantage of having an unbiased decision maker. Athey and Roberts (2001) consider performance-based incentive contracts that must be designed to balance the dual goals of effort provision and efficient decision making. They assume that an unbiased decision maker can ob-
tain all relevant information at a fixed cost. In our model, in contrast, the CEO must obtain this information from strategically communicating managers. While centralization gives rise to information rents, we show that those rents are lower than the costs of putting a biased division manager in charge.

**Capital budgeting and internal capital markets:** Most agency-based research on intra-firm resource allocation is part of a large literature on capital allocation within firms. Although capital and the resources in our model differ in terms of what contracts about their use can be written, there are many parallels between our paper and this corporate finance literature.

Stein (1997) was the first to formalize Williamson’s (1975) conjecture that internal capital markets create value by channeling capital to the most productive divisions. He showed that because of the “winner picker” role of headquarters, integration provides benefits even if it does not relax overall credit constraints, and even if managers are empire builders. In our model, the production technology, and the way in which the pooling of resources allows for their more efficient use, are adopted straight from Stein. However, our informational and contracting assumptions, and ultimately the reason for firms to integrate, are very different from his paper:

In Stein’s model, integration leads to better information about investment opportunities. Although this is an assumption in his model, the idea is that ownership of assets gives the firm’s headquarters an incentive to monitor its business units actively; for a formalization, see Gertner, Scharfstein and Stein (1994). Without the principal’s ability to monitor, integration would not serve any purpose. In our model, in which resources are not contractible, integration under a central authority is necessary to make an efficient resource allocation possible. Integration does not per se lead to better information; on the contrary, the core problem in our theory is that the CEO cannot monitor but must obtain information from division managers who are endogenously biased and communicate strategically. Moreover, in our model the CEO will want to shift all resources to one division only if the information that doing so is optimal is sufficiently reliable. Thus, our conclusions still hold if monitoring is possible, as long as the associated cost of obtaining reliable information is sufficiently high.

In Scharfstein and Stein (2000), inefficiencies in an internal capital market result from division managers’ influence activities to increase their capital allocation, similar to Milgrom and Roberts’ approach discussed in the Introduction. In contrast, we do not model influence activities as a separate task; in our model, influence stems from the ability to misrepresent the very information that is essential to realize the benefits of integration.

Stein (2002) and Inderst and Laux (2005) extend the analysis of Stein (1997); in both papers project qualities are determined by the agents’ effort. Inderst and Laux assume that
headquarters is perfectly informed; they show that with symmetric divisions, an internal capital market improves the managers’ effort incentives. With asymmetric divisions it may not, possibly leading to an overall loss from integrating two production units. Stein (2002) makes weaker informational assumptions, in considering “hard” information that can be withheld but not misrepresented. Among other results, he too shows that creating an internal capital market with a winner-picking headquarters has a positive effect on division managers’ incentives. We build on these papers in modeling project qualities as endogenous; that is why under perfect information, centralized resource allocation lowers the cost of inducing effort. The key assumption of our model, however, is that the CEO must obtain information about the divisions from managers who communicate strategically and can lie about their projects. We show that the associated information rent effect always cancels, and often dominates, the competition effect identified by Stein and by Inderst and Laux.

Several papers studying capital-budgeting processes emphasize the information problem, but are not concerned with divisional competition, the winner-picking role of headquarters, or firm scope. Early papers are those of Harris and Raviv (1996, 1998). Bernardo, Cai and Luo (2001) introduce incentive contracting into a framework similar to that of Harris and Raviv. Incentive contracts induce both effort and take care of the agent’s tendency to overstate the profitability of his project; the structure of the optimal contract is similar to the optimal mechanism in our benchmark case of Section 4.3. Bernardo, Cai and Luo (2004) extend this analysis to a two-division firm. Neither paper is concerned with firm scope; both examine whether investment is efficient, under the assumption that the firm’s access to capital is unlimited. Our paper, in contrast, studies the benefit and cost of integration, both of which hinge on the scarcity of resources: the benefit is their more efficient use, and the cost the agency problem resulting from the managers’ incentives to increase their own share.

Only few papers study internal capital markets with localized information, and thus combine the major themes of Scharfstein and Stein and of Harris and Raviv. Ozbas (2004) examines the interaction between strategic communication and managers’ career concerns in a signaling model. Wulf (2005) studies capital allocation in a model with asymmetric divisions in which division managers can distort private information. Neither paper considers effort provision. Inderst and Klein (2006) study a model in which (like in ours) incentive contracts must balance the goals of inducing effort and establishing truthful communication, and endogenously give rise to “empire-building” behavior. Like Levitt and Snyder (1997) and Bernardo et al. (2001, 2004), Inderst and Klein assume that managers’ wages can be made contingent on whether their projects are financed, which we rule out. They also examine competition between divisions for capital, but
are concerned with the efficiency of investment rather than with optimal firm scope.

More generally, what distinguishes our paper from the finance literature is its focus on the scope of the firm and on hierarchical coordination, rather than on capital-budgeting processes and distortions in investment behavior. Another difference concerns the assumptions about the underlying agency problems. Almost the entire literature discussed here (except for Inderst and Klein, 2006) assumes that managers are “empire builders”; i.e. that they derive utility directly from the size of their budget or their division. In some models, the managers’ empire-building tendencies can be mitigated by the use of incentive contracts. In our model, an empire-building motive emerges endogenously; it is caused by incentive contracts that place a large weight on individual performance.

Aside from its greater parsimony, this modeling approach avoids potential inconsistencies. After all, theoretical reasoning and empirical evidence suggest that empire-building behavior results from ulterior motives such as higher compensation or an enhanced reputation in the market for executives (see e.g. Jensen, 1989, or Avery et al., 1998), rather than for purely behavioral reasons. These motives may interact with the question investigated. To avoid this problem, we rely on the assumptions of standard incentive theory, in which agents like money and dislike effort. Our approach is applicable to other situations, and may help to bridge the economics and finance literatures on organizations, which often differ according to whether they employ an empire-builder assumption.

3 Model

There are two identical production units, 1 and 2. They can be run as independent firms, or as divisions of a single integrated firm headed by a CEO. The goal of our analysis is to determine which organizational form will maximize shareholder value and will thus emerge in the equilibrium of a market for ownership of the units.

To facilitate the comparison, we assume that in both cases ownership and control are separated (cf. Stein 1997): those capable of managing the divisions have no wealth of their own to finance the necessary assets, and those able to provide money lack the necessary managerial skills. This assumption is not essential, however. In our integrated firm, the owner could also be the CEO. The non-integrated firms, too, could be run by owner-managers, in which case the analysis would be slightly different but the results largely the same; see Section 4.6.

In contrast to property-rights theory, owners of firms are residual claimants, but do not necessarily play any active managerial role. It is therefore not useful to define integration or non-integration merely according to who owns the units. After all, businesses that have a common
owner but are not linked organizationally in any way are usually considered independent. In line with this common-sense use of language, we shall therefore speak of integration of the two units only if it involves a reallocation of decision rights or a change in incentive contracts.

3.1 Basic Ingredients

**Resources:** We make two assumptions about the resources in our model:

First, the resources are specific to a firm; they cannot be obtained in an external market. However, they are not necessarily specific to a particular use. The resources we have in mind may be physical assets. More importantly, they also include strategic, organizational or procedural capabilities of the firm’s personnel, capabilities that do not reside within individuals (who could be hired away) but instead are embedded in the routines of the organization. Put differently, the rents the resources generate accrue to the firm, not to individuals in the firm. Although firm-specific, the resources have multiple potential uses; within a firm, they can be moved across product lines, projects, and even industries. Our notion of resources is perhaps uncommon in the economics literature (for an exception, see Matsusaka 2001), but is very similar to the notions of “organizational capabilities” in Chandler (1990) and of “unique resources and capabilities” in the resource-based view of the firm that dominates the strategic management literature. In our model, we assume that each production unit is endowed with an amount of resources $K = 1$. The units do not have access to any additional amounts, unless they merge, in which case they can pool their resources.

Second, it is impossible to write contracts about the precise use of resources. This assumption rules out both bilateral spot contracting between the two units, as well as compensation contracts for managers that are conditioned on the use of the resources. The idea here is that the resources, or the conditions of their use, are too difficult to describe in any contract. What can be contracted upon, however, is the **authority** over how to use resources. Also, while spot contracting is impossible, resources can be transferred from one firm to another by merger. In making these assumptions, we follow a recent literature which assumes that only the authority over actions, but not actions themselves, is contractible, see Aghion et al. (2004) and the survey of Dewatripont (2001). This has proven to be a simple and tractable way to model frictions.

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8 The exact cause of contracting difficulties can take different forms. For example, suppose firm A has a unique capability in say, engineering. Even if one could attribute this capability to a particular department within the firm (as argued above, capabilities of individuals would not count as firm-specific), the firm would be reluctant to rent the department to another firm B for fear of leakage of know-how or ideas. On the other hand, the usefulness of firm A’s capability in firm B’s business may well be a reason for firms A and B to merge.
in the marketplace, without which firms would not need to exist. In contrast to most of this literature, however, we allow for incentive contracts; more on those below.

**Effort, Projects and Payoffs:** Each production unit is run by a manager, whose job is to create profitable investment opportunities, or “projects”. Once a project has been created, it requires resources to be carried out; the resulting payoff depends on the quality of the project and the resources invested.

Unit i’s manager can exert high effort ($e_i = 1$) or low effort ($e_i = 0$), which is unobservable. We normalize the manager’s cost of low effort to zero; the cost of high effort is $c > 0$. By exerting high effort, a manager comes up with a good project (type $\theta_i = G$) with probability $p$ and with a bad one (type $\theta_i = B$) with probability $1 - p$. If he exerts low effort instead, the project is good with probability $q$ and bad with probability $1 - q$, where $q < p$. Let $\theta = (\theta_1, \theta_2)$.

Our assumptions and most notation for the production technology are the same as in Stein (1997), except that we also introduce some uncertainty. We consider a binary version of a technology with linear returns to project quality and decreasing returns to resource investment. The resource investment in any project can be either 1 or 2; a zero investment has a zero return. If an amount of $k_i \in \{1, 2\}$ is invested in a bad project in unit $i$, the resulting payoff expected payoff $z_i$ is $y_{k_i}$ with $y_2 > y_1 > 0$. A good project has an expected payoff of $\varphi y_{k_i}$ for $\varphi > 1$:

$$z_i(k_i, \theta_i) = \begin{cases} \varphi y_{k_i} & \text{if } \theta_i = \text{“G”} \\ y_{k_i} & \text{if } \theta_i = \text{“B”} \end{cases} \quad \text{for } k_i = 1, 2 \quad (1)$$

The actual payoff for each unit, denoted $\tilde{z}_i$, is either $\mu$ or 0; let $\tilde{z} = (\tilde{z}_1, \tilde{z}_2)$. The probability of the event $\tilde{z}_i = \mu$ is given by $z_i(k_i, \theta_i)/\mu$, where $\mu$ is assumed to be large enough so that all $z_i/\mu$ are less than 1. It follows that $[z_i(k_i, \theta_i)/\mu] \mu + [1 - z_i(k_i, \theta_i)/\mu] 0 = z_i(k_i, \theta_i)$ is indeed the expected payoff. The purpose of introducing uncertainty with a “full support” property is to ensure that no direct inferences can be made about $k_i$ or $\theta_i$ from the observed $\tilde{z}_i$. We will get back to this point below when we discuss feasible contracts.

We make two assumptions about $y_1$, $y_2$, and $\varphi$, again following Stein (1997):

**Assumption 1** $1 < y_1 < y_2 < 2$.

**Assumption 2** $\varphi(y_2 - y_1) > y_1$.

Assumption 1 implies that if a project is bad, then the gross profit from investment (expected payoff minus resources invested, but not including wage payments) is maximized with an investment of 1. Another consequence is that $y_2/y_1 < 2$, which means that the returns to the resources invested are decreasing. Hence, if an amount of 2 can be invested and two equally
good or bad projects are available (as might be the case in an integrated firm), then it is strictly better to invest 1 in each project instead of 2 in one of them, since $2y_1 > y_2$.\textsuperscript{9}

Assumption 2 can be stated equivalently as $\varphi y_2 > \varphi y_1 + y_1$. Because of $y_1 > 1$, it follows that $\varphi y_2 - 2 > \varphi y_1 - 1$, which means that the gross profit from investment is maximized with an investment of 2. Assumption 2 also guarantees that if an integrated firm has both a good and a bad project to invest in, it strictly prefers to invest 2 in the good project and zero in the bad one over investing 1 in each project. This is essential for any benefit from integration to exist.

Both assumptions together imply that the efficient way to allocate two units of resources between the two production units is given by

$$
k^*(\theta) = \begin{cases} 
  k_1 = k_2 = 1 & \text{if } \theta_1 = \theta_2 = \text{“G”} \text{ or if } \theta_1 = \theta_2 = \text{“B”} \\
  k_1 = 2, k_2 = 0 & \text{if } \theta_1 = \text{“G”} \text{ and } \theta_2 = \text{“B”} \\
  k_1 = 0, k_2 = 2 & \text{if } \theta_1 = \text{“B”} \text{ and } \theta_2 = \text{“G”}
\end{cases}
$$

Managers’ preferences: Unit $i$’s manager is risk-neutral but protected by limited liability. The manager’s utility is given by $U_i(w_i, e_i) = w_i - ce_i$, where $w_i$ is the monetary wage and $ce_i \in \{0, c\}$ is the disutility of effort. We make no assumptions about the managers’ reservation wages except that they are low enough such that in equilibrium, the managers’ participation constraints are not binding.

In contrast to most of the literature on capital allocation in firms (see Section 2), managers in our model are not empire builders by assumption; that is, they do not derive utility directly from their division’s resource allocation or payoff. Rather, we show that empire-building motives can result endogenously from the design of incentives.

Contracts: We assume that the managers’ wages can be contingent on both production units’ realized payoffs $\tilde{z}_1$ and $\tilde{z}_2$. With independent firms, there is no reason for manager $i$’s wage to depend on firm $i \neq j$’s payoff (although we allow for this, for symmetry). In the integrated firm, however, “team” incentives of this sort play an important role, as we will see. For simplicity, we restrict the analysis to contracts that are symmetric for both managers.

There are no other verifiable measures to base wages on. As we already discussed, resources are not contractible. Following Crawford and Sobel (1982) and Dessein (2002), we also assume that any communication from a manager to his boss (the owner or CEO of his firm) is cheap.

\textsuperscript{9} For the same reason, if the projects are unknown but have the same expected quality, then it is best to invest the resources equally instead of putting all in one of the production units. This means that a potential gain from integrating the two production units into one firm exists only if the person with authority over resources is informed about the project qualities; there is no gain from allocating resources randomly.
talk. That is, a manager’s messages do not cost the manager anything, nor can any contracts be conditioned on them. In order to understand the role of this assumption, we examine message-contingent contracts as a benchmark case in Section 4.3.

In the most general formulation, performance-based contracts specify a wage for every possible combination of the divisions’ payoffs. With two possible payoffs for each unit, there are four possible outcomes overall, and so each manager’s contract can be described by a quadruple of wages. The complete analysis of this general case is contained in Appendix B. Almost all of our main results are the same, however, if we require a manager’s wage to be separable in his own and the other division’s payoff; i.e., if we rule out wages that are contingent on joint realizations of the divisions’ payoffs. With binary division payoffs, separability implies that expected wages and profits can be expressed as linear functions of the expected payoffs $z_i$ in (1), which significantly simplifies the analysis and exposition. For this reason, we will focus on contracts with separable wage functions in the main text, and report the corresponding results for “nonlinear” contracts; see Appendix B for details.

For our main analysis, therefore, we consider wage contracts of the form $\tilde{w}_i(\tilde{z}_1, \tilde{z}_2) = \alpha + \beta \tilde{z}_i + \gamma \tilde{z}_j$ for $i = 1, 2$ and $j \neq i$. Expected wages are then given by $w_i(z_1, z_2) = \alpha + \beta z_i + \gamma z_j$.\(^\text{10}\) With risk-neutral agents, limited liability and a non-binding participation constraint, the limited-liability constraint must be binding when contracts are optimal. Since the $\tilde{z}_i$ can be zero, $\alpha$ should be set to zero, and since the $\tilde{z}_i$ can be positive, $\beta$ and $\gamma$ must be nonnegative given that $\alpha = 0$. Optimal contracts are then completely characterized by the parameters $\beta$ and $\gamma$.

### 3.2 Independent Firms

When the two units are run as independent firms, each firm $i$ can only use its own resources $K = 1$; hence its investment $k_i$ is constrained by $k_i \leq 1$.

It is weakly optimal for firm $i$’s owner to give her manager decision rights over the firm’s resources, since under our assumptions the manager will then maximize the owner’s net profit. Profit maximization involves investing all of the firm’s resources, since any project has a positive NPV if an amount $k = 1$ is invested (i.e. the bad project is bad only relative to a good one). Although the manager cares only about his wage and not the owner’s profit, it is easy for the owner to align the manager’s interests with her own. To induce low effort, the owner can simply pay the manager his reservation wage, whereas inducing high effort requires incentives that reward the manager for a high output. In either case, the manager weakly prefers investments

\(^{10}\) We refer to $\beta$ and $\gamma$ as “bonuses” since the $\tilde{z}_i$ are binary, but technically $\beta$ and $\gamma$ are shares of the units’ payoffs, akin to piece rates.
that maximize the expected value of the profit $z_i - k_i$. The timing of events is as follows:

1. The owner of firm $i$ offers her manager a wage contract, which he accepts or rejects.

2. Manager $i$ exerts effort $e_i \in \{0, 1\}$. The manager thereupon learns the profitability of his project $\theta_i \in \{G, B\}$, which is his private information.

3. Manager $i$ invests $k_i = 1$ in his project.

4. The payoff $\tilde{z}_i$ is realized, and the manager is paid $\beta \tilde{z}_i$.

Although firm $i$ can base its manager’s wage on $\tilde{z}_j$ as well as on $\tilde{z}_i$, there is no reason to do so, since $\tilde{z}_j$ contains no information about manager $i$’s effort. It therefore suffices to pay manager $i$ a bonus $\beta \geq 0$ (expressed as share of the payoff) if $\tilde{z}_i = \mu$.\(^\text{11}\) For each firm $i$, the optimal contract that induces its manager to exert high effort solves the problem

$$\max_{\beta} (1 - \beta) E_{\theta_i}[z_i(1, \theta_i)|e_i = 1] \quad \text{s.t.}$$

\[
\begin{align*}
(\text{IC-e}) & \quad \beta E_{\theta_i}[z_i(1, \theta_i)|e_i = 1] - c \geq \beta E_{\theta_i}[z_i(1, \theta_i)|e_i = 0], \\
(\text{LL}) & \quad \beta \geq 0.
\end{align*}
\]

### 3.3 Integrated Firm

In the integrated firm, the production units are jointly owned; all assumptions made in Section 3.1 apply without change. The potential benefit of integration is that the units’ resources can be reallocated according to the profitability of the units’ investment opportunities. Formally, instead of the previous resource constraints $k_i \leq 1$, we now have only a joint constraint $k_1 + k_2 \leq 2$, $k_i \in \{0, 1, 2\}$ for investment in the two divisions.

Since contracting on resources is infeasible, someone must be given the authority to allocate the firm’s resources. A natural solution on which we focus in most of our analysis is for a manager at the top, whom we call the CEO, to allocate the firm’s resources. The alternative — to have one of the division managers allocate resources — is examined in Section 5.

The CEO could be the owner herself, or a hired agent. The distinction makes no difference since we are not concerned with agency problems at the CEO level. Instead, we want to understand what the limits to integration are when top management always pursues value-maximizing actions; cf. our discussion in the Introduction. Thus, suppose a hired CEO must exert effort to evaluate information obtained from division managers and to allocate the firm’s resources, but

\(^{11}\) Once again, the manager’s actual wage is $\beta \tilde{z}_i \in \{0, \beta \mu\}$; the expected wage then is $\beta \tilde{z}_i$. 

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that this effort is observable. The owner can then simply pay the CEO wage \( \bar{w}_0 \) for his effort, plus a very small share of total profits to give her an incentive to allocate resources optimally. As there is no interesting interaction between \( \bar{w}_0 \) and the tradeoff between coordination and incentives that is our main concern, we can simplify further by setting \( \bar{w}_0 = 0 \).

The problem is that while the CEO allocates the firm’s resources, only the division managers know the profitability of their investment projects. We assume that after exerting effort and learning about his project, each manager communicates his project type to the CEO, which is cheap talk; see our discussion in Section 3.1. Depending on the incentives provided, the managers may have an incentive to lie about their projects.

After receiving messages about the projects from the division managers, the CEO allocates the firm’s resources between both divisions. As the resources are not contractible, the CEO cannot commit herself in advance to any allocation rule, and therefore allocates them to maximize the firm’s profit net of the managers’ wages. Integration thus differs from non-integration in what happens at stage 3 of the timing explained in the previous subsection:

3a. The division managers simultaneously send costless and unverifiable messages \( \hat{\theta}_i \) about their projects to the CEO. Let \( \hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2) \).

3b. The CEO allocates resources to the two divisions, subject to the constraint \( k_1 + k_2 \leq 2 \) and \( k_i \in \{0, 1, 2\} \).

If the owner wants the managers to exert low effort, she can simply pay them their reservation wage. Doing so also induces truth-telling because the managers have no reason to misrepresent their projects. If the firm wants to implement high effort, wages must be be conditioned on the division payoffs, and additional constraints must be satisfied for the managers to report their project types truthfully, i.e., for a separating equilibrium to exist. The utility functions of managers with good and with bad projects are identical, which would make separation infeasible in most cheap-talk games. Here, in contrast, separation is possible because the managers differ in their ability to generate profits for the firm.\(^\text{12}\) Note also that although the CEO could be principal herself, her inability to commit to resource allocations by contract means that her optimal response to the managers’ messages becomes part of the contracting problem.

Below, we state the owner’s optimization problem for an unspecified set of feasible contracts \( C \). In subsequent sections, we solve this problem for different assumptions about \( C \). For any contract \( \zeta \in C \), denote by \( \bar{w}_i(\theta, \hat{\theta}, \zeta) \) manager \( i \)’s expected wage at stage 3a of the game if his project is of type \( \theta \) and he reports it to be of type \( \hat{\theta} \), under the assumptions that manager \( j \)

\(^{12}\) See Fingleton and Raith (2005) for a model of delegated bargaining that exhibits the same properties.
exerts high effort and reports his type truthfully, and that the CEO allocates resources according to the efficient rule \( k^* \) (we occasionally suppress the argument \( \zeta \) where no confusion can arise).

The firm’s optimal contract for each manager that induces high effort and truth-telling on part of the managers, and an efficient allocation of resources by the CEO, then solves the following problem:

\[
\max_{\zeta,e_1,e_2,\theta_1,\theta_2,k_1,k_2} E_\theta \left\{ [z_1(k_1, \theta_1) + z_2(k_2, \theta_2)] - \bar{w}_1(\theta_1, \theta_1, \zeta) - \bar{w}_2(\theta_2, \theta_2, \zeta) \mid e_1 = e_2 = 1 \right\}
\]

s.t.

\( (\text{IC-e}) \quad p \bar{w}_i(G, G, \zeta) + (1 - p) \bar{w}_i(B, B, \zeta) - c \geq q \bar{w}_i(G, G, \zeta) + (1 - q) \bar{w}_i(B, B, \zeta) \quad \text{for } i = 1, 2 \)

\( (\text{IC-G}) \quad \bar{w}_i(G, G, \zeta) \geq \bar{w}_i(B, G, \zeta) \quad \text{for } i = 1, 2 \)

\( (\text{IC-B}) \quad \bar{w}_i(B, B, \zeta) \geq \bar{w}_i(B, G, \zeta) \quad \text{for } i = 1, 2 \)

\( (\text{RA}) \quad k = k^*(\tilde{\theta}) = \arg \max_{k_1^*, k_2^*} [z_1(k_1^*, \theta_1) + z_2(k_2^*, \theta_2) - E[\tilde{w}_1(\tilde{z}|k_1^*, \theta_1), \zeta] - E[\tilde{w}_2(\tilde{z}|k_2^*, \theta_2), \zeta]] \)

s.t. \( k_1 + k_2 \leq 2, k_i \in \{0, 1, 2\} \)

\( (\text{LL}) \quad \tilde{w}_i(\tilde{z}|k_i, \theta_i, \zeta) \geq 0 \quad \text{for } i = 1, 2. \) (4)

Condition (IC-e) must be satisfied for manager \( i \) to exert high effort. The ancillary assumptions that agent \( j \neq i \) also exerts high effort, that both managers report truthfully, and that resources are allocated according to \( k^* \), are embodied in the definition of \( \bar{w}_i \) above. Conditions (IC-G) and (IC-B) ensure that manager \( i \) reports his project type truthfully, depending on whether his project is good or bad. Finally, (RA) states as condition that the CEO allocates resources to maximize the firm’s net profit, and in doing so implements the efficient allocation rule \( k^* \).

### 4 Benefits and Costs of Integration

Our analysis of the benefits and costs of integration proceeds in several steps. In Sections 4.1 and 4.4, we determine the optimal contracts for independent firms and for the integrated firm with cheap-talk communication, respectively. In between, however, it is useful to look at two benchmark cases (see Sections 4.2 and 4.3) in order to understand how our results for the integrated firm depend on the information structure and on what contracts are feasible. Our ultimate goal is to determine which organizational form is most likely to prevail in a market for ownership of the production units. We turn to this question in Section 4.5 for our main case in which the independent firms are run by agents of the firms’ owners, and briefly discuss the case in which they are run by the owners themselves in Section 4.6.
4.1 Independent Firms

Determining each firm’s optimal incentive contract for its manager is straightforward:

**Lemma 1** With independent firms, the optimal contract for each division manager that leads to high effort is given by

\[ \beta_{ni} = \frac{c}{(p-q)(\varphi - 1)y_1} \text{ and } \gamma_{ni} = 0. \]  

(5)

For all proofs, see the Appendix.

We already discussed the optimality of \( \gamma_{ni} = 0 \). The optimal bonus for own output, \( \beta_{ni} \), is increasing in the cost of effort \( c \), and decreasing in the marginal effectiveness of high effort in generating a good project \( (p-q) \) and the difference in the marginal profitabilities between a good and a bad project \( ((\varphi - 1)y_1) \). Linearity of the wage function is no restriction under non-integration; the optimal contract is the same in the more general case.

It is customary in models with two effort levels to assume that high effort is optimal under first-best conditions, for otherwise there would be no interesting agency problem. However, our interest is not per se in what agency problems might exist in independent firms, but rather in what new agency problems are created by integration. We therefore make the slightly stronger assumption that with non-integrated firms, wage contracts that induce high effort are optimal under second-best conditions. Otherwise, if low effort were optimal with independent firms, it could be induced at the same cost in an integrated firm, and since with a flat wage, managers would weakly prefer to report their projects truthfully, integration would trivially be preferred.

For a reservation wage of zero, the relevant condition is stated in the next result:

**Lemma 2** With a zero reservation wage, the contract of Lemma 1 is optimal if

\[ \frac{(p-q)^2(\varphi - 1)^2y_1}{p(\varphi - 1) + 1} > c. \]  

(6)

4.2 First Benchmark: Integration With Perfect Information

Suppose that the CEO can perfectly observe the divisions’ project types \( \theta_i \), as in Stein (1997) or Inderst and Laux (2005), while effort remains unobservable. We then obtain the following result; cf. Proposition 1 in Inderst and Laux (2005):

**Proposition 1** In an integrated firm in which the CEO has perfect information about \( \theta \), the optimal linear contract for each division manager is given by

\[ \beta_{pi} = \frac{c}{(p-q)\{(1-p)[\varphi(y_2 - y_1) - y_1] + \varphi y_1\}} \text{ and } \gamma_{pi} = 0, \]

where \( \beta_{pi} < \beta_{ni} \).
With perfect information, it is again optimal to provide individual incentives only, i.e. $\beta > 0$ and $\gamma = 0$. In contrast to the case of independent firms, the divisions are now linked through the allocation of the pooled resources, but manager $i$’s effort has a negative effect on $z_j$: the higher $e_i$, the more likely it is that division $i$ will find a good project, which leads to a lower expected resource investment in division $j$ and hence a lower expected output $z_j$. Paying manager $i$ a reward $\gamma > 0$ for division $j$’s output would therefore only reduce $i$’s incentive to exert effort, whereas $\gamma < 0$ is not feasible because of limited liability.

The more important part of Proposition 1, however, is that $\beta^{pi} < \beta^{ni}$, which means that compared with independent firms, integration with perfect information leads to better incentives for managers in the sense that effort is less costly to induce. The intuition is that while the resource investment always equals 1 in an independent firm, the expected investment under integration is larger than 1 if a division has a good project and smaller than 1 if it has a bad project. Since a division’s expected performance depends on the resources invested and its manager is in turn rewarded for performance, creating a good project that warrants a large investment becomes more valuable to the manager.\(^{13}\) We shall refer to this effect as the competition effect of centralized resource allocation on managerial incentives.

Proposition 1 stands in contrast to the received view in the economic literature on authority in organizations, see our discussion in the Introduction. According to this view, giving authority over some decision to one party improves that party’s incentives but reduces the incentives of others affected by the decision, which results from the assumption that players have different preferences over decisions to take. No such differences are assumed here; neither the CEO nor the managers care directly about how resources are invested. With perfect information, shifting authority from division managers to the CEO then improves the managers’ incentives. We believe it is important not to lose sight of this quintessential role of managers as coordinators, as opposed to a prevailing view that envisions managers as preoccupied with pet projects.

Critically, though, Proposition 1 rests on the assumption that the CEO has perfect information about project qualities. We show next what happens when she needs to obtain this information from the managers.

\(^{13}\) As discussed in Section 2, Stein (2002) obtains a similar result under weaker informational assumptions. See also Marino and Zabojnik (2004), who show that competition for corporate resources can significantly strengthen agents’ incentives in firms that would otherwise be plagued by free-rider problems.
4.3 Second Benchmark: Integration With Private Information and Message-Contingent Contracts

Now assume that the information about the divisions’ projects is the managers’ private information, and that wage contracts can be written contingent on the units’ payoffs and on any messages sent by the two managers. Formally, the owner’s optimization problem is essentially the same as (4), where $C$ is the set of all payoff- and message-contingent contracts.

A first observation is that for an independent firm, the contract of Lemma 1 remains optimal; that is, a message-contingent contract cannot improve upon a purely performance-based contract. The reason is that even a bad project is worth carrying out because its NPV is positive (in contrast to e.g. Levitt and Snyder, 1997); the manager’s information is therefore not decision-relevant. Reducing the manager’s income risk by using a message-contingent contract is also irrelevant since the manager is risk-neutral.

In the integrated firm, it is a priori unclear whether centralized resource allocation with privately informed division managers leads to higher or lower wage costs for the firm. There are two countervailing effects. One is the competition effect that we discussed earlier, which makes inducing effort less costly compared with non-integration. However, wage contracts must also be designed to get the managers to report their project types truthfully. The resulting information rent effect raises the wage costs to the firm.

With message-contingent contracts, these two effects exactly cancel each other:

**Proposition 2** Consider the contract $(w^1, w_B)$ given by

$$w^1 = \frac{c\mu}{(p-q)(\varphi-1)[py_1 + (1-p)y_2]}$$

and

$$w_B = \frac{c}{(p-q)(\varphi-1)},$$

where $w_B$ is the payment to a manager who reports a bad project, and $w^1$ is the payment to manager who reports a good project and whose unit has a high payoff (if the unit’s payoff is zero, the manager gets zero). If $w^1 \leq \mu(1 - \frac{y_1}{\varphi(y_2-y_1)})$, then $(w^1, w_B)$ is an optimal payoff- and message-contingent contract that leads to high effort and an efficient resource allocation (otherwise, it is not a feasible solution of (4)). The resulting expected wage payments are the same as under non-integration.

One could prove this result step by step by solving for an optimal contract. Instead, we first provide a general argument to show that with private information, any contract inducing truth-telling must lead to a wage bill at least as high as that under non-integration. We then show that the contract stated satisfies the managers’ effort and truth-telling constraints. The upper bound on $w^1$ ensures that the CEO never has an incentive to misallocate resources simply...
to save the firm payments of $w^1$. Optimality of the contract then follows from the first part of the proof, by showing that the associated wage bill is the same as under non-integration.

What plays a key role in the first part of the proof is the strong incentive for a manager with a bad project to claim that his project is good. Specifically, we show that the expected wage of a manager who falsely claims to have a good project, $\bar{w}_1(B, G)$, is always at least $1/\varphi$ times the expected wage of a manager who indeed has a good project, i.e. $\bar{w}_1(G, G)$. The incentive constraints for truth-telling and effort provision then jointly imply that the firm must pay the managers at least as much under integration with truthful upward communication as they receive in non-integrated firms.

The proof also makes clear that the critical fraction $1/\varphi$ is tight: whether the wage bill under integration is the same as or greater than under non-integration hinges on whether the inequality $\bar{w}_1(G, G) \leq \varphi \bar{w}_1(B, G)$ holds with equality or is strict. An example of the former case is the contract stated in the proposition. The latter case arises when the set of feasible contracts is more restricted. Specifically, getting a manager with a bad project to report truthfully may require paying him a bonus for the other division’s payoff. (Notice that the contract characterized in the proposition does not entail such payments.) In that case, the relation $\bar{w}_1(G, G) < \varphi \bar{w}_1(B, G)$ holds strictly (see the proof of Proposition 2), and hence the resulting wage bill is strictly higher than under non-integration. This is generally the case when only output-contingent contracts are feasible, as we show next.

4.4 Integration With Strategic Information Transmission

When the managers’ communication is cheap talk, only performance-based contracts are feasible. The managers’ bonuses for each division’s payoff must then be structured to ensure that managers report their projects truthfully:

**Lemma 3** For any contract $(\beta, \gamma)$, and assuming the CEO believes that the division managers’ reports are truthful and allocates the firm’s resources according to $k^*$, a manager with a bad project has an incentive to report his type truthfully if and only if

$$\frac{\gamma}{\beta} \geq \frac{p(2y_1 - y_2) + y_2 - y_1}{y_1 + p[\varphi(y_2 - y_1) - y_1]},$$

and a manager with a good project has an incentive to report his type truthfully if and only if

$$\frac{\gamma}{\beta} \leq \frac{\varphi[y_2 - y_1 + p(2y_1 - y_2)]}{(1 - p)y_1 + p\varphi(y_2 - y_1)}.$$  

The right-hand side of (7) is between 0 and 1, and the right-hand side of (8) is greater than 1.
Condition (7) implies, in particular, that individual incentives alone (i.e. $\beta > 0$ and $\gamma = 0$) can never elicit truthful reports, since a manager with a bad project always has an incentive to claim that his project is good, in order to receive resources. In this sense, the provision of performance incentives endogenously generates “empire-building” behavior, even though the managers do not derive any intrinsic utility from the resources they receive. Our model thus exhibits an extreme case of a tradeoff between effort and truthtelling incentives. What drives this result is that only payoffs are contractible, and that the (expected) payoff increases with the resources invested.\textsuperscript{14} The main result of this section is the following.

**Proposition 3** In an integrated firm, the optimal contract for each division manager that leads to high effort, truthful reports about projects, and an efficient resource allocation, is given by

$$
\beta^{\text{int}} = \frac{c}{(p-q)(\varphi - 1)(1-p)y_2 + py_1}
$$

and

$$
\gamma^{\text{int}} = \frac{c[p(2y_1 - y_2) + y_2 - y_1]}{(p-q)(\varphi - 1)(1-p)y_2 + py_1[p\varphi(y_2 - y_1) + (1-p)y_1]},
$$

where $\beta^{\text{int}} \in (\beta^p, \beta^n)$ and $\gamma^{\text{int}} > 0$. The expected wage per agent is strictly higher than under non-integration.

As stated in Lemma 3, getting division manager $i$ to report $\theta_i$’s truthfully requires paying him partly based on division $j$’s output. Proposition 3 states that this is in fact possible. Although a manager with a bad project stands to benefit from resources invested in his division, he also knows that the firm’s funds can be more profitably invested in the other division than in his own. The key is then to let the manager participate in this gain from a better use of resources. While the firm cannot reward the manager directly based on the report of a bad project or the allocation of resources, it can reward him indirectly in the form of incentive pay based on the other division’s output (or equivalently, the firm’s profit).

The downside is that rewarding manager $i$ for division $j$’s good performance creates a free-rider problem, and reduces his incentives to exert effort. Thus, as $\gamma$ is raised from zero to satisfy the manager’s truth-telling constraint, $\beta$ must be raised (starting from $\beta^p$) as well to maintain the manager’s incentive to exert high effort. Although the resulting optimal $\beta^{\text{int}}$ is still lower than the bonus $\beta^{\text{int}}$ required under non-integration, having to pay $\gamma^{\text{int}} > 0$ for the other division’s good performance leads to an expected wage bill for the firm that is strictly higher.

\textsuperscript{14} Lemma 3 stands in contrast to Levitt and Snyder (1997), in whose model the agent can be brought to report bad news even if communication is cheap talk. This result rests on the assumption, however, that the agent can be paid conditional on termination of the project, which is not possible in our model.
than under non-integration. In other words, the competition effect of centralized coordination on the managers’ incentives is more than offset by the information rent effect.

Propositions 1 and 3 and Lemma 3 are illustrated in Figure 1. Lemma 3 characterizes a cone (shaded in light gray) in which $\gamma/\beta$ must lie to induce truthful reports about project types. The effort-incentive constraint (IC-e) in (4) defines feasible combinations of $\beta$ and $\gamma$ that induce high effort (shaded in medium gray), conditional on truthtelling by both managers. The dashed line represents one of the firm’s isoprofit curves, which have a slope of $-1$; the lower the curve, the higher the profit. The isocurves for expected wages look the same but are ordered in the opposite direction.

Under non-integration, the optimal contract is given by $(\beta_{ni}, \gamma_{ni})$; the incentive constraint for effort (not depicted) is a vertical line through that point. Under integration with perfect information, the effort incentive constraint changes to the line IC-e depicted, as a result of the competition effect. The profit-maximizing contract in this case is $(\beta_{pi}, \gamma_{pi})$. With strategic communication, the contract must lie in the dark-shaded area in order to satisfy both effort and truthtelling constraints. The profit-maximizing point in that area is $(\beta_{int}, \gamma_{int})$, which is associated with a higher wage bill than the contract $(\beta_{ni}, \gamma_{ni})$ under non-integration.

Figure 1: Truthtelling and effort incentive constraints as functions of $\beta$ and $\gamma$. 

25
When nonlinear contracts are feasible (see Appendix B for details), contacts can be characterized by \((\beta, \gamma, \delta)\), where manager \(i\) gets paid \(\beta\) (as before, as a share of \(\mu\)) if only \(\tilde{z}_i = \mu\), \(\gamma\) if only \(\tilde{z}_j = \mu\), and \(\delta\) if both units have a high payoff (as before, it is optimal to pay zero if neither does). In addition, new incentive constraints come into play: with linear contracts, the CEO’s incentive to allocate resources efficiently never poses a binding constraint because the firm’s net profit is simply \(1 - \beta - \gamma\) times expected total payoff. With nonlinear contracts that is no longer the case. For instance, with a contract that stipulates a large bonus \(\delta\) to be paid if \(\tilde{z}_1 = \tilde{z}_2 = \mu\), the CEO might be tempted to allocate all resources to one division — leaving the other with no investment to make — even when doing so is inefficient, simply to prevent an outcome in which both units have high payoff. Likewise, if \(\beta\) is large but \(\delta\) small, the CEO might be tempted to allocate the resources equally even when the projects are not equally good, to influence the outcome towards one where both divisions have a high payoff rather than only one.

The resulting solutions for an optimal contract are either interior solutions with \(\beta, \gamma, \delta > 0\), or have one of the variables set to zero. There are two main cases to distinguish. As is explained in greater detail in Appendix B, if \(p < 1/(1 + \varphi)\), then \(\delta\) has a positive effect on the truth-telling constraint of a manager with a bad project. It is then possible to induce truth-telling by paying a large enough \(\delta\), whereas high effort can be induced with \(\beta > 0\). The optimal \(\gamma\) in this case is zero; it is therefore not necessary to pay a manager whose unit does not produce. Consequently, the managers’ information can be elicited without additional cost, implying that integration is always optimal, cf. the discussion in Section 4.3. If \(p > 1/(1 + \varphi)\), in contrast, then \(\delta\) has a negative effect on the truth-telling constraint of a manager with a bad project. In this case, inducing truth-telling requires setting \(\gamma > 0\), and the resulting wage bill is strictly higher than under non-integration.

To conclude, inducing high effort and truth-telling in general requires higher wages than are needed in independent firms, which may or may not be worth paying from the perspective of the firm’s owner. That is, even when high-powered incentives (here, incentives that lead to high effort) are optimal in independent firms, low-powered incentives (here, a constant wage that induces low effort) may be optimal in an integrated firm. In a model with continuous instead of binary effort, the owner’s response to a higher cost of inducing effort under integration would be to provide lower-powered incentives in any case.\(^{15}\)

\(^{15}\) We have studied a variant of our model in which effort is continuous. For the model to be tractable, we then need to assume that the project qualities are exogenous rather than determined by effort, and that payoffs depend additively on project quality and effort, similar to Dessein et al. (2005). In that modified model, our conjecture above indeed holds; incentives are always lower-powered (in the sense of a lower reward for a high payoff) than under non-integration. But although this result is appealing, overall the modified model leads to
4.5 Optimal Choice of Organizational Form

In a market for ownership of the production units, the organizational form most likely to prevail is the one that leads to the highest joint profits for the firms’ owners. For instance, for integration to be the equilibrium outcome when the two production units could be independently owned, the profits for the owner of the integrated firm must be large enough to compensate the owners of the independent firms for their forgone profits if they sell. Conversely, the owner of an integrated firm will be willing to split the firm and sell her assets to two independent owners if the sum of the payments she receives exceeds the integration profits, which means that the total profits under non-integration must exceed integration profits. In other words, we assume that the organizational form most likely to emerge in equilibrium is the one that maximizes total firm value, which in our model without debt is the same as shareholder value.

With separate ownership and control, the profit of an independent firm’s owner is the expected payoff minus the manager’s wage. If the firm were run by its owner instead, the relevant profit would be the total surplus created, see Section 4.6. For the integrated firm, the separation of ownership and control does not matter for the calculation of profit, since we disregard the CEO’s compensation. Thus, whether the CEO is the owner herself or an agent, the relevant profit is the firm’s expected total payoff minus the division managers’ wages. The rents that the managers receive are not to be included in the firm value. If they were — amounting to some kind of stakeholder value calculation —, integration would always be optimal since total surplus can only increase when resource allocation is improved while effort is held at a high level. But that would also amount to assuming away agency problems, since the tradeoff between efficiency and rent extraction is the essence of any agency problem when agents have limited liability.

Recall that we ruled out non-integration with low-powered incentives, cf. Section 4.1. Also recall that we reserve the term “integration” for a firm with centralized resource allocation, rather than mere joint ownership of the units.\(^\text{16}\) Depending on the parameters of the model, then, one of the following three solutions is optimal:

1. Integration with high effort and truthtelling according to Proposition 3,

2. Integration with low-powered incentives \((\beta = \gamma = 0)\) and truthtelling,

3. Non-integration with high-powered incentives according to Lemma 1.

less rich implications than our main model because of the assumed exogeneity of project qualities.

\(^{16}\) If, in contrast, there are unmodeled reasons for the two units to be integrated, then the tradeoffs emphasized in this paper imply a choice between centralized decision-making with lower-powered incentives, and decentralized decisions with high-powered incentives.
That all three solutions can be optimal can be seen by example. Assume the managers’ reservation wage is zero, and fix the values \( q = 0.2, y_1 = 1.01, y_2 = 1.9, \) and \( c = 0.2; \) and let \( p \) vary between \( q \) and 1, and \( \varphi \) between 1 and 3. Next, determine the set of \((p, \varphi)\) pairs for which both the parameter assumptions of Section 3.1 and condition (6) hold. Within this set, finally, compute the integrated firm’s profit for cases 1 and 2 listed above, and the independent firms’ joint profits, and determine which solution leads to the highest profit. The result is depicted in Figure 2.

Figure 2: Optimal organizational form as function of \( p \) and \( \varphi \). Light gray: non-integration; medium gray: integration with zero wages (low effort); dark gray: integration with team incentives (high effort);

The union of the shaded areas in Figure 2 is the set of \((p, \varphi)\) pairs for which (6) holds, i.e., for which independent firms want to implement high effort. Intuitively, for a non-integrated firm, high effort is better the larger the resulting probability of having a good project \((p)\), or its value \((\varphi)\). Integration with high effort and truthtelling (the dark-gray area) is optimal when \( \varphi \) and \( p \) are sufficiently large but \( p \) is not too large. For these parameters, both high effort and the centralized allocation of resources are sufficiently valuable for the firm to pay the high wages that make both possible. Integration with low effort is optimal for parameters towards...
the lower border of the relevant set defined by (6), cf. the medium-gray area. At the border, non-integration with high and with low effort are equally profitable, but we know that non-integration with low effort is strictly dominated by integration with low effort. It follows by continuity that integration with low effort must dominate non-integration with high effort over some range of parameters (this argument remains valid if the reservation wages are positive, although condition (6) would of course have to be modified). Finally, for high \( p \), non-integration must dominate again since if both agents exert high effort and get a good project with high probability, the benefit from reallocating resources is small, and hence the benefit of paying agents for admitting that they have bad projects is small, too.\(^{17}\)

### 4.6 Entrepreneurial Firms

We have assumed a separation of ownership and control even for independent firms, to facilitate the comparison with the integrated firm. However, the presence of agency problems in independent firms is clearly not an important part of our story.

Suppose, instead, that under non-integration both units are run by owner-managers. For integration, i.e. the creation of a hierarchy with two divisions, to be the equilibrium outcome, the owner of the integrated firm must be able to compensate the owner-managers for their forgone total surplus (expected output minus cost of effort) for them to be willing to sell. Since this is a higher amount than the compensation necessary in the analysis above (where the owner’s profit is reduced by the rent paid to the manager), we would expect integration to be less likely. We can distinguish two possible arrangements.

One arrangement is that upon integration, the previous owner-managers become the division managers as employees of the new owner. Somewhat paradoxically, in this case integration is always optimal. To see why, observe that the owner-managers can be compensated for selling their firms partly through their wages as managers in the integrated firm. This means that for integration to be profitable, the total surplus created by the integrated firm must be higher than total surplus created by the independent firms, which (given the same high level of effort) is always the case. The intuition here is that the agency costs of integration consist of the rents that must be paid to wealth-constrained agents. But if the owner-managers who sell their firms can be paid off with the rents they receive as agents of the integrated firm, it works to the advantage of the owner of the integrated firm, who can pocket the remainder of the total

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\(^{17}\) With continuous instead of binary effort, the high- and low-effort regions for integration would merge into one region, in which incentives are always lower-powered than under non-integration but still vary with \( p \) and \( \phi \) for the same reasons as described above.
surplus as profit.

A second arrangement is where the owner of the integrated firm hires new managers. In this case, the owner of the integrated firm must pay the new managers’ wages on top of the compensation of the previous owner-managers for their forgone profit. Integration is then less likely to be profitable, but can still emerge as an optimal organizational form, as long as the benefit from centralized resource allocation is large enough. Replicating the analysis of Section 4.5 for entrepreneurial instead of managerial firms leads to the same results as are depicted in Figure 2, except that larger values of $\varphi$ are required for integration to be optimal.

5 Optimality of a Symmetric Hierarchy

In our theory, coordinating requires that someone has authority over the non-contractible resources. It is not obvious, though, why that person should be a third player, the CEO. An alternative solution would be a two-agent “hybrid” organization in which one of the division managers, say manager 1, has the authority over resource allocation in addition to his normal task of running a division. In this section, we show that the three-agent, “CEO” hierarchy strictly dominates this hybrid organization in terms of the incentive constraints involved.

We have been ignoring the agents’ participation constraints, and will continue to do so here. It is rather clear how the managers’ participation constraints affect the owner’s choice between the two organizational forms: If the managers earn substantial rents in the CEO hierarchy, then the owner might be able to assign the task of allocating resources — for which we assumed the CEO needed to be paid $\bar{w}_0$ — to one of the managers without paying him any more than before. At the opposite extreme, if the managers’ reservation wages are such that their rents in the CEO hierarchy are close to zero, the owner would have to pay a manager close to $\bar{w}_0$ on top of his previous compensation. This could be done with a salary component, and there would be no interesting interaction with the managers’ incentives. Thus, we will focus on the managers’ incentive constraints in what follows.

Notice that in the hybrid organization, managers no longer have the same jobs. It therefore no longer makes sense to require their contracts to be the same. Instead, the objective now is to design manager 1’s contract in a way that reconciles his two tasks of running a division and allocating resources. The modified timing of events is as follows:

1. The firm’s owner offers each manager $i$ a contract $(\beta_i, \gamma_i)$, which he accepts or rejects.
2. The managers simultaneously exert effort $e_i \in \{0, 1\}$. Each manager then learns the profitability of his project $\theta_i \in \{G, B\}$, which is his private information.
3. Manager 2 sends a costless and unverifiable message $\tilde{\theta}_2$ about $\theta_2$ to manager 1.

4. Manager 1 allocates resources to the two divisions, subject to the constraint $k_1 + k_2 \leq 2$ and $k_i \in \{0, 1, 2\}$.

5. The payoffs $z_1$ and $z_2$ are realized, and the managers are compensated.

Like in the CEO hierarchy, integration is beneficial only if it leads to a more efficient resource allocation, which in turn requires that the managers’ information is aggregated accurately. Hence, we want to determine optimal wage contracts that satisfy manager 2’s truthtelling constraints. In addition, in line with condition (6) and our previous restriction to symmetric contracts, we want the contracts to ensure that both managers exert high effort.

As manager 1 does not communicate to any superior, no truthtelling constraint for him needs to be considered. But two new constraints on $(\beta_1, \gamma_1)$ need to be satisfied for him to have an incentive to allocate the firm’s resources efficiently according to $k^*(\theta)$. One condition ensures that manager 1 distributes the resources equally if both divisions’ projects are equally good or bad, instead of allocating all of it to his own division. The other condition ensures that he allocates all resources to division 2 if it has a good project and his own division 1 a bad one. (There are more conditions, but they are non-binding in equilibrium.) Both conditions lead to lower bounds on $\gamma/\beta$. We can then show the following.

**Proposition 4** Assume that the owner of an integrated firm wants the managers to exert high effort, and wants resources to be allocated efficiently. Then the incentive constraints in the CEO hierarchy are unambiguously less restrictive than those in the hybrid organization.

The result is precisely the same with nonlinear contracts; see Appendix B. To understand the result, notice first what is not different between the two scenarios. If manager 2 assumes that manager 1 exerts high effort and allocates resources efficiently, then 2’s truthtelling and effort incentive constraints are the same as before. As the managers’ contracts can be different, manager 2’s optimal contract can be determined independently from manager 1’s; the solution is given by Proposition 3. Second, if manager 1 assumes that manager 2 exerts high effort, and that manager 1 himself then allocates resources efficiently, his own effort incentive constraint is the same as before, too. The only difference between the firm with CEO and the one without, then, is that manager 1’s truthtelling constraint is replaced with incentive constraints that induce him to allocate resources efficiently.

The economic intuition for the proposition rests on an equivalence between lying to the CEO about one’s project to manipulate the resource allocation in one scenario, and misallocating
resources directly in the other. To be more concrete, suppose that in the two-agent firm, manager 1 has a bad project and learns that manager 2 has a good project. For manager 1 to allocate resources efficiently, he must prefer allocating all resources to division 2 rather than dividing the resources equally. (The equivalent situation in the firm with CEO is a manager 1 with a bad project who — contrary to our assumptions — would happen to know that manager 2 has a good project.) For manager 1 to report his type truthfully in this situation, he has to be weakly better off admitting to a bad project, upon which the CEO will allocate all resources to division 2, than by claiming to have a good project, which would lead to an equal division of resources. Since the outcomes in each case are the same, the resulting constraints for manager 1 are the same too. The same reasoning applies to the case where manager 2 has a bad project.

The critical difference between the two scenarios is that in the CEO hierarchy, manager 1 reports his type without knowing manager 2’s, which means his truthtelling constraint is a weighted average of the constraints for each type of manager 2. In contrast, in the hybrid organisation manager 1 allocates resources after learning 2’s project type, which means that each constraint must be satisfied.\textsuperscript{18}

The general logic of Proposition 4 is reminiscent of the main result of Dewatripont and Tirole (1999). They show that when an output (such as a justified conviction of a defendant in a trial) depends positively on one input (plaintiff’s evidence) and negatively on another input (defense’s evidence), then it is best to assign two different agents as “advocates” of each task. Similarly, in our model, the incentives of a division manager and a coordinator are not directly opposed, but are sufficiently misaligned to warrant separation into two jobs. Unlike in Dewatripont and Tirole, however, here the interaction with a third agent (manager 2) plays a critical role.

Proposition 4 suggests the superiority of a symmetric hierarchy with a manager (CEO) who specializes in coordinating the activities of others, over an alternative asymmetric structure in which one agent produces and coordinates. We discussed other explanations for the hierarchical structure of firms in Section 2. They include Athey and Roberts (2001) and Alonso et al.

\textsuperscript{18} Although Proposition 4 may appear specific to our binary model setup, our conjecture is that it is quite general. First, what drives the result is the difference in the timing between the two organizational structures rather than the binary project types; hence the same argument should apply to more general type spaces. Second, the binary nature of effort matters insofar as Proposition 4 assumes that the owner wants both managers to exert high effort. In a model with continuous effort (see also Footnote 15), we would expect the owner to offer lower-powered incentives to manager 1 than to manager 2, precisely because his resource allocation incentive constraints are more difficult to satisfy than the corresponding truthtelling constraint in the firm with CEO (which is the same constraint that determines manager 2’s optimal contract). But this result would not affect the statement of Proposition 4 and thus the main point of this section.
(2006), who also emphasize the value of having an unbiased decision maker. But neither paper considers the interaction between incentives and communication that is central to our model. In Alonso et al. (2006), the alternative to centralization is to leave decisions to division managers whose bias is exogenous. Athey and Roberts (2001), in turn, explicitly model incentives but not communication; in their model the decision maker can acquire all relevant information at a fixed cost.

6 Concluding Remarks

The purpose of our paper is to shed light on the benefits and costs of integration by focusing on one of the key tasks of managers, the allocation of resources. We have shown that this task is associated with incentive conflicts that do not exist in non-integrated firms. More generally, our focus reflects a belief that has its origins in the work of Coase, Barnard, Simon, Williamson and others: understanding hierarchies, and the role of managers in them as people who coordinate others’ actions or resolve conflicts through the use of authority, is key to understanding why firms exist and what determines their boundaries.

Methodologically, we borrow from both the incentive-system and the property-rights theories of the firm. But while each theory emphasizes either incentive contracts or control, in ours the two are inseparable, and are part of the same organizational design problem. We add an element that plays a prominent role of much recent organizational economics, but that has been missing in the theory of the firm: the dispersion of information in a firm, the need to communicate critical information to decision makers, and the resulting incentive problems involving the agents who are the sources of information. In our view, this Hayekian perspective is essential to understanding not only how firms work, but why they exist.

It may be considered a limitation of our analysis that we do not consider agency problems between shareholders and top management. Bolton and Scharfstein (1998) discuss how to integrate these problems into the theory of the firm. It seems intuitive, however, that there are limits to integration and thus firm size even in the absence of shareholder-manager conflicts. While the separation of ownership and control creates its own agency problems, “managerial diseconomies of scale” — a subject of much discussion in the economics literature since at least the 1930s — most likely also exist in firms that are run by their owners. As we have seen, focusing the spotlight on agency problems at lower levels in a firm, while assuming that top management is benevolent, leads to many new insights into this old problem.
Appendix A: Proofs

Proof of Lemma 1: \( E_\theta[z_i(1, \theta_i)|e_i = 1] \) in (3) is given by \( p\varphi y_1 + (1-p)y_1 \), while \( E_\theta[z_i(1, \theta_i)|e_i = 0] \) equals \( q\varphi y_1 + (1-q)y_1 \). The manager’s incentive constraint can thus be rephrased as \( \beta(p-q)(\varphi-1)y_1 \geq c \). Under an optimal contract, this condition must be binding, which leads to the bonus \( \beta \) stated in the Lemma.

Proof of Lemma 2: If owner \( i \) wants to implement high effort, she needs to pay the bonus \( \beta^{ni} \) given by Lemma 1, and the resulting profit is \( (1-\beta^{ni})[p\varphi + 1-p]y_1 \). If the owner wants to implement low effort, no wage needs to paid (under the assumption that the manager’s reservation wage is zero), and the resulting profit is \( [q\varphi + 1-q]y_1 \). It is straightforward to show that the difference between these two expressions is positive if and only if (6) holds.

Proof of Proposition 1: Under perfect information, if manager 1 has a good project, then with probability \( p \) manager 2 has a good project as well, and each is allocated one unit of resources. Manager 1 then earns \( \beta \mu \) if division 1 has a high payoff and \( \gamma \mu \) if division 2 does; the probability of each event is \( \varphi y_1/\mu \). With probability \( 1-p \), manager 2 has a bad project. All resources are then allocated to division 1, and manager 1 earns \( \beta \mu \) with probability \( \varphi y_2/\mu \). Manager 1’s expected wage from having a good project is thus

\[
W(G) = \varphi[p(\beta + \gamma)y_1 + (1-p)\beta y_2].
\] (11)

Similarly, if manager 1 has a bad project, then with probability \( p \) manager 2 has a good one, and all resources go to division 2; whereas with probability \( 1-p \) manager 2 has a bad project as well. Manager 1’s expected wage from having a bad project then is

\[
W(B) = p\gamma \varphi y_2 + (1-p)(\beta + \gamma)y_1.
\] (12)

These expressions are the same for manager 2, and so each manager will exert high effort if \( pW(G) + (1-p)W(B) - c \geq qW(G) + (1-q)W(B) \), or equivalently,

\[
(p-q)[(1-p)(\varphi y_2 - y_1) + p\varphi y_1]\beta - [p\varphi(y_2 - y_1) + (1-p)y_1]\gamma \geq c.
\] (13)

The term in brackets before \( \gamma \) in (13) is positive; hence, \( \gamma \) has a negative effect on effort incentives.

Next, let us determine the firm’s net profit. With linear contracts and for any division \( i \), the firm pays \( \beta \) (expressed as share of \( \mu \)) to manager \( i \) and \( \gamma \) to manager \( j \neq i \) if division \( i \) has a high payoff. The firm’s net profit therefore is given by \( 1 - \beta - \gamma \) times the expected total payoff in the two divisions, where the latter is given by

\[
2p^2\varphi y_1 + 2p(1-p)\varphi y_2 + 2(1-p)^2 y_1.
\] (14)

This expression is obtained as follows: with probability \( p^2 \), both divisions have a good project and receive one unit of resources each. The resulting expected payoff in each division then is \( \varphi y_1 \), which leads to the first term in (14). With probability \( 2p(1-p) \), one division has a good project and the other a bad one. All resources are allocated to the good project, whose expected payoff is \( \varphi y_2 \). This is the second term in (14). The third term is obtained in the same way as the first. The firm’s expected net profit therefore is

\[
2(1-\beta-\gamma)(p^2 \varphi y_1 + p(1-p)\varphi y_2 + (1-p)^2 y_1),
\] (15)

which is decreasing in \( \beta \) and \( \gamma \).

Since \( \gamma \) has a negative effect on profit and on effort incentives, it is optimal to set \( \gamma = 0 \). The optimal \( \beta \) is then obtained by solving (13) as equality for \( \beta \), setting \( \gamma = 0 \). The result is \( \beta^{ni} \) as stated in the Proposition.

To show that \( \beta^{ni} < \beta^{ni} \), we need to compare \( (1-p)[\varphi(y_2 - y_1) - y_1] + \varphi y_1 \) in the denominator of \( \beta^{ni} \) with \( (\varphi - 1)y_1 \) in the denominator of \( \beta^{ni} \) in (5). The difference between these two expressions equals \( p y_1 + (1-p)\varphi y_2 - y_1 \), which is positive, proving our claim.
Proof of Proposition 2: Instead of deriving the optimal mechanism directly, we first show that no contract can lead to a wage bill that is lower than under non-integration. Then, we show that the wage bill associated with the contract stated equals that under non-integration. Since the principal’s profit is expected total payoff minus the expected wage bill, it then follows that the stated contract must be an optimal (though not unique) contract. This indirect strategy of proof makes precise which assumptions of the model drive the result.

If manager 1 exerts high effort and communicates truthfully, his expected wage is

\[ pw_1(G, G) + (1 - p)\bar{w}_1(B, B) = p[\bar{w}_1(G, G) - \bar{w}_1(B, B)] + \bar{w}_1(B, B), \]

while his effort incentive constraint can be written as

\[ \bar{w}_1(G, G) - \bar{w}_1(B, B) \geq \frac{c}{p - q}. \]

Since (17) is binding under non-integration, the difference \( \bar{w}_1(G, G) - \bar{w}_1(B, B) \) in (16) cannot decrease under non-integration, and thus the firm’s expected wage payment to manager 1 must increases whenever \( \bar{w}_1(B, B) \) is higher than a bad manager’s expected wage under non-integration, which is given by \( \beta y_1 = c/[(p - q)(\varphi - 1)] \), cf. (5).

The key property that our general argument relies on is that

\[ \varphi \bar{w}_1(B, G) \geq \bar{w}_1(G, G) \quad \text{or} \quad \frac{\bar{w}_1(B, G)}{\bar{w}_1(G, G)} \geq \frac{1}{\varphi}. \]

To see why, notice that since the resource allocation depends only on the manager’s reported type, both a truthful manager with a good project and a lying manager with a bad project receive the same resources. To see this, notice that since the resource allocation depends only on the manager’s reported type, both a truthful manager with a good project and a lying manager with a bad project receive the same resources.

The truth-telling constraint for a manager 1 with a bad project is given by

\[ \bar{w}_1(B, B) \geq \bar{w}_1(B, G). \]

Then, (19), (18) and (17), respectively, imply that

\[ (\varphi - 1)\bar{w}_1(B, B) \geq (\varphi - 1)\bar{w}_1(B, G) \geq \varphi \bar{w}_1(B, G) - \bar{w}_1(B, B) \geq \bar{w}_1(G, G) - \bar{w}_1(B, B) \geq \frac{c}{p - q}, \]

and hence \( \bar{w}_1(B, B) \geq c/[(p - q)(\varphi - 1)] \). We have thus shown that \( \bar{w}_1(B, B) \) must be weakly greater than a bad manager’s expected wage under non-integration. Notice also that the condition \( \varphi \bar{w}_1(B, G) \geq \bar{w}_1(G, G) \) is tight: if \( \varphi \bar{w}_1(B, G) = \bar{w}_1(G, G) \), the inequality signs in (20) can normally be replaced by equality signs, provided the contract space is large enough (i.e. allows for enough wage variables to specify) to make the effort and truth-telling constraints binding.

Let us now verify that the contract stated in the proposition is a solution to the problem (4). Consider first manager 1’s incentive to choose high effort, assuming that manager 2 does too. If manager 1 has a good project, then with probability \( p \) manager 2 does too, and both get one unit of resources. With probability \( 1 - p \), manager 2’s project is bad, and manager 1 gets all resources. If manager 1 has a bad project, he gets \( w_B \) for sure. Manager 1 then chooses high effort if (cf. 13)

\[ (p - q) \left[ p\varphi \frac{y_1}{\mu} w^1 + (1 - p)\varphi \frac{y_2}{\mu} w^1 - w_B \right] \geq c. \]

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Next, consider manager 1’s incentive to report his project type truthfully if his project is bad. If he reports a bad project, he gets \( w_B \) for sure. If instead he reports to have a good project, then he receives one unit of resources if manager 2 has a good project (which occurs with probability \( p \)), and all resources if if manager 2 has a bad project (with probability \( 1 - p \)). Manager 1 then reports truthfully if

\[
 w_B \geq p \frac{y_1}{\mu} w^1 + (1 - p) \frac{y_2}{\mu} w^1 .
\]  

(22)

Similarly, if manager 1 has a good project, he will report his type truthfully if

\[
 p \frac{\varphi y_1}{\mu} w^1 + (1 - p) \frac{\varphi y_2}{\mu} w^1 \geq w_B .
\]  

(23)

The expressions for \( w^1 \) and \( w_B \) stated in the proposition are the unique solution of (21) and (22) as equalities. It is easy to verify that this contract also satisfies (23). To complete the proof that the contract is feasible, we need to check the CEO’s resource allocation constraint (RA) in (4). If both projects are equally good or bad, the CEO allocates the resources equally as is efficient (for two good projects, this follows from \( 2y_1 > y_2 \), cf. Assumption 1). Suppose now that project 1 (say) is good and project 2 bad. If the CEO allocates all resources to division 1, then the firm’s resulting expected profit is \( \varphi \frac{y_1}{\mu} (\mu - w^1) - w_B \), where \( w^1 \) is paid to manager 1 if \( z_1 = \mu \) and \( w_B \) is paid to manager 2 with certainty. If instead the CEO allocates the resources equally, then the firm’s resulting expected profit is \( \varphi \frac{y}{\mu} (\mu - w^1) + \frac{\mu}{2} \). It is then straightforward to show that for the CEO to allocate all resources to division 1, \( w^1 \) must not exceed the stated upper bound.

Under non-integration, the expected wage bill per firm is \( \beta^{\mu} \) times the expected payoff \( [p\varphi + (1 - p)]y_1 \), which simplifies to

\[
\frac{c[p\varphi + (1 - p)]}{(p - q)(\varphi - 1)} \]

(24)

The expected wage bill per manager in the integrated firm is given by

\[
\frac{1}{2} \left[ p^2 2 \varphi \frac{y_1}{\mu} w^1 + (1 - p)^2 2 w_B + 2p(1 - p) \left( \varphi \frac{y_1^2}{\mu} w^1 + w_B \right) \right]
\]

which upon substituting the expressions in the proposition simplifies to (24), i.e. the same as under non-integration. As argued above, it follows that the contract \((w^1, w_B)\) must be optimal.

**Proof of Lemma 3:** For a manager 1 with a *good* project, the expected payoff from reporting truthfully, and under the assumption that manager 2 reports truthfully too, is given by \( \bar{w}_1(G, G) = W(G) \) as given by (11). Suppose manager 1 reports “B” instead. Then with probability \( p \), manager 2 has a good project, in which case all resources go to division 2 (given that manager 2 reports truthfully and that the CEO assumes truth telling on part of both managers). With probability \( 1 - p \), manager 2 has a bad project, and each division is allocated one unit of resources. The resulting expected wage for manager 1 thus is

\[
\bar{w}_1(G, B) = p\gamma y_2 + (1 - p)(\beta \varphi y_1 + \gamma y_1).
\]  

(25)

The truth telling constraint (IC-G) given by \( \bar{w}_1(G, G) \geq \bar{w}_1(G, B) \) can therefore be expressed as

\[
\varphi[y_2 - y_1 + p(2y_2 - y_1)] \beta \geq [(1 - p)y_1 + p\varphi(2y_1 - y_2)] \gamma,
\]  

(26)

which is equivalent to (8). The difference between the numerator and denominator on the right-hand side of (8) simplifies to \( (1 - p)[\varphi(y_2 - y_1) - y_1] + p\varphi(2y_1 - y_2) > 0 \), which means the fraction is greater than 1.

For a manager 1 with a *bad* project, the expected payoff from reporting truthfully (under the same ancillary assumptions as above), is \( \bar{w}_1(B, B) = W(B) \) as given by (12). Suppose manager 1 reports “G” instead. Then with probability \( p \), manager 2 has a good project too, in which case each division gets one
unit of resources. With probability $1 - p$, manager 2 has a bad project, and all resources go to division 1. The resulting expected wage for manager 1 is
\[
\bar{w}_1(G, B) = p(\beta + \varphi \gamma)y_1 + (1 - p)\beta y_2.
\] (27)
The truth-telling constraint (IC-B) given by $\bar{w}_1(B, B) \geq \bar{w}_1(B, G)$ can therefore be expressed as
\[
\{p[\varphi(y_2 - y_1) - y_1] + y_1\} \gamma \geq \{p(2y_1 - y_2) + y_2 - y_1\} \beta,
\] (28)
which is equivalent to (7). The difference between the numerator and denominator on the right-hand side of (7) simplifies to $- (1 - p)(2y_1 - y_2) - p[\varphi(y_2 - y_1) - y_1] < 0$, which means the fraction is smaller than 1 (but positive, since both numerator and denominator are).

**Proof of Proposition 3:** The optimal linear contract maximizes (15) with respect to $\beta$ and $\gamma$, subject to (i) the effort incentive constraint (13), (ii) the truth-telling constraints (8) and (7), (iii) the constraint (RA) that it must be (ex post) optimal for the CEO to allocate resources efficiently, assuming high effort and truth-telling on part of both managers, and (iv) nonnegativity constraints $\beta, \gamma > 0$. This optimization problem is a linear program, and therefore the optimal solution must be a corner point of the parameter set defined by the constraints.

The resource allocation constraint (RA) is never binding with linear contracts. This follows immediately from the fact that the firm’s profit is $1 - \beta - \gamma$ times expected total payoff, which by definition is maximized if resources are allocated efficiently, cf. the discussion in Section 3.1 on the role of our parameter constraints. Thus, for $\beta + \gamma < 1$, ex-post profit maximization on part of the CEO leads to an efficient resource allocation, whereas $\beta + \gamma > 1$ would never be chosen since the resulting profit would be negative.

The relevant constraints are (IC-e) and (IC-B), whereas (IC-G) is redundant. To see why, observe first that the effort constraint (13) must be binding, for otherwise truth-telling would be optimally achieved by setting $\beta = \gamma = 0$, when (13) is clearly violated. Second, we already know from Proposition 1 the solution to the relaxed problem in which (IC-G) and (IC-B) are not imposed, and from Lemma 3 we know that it satisfies (IC-G) but not (IC-B). Hence (IC-B) must be binding. Finally, given Lemma 3, any $(\beta, \gamma)$ that satisfies (IC-B) also satisfies (IC-G). The optimal contract is therefore given by solving (7) and (13) as equalities for $\beta$ and $\gamma$: the solution is stated in the Proposition.

Since both $(\beta^e, 0)$ and $(\beta^{int}, \gamma^{int})$ solve (13) with equality and since (13) is increasing in $\beta$ and decreasing in $\gamma$, it follows that $\beta^{int} > \beta^e$, see Figure 2. That $\beta^{int} > \beta^e$ follows from comparing the expressions for both variables, where $p y_1 + (1 - p) y_2$ in the denominator of $\beta^{int}$ is greater than $y_1$ in the denominator of $\beta^e$, and the expressions are otherwise the same.

The integrated firm’s total wage bill is given by $\beta + \gamma$ times total expected output as given by (14), while the total wage bill for two independent firms is given by $\beta$ times $2[p\varphi + (1 - p)y_1].$ Upon substituting $(\beta^{int}, \gamma^{int})$ into the wage bill of the integrated firm and $\beta^e$ into that of the non-integrated firms, the difference between the two simplifies to
\[
\frac{2 - p\varphi y_1}{(p - q)(\varphi - 1)} \cdot \frac{y_2 - y_1 + p(2y_1 - y_2)}{py_1 + (1 - p)y_2} = \frac{y_2 - y_1 + p(2y_1 - y_2)}{py_1 + (1 - p)y_2} \cdot \frac{2 - p\varphi y_1}{(p - q)(\varphi - 1)},
\]
which under Assumptions 1 and 2 is strictly positive.

**Proof of Proposition 4:** Manager 2 must be willing to report his project truthfully, under the assumption that manager 1 exerts high effort, and that manager 1 allocates resources according to $k^*$. But these constraints for manager 1 are the same as for the CEO hierarchy, and are given by (8) and (7). Hence manager 2’s truth-telling constraints is the same as in the CEO hierarchy. Similarly, manager 2 must be willing to exert effort, again under the same assumptions as above. Since the managers’ contracts can be different, we can solve for manager 2’s optimal contract independently from manager 1’s. Thus,
the arguments of Proposition 3 apply without change for manager 2, and his optimal contract \((\beta_2, \gamma_2)\) is given by Proposition 3.

For manager 1, the effort incentive constraint is likewise the same as before. In place of a truth-telling constraint, there are now incentive constraints for manager 1 to be willing to allocate resources efficiently. There are two relevant new constraints; suppose in both cases that manager 1 has a bad project. First, if manager 2’s project is bad as well, allocating the resources equally (which is the efficient choice) leads to an expected wage of \((\beta + \gamma_1)y_1\) for manager 1, while allocating all resources to division 1 leads to an expected wage of \(\beta_1y_2\). Manager 1 therefore allocates resources efficiently if

\[(\beta_1 + \gamma_1)y_1 - \beta_1y_2 \geq 0\]  

or equivalently \(\gamma_1/\beta_1 \geq (y_2 - y_1)/y_1\). Second, if manager 2’s project is good, then allocating all resources to division 2 (the efficient choice) leads to a payoff of \(\gamma y_2\) for manager 1, whereas allocating the resources equally instead leads to a wage of \((\beta + \varphi\gamma)y_1\). Manager 1 therefore allocates resources efficiently if

\[\gamma_1\varphi y_2 - (\beta_1 + \gamma_1)y_1 \geq 0\]  

or equivalently \(\gamma_1/\beta_1 \geq y_1/[(\varphi(y_2 - y_1)]\). There are more constraints, in particular since there are other ways to allocate resources inefficiently, but all of them are equivalent to or less restrictive than (29) or (30), and therefore need not be considered.

To prove the proposition, we show that (29) and (30) are jointly more restrictive than the truth-telling constraint (28) in the symmetric hierarchy that they replaced. This follows simply from the fact that if we write (28) in this form,

\[
[p\varphi(y_2 - y_1) - y_1]\gamma - [p(2y_1 - y_2) + y_2 - y_1]\beta \geq 0,
\]

the left-hand side is equal to \(1 - p\) times the left-hand side of (29) plus \(p\) times the left-hand side of (30), as is easy to verify. Thus, in the hybrid organization, the more restrictive condition of (29) and (30) must hold, whereas in the symmetric hierarchy only a weighted average of the two constraints must hold, which is an unambiguously weaker constraint.

**Appendix B: Nonlinear Contracts**

General nonlinear contracts specify a wage for each possible realization of \((\tilde{z}_1, \tilde{z}_2)\). For a non-integrated firm, the contract given by Lemma 1 remains optimal when non-linear contracts are allowed, since there is no reason to condition wage payments on the other firm’s payoff, and since in each firm realized payoff is only \(\mu\) or zero, requiring only one non-zero wage variable.

We therefore need to study nonlinear contracts only for the integrated firm. As before, with limited liability but a non-binding participation constraint, it is optimal to pay each manager zero if both divisions have a zero payoff. For this reason, it is without loss of generality to describe each manager’s wage as a share of \(\mu\) as in the linear case. The managers’ (symmetric) contracts can then be described by the triple \((\beta, \gamma, \delta)\), where manager \(i\) is paid \(\beta\) (as a share of \(\mu\)) if only \(\tilde{z}_i = \mu\), \(\gamma\) if only \(\tilde{z}_j = \mu\), and \(\delta\) if both units have a high payoff. The case of linear contracts discussed in the main text is a special case of this more general setting, and corresponds to the restriction \(\delta = \beta + \gamma\). Our first result generalizes Proposition 1:

**Proposition 5** In an integrated firm in which the CEO has perfect information about \(\Theta\) and allocates resources efficiently, the optimal contract for each division manager is given by \(\gamma = 0\), and

\[
\begin{align*}
\text{if } p &< \frac{y_2}{(\varphi - 1)y_1 + y_2}; \quad \beta^{\mu} = \frac{(p - q)[(1 - p)(\varphi y_2 - y_1) + p\varphi y_1\mu - (p\varphi^2 - 1 + p)y_1^2]}{\frac{\varphi y_2 - y_1}{y_1^2}} \\
\delta^{\mu} & = 0, \quad \text{and} \quad \text{and} \quad \delta^{\mu} = \frac{\varphi y_2 - y_1}{y_1^2}
\end{align*}
\]

\[\text{if } p > \frac{y_2}{(\varphi - 1)y_1 + y_2}; \quad \beta^{\mu} = 0 \quad \text{and} \quad \delta^{\mu} = \frac{\varphi y_2 - y_1}{y_1^2}.
\]

In either case, the expected total wage bill is lower under integration than under non-integration.
Proof: Under perfect information, if manager 1 has a good project, then with probability $p$ manager 2 has a good project as well and each is allocated one unit of resources. Manager 1 can then earn either $\delta \mu$, $\beta \mu$ or $\gamma \mu$ (with appropriate probabilities), depending on which of the two divisions has a high payoff. With probability $1-p$, manager 2 has a bad project, all resources are allocated to division 1, and manager 1 earns $\beta \mu$ with probability $\varphi y_2/\mu$. Overall, manager 1’s expected wage from having a good project is

$$W(G) = \left\{ p \left[ \frac{\varphi^2 y_1^2}{\mu^2} \delta + \frac{\varphi y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) (\beta + \gamma) \right] + (1-p) \frac{\varphi y_2}{\mu} \beta \right\} \mu. \tag{32}$$

If manager 1 has a bad project, then with probability $p$ manager 2 has a good one, and all resources go to division 2; whereas with probability $1-p$ manager 2 has a bad project as well. Manager 1’s expected wage from having a bad project then is

$$\left\{ p \frac{\varphi y_2}{\mu} \gamma + (1-p) \left[ \frac{y_2^2}{\mu^2} \delta + \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) (\beta + \gamma) \right] \right\} \mu. \tag{33}$$

By symmetry, these expressions are the same for manager 2, and so each manager will exert high effort if $pW(G) + (1-p)W(B) - c \geq qW(G) + (1-q)W(B)$, or equivalently,

$$(p-q) \left\{ (p\varphi^2 - 1 + p) \frac{y_1^2}{\mu^2} \delta + \left[ (1-p) \left( \varphi y_2 - y_1 \left( 1 - \frac{y_1}{\mu} \right) \right) \right] + p\varphi y_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) \right\} \beta
- \left[ p\varphi \left( y_2 - y_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) \right) \right] + (1-p) y_1 \left( 1 - \frac{y_1}{\mu} \right) \gamma \right\} \geq c. \tag{34}$$

As in the linear case, the left-hand side of (34) is decreasing in $\gamma$, meaning that $\gamma$ has a negative effect on effort incentives. The firm’s expected net profit is $\mu$ times

$$[p^2 \varphi^2 + (1-p)^2] \frac{y_1^2}{\mu^2} (2 - 2\delta) + 2(1 - \beta - \gamma) \left[ p \frac{\varphi y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) + (1-p)^2 \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) + p(1-p) \frac{y_2}{\mu} \right]. \tag{35}$$

This expression is obtained as follows. If both divisions have a high payoff, the firm’s profit is $(2-2\delta)\mu$. For this to occur requires that both projects are either good (probability $p^2$) or bad (probability $(1-p)^2$), which leads to the first term in (35). Otherwise, if only one division has high payoff, the firm’s profit is $(1-\beta-\gamma)\mu$; cf. the coefficient of the second term in (35). This occurs if both divisions have equally good or bad projects, and both get resources, but only one has a high payoff (the first two terms in $[\cdot]-brackets in (35))$, or if only one division has a good project and gets all resources (the last term in $[\cdot]-brackets in (35))$.

Since both (35) and the left-hand side of (34) are decreasing in $\gamma$, it is optimal to set $\gamma = 0$. This leaves two variables to choose but only one constraint. Since both $\beta$ and $\delta$ decrease the firm’s profit and the slopes of (35) and the left-hand side of (34) are different, it is optimal to set $\delta$ or $\beta$ to zero and solve (34) for the other variable. The solutions for each case are stated in the proposition.

Which of these two cases is optimal depends on the slopes of (35) and the left-hand side of (34) in $\beta, \delta$-space. Choosing $\delta > 0$ and $\beta = 0$ is optimal if the iso-profit curves are steeper than the (34)-line. The formal condition can be expressed as $-\frac{\partial \Pi}{\partial \beta} / \frac{\partial \Pi}{\partial \delta} < -\frac{\partial IC-e}{\partial \beta} / \frac{\partial IC-e}{\partial \delta}$, or equivalently $\frac{\partial \Pi}{\partial \beta} / \frac{\partial \Pi}{\partial \delta} > \frac{\partial IC-e}{\partial \beta} / \frac{\partial IC-e}{\partial \delta}$, with $\Pi$ given by (35) and “IC-e” given by the left-hand side of (34). The difference between the two slopes, $\frac{\partial \Pi}{\partial \beta} / \frac{\partial \Pi}{\partial \delta} - \frac{\partial IC-e}{\partial \beta} / \frac{\partial IC-e}{\partial \delta}$, can be shown to be equal to

$$\mu (1-p) \varphi y_2 \left[ y_1^2 \beta - (1-p) y_2 \frac{y_1}{(1-p)^2 + p^2 \varphi^2} (p \varphi^2 - 1 + p) \right]. \tag{36}$$

The last expression in the denominator of (36) must be positive since it appears in the denominator of $\delta$ in (31), and so must be positive for this solution to be feasible at all. All other terms in (36) are
unambiguously positive, except for \( p(\varphi - 1)y_1 - (1 - p)y_2 \), which is positive if and only if \( p > \frac{y_2}{(\varphi - 1)y_1 + y_2} \).

This leads to the condition for the case distinction stated in the proposition.

Finally, since \( \beta^p < \beta^u \) in Proposition 1, the optimal linear contract under perfect information already leads to a lower expected wage bill for the firm. Since the set of linear contracts is a strict subset of the set of nonlinear contracts, it follows that the wage bill must also be lower with an optimal non-linear contract.

The optimal solutions stated in Proposition 5 look quite different from the contract in Proposition 1 for the linear case. Given that \( \gamma = 0 \) is optimal, linearity imposes the restriction \( \delta = \beta \) on the contract. Without this restriction, it is optimal to set one of \( \beta \) or \( \delta \) to zero (this is also why the expressions for \( \beta \) in Propositions 1 and 5 look very different).

The main conclusion from Proposition 1, however, remains intact when nonlinear contracts are feasible: with perfect information, the competition effect of centralized resource allocation improves effort incentives relative to non-integration, as reflected in a lower wage bill.

Let us now turn to the case where project types are communicated strategically by the managers. As in Section 4.4, additional constraints come into play. In the following, we first derive these constraints formally, and then generalize Proposition 3.

First, it must be optimal for each manager to report his type truthfully. For a manager 1 with a good project, the expected payoff from reporting truthfully, and under the assumption that manager 2 reports "G" instead. Then with probability \( p \), manager 2 has a good project, in which case all resources go to division 2 and manager 1 earns \( \gamma \) if division 2 has high payoff. With probability \( 1 - p \), manager 2 has a bad project, each division is allocated one unit of resources, and the manager can earn \( \delta, \beta \) or \( \gamma \) times \( \mu \), depending on both divisions' payoffs. The resulting expected wage for manager 1 is

\[
\bar{w}_1(G, B) = \left\{ p \frac{\varphi y_2}{\mu} \gamma + (1 - p) \left[ \frac{y_2^2}{\mu^2} \delta + \frac{\varphi y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) \beta + \left( 1 - \frac{\varphi y_1}{\mu} \right) \gamma \right] \right\} \mu. \tag{37}
\]

The truth telling constraint (IC-G) given by \( \bar{w}_1(G, G) \geq \bar{w}_1(G, B) \) can therefore be expressed as

\[
\varphi \frac{y_1^2}{\mu} (p\varphi - 1 + p) \delta + \varphi \left[ (1 - p)y_2 - (1 - p)y_1 \left( 1 - \frac{y_1}{\mu} \right) \beta + \left( 1 - \frac{\varphi y_1}{\mu} \right) \gamma \right] \beta 
+ \left[ (p\varphi - 1 + p)y_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) - p\varphi y_2 \right] \gamma \geq 0. \tag{38}
\]

For a manager 1 with a bad project, the expected payoff from reporting truthfully is \( \bar{w}_1(B, B) = W(B) \) as given by (33). Suppose manager 1 reports "G" instead. Then with probability \( p \), manager 2 has a good project too, in which case each division gets one unit of resources, and manager 1 can earn \( \delta, \beta \) or \( \gamma \) times \( \mu \) depending on the divisions' payoffs. With probability \( 1 - p \), manager 2 has a bad project, and all resources go to division 1. The resulting expected wage for manager 1 is

\[
\bar{w}_1(B, G) = \left\{ p \left[ \frac{\varphi y_2}{\mu} \delta + \left( 1 - \frac{y_1}{\mu} \right) \varphi y_1 \gamma + \frac{y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) \beta \right] + (1 - p) \frac{y_2}{\mu} \right\} \mu. \tag{39}
\]

The truth telling constraint (IC-B) given by \( \bar{w}_1(B, B) \geq \bar{w}_1(B, G) \) can then be expressed as

\[
-(p\varphi - 1 + p) \frac{y_2}{\mu} \delta - \left[ (1 - p) \left( y_2 - y_1 \left( 1 - \frac{y_1}{\mu} \right) \right) + py_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) \beta \right] 
+ \left[ p\varphi \left( y_2 - y_1 \left( 1 - \frac{y_1}{\mu} \right) \right) + (1 - p)y_1 \left( 1 - \frac{y_1}{\mu} \right) \right] \gamma \geq 0. \tag{40}
\]

Second, it must be optimal for the CEO to allocate resources efficiently if she assumes that project types are reported truthfully. This constraint was never binding with linear contracts, but may become
binding with nonlinear contracts. Suppose first that both projects are good. If the CEO allocates one unit of resources to each division (the efficient allocation), the expected profit for the firm is 
\[
2(1 - \delta)\frac{\varphi y_i^2}{\mu^2} + 2(1 - \beta - \gamma)\frac{\varphi y_i}{\mu} \left(1 - \frac{\varphi y_i}{\mu}\right)
\]
(times \(\mu\) (in the following next equations, all profit expressions are stated as shares of \(\mu\)). If instead the CEO were to allocate all resources to one division, then the expected profit would be 
\[
(1 - \beta - \gamma)\frac{\varphi y_2}{\mu}.
\]
For the CEO to choose the efficient allocation requires that (41) be at least as large as (42), or 
\[
2(\beta + \gamma - \delta)\frac{\varphi y_2^2}{\mu} + (1 - \beta - \gamma)(2y_1 - y_2) \geq 0.
\]
By similar reasoning, it can be shown that the condition for the CEO to allocate resources efficiently if both projects are bad is given by 
\[
2(\beta + \gamma - \delta)\frac{\varphi y_2^2}{\mu} + (1 - \beta - \gamma)(2y_1 - y_2) \geq 0.
\]
Finally, suppose that division 1’s project is good and division 2’s bad. If the CEO allocates all resources division to 1 (the efficient allocation), the firm’s expected profit is 
\[
(1 - \beta - \gamma)\frac{\varphi y_2}{\mu}
\]
If instead the CEO were to allocate the resources equally, then the expected profit would be 
\[
2(1 - \delta)\frac{\varphi y_2^2}{\mu^2} + (1 - \beta - \gamma)\left[\frac{\varphi y_1}{\mu} \left(1 - \frac{y_1}{\mu}\right) + \left(1 - \frac{\varphi y_1}{\mu}\right) \frac{y_1}{\mu}\right].
\]
For the CEO to choose the efficient allocation requires that (45) be at least as large as (46), or equivalently 
\[
(1 - \beta - \gamma)[\varphi(y_2 - y_1) - y_1] - 2(\beta + \gamma - \delta)\frac{\varphi y_2^2}{\mu} \geq 0.
\]
Of these three constraints, (44) is redundant. To see why, notice that since both (43) and (47) must hold, the sum of their left-hand sides, which yields \(\mu(\varphi - 1)(y_2 - y_1)(1 - \beta - \gamma)\), must be positive. This in turn requires that \(\beta + \gamma < 1\). Next, given the last result, both (43) and (44) can be binding only if \(\delta > \beta + \gamma\); but in that case (42) is clearly the more restrictive condition. We can therefore ignore (44).

The optimization problem (4) can hence be stated more precisely as the problem of maximizing (35) with respect to \(\beta, \gamma\) and \(\delta\), subject to the effort incentive constraint (34), the truth-telling constraints (38) and (40), the resource allocation constraints (43) and (47), and the nonnegativity constraints \(\beta, \gamma, \delta \geq 0\).

With three variables to specify and eight linear constraints, there are as many as \(8!/(3! \cdot 5!) = 56\) different corner points as possible candidates for an optimal solution. The following result, which generalizes Proposition 3, narrows the number of possible solutions down to only a few:

**Proposition 6** In an integrated firm, the optimal nonlinear contract for each division manager that leads to high effort, truthful reports about investment projects, and an efficient resource allocation, depends on \(p\) as follows:

(a) If \(p < 1/(1 + \varphi)\), then an optimal contract is given by

\[
\begin{align*}
\beta &= c \frac{p\varphi - 1 + p}{(1 - p)(p - q)(\varphi - 1)[p(\varphi - 1)y_1 - (1 - p)y_2]} \\
\gamma &= 0, \\
\delta &= c \frac{\mu - \varphi y_1}[p(2y_2 - y_1) + y_2 - y_1] + (1 - p)y_1[\varphi(y_2 - y_1) + y_1]}{(1 - p)(p - q)(\varphi - 1)y_2^2[(1 - p)y_2 - p(\varphi - 1)y_1]}.
\end{align*}
\]

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The resulting expected wage per agent is the same as under non-integration.

(b) If \( p > 1/(1 + \varphi) \), then the optimal contract is given by the unique solution for \( \beta, \gamma \) and \( \delta \) of one of six different systems of conditions (with inequalities interpreted as equations): in four of the possible solutions, (34) and (40) hold, and in addition either \( \beta = 0, \gamma = 0, (43), \) or (47). In the remaining two possible solutions, both (43) and \( \beta = 0 \) hold, and in addition either (34) or (40). In each of these solutions, \( \gamma \) is strictly positive, and the resulting expected wage per agent is strictly higher than under non-integration.

Proof: Part (a): The solution stated is the unique solution for which \( \gamma = 0 \) and both (34) and (40) are binding. Feasibility of this solution requires \( \beta, \delta \geq 0 \). Since the numerator of \( \delta \) in (48) is positive, we need \( (1 - p)y_2 > p(\varphi - 1)y_1 \) for the denominator of \( \delta \) to be positive. But if the latter condition holds, the denominator of \( \beta \) in (48) is negative, which means that for \( \beta \) to be positive, we need \( 1 - p > p\varphi \) for the numerator to be negative as well. Conversely, if \( 1 - p > p\varphi \), then it follows that \( (1 - p)y_2 > p\varphi y_1 > p(\varphi - 1)y_1 \), i.e. the same condition we started with. We can conclude that \( 1 - p > p\varphi \) or \( p < 1/(1 + \varphi) \) is both necessary and sufficient for the stated solution to be feasible (although we haven’t established optimality yet).

For general \( \beta, \gamma \) and \( \delta \), the total expected wage bill for the integrated firm is \( \mu \) times

\[
2\delta \left[ \frac{\varphi^2}{\mu^2} + (1 - p)^2 \right] \frac{y_1}{\mu^2} + 2(\beta + \gamma) \left[ \frac{\varphi y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) + p(1 - p) \frac{\varphi y_2}{\mu} + (1 - p)^2 \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) \right],
\]

cf. the expression for the firm’s net profit in (35). Substituting the contract (48) into (49) and simplifying leads to \( 2p(\varphi - 1 + p)/[(p - q)(\varphi - 1)] \), which is the same as the total wage bill for both firms under non-integration, cf. (24). Optimality of the solution (48) then follows from Proposition 2 (the contract (48) need not be the unique optimal contract, though).

Part (b): If \( p > 1/(1 + \varphi) \) or \( p\varphi - 1 + p > 0 \), then the truth-telling constraint (40) is decreasing in \( \delta \). Since it is also decreasing in \( \beta \), the only way to satisfy (40) is to set \( \gamma > 0 \). Next, recall from the proof of Proposition 2 that the wage bill under integration is strictly higher than under non-integration if \( \varphi w(B, G) > \bar{w}(G, G) \). Evaluating the difference \( \varphi w(B, G) - \bar{w}(G, G) \), using the expressions in (32) and (39), simplifies to \( p\varphi(\varphi - 1)y_1\gamma \), which is positive whenever \( \gamma \) is. Both results together imply that any solution for the case \( p > 1/(1 + \varphi) \) leads to a strictly higher wage bill than under non-integration, as stated.

To sort through the numerous possible candidates for an optimal contract, let us proceed in order of the number of variables that are set to zero. First, \( \beta = \gamma = \delta = 0 \) is clearly not a feasible solution since it violates the effort constraint (34). Second, given the requirement \( \gamma > 0 \), the only candidate for a solution with two variables equal to zero has \( \beta = \delta = 0 \) and \( \gamma > 0 \); but in this case (34) is again violated.

Third, consider solutions in which exactly one variable is zero; this can only be either \( \beta \) or \( \delta \). The other two variables are then determined by any two of (34), (40), (42) and (47) as binding conditions. Under an optimal contract, the resource allocation constraints (42) and (47) will never simultaneously be binding, for this would require \( \beta + \gamma = \delta = 1 \) and would lead to a profit of zero. (Intuitively, the CEO is indifferent between all resource allocations only if the entire division payoffs are paid out to the managers.)

Suppose first that \( \delta = 0 \). One solution is given where both (34) and (40) are binding. It satisfies the nonnegativity and resource allocation constraints for some but not all parameters, and therefore belongs to the set of possible solutions. Next, the resource allocation constraint (43) is always satisfied if \( \delta = 0 \); therefore only (47) can possibly be binding. This leaves two remaining potential solutions in this group, where (47) and either (34) or (40) are binding. Neither can be optimal, however: if (34) and (47) are to be binding, it would be optimal to set \( \gamma = 0 \) since both constraints are decreasing in \( \gamma \), which we already know would violate (40). And if (40) and (47) are to be binding, it would be optimal to set \( \beta = 0 \) since both constraints are decreasing in \( \beta \), but doing so would violate our assumption that only one variable
is set to zero. Hence, the only contract with \( \delta = 0 \) that can be optimal is the one where (34) and (40) are binding.

Now, suppose that \( \beta = 0 \). Again, one possible solution is given where both (34) and (40) are binding. Next, we show that (47) can never be binding: with \( \beta = 0 \), (47) could bind only if \( \delta < \gamma \). But if \( \beta = 0 \) and \( \delta = \gamma \), then (34) is definitely violated, as is easy to verify, and since (34) is increasing in \( \delta \) in this range of \( p \), the same must be true for even smaller \( \delta \). It follows that out of the two resource allocation constraints, only (43) can possibly be binding. This leaves two remaining potential solutions in this group, where (43) and either (34) or (40) are binding. Both are feasible for some parameters, and since we have not been able to rule out their optimality, they too are possible solutions.\(^{19}\)

Fourth, consider strictly interior solutions, which are obtained by making any three out of (34), (40), (42) and (47) bind. Since we already showed that (42) and (47) will never simultaneously be binding, this leaves two possible solutions, in which both (34), (40), and either of (42) or (47) are binding. Both solutions are feasible, and can be shown to be optimal for certain parameters.

Overall, we have narrowed the range of possible solutions for case (b) down to the six stated in the proposition.

Proposition 6 states that if \( p > 1/(1 + \varphi) \) (part b), then the conclusions of Proposition 3 carry over to the nonlinear case: any feasible solution entails \( \gamma > 0 \), and from the proof of Proposition 2 it follows that the wage bill must be higher than under non-integration. If \( p \leq 1/(1 + \varphi) \), on the other hand, the managers’ information can be elicited without any additional cost relative to the case of non-integration, similar to the message-contingent contract of Proposition 2.

The condition on \( p \) that distinguishes the two cases results from the truth-telling constraint (40) for a manager with a bad project, which in turn is derived from (33) and (39). Specifically, the condition follows from the derivative of the left-hand side of (40) with respect to \( \delta \): case (a) of the Proposition applies when \( \delta > 0 \) increases the manager’s incentive to report truthfully, and otherwise case (b) applies. To understand the sign of the derivative, observe first that a manager can earn \( \delta \) only if both divisions have high payoffs, which requires that the CEO allocates the resources equally between the divisions (if one division has no resources, it cannot attain a high payoff). If manager 1 has a bad project and reports truthfully, he earns \( \delta \) if manager 2 also has a bad project (the probability of which is \( 1 - p \)), and both have high payoff, which occurs with probability \( y_1^2/\mu^2 \). In contrast, if manager 1 claims to have a good project, he earns \( \delta \) if manager 2 has a good project (the probability of which is \( p \)), and both have high payoff, which occurs with probability \( \varphi y_1^2/\mu^2 \). Thus, the effect of \( \delta \) to report truthfully is given by \( (1 - p - p\varphi)y_1^2/\mu^2 \), which is positive if and only if \( p \leq 1/(1 + \varphi) \).

Our last result generalizes Proposition 4. Like in Section 5, we allow for asymmetric contracts for the managers. The managers’ wages for the different possible payoff outcomes can therefore be described by \( \delta_1, \beta_1, \gamma_1 \) and \( \delta_2, \gamma_2, \beta_2 \), respectively.

**Proposition 7** Assume that the owner of an integrated firm wants the managers to exert high effort, and wants resources to be allocated efficiently. Then the incentive constraints in the CEO hierarchy are unambiguously less restrictive than those in the hybrid organization.

**Proof:** As in the linear case, in the hybrid organization all incentive constraints for manager 2, as well as the effort incentive constraint for manager 1, are the same as in the CEO hierarchy, cf. the proof of Proposition 5. It remains to show how the resource allocation constraints for a manager 1 with a bad project compare to his truth-telling constraint in the hierarchy with CEO. Suppose manager 1 has a bad project. If manager 2’s project is bad too, and manager 1 allocates the resources equally as would be efficient, his expected wage is

\[
\left( \frac{y_1^2}{\mu^2} \delta_1 + \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) (\beta_1 + \gamma_1) \right) \mu.
\]

\(^{19}\) In numerical simulations, though, these two solutions, where feasible, have turned out to be inferior to the one in which \( \beta = 0 \) and (34) and (40) are binding.
If instead he allocates all resources to himself, his expected wage is $\beta_1 y_2$. Manager 1 will therefore allocate resources efficiently if

$$\left[ \frac{y_2^2}{\mu^2} \delta_1 + \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) (\beta_1 + \gamma_1) \right] \mu - \beta_1 y_2 \geq 0. \tag{50}$$

If manager 2’s project is good and manager 1 allocates all resources to division 2 as would be efficient, his expected wage is $\gamma_1 \varphi y_2$. If instead he allocates the resources equally, his expected wage is

$$\left[ \frac{\varphi y_1^2}{\mu^2} \delta_1 + \frac{y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) \beta_1 + \left( 1 - \frac{y_1}{\mu} \right) \frac{\varphi y_1}{\mu} \gamma_1 \right] \mu.$$  

Manager 1 will therefore allocate resources efficiently if

$$\gamma_1 \varphi y_2 - \left[ \frac{\varphi y_1^2}{\mu^2} \delta_1 + \frac{y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) \beta_1 + \left( 1 - \frac{y_1}{\mu} \right) \frac{\varphi y_1}{\mu} \gamma_1 \right] \mu \geq 0. \tag{51}$$

It can then be shown that left-hand side of (40) is equal to $(1 - p)$ times the left-hand side of (50) plus $p$ times the left-hand side of (51), which completes the proof (see the proof of Proposition 5 for further details).

References


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