

Schlicht, Ekkehart

**Working Paper**

## Economic surplus and derived demand

IZA Discussion Papers, No. 2159

**Provided in Cooperation with:**

IZA – Institute of Labor Economics

*Suggested Citation:* Schlicht, Ekkehart (2006) : Economic surplus and derived demand, IZA Discussion Papers, No. 2159, Institute for the Study of Labor (IZA), Bonn

This Version is available at:

<https://hdl.handle.net/10419/33979>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

IZA DP No. 2159

## Economic Surplus and Derived Demand

Ekkehart Schlicht

June 2006

# Economic Surplus and Derived Demand

**Ekkehart Schlicht**

*University of Munich  
and IZA Bonn*

Discussion Paper No. 2159  
June 2006

IZA

P.O. Box 7240  
53072 Bonn  
Germany

Phone: +49-228-3894-0  
Fax: +49-228-3894-180  
Email: [iza@iza.org](mailto:iza@iza.org)

Any opinions expressed here are those of the author(s) and not those of the institute. Research disseminated by IZA may include views on policy, but the institute itself takes no institutional policy positions.

The Institute for the Study of Labor (IZA) in Bonn is a local and virtual international research center and a place of communication between science, politics and business. IZA is an independent nonprofit company supported by Deutsche Post World Net. The center is associated with the University of Bonn and offers a stimulating research environment through its research networks, research support, and visitors and doctoral programs. IZA engages in (i) original and internationally competitive research in all fields of labor economics, (ii) development of policy concepts, and (iii) dissemination of research results and concepts to the interested public.

IZA Discussion Papers often represent preliminary work and are circulated to encourage discussion. Citation of such a paper should account for its provisional character. A revised version may be available directly from the author.

## **ABSTRACT**

### **Economic Surplus and Derived Demand<sup>\*</sup>**

Most demand – especially labor demand – is derived from the demand for some other product. This note demonstrates that the usual analysis of economic rent, as typically explained for the case of consumers' surplus, carries over to the case of derived demand.

JEL Classification: D00, D61

Keywords: derived demand, indirect demand, consumers' surplus, economic rent

Corresponding author:

Ekkehart Schlicht  
Institutional Economics Group  
Department of Economics  
University of Munich  
Schackstr. 4  
80539 Munich  
Germany  
Email: [schlicht@lmu.de](mailto:schlicht@lmu.de)

---

<sup>\*</sup> I thank Robert Schlicht for a simplifying suggestion.

# Economic Surplus and Derived Demand

## 1 Introduction

ALFRED MARSHALL (1920, v.vi.4) has pointed out that most demand – especially labor demand – is derived from the demand for some other product. He wrote: “To take another illustration, the direct demand for houses gives rise to a joint demand for the labour of all the various building trades, and for bricks, stone, wood, etc. which are factors of production of building work of all kinds, or as we may say for shortness, of new houses. The demand for any one of these, as for instance the labour of plasterers, is only an indirect or derived demand.”

This note demonstrates that the usual analysis of economic rent, as typically explained for the case of consumers’ surplus, carries over to the case of derived demand. The assertion comes as no surprise; rather such is typically presupposed in all kinds of cost-benefit considerations involving derived demand, yet there is, to the best of my knowledge, no demonstration to be found in the textbooks or the literature. This note is intended to fill the gap.

## 2 Derived Demand

Consider a market for a consumer good with the falling demand curve (inverse demand function)

$$p = p(x) , \quad p' < 0 \tag{1}$$

where  $x$  denotes the quantity demanded at price  $p$ .

Assume that the commodity is produced by many firms by means of some input  $n$  (labor, for instance) according to a production function

$$x_i = f_i(n_i) , \quad f'_i > 0 , \quad f''_i < 0 .$$

where the index  $i$  refers to firm number  $i$ .<sup>1</sup>

Profits of firm  $i$  are  $p_i f_i(n_i) - w n_i$ , and the profit maximizing level of production is characterized by the marginal productivity condition

$$p f'_i(n_i) = w. \quad (2)$$

Denote by  $n = \sum_i n_i$  the aggregate labor input and define the aggregate production function  $f$  as

$$f(n) := \max \left\{ \sum_i f_i(n_i) \mid \sum_i n_i = n \right\}. \quad (3)$$

Assuming a unique interior maximum, we obtain from (2) and (3)

$$p f'(n) = w. \quad (4)$$

For any given product price  $p$ , this equation gives the ordinary (inverse) demand curve for the input (*e.g.* labor). Its slope is equal to  $p f'' < 0$ . The sum of profits accruing to all firms together is  $\sum_i (p f_i(n_i) - w n_i)$  and hence

$$\pi = p f(n) - w n. \quad (5)$$

Any level of production  $x$  is, according to (1), uniquely related to the product price  $p$  that is necessary to clear the market. These interrelations can be incorporated within the analysis by inserting (1) and (3) into (4), yielding a relationship between factor input and input price:  $p(f(n)) f'(n) = w$ . This gives rise to the *indirect input demand curve*

$$w(n) := p(f(n)) f'(n). \quad (6)$$

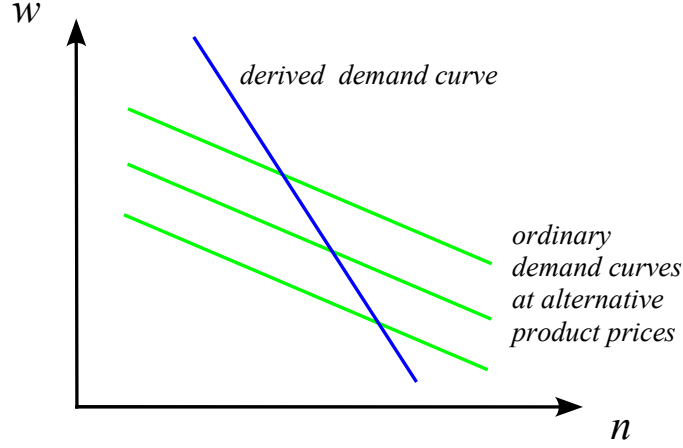
Its slope is  $w' = p f'' + p'(f')^2 < p f''$ . It is, therefore, steeper than the ordinary demand curve (Figure 1).<sup>2</sup>

Consider a wage reduction from  $w_0$  to  $w_1$  that goes along with an employment increase from  $n_0$  to  $n_1$ , an output increase from  $x_0$  to  $x_1$ , and a price reduction from  $p_0$  to  $p_1$ . These quantities relate as follows:

$$\begin{aligned} x_0 &= f(n_0), & p_0 &= p(x_0) \\ x_1 &= f(n_1), & p_1 &= p(x_1). \end{aligned} \quad (7)$$

---

<sup>1</sup> In the simplest case, only one input (such as labor) is needed. For reasons of simplicity this is assumed in the following exposition. The more general case of many inputs can be covered by re-interpreting  $f_i(n_i)$  as the maximum value added obtainable for firm  $i$  if factor input  $n_i$  is costlessly provided, *etc.* <sup>2</sup> See PINDYCK and RUBINFELD (2005, 521). Note that some authors use the concept in a different sense; see VARIAN (1996, 338), for instance.



**Figure 1:** The derived input demand curve is steeper than the ordinary input demand curves.

Consider the area under the demand curve (1) between  $x_0$  and output level  $x = f(n)$  belonging to some employment level  $n$ :

$$P(n) = \int_{x_0}^{f(n)} p(x) dx. \quad (8)$$

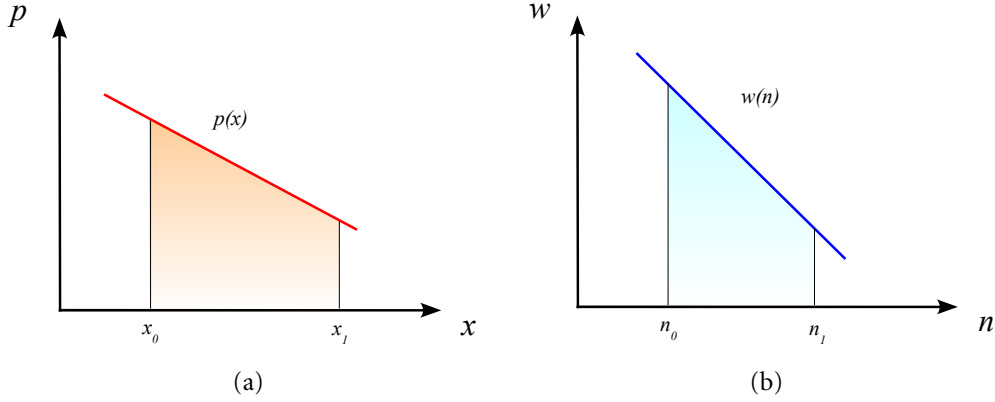
Differentiation of (8) with respect to  $n$  yields

$$\frac{\partial P}{\partial n} = p(f(n)) f'(n) = w(n)$$

which implies

$$P(n_1) = \int_{n_0}^{n_1} w(n) dn.$$

This is just the area under the derived input demand curve. Hence the corresponding areas under the product demand curve and the indirect input demand curve are identical in size (Figure 2).



**Figure 2:** The area under the demand curve for output (a) and the area under the derived demand curve for input (b) between corresponding values of input and output are identical.

### 3 Derived Surplus

Consider now the change in consumer's surplus that results from a price change from  $p_0$  to  $p_1$ . It is

$$\Delta cs = p_0 x_0 - p_1 x_1 + \int_{x_0}^{x_1} p(x) dx \quad (9)$$

and is depicted in Figure 3. According to the previous argument, the integral over the demand curve equals the corresponding integral over the indirect demand curve and we have  $\int_{x_0}^{x_1} p(x) dx = \int_{n_0}^{n_1} w(n) dn$ . The profits associated with the different levels of production are

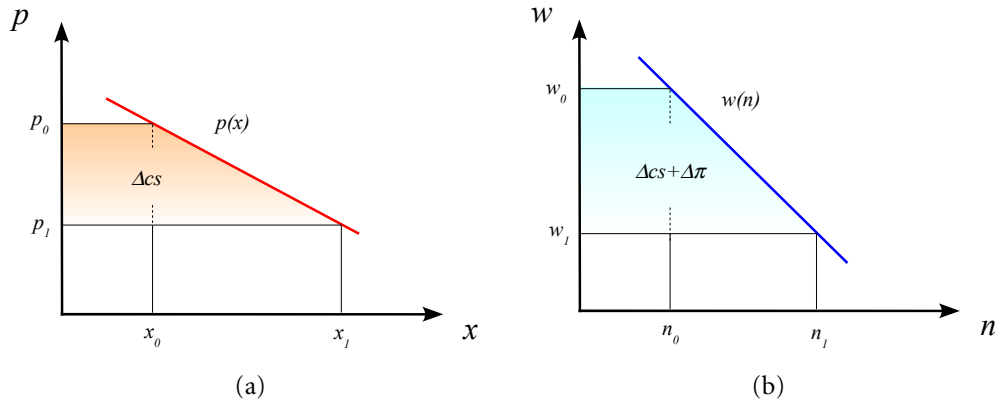
$$\begin{aligned} \pi_0 &= p_0 x_0 - w_0 n_0 \\ \pi_1 &= p_1 x_1 - w_1 n_1 \end{aligned}$$

and the change in profits is

$$\Delta \pi = \pi_1 - \pi_0.$$

Hence (9) can be written as

$$\Delta cs + \Delta \pi = w_0 n_0 - w_1 n_1 + \int_{n_0}^{n_1} w(n) dn. \quad (10)$$



**Figure 3:** The change in consumers' surplus  $\Delta cs$  is given by the shaded area left of the final demand curve (a). The analogous area left of the derived demand curve (b) gives the change in economic rent as the sum of the changes in consumers' surplus  $\Delta cs$  and profits  $\Delta \pi$ .

The left-hand side gives the change in economic rent as the sum of the changes in consumers' surplus  $\Delta cs$  and profits  $\Delta \pi$ . The right-hand side gives the surplus area in the derived demand diagram, which is, thus, a measure for the increase in economic rent induced by a price decrease of input from  $w_0$  to  $w_1$  and an entailed price decrease of the final product from  $p_0$  to  $p_1$ . (Figure 3).

## References

- MARSHALL, A. 1920, *Principles of Economics*, 8th ed., London: Macmillan, online at <http://www.econlib.org/library/Marshall/marPtoc.html>, reprint 1949, first edition 1890.
- PINDYCK, R. S. and D. L. RUBINFELD 2005, *Microeconomics*, 6th ed., Pearson, Upper Saddle River, New Jersey, first edition 1995.
- VARIAN, H. R. 1996, *Intermediate Microeconomics*, fourth ed., Norton, New York, first edition 1987.