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Endogenous R&D elasticities of productivity functions and BERD bias: Explorations of functional coefficient regressions

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Endogenous R&D elasticities of productivity functions and BERD bias: Explorations of functional coefficient regressions

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Abstract Macroeconomic productivity is modelled as a Cobb-Douglas function of private and public R&D stocks in recent literature. The slope parameters of a growth rate version may change over time and with circumstances. Using the method of functional-coefficient regression, we show that human capital, GDP (per worker), services and defence R&D (both % GDP), lags of domestic and foreign private and public R&D, and lagged labour-augmenting technical change, all in growth rates, change the elasticities of productivity. The result is a panel data set of regression coefficients representing elasticities of productivity. Eventually, the panel average of the productivity elasticities of domestic and foreign private R&D goes to constant values; elasticities of public R&D go down slightly. This may contribute to an explanation of the productivity slowdown and why private R&D has been expanded relative to public R&D in recent years. JEL codes: O33; O47.

Keywords: technical change; private and public R&D substitution; panel data; changing elasticities of productivity; functional coefficient regression; productivity slowdown.

1. Introduction

The literature on the relation between private and public R&D agrees with some exceptions on a positive link with productivity, with some exceptions of papers finding no effect of public R&D and some exceptions finding no effect of private R&D (See Soete et al. 2022; Ziesemer 2021a, 2024a). These are extreme cases of the more general result that papers do not find numerically similar regression coefficients in similar approaches (see surveys of Hall et al. 2010, OECD 2017, Ziesemer 2021a).

If countries had similar elasticities of productivity of public and private R&D, we should expect similar private/public R&D stock or flow ratios. However, countries with higher incomes have higher private and public R&D (% GDP), and relatively more private R&D (Coccia 2010). Moreover, the private/public R&D ratio going from 1 to 3 during the period 1989- 2019 for USA (Ciaffi et al. 2024) signals a shift of expected elasticities of productivity from public to private. These aspects can all be read as issues potentially questioning the assumption of fixed R&D elasticities of productivity.

A productivity function is a condensed version of the view how knowledge from R&D translates into productivity. Institutions of R&D may be separated from those of production that use the knowledge from R&D. The transfer of knowledge from R&D institutions to production organizations may therefore depend on the public and business environment and it may change over time and across countries. One of these may be technical changes in the

¹ I am grateful to Mercedes Menéndez De Medina and Bart Verspagen for useful comments. Responsibility is entirely mine.

productivity function. Another one may be changes in the parameters, which literature has assumed to be constant.

In this paper we try to find evidence for changes in R&D elasticities of productivity and the underlying causes as well as indications of (in-)efficiencies and/or technical change in the productivity function, which may be neutral as in the case of technical change in a Cobb-Douglas function or biased in case of more general CES or VES productivity functions.

We test the effects on elasticities of productivity from several variables. Having more country-wide human capital in the country was suggested by Schultz (1964) as a source of more productive private and public R&D. Having higher recent growth rates of GDP, and past growth rates of GDP per worker may create changes in incentives explaining Coccia's observation.² As services (% GDP) have lower productivity growth rates (Herrendorf et al. 2014, Figure 6.10; Comin et al. 2025), countries with a higher share of services may have lower growth rates of productivity for the same R&D inputs at the macroeconomic level; this may be reflected in higher or lower macroeconomic R&D elasticities of productivity for private and public R&D when the share of services changes.³

Finally, the literature on defence R&D at times has doubted that it contributes to productivity (Mowery and Rosenberg 1989). Similarly, for OECD panel data, Guellec and van Pottelsberghe De La Potterie (2003) find a negative effect of defense R&D on business R&D, and “the defence component of government-performed research has a negative impact on business-funded R&D, civilian R&D has no impact” (Jaumotte and Pain (2005, p.38), which may reduce productivity growth. Fieldhouse and Mertens (2023) also do not find positive effects of US defence R&D. In contrast, Goel *et al.* (2008) find a higher rate of return for US defense compared to non-defense R&D. Deleidi and Mazzucato (2021) find positive effects of mission-oriented military R&D on private R&D and GDP growth of US defense R&D; Boysen-Hogrefe (2025) shows that using data only since 1985 leads to a crowding-out result. Both results together suggest changing coefficients. Moretti et al. (2025) find a positive effect of defence R&D on business R&D for a country-industry year-level dataset for OECD countries and a firm-year-level dataset for France. These diverse results suggest investigating how military expenditure on R&D affects the elasticities of productivity of public and private R&D. In addition, we include effects from lagged variables of the productivity functions; we explain this in detail below.

For the analysis of changing elasticities of productivity, we combine the theoretical concept of Cobb-Douglas, CES or VES functions for productivity depending on R&D variables (see section 2) and the empirical method of functional coefficient regression (Cai et al. 2000), which makes regression coefficients dependent on the arguments of the previous paragraph called functional coefficient series (see section 3). Section 4 describes the data including their

² Khan and Luintel (2006) investigate the effect of interactions of human capital with some variables on countries' productivity, but these do not include public and private R&D capital except for their weighting with country-specific mean stocks.

³ Similarly, Boppart and Weiss (2013) show that a one percentage point larger share of 30 industries generates a 0.3 higher growth rate of TFP in five years in the USA. Up to changes in the primary sector, the role of industry is the mirror image of the role of services.

time trends and panel unit root analyses as the method of functional coefficient regression as used requires stationarity (Cai et al. 2000). The results in section 5 show how and which functional coefficient series or smoothing factors have an impact on the regression coefficients when used separately and then puts all effects together to see how regression coefficients or their combination for the direct and indirect effects of public R&D change over time. As this is an unusual flood of information with a not yet well-known method, we summarize results more extensively than usual in section 5.3 for the single variables, and in section 6 for the overall results and provide suggestions for policy and future research.

2. Theoretical Model

Herzer (2022) and Huang, Lai, and Peretto (2025), in a theoretical endogenous growth paper, interpret the empirical literature as a Cobb-Douglas function of productivity depending on domestic business and government R&D stocks denoted as B(ERD) and P(ublic) respectively. This is the standard interpretation found in earlier surveys (Hall et al. 2010; OECD 2017). Van Elk et al. (2015, 2019) add foreign R&D, and Soete et al. (2022) add foreign private and public R&D separately and the lagged level of productivity. Equation (1) states this assumption formally.

$$T = B^\alpha P^\beta \prod_i X_i^{\gamma_i} \quad (1)$$

The first two terms are the direct effects of business and government R&D on productivity. X abbreviates for other variables such as foreign R&D variables, and time trends mimicking technical change in the log-version of the productivity function (1). Optimal growth requires equal marginal products of private and public R&D (Ziesemer, 2021b) as formulated in (2) for the Cobb-Douglas case:

$$\alpha \frac{T}{B} = \beta \frac{T}{P} \quad (2)$$

Technical change would drop out in (2) because T can be cancelled. Allowing for inefficiency in the sense of unequal marginal products, (2) implies (2')⁴ (a 'hat' indicating a percentage growth rate):

$$\frac{\hat{B}}{\hat{P}} = A(t) \frac{\alpha}{\beta} \quad \text{or} \quad \hat{B} - \hat{P} = \hat{A} \quad (2')$$

In case of efficiency, $A = 1$, and constant. Under inefficiency, A does not equal unity and may go to unity and thereby towards efficiency, or away from it leading to increasing inefficiency. (2) and (2') are the theoretical basis to think of an indirect effect of public R&D: higher public R&D may be seen as an incentive for firms to invest into higher private R&D, and higher private R&D may be an incentive for governments to think of more public R&D. This can happen independently or through some policy coordination as known from OECD and EU recommendations and national policies. The model is a highly stylized formulation of

⁴ More generally, if the function would be CES with other elasticity of substitution than unity, or VES, the external derivative would drop out.

these coordinations and adjustments.⁵ In particular, the growth rate of inefficiencies in the Cobb-Douglas case is as large as the differences of the growth rates of private and public R&D. Suppose instead that (1) is replaced by the more general VES function⁶ (3).

$$T(t) = \{[D(t)B(t)]^\delta + [F(t)P(t)]^\varepsilon\}^{\theta/\gamma} \quad (3)$$

$D(t)$ and $F(t)$ are exogenous technical change functions of time. A special case of (3) is the CES function for $\gamma = \delta = \varepsilon$, with or without scale parameter $\theta = 1$. If these parameters go to zero, we go to the Cobb-Douglas function (1) again (requiring L'Hôpital's theorem).

Equal marginal products of B and P lead to (4), where the external derivative has dropped out):

$$\delta[D(t)B(t)]^{\delta-1}D(t) = \varepsilon[F(t)P(t)]^{\varepsilon-1}F(t) \quad (4)$$

Rewriting (4) in terms of growth rates we get (4'):

$$\hat{B} = \frac{(\varepsilon-1)}{\delta-1} \hat{P} + \frac{\varepsilon}{\delta-1} \hat{F} - \frac{\delta}{\delta-1} \hat{D} \quad (4')$$

As D and F are exogenous by assumption, the sum of the last two terms is the intercept of the equation. If this sum is a positive growth rate, we call this BERD biased technical change. In the special case of a Cobb-Douglas function it is zero because we have dropped the idea of inefficiency $A(t)$ through unequal marginal products, and F and D would drop out from (2') if we had included them in (1), where a log version can replace the corresponding sum by a time trend, and a growth rate version has them as a constant. The slope in (4') depends on the VES parameters.

In the Cobb-Douglas model, α and β are constant elasticities of productivity. The major purpose of this paper is to go one step beyond this basic standard model to show that these elasticities may depend on the circumstances prevailing in the economy when trying to turn R&D experience into productivity. A second important aspect is that the unit elasticities in (2') are unlikely to be fulfilled. (4') then offers a more realistic alternative; the CD version can be remarkably close to these though. We will relate the empirical works introduced next to these functions without adhering strictly to one of them or trying to clarify which is more realistic.

3. Empirical Methodology

Functional coefficient regression is a semi-parametric method. Regression coefficients a and b in $Y = a + bX + u$ are local polynomials of degree l , $\gamma_j(Z) = \sum c_{jl} (Z - z_0)^l$ (or other nonlinear functions)⁷ in third variables Z , called functional coefficient series, leading to $a(Z)$, $b(Z)$. The polynomials are Taylor series approximations of the Z variables (Cai et al. 2000). The variables Z are for example current or lagged regressors: $Z = X, X(-1)$ (Chen and Tsay

⁵ Park (1998) and Herzer (2022) specify slightly different models of direct and indirect effects.

⁶ Ziesemer (2021b) relates this function to the literature on utility and production functions.

⁷ The literature of this paragraph discusses other nonlinear functions as alternatives to polynomials, such as $(-0.6-1.25x)e^{-4x^2}$. Eviews selects only the polynomials as procedure, which is based on Cai et al. (2000).

1993), time trends: $Z = t$ (Hastie and Tibshirani 1993; Cai 2003), or lagged dependent variables, $Z = Y(-1)$, (Huang/Shen 2004), or other lagged variables, $Z = V(-1)$ (Huang/Shen 2004). Variables Z have to be determined according to economic context based on a priori knowledge of the researcher.⁸ Polynomials $a(Z)$ and $b(Z)$ - where b , in case of several regressors, and Z can be vectors - are then inserted resulting in $Y = a(Z) + b(Z)X + u$. Residuals are weighed with kernel functions and squared. The estimation method then is penalized least squares in combination with non-parametric procedures to find kernel functions, which balance low bias with high variance under a small bandwidth, and high bias with low variance under a large bandwidth (Ullah and Pagan 1999). We should choose the Taylor approximation with the highest degree that yields a smooth curve (Chen and Tsay 1993).⁹ Estimation bias and confidence intervals are found by way of comparison with using two (or more) additional degrees of the Taylor approximation. Using a lower degree of the Taylor approximation than the highest resulting in a smooth curve therefore leads to a larger bias. Statistically competing models are linear autoregressive models with lag p , $ar(p)$, and threshold models (transition, smooth) (see Cai et al. 2000) and it is a matter of testing which work better. Our results therefore should not be misunderstood as criticism of (v)ar, threshold, or spline models. We do not compare our results to these alternatives, because they are related to different economic research questions. Thresholds and to some extent lags are statistical steps, whereas the functional coefficient series bring also more economics into the analysis.

We estimate the equations in growth rates as the data described below have unit roots.

Production function (1) in growth rates with $c(i)$ as regression coefficients is (1'):

$$DLT = C(1) + C(2)DLT(-1) + C(3)DLB + C(4)DLP + C(5)DLB^* + C(6)DLP^* \quad (1')$$

D indicates a difference. L is a natural logarithm. T is the labour-augmenting technology level, $B(ERD)$ is the private R&D stock, P the public R&D stock, and a '*' indicates the corresponding variables of foreign countries. $C(1)$ is exogenous technical change.

The regression coefficients of (1') are estimated as functions of growth rates of human capital, $\log(hc/hc(-5))$, lagged GDP per worker in the labour force, $d(\log(gdp(-5)/(lf(-5))))$, current share of services in GDP, $d(\log(srvy))$, lags of defence R&D as share of GDP, $d(\log(0.00000001 + defrdy(-5)))$, and lags of variables in (1'), $dlth07(-2)$, $dlberdst(-5)$, $dlpubst(-3)$,¹⁰ as suggested by the FAR model in Cai et al. (2000) and others.¹¹ Again, an L

⁸ Interesting economic examples are examples: Average rate of return over some periods ((Huang/Shen 2004; example 2)); distance from TFP frontier, $A - A^i < 0$ (Goñi and Maloney 2017).

⁹ $b(Z)X$ therefore has some similarity with interaction effects in parametric methods. There, quadratic and cubic polynomials would often run into the 'near singular matrix' warning though.

¹⁰ $dlfpubst(-3)$ has constant coefficients when using a polynomial of degree 4, but not for a linear one.

¹¹ For countries without defence R&D the log does not exist. Therefore, we have added a small number to the defence R&D variable. An alternative to adding $1/10^j$, $j = \dots, -2, -1, 0, 1, 2, \dots$ is the formula $\log(x + (x^2 + 1)^{0.5})$ taken from Busse and Hefeker (2007); replacing 1 by $1/10^j$ in this formula still allows this formula to work and it is thereby also not unique. The variables have been postulated on the basis of the economic knowledge of the author. There is no natural large data set from which variables have to be selected using variable selection methods in order to avoid computational problems (Ghosal et al. 2020).

indicates a natural logarithm. The d indicates a difference chosen to ensure stationarity of the variables as required by Cai et al. (2000). Without it variables may have unit roots.¹²

Juhl (2005) argues that results with nonstationary variables still make sense if the residuals are stationary implying cointegration. This can easily be specified but is not guaranteed empirically (Wang and Phillips, 2025). Tu and Wang (2022) point out that cointegration testing for functional coefficient regression is just in its infancy. Therefore, we use only stationary variables. Cai et al. (2009) analyze the case of nonstationary functional coefficient series Z in terms of statistical theory. Their two-step procedure to include variables with unit roots is not yet integrated in Eviews14. One economic reason may be that with a unit root, Z would change only with accumulated residuals once $Z(0)$ is given. This would imply that coefficients are explained by random walks, which could be of limited economic interest, unless emphasis is on an included time trend or the cross-section differentiation of $Z(0)$. Using cointegrating relations may change results in future research.

Another econometric issue is endogeneity, which is not yet addressed in the EViews14 tool for functional coefficient regression and the corresponding manual chapter and the underlying article by Cai et al. (2000). Cai et al. (2015) are dealing with it but EViews has not yet integrated this. Our results are therefore based on the assumption that a potential endogeneity bias is small (see Nakamura and Nakamura 1998) and having functional coefficients is quantitatively more important. This makes the results somewhat preliminary and future research may lead to modifications. Nevertheless, we provide a lot of information in regard to functional coefficient series, which sometimes are called factors (Herwartz and Walle 2014) or smoothing variables (Cai et al. 2015). It should be noted that the definition of bias in comparison with polynomials two degrees higher used in functional coefficient analysis is different from the standard endogeneity or serial correlation bias. Logs and lags of the functional series changing coefficients have been found by way of experimentation. In the first phase we use each series alone and test that coefficients are not constant; otherwise, variables could seem to produce smooth results, which are caused only by covariates used simultaneously. For every given value of a functional series, we have implicitly a cross-section regression estimating the regression coefficients. Then the method shifts to the next value and estimates the regression coefficients again which should generate similar values in order to get smooth series of the regression coefficients. In the second phase we have used the previously significant variables jointly. Testing for linear, quadratic, and cubic approximation and in some cases polynomial degree 4 and higher (see notes to figures and tables), we find the highest degree yielding a smooth curve.

The effect of public on private R&D is as in (2''):

$$DLB = C(7) + C(8)DLP \quad (5)$$

The constant can be interpreted as the growth rate of the BERD-bias indicator in (4'), (4'') or as inefficiency growth as in (2') or a combination of both, because they cannot be distinguished empirically. The symbols are explained above. We have experimented with the same functional coefficient series as for (1'). We find smoothing effects for five-year changes of human capital, $\log(hc/hc(-5))$, lags of services (% GDP), $d(\log(srvy(-3)))$, $d(\log(srvy(-4)))$, $d(\log(srvy(-5)))$, defence R&D, $d(\log(0.00000001+defrdy(-2)))$, $d(\log(berdst(-3)))$, and the

¹² An alternative for avoidance of unit roots is the standard normalization of a variable for each given point in time (Herwartz and Walle 2014). The economic interpretation is of course quite different because that version of a variable emphasizes the deviation from the temporary mean rather than the change of the size of the variable. Which version of a variable is more suitable depends on the research question.

average of GDP growth rates of the last four years, sumg4 (using degree 4), and lags of variables in (5), here only $\text{dlberdst}(-2)$.

For (1') and (5), testing for constant coefficients (not varying with functional series) may run into the problem of 'attempt to raise a negative number to a non-integer power'. In these cases, we try to avoid the problem by using a polynomial of a lower degree. Later, when calculating the coefficients $C(1)$ - $C(8)$ we go back to the smooth curve from the polynomial with the highest degree. The estimation bias is calculated through comparison with a polynomial of (at least) two degrees more, on which also the confidence intervals are based, which test whether coefficients are different from zero as proven by confidence intervals being on the same side of the horizontal axis (same sign for upper and lower bounds).

For public R&D, the direct effect then is $c(4)$, and the indirect effect is $c(3)*c(8)$ from (1') and (5). The total effect of a one percent change of public R&D then is $c(4) + c(3)*c(8)$. This simple model ignores all impacts and feedback on and from foreign R&D, and feedback from private on public R&D as it was considered using vector-error correction models by Soete et al. (2020, 2022) and Ziesemer (2021b). The method of functional regression coefficients is a single equation method serving well the purpose of this paper, but it is not a system method.

4. Data

We construct a data set similar to that of van Elk et al. (2015, 2019) and Soete et al. (2022) (extended later to 2017) and extend it to 2021 for R&D data for all 44 countries of OECD MSTI, where explanations are given in detail in Ziesemer (2025). Moreover, labour-augmented technical change data from Ziesemer (2023) are updated until 2019; we use here the one based on a CES production function under an elasticity of substitution of 0.7. These data are explained in detail in Ziesemer (2023, 2025). The required human capital data are taken from PWT 10.01; they are weighted-average data from years of schooling in Barro-Lee.com, defined on a scale from 1 to 5. Data for GDP, labour force, and services (% GDP) are taken from World Development Indicators.¹³ R&D expenditure on defence are taken from OECD MSTI.

The data are divided into two subsamples. For the richer countries there are longer data series. For the emerging economies there are shorter data series. In this paper we use only the longer data series for the richer 21 countries: Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Ireland, Iceland, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Sweden, United States.

Table 1 shows the standard information of the data. The human capital value is defined as between 1 and 5, and therefore all values and their logs are positive. We also add panel unit root tests. We mostly use Pesaran's CIPS tests, which are a weighted statistic of country-specific ADF tests. In some cases, we add the Bai-Ng test in the Ahn-Horenstein manner to find common factors and test for their unit roots besides those in the residuals. When there is too low a number of observations for using these tests, we use those assuming no cross-section dependence. After all, the growth rate of public R&D has a fairly high probability of unit root. Here we decide to stick to the theoretical model and assume that it is stationary.

¹³ Data for service/GDP ratios of Taiwan are from <https://eng.stat.gov.tw/>.

Where the log levels have a unit root, we use its difference in the estimation below. The differenced variables without unit roots are growth rates. Overall, they have a strong variation in terms of difference between maximum and minimum and in terms of standard deviations.

In the pre-column the variables are presented immediately as constant and time trend. All growth rates have a negative trend in the sample data. Therefore, we will formulate most interpretations below for falling growth rates going from right to left when reading the figures below. This leaves the cross-section interpretation to the reader to save space, except for defence R&D, which has no trend. The growth rates for domestic and foreign public R&D are more negative than all other variables in Table 1, a fact recently discussed by Archibugi and Filippetti (2018) and resumed below.

Table 1 Data description for a sample of 21 rich countries with long data series

Statistics Variable, trend (f) ↓	Mean	Median	Maximum	Minimum	Std dev	Pesaran CADF (a), CIPS: p (unit root)
$dlth07 = 0.0268 - 0.00043t$	0.0119	0.011	0.234	-0.105	0.025	< 0.01
$dLB = 0.08 - 0.00073t$	0.055	0.047	0.71	-.1625	0.053	< 0.01
$dLP = 0.0785 - 0.001t$	0.043	0.036	2.76	-.1625	0.089	< 0.01
$dLB^* = 0.066 - 0.00046t$	0.051	0.044	0.99	-.0046	0.051	> 0.1; (b):.011; .002
$dLP^* = 0.087 - 0.0012t$	0.047	0.035	1.08	-0.032	0.062	> 0.1; (b):0.000;0.21
$Log(hc/hc(-5)) = 0.046 - 0.00042t$	0.0309	0.0311	0.1	-0.024	0.0152	< 0.01
$d(log(gdp(-5)/(lf(-5)))) = 0.032 - 0.000387t$	0.011	0.0127	0.207	-0.113	0.0252	< 0.05
$Lsry = 3.95 + 0.0041t$	4.128	4.139	4.345	3.73	0.102	(c): 0.756; 0.012
$d(log(sry)) = 0.012 - 0.00019t$	0.0039	0.0034	0.18	-0.214	0.021	0.0000; 0.0000 (c)
$L(0.00000001+defrdy) = -18.32 - 0.00093t$	-18.36	-18.4	-17.8	-18.4	0.11	(e) 0.01-0.18
$d(log(0.00000001+defrdy)) = -0.00126 (d)$	-.00126	-8.65e-06	0.0726	-0.134	0.013	(e) 0.0000 f. all tests
$Sumg4 = 0.0473 - 0.00057t$	0.0276	0.0258	0.11	-.0235	0.019	< 0.01

(a) Lag length criterion: AIC; max 8 lags. (b) Bai-Ng test; factor selection method: Ahn-Horenstein (2013); 1 factor max and selected; p-values for unit root in common factor and residuals. (c) As (b), for one common factor. (d) $e-x$ or $e-\theta x$ means that the dot should be shifted x decimals to the left. (e) Insufficient observations for unit root tests with cross-section dependence (csd); for all tests without csd, p between 0.01 and 0.18. (f) Panel EGLS estimate with cross-section weights and country fixed effects; all p = 0.0000.

5. Results

Besides variables in Table 1 we have also tested ‘Internet per head of the population’ in several variants of $\log(1+\text{intpop}*\text{hc})$ or modifications; they do not yield smooth results for the long, rich sample for any lag. Therefore, we do not use an internet variable.

In the following figures, the functional coefficient series in the Taylor approximation polynomial are listed below the graphs. The graphs show the impact of only one of the functional coefficient series. Adding the other functional coefficient series leads more often than not to the warning ‘attempt to raise a negative number to a non-integer power’ and thereby no results for the test of constant coefficients. From the perspective of an economic interpretation, it is clear then that the effects in the figures below stem from one variable only and not from including other functional coefficient series from Table 1 with lags. When we want to calculate the coefficients of (1') later, we use all functional coefficient series simultaneously, but we do not need the test for constant coefficients then. To show all the impacts, we get inevitably as many figures for the productivity function (1') as we have functional coefficient series with each one graph per regression coefficient in (1'). We discuss only the main impacts; at the very left or very right of the graphs there are often results based on small numbers of observations. As most observations are in the middle of the range, we mainly discuss the effects around the mean and median shown in Table 1.

5.1 Partial effects on the regression coefficients of the productivity function

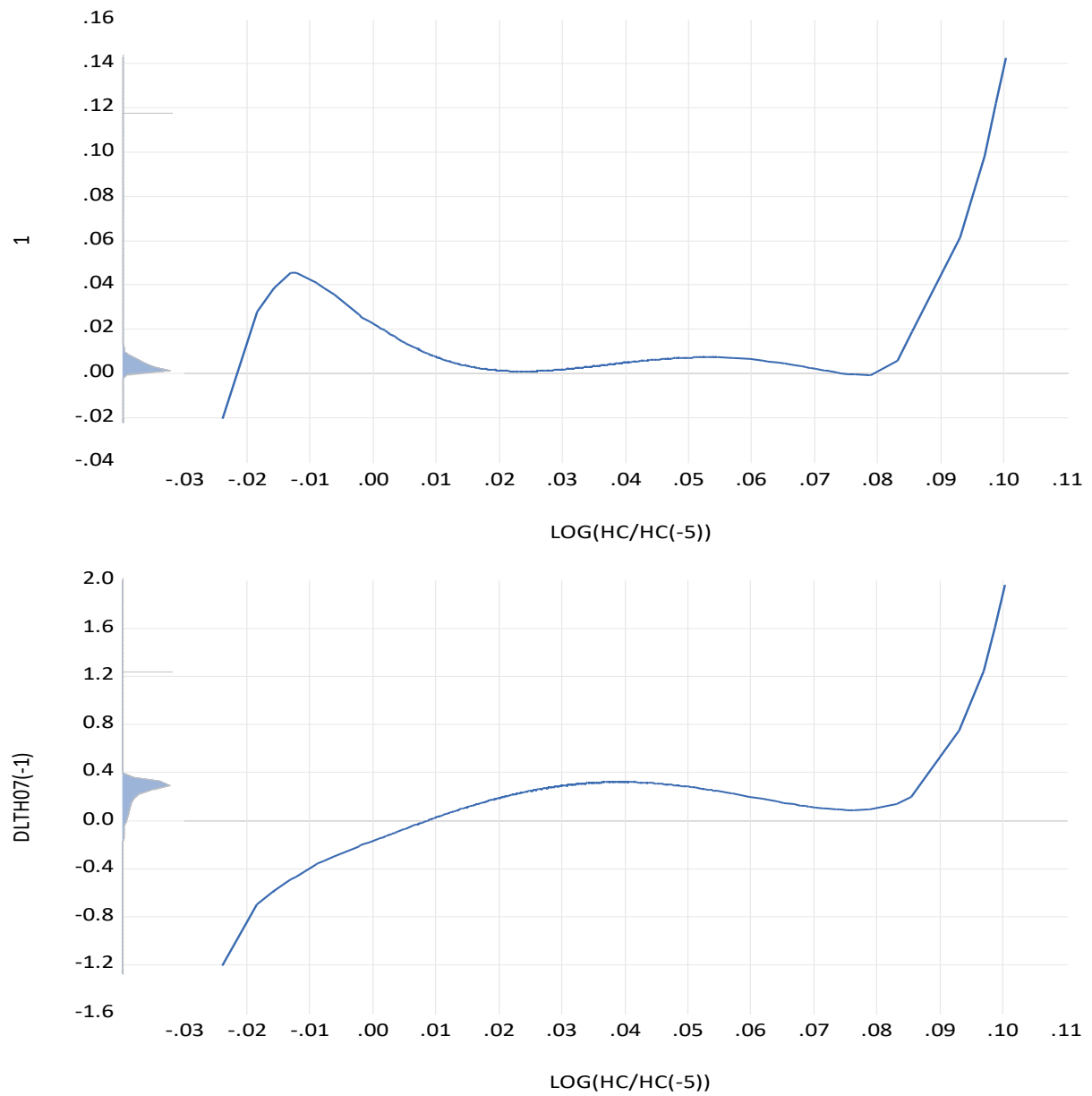
The horizontal axis of the graphs of the figures has the values of the functional coefficient series, which changes the regression coefficients. We have its contribution to the regression coefficients on the vertical axis belonging to the variable denoted there. For example, if the functional coefficient series on the horizontal axis is $\log(\text{hc}/\text{hc}(-5))$, the vertical axis has $a_1\log(\text{hc}/\text{hc}(-5))$ in the linear case and $a_1\log(\text{hc}/\text{hc}(-5)) + a_2\log(\text{hc}/\text{hc}(-5))^2$ in the quadratic case, which are the contributions of $\log(\text{hc}/\text{hc}(-5))$ to the coefficients of (1').¹⁴ Later we will add up these contributions from all functional coefficient series to get the coefficient per country-year observation.

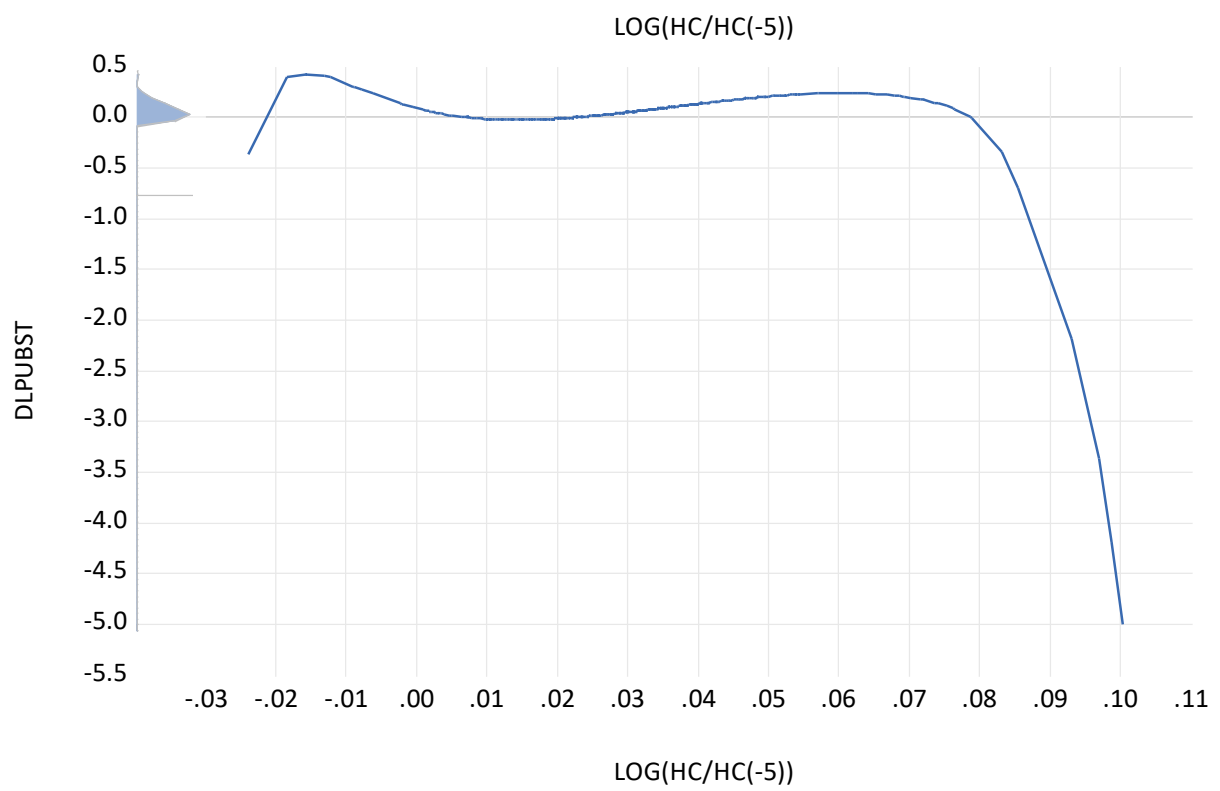
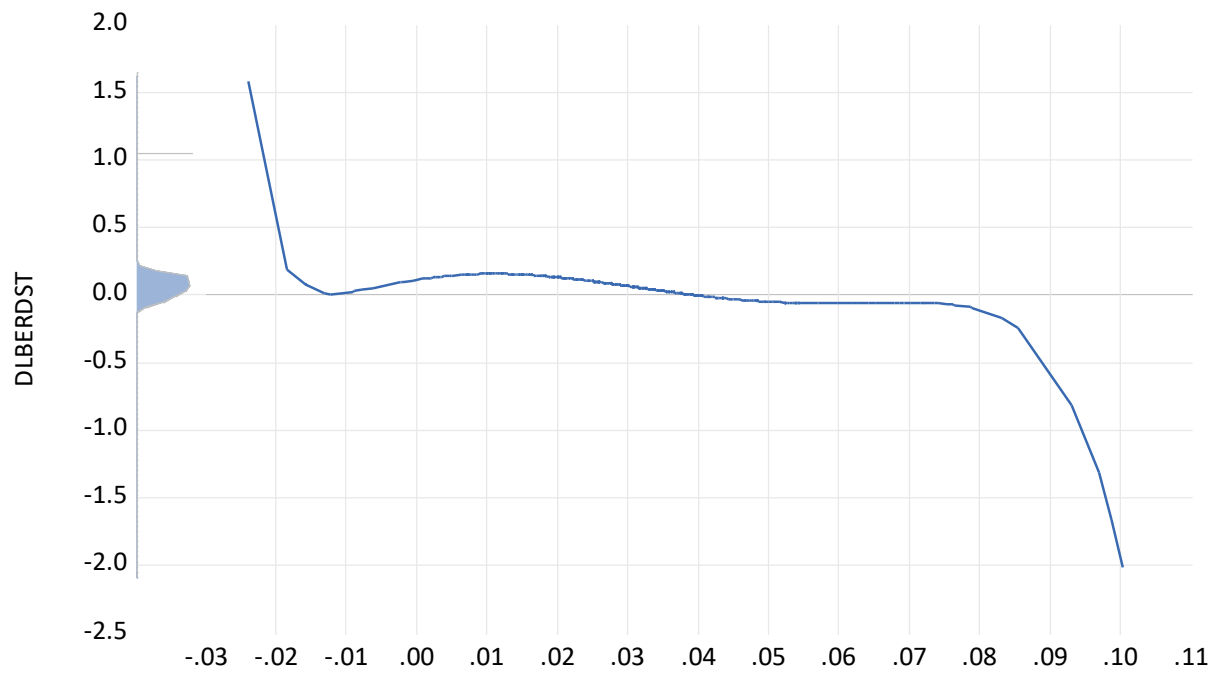
5.1.1 Partial contributions of human capital growth to the regression coefficients of the productivity function

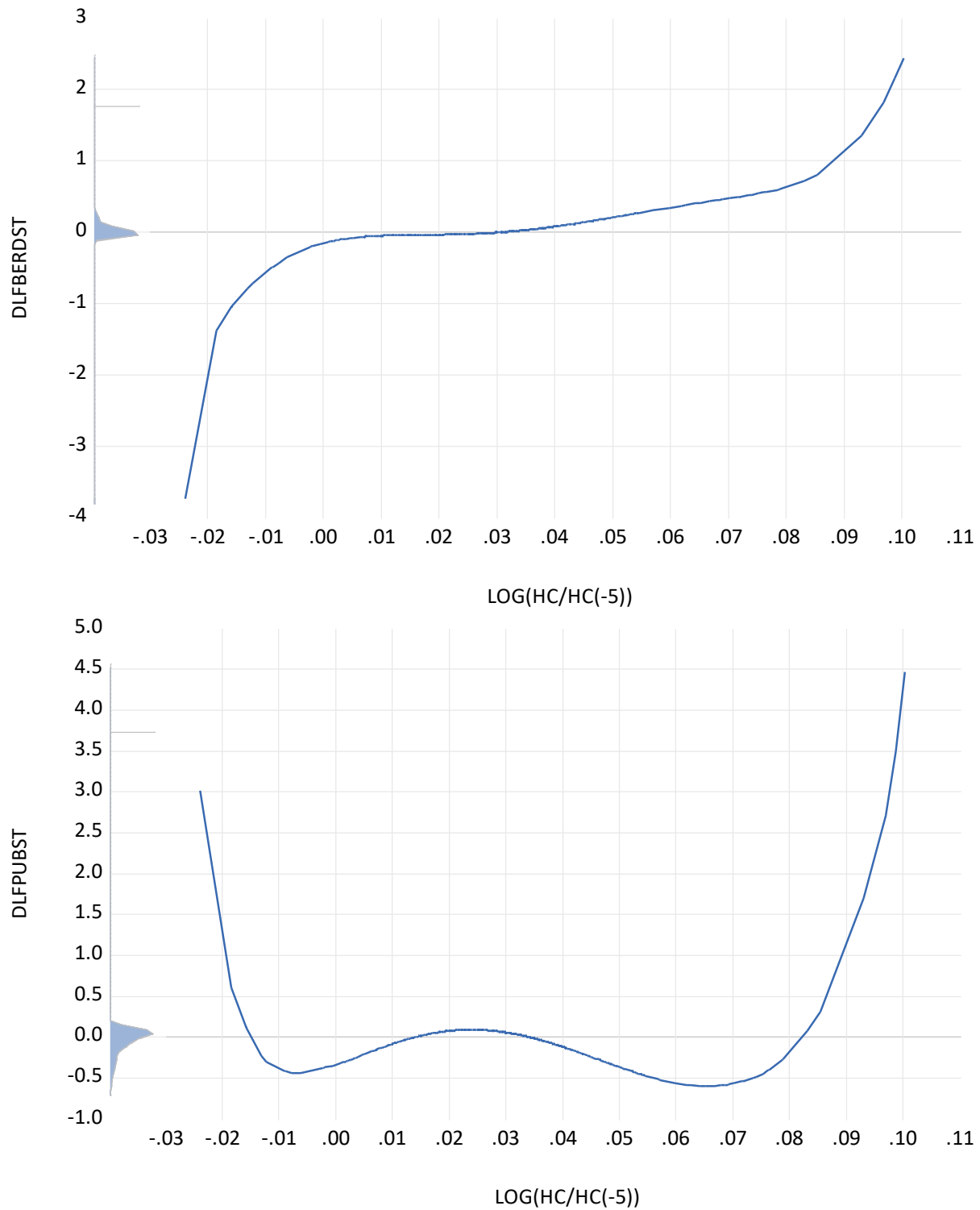
Figure 1 shows the effects of the five-year human capital growth rate, $\log(\text{hc}/\text{hc}(-5))$, on the regression coefficients. The first graph in Figure 1 shows the intercept of (1') as a relatively flat curve of human capital growth, $\log(\text{hc}/\text{hc}(-5))$, or alternatively, inverted u-shapes changing with u-shapes.

¹⁴ See Cai et al. (2000). These values can be downloaded. In the quadratic case, for example, we get one number per country-year combination for each of the regressors of (1'). It is the linear-quadratic combination of coefficients with linear and quadratic variables.

Figure 1: Contributions of five-year human capital growth to the regression coefficients of the productivity function







Notes: Dependent Variable: DLTH07 . Method: Functional Coefficients. Sample (adjusted): 1976-2019. Included observations: 1023 after adjustments. Functional coefficient series: $\text{LOG}(\text{hc}/\text{hc}(-5))$. Kernel: Epanechnikov. Local polynomial degree: 4. Bandwidth: 0.1 (automatic: CV). Localization grid: functional coefficient series.

Table 2 has some statistics dealing with the uncertainties. Columns 2 and 3 suggest that only for the lagged dependent variable there is a statistically significant impact of changing human

capital growth. The coefficient of the lagged dependent variable is decreasing with the fall in the growth rate of human capital in its range from 0.01 to 0.04. Statistical significance compared to a zero coefficient for this goes from 1.6% to 6% as reported in column 4 of Table 2. *Dlberdst* would have a statistically significant range from 1.1% to 3.3%, but here the effect of lower human capital growth increases the coefficient of BERD growth only statistically insignificantly. There are also ranges with positive confidence intervals and positive effects for *dlpubst* and *dlfberdst* indicating that coefficients differ from zero, but in both cases, the probability for constant coefficient tests is larger than 10%. Having no impact on the coefficients of foreign R&D variables may be surprising from an adoption perspective, which would suggest improved adoption with more human capital growth; it could perhaps be different for other human capital indicators. The test for constant coefficients, indicating weak impacts, is a bit more critical of human capital growth effects than the impression from confidence intervals with two standard deviations, which indicates statistically significant difference from zero of the estimated line when using polynomials of two degrees higher than the estimated line. The minimum and the maximum value of the estimation bias as measured in comparison with a polynomial two degrees higher are extremely low, which is hardly surprising because Figure 1 is an estimate using degree 4, which is very lenient. Overall, the main effect of falling human capital growth is to make the self-perpetuating effect of productivity growth weaker in the range of 1 to 4% in five years growth.

Table 2: Statistics for the contributions of human capital growth to regression coefficients (a)

<i>Variable</i>	<i>Hypothesis, statistic, p value (b)</i>			<i>Statist. sign. range of log(hc/hc(-5)) (c)</i>	<i>Range of bias (e): min, max</i>
<i>constant</i>	C(1) = c	0.43	>= .10	none	-0.0000002, 0.0000055
<i>dlth07(-1)</i>	C(2) = c	4.81	< .05	+: 0.016-0.06	-0.000114, 0.000007
<i>dlberdst</i>	C(3) = c	1.26	>= .10	- : 0.011-0.033	-0.000029, 0.000063
<i>dlpubst</i>	C(4) = c	1.32	>= .10	+: 0.036 – 0.055	-0.000001, 0.000211
<i>dlfberdst</i>	C(5) = c	0.91	>= .10	+: 0.042 – 0.061	-0.000017, 0.000025
<i>dlfpbst</i>	C(6) = c	2.20	>= .10	none	-0.00064, 0.00001

(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 4; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 1, on the same side of the x-axis as coefficient for the range expressed in column 4 and the regression line in the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 6. Pilot stage bandwidth: 0.1611 (automatic: CV). Type: +- 2SE Bands. Bias: Included. (d) Functional Coefficient Bias Curves. Pilot stage local polynomial degree: 6. Pilot stage bandwidth: 0.1611 (automatic: CV).

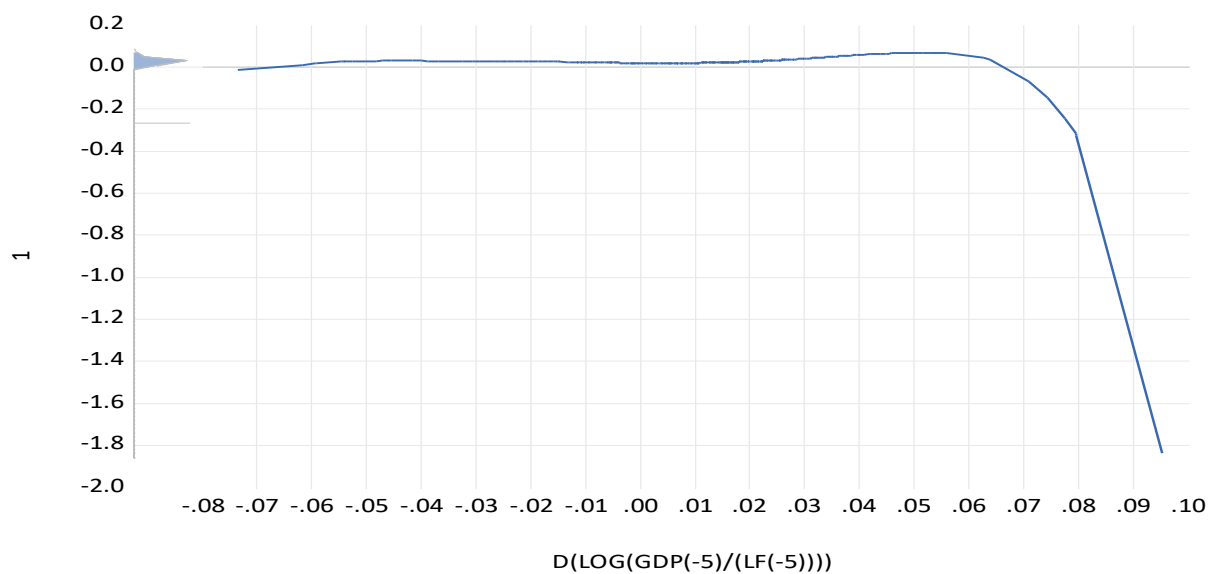
5.1.2 *Partial effects of past GDP per worker growth rates on the regression coefficients of the productivity function*

The analysis of Coccia's ideas is beleaguered by the problem of unit roots in GDP per worker variables. Therefore, we look at the five-year-lagged growth rate, which is falling and will lead to a lower level of GDP per worker later. This makes the analysis of Coccia's ideas a loose one.¹⁵

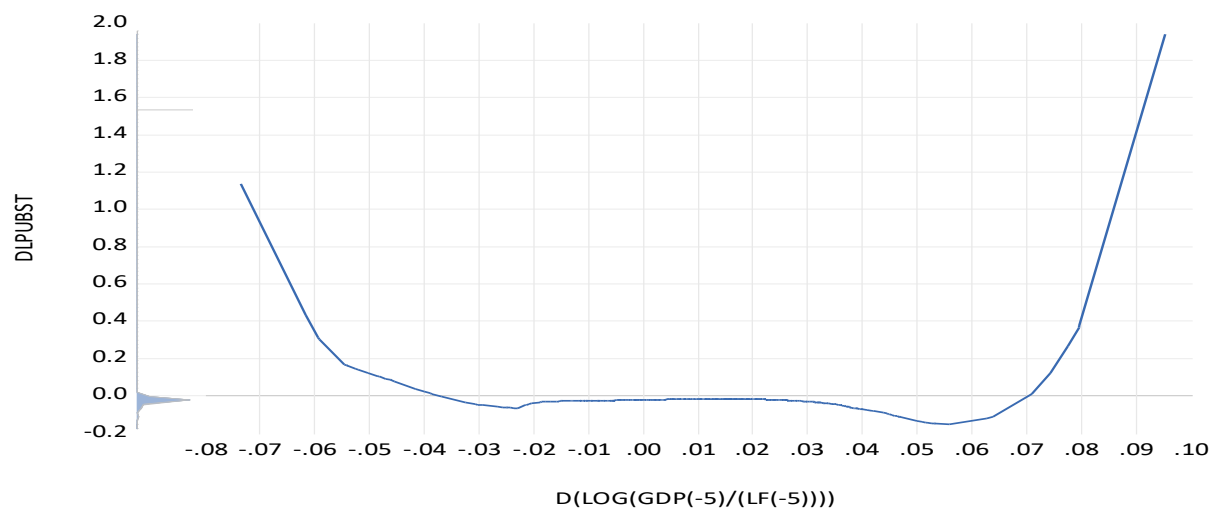
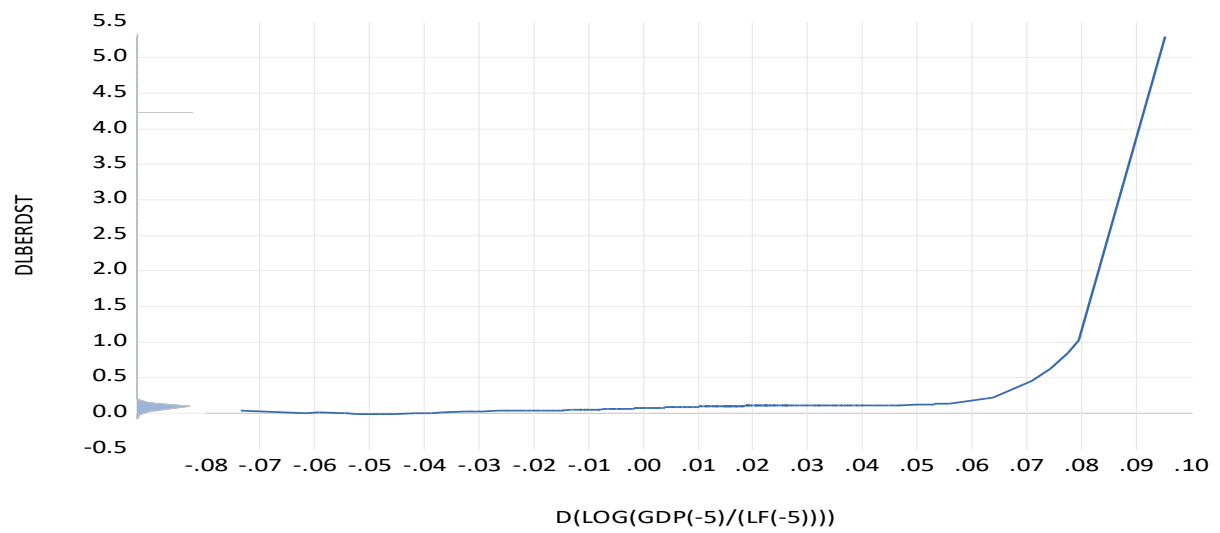
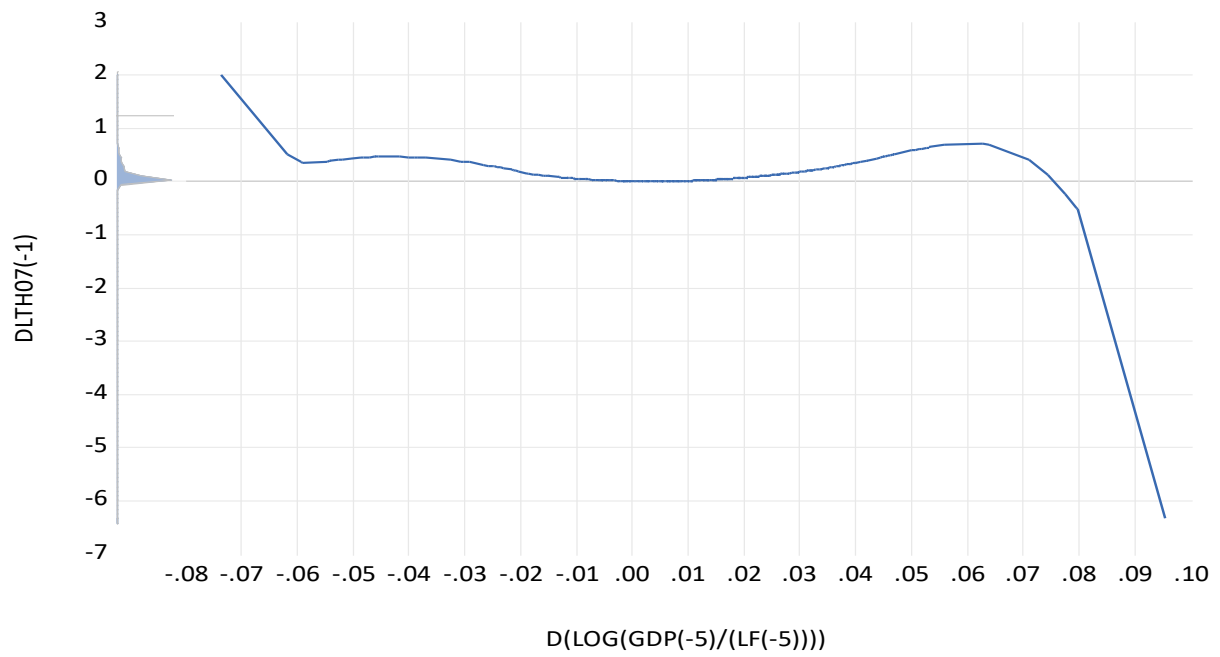
Figure 2 shows the contributions of the falling GDP per worker growth rate as of five years ago to the regression coefficients. The first graph in Figure 2 shows that the intercept is driven down by falling values of growth rates between 2 and 5.5%, thereby leading to lower exogenous technical change. For the lagged dependent variable, we see a negative impact of the falling growth rate between 1 and 6% GDP per worker growth. For falling BERD stock growth we see a negative effect on the coefficient for growth rates from -3% to +9.5%. Table 3 tells us that only the coefficient of public R&D does not change significantly, and its coefficient is therefore approximately constant. With elasticities of productivity for BERD stock (growth) falling and for public R&D constant, there is less of an incentive to have increasingly higher BERD stocks (at least relatively) in line with Coccia's idea, based here on higher past growth rates instead of lower current levels.

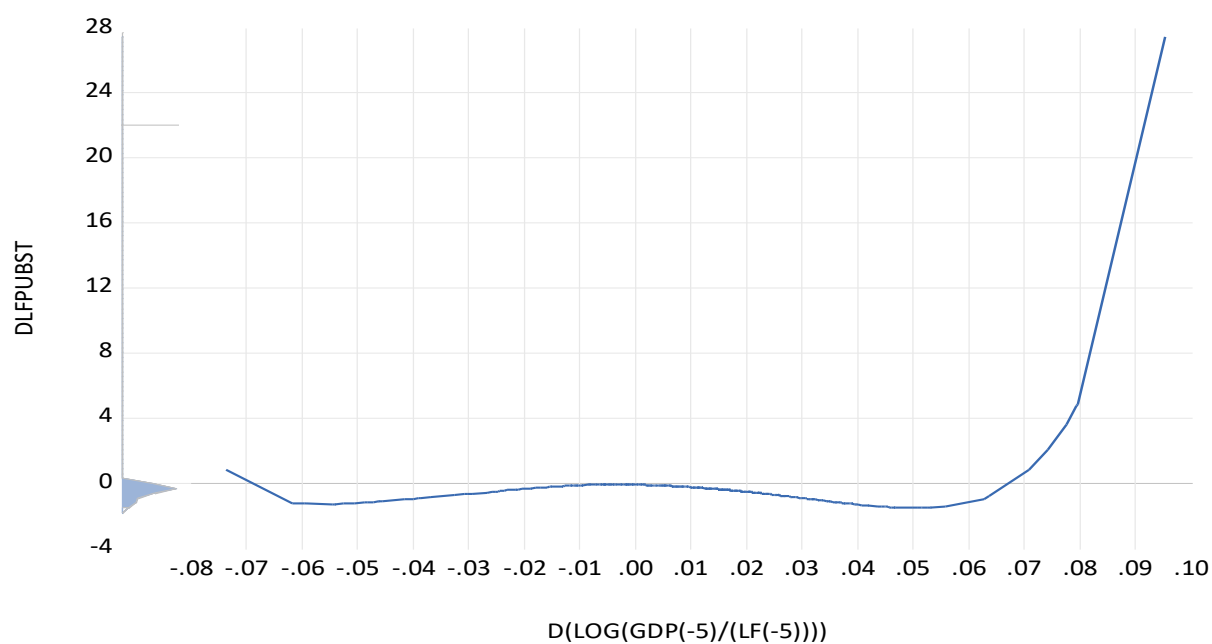
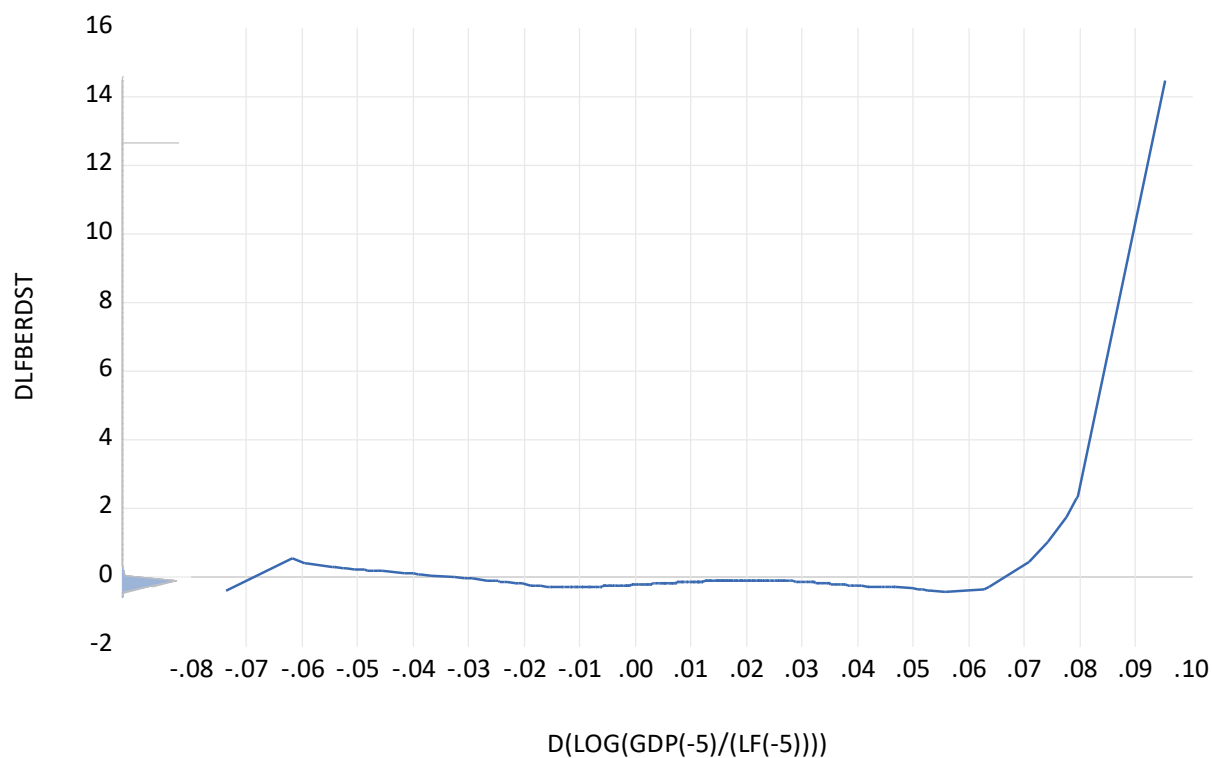
Confidence intervals in Table 3, column 4, show an overlap in the range of growth rates with the effects of Figure 2. Biases in the last column from comparison with two polynomial degrees more are small according to Table 3, last column. Finally, using degree 4 for Figure 2 is possible with remarkably comparable results also for Table 3, column 3, but the extremely large values at the right end of the figure make results even less well visible.

Figure 2: Contributions of past growth rates GDP per worker to the regression coefficients of the productivity function



¹⁵ We also have explored the effect of the average growth rates of the last four years, but this has an effect only on the indirect effect shown below.





Notes: Dependent Variable: DLTH07. Method: Functional Coefficients. Sample (adjusted): 1996 – 2019. Included observations: 504 after adjustments. Functional coefficient series: $D(\text{LOG}(\text{GDP}(-5)/(\text{LF}(-5))))$. Kernel: Epanechnikov. Local polynomial degree: 3. Bandwidth: 0.1000 (automatic: CV). Localization grid: functional coefficient series.

Table 3: Statistics for the contributions of five-year lagged GDP per worker growth rates to regression coefficients (a)

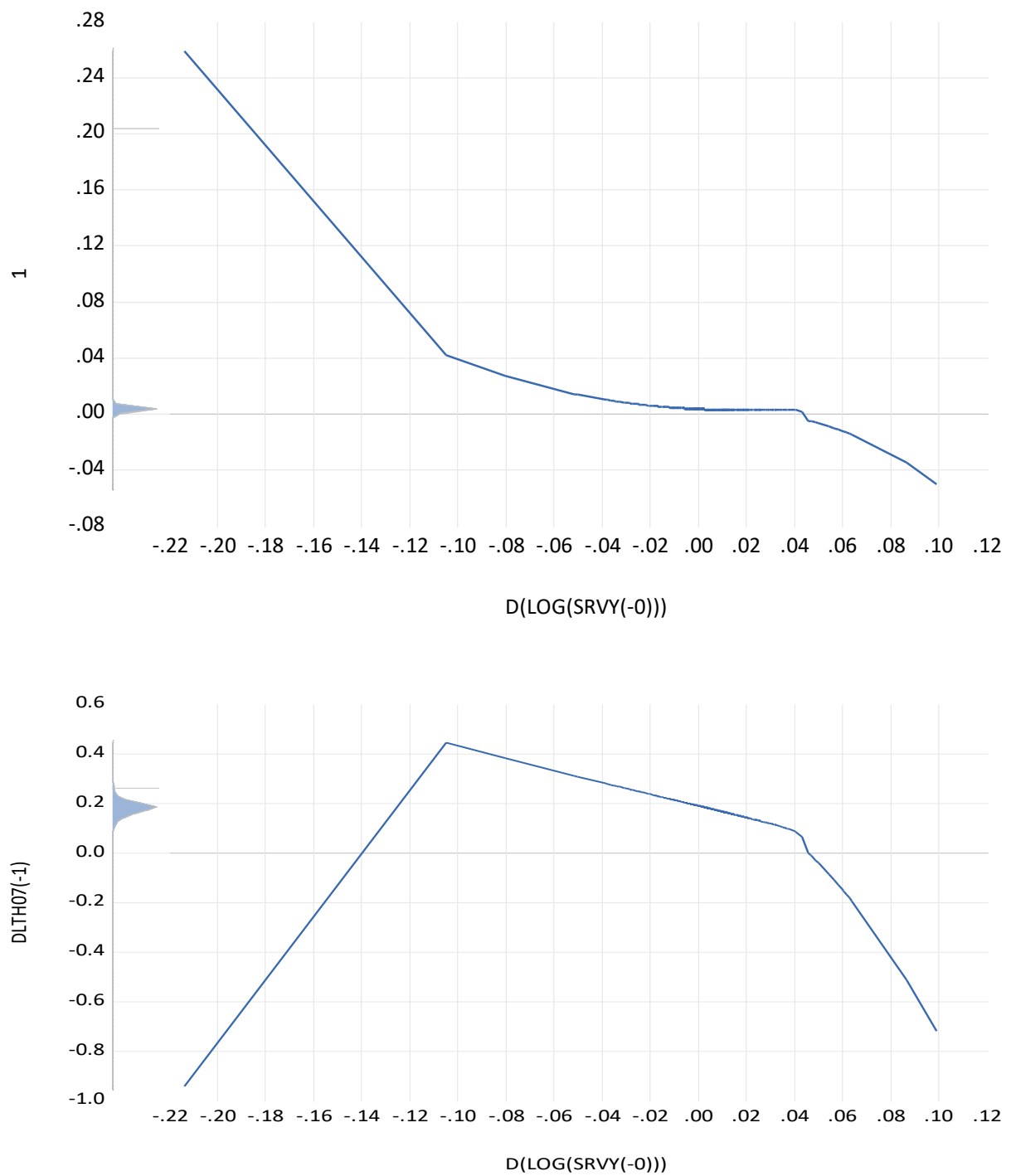
<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. range of $d(\log(gdp(-5)/lf(-5)))$ (c)</i>	<i>Range of bias, min, max (d)</i>
<i>constant</i>	$C(1) = c$	3.96	< .05	+: 0.012 - 0.042; -: 0.056 < g	0, 0.005
<i>dlth07(-1)</i>	$C(2) = c$	6.70	< .01	+: 0.028 - 0.064; -: 0.071 < g	-0.00091, -0.00229
<i>dlberdst</i>	$C(3) = c$	6.15	< .01	+: -0.003 – 0.024; +: .051 – .095	-0.00146, 0.00017
<i>dlpubst</i>	$C(4) = c$	2.34	$\geq .10$	+: 0.08 – 0.095	-0.00052, 0.00001
<i>dlfberdst</i>	$C(5) = c$	5.21	< .05	+: 0.05 – 0.095	-0.0041, 0.0049
<i>dlfpbst</i>	$C(6) = c$	6.45	< .01	-: 0.018 -0.05; +: .063-.095	-0.0071, 0.0008

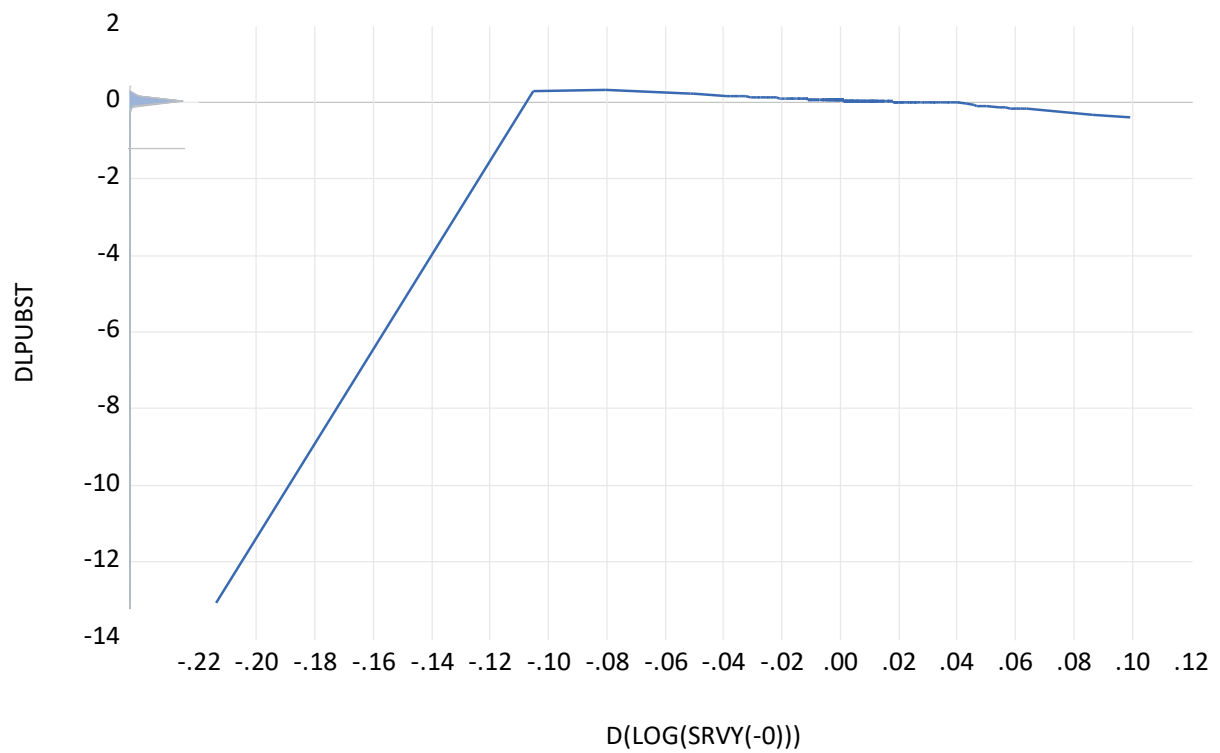
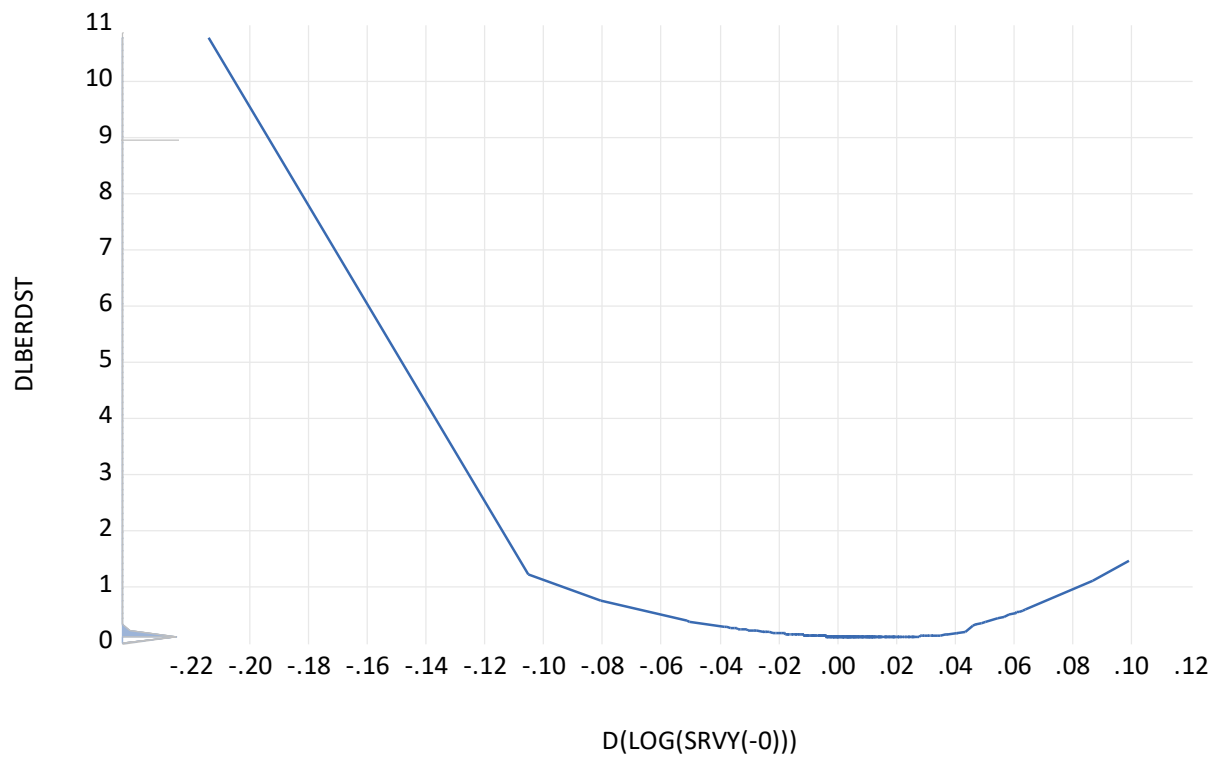
(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 3 with critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 2, on the same side of the x-axis as regression coefficient for the range expressed in column 4; the regression line within the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 1.0835 (automatic: CV). Type: +- 2SE Bands. Bias: Included. (d) From functional coefficient bias curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 1.0835 (automatic: CV).

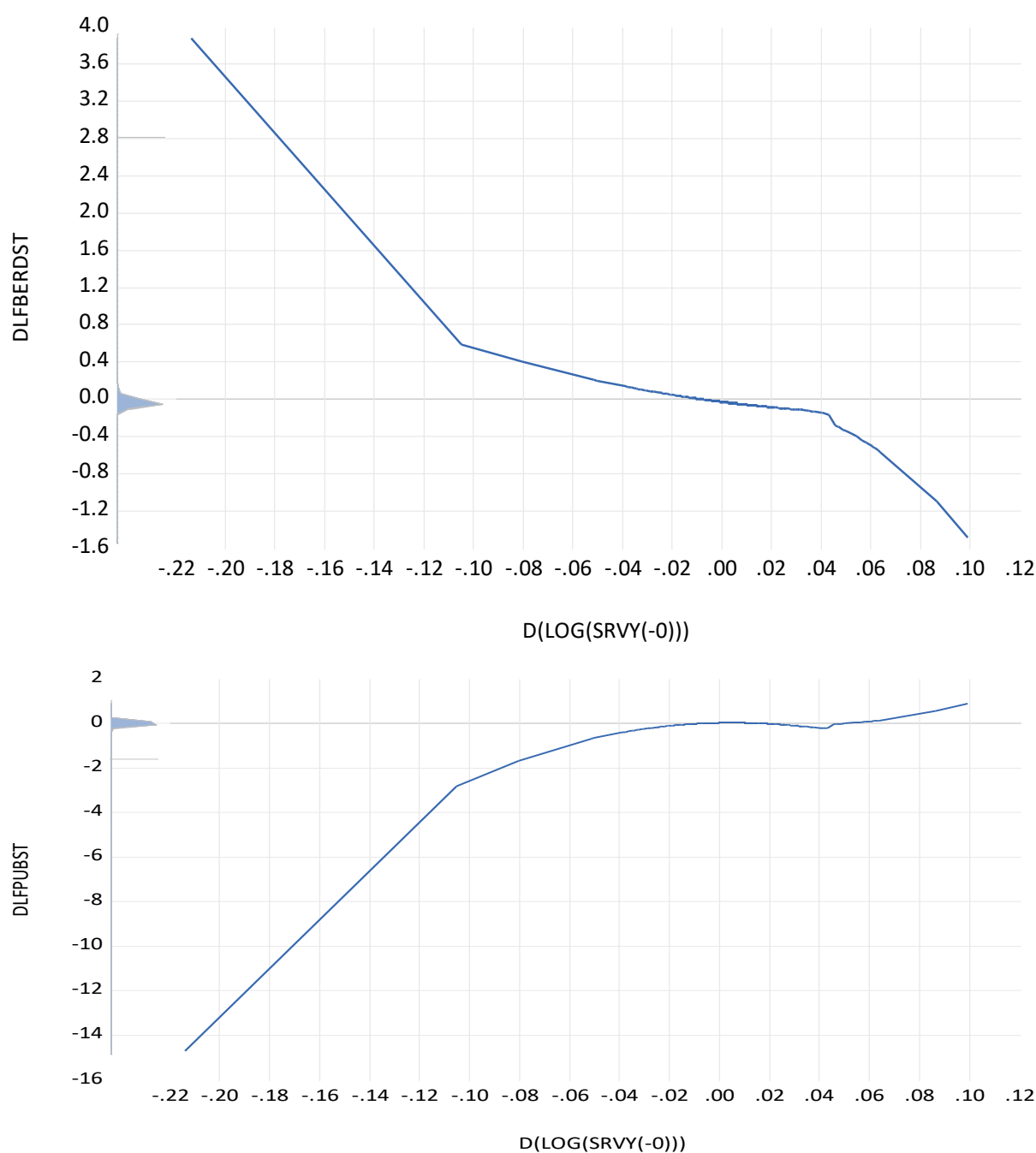
5.1.3 Partial effects of services (% GDP) growth on the regression coefficients of the productivity function

Figure 3 shows that with less service (% GDP) growth exogenous technical change as represented by the intercept is increasing. In the relevant range of growth rates larger than -10 percent, the self-perpetuating character of technical change as represented by the coefficient of the lagged dependent variable has an increasing coefficient with lower service growth. The coefficient of BERD growth looks like a u-shape in Figure 3 but it is constant according to Table 4. The coefficient of public R&D growth is increasing with less services growth, which is confirmed in the sense of rejection of the hypothesis of a constant coefficient in Table 4. Foreign private R&D has an increasing coefficient, and foreign public R&D has a decreasing one with falling service growth, both statistically significant according to Table 4.

Figure 3: Contributions of services (% GDP) growth to the regression coefficients of the productivity function







Notes. Dependent Variable: DLTH07. Method: Functional Coefficients. Sample (adjusted): 1991-2019. Included observations: 700 after adjustments. Functional coefficient series: $D(\text{LOG}(\text{SRVY}))$. Kernel: Epanechnikov. Local polynomial degree: 2. Bandwidth: 0.2594 (automatic: CV). Localization grid: functional coefficient series.

In Table 4, we show more test results. Only for BERD stock growth does the test suggest a constant coefficient. Confidence intervals are hard to read and do often not show clear information when all lines are visually close to the horizontal axis; this may lead to slight errors in Table 4. The increase in the coefficient for public R&D with falling service growth is clear though, because the first part of the confidence interval is in the positive area and the

second part is in the negative area. Here is a partial incentive against the shift from public to private R&D.

Table 4: Statistics for the contributions of service (% GDP) growth to regression coefficients (a)

<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. range of service (% GDP) growth $\equiv g$ (c)</i>	<i>Range of bias min, max (d)</i>
<i>constant</i>	C(1) = c	55.8	< .01	+: -0.214 - (-0.08); +: 0.058-0.099	-.0013, 0.0001
<i>dlth07(-1)</i>	C(2) = c	4.73	< .05	-: g < 0.051; +: 0.087 < g	-0.006, 0.004
<i>dlberdst</i>	C(3) = c	2.93	>= .10	-: -0.105; +: -0.039- (-0.023); +: 0.026-0.064	0.001, 0.0023
<i>dlpubst</i>	C(4) = c	6.65	< .01	+: -0.21 - (-0.033); -: .049-.099	-0.002, 0.001
<i>dlfberdst</i>	C(5) = c	4.84	< .05	-: -0.214-(-0.08); -: 0.043-0.099	0.002, 0.006
<i>dlfpbst</i>	C(6) = c	6.84	< .01	-: -0.21-(-0.08);	0.00, 0.01

(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 2; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 3, on the same side of the x-axis as regression coefficient for the range expressed in column 4; the regression line within the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 4. Pilot stage bandwidth: 0.2853 (automatic: CV). Type: +- 2SE Bands. Bias: Included. (d) From functional coefficient bias curves. Pilot stage local polynomial degree: 4. Pilot stage bandwidth: 0.2853 (automatic: CV). Table info for growth rates algebraically larger than -0.08 and smaller than 0.08.

In the data, the share of services goes from positive to zero growth rates in a panel average (not shown). The four negative slopes of Figure 3 would then imply that the development goes to larger coefficients. If these effects dominate, the out phasing of the trend to services implies higher productivity growth. Although other factors may be involved, a dynamic fixed effects regression may be helpful although it is very preliminary (t-values in parentheses)¹⁶:

$$DLTH07 = 0.009 + 0.147DLTH07(-1) - 0.366D(LOG(SRVY)) + 0.088D(LOG(SRVY(-1)))$$

(10.14) (2.75) (-5.78) (1.925)

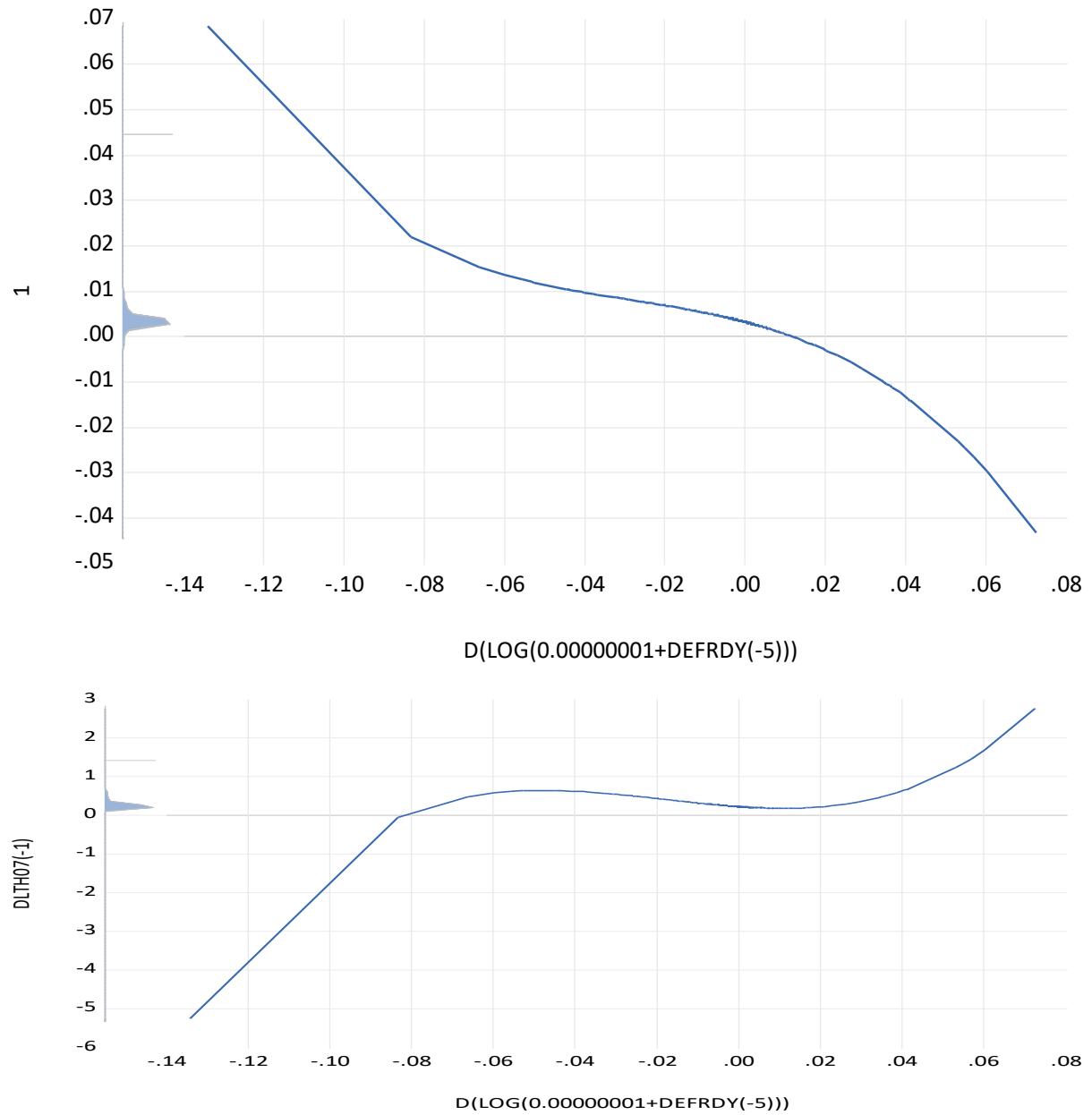
¹⁶ Method: Panel Least Squares. Sample (adjusted): 1968 2019. Periods included: 52. Cross-sections included: 21. Total panel (unbalanced) observations: 680. Period SUR (PCSE) standard errors & covariance (d.f. corrected). Cross-section fixed (dummy variables). Period fixed (dummy variables). The corresponding p-values are 1.989e-22; 0.006; 1.169e-08; 0.05469054312644774. Adj. R-sq.: 0.36. Durbin-Watson stat: 2.029. Adj R-sq is lower when cross-section GLS weights are used instead of period fixed effects. When service shares are constant, the long run rate of technical change is obtained as 0.009/(1-0.147) = 0.011.

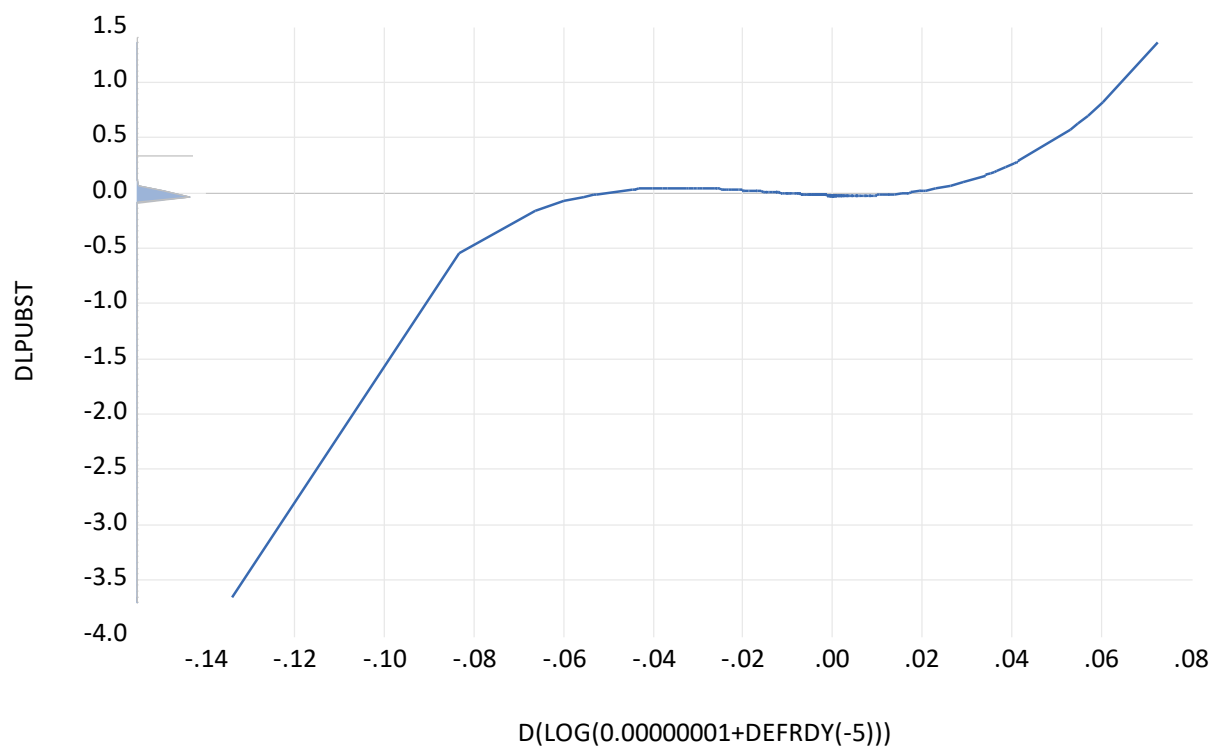
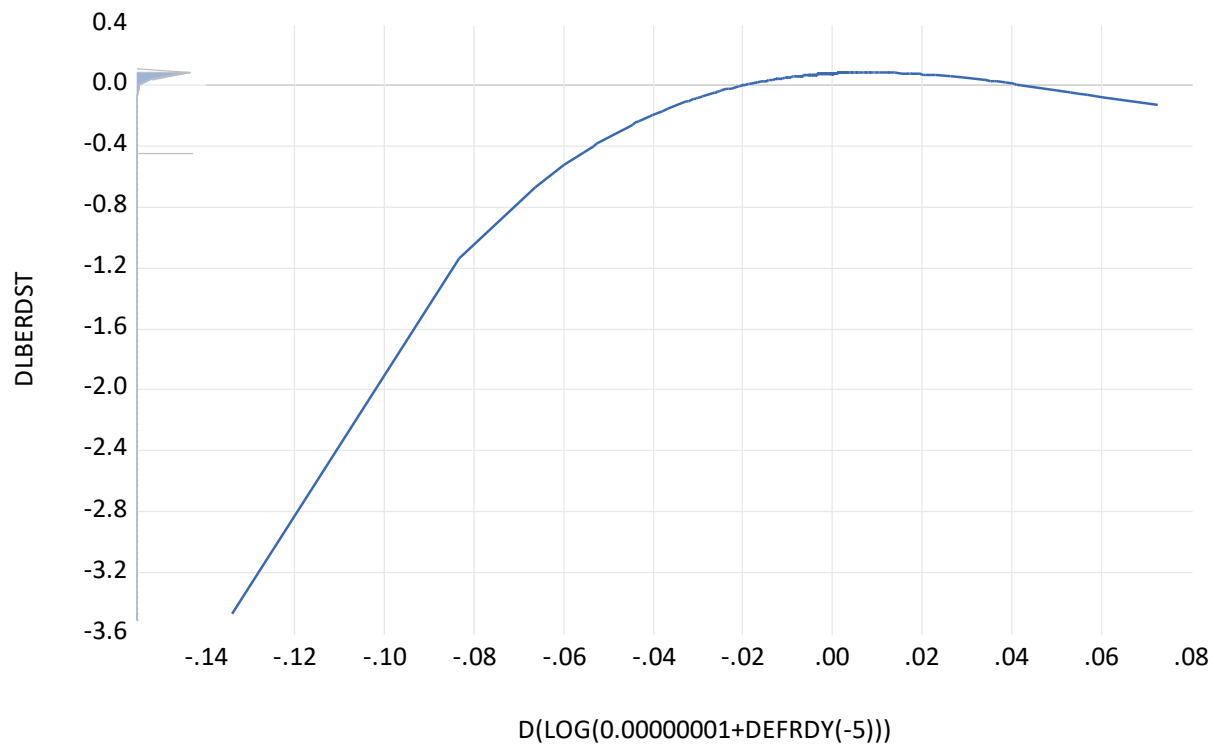
The regression suggests that the growth rate of the share of services going from positive to zero increases the rate of technical change. In spirit, this result is in line with the finding of Boppart and Weiss (2013) that a higher share of industry leads to a higher rate of technical change, ignoring the changing share of the primary sector. Here it suggests that the four falling coefficients in Figure 3 are dominating.

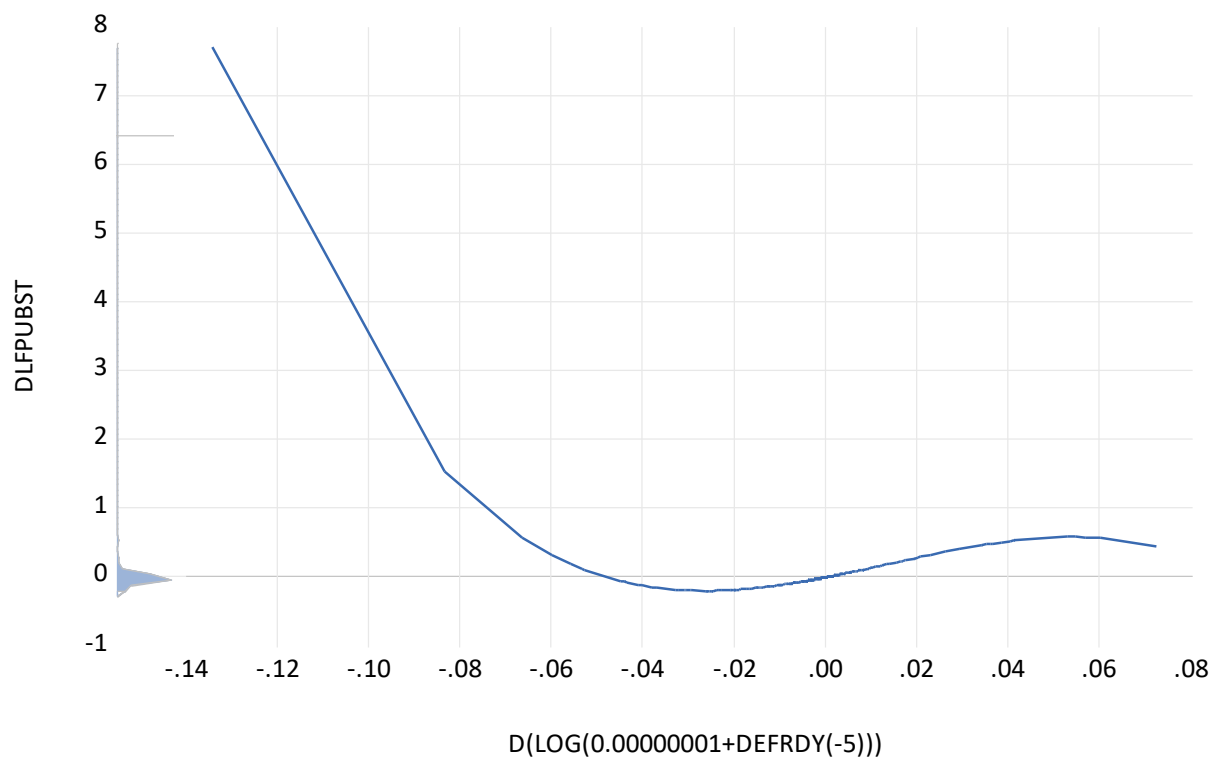
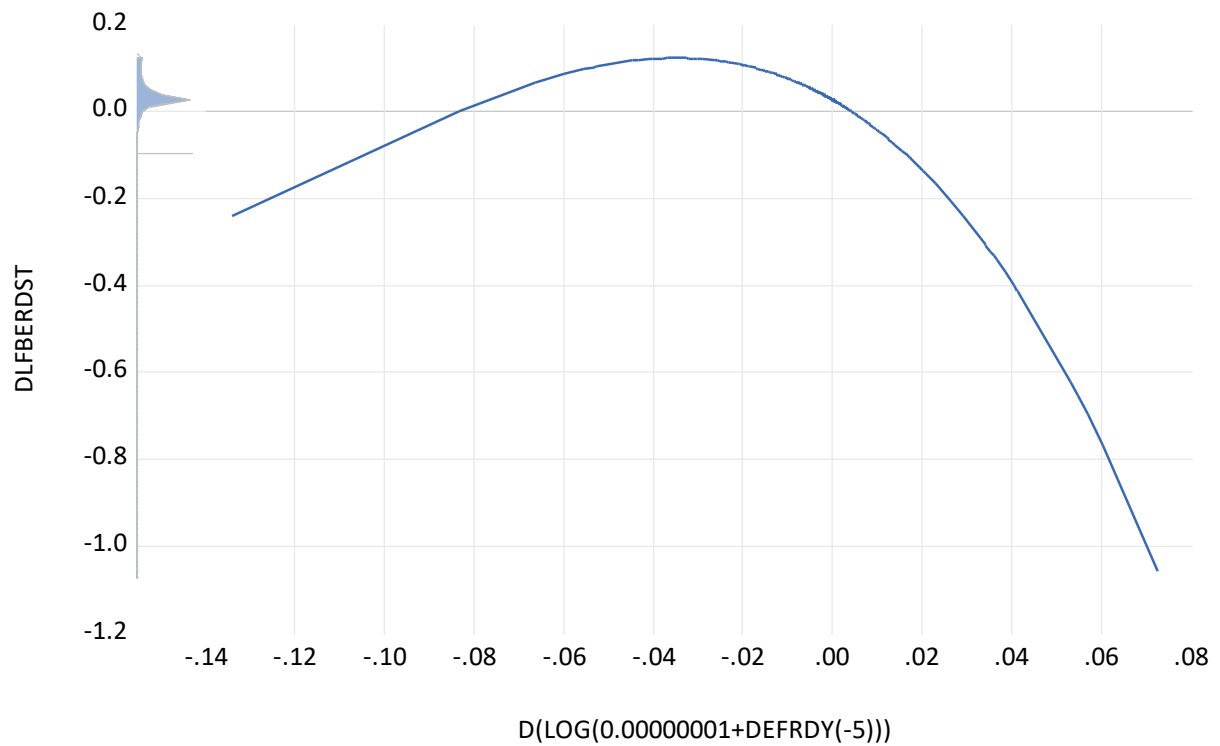
5.1.4 Partial effects of defence R&D (% GDP) growth on the regression coefficients of the productivity function

For defence R&D growth we get an impact on the regression coefficients with lag five. At all other lags, we get a probability larger than ten percent for constant coefficients. Polynomials with degree 5 have constant coefficients with $p > 0.1$, but it is not smooth; degree four does not have constant coefficients (see Table 5), but the corresponding figure has so high values on the vertical axes that the curvature is invisible. In Figure 4 we show results for polynomial degree 3. As the growth rate is constant over time, the interpretations hold in the cross-section dimension. Countries with higher (lower) growth in defence R&D have lower (higher) exogenous technical change. The coefficient of the lagged dependent variable first goes to a local maximum and then to a local minimum and up again; near the mean and median of negative zero, countries with higher defence R&D growth have less self-perpetuating technical change effects. The values of the contribution to the coefficients are positive when growth rates are larger than minus eight percent. The coefficient of private R&D growth peaks at around one percent growth of defence R&D; near the mean and median of negative zero, countries with higher defence R&D growth have stronger effects of private R&D. The coefficient of public R&D goes first to a maximum at -0.03 and then to a minimum around 0.01 of defence R&D growth, and from there further upward; near the median, countries with larger defence R&D growth have lower effects of public R&D. The coefficient of foreign BERD stock growth has a peak around -0.03, where the foreign public R&D coefficient has a minimum; near zero growth of defence R&D higher values lead to lower coefficients of foreign private R&D and to higher coefficients of foreign public R&D. Table 5 confirms that the coefficients are not constant, but the confidence intervals suggest that they are not significantly different from zero around mean and median with the exception of the lagged dependent variable.

Figure 4: Contributions of defence R&D (% GDP) growth to the regression coefficients of the productivity function without controls from functional coefficient series







Notes. Dependent Variable: DLTH07. Method: Functional Coefficients. Sample (adjusted): 1987 – 2019. Included observations: 616 after adjustments. Functional coefficient series: $D(\text{LOG}(0.00000001 + \text{DEFRDY}(-5)))$. Kernel: Epanechnikov. Local polynomial degree: 3. Bandwidth: 1.0835 (automatic: CV). Localization grid: functional coefficient series.

Table 5: Statistics for the contributions of defence R&D (% GDP) growth to regression coefficients from functional coefficient series (a)

<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. range of defence R&D (% GDP) growth rate g (c)</i>	<i>Range of bias near zero (d)</i>
<i>constant</i>	$C(1) = c$	5.75	< .01	$-\therefore g < -0.066; +: 0.043 < g$;	$-0.07 < g < 0.05$
<i>dlth07(-1)</i>	$C(2) = c$	30.69	< .01	$-\therefore g < -0.066; +: -0.039 < g < 0.004; +: 0.019 < g < 0.039; -: 0.056 < g$	$-0.06 < g < 0.05$
<i>dlberdst</i>	$C(3) = c$	26.29	< .01	$-\therefore g < -0.066;$	$-0.06 < g < 0.06$
<i>dlpubst</i>	$C(4) = c$	6.182	< .01	$-\therefore g < -0.083; +: \text{at } -0.066; -: 0.056 < g$	$0.06 < g < 0.05$
<i>dlfberdst</i>	$C(5) = c$	12.83	< .01	$+: g < -0.083; -: -0.066 < g < -0.06;$	$-0.06 < g < 0.05$
<i>dlfpubst</i>	$C(6) = c$	52.11	< .01	$-\therefore g < -0.066; -: 0.018 - 0.023$	$-0.06 < g < 0.05$

(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 3; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 4, on the same side of the x-axis as coefficient for the range expressed in column 4, and the regression line in the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 0.1210 (automatic: CV). Type: SE Band. Additional levels: 2. Bias: Included. (d) From functional coefficient bias curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 0.1210 (automatic: CV).

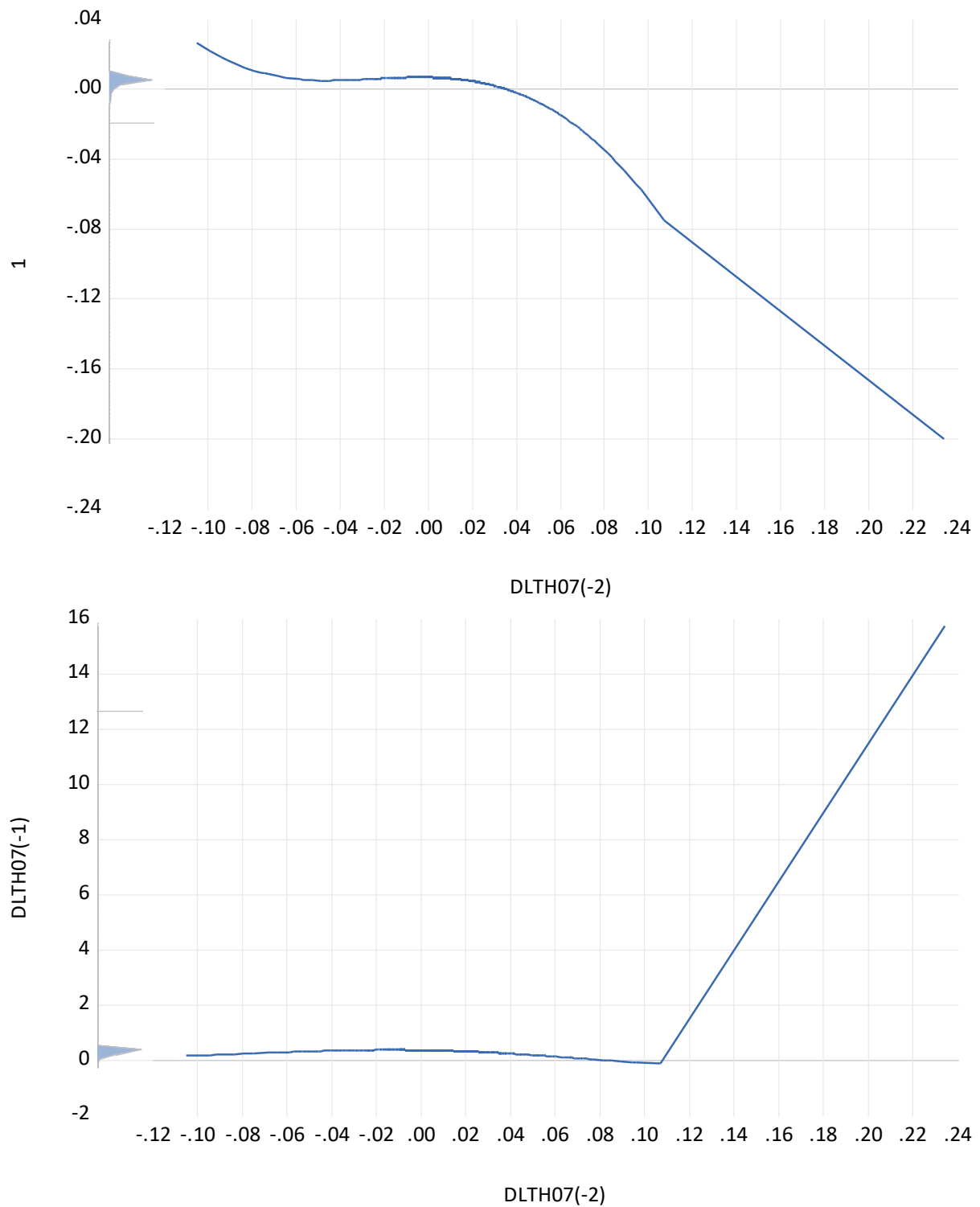
5.1.5 Partial effects of the twice lagged labour-productivity growth, *dlth07(-2)*, on the regression coefficients of the productivity function

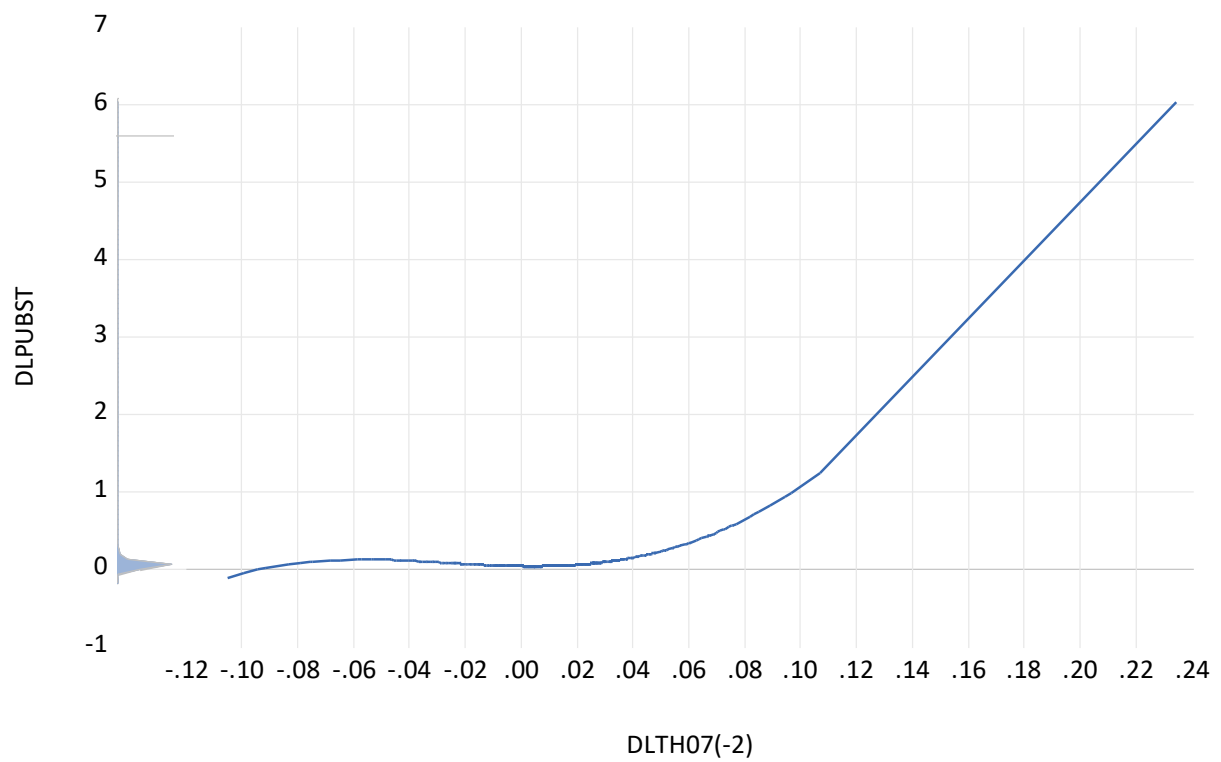
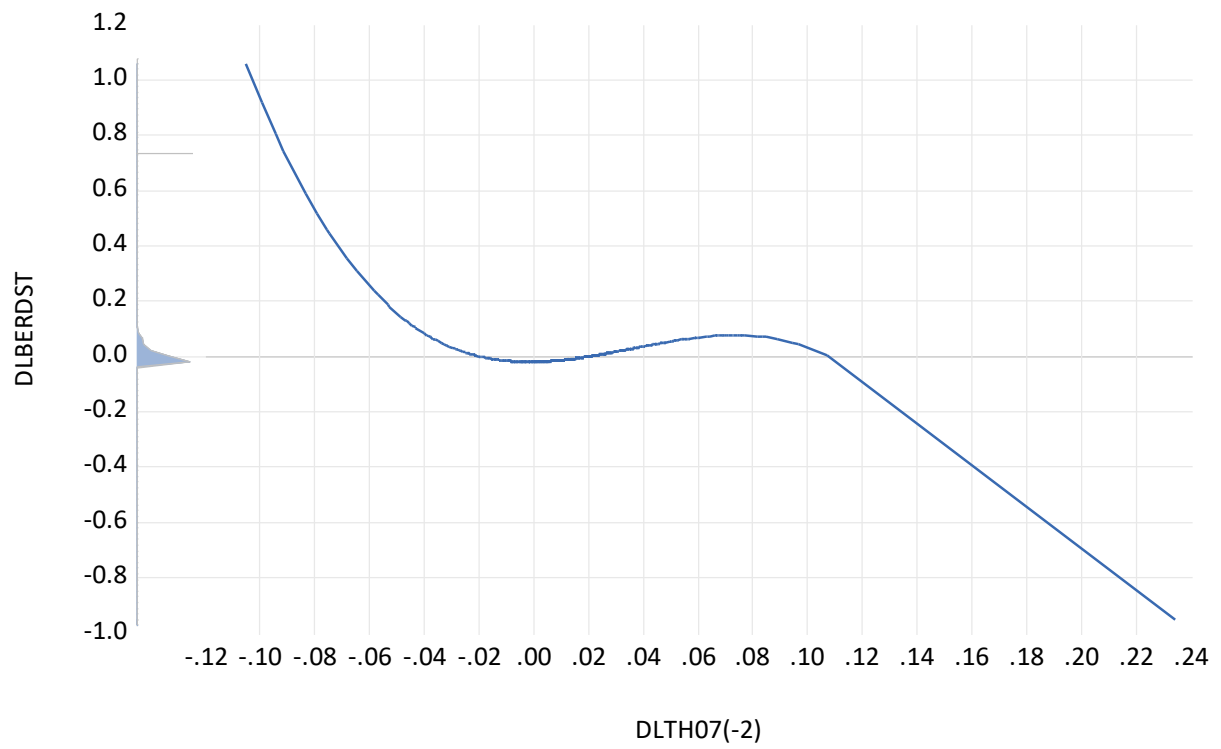
Figure 5 shows that decreasing rates of past labour-productivity growth slightly increase the positive rate of exogenous technical change, the intercept of (1'). The regression coefficient of the lagged dependent variable has an inverted u-shape through a twice lagged value from (-0.1) to 0.1; the curve is flat near mean and median of 1.1 percent. BERD stock growth has an increasing coefficient from zero to seven percent growth rate of *lth07(-2)*, and a falling one before and after this interval. Public R&D has increasing coefficients from one to 22% of *dlth07(-2)*, and an inverted u-shape before this interval. Correspondingly, a growth slowdown reduces both coefficients in the neighborhood of mean and median of one percent for *dlth07(-2)*. The coefficient of foreign BERD stock is increasing throughout (falling with falling growth rates), with a flat part between -0.06 to zero percent growth of *th07(-2)*. Foreign public R&D has coefficients of -0.0 until *dlth07(-2) = 0.09*; then it starts falling quickly. These results hold for a polynomial of degree 3.

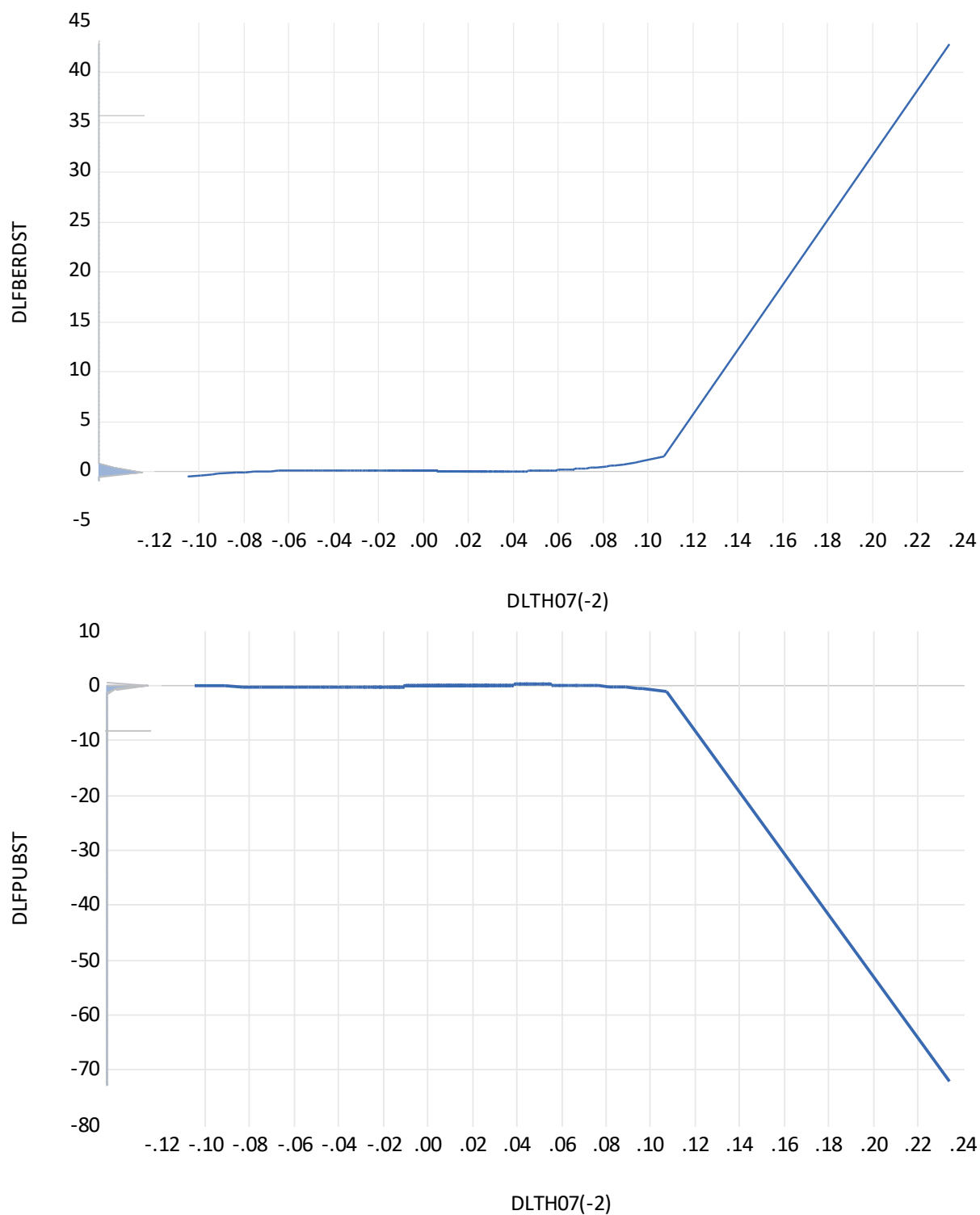
Table 6 shows that the probability for constant coefficients is below five or one percent. There are many parts of the curve where the confidence interval is on one side of the axis indicating statistical significance, here taken for polynomial of degree 4 for estimates of

degree 2, because degree 5 results are not visible because of huge values on the vertical axis. The bias in comparison with degree 5 is below (-) 0.001 over almost the entire range for $dlth07(-2)$ from minus to plus ten percent.

Figure 5: Contributions of twice lagged labour-productivity growth to the regression coefficients of the productivity function







Notes. Dependent Variable: $DLTH07$. Method: Functional Coefficients. Sample (adjusted): 1976 – 2019. Included observations: 1005 after adjustments. Functional coefficient series: $DLTH07(-2)$. Kernel: Epanechnikov. Local polynomial degree: 3. Bandwidth: 0.2853 (automatic: CV). Localization grid: functional coefficient series.

Table 6: Statistics for the contributions of twice lagged labour-productivity growth to regression coefficients (a)

<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. range of $dlth07(-2) \equiv g$ (c)</i>	<i>Range of bias ≤ 0.001 (d)</i>
<i>constant</i>	$C(1) = c$	4.236	< .05	$-: g < -0.094; +: -0.068 < g < -0.019; g < 0.042$ unclear	-.105 – 0.107
<i>dlth07(-1)</i>	$C(2) = c$	73.15	< .01	$+: g < -0.099; -: -0.065 < g < -0.053; +: -0.03 < g < 0.047; -: 0.064 < g < 0.085; +: 0.097 < g$	-.105 – 0.107
<i>dlberdst</i>	$C(3) = c$	19.20	< .01	$-: g < -0.091;$	-.105 – 0.107
<i>dlpubst</i>	$C(4) = c$	57.67	< .01	$-: g < -0.094; +: -0.068 - (-0.025; +: 0.031 < g$	-.105 – 0.107
<i>dlfberdst</i>	$C(5) = c$	57.82	< .01	$-: g < -0.068; +: -0.035 < g < -0.008; +: 0.089 < g$	-.105 – 0.107
<i>dlfpubst</i>	$C(6) = c$	12.52	< .01	$+: g < -0.083; -: -0.057 < g < -0.007; -: 0.077 < g$	-0.079– 0.107

(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 3; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 5, on the same side of the x-axis as coefficient for the range expressed in column 4, and the regression line in the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 4 (degree 5 hardly visible). Pilot stage bandwidth: 0.1772 (automatic: CV). Type: SE Band. Additional levels: 2. Bias: Included. (d) Functional Coefficient Bias Curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 0.2853 (automatic: CV).

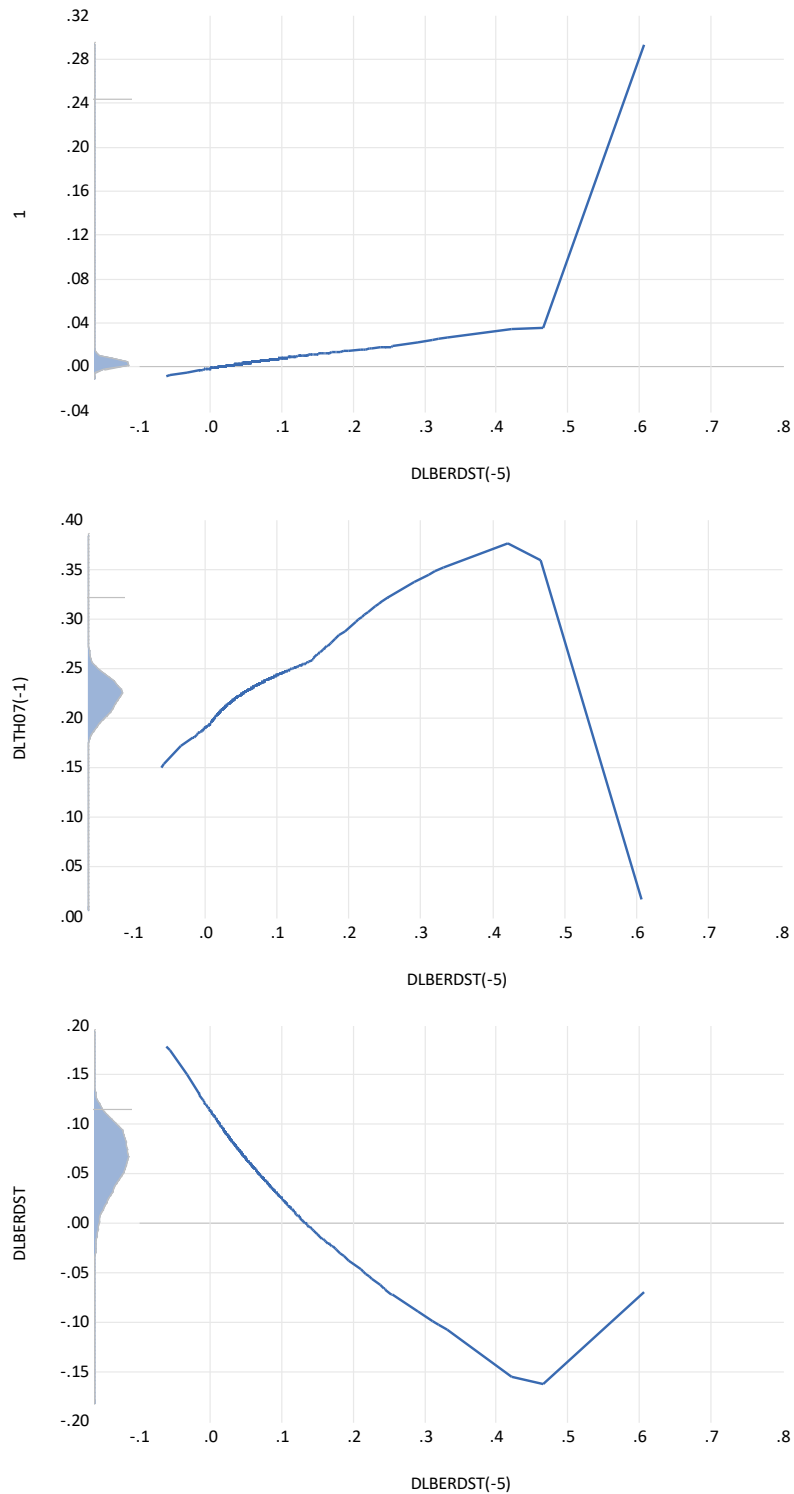
5.1.6 Partial effects of five-year lagged BERD stock growth on the regression coefficients of the productivity function

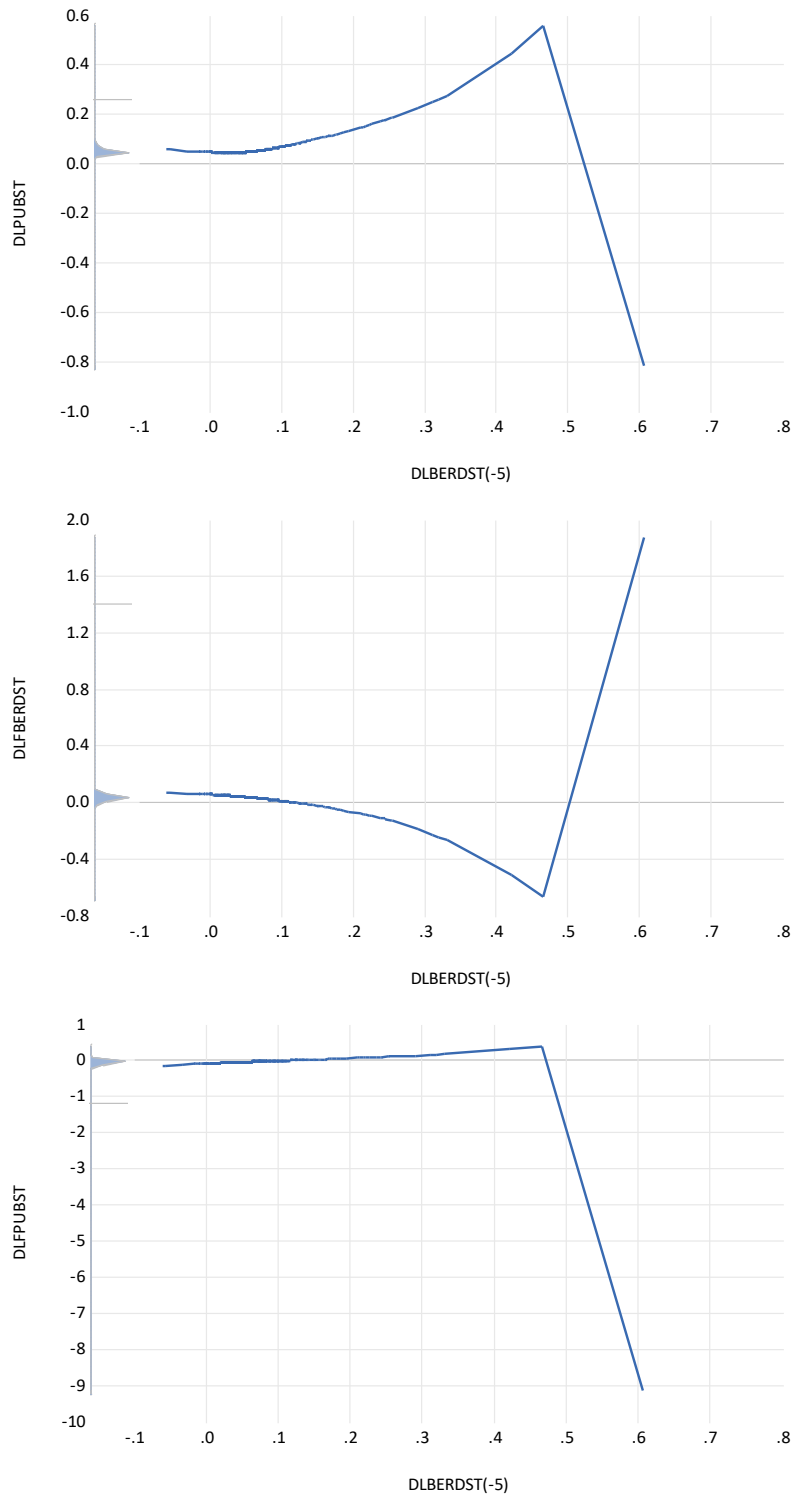
The first graph of Figure 6 shows that exogenous technical change is going down with falling lagged growth of BERD stocks. The coefficient of the lagged dependent variable also goes down for growth rates below 40 percent. The coefficients for private R&D growth go up to positive coefficients with falling *dlberdst(-5)* and those for public R&D go down, both from growth rates of 40 percent again to lower ones. The qualitatively same results seem to hold for foreign R&D variables.

However, Table 7 tells us that the coefficients of the lagged dependent variables and foreign private R&D growth are constant with a probability of more than 10 percent. The hypothesis of constant coefficients is not rejected for these two variables' coefficients, but for domestic private and public R&D coefficients are affected according to column 3 of Table 7. All variables have significant ranges of BERD stock growth supporting the sign of Figure 6 in column 4 of Table 7. Again, here is a contribution to an incentive to shift from public to

private R&D. The bias is a small amount of the coefficients or a broad range of the growth rate of BERD stock.

Figure 6: Contributions of *dlberdst(-5)* to the regression coefficients of the productivity function go up





Notes. Dependent Variable: DLTH07. Method: Functional Coefficients. Sample (adjusted): 1981 – 2019. Included observations: 992 after adjustments. Functional coefficient series: DLBERDST(-5). Kernel: Epanechnikov. Local polynomial degree: 1 (degree 2 is not smooth; degree 3 does not give constant coefficient tests). Bandwidth: 0.4595 (automatic: CV). Localization grid: functional coefficient series.

Table 7: Statistics for the contributions of *dlberdst(-5)* to regression coefficients (a)

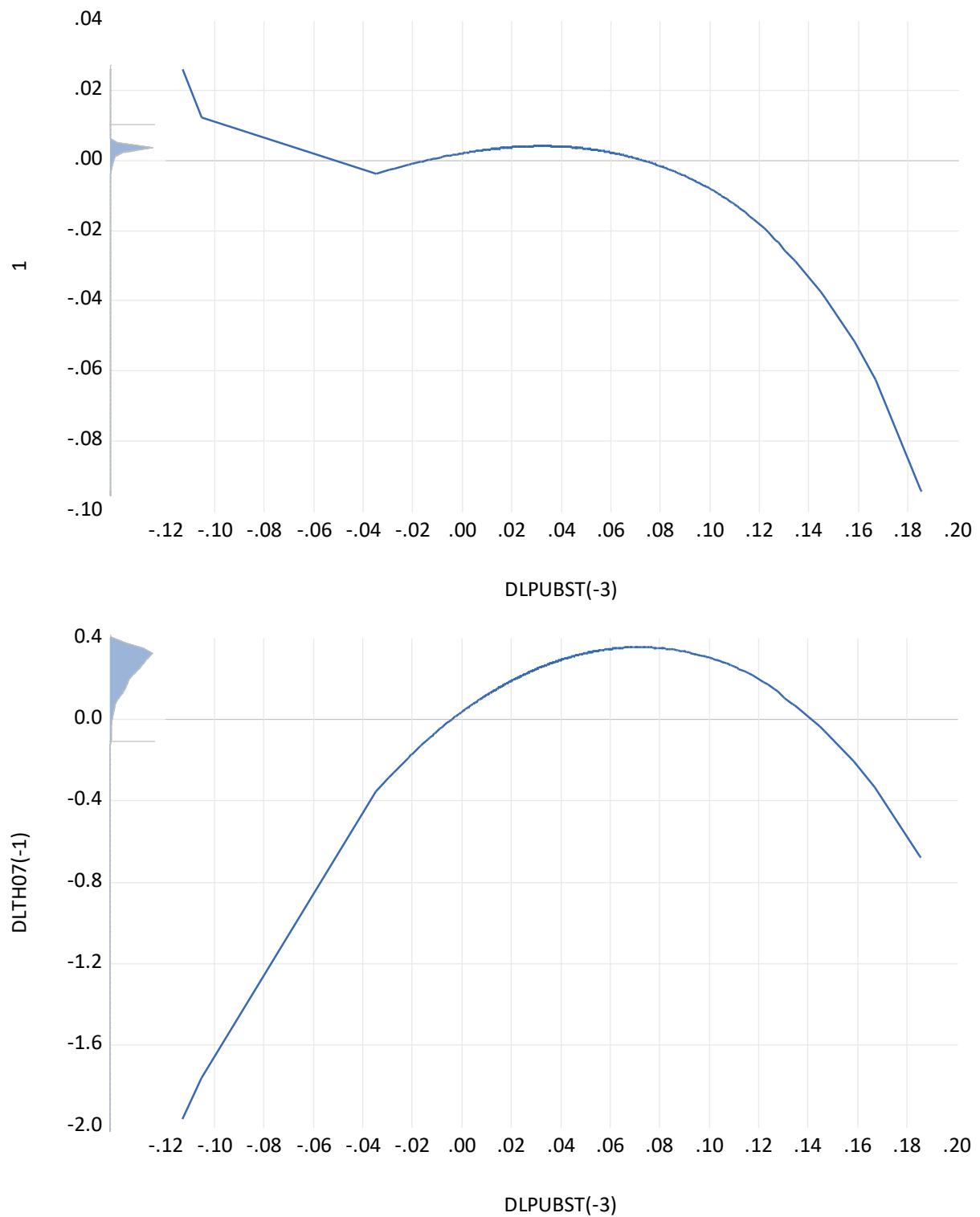
<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. range of $dlberdst(-5) \equiv g$ (c)</i>	<i>Range of bias ≤ 0.001 (d)</i>
<i>constant</i>	$C(1) = c$	3.914	$< .05$	$+$: -0.06 - (-0.03); unclear in betw; $-$: $0.25 < g$	-0.06 – 0.33
<i>dlth07(-1)</i>	$C(2) = c$	2.766	$\geq .10$	$+$: -0.06 - 0.25; $-$: $0.33 < g$	-0.06 – 0.25
<i>dlberdst</i>	$C(3) = c$	15.05	$< .01$	$-$: -0.06 - (-0.03); $-$: $0.17-0.31$; $+$: $0.35 < g$	-0.02 – 0.33
<i>dlpubst</i>	$C(4) = c$	13.33	$< .01$	$+$: -0.06 - (-0.02); unclear in betw.; $+$: $0.17- 0.33$. $-$: $0.42 < g$	-0.005 – 0.15; 0.35-0.36
<i>dlfberdst</i>	$C(5) = c$	2.185	$\geq .10$	$-$: $0.25 - 0.47$	-0.06 – 0.24
<i>dlfpbst</i>	$C(6) = c$	16.37	$< .01$	$-$: -0.06 - (-0.01); $+$: $0.29 < g$	-0.01 – 0.0022

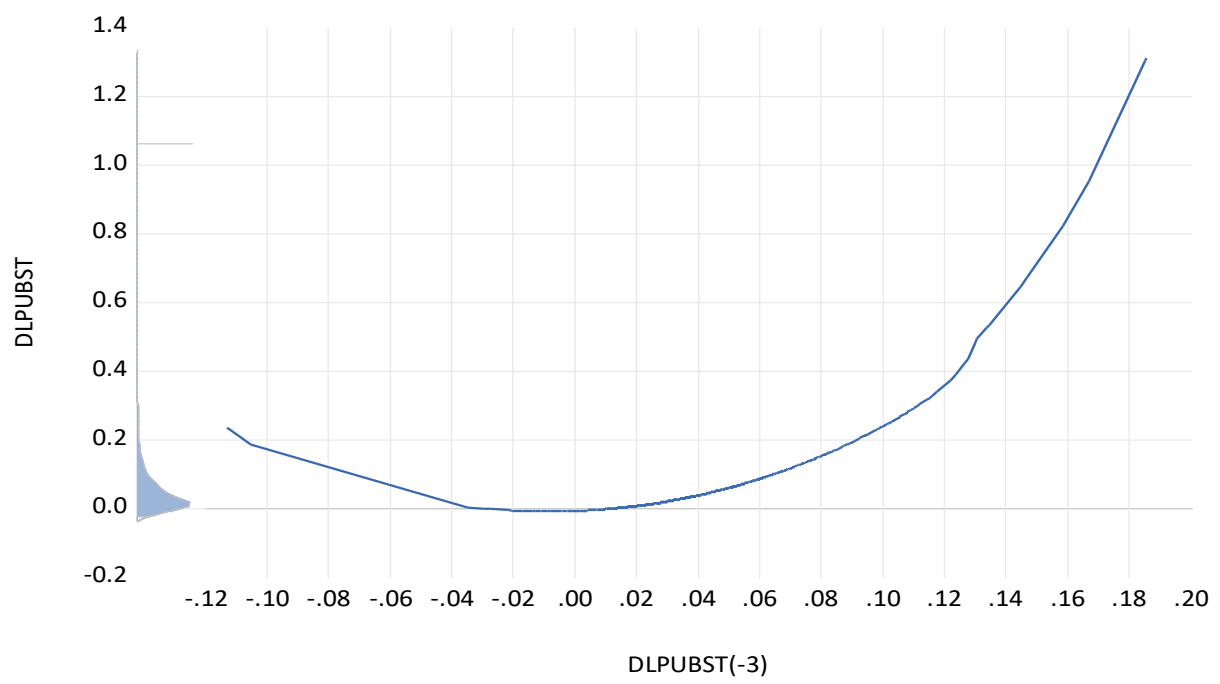
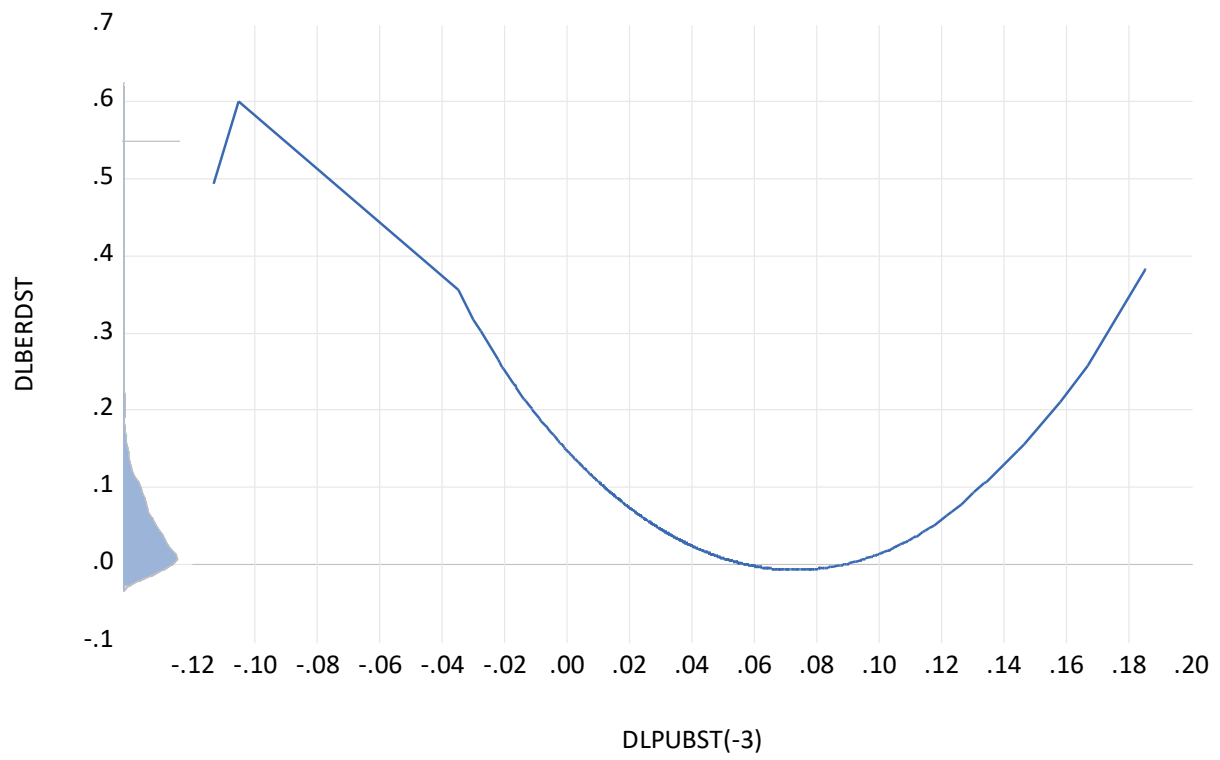
(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 1; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 6, on the same side of the x-axis as regression coefficient for the range expressed in column 4; the regression line within the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 3. Pilot stage bandwidth: 1.0835 (automatic: CV). Type: $\pm$ 2SE Bands. Bias: Included. (d) Functional Coefficient Bias Curves. Pilot stage local polynomial degree: 3. Pilot stage bandwidth: 1.0835 (automatic: CV).

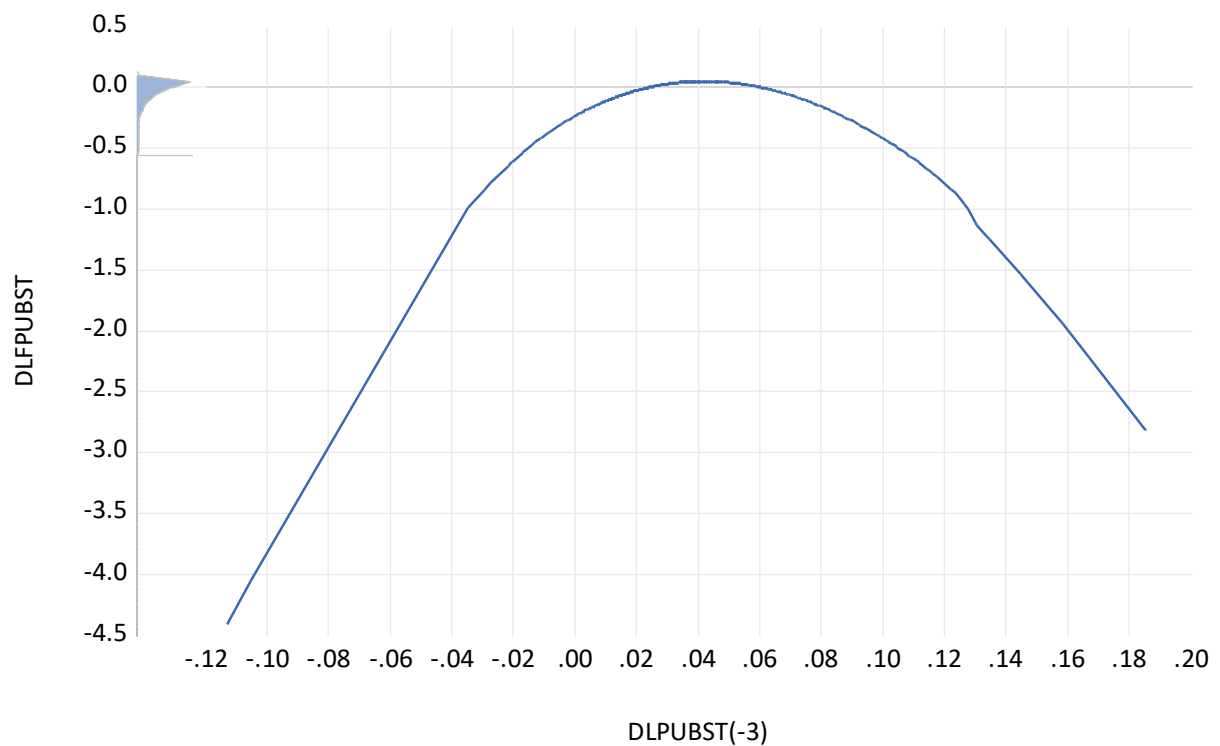
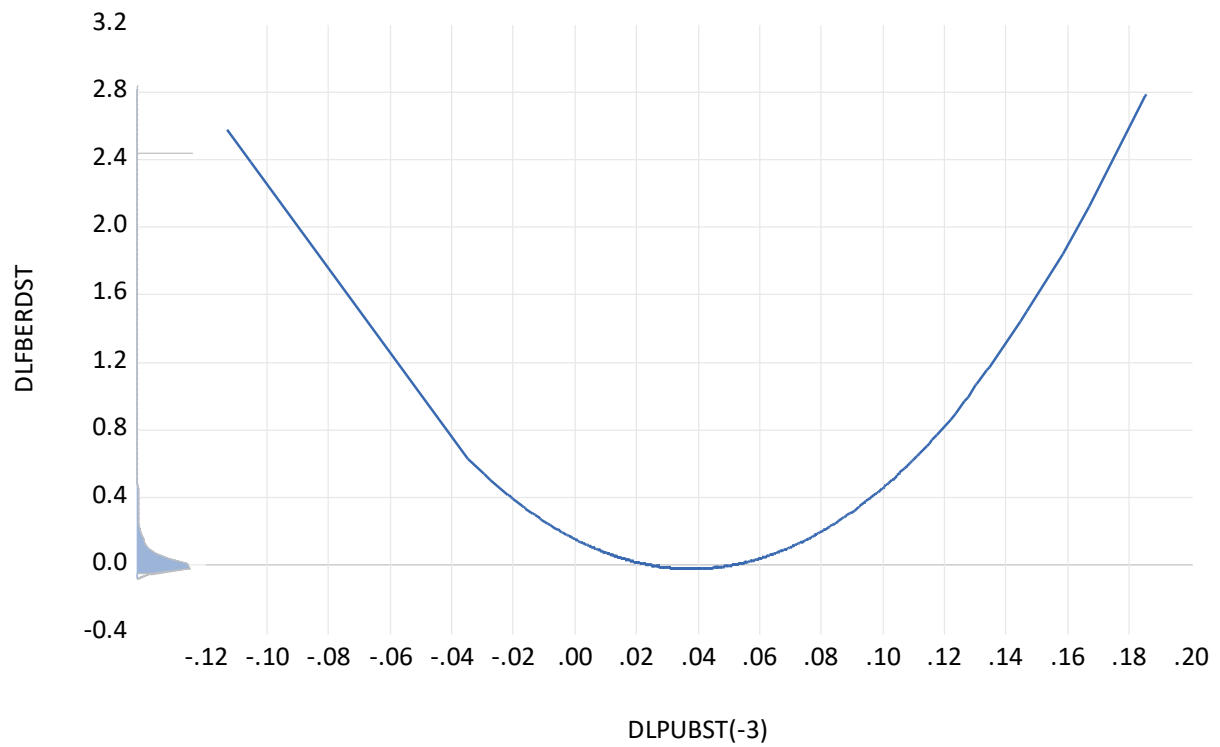
5.1.7 Partial effects of public R&D stock growth, *dlpubst(-3)*, on the regression coefficients of the productivity function

For public R&D we get smooth results and rejected ‘constant coefficient’ tests only when using lag three in Figure 7. The contribution to exogenous technical change is around 0.003 for *dlpubst(-3) < 0.08* and slightly increasing when going to the left from the mean and median range of 4.4 to 5.1 percent. The coefficient of the lagged dependent variable has inverted u-shape and peaks at *dlpubst(-3) = 0.07* with probably constant coefficients (see Table 8). Coefficients of BERD stock growth have a u-shape with a minimum of zero at *dlpubst(-3) = 0.07*, implying that they are increasing near mean and median for falling growth rates of public R&D. Coefficients of public R&D have u-shape with a minimum at *dlpubst(-3) = -0.01*, implying that they are decreasing near mean and median for falling growth rates. Again, here is a contribution to an incentive to shift R&D expenditure from public to private. Coefficients of foreign private and public R&D have u-shape and inverted u-shape respectively with minimum and maximum near mean and median, *dlpubst(-3) = 0.04*. However, for the foreign R&D variables, Table 8 says that the probability of constant coefficients is larger than ten percent on the basis of polynomials of degree 4 (using polynomials of degree 3, which is slightly less smooth, and 5 instead we can argue that the probability is below 0.01 for all coefficients).

Figure 7: Contributions of public R&D stock growth, $dlpubst(-3)$, to the regression coefficients of the productivity function







Notes. Dependent Variable: DLTH07. Method: Functional Coefficients. Sample (adjusted): 1979 – 2019. Included observations: 1002 after adjustments. Functional coefficient series: DLPUBST(-3). Kernel: Epanechnikov. Local polynomial degree: 2. Bandwidth: 0.2358 (automatic: CV). Localization grid: functional coefficient series.

Table 8: Statistics for the contributions of public R&D stock growth, $dlpubst(-3)$, to regression coefficients (a)

<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. range of $dlpubst(-3)$ (c)</i>	<i>Range of bias < 0.001 (d)</i>
<i>constant</i>	C(1) = c	13.63	< .01	+: -0.113 - (-0.013); -: 0.107 - 0.145	-0.075 – 0.185
<i>dlth07(-1)</i>	C(2) = c	2.046	>= .10	-: -0.113 - (-0.017); +: 0.011 - 0.116; -: 0.158 - 0.185	-0.046 – 0.18
<i>dlberdst</i>	C(3) = c	4.555	< .05	-: -0.113 – (-0.03); +: 0.105 - 0.185	-0.035 – 0.186
<i>dlpubst</i>	C(4) = c	2.966	< .10	+: -0.113 - (-0.02); +: 0.069 - 0.186	-0.113 – 0.185
<i>dlfberdst</i>	C(5) = c	1.927	>= .10	-: -0.113 – (-0.035); +: 0.075 - 0.111; -: 0.158 - 0.186	-0.035 – 0.129
<i>dlfpbst</i>	C(6) = c	1.758	>= .10	-: -0.05 – 0.004; -: 0.076 – 0.127; +: 0.185 – 0.186	-0.052 – 0.137

(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 2; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 7, on the same side of the x-axis as regression coefficient for the range expressed in column 4; the regression line within the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 4. Pilot stage bandwidth: 1.0835 (automatic: CV). Type: SE Band. Levels: 2. Bias: Included. (d) Functional Coefficient Bias Curves. Pilot stage local polynomial degree: 4. Pilot stage bandwidth: 1.0835 (automatic: CV).

The partial effects give nice insights into the driving forces changing the coefficients. However, they do not give a clear picture of the total trend of the coefficients, as for all coefficients just discussed there are forces driving them up and others driving them down as shown above and restated in a summary below.

In Figure 1 to 7 the contributions to coefficients for public R&D always have ranges with negative or zero coefficients (only little in Figures 5 to 7). This problem was found earlier by Park (1995) and Sveikauskas (2007), who suggested that public R&D works mainly through its indirect effect on private R&D and from there only indirectly on productivity (see section 5.2 below). Similarly, Soete et al. (2022) find three cases of countries where additional public R&D has negative effects on productivity and others where they are weak in 17 country-specific vector-error correction models. Their diagnosis shows that the effect on productivity is weak or absent when public R&D does not stimulate private R&D for a number of reasons revealed by discriminant analysis. Therefore, we look at the impact of public on private R&D from the perspective of changes in functional coefficients.

5.2 Partial effects on the regression coefficients of the first-order conditions

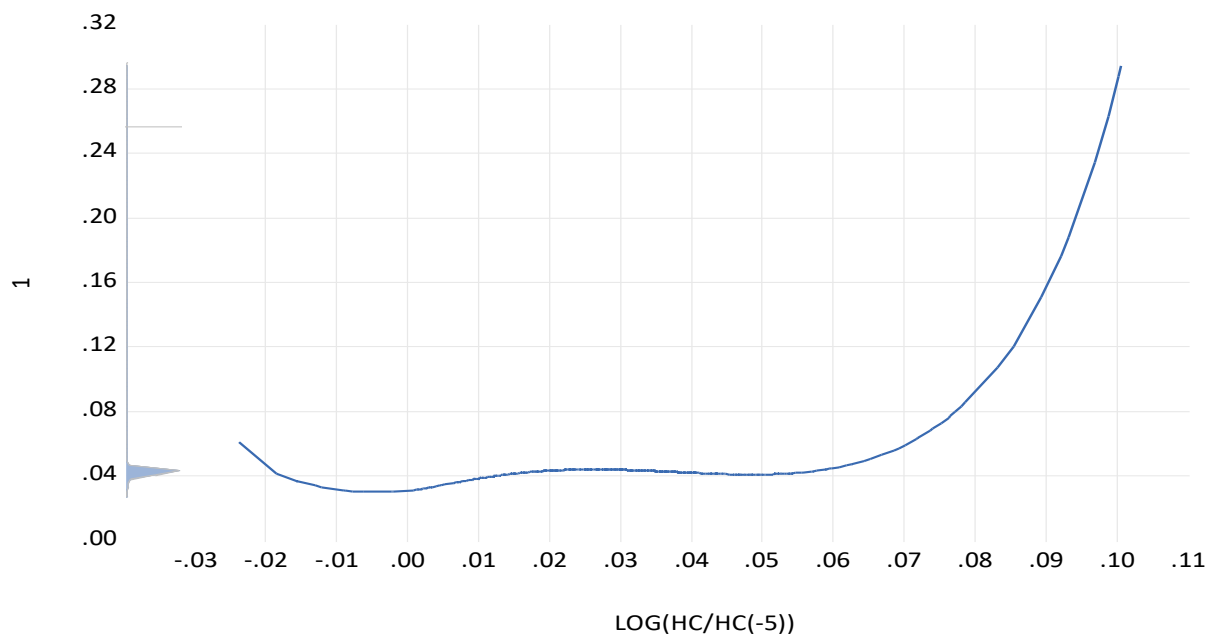
The weak effect of public R&D on productivity in some regressions of section 5.1 makes it important to figure out whether the indirect effect is positive and dominating. For the estimation of (5) we find that the following functional coefficient series tested also above seem to matter: five-year changes of human capital, $\log(hc/hc(-5))$, lags of growth in services (% GDP), $d(\log(srvy(-3)))$, $d(\log(srvy(-4)))$, $d(\log(srvy(-5)))$, growth of defence R&D, $d(\log(0.00000001+defrdy(-2)))$, lags of variables in (1'), only $dlfberdst(-3)$, lags of variables in (5), only $dlberdst(-2)$, and the average of GDP growth rates of the last four years, $sumg4$.

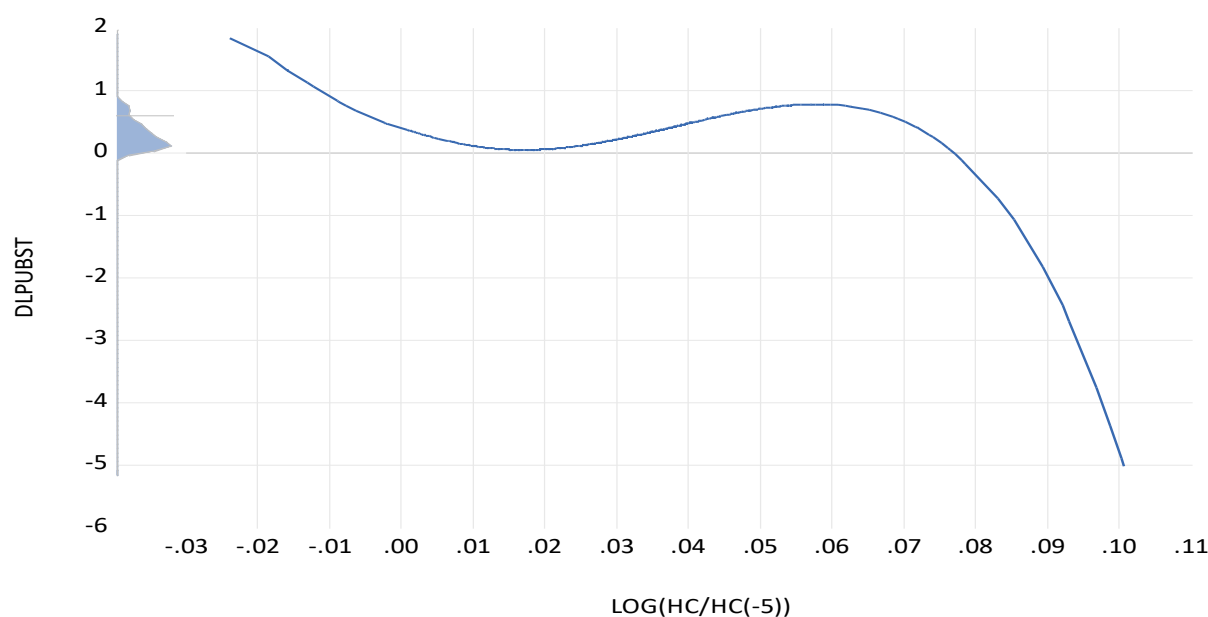
They generate smooth contributions to correlation coefficients mostly for cubic polynomials. However, testing for constant coefficients runs again into the problem of 'raising a negative number to a non-integer power'. Polynomials with a lower or sometimes higher degree show similar trends in the coefficients and therefore for showing the partial contributions to the coefficients we always use the version for which we can also run the test of constant coefficients in this sub-section.

5.2.1 The impact of human capital growth on the private-public regression coefficients

Figure 8 shows that the falling five-year growth rate of human capital contributes to a positive but decreasing slope coefficient of (5), which is mostly below unity, its value in case of a Cobb-Douglas productivity function. It is decreasing with the human capital growth rate

Figure 8 The partial contribution of 5-year growth rate of human capital to the regression coefficients of private R&D depending on public R&D





Notes: Dependent Variable: DLBERDST. Method: Functional Coefficients. Sample (adjusted): 1976 – 2019. Included observations: 1115 after adjustments. Functional coefficient series: LOG(HC/HC(-5)). Kernel: Epanechnikov. Local polynomial degree: 3. Bandwidth: 0.1000 (automatic: CV). Localization grid: functional coefficient series.

between 0.017 and 0.057, where the mean and median are around 0.03; before and after this range it has a negative slope – all for a polynomial of degree 3. There is a positive intercept at about 0.04, which means that it supports the idea of a CES or VES function with either increasing inefficiency or BERD biased technical change in the productivity function.

Table 9 Statistics for the partial contribution of the 5-year growth rate of human capital to the effects of public R&D on private R&D (a)

<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. range of log(hc/hc(-5)) (c)</i>	<i>Range of bias: min, max (d)</i>
<i>constant</i>	C(1) = c	2.364	$\geq .10$	+: -0.017 - 0.056; -: 0.07 - 0.077	-0.000009, 0.00004
<i>dlpubst</i>	C(2) = c	4.468	$< .05$	+: 0.03 - 0.082;	-0.0012, 0.00028

(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 3; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 8, on the same side of the x-axis as regression coefficient for the range expressed in column 4; the regression line within the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 0.1100 (automatic: CV). Type: +- 2SE Band. Bias: Included. (d) Functional Coefficient Bias Curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 0.1100 (automatic: CV).

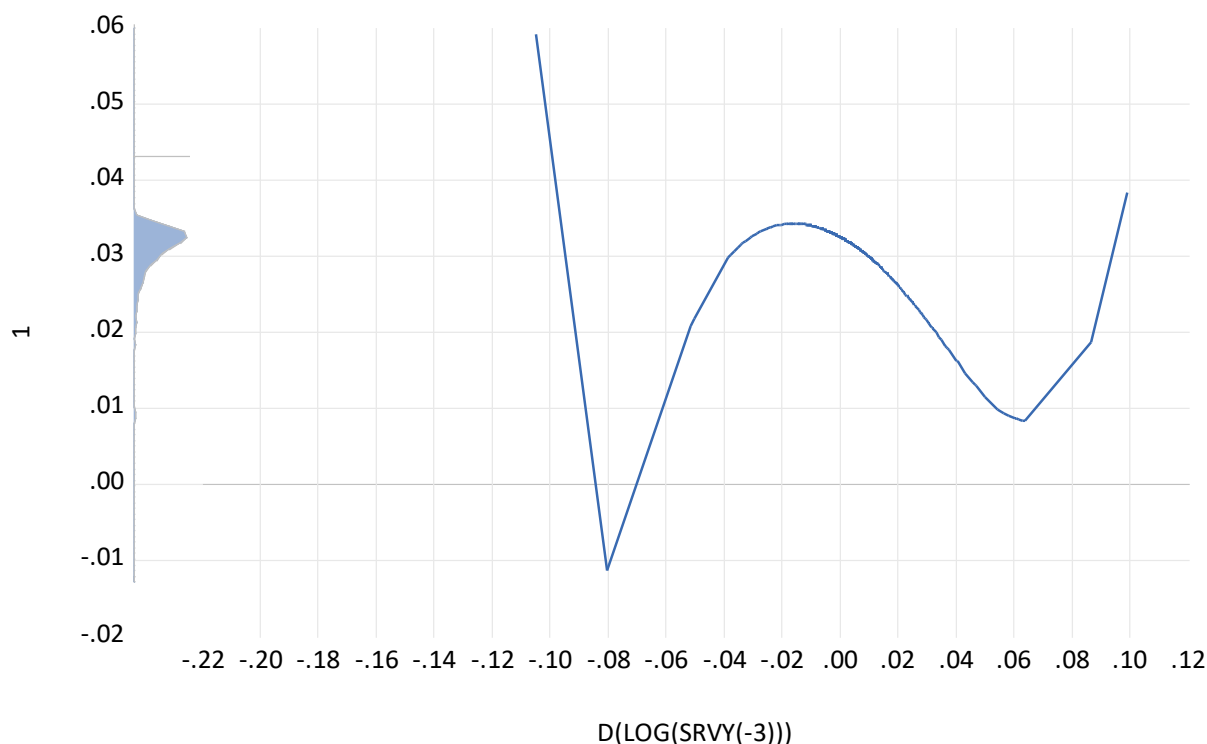
However, Table 9, in the first column, suggests that the intercept may be constant with a probability larger than 10 percent. As this intercept would be significantly positive from 0.017 to 0.056 percent growth of human capital, we get BERD biased technical change at a possibly constant rate. The slope coefficient for public R&D is constant with a probability lower than 5 percent.

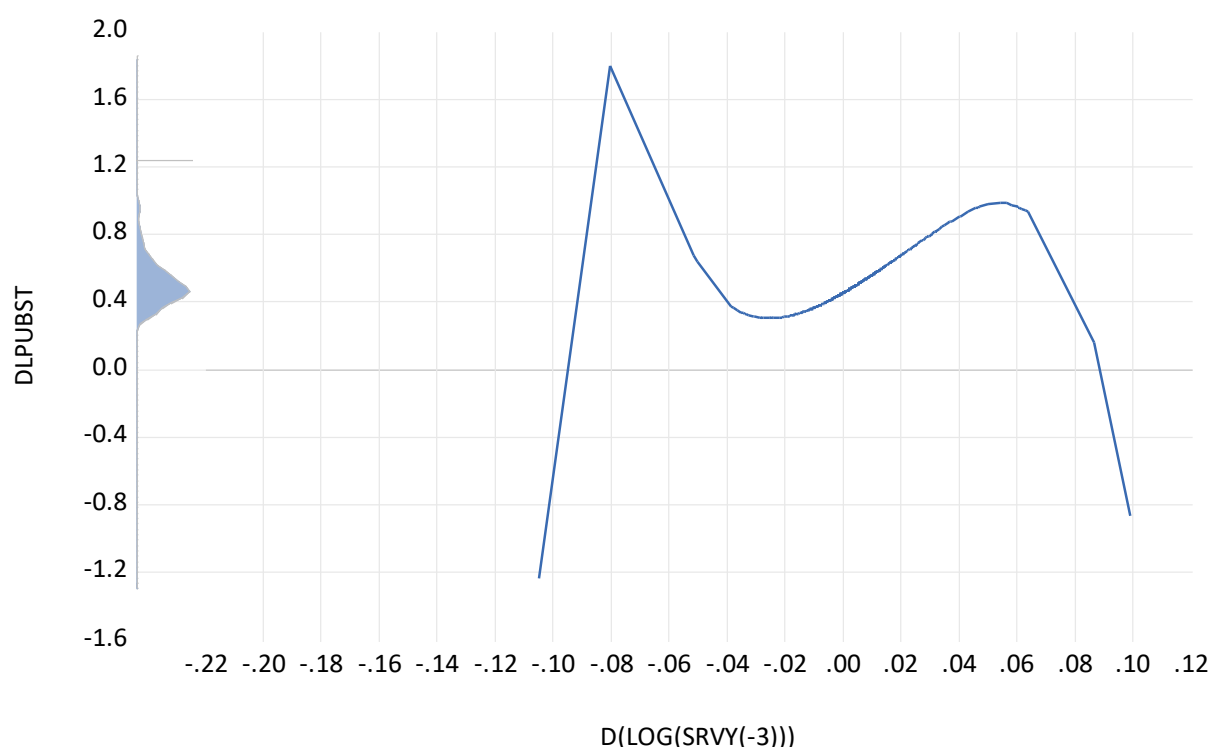
5.2.2 The impact of service/GDP growth on the private-public regression coefficients

The impact of the growth rate of services(-3) (% GDP(-3)) on the regression coefficients for private on public R&D are shown in Figure 9. The intercept is an inverted u-shape function in service growth between -0.08 and 0.064; when service growth is zero, the intercept representing BERD biased technical change or inefficiency, is 0.035. The slope coefficient has u-shape from -0.08 to 0.054 with a value of 0.44 when services reach a constant share of GDP.

Table 10 suggests that BERD biased technical change does not react to the share of services with probability larger than 10 percent. However, the slope does react. When the growth rate of the share of services falls from positive values to zero the slope coefficient falls to 0.44. Confidence intervals support Figure 9 for a broad range of growth of services (% GDP). Biases are minor compared to the size of the coefficients.

Figure 9 The partial contribution of growth of services(-3) (%GDP(-3)) to the regression coefficients of private R&D depending on public R&D





Notes. Dependent Variable: DLBERDST. Method: Functional Coefficients. Sample (adjusted): 1994 – 2021. Included observations: 695 after adjustments. Functional coefficient series: $D(\text{LOG}(\text{SRVY}(-3)))$. Kernel: Epanechnikov. Local polynomial degree: 2. Bandwidth: 0.1100 (automatic: CV). Localization grid: functional coefficient series.

Table 10 Statistics for the partial contribution of growth of services(-3) (%GDP(-3)) to the effects of public R&D on private R&D (a)

<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. range of dlogsrvy(-3) (c)</i>	<i>Bias range, min, max (d)</i>
<i>constant</i>	$C(1) = c$	0.986	$\geq .10$	+: -0.05 - 0.03;	-.000044, .000008
<i>dlpubst</i>	$C(2) = c$	4.612	$< .05$	+: -0.024 – 0.064	-0.00029, 0.00197

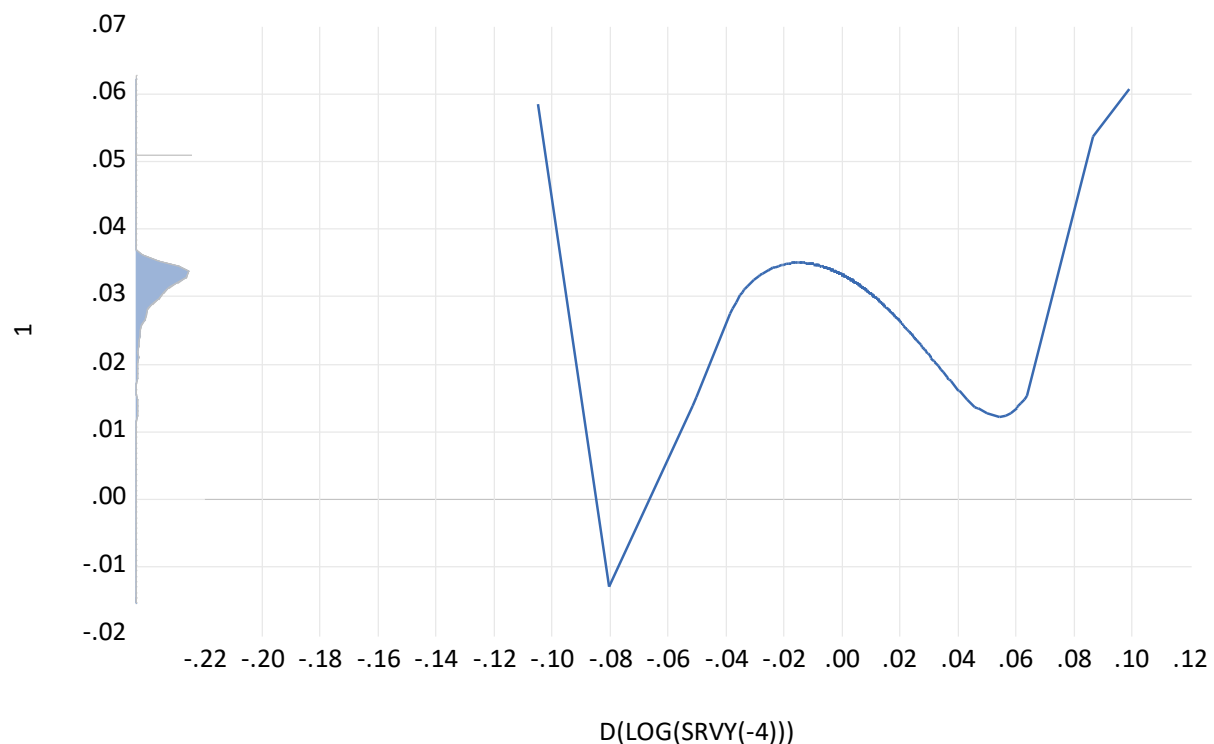
(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 2; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 9, on the same side of the x-axis as regression coefficient for the range expressed in column 4; the regression line within the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 4. Pilot stage bandwidth: 1.0835 (automatic: CV). Type: $\pm 2\text{SE}$ Band. Bias: Included. (d) Functional Coefficient Bias Curves. Pilot stage local polynomial degree: 4. Pilot stage bandwidth: 1.0835 (automatic: CV).

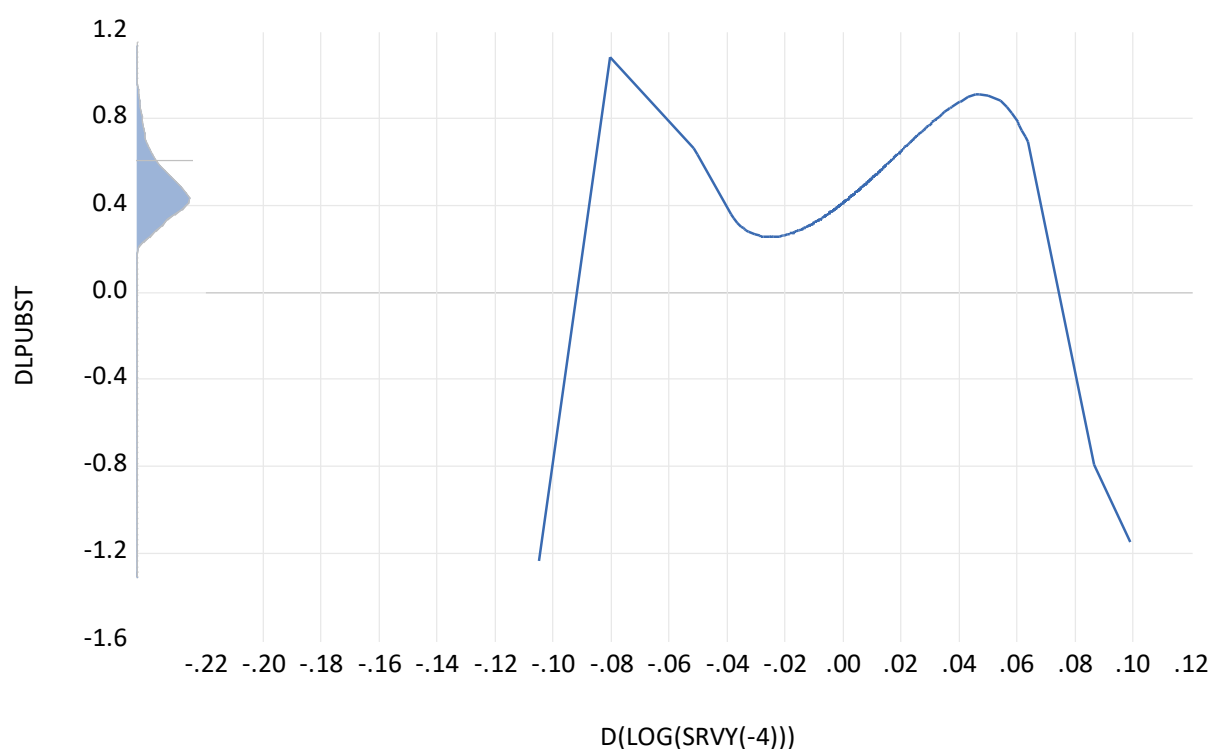
The impact of four-year lagged services (% GDP), $d(\log(\text{srvy}(-4)))$, on the regression coefficients for private on public R&D are shown in Figure 10. As for lag three, we have an

inverted u-shape for the intercept and a u-shape for the slope. When the growth rate of the service share goes to zero from higher values, the intercept as BERD biased technical change or inefficiency grows and then leaves them constant at a positive BERD biased technical change value of 0.034. The impact of public on private R&D goes down to 0.4 from higher values.

A major difference with Figure 9 and Table 10 for the third lag of the service/GDP ratio is that Table 11 confirms that the intercept does react to 4-year lagged growth of the share of services statistically significantly, and the coefficient of public R&D also goes down statistically significantly for positive to zero growth of the share of services. Confidence intervals confirm statistical significance, and biases are extremely low. For Figure 9 and 10 we have used polynomials of degree 2 because for higher ones we did get ‘attempt to raise a negative number to an integer power’ when trying to test for the probability for constant coefficients.

Figure 10 The partial contribution of services(-4) (%GDP(-4) growth to the regression coefficients of private R&D depending on public R&D





Notes. Dependent Variable: DLBERDST. Method: Functional Coefficients. Sample (adjusted): 1995 2021. Included observations: 674 after adjustments. Functional coefficient series: D(LOG(SRVY(-4))). Kernel: Epanechnikov. Local polynomial degree: 2. Bandwidth: 0.1000 (automatic: CV). Localization grid: functional coefficient series.

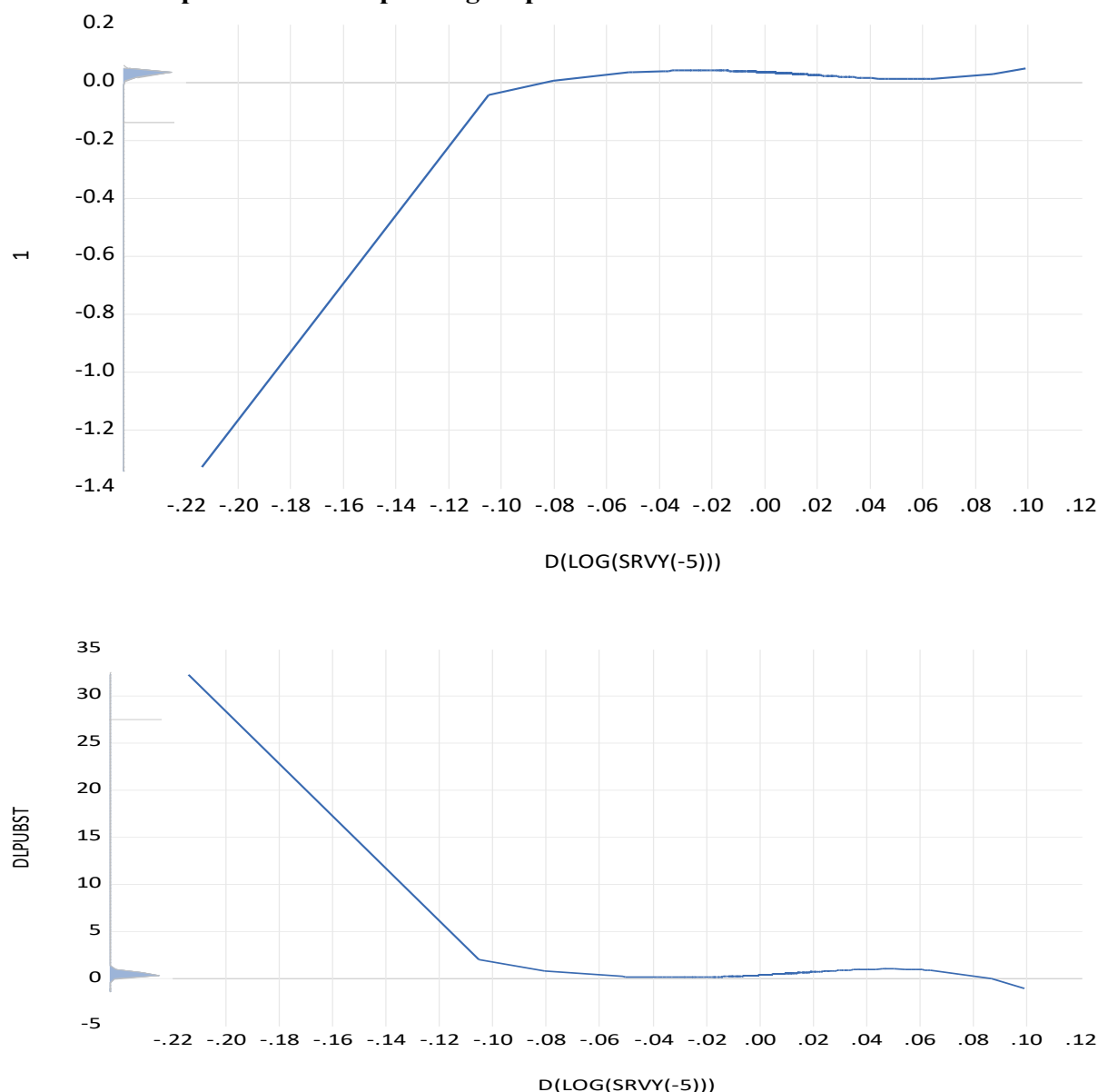
Table 11 Statistics for the partial contribution of services(-4) (%GDP(-4) to the effects of public R&D on private R&D (a)

<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. range of dlogsrvy(-4) (c)</i>	<i>Bias near zero (in range 3.85-4.34 (d)</i>
<i>constant</i>	C(1) = c	5.403	< .01	+: -0.039 -0.029; -: 0.064 – 0.099	-0.000305 - 0.000017
<i>dlpubst</i>	C(2) = c	11.31	< .01	+: -0.051- 0.062; -:0.087-0.099	-0.0005 - 0.0078

(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 2; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 10, on the same side of the x-axis as regression coefficient for the range expressed in column 4; the regression line within the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 4. Pilot stage bandwidth: 0.3138 (automatic: CV). Type: +- 2SE Band. Bias: Included. (d) Functional Coefficient Bias Curves. Pilot stage local polynomial degree: 4. Pilot stage bandwidth: 0.3138 (automatic: CV).

The impact of five-year lagged services (% GDP), $d(\log(srvy(-5)))$, on the regression coefficients for private on public R&D are shown in Figure 11 and Table 12. Here we could use degree 3 for the polynomial. Values of Figure 11 on the vertical axis are much higher leading to a different visual impression. However, we have again (inverted) u-shapes roughly from -0.08 to +0.05. Service growth from positive to zero leads again to higher intercepts as BERD biased technical change and to lower slopes of public R&D growth.

Figure 11 The partial contribution of services(-5) (%GDP(-5) growth to the regression coefficients of private R&D depending on public R&D



Notes. Dependent Variable: DLBERDST. Method: Functional Coefficients. Sample (adjusted): 1996 – 2021. Included observations: 653 after adjustments. Functional coefficient series: $D(\text{LOG}(\text{SRVY}(-5)))$. Kernel: Epanechnikov. Local polynomial degree: 3. Bandwidth: 0.1949 (automatic: CV). Localization grid: functional coefficient series.

Table 12 shows now a probability for a constant slope that is higher than in Table 11 and 10 for service shares lagged three or four periods, but it is still below 10 percent. Confidence intervals have lower and upper bounds, with positive medians (after elimination of one outlier to get better visibility). The lower bounds have some negative values interrupting the statistically significant intervals. Biases are lower than 0.001 in absolute values, over the entire range to the right of -0.105 growth of the service share.

Table 12 Statistics for the partial contribution of services(-5) (%GDP(-5) to the effects of public R&D on private R&D (a)

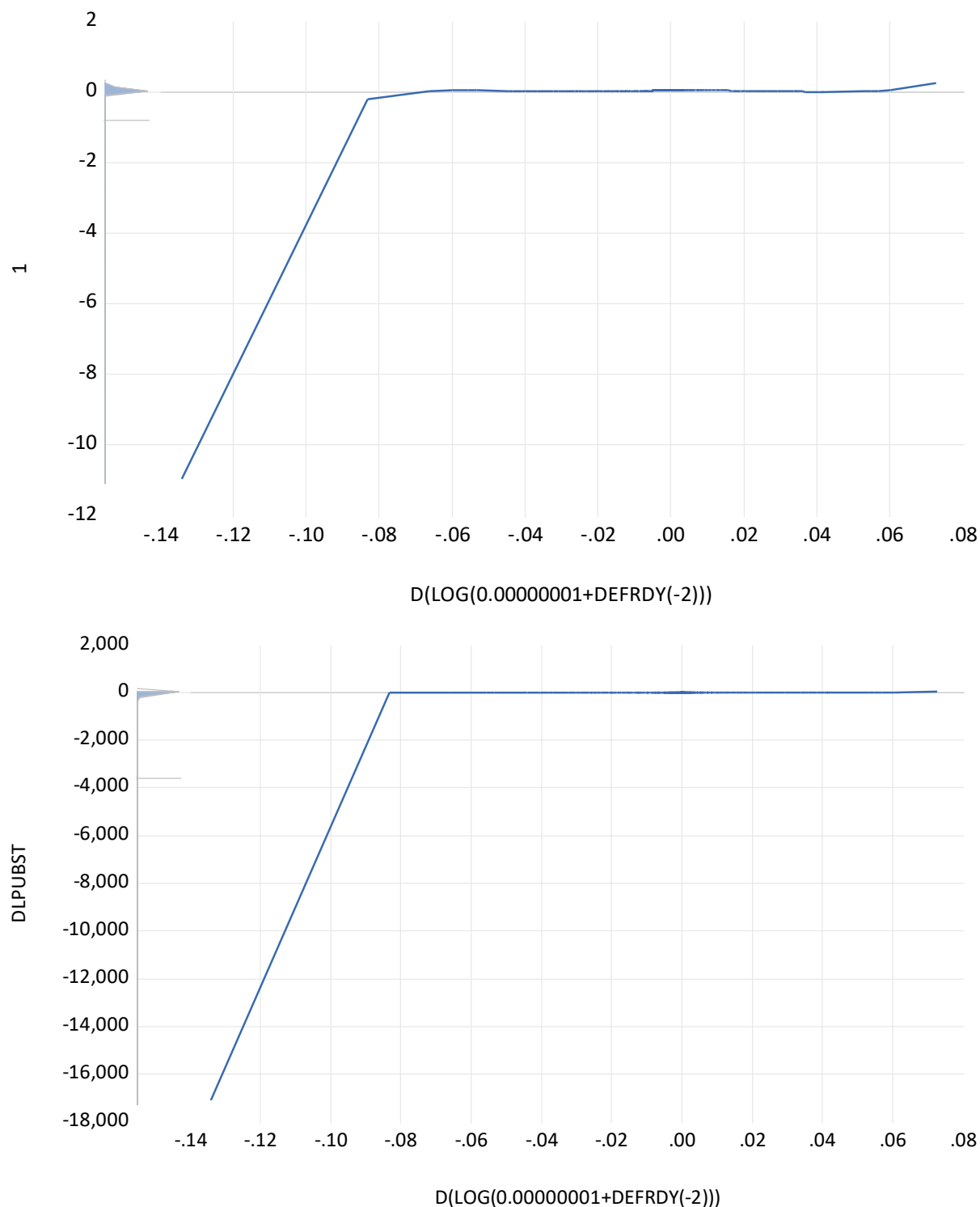
<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. range of growth rates (c)</i>	<i>Bias ≤ 0.001 in range (d)</i>
<i>constant</i>	C(1) = c	19.52	< .01	mean & median for lower and upper bound positive	-0.105 - 0.099
<i>dlpubst</i>	C(2) = c	3.512	< .10	mean & median for lower and upper bound positive	-0.105 - 0.099

(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 3; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 11, on the same side of the x-axis as regression coefficient for the range expressed in column 4; the regression line within the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 0.2594 (automatic: CV). Type: +-2SE Band. Bias: Included. Values for observation 369 cancelled as outlier makes median of lower bound for intercept positive; graph has many intervals of just (in-)significant because of changing sign of lower bounds. (d) Functional Coefficient Bias Curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 0.2594 (automatic: CV).

5.2.3 The impact of defence R&D (% GDP) growth on the private-public regression coefficients

The impact of defence R&D (% GDP) growth on the regression coefficients for private on public R&D is shown in Figure 12. Effects are hard to see because of some huge value, which we cannot eliminate in the standard set up used so far, but the tests in Table 13 confirm that coefficients are not constant. In Figure 13 we plot the contributions to the regression coefficients in panel format. Most of the slope coefficients are near 0.5, and the intercept is mostly zero. Figure 12 does have negative slopes for the intercept for growth rates below -0.08 and higher than 0.06, limiting the occurrence of contributions to inefficiency or BERD biased technical change to extremely high or low values.

Figure 12 The partial contribution of defence R&D (%GDP) growth to the regression coefficients of private R&D depending on public R&D



Notes. Dependent Variable: DLBERDST. Method: Functional Coefficients. Sample (adjusted): 1984 – 2021. Included observations: 715 after adjustments. Functional coefficient series: $D(\text{LOG}(0.00000001 + \text{DEFRDY}(-2)))$. Kernel: Epanechnikov. Local polynomial degree: 7. Bandwidth: 0.1331 (automatic: CV). Localization grid: functional coefficient series.

Figure 13 The partial contribution of defence R&D (%GDP) growth to the regression coefficients of private R&D depending on public R&D in country-year format

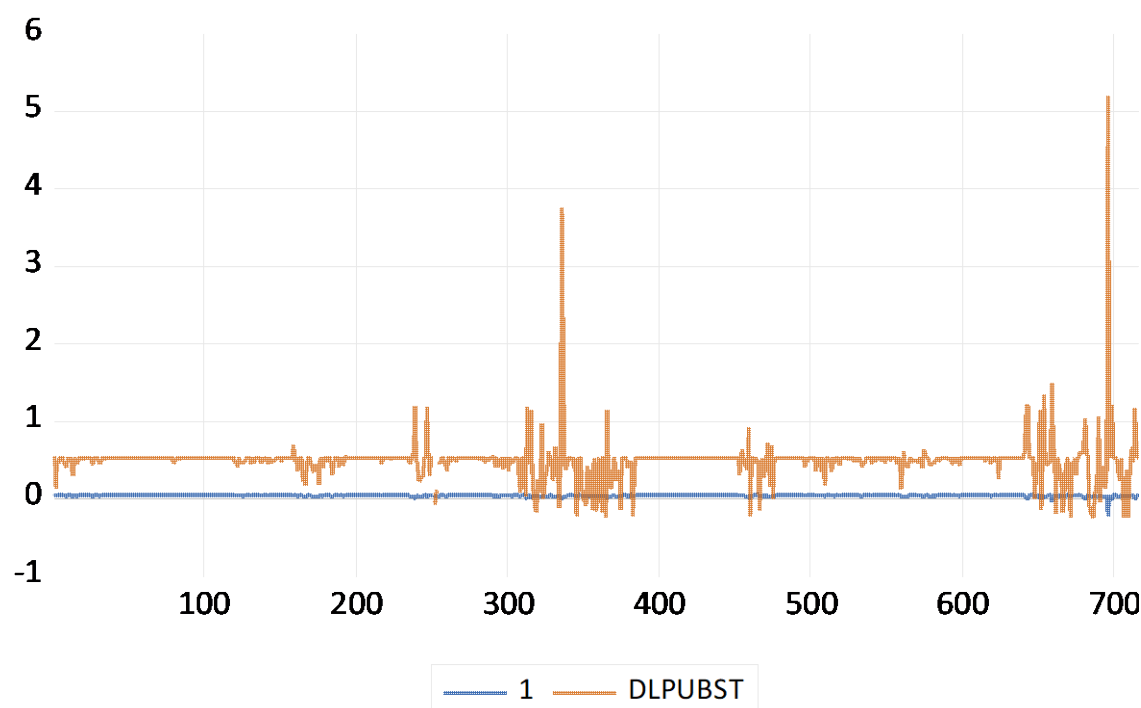


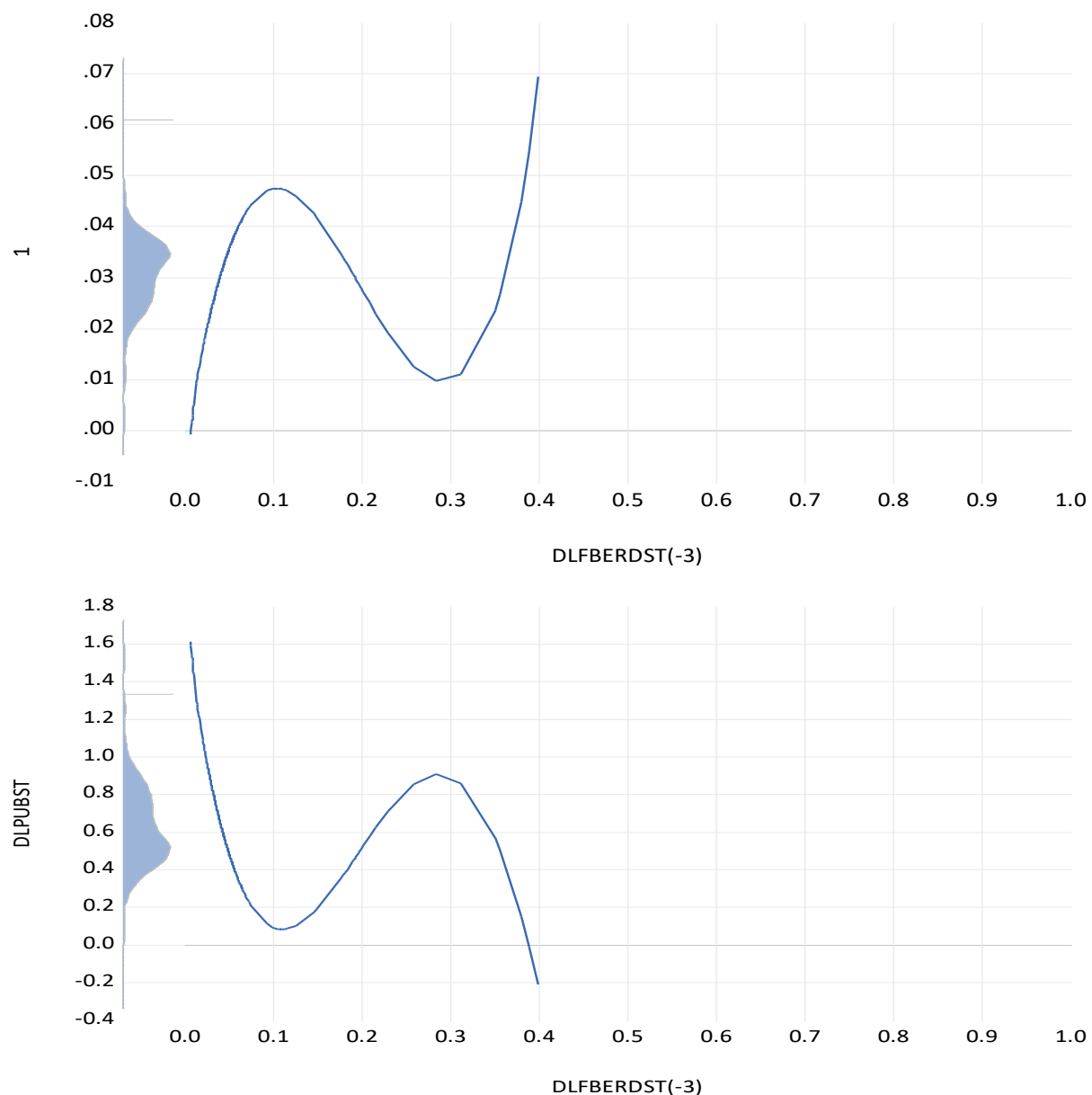
Table 13 Statistics for the partial contribution of defence R&D (%GDP) growth to the effects of public R&D on private R&D (a)

<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. of growth rates (c)</i>	<i>Bias near zero (d) min; max</i>
<i>constant</i>	$C(1) = c$	75.61	< .01	mean & median of lower & upper bound positive	-6.99E-08; 1.36E-07
<i>dlpubst</i>	$C(2) = c$	3.757	< .05	mean & median of lower & upper bound positive	-0.27E-05; 0.11E-05

(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 7; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 12, on the same side of the x-axis as regression coefficient for the range expressed in column 4; the regression line within the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 9. Pilot stage bandwidth: 0.2144 (automatic: CV). Type: +2SE Band. Bias: Included. (d) From functional coefficient bias curves. Pilot stage local polynomial degree: 9. Pilot stage bandwidth: 0.2144 (automatic: CV).

The slope of Figure 12 is falling for values below -0.02 of the seemingly flat part and is increasing for growth rates of defence R&D above 0.02 but is flat near a growth rate of zero. Visibility in Figure 13 is achieved by way of dropping two outliers. Confidence intervals using degree 9 are both in the positive area, and biases are near zero.

Figure 14 The partial contribution of foreign BERD stock growth to the regression coefficients of private R&D depending on public R&D in country-year format



Notes. Dependent Variable: DLBERDST. Method: Functional Coefficients. Sample (adjusted): 1976 – 2021. Included observations: 1135 after adjustments. Functional coefficient series: DLFBERDST(-3). Kernel: Epanechnikov. Local polynomial degree: 3. Bandwidth: 0.3797 (automatic: CV). Localization grid: functional coefficient series.

5.2.4 The impact of foreign BERD stock growth on the private-public regression coefficients

Figure 14 shows that the contribution of foreign BERD stock growth to the intercept and the slope has (inverted) N-shape. The first part goes until 0.1, which is about mean or median plus one standard deviation (see Table 1). The positive intercept representing BERD biased technical change is increasing with foreign BERD stock growth. The slope is decreasing through higher BERD growth in the past, which means that it reduces the impact of public on private R&D. Only when the growth rates go beyond ten percent do we get the opposite effects. Beyond 28 percent we get the slopes of the first part again.

Table 14 shows the lowest probability for constant coefficients compared to all other tables. Mean and median of the values for lower and upper bounds are positive and statistically different from zero, implying that coefficients are significantly different from zero. Biases are again small.

Table 14 Statistics for the partial contribution of foreign BERD stock growth, $dlfberdst(-3)$, to the effects of public R&D on private R&D (a)

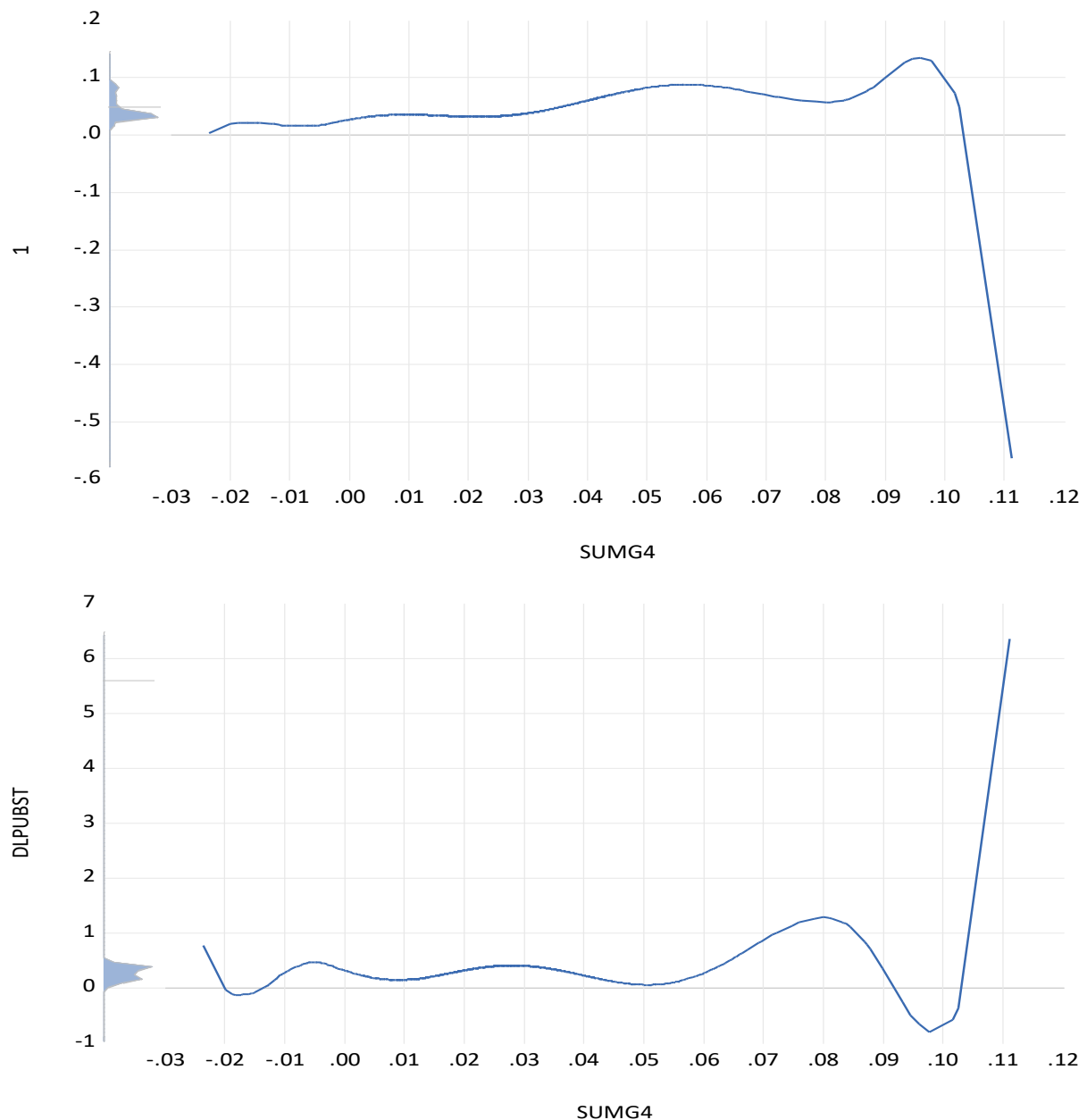
<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. of $dlfberdst(-3)$ (c)</i>	<i>Bias near zero (d) min; max</i>
<i>constant</i>	C(1) = c	-142.8	< .01	mean & median of lower & upper bounds positive	-0.00002; 0.0000083
<i>dlpubst</i>	C(2) = c	2789.8	< .01	mean & median of lower & upper bounds positive	-0.000104; 0.000155

(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 3; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 14, on the same side of the x-axis as regression coefficient for the range expressed in column 4; the regression line within the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 0.8140 (automatic: CV). Type: +- 2SE Band. Bias: Included. (d) Functional Coefficient Bias Curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 0.8140 (automatic: CV).

5.2.5 The impact of the four-year average growth rates of GDP on the private-public regression coefficients

When $sumg4$ goes from 0.095 to -0.024 in Figure 15, the intercept representing BERD-biased technical change goes from 0.13 to zero. The slope coefficient has four internal local minima and three internal local maxima. The middle peak is near mean and median of 2.7%;

Figure 15 The partial contribution of GDP four-year average growth rates to the regression coefficients of private R&D depending on public R&D in country-year format



Notes. Dependent Variable: DLBERDST. Method: Functional Coefficients. Sample (adjusted): 1976 2021. Included observations: 1155 after adjustments. Functional coefficient series: SUMG4. Kernel: Epanechnikov. Local polynomial degree: 7. Bandwidth: 0.1000 (automatic: CV). Localization grid: functional coefficient series.

growth rate deviations from mean and median within one standard deviation therefore lead to lower slope coefficients. According to Table 15 these changes are statistically significant. The confidence intervals indicated in the fourth column of Table 15 show that the coefficients

are mostly significantly above zero. Biases are extremely low, which is not surprising because a polynomial of degree 7 is used.

Table 15 Statistics for the partial contribution of GDP four-year average growth rates, *sumg4*, to the effects of public R&D on private R&D (a)

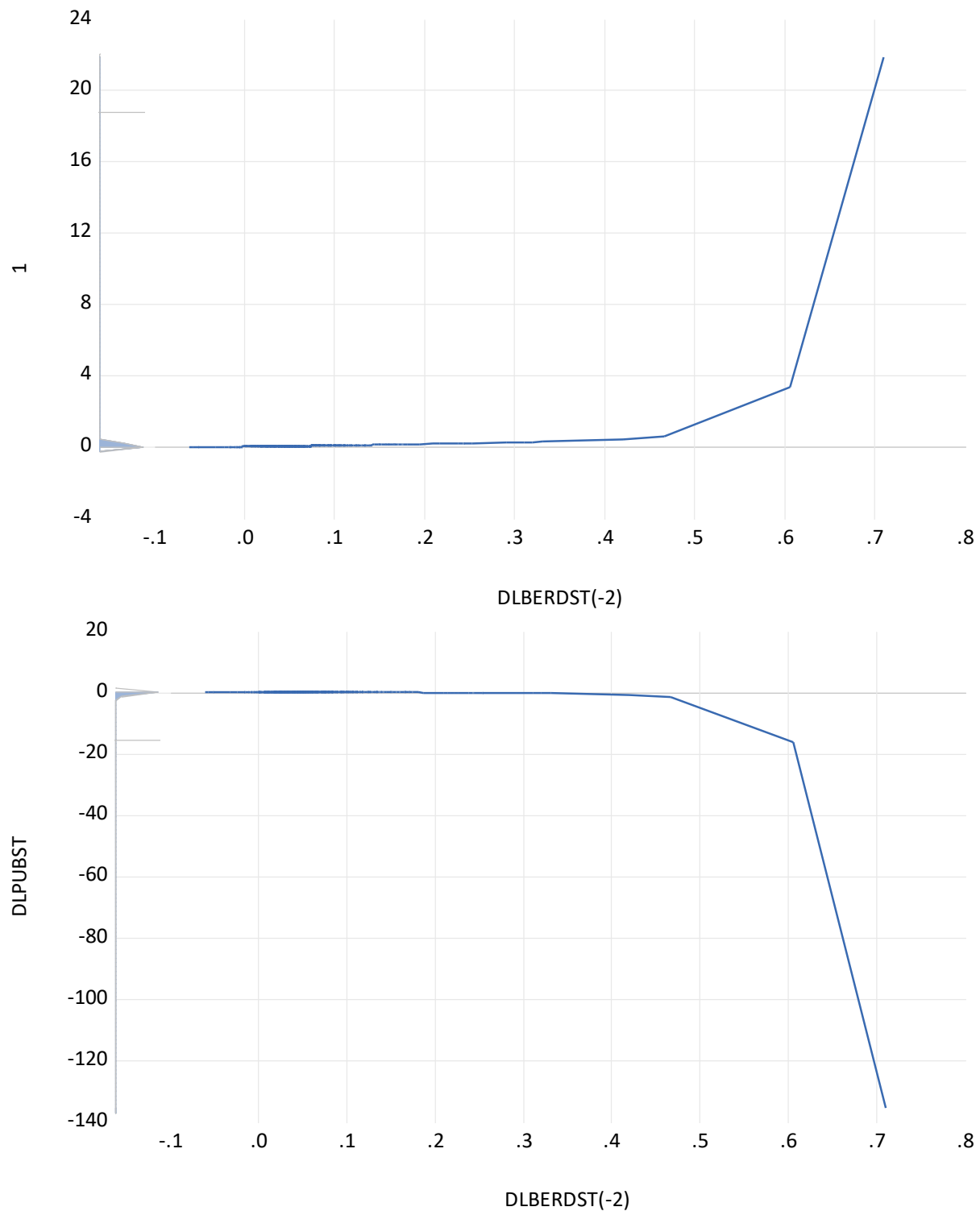
<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. of sumg4 (c)</i>	<i>Bias near zero (d) min; max</i>
<i>constant</i>	C(1) = c	5.1860	< .05	+: -0.016 - (-0.012); +: -0.002 - 0.072; +: 0.08 - 0.098	-5.59E-10; 2.86E-09
<i>dlpubst</i>	C(2) = c	3.3904	< .10	+: -0.022 - (-0.024); +: -0.008 - 0.008; +: 0.021 - 0.043; +: 0.058 - 0.084	-7.2E-08; 1.4E-08

(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 7; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 15, on the same side of the x-axis as regression coefficient for the range expressed in column 4; the regression line within the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 9. Pilot stage bandwidth: 0.3797 (automatic: CV). Type: +- 2SE Band. Bias: Included. (d) Functional Coefficient Bias Curves. Pilot stage local polynomial degree: 9. Pilot stage bandwidth: 0.3797 (automatic: CV).

5.2.6 The impact of lagged BERD stock growth, *dlberdst(-2)*, on the private-public regression coefficients

Among the BERD stock growth rates only the second lag, *dlberdst(-2)*, does have a statistically significant impact on the coefficients of first-order condition (5). Table 16, column 3, confirms that the contributions to the coefficients shown in Figure 16 are not constant. The intercept, possibly representing BERD-biased technical change, falls when *dlberdst(-2)* gets lower over time or across-countries. For the slope, the opposite holds. Table 16, column 4, shows that the slope coefficient changes repeatedly from statistically positive to negative. The intercept is significantly positive. Biases are extremely low from -0.06 to 0.42 (slope 0.47).

Figure 16 The partial contribution of lagged BERD stock growth, *dlberdst(-2)*, to the regression coefficients of private R&D depending on public R&D in country-year format



Notes. Dependent Variable: DLBERDST. Method: Functional Coefficients. Sample (adjusted): 1978 – 2021. Included observations: 1125 after adjustments. Functional coefficient series: DLBERDST(-2). Kernel: Epanechnikov. Local polynomial degree: 3. Bandwidth: 0.4595 (automatic: CV). Localization grid: functional coefficient series.

Table 16 Statistics for the partial contribution of lagged BERD stock growth, *dlberdst*(-2), to the effects of public R&D on private R&D (a)

<i>Variable</i>	<i>Hypothesis, statistic, p val (b)</i>			<i>Statist. sign. of dlberdst (c)</i>	<i>Bias near zero (d) interval</i>
<i>constant</i>	C(1) = c	20.28	< .01	Upper & lower bounds positive with few exceptions	-0.06 – 0.42
<i>dlpubst</i>	C(2) = c	5.793	< .01	upper bound positive, lower bound changing half pos., half neg. for 2 SE	-0.06 – 0.42 (below 0.0003)

(a) Multi cross validation and updating of local bandwidth used for all statistics as proposed by Cai, Fan, and Yao (2000). (b) Functional Coefficient Equality Hypothesis Tests. Statistics and p values for polynomial of degree 3; critical values: for level 1%, 5%, 10%, value 5.29, 3.66, 2.94. Reject null when absolute value of statistic exceeds critical value. (c) Confidence interval in line with the sign of Figure 16, on the same side of the x-axis as regression coefficient for the range expressed in column 4; the regression line within the confidence interval. Functional Coefficient Confidence Interval Curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 1.0835 (automatic: CV). Type: SE Band. Levels: 2. Bias: Included. (d) Functional Coefficient Bias Curves. Pilot stage local polynomial degree: 5. Pilot stage bandwidth: 1.0835 (automatic: CV).

5.3 A two-way summary of functional coefficient impact results

Table 17 summarizes from the perspective of the functional coefficient series. This is inevitably a bit rough and purely serving the purpose of getting an overview of the many results. For detailed information it is better to go back to Figure 1 to 16 and Table 2 to 16.

Less positive human capital growth reduces only the coefficients of the lagged dependent variable in (1') in the neighborhood of the mean and median; this reduces all long-run effects (Figure 1) as long as human capital growth is increasing. It also decreases the impact of public on private R&D, which could be turned around by education policy if it is strong enough to bring back increasing growth rates. Both effects are a contribution to the productivity slowdown.

Lower GDP per worker growth is driving the coefficients of private R&D down, leaving that of public R&D unaffected (Figure 2). Growth in GDP per worker contributes to this as suggested by Coccia (2010), and it contributes to the productivity slowdown.

Lower services (% GDP) growth leaves the coefficients of private R&D unchanged near zero growth of services and drives those for public R&D up (Figure 3). Services growth has a different impact when going away from the neighborhood of zero. The coefficients of private R&D go down with the fall in the growth rate of service (% GDP) if the latter is larger than

two percent. The coefficient of public R&D goes up with the fall in the growth rate of the share of services if service growth is not below minus 10%. For foreign R&D we find the inverse role: when service growth goes from positive to zero, the impact of private foreign R&D goes from negative to positive and that of foreign public R&D from positive to negative. All effects link the sectors to macroeconomic productivity and can be seen as complementary to each other. Moreover, the coefficient of the lagged dependent variable and exogenous technical change go up with a lower growth rate of the share of services. These are strong indications for the role of services compared to industry, especially if the four negative effects dominate. The impact of public on private R&D is reduced through the growth of the service share from positive values to zero. A dynamic panel regression in section 5.1.3 suggests that the move from positive to zero growth of the share of services enhances the growth rate of labour augmenting technical change showing that the four effects going in the same direction are dominating. This effect works against the productivity slowdown.

Higher (lower) defence R&D growth increases (decreases) the coefficients of private and decreases (increases) those of public R&D near mean and median, but they are not statistically different from zero in this area (Figure 4; Table 5). Defence R&D growth has an impact on direct effects in the productivity function (1') but leaves the impact on the coefficients insignificantly different from zero in the relevant range with the exception of the lagged dependent variable; it stays between the contributions from literature, which shows contradictory results. It strengthens the impact of public on private R&D (Figure 13) and therefore works against the fall in the indirect and total effects of public R&D (see below) and thereby against the productivity slowdown.

Lower past productivity growth, $dlth07(-2)$, reduces the coefficients of private and public R&D in the neighborhood of the mean and media (Figure 5). By implication, the productivity slowdown is self-perpetuating by decreasing the coefficients with a lag of two periods. This takes place in a statistically significant area, about four percentage points below and above the mean (see Table 6).

With actually falling $dlberdst(-5)$, the coefficients for private R&D growth go up and those for public R&D go down (Figure 6) and therefore stimulates a shift from public to private R&D.

With falling $dlpubst(-3)$, coefficients of private R&D are increasing and those of public R&D are decreasing. These latter two effects support the idea that the coefficients of private R&D growth go up and those of public R&D growth go down. Again, we find here an increased recent incentive to move to more private R&D. However, this effect also suggests a possibility or problem for public R&D policies if they enhance the growth rate of public R&D: It will not only increase the growth rate of private R&D for given coefficients according to (5), and of the total effect of public R&D according to (see below) Table 18, Figure 17 and 20 where we see positive coefficients of the total effect, but it will decrease the coefficient of $dlberdst$, which is a direct disadvantage, and it will increase that of public R&D

growth, both in (1'),¹⁷ and it will increase also the coefficient of exogenous technical change, and the coefficient of the lagged dependent variable. The net result may be positive or negative, but policy becomes more powerful in the sense of increasing the coefficients of public R&D in (1') at the cost of weakening that of private R&D.

Lags of all variables in the productivity function (1') have an effect either on the coefficients of (1') or on those of (5) but never on both. The fall in *dlfberdst(-3)* increases the impact of public on private R&D in (5) and decreases BERD biased technical change in the first-order condition (5). Falling *dlberdst(-2)* increases the impact of public on private R&D and diminishes exogenous BERD biased technical change. *dlfpubst* is the only regressor that does not affect any of the coefficients in (1') and (5) for any chosen lag (zero to five) or degree of polynomial (one to three).

Table 17 Two-way summary: Effects of smoothing factors on functional coefficients (a)

Funct.coef.(b). factors	c(1)	c(2)	c(3)	c(4)	c(5)	c(6)	c(7)	c(8)	Remarks
<i>log(hc/hc(-5))</i>	0	+	0	0	0	0	0	+	
<i>d(log(GDP(-5)/LF(-5))</i>	+	+	+	0	-	-	0	0	
<i>d(log(srvy))</i>	-	-	0	-	-	+	-	+	(c)
<i>d(log(defrdy))</i>	(-)	-, +	(+)	(-)	(-)	(+)	0	+	(d) for (.); (g) for c(2)
<i>dLTH07(-2)</i>	-	+, -	+	+	(+)	(+, -)	0	0	(d) for (.); (f) for c(2)
<i>dlberdst(-5)</i>	+	0	-	+	0	+	0	0	
<i>dlpubst(-3)</i>	(+), -	0 (+)	-	+	0 (-)	+, -	0	0	(e); (f) for c(6)
<i>dlfberdst(-3)</i>	0	0	0	0	0	0	+	-	-
<i>dlfpubst</i>	0	0	0	0	0	0	0	0	
<i>dlberdst(-2)</i>	0	0	0	0	0	0	+	-	
<i>sumg4</i>	0	0	0	0	0	0	+	+	

(a) Collection of information from Table 2 to 16, and Figure 1 to 16. Signs as slopes of figures. (b) Coefficient c(1) - c(6) of equation (1'); coefficient c(7) and c(8) of equation (5). (c) signs near zero growth of services. (d) Effect in parentheses when coefficients remain insignificantly different from zero in relevant range. (e) d(c(1)) positive near zero value of *dlpubst(-3)*, negative near mean and median. (f) falls with deviation from mean and median. (g) positive from zero growth until 0.01.

Collecting results from the perspective of the coefficients of (1') and (5) we can give the following interpretation of the results in Table 17, using mostly falling growth rates again.

¹⁷ Note the similarity with the Lucas' critique in macroeconomics.

Factor strengthening the coefficient of private relative to public R&D in (1') are the fall of BERD stock growth rates, $dlberdst(-5)$, and the fall of the lagged public R&D stock growth, $dlpubst(-3)$.

Factors weakening the coefficient of private relative to public R&D are the fall of the five-year lagged growth of GDP per worker, service (% GDP) growth rates when going from positive to zero (while the share is still increasing).

Exogenous technical change, the intercept of the productivity function, is enhanced through falling service (% GDP) growth rates, falling growth rates of twice lagged labour productivity growth, falling (from the mean and median) three-year lagged public R&D growth rates, for countries with lower defence R&D growth compared to other countries.

Exogenous technical change, the intercept of the productivity function, is reduced through falling growth rates of GDP per worker, falling growth rates of five-year lagged BERD stock, falling (near zero) three-year lagged public R&D growth rates, and for countries with higher defence R&D growth compared to other countries as since 2014.

The coefficient of lagged dependent variable, $dlth07(-1)$, falls with the falling growth rates of human capital, GDP per worker five years lagged, second lag, $dlth07(-2)$, falling while being below mean and median, $dlpubst(-3)$ near mean and median but insignificantly so.

The coefficient of lagged dependent variable, $dlth07(-1)$, is higher with the lower growth rate of service (% GDP), with lower (higher) defence R&D growth near zero mean (larger than 0.01), and second lag, $dlth07(-2)$, falling while being above mean and median.

The coefficient of foreign private R&D growth, $dlfberdst$, increases with falling growth rates of GDP per worker lagged five years near mean and median, services (%GDP), when $dlpubst(-3)$ falls away from mean and median but insignificantly so.

The coefficient of foreign private R&D growth, $dlfberdst$, is lower for lower growth rates of $dlth07(-2)$, higher growth rates of defence R&D (but coefficient remains insignificant).

The coefficient of foreign public R&D growth, $dlfpubst$, is higher with lower growth rates of GDP per worker for five years lagged near mean and median, and with higher growth rates of defence R&D.

The coefficient of foreign public R&D growth, $dlfpubst$, is lower with lower growth rates of service (% GDP), $dlberdst(-5)$, with deviation from mean and median of $dlpubst(-3)$.

The impact of public on private R&D is reduced through falling human capital growth, service (% GDP) growth rates with lag three to five, when going from positive to zero, falling $sumg4$ (near mean, and with ups and downs elsewhere), and for countries with lower defence R&D growth compared to others.

The impact of public on private R&D is increased through lower growth rates of $dlfberdst(-3)$, and of $dlberdst(-2)$ and for countries with higher defence R&D growth compared to others.

BERD biased technical change is supported through four and five-year lagged services (% GDP), $dlogsrvy(-4)$, $dlogsrvy(-5)$ going from positive to zero, and diminished by lower $dlfberdst(-3)$, $dlberdst(-2)$, lower $sumg4$.

In Table 17, under all coefficients we find plus signs, minus signs, and zeros. There is no single dominating factor that could be found at this stage.

5.4 From partial contribution to total functional regression coefficients

The values on the vertical axis of Figure 1 to 16 do not give us the values of the regression coefficients of equation (1') and (5) but only the partial contributions of the functional series denoted on the horizontal axis. In order to get the coefficients, we rerun the regressions using all functional series simultaneously and select the smooth polynomial with the highest degree. As part of the regression output, we get the contribution of the first listed functional coefficient series to the regression coefficients of (1') and (5). We can then save the contribution of the first listed variable for each coefficient in (1') or (5) for each country-year observation. In the quadratic case, for example, this is a coefficient derived from the form $c+aZ+bZ^2$.¹⁸ For (1') this is a matrix with six columns in stacked panel format (one for each coefficient in (1')) and NT rows. For (5) this is a matrix with two columns and NT rows. We repeat the procedure for the next listed variables. We get such a matrix for each first-listed functional coefficient series. Adding up the matrices belonging to (1') gives us the matrix of coefficients with six columns for $c(1)$ to $c(6)$. Adding up the matrices belonging to (5) gives us the matrix of coefficients with two columns for $c(7)$ and $c(8)$. The results for these coefficients are now presented in Figure 17. In Table 18 we show the standard statistics for the functional regression coefficients. The intercept has a mean and median at about 0.16, which may seem a bit high for the Cobb-Douglas interpretation of exogenous technical change. However, this value may be driven up by negative effects of foreign private and public R&D, which we have added to the function used in economic theory (Huang et al., 2025). These negative effects suggest that foreign R&D variables have more of a competitive effect than that of a knowledge spillover (Luintel and Khan, 2004). The lagged dependent variable has on average roughly a unit coefficient although the growth rate version of the variable in the data is stationary (see Table 1). Private R&D in (1') has mainly positive coefficients around 0.43 with a standard deviation that is more than twice as high. Direct public R&D in (1') has a positive mean and a negative median, and also a relatively large standard deviation. The foreign R&D variables have low standard deviations relative to mean and median. Overall, the method of functional coefficients in our case produces a huge variability in the estimated coefficients. The intercept of the indirect effect, $c(7)$, has very high growth rates compared to what one might expect as BERD-biased technical change or growth of inefficiency; the separability of CD, CES, or VES functions may be worth reconsidering.¹⁹ The coefficient of public R&D, with mean 3.33, median 3.05, and standard deviation of 2.51, would suggest that each percentage point higher growth rate of public

¹⁸ See Cai et al. (2000) for an exact but lengthy and complicated formulation.

¹⁹ A highly nonlinear function that could be tested is $T = F(G(B, B^*), H(P, P^*))$ where F , G , and H are all CES functions. The equality of marginal products of B and P would leave foreign R&D variables in the first-order equation. This would require a paper on its own though as the current one is already extraordinarily long.

Figure 17 Functional regression coefficients for productivity function and first-order conditions (a)



(a) Functional regression coefficients of productivity function (1') and, last line, first-order condition (5)

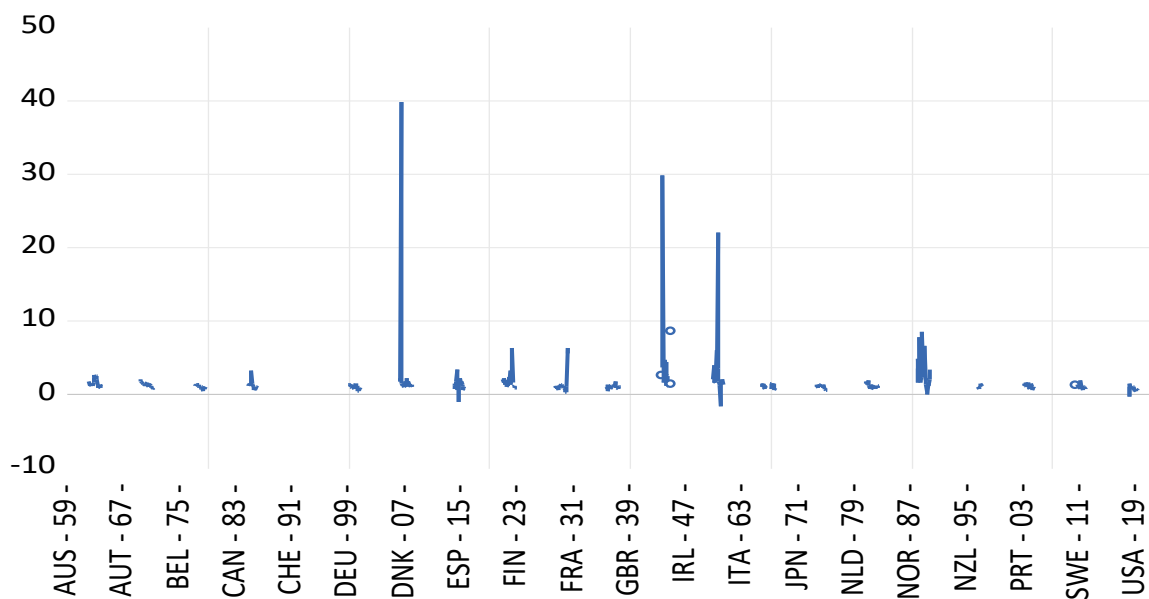
Table 18 Basic statistics of functional regression coefficients of productivity function and first-order conditions (a)

Statistic → Variable ↓	Mean	Median	Maximum	Minimum	Std dev
<i>intercept, c(1)</i>	0.161	0.166	0.955	-0.924	0.140
<i>dlth07(-1), c(2)</i>	0.972	1.066	9.559	-15.69	1.534
<i>dlberdst, c(3)</i>	0.495	0.368	12.75	-1.973	1.072
<i>dlpubst, c(4)</i>	0.071	-0.027	4.78	-6.681	0.802
<i>dlfberdst, c(5)</i>	-1.560	-1.26	17.09	-24.17	2.494
<i>dlfpubst, c(6)</i>	-2.279	-2.605	24.99	-9.221	3.755
<i>intercept, c(7)</i>	0.248	0.257	0.357	-2.61	0.139
<i>dlpubst, c(8)</i>	3.276	2.965	46.35	-1.067	2.532
<i>c(4)+c(3)*c(8), (b)</i>	1.566	1.125	39.8 (332.6)	-1.675	2.79

(a) Statistics of functional regression coefficients of productivity function (1'), c(1)-c(6), and first-order conditions (5), c(7) and c(8). 452 observations. (b) total effect of public R&D shown in Figure 18.

R&D turns into 3 percentage points higher growth of private R&D, which is a strong effect of public-private interaction. Insertion of (5) into (1') yields $c(4) + c(3)*c(8)$ as total effect of public R&D on technical change shown in Figure 18. The corresponding standard statistics are shown in the last row of Table 18. The maximum of 332.9 is deleted in Figure 18 to make the other values more clearly visible. The positive indirect effect clearly outweighs the often negative direct effect in Figure 17 and thereby supports the idea of Park (1995) and Sveikaukas (2007) supported by the analysis of Soete et al. (2022).

**Figure 18 The sum of direct and indirect effect of public R&D on technical change
C_DLPUBST_TOT**



Having shown the functional coefficient results in Figure 1 to 18, the question is now how this relates to the productivity slowdown. From the perspective of functional coefficient regressions, the question is what the trends in the coefficients of Figure 17 are. To see this, we regress each coefficient series on its own one or two lags, a time trend, and autoregressive processes correcting for serial correlation. In case of doubts about the best specification we use the covariance proportion of the Theil inequality coefficient, which consists of the bias proportion, the uncertainty proportion, and the covariance proportion as unexplained ‘rest’, which should be high. Table 19 reports these results. For the interpretation it seems most useful to imagine the standard graph of a difference equation with $x(-1)$ and x on the axes.

Table 19 Dynamic trend analysis for functional coefficient estimates (a)

Funct. coef. (b); regressors	$C(1)$	$c(2)$	$c(3)$	$c(4)$	$c(5)$	$c(6)$	$c(7)$	$c(8)$	$c(4)+c(3)*c(8)$
<i>constant</i>	0.263	1.41	0.393	0.48	-1.115	-2.7	0.077	3.942	2.244
<i>dep.lag 1</i>	-	0.2	0.096	0.225	0.269	0.420	0.963	0.721	0.184
<i>dep.lag 2</i>	-0.041	-	0.046	-	-	-	-0.280	-0.335	-
<i>dep.lag 3</i>	-	-	-	-	-	-	-	-0.148	-
<i>dep.lag 4</i>	-	-	-	-	-	-	-	0.051	-
<i>trend</i>	-0.0017	-0.0127	-	-0.007	-	0.026	-	-0.0335	-0.0196
<i>ar(1)</i>	-	-	-	-	-	-0.296	-0.219	-	-
<i>ar(2)</i>	-	-	-	-	-	-	-	-	-
<i>ar(3)</i>	-	-	-	-	-	-	-	-0.136	-
<i>ar(4)</i>	-0.03	-	-	0.057	-	-	-	0.046	-
<i>ar(5)</i>	-	-	-	-0.088	-	-	-	-	-
<i>Periods, tot obs.</i>	18, 303	23, 425	22, 399	18, 303	23, 425	22, 399	33, 434	28, 323	23, 377
<i>Adj. R²</i>	0.182	0.118	0.095	0.313	0.162	0.176	0.610	0.694	0.291
<i>DW (c)</i>	1.987	2.004	2.07	2.05	1.967	2.125	1.97	2.181	2.080

(a) Common information for the regressions: Method: Panel EGLS (cross-section weights and fixed effects). Iterate coefficients after one-step weighting matrix. Period SUR (PCSE) standard errors & covariance (d.f. corrected). 20 countries. Statistical significance at least at the ten percent level. (b) Coefficients as indicated in equations (1') and (5). (c) Durbin-Watson statistic.

In all cases we do have a (sum of) slope coefficients below unity and larger than minus unity, which would ensure stability. However, in five (one) of our nine cases the time trend shifts the equation line down (up) and along the 45-degree line coefficients then fall (increase). For *dlberdst*, $c(3)$, and *dlfberdst*, $c(5)$ there is no such trend, and because of the intercepts, stability leads in the long run to a negative coefficient for foreign and a positive coefficient for domestic private R&D, meaning that foreign competition matters more than foreign spillovers from private R&D. The coefficient of *dlfpbst*, $c(6)$, has a strongly positive trend

and therefore will increase unless a structural break stops it. Foreign public R&D therefore has strongly positive spillovers in the future when the negative intercept is dominated by the positive trend. A zero trend is in $c(7)$, the intercept of (5); with positive intercept and slope below unity it converges to a positive constant value. Inefficiency or BERD biased technical change has positive and constant growth rates. A negative time trend also applies to the sum of direct and indirect effect of public R&D in the last column of Table 19.

Figure 19 shows all the univariate trends of the functional coefficients of (1') together with their two-standard-errors bounds. The purpose here is only to make the trend in the coefficients visible. It is not to give an explanation again, and therefore the functional coefficient series are not included again. As suggested by the interpretation of the regressions of Table 19, only domestic and foreign private R&D variables go towards having constant coefficients. Foreign public R&D has increasing coefficients. All other variables have decreasing coefficients; a decreasing intercept suggests falling exogenous technical progress in the productivity growth function (1').

Figure 19 Autoregressive forecasts of functional regression coefficients of the productivity function

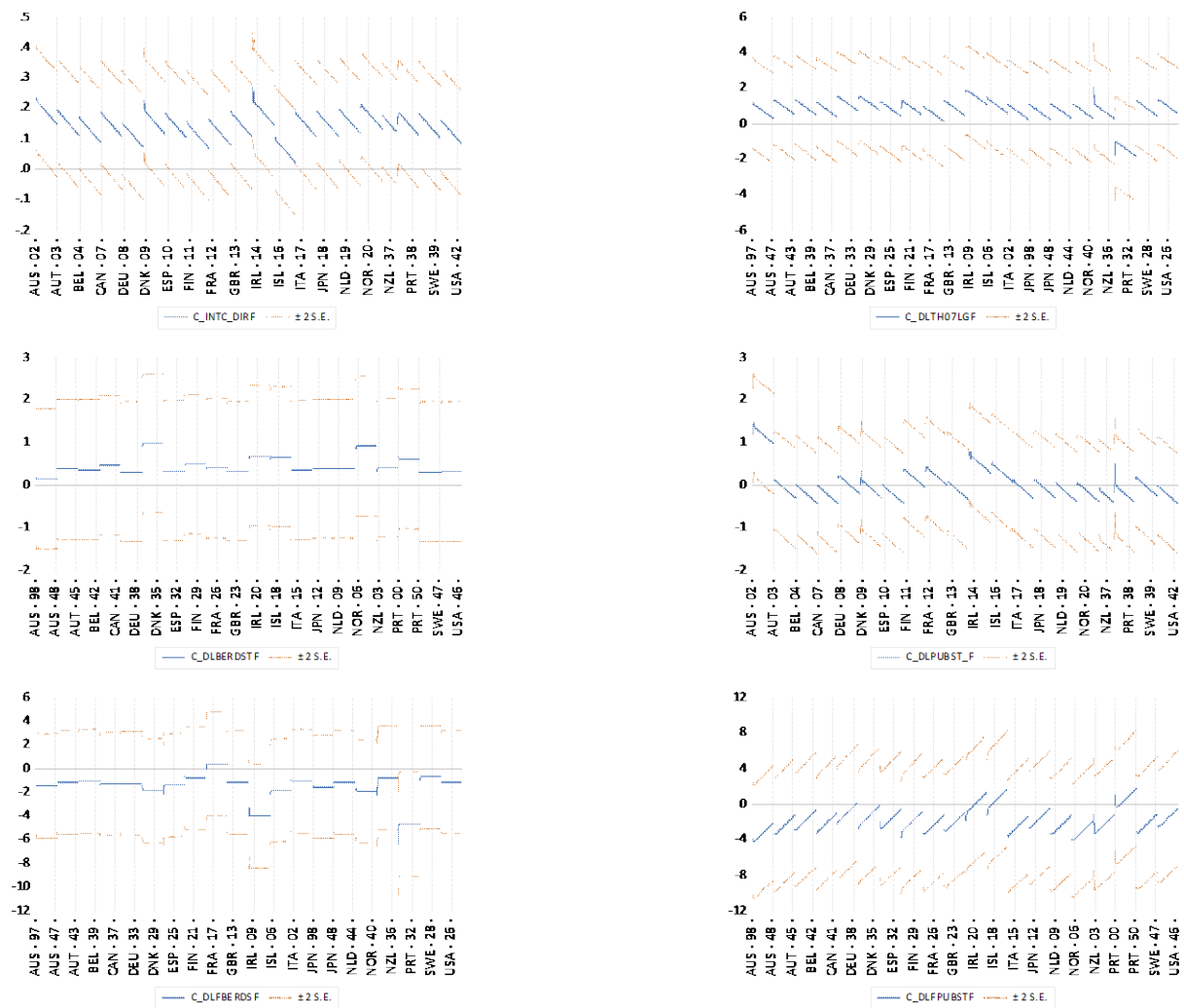
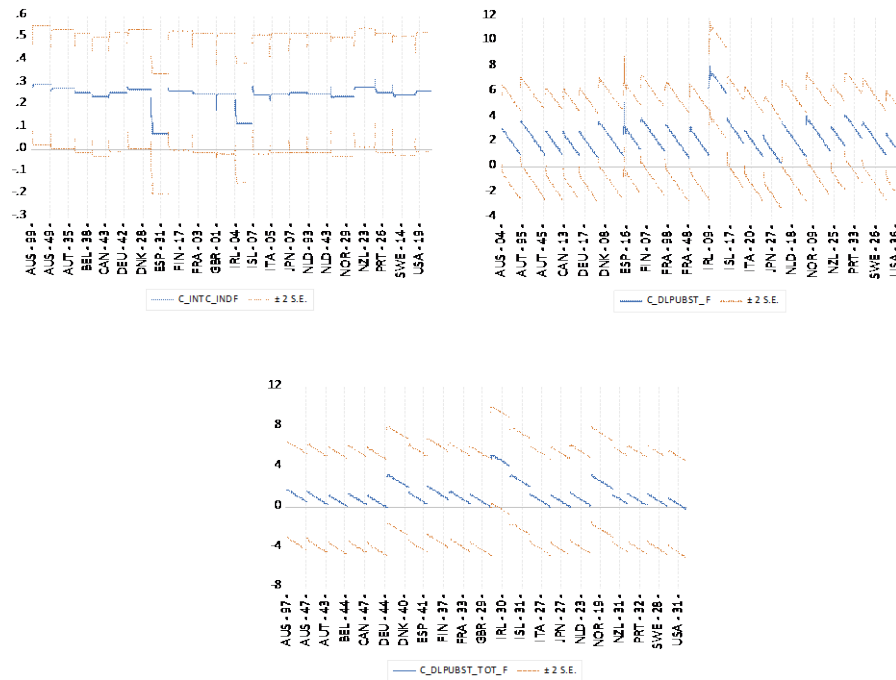


Figure 20 shows that the intercept of (5) goes to a constant value and thereby indicating a constant BERD biased technical change or/and increasing inefficiency, whereas the slope coefficient goes down. The total effect of public R&D goes down in line with the discussion of the last column of Table 19. The low statistical significance obviously comes from the lack of adding explanatory variables again.

Figure 20 Autoregressive forecasts of functional regression coefficients of the first-order conditions and the total effect of public R&D



6 Summary and conclusion

This paper has shown that regression coefficients for R&D variables in countries' productivity functions are not constant and depend on functional coefficient series (smoothing variables).

The overall conclusion from putting all these partial effects on regression coefficients together is as follows. Panel average values of coefficients for domestic and foreign private R&D, $dlberdst$ and $dlfberdst$, go to constant values as time passes. As direct, indirect, and total coefficients of public R&D go down, we find an important aspect of the productivity slowdown and it is plausible that investment in R&D has shifted from public to private R&D in recent decades. This does not necessarily justify the extent to which this has taken place. In the raw data, public R&D flows as share of GDP is mostly constant (not shown; see OECD MSTI), with some exceptions like Germany where they are increasing. The fall in the growth rates of public R&D flows therefore is roughly the same as that in GDP, which is caused by the productivity slowdown and some recent crises (Fernald et al. 2025). Falling regression coefficients add to this.

With changing coefficients, it is much harder to calculate the advantages of R&D investments, which is also an innovative insight of this paper with relevance for the administration of innovation policy. The method allows us to find the coefficients for the whole panel with values differing for each country-year pair.

In regard to hoping to overcome the productivity slowdown we can only hope for a structural break ahead, phasing out the negative trends. This could perhaps be that the share of services becomes constant; this would stop reducing the effect of public on private R&D. Moreover, technical change could get directed towards services or/and its inputs as suggested by the logic of induced sectoral technical change in Boppart and Weiss (2013) and Comin et al. (2025). According to Boppart (2014) this redirection of technical change has not yet happened because the prices of industry relative to those of service are still falling in his data. If it happens in the future, it could take a lot of time though as in the data of Boppart and Weiss (2013) for durable and non-durable goods where the delay is thirty years. Similarly, Comin et al. (2025) point out that technical change in services accelerated in the 1980s ‘and achieved the fastest growth rate in the last decade’. However, this is still not enough to get rid of the productivity slowdown. They expect the recovery of TFP growth through innovation in services in 2100. Their model has only private R&D.

Besides waiting for market driven sectoral effects, education policy could enlarge human capital growth in order to improve the effects of public on private R&D that we have shown to depend on human capital. A stronger positive effect on the lagged dependent variable can also be achieved through more human capital growth. This may require targeting education to areas able to link technical change and R&D. Improved education could go together with mission-oriented and public R&D, which both increase TFP and GDP growth (Ziesemer 2021c, 2024b; Soete et al. 2022). The reduced incentive through falling coefficients does not necessarily exclude the possibility to go against this with higher expenditure.

An outlook from here to future research suggests the following issues. Regarding modelling, specifying nested CES functions may lead to richer first-order equations than just linking domestic private and public R&D, but they would also be highly nonlinear. Other functional coefficient series may be found by authors with a different background knowledge; this may lead to more or less different shifts in the coefficients.²⁰ Progress in econometrics in the area of functional coefficients is going through all sub-areas of econometrics that dynamic panel data analysis has gone through (Nickel bias, endogeneity, instrument choice, Hansen-Sargan statistics, cross-section dependence, unit root and cointegration analysis); it will take some time until teams of package providers have taken decisions, which of the new elements can be put together meaningfully.

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²⁰ Bart Verspagen has pointed out that Khan and Luintel (2006) may contain interesting candidate variables. This would lead us more into the realm of international economics.

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