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The “Mission Impossible” of a Steady State, Value-Neutral Investments, and Value Maximization in Terminal Value Calculations

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Abstract Assumptions about growth and profitability in terminal values significantly influence corporate valuations but are inherently uncertain and challenging to predict. To mitigate potential manipulation, theory and valuation practice often recommend two premises: 1) the firm operates in a steady state, justifying a perpetual constant growth rate, and 2) net investments in the terminal value calculation are value-neutral, yielding returns equal to the firm’s cost of capital. This paper evaluates the consistency of these assumptions with a value-maximizing investment policy. By modeling the firm as a series of investment projects under a constant investment technology with decreasing returns to scale, we find that value-maximizing investment levels remain steady over time, leading to zero growth. Achieving a steady state with positive nominal growth requires continuous productivity improvements in the investment technology. When such improvements occur, optimal investment scales can increase at a constant rate, necessitating value-enhancing net investments. In this scenario, value-neutral net investments misalign with the optimal strategy, resulting in suboptimal decisions and reduced terminal value. Our findings indicate that, under standard investment technology assumptions, positive nominal growth, value-neutral net investments, and value-maximizing behavior cannot coexist in a steady state. Consequently, standard setters and practitioners must choose between adhering to investment theory by abandoning the value-neutrality assumption—accepting

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increased ambiguity and uncertainty in growth projections—or retaining the value-neutrality assumption at the expense of theoretical soundness in corporate valuation.

Keywords Corporate valuation · Terminal value · Perpetuity phase · Steady state · Growth · Return

JEL Classification G32 · G31 · D25 · M41

1 Introduction

Corporate valuation serves multiple purposes (Moxter 1983, p. 5; Matschke et al. 2023, p. 54). First, it determines the reservation price — the maximum price a buyer is willing to pay or the minimum price a seller is willing to accept — at which the transaction’s net present value equals zero, making both parties indifferent to proceeding. When both sides are aware of their respective reservation prices, a valuation can facilitate negotiations to settle on a mutually beneficial price. Additionally, both buyer and seller will use corporate valuation to support their arguments in favor for a lower (buyer) or higher (seller) price.

The intrinsic value of a company is the present value of all expected future cash flows, based on the strategy of the current or potential owner. Valuation thus requires projecting the corresponding future cash flows; this projection is generally divided into two stages (Koller et al. 2020, pp. 259–260, 360; Drukarczyk and Schüler 2021, p. 136; Kuhner and Maltry 2022, p. 1035): Phase one involves the detailed projection of relevant earnings, balance sheet and cash flow figures for the near-term business plan with a usual time span of three to five years, while phase two represents the more distant future and assumes a perpetuity phase with an infinite lifetime of the firm. During this phase, the “terminal value” is calculated as the present value of cash flows growing at a constant rate to infinity.

As the discounted terminal value often accounts for more than half of the corporate value, the choice of the nominal perpetual growth rate is a critical determinant. At the same time, it is difficult to project and subject to high uncertainty. Given the conflicting interests of both parties, the terminal growth rate is a standard “pain point” in negotiations between potential buyers and sellers. As a result, both academic literature and standard setters have long sought ways to rationalize growth and related profitability assumptions in valuation reports, aiming to limit the potential leeway for manipulating valuation results.

One standard assumption to justify the perpetual constant growth rate is that the firm to be valued is in a “steady state” when it enters the second phase (Lobe 2006, p. 22; Tinz 2010, p. 198; Damodaran 2012, pp. 306–317; Penman 2013, pp. 111–118; Drefke 2016, p. 111; Koller et al. 2020, pp. 285–289; Kuhner and Maltry 2022, p. 1035; Palepu et al. 2022, pp. 257, 333–336).¹ The second important assumption relates to the profitability of the firm’s (net) investments and the firm’s

¹ See also German and Austrian valuation standards (IDW S 1 2016, para. 78, 98; KFS/BW 1 2014, para. 63, 115).

ability to earn excess returns above the cost of capital over an infinite time horizon: It is frequently suggested that the firm can only invest in “value-neutral” investment opportunities earning its cost of capital (Penman 2013, p. 142; Pinto et al. 2015, p. 468; Koller et al. 2020, p. 294; Palepu et al. 2022, pp. 289–290).

In this paper, we analyze whether the assumptions of a steady state and of value neutrality for terminal value calculations are compatible with a policy of maximizing firm value. We apply the most common definition of the required steady state and model the firm as a sequence of investment projects in a given technology exhibiting decreasing returns to scale as a standard assumption of investment theory.

Our work builds on a model of an investment technology with decreasing marginal returns to study the interrelationships between steady state, growth, and associated inflationary effects (Schwetzler 2018). This paper focuses on the compatibility of the assumption of value-neutral net investments, nominal growth and a policy maximizing the firm value, a topic that has received surprisingly little attention in the academic literature so far.²

As a first result, our analysis reveals that combining the described investment technology with uncertainty of future operating cash flows and corresponding (re-)investments rules out the possibility of reaching a steady state for further analysis. Continuing our examination under the assumption of certainty, we find that with an exogenous constant investment technology, the realization of the optimal investment scale yields a perpetuity model with zero net investments and no growth. Positive net investments result in either a deviation from a steady state (financial assets) or in a deviation from a steady state combined with a suboptimal investment scale (operating assets). Therefore, combining the investment technology with a steady state and assuming a constant positive nominal growth rate requires a specific type of technological improvement of the investment technology. If this technological requirement holds, we find two contradictory perpetuity-phase scenarios:

- If the firm realizes the optimal, value-maximizing investment scale, it maintains a steady state. Notwithstanding, net investments are not value-neutral and carry a positive net present value.
- If the firm realizes value-neutral net investments, it fails to meet the steady-state requirement and realizes a suboptimal investment scale by either over- or underinvesting in its operating assets, depending on whether the net investments are made in operating or financial assets.

We thus conclude that the combination of

1. a steady state with a constant positive growth rate,
2. value neutrality of net investments, and
3. a value-maximizing investment strategy

is impossible—even under certain cash flows—when assuming an investment technology displaying decreasing returns to scale.

² Knoll (2016, p. 34) points out that this assumption leads to a situation of the firm’s market value equaling its book value. See the following discussion in Sect. 3.4.

This paper makes several important contributions to the literature:

- First, it combines basic principles of investment theory with core assumptions of terminal value models in corporate valuation, shedding light on potential inconsistencies.
- Second, it provides a deeper understanding of the economic requirements for a steady state. While some studies analyze the steady-state condition for a firm with multiple business segments³ or investment projects⁴, we focus on modeling a single, homogenous investment opportunity and demonstrate in depth that a perpetual constant growth rate requires a constant, exogenous change in the firm's productivity, which cannot be achieved with investments in financial assets. As the results obtained for this investment technology also apply when a firm operates with several of them, we enhance the current body of steady state literature.
- Third, our work contributes to the general discussion around the growth assumptions in terminal value models and highlights a particular dilemma for standard setters: Given decreasing returns to scale as a standard property for a firm's investments, standard setters and practitioners must choose between the value-neutrality and value-maximization principle.

The subsequent sections are organized as follows. Section 2 reviews the perpetuity phase assumptions discussed by the literature and standard setters. In Sect. 3, we introduce our model of a firm and derive common terminal value and steady state definitions. Section 4 further outlines a simple microeconomic model of a basic investment technology. Then, Sect. 5 analyzes several perpetuity scenarios involving net investments in operating or financial assets with respect to their feasibility and consequences. Finally, Sect. 6 summarizes our findings and their implications.

2 Review of the Literature On Terminal Value Assumptions

Academic literature, standard setters, and valuation practice all recognize the difficulty in deriving reasonable perpetuity assumptions for terminal value calculations. They have developed guidelines and recommendations addressing two key questions:

1. Which conditions are necessary for a stable development of the firm's earnings and cash flow, thus justifying a perpetual valuation?

The corresponding recommendations in academic literature emphasize the importance of the firm being in a "steady state" during the terminal value phase (Tinz 2010, p. 198; Damodaran 2012, pp. 306–317; Penman 2013, pp. 111–118; Drefke 2016, p. 111; Koller et al. 2020, pp. 285–289; Kuhner and Maltry 2022, p. 1035; Palepu et al. 2022, pp. 257, 333–336). This steady state is commonly defined by "all

³ Dierkes and Schäfer (2021) discuss difficulties of steady state assumptions for different business lines and investment opportunities with different profitability and growth prospects, including internal investment as well as financing relationships, but do not address properties of investment technologies.

⁴ Meitner (2013) analyzes the impact of two different types of investments on the steady state.

financial figures” of the firm growing at the same rate as the free cash flow (Lobe 2006, p. 22; Damodaran 2012, p. 324; Koller et al. 2020, pp. 259–260, 360; Kuhner and Maltry 2022, p. 1035; Palepu et al. 2022, p. 338).⁵

Valuation standards also support this definition. The International Valuation Standards require “a stabilized level of growth and profits” for any asset to be valued, also pointing to constant returns as a consequence (IVSC 2024, para. A20.09 (c), A20.23). The US AICPA standards for goodwill impairment similarly emphasize constant returns and allow for a differentiation between assets in place and net investments (US AICPA 2013, para. 4.30).⁶ Lastly, German and Austrian valuation standards mandate a steady state when entering the terminal value phase as well (IDW S 1 2016, para. 78, 98; KFS/BW 1 2014, para. 63, 115).

2. What are reasonable assumptions for the firm’s profitability during the perpetuity phase?

Recommendations also focus on the question of whether a firm can permanently earn returns on invested capital (ROIC) above its cost of capital. Most authors are skeptical, citing competitive forces that would erode such excess returns over time (Penman 2013, p. 17; Pinto et al. 2015, p. 479; Palepu et al. 2022, p. 334). “In a perfectly competitive economy, ROIC higher than the cost of capital get competed away,” (Koller et al. 2020, p. 139) and only unique competitive advantages or entry barriers could sustain such returns (Damodaran 2012, p. 291; Brealey et al. 2023, p. 309).⁷ Literature describes these excess returns as “abnormal” or “supernormal” (Penman 2013, p. 142; Pinto et al. 2015, p. 468; Koller et al. 2020, p. 294; Palepu et al. 2022, pp. 289–290) and frequently assumes that such a “competitive advantage period” (Damodaran 2012, p. 309; Penman 2013, p. 508) cannot be maintained indefinitely. Instead, a “competitive equilibrium assumption” (Palepu et al. 2022, pp. 333–334) of value-neutral investments without excess returns should be considered, referring to the entire invested capital, or alternatively, only to net investments of the perpetuity phase and their accumulating reinvestments (Penman 2013, p. 154; Pinto et al. 2015, p. 480; Koller et al. 2020, p. 288; Palepu et al. 2022, pp. 333–335).⁸

Compared to academic literature, international standard setters rarely address return assumptions for the perpetuity phase. Most standards do not specify appropriate assumptions on returns.⁹ However, an exception is found in German-speaking

⁵ For a detailed discussion of the requirements, see Levin and Olsson (2000).

⁶ The US AICPA (2019, para. 5.83) guide for a valuation of VC & PE portfolio companies only mentions the steady-state requirement.

⁷ Penman (2013, p. 561) points to measurement problems as a potential reason for positive excess returns. Investments for R&D may not be included in the invested capital reported in the balance sheet, yielding an upwardly biased ROIC above the cost of capital in accounting terms. See also Sect. 3.4.

⁸ Differentiating between assets in place and accumulating net investments allows for the assumption that the former still earn excess returns, while the latter do not (Koller et al. 2020, p. 294; Palepu et al. 2022, p. 334). However, as the return on all assets slowly converges to the cost of capital under this condition, this does not allow for a steady state.

⁹ See, for example, IVSC (2024) or IPEV (2022) for renowned international standards and RICS (2024) (United Kingdom), CBV (2009/2010) (Canada), or NACVA (2023) and ASA (2022) (United States) for

countries: IDW S 1 principles of the German CPA association (IDW) require retained earnings to be invested value-neutral in the perpetuity phase (IDW S 1 2016, para. 37; IDW ES 1 2024, para. 84).¹⁰ Related literature states the firm should “earn its cost of capital” in a steady state (Wagner et al. 2004, p. 897; Tschöpel et al. 2010, p. 407; IDW 2018, ch. A, para. 442). Austrian valuation standards also set value neutrality as the standard but allow for perpetual excess returns above the cost of capital in exceptional cases (KFS/BW 1 2014, para. 64; KFS/BW 1 E 4 2015, para. 23–38).

In summary, valuation literature and standard setters uniformly require a firm to be in a steady state when entering the terminal value phase to justify a constant perpetual growth rate. They recommend that the firm shall earn a return equal to its cost of capital, which results in value-neutral net investments in terminal value models. Exceptions are made only in extraordinary cases.

While not explicitly mentioned in valuation standards, an obvious final requirement of the common assumption set to be analyzed is that a firm and its management should strive to maximize shareholder value and pursue a corresponding strategy of realizing investments carrying a positive net present value (Brealey et al. 2023, pp. 8–14; Berk and DeMarzo 2024, pp. 100–109).

3 Steady State, Growth, and Profitability in Terminal Value Models

3.1 The Firm

We consider a firm with one operating business financed by debt and equity, maintaining a constant optimal leverage ratio and adjusting its level of debt according to the realized value of the firm.¹¹ Fig. 1 illustrates the firm’s balance sheet at the end of period t and its income statement for period t :

The firm’s net investments are mainly driven by the volume of capital expenditure (CAPEX) and are financed by a constant retention rate (q) in percentage of the firm’s NOPAT in period t :

$$NI_t = \text{CAPEX}_t - D\&A_t \pm \Delta\text{NWC}_t = I_t - I_{t-1} = q \cdot \text{NOPAT}_t. \quad (1)$$

Accordingly, free cash flow (FCF) for period t in terms of operating accounting figures is given by

$$\text{FCF}_t = \text{NOPAT}_t - NI_t = \text{NOPAT}_t (1 - q), \quad (2)$$

local standards. As one of the few exceptions for an association, TEGOVA (2020, para. 6.3.1.5) explicitly formulates the possibility of a ROIC above or equal to the cost of capital for the steady state of a business.

¹⁰ As these standards prefer the DCF equity approach (“Ertragswert” in German), the neutrality requirement refers to the equity fraction of net investments that is financed by retained earnings after taxes.

¹¹ The debt policy in this case is frequently referred as “active.” See Miles and Ezzell (1980) or Arnold et al. (2019).

and the return on invested capital¹² equals

$$\text{ROIC}_{t+1} = \frac{\text{NOPAT}_{t+1}}{I_t}. \quad (3)$$

In period t , the firm holds no excess cash or financial assets, and its corporate tax rate τ is constant.

3.2 The Terminal Value of the Firm

The standard equation for a terminal value under uncertainty allowing for a constant perpetual growth of expected free cash flows is

$$\text{TV}_t = \frac{E(\text{FCF}_{t+1})}{r - E(g)}. \quad (4)$$

With: TV_t : terminal value at the end of period t , $E(\text{FCF}_{t+1})$: expected free cash flow in $t + 1$, $E(g)$: expected constant growth rate of the free cash flow,¹³ and r : the firm's constant cost of capital.¹⁴

Panel A: Balance sheet

Fixed assets _{<i>t</i>} (FA _{<i>t</i>})	...
Current assets _{<i>t</i>} (CA _{<i>t</i>})	...
Total assets	...
<hr/>	
Equity _{<i>t</i>} (E _{<i>t</i>})	...
Debt _{<i>t</i>} (D _{<i>t</i>})	...
Current liabilities _{<i>t</i>} (CL _{<i>t</i>})	...
Total equity & liabilities	...

Panel B: Income statement

Sales_{<i>t</i>}	...
– Operating costs _{<i>t</i>}	...
= EBITDA_{<i>t</i>}	...
– D&A _{<i>t</i>}	...
= EBIT_{<i>t</i>}	...
– Corporate taxes _{<i>t</i>}	...
= NOPAT_{<i>t</i>}	...

Fig. 1 Financial statements in period t . With: $\text{NWC}_t = \text{CA}_t - \text{CL}_t$: net working capital at the end of period t , $I_t = \text{FA}_t + \text{NWC}_t = E_t + D_t$: invested capital at the end of period t , EBITDA_t : earnings before interest, taxes, and depreciation & amortization in period t , $\text{EBIT}_t = \text{EBITDA}_t - \text{D\&A}_t$: earnings before interest and taxes in period t , and $\text{NOPAT}_t = \text{EBIT}_t(1 - \tau)$: net operating profit after taxes, with τ as corporate tax rate

¹² We discuss accounting returns as potential measures of economic success in Sect. 3.4.

¹³ \tilde{g} as uncertain growth rate is assumed to be independently and identically distributed, i.e., $(\tilde{g}_{t+1} | F_t) = \tilde{g} \forall t$.

¹⁴ Our model and the results derived do not require a particular assumption about the model behind the firm's cost of capital, such as the "CAPM" model for the cost of equity. The only requirement necessary for our analysis is that r is an expected one-period rate of return that reflects the risk of cash flows and is constant over time.

Setting the expected net investments in $t + 1$ equal to the nominal expected growth of I_t leads to

$$\begin{aligned} E(NI_{t+1}) &= E(I_{t+1}) - I_t \\ &= E(\text{CAPEX}_{t+1} - \text{D\&A}_{t+1} \pm \Delta\text{NWC}_{t+1}) = E(g) \cdot I_t. \end{aligned} \quad (5)$$

Combining this definition with its financing through retained earnings from Eq. 1 yields

$$q \cdot \frac{E(\text{NOPAT}_{t+1})}{I_t} = E(g), \quad (6)$$

and, finally, relates the expected growth of the perpetuity phase to its return of $E(\text{ROIC})$:

$$q \cdot E(\text{ROIC}) = E(g). \quad (7)$$

Extending the standard growth model of Eq. 4 by relationships (2) and (7) yields the following equation for the calculation of the terminal value (Koller et al. 2020, pp. 49–51):

$$\text{TV}_t = \frac{E(\text{FCF}_{t+1})}{r - E(g)} = \frac{E(\text{NOPAT}_{t+1})(1 - q)}{r - q \cdot E(\text{ROIC})}. \quad (8)$$

Equation 8 is well known as the Gordon-Shapiro model (Gordon and Shapiro 1956, p. 105) and allows for an analysis of the implicitly assumed profitability and growth assumptions of the perpetuity phase.¹⁵ This becomes evident when considering a value-neutral total invested capital and assuming $E(\text{ROIC}) = r$. By substituting into formula (8), the terminal value calculation further simplifies, and growth becomes value-neutral with no effect of $E(g)$ on the terminal value:¹⁶

$$\text{TV}_t = \frac{E(\text{NOPAT}_{t+1})}{r} \quad (9)$$

Does Eq. 8 account for inflation? Basic valuation principles require future nominal cash flows measured in the prevailing purchasing power to be discounted with nominal cost of capital containing a premium as compensation for the loss in purchasing power caused by inflation (e.g., Ballwieser 1981, p. 97). Since all variables

¹⁵ For their “value driver model,” Koller et al. (2020, pp. 49–51) demonstrate that the same relationship holds when only considering a constant return on all net investments and their accumulating reinvestments, which they refer to as new invested capital. However, we do not focus on this model, as return differences between old and new assets do not allow for a steady state. See also Footnote 8.

¹⁶ By introducing personal income taxes and assuming, in accordance with the German guidelines IDW S 1, an “equivalent” payout policy of the firm being valued and the alternative investment reflected in the discount rate, the same result can be obtained (Schwetzler 2021, pp. 214–215).

in Eq. 8 are nominal figures, this model fully meets this requirement and thus reflects any potential impact of inflation on corporate value.¹⁷

3.3 The Steady-state Requirement

Next, we examine the assumption of a “steady state” for terminal value calculations that requires all financial figures to increase at a constant and uniform growth rate g . As the stochastic properties necessary for multiperiod discounting under uncertainty are attached to the firm’s free cash flow,¹⁸ an intuitive starting point to analyze the requirements for a steady state under uncertainty is the free cash flow under a constant perpetual growth rate. We recognize that the subsequent free cash flow in $t + 1$ is subject to uncertainty after period t is realized, resulting in an uncertain free cash flow \widetilde{FCF}_{t+1} and an expected free cash flow $E(FCF_{t+1})$ of

$$\widetilde{FCF}_{t+1} = FCF_t (1 + \tilde{g}) \quad (10)$$

and

$$E(FCF_{t+1}) = FCF_t (1 + E(g)). \quad (11)$$

As the firm’s operating earnings NOPAT in $t + 1$ are linked with the free cash flow via a state-independent and constant retention rate¹⁹ with

$$\widetilde{NOPAT}_{t+1} = \frac{\widetilde{FCF}_{t+1}}{(1 - q)}, \quad E(NOPAT_{t+1}) = \frac{E(FCF_{t+1})}{(1 - q)}, \quad (12)$$

it grows at the same expected rate:

$$E(NOPAT_{t+1}) = NOPAT_t (1 + E(g)). \quad (13)$$

Given Eq. 1 above, invested capital I_t must grow at the same expected rate as NOPAT with

$$E(I_{t+1}) = I_t (1 + E(g)). \quad (14)$$

¹⁷ An alternative terminal value model by Bradley and Jarrell (2008) includes an additional growth factor aiming at reflecting the impact of inflation. Unlike additional earnings from net investments, the additional growth in this model stems from an inflation-related increase in the value of existing assets. The model implicitly assumes that these write-ups, when multiplied by a rate of return, generate additional earnings and cash flows. As we do not consider asset write-ups to be a viable source for growth of earnings and cash flows, we abstain from using this model for our analysis. For a critique on this model, see Friedl and Schwetzler (2010, p. 417; 2011, p. 88).

¹⁸ See, e.g., Fama (1977) regarding the multiperiod “CAPM” and Richter (2001) for a multi-period binomial model.

¹⁹ A constant and state-independent retention rate assures that the only random variable driving the development of all “relevant figures” in a steady state is the uncertain nominal growth rate \tilde{g} .

Combining properties (13) and (14) shows that the steady-state requirement regarding E(FCF) yields

$$\begin{aligned} E(\text{ROIC}_{t+2}) &= \frac{E(\text{NOPAT}_{t+2})}{E(I_{t+1})} \\ &= \frac{E(\text{NOPAT}_{t+1})(1 + E(g))}{I_t(1 + E(g))} \\ &= E(\text{ROIC}_{t+1}) = E(\text{ROIC}) \quad \forall t. \end{aligned} \quad (15)$$

Thus, the steady state definition widely assumed in the literature rests on a state where cash flows, earnings, and invested capital grow at a consistent rate, implying a constant expected return on invested capital.²⁰ To effectively examine the prevalent terminal value assumptions endorsed by standards and practitioners, we will base our following analysis on this most common definition and assume that a steady state refers to all accounting and cash flow figures either remaining constant or growing at a uniform perpetual rate.²¹

3.4 The Economic Return Requirement

In both the Gordon-Shapiro model and value driver model of Koller et al., the profitability of invested capital is measured by accounting returns. Critics express their concern against using accounting figures and returns as a measure for the company's profitability in investment decisions: Equity or capital employed is measured by book values and thus may not reflect the true value of capital as an input variable (IDW 2018, ch. A, para. 435). As accounting standards do not allow to fully capitalize investments in intangible assets, the invested capital measured by book values tends to understate the value of I_t , yielding in turn an overstatement of calculated returns (e.g., Penman 2013, p. 561).²² According to critics, profitability should therefore be measured based on "economic returns" related to cash flows instead of accounting returns (Tschöpel et al. 2010, pp. 349–350; IDW 2018, ch. A, para. 435). Additionally, the depreciation schemes required by accounting standards often do not reflect the true value decline of the invested capital, thus also distorting accounting-based returns²³ and via the effects of the accounting-based depreciation on tax burden, cash-flow-based returns as well. Along these arguments, sources related to the German CPA Association reject accounting returns as a measure for the implied profitability linked to growth assumptions in a terminal value calculation.²⁴

²⁰ Note that assuming a constant capital structure combined with a periodical rebalancing of the absolute debt level, the values of debt and equity shall also grow with the same expected rate $E(g)$.

²¹ See also the steady state definition implying constant margins and returns of Levin and Olsson (2000, p. 7).

²² Ballwieser and Hachmeister (2021, p. 47) also refer to non-capitalized research and development as the main reason for these differences.

²³ For a sound analysis on the relationship between internal rate of return (IRR) and accounting-based rate of return (ROI), see Rajan et al. (2007).

²⁴ "In particular, no assessment of a possible sustainable absolute excess profitability of the valuation object can be made on its basis." (IDW 2018, ch. A, para. 435). In contrast, the Austrian principles for

However, there are ways to significantly reduce the gap between accounting-based and cash-flow-based returns, when analyzing the profitability of invested capital. Concepts of value-based management allow for the additional capitalization of intangible assets and by doing so significantly reduce the distortion of accounting-based performance measurement in. These concepts are widely used in practice and are also applied in corporate valuation (e.g., Young and O’Byrne 2000; Koller et al. 2020, p. 577).²⁵

Given the chosen structure of our model, the described distortions caused by accounting-based depreciation rules do not affect our analysis: Our model assumes an investment project with a lifetime of one period in order to reduce any complexity not necessary for our fundamental argumentation. Following a standard approach in the academic literature, we define the firm as a sequence of “typical” investment opportunities (e.g., Rajan et al. 2007; Rajan and Reichelstein 2009); in each period of the perpetuity phase, the firm under consideration can realize one investment project. For our further analysis, we continue to assume that

1. the investment outlay fully includes all intangible assets and that
2. the useful life of all assets of the “typical” investment project is one period.

While those are significant limitations, they offer several important benefits for our analysis. First, it removes any potential distortion caused by accounting-based depreciation schemes from our return figure. As the accounting return under this condition is equal to the economic return (IRR), it provides a sound basis for the investment decision and optimal investment scale by comparing it against the firm’s cost of capital.²⁶ Second, there is no distortion caused by an overlap of depreciations on older assets in place of investments that have been made under potentially different conditions. Third, setting $D\&A_{t+1} = I_t$ avoids any negative side effects of depreciation schemes on tax payments.

As the cash cycle of the representative project is just one period, note that ΔNWC is equal to zero and it follows that $CAPEX_t$ strictly equals the project investment I_t in period t . Without loss of generality, the assumptions allow us to link accounting figures with CF_t as the operating cash flow after taxes:

$$NOPAT_t = EBITDA_t - \tau \cdot EBIT_t - D\&A_t = CF_t - D\&A_t = CF_t - I_{t-1}. \quad (16)$$

corporate valuation of KFS/BW are open to use accounting-based returns for plausibility checks: While the guidelines also prefer economic rates of return like the IRR over accounting returns, they allow for accounting-based rates of return on equity as simplification (KFS/BW 1 E4 2015, para. 10). See also Rabel (2016, p. 280).

²⁵ The corresponding concepts of “value-based management” are also part of the range of services offered by the majority of professional advisors that provide corporate valuation services.

²⁶ To derive the optimal investment scale, we compare the economic return of the invested capital against the capital market return; as this comparison yields the same result as maximizing the projects net present value the standard critique on using IRR for investment decisions and analysis does not apply.

Under these conditions, ROIC definition (3) also measures the economic return and can be expressed in terms of the operating cash flow after taxes CF_{t+1} :

$$ROIC_{t+1} = \frac{NOPAT_{t+1}}{I_t} = \frac{CF_{t+1}}{I_t} - 1. \quad (17)$$

Accordingly, we can relate the ROIC as perfect proxy for the economic return to the cost of capital in the following analysis.

4 The Investment Technology of the Firm

4.1 Decreasing Marginal Returns and Certainty as Necessary Assumptions

For a given period t , we assume the investment technology $F_t(I)$ to be state-independent and relate the invested amount I_t at period's end to a resulting operating cash flow after taxes in the following period. We define this exogenous investment technology²⁷ in period t as

$$F_t(I) = \widetilde{CF}_{t+1}. \quad (18)$$

Both cash investment in period t and uncertain cash inflow in period $t + 1$ are nominal and measured in the prevailing purchasing power. The investment technology $F_t(I)$ includes potential measures of the firm and its management as a reaction to increasing prices for certain input factors.²⁸

$F_t(I)$ exhibits decreasing marginal returns and thus meets a central requirement of a neoclassical production function.²⁹

$$\frac{dF_t(I)}{dI} > 0, \quad \frac{d^2F_t(I)}{d^2I} < 0. \quad (19)$$

Departing from the previous sections, we now follow Rajan et al. (2007, p. 329) and assume certainty: Given an investment of I_t , the subsequent operating cash flow after taxes CF_{t+1} resulting from the investment technology $F_t(I)$ is known with certainty in period t for our analysis. This is a significant simplification that requires a detailed justification. The reason for this choice is not the risk inherent in the investment opportunity itself affecting CF_{t+1} , as the analysis of a certain investment outlay yielding uncertain future cash flows is a standard case in investment textbooks. Instead, the rationale for assuming certainty stems from the substantial difficulties

²⁷ In our Appendix, we derive key results for a more concrete exemplary investment technology.

²⁸ The term “investment technology” describes the combination of the cash outflow in period t with the corresponding cash inflow after taxes in period $t + 1$. It does not relate to any physical assets, inputs, or processes for a good or a service but adds a layer of monetary price effects to such production functions.

²⁹ Note that returns to scale have a slightly different meaning than marginal return. Returns to scale refer to the effect of increasing all business inputs, while marginal returns refer to increasing only one. We can use both terms in our context because a firm cannot have decreasing returns to scale without experiencing diminishing marginal returns on capital input as one of its inputs (e.g., Pindyck and Rubinfeld 2018, pp. 218, 224–233).

in modeling the firm as a sequence of investments with the above properties under uncertainty. Consider the relationship between the uncertain operating cash flow after corporate taxes \widetilde{CF}_{t+1} and the cash flow before taxes, which effectively equals \widetilde{EBITDA}_{t+1} , in our model:

$$\widetilde{CF}_{t+1} = \widetilde{EBITDA}_{t+1} - \tau \cdot \widetilde{EBIT}_{t+1} = \widetilde{EBITDA}_{t+1} (1 - \tau) + \tau \cdot \widetilde{D\&A}_{t+1}. \quad (20)$$

Due to $\widetilde{D\&A}_{t+1} = I_t$, it follows that

$$\widetilde{CF}_{t+1} = \widetilde{EBITDA}_{t+1} (1 - \tau) + \tau \cdot I_t. \quad (21)$$

As $\widetilde{NOPAT}_{t+1} = \widetilde{CF}_{t+1} - I_t$ and $\widetilde{NI}_{t+1} = \widetilde{NOPAT}_{t+1} \cdot q = I_t \cdot \tilde{g}$, the firm's future net investments in $t + 1$ also turn into a random variable. Consequently, the firm's total invested capital in $t + 1$, \tilde{I}_{t+1} is uncertain and state-dependent, as $\tilde{I}_{t+1} = \widetilde{NOPAT}_{t+1} \cdot q + I_t = I_t (1 + \tilde{g})$. As the state-independent investment technology $F_t(I)$ yields a unique and state-independent optimal investment scale,³⁰ random and state-dependent future investments \tilde{I}_{t+1} prompt the firm to deviate from its optimal investment policy, resulting in a suboptimal investment program. Even if the expected value of the prevailing investment outlay in $t + 1$, given by $E(I_{t+1})$, is equal to the state-independent and certain optimal investment scale I_{t+1}^* , Jensen's inequality reveals that the firm cannot maintain the required steady state. This is particularly evident when assuming the case of a perpetual free cash flow with zero growth, $E(g) = 0$ and $E(NI_{t+1}) = 0$. In this scenario, a steady-state condition without growth would require that the firm realizes its optimal investment scale every period, $I_{t+1}^* = I_{t+2}^* = I^* \forall t$ and $E(CF_{t+1}) = E(CF_{t+2}) = E(CF) \forall t$. However, decreasing returns to scale and Jensen's inequality in combination imply that

$$E(F_{t+1}(I)) < F_{t+1}(E(I)) = F_t(I^*). \quad (22)$$

The interaction between realized cash flow and the resulting state-dependent investment in $t + 1$ results in $E(CF_{t+2}) < E(CF_{t+1})$. Depending on the realized operating cash flow CF_{t+1} , the firm would either overinvest (in cases of $I_t^* (1 + \tilde{g}) > I_{t+1}^*$) or underinvest (in cases of $I_t^* (1 + \tilde{g}) < I_{t+1}^*$). Thus, the firm fails to satisfy the necessary conditions of a steady state as soon as we consider uncertainty after the starting period t for the defined and state-independent investment technology.³¹

While excluding uncertainty in the following sections represents a significant limitation on the assumptions of our analysis, we think that the issues outlined above provide valid reasons for this restriction and it does not limit the significance of our results. The conclusions obtained under this simplified condition still highlight

³⁰ See the Appendix or next section for more details.

³¹ Eq. 22 also shows that assuming uncertainty and combining it with risk-neutral investors would not solve the problem; uncertain investments with decreasing returns to scale would still not allow for a steady state.

potential inconsistencies in standard assumptions of corporate valuation that are widely used in practice.

4.2 Optimal Investment Scale

In this section, we analyze the firm's first investment decision into $F_t(I)$. All figures in t and $t + 1$ are nominal and measured in terms of the prevailing purchasing power, so that any inflationary effects are fully reflected. We assume that the firm has nominal cost of capital of rate r after corporate taxes corresponding to the risk of the cash flows generated by the investment technology.

Given the net present value NPV of the project realized in period t of

$$NPV_t(I_t) = \frac{CF_{t+1}}{1+r} - I_t, \quad (23)$$

the investment optimum I_t^* maximizing the project's NPV is characterized by

$$\frac{dF_t(I)}{dI_t} = 1 + r. \quad (24)$$

When extrapolating other key financial figures based on the optimal investments scale, note that the firm's operating earnings after taxes τ in period $t + 1$ at the optimal scale are given by

$$NOPAT_{t+1}^* = (EBITDA_{t+1}^* - D\&A_{t+1}) (1 - \tau), \quad (25)$$

and net investments in the optimum can be derived as

$$NI_t^* = I_t^* - D\&A_t = I_t^* - I_{t-1}^*. \quad (26)$$

The firm's free cash flow in $t + 1$ under optimal investment is then

$$FCF_{t+1}^* = NOPAT_{t+1}^* - NI_t^*. \quad (27)$$

Given relationship (24) and the definition of the marginal rate of return in $t + 1$ as

$$MROIC_{t+1} = \frac{dF_t(I_t)}{dI_t} - 1, \quad (28)$$

the marginal return $MROIC_{t+1}^*$ at the optimal investment scale I_t^* is equal to the firm's cost of capital with $MROIC_{t+1}^* = r$ (e.g., Berk and DeMarzo 2024, pp. 249–257). Combined with the property of decreasing returns to scale of Eq. 19, the average return $ROIC_{t+1}^*$ at the optimal investment scale exceeds the expected marginal return and thus also the cost of capital r :

$$ROIC_{t+1}^* > MROIC_{t+1}^* = r. \quad (29)$$

Finally, the assumed one-period lifetime of the project allows us to define RONIC_{t+1} as the average return on net investments NI_t made in period t . It is calculated by relating the incremental earnings compared to the earnings under zero net investments ($\text{NI}_t = 0$) to the net investments in the preceding period t :

$$\text{RONIC}_{t+1} = \frac{\text{NOPAT}_{t+1} - \text{NOPAT}_{t+1}^{\text{NI}=0}}{I_t - I_{t-1}} = \frac{\text{NOPAT}_{t+1} - \text{NOPAT}_{t+1}^{\text{NI}=0}}{\text{NI}_t}. \quad (30)$$

Again, due to decreasing returns to scale, the average return on net investments—both at the optimal and any other investment level—is lower than the return on total investment with $\text{RONIC}_{t+1} < \text{ROIC}_{t+1}$.

In the following, we will assume that the company realizes this optimal investment volume and the corresponding return when it enters the perpetuity phase with the latest investment I_t^* of phase 1, and hence the assumption of rational and value-maximizing behavior should already hold in the detailed planning phase.³²

4.3 Definition of Value Neutrality

Under decreasing returns to scale for the investment technology, the requirement of zero excess returns and “the firm earning its cost of capital” is ambiguous and allows for different interpretations. It may refer to the entire invested capital and thus the average return ROIC_{t+1} as the relevant return figure, to net investments (retained earnings) with a return of RONIC_{t+1} only, or even to the last incremental unit invested with a marginal return of MROIC_{t+1} . As the academic discussion mainly centers around value-neutral earnings retention, we will focus on the compatibility of assuming value-neutral earnings retention, we will focus on the compatibility of assuming value-neutral net investments.³³ Our relevant return figure is thus RONIC_{t+1} , as the average rate of return on net investments NI_t ; we define value-neutral net investments to have an average return equal to the cost of capital with $\text{RONIC}_{t+1} = r$.

Note that assuming an efficient capital market, net investments in financial assets—in contrast to net investments in operating assets—are always yielding a return equal to the cost of capital and thus by definition satisfy the value-neutrality requirement. Consequently, we will also study the feasibility of such investments in combination with the steady-state requirement and a value-maximizing firm.

³² An entry above or below the optimum would imply irrational investment behavior in earlier periods.

³³ Knoll (2016, p. 34) argument pointing towards the firm’s market value being equal to its book value is related to the case where the firm’s average return on the entire invested capital is equal to its cost of capital.

5 Perpetuity Scenarios

5.1 Net Investments Under Constant Technology

We first consider the perpetuity case of constant free cash flows to infinity. Combined with the steady state definition, this requires the optimal investment scale in $t + 1$ to be equal to the realized nominal optimal scale in t :³⁴ $I_t^* = I_{t+1}^*$. We assume that the investment technology $F_t(I)$ from the previous section is constant during the perpetuity phase.³⁵ As a direct consequence, the optimal scale of the investment I_t^* is also constant over time—namely, $I_t^* = I_{t+1}^* = I^* \forall t$. Since the lifetime of the representative investment project is one period and $\Delta NWC_t = 0$, it follows that

$$D\&A_{t+1}^* = I_t^* = I^* = D\&A^* \forall t. \quad (31)$$

Consequently, the operating cash flow under optimal investment scale is constant over time. As an earnings retention for net investments is not necessary, NI_t^* is equal to zero:

$$NI_t^* = I_t^* - D\&A_t^* = I_t^* - I_{t-1}^* = 0 \forall t. \quad (32)$$

and the free cash flow is equal to NOPAT:

$$CF_t^* = CF_{t+1}^* = CF^* = F(I^*) \forall t, \quad (33)$$

$$NOPAT_t^* = CF^* - I^* = NOPAT^* \forall t, \quad (34)$$

$$FCF_t^* = CF^* - I^* = FCF^* \forall t, \quad (35)$$

The average return at the optimal investment scale is constant as well with

$$ROIC_{t+1}^* = \frac{CF_{t+1}^* - D\&A_{t+1}^*}{I_t^*} = \frac{CF^* - I^*}{I^*} = \frac{NOPAT^*}{I^*} = ROIC^* \forall t. \quad (36)$$

Since net investment is zero, the growth rate of invested capital—and hence the overall growth rate of the steady state—is zero as well. All these figures are nominal and fully reflect the anticipated impact of inflation including the firm's response to it.

Without nominal growth and under certainty, the valuation Eq. 4 turns into

$$TV_t = \frac{FCF_{t+1}}{r} = \frac{NOPAT_{t+1}}{r}. \quad (37)$$

How would this steady state be affected if we were to assume positive net investments in perpetuity? If we assume the firm's invested capital to grow by g per cent

³⁴ The constant nominal cash flows and CAPEX duly reflect declining real values when adjusted for inflation.

³⁵ Again, this statement does not relate to technological properties of the investment; it just states that the relationship between nominal cash outflows in t and cash inflows in $t + 1$ remains constant.

every period and start with the optimal investment scale I_t^* in the last period t prior to the perpetuity phase, we obtain an investment of $I_{t+1} = I_t^* (1 + g)$; net investments in period $t + 1$ are thus

$$NI_{t+1} = I_{t+1} = I_{t+1} - D\&A_{t+1} = I_t^* \cdot (1 + g) - I_t^* = I_t^* \cdot g, \quad (38)$$

and are funded by a respective positive retention of the firm's NOPAT.

Given the constant investment technology, the additional net investment of $I_t^* \cdot g$ in $t + 1$ does not translate into additional operating cash flows after taxes of $CF_{t+1}^* \cdot g$ in $t + 2$. Under the assumed decreasing marginal returns, Jensen's inequality dictates that

$$F_t(I^* (1 + g)) < F_t(I^*) (1 + g). \quad (39)$$

With $F_t(I)$ being a concave function, the growth rate of cash flows is lower than g as

$$\frac{CF_{t+1}}{CF_t^*} - 1 = \frac{F_t(I_t^* (1 + g))}{F_t(I_t^*)} - 1 < g. \quad (40)$$

Operating cash flow, NOPAT, and free cash flow in $t + 1$ would not be growing at the same rate g as the invested capital. The following Fig. 2 illustrates this result:

In this figure, the X-axis shows the invested capital in periods t and $t + 1$ and the Y-axis reflects the operating CF after taxes resulting from the concave investment technology $F_t(I)$.

The optimal investment scale I^* is located where the slope of the black line of the investment technology $F_t(I)$, given by the marginal return of $1 + MROIC_{t+1}$, is equal to the cost of capital, depicted by the red line with a slope of $1 + r$. The areas under the investment technology curve are shaded green up to the optimal scale and light red beyond it. These areas illustrate the ranges where a marginal investment earns an MROIC higher or lower than the cost of capital, respectively, due to the diminishing marginal returns described by property (19).

Figure 2 highlights that any net investment in $F_t(I)$ beyond the optimal scale I_t^* leads to the firm realizing an unfavorable investment scale. This is exemplified by point A, where the marginal return falls below the cost of capital. Beyond the optimal investment scale, the slope of the investment technology $F_t(I)$ is lower than the slope of the red cost of capital tangent, $1 + r$. Point B of an alternative cash flow for value-neutral net investments with a return equal to the firm's cost of capital is above point A of the cash flow actually realized. Overall, the average return on net investments ($RONIC_{t+1}$) is lower than the cost of capital; the additional net investment $I_t^* \cdot g$ has a negative NPV and destroys corporate value. The firm is overinvesting.

Accordingly, a steady state is not achievable under these conditions: In subsequent periods, the total average return ROIC of the firm's invested capital would continue to decrease with additional net investments in each period. In Fig. 2, the beginning

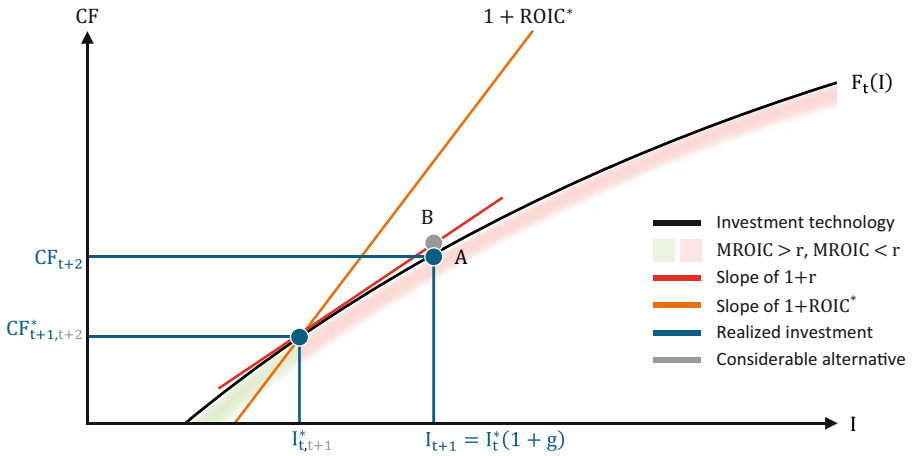


Fig. 2 Net investments in operating assets under constant technology

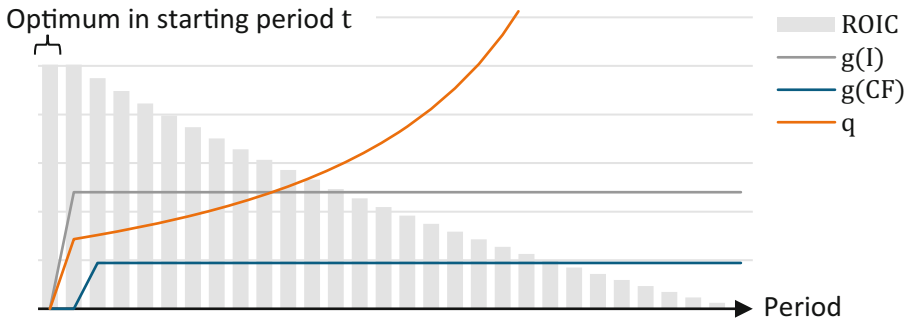


Fig. 3 Further development of net investments in operating assets under constant technology

of this process is represented by the fact that the cash flow in $t + 2$ of point A is below the orange line of initial profitability and a slope of $1 + ROIC^*$.

Figure 3 highlights the long-term development of key financial figures for the assumption scenario.³⁶

Starting from an optimum of zero net investments, the firm engages in constantly growing its invested capital. As the rate of return ROIC declines over time due to the increasing overinvestment, the firm needs to retain an increasing fraction q of its NOPAT to maintain the constant growth rate $g(I)$ of its invested capital. Although a constant operating cash flow growth can be realized, free cash flow decreases as the cash outflow for investments grows faster than the cash inflow.

What is the impact of considering net investments in financial assets instead of operating assets? In an efficient capital market, any financial asset earns a return equal to its cost of capital and provides constant returns to scale. If we assume

³⁶ Fig. 3 and similar following graphs are generated by a calculation based on an exemplary investment technology with the derived requirements. The respective Excel calculations are available upon request.

without loss of generality that the financial assets chosen by the firm carry the same risk properties as its operating assets and thus earn a RONIC equal to the firm's cost of capital r , then Eq. 9 from above shows that neither the net investments nor the growth generated by it have any impact on the terminal value. If an amount equal to the depreciation of the operating assets is constantly reinvested at $\text{ROIC}_{t+1}^* > r$, the existing nominal operating assets and the expected earnings generated by them stay constant. However, as an increasing share of financial net investments and their accumulating reinvestments are yielding $\text{RONIC} = r$, one can observe a slow deterioration of average return towards the cost of capital. Thus, investments into financial assets do not allow for a steady state.

For the case of a constant investment technology, we obtain the following overall results:

- The optimal investment scale results in a positive average excess return on all assets of $\text{ROIC}_{t+1}^* = \text{ROIC}^* > r$ and a positive net present value ($\text{NPV} > 0$).³⁷ In this scenario, net investments and overall growth are equal to zero, implying that a constant investment technology allows for a steady state and value maximization only if no nominal growth ($g = 0$) is assumed for the perpetuity phase.
- Positive net investments in operating assets beyond the optimal investment scale yield both marginal and average returns below r . Thus, they are not value-neutral but **value-destroying**. The firm realizes **suboptimal overinvestments** and does not maximize its value.
- Under positive net investments, no **steady state is attainable**, as earnings and cash flows do not grow at the same rate as the invested capital. The firm's average return ROIC does not remain constant over time but declines under positive net investments.
- Net investments in financial assets are value-neutral and do not create any valuable growth. Due to $\text{ROIC}_{t+1}^* > r$ for existing operating assets, accumulating net investments in financial assets cause the average return to converge to r . Thus, the setting does **not allow for a steady state**.

5.2 Optimal Net Investments Under Improving Technology

The established definition of a steady state requires earnings, cash flows, and invested capital to grow at the same rate g , thus resulting in the average return on the firm's optimal investment scale to remain constant over time. Figure 4 below illustrates that given the assumption of decreasing marginal returns, this requirement is only met under a specific change of the investment technology $F_t(I)$ (Schwetzler 2018, pp. 83–85):

For achieving a steady state with positive growth, invested capital at the optimal scale I_t^* and cash flow CF_{t+1}^* must grow at the same rate g to point A, yielding a constant average rate of return of $\text{ROIC}_{t+2}^* = \text{ROIC}_{t+1}^* > r$. Under the assumption of a sequentially optimal investment policy, this is only possible if the

³⁷ Note that the total investment is not value-neutral, graphically visible in Fig. 2 by the steeper slope and higher marginal returns of the concave curve of $F_t(I)$ up to I_t^* in comparison to $1 + r$.

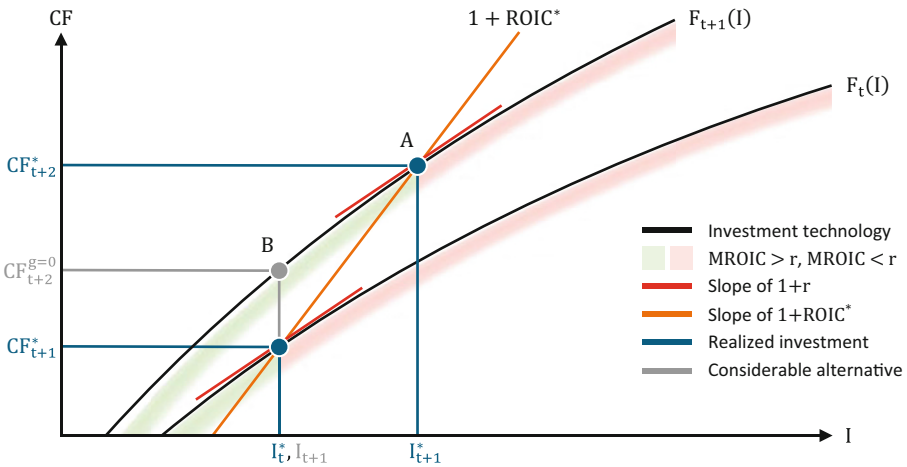


Fig. 4 Optimal net investments under improving technology

investment technology changes to higher profitability levels, so that the point of marginal $MROIC_{t+1}^* = r$ is shifted and the optimal level of investment I_t^* can grow with a rate of g . The new investment technology $F_{t+1}(I)$ in Fig. 4 ensures that both the optimal investment volume I_{t+1}^* and the cash flow CF_{t+2}^* generated by it grow with the same rate of g . The realized profitability is again located on the line of the previous average return with $1 + ROIC_{t+2}^* = 1 + ROIC^*$.³⁸ The marginal return $MROIC_{t+2}^*$ of the new technology $F_{t+1}(I)$ at the optimal investment volume $I_t^*(1 + g)$ is still equal to r . In Fig. 4, this is illustrated by the parallel red tangents, both with a slope of $1 + r$.

Given constant cost of capital, the investment technology $F_{t+1}(I)$ must adjust in comparison to $F_t(I)$ of the previous period by satisfying the two following conditions:

- The average return on the increased optimal investment volume $I_{t+1}^* = I_t^*(1 + g)$ remains constant: $ROIC_{t+2}^* = ROIC_{t+1}^*$, as indicated by the orange line in Fig. 4, and
- the marginal return on the increased optimal investment volume $I_{t+1}^* = I_t^*(1 + g)$ is equal to the firm’s cost of capital: $MROIC_{t+2}^* = r$.

The first condition can also be written as

$$\frac{F_{t+1}(I_{t+1}^*)}{I_{t+1}^*} = \frac{F_{t+1}(I_t^*(1 + g))}{I_t^*(1 + g)} = \frac{F_t(I_t^*)}{I_t^*} \forall t, \tag{41}$$

or

$$F_{t+1}(I_t^*(1 + g)) = F_t(I_t^*) \cdot (1 + g) \forall t. \tag{42}$$

³⁸ Setting $I_{t+1} = I_t^*$ and assuming zero net investment leads the firm to outcome B and an underinvestment, because marginal investments would still yield returns exceeding the cost of capital until point A is reached.

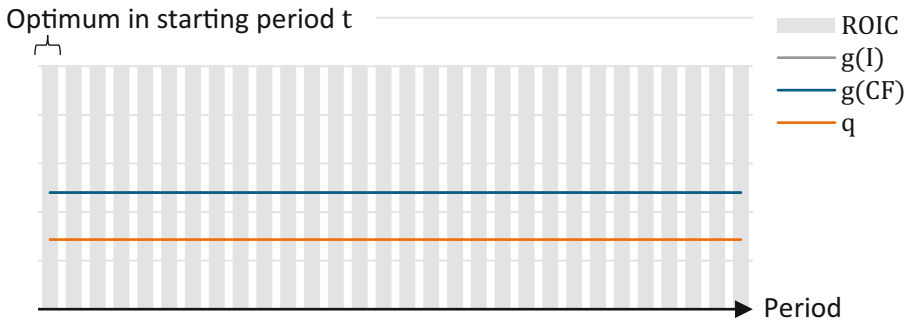


Fig. 5 Further development of optimal net investments under improving technology

Equation 42 demonstrates that in every period the cash flow generated by the adjusted technology of the increased optimal investment volume $I_{t+1}^* = I_t^* (1 + g)$ on the left-hand side must match the cash flow generated by the previous period's investment volume I_t^* , increased by the growth factor $(1 + g)$. This equation reveals that a specific exogenous change in the firm's investment technology is necessary for the resulting nominal cash flow and NOPAT to also grow at the constant rate g . Only if the technical requirement in Eq. 42 is met, the firm maintains its average ROIC* at optimal investment levels in perpetuity and reaches a steady state with a positive and constant growth rate.³⁹

For the required recurrent change in investment technology there may be several sources, such as technical innovations increasing productivity, changing exogenous conditions for input compositions, or possible distribution and pricing policies. As the above model is based on nominal figures, the required technological change may also reflect the impact of inflation and the firm's response to it. We refer to this case as “constant improvement” in technology.

Figure 5 highlights the long-term development of key financial figures for the assumed scenario:

Figure 5 illustrates that “all relevant figures” as cash flow, invested capital, and the resulting free cash flow grow at an identical rate,⁴⁰ yielding a constant rate of return ROIC over time.

We obtain the following results for the development of optimal investment scales and a steady state:

- The invested capital matches the optimal investment scale in each period and grows at a constant rate g , with positive net investments. These net investments have a **positive NPV** with $RONIC^* > r$. Thus, the requirement of **value neutrality is not fulfilled**.
- The firm realizes its **optimal investment scale** in every period and thus pursues a value-maximizing investment policy.

³⁹ Note that this requirement is already necessary under our assumption of certainty.

⁴⁰ In Fig. 5, the grey line of investment growth lies behind the blue line and is therefore not directly visible.

- If the investment technology used by the firm constantly improves with the condition described in Eq. 42, a **steady state** with a unique sustainable growth rate g is **feasible**.

5.3 Value-neutral Net Investments Under Improving Technology

a) Net investments in operating assets

We continue to assume that the defined improvements of the investment technology to enable a steady state are realized. Based on this premise, we proceed to examine value-neutral net investments in this technology. It is important to recall that we define “value neutral” as the net present value of the firm’s net investments being equal to zero. Given the characteristics of the investment technology, this implies that the average return on net investments, RONIC, is equal to the firm’s cost of capital r .

Since the optimal investment scale features positive net investments with average returns above the firm’s cost of capital, value-neutral net investments in the given technology are only possible if the firm expands its investment scale beyond the optimal scale I_{t+1}^* and thus overinvests. In this case, the negative excess returns for the investment beyond the optimal scale $I_{t+1} - I_{t+1}^*$ offset the positive excess returns on the optimal net investments $I_t^* \cdot g$. Figure 6 below highlights this result:

First, we consider for $t + 1$ that the firm initially reinvests the previous investment scale I_t^* of the preceding period t into the improved technology $F_{t+1}(I)$. This yields a hypothetical $CF_{t+2}^{g=0}$ instead of the CF_{t+1}^* that could be generated under the previous, less productive technology $F_t(I)$. Under the requirement of value neutrality, the additional net investment NI_{t+1} shall also be invested in technology $F_{t+1}(I)$ and yield an average RONIC_{t+2} equal to the firm’s cost of capital r . This is only possible if the firm extends its net investment significantly beyond the optimal scale I_{t+1}^* into the area of marginal returns below the cost of capital r , reaching up to I_{t+1}^{VN} . In Fig. 6, this investment is located at point A, where the value-neutral red line with a slope of $1 + r$ — starting from the investment of I_t^* and its cash flow without additional net investments $CF_{t+2}^{g=0}$ — intersects the curve $F_{t+1}(I)$ of the improved investment technology. As $I_{t+1}^{\text{VN}} > I_{t+1}^*$, the firm significantly overinvests, resulting in a suboptimal investment scale and the destruction of value. Realizing the optimal investment scale I_{t+1}^* and investing the remaining net investments up to I_{t+1}^{VN} in an asset simply earning the cost of capital would have yielded a higher cash flow in $t + 1$, as represented by point B.

Figure 6 also highlights that, due to the overinvestment, the realized initial growth rate of invested capital $I_{t+1}^{\text{VN}} - I_t^*$ is significantly above the growth rate for a steady state g imposed by the realization of the net investments at optimal investment scale of $I_{t+1}^* - I_t^*$. In addition, the average return of the firm’s invested capital declines due to the suboptimality of the total investment. In Fig. 6, this is highlighted by point A lying below the orange line $1 + \text{ROIC}^*$ of previous profitability.

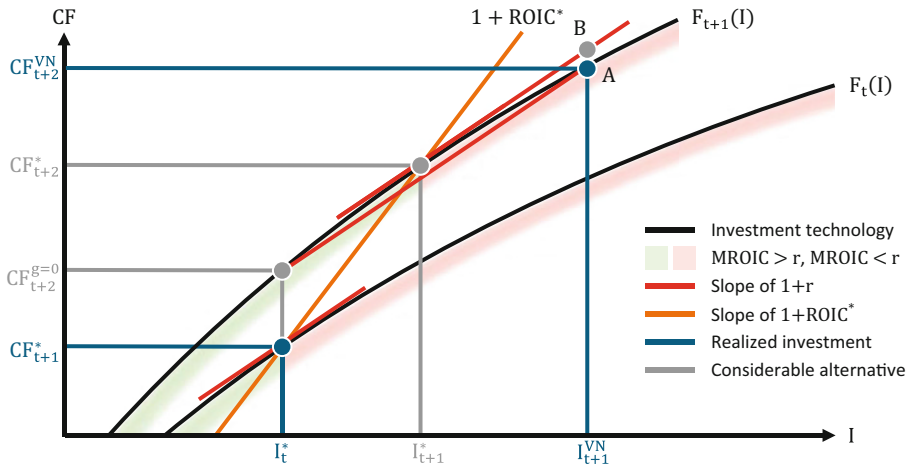


Fig. 6 Value-neutral net investments under improving technology

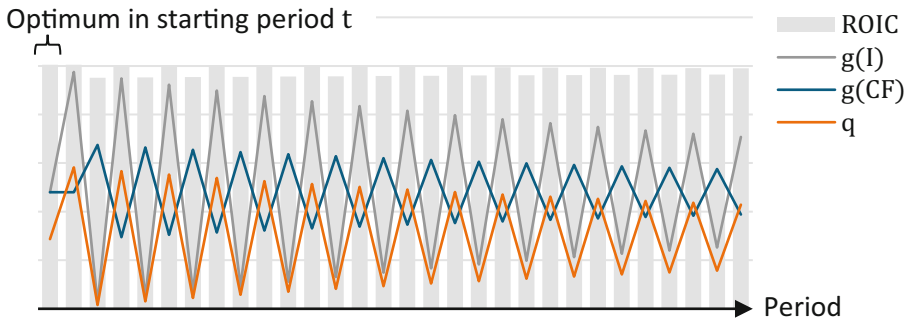


Fig. 7 Further development of value-neutral net investments under improving technology

Figure 7 highlights the long-term development of key financial figures for the assumption scenario:

Figure 7 illustrates an interesting property of relevant growth rates fluctuating over time that requires a more detailed explanation: The alternative optimal investment scale still increases with a constant rate g and is surrounded by the positive and negative marginal excess returns to be neutralized. As our initial I_{t+1}^{VN} includes an overinvestment for achieving value neutrality in $t + 2$, it grows by more than g with $NI_{t+1}^{VN} > I_t^* \cdot g$. However, note that the investment is then much closer to the next optimum I_{t+2}^* than I_{t+1}^* would have been. In the following period, the net investment beyond I_{t+2}^* necessary to neutralize any positive marginal excess returns up to I_{t+2}^* is thus significantly lower as in the previous period and thus $NI_{t+2}^{VN} < I_{t+1}^* \cdot g$. This results in a fluctuation of alternating periods of higher and lower required investment growth. Figure 7 also displays a fluctuating ROIC over time that remains below $ROIC^*$, the level associated with an optimal investment policy.

We derive the following results for the case of value-neutral net investments in operating assets:

- The realization of **value-neutral net investments** in operating assets is **generally feasible**.
- The firm overinvests and realizes a **suboptimal investment scale** that does not maximize firm value.
- **No steady state** is possible under these conditions. As Fig. 7 highlights, there are no stable growth rates for any of the relevant financial figures.

b) Net investments in financial assets

In an alternative perpetuity scenario involving net investments in financial assets under an improving operating technology, we assume that the operating assets at the end of the detailed planning phase remain constant in nominal terms via reinvestments equal to their D&A. All additional net investments from earnings retention and their accumulating reinvestments are allocated to financial assets. As postulated above, financial investments earn a return equal to the cost of capital. These investments are value-neutral and have a NPV of zero for any amount invested. Figure 8 illustrates the financial consequences for this scenario:

By realizing value-neutral net investments in financial assets, the firm now departs from the improved investment technology $F_{t+1}(I)$ for operating assets after realizing I_t^* and undertakes financial investments graphically represented by the red line of $1 + r$ starting from the intersection of I_t^* and $CF_{t+2}^{g=0}$. Although the firm could theoretically invest any amount in financial assets at cost of capital, we assume it to reinvest the identical amount of retained earnings as in previous optimal cases until the total investment I_{t+1}^{VN} equals the optimal investment amount I_{t+1}^* completely invested in operating assets with $I_{t+1}^{VN} = I_t^* (1 + g)$. The firm then realizes a cash flow of CF_{t+2}^{VN} as indicated by point A. As a consequence of this strategy, the firm does not fully exploit the benefits of the improved operating investment technology: By realizing the optimal investment scale in operating assets, the firm could have realized a higher cash flow CF_{t+2}^* in $t + 2$ as represented by point B; the difference

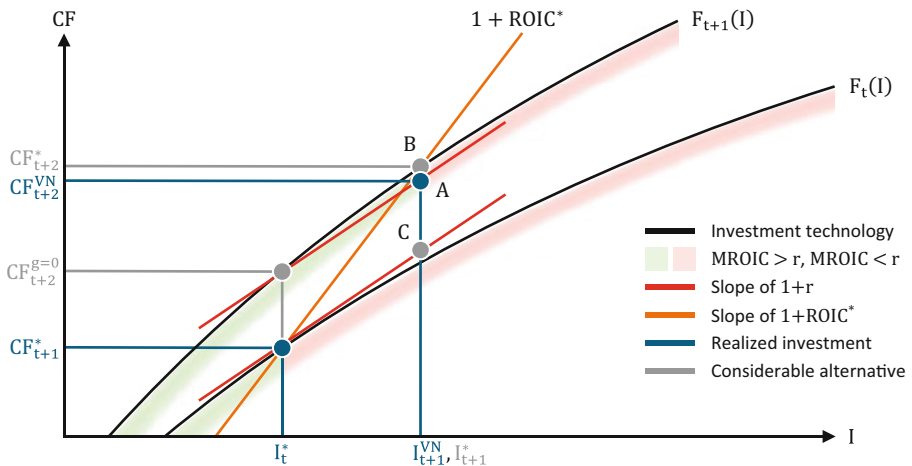


Fig. 8 Value-neutral net investments in financial assets under improving technology

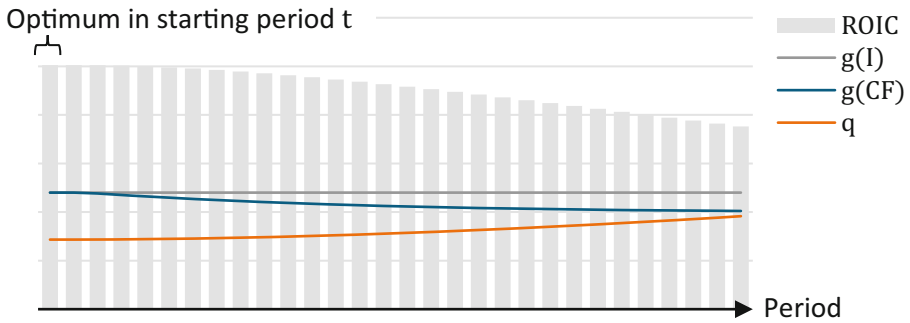


Fig. 9 Further development of net investments in financial assets under improving technology

between CF_{t+2}^* and CF_{t+2}^{VN} for the same investment scale illustrates the loss in cash flow. Under the given improving investment technology, net investments in financial assets yield underinvestment and a sub-optimal firm value.

Figure 9 highlights the long-term development of key financial figures for the assumption scenario:

As the potential increase in the optimal investment is substituted by value-neutral financial assets, the combined investment base grows at a constant rate $g(I)$, but its average return ROIC on all assets declines and slowly converges to rate r . We also observe a constant decline of the cash flow growth, coupled with an increase of retention rate q . For financing the identical asset growth with cash flows from a suboptimal investment strategy, the firm must retain an increasing fraction q of its NOPAT.

We arrive at the following results for a scenario of value-neutral net investments in financial assets:

- The requirement of **value-neutral net investments** is met, as by definition financial investments are value-neutral under the assumption of efficient capital markets.
- The firm realizes a **suboptimal investment program**; as net investments in operating rather than financial assets would yield higher returns, the firm does not pursue a policy of maximizing its value.
- A **steady state is not feasible**, as cash flow and earnings grow slower than the invested capital, and the average ROIC converges to the cost of capital.

6 Conclusions

Growth assumptions in terminal value calculations have a significant impact on the overall corporate value. However, their projection is subject to high uncertainty. Therefore, academics, standard setters, and practitioners have sought ways to identify and prevent unreasonable growth and profitability assumptions for the perpetuity phase. One key proposal has been the requirement that a firm's net investments in the perpetuity phase shall be "value-neutral."

Table 1 Summary of possible perpetuity phase scenarios

Investment technology	Assumptions		Implications	
	Net investments	Value neutrality	Steady state	Value maximization
Constant	No	n/a	Yes	Yes
Constant	Operating assets	Yes	No	No
Constant	Financial assets	Yes	No	Yes
Improving	Operating assets	No	Yes	Yes
Improving	Operating assets	Yes	No	No
Improving	Financial assets	Yes	No	No

Using a simple model of a firm under certainty with positive net investments in a technology featuring decreasing returns to scale, we demonstrate that a steady state, value-neutral net investments, and rational value-maximizing behavior cannot be achieved simultaneously. Table 1 summarizes the results for each set of assumptions considered in our analysis:

Our results highlight a particular dilemma regarding growth assumptions in terminal value calculations:

- Assuming a constant investment technology, value-neutral net investments in operating assets are not feasible. Any net investment would yield increasingly sub-optimal investment scales and a deviation from a steady state. Value-neutral net investments in financial assets are feasible, but do not create any value-enhancing growth with no impact on the terminal value.
- Assuming an improving investment technology, value-neutral net investments cause the firm to deviate from a steady state and to not realize a value-maximizing investment policy. In case of value-neutral net investments in operating assets (financial assets), the firm overinvests (underinvests) in the operating investment technology.
- Under a constantly improving technology allowing for a steady state, the firm can realize its optimal investment scale and a positive constant growth rate but does not meet the requirement of value-neutral net investments when following a value-maximizing policy.

For standard setting bodies and valuation practice, those considerations highlight a pivotal trade-off: balancing the limitation of discretion in projecting future cash flows with strict adherence to the theoretical principles underpinning corporate valuation and economics. The issue has substantial implications, as it underscores the importance of carefully incorporating growth into terminal value calculations—a process necessitating a deliberate choice between these two approaches.

From a theoretical perspective, we recommend re-evaluating the requirement of value-neutral net investments. Instead, the valuator should be free to choose between the following two options:

- Assume a positive perpetual growth rate, which implies a perpetual positive spread of RONIC exceeding the cost of capital (i.e., $\text{RONIC} - r > 0$) for a rationally investing firm;⁴¹ or
- Voluntarily allow for value-neutral net investment and rational behavior simultaneously by setting the perpetual growth rate to zero.

Only this approach would ensure general assumption compatibility for the terminal value calculation.

Our findings also carry implications for the IDW S 1 German valuation guidelines, which assume value-neutral net investments (IDW S 1 2016, para. 37) and still suggest this assumption in their recent draft of a guideline update (IDW ES 1 2024, para. 84). Since these guidelines require personal income taxes to be included in the valuation calculus, discrepancies between effective tax rates on capital gains and dividends can affect valuation outcomes. They thus propose an “equivalent” payout policy of the firm and alternative investments reflected in the discount rate.⁴² Assuming value-neutral net investments under these conditions yields the personal income tax to cancel out of the valuation equation.

However, allowing for value-increasing net investments under these assumptions would slightly shift the tax burden towards capital gains in favor of the firm, disrupting the required tax equivalence. Despite this inconsistency, we would still argue in favor of abandoning the value neutrality requirement and accepting this deviation. Allowing an implied positive excess return above the cost of capital in terminal value calculations would not create a new unfair advantage for shareholders selling their shares or being squeezed out in a dominated transaction: the current guidelines allow for adding an “inflation-related” growth rate on top of the “retention-related” growth and by doing so, implicitly and inconsistently already assume positive excess returns. Ultimately, sticking to the value neutrality requirement in the German guidelines means to assume an irrational and value-destructive investment policy that deviates from the optimal investment scale to address an issue indirectly caused by the standard setter itself—the requirement to include personal income taxes in valuation.⁴³

We conclude this paper by addressing some limitations of our work and proposing directions for future research. First, there are technical challenges associated to the assumption of a state-independent investment technology. Any future refinement of the model to incorporate uncertainty will need to adapt the investment technology to be state-dependent. This alignment ensures that the stochastic properties of operating cash flows and resulting state-dependent (re)investment outlays support the realization of an optimal investment scale across varying economic conditions.

⁴¹ This would still require the special improvement of the investment technology described in Sect. 4.

⁴² For avoiding “inappropriate, tax-induced influences.” (Wagner et al. 2004, p. 897; IDW S 1 2016, para. 37).

⁴³ The straightforward solution to this dilemma is evident: The removal of the assumed relevance of personal income taxes from the valuation guidelines of the German CPA Institute. See Richter (2004, p. 20) and Schwetzler (2007, p. 2) for related contributions.

Second, extending the project's lifespan across multiple periods requires a depreciation scheme that maintains a constant economic return, facilitating the determination of the optimal investment scale. This also necessitates handling overlapping cash flows and depreciations from earlier investments made under different economic conditions. Such conditions could include varying cost of capital or shifts in the firm's investment opportunity set reflecting its lifecycle stage.

Finally, we emphasize that our conclusions and identified inconsistencies rely on the prevailing interpretation of the steady-state assumption — specifically, that it implies a constant average return on total invested capital. Future research could explore terminal value assumptions under more flexible growth conditions, without requiring uniform growth across all financial metrics.

7 Appendix

7.1 Definition of the Investment Technology

In the appendix below, we mathematically derive key results of our analysis using an exemplary investment technology. Building on previous work that derived the optimal investment scale under decreasing returns to scale (Schwetzler 2018), we extend these considerations by also identifying the return at the optimum, the necessary technological progress, and the value-neutral investment scale. We employ a basic investment technology defined as follows:

$$F_t(I) = CF_{t+1} = \beta_t \cdot I_t^\alpha, \quad (A1)$$

with β_t : productivity factor of technological efficiency in period t , α : output elasticity of capital input with $\alpha \in (0,1)$ for diminishing marginal returns.⁴⁴

This fundamental investment technology fulfills the requirement of decreasing returns to scale due to $(1 - \alpha) < 0$ and the other variables necessarily being positive in

$$\frac{d^2 F_t(I)}{d^2 I} = \frac{\beta_t \cdot \alpha \cdot (1 - \alpha)}{I_t^{2-\alpha}} < 0. \quad (A2)$$

Note that the basic structure of this investment technology is similar to known microeconomic production functions such as the Cobb-Douglas function (Cobb and Douglas 1928), where input factors such as capital input are exponentiated with their output elasticity and determine output jointly with a productivity factor.⁴⁵

⁴⁴ Note that this property of diminishing marginal returns is the reason why we do not assume that α changes over time or can even be the exogenous driver of a perpetual growth g . It cannot be allowed to reach a value of one, which is why we identify the unrestricted β_t as only possible driver of growth.

⁴⁵ Microeconomic production functions often add more input factors like labor input (Cobb-Douglas) or energy input and define output volume or sales instead of cash flow after taxes as function output. However, it can be shown mathematically that first deriving the cash flow from those initial figures does not change the decisive properties of decreasing returns to scale in Eq. 19. For the convenience of the

7.2 Optimal Investment Scale

By inserting our exemplary investment technology for the cash flow in the NPV calculation of

$$\text{NPV}_t(I_t) = \frac{\text{CF}_{t+1}}{1+r} - I_t = \frac{\beta_t \cdot I_t^\alpha}{1+r} - I_t, \quad (\text{A3})$$

and solving Eq. 24 of the first derivative condition for the investment volume, we can mathematically determine the optimal investment scale:

$$I_t^* = \left(\frac{\alpha \cdot \beta_t}{1+r} \right)^{\frac{1}{1-\alpha}}. \quad (\text{A4})$$

This equation reveals that the optimal investment scale I_t^* increases with output elasticity α and productivity β_t . However, when the cost of capital is higher, less profitable investments and thus the total invested capital must be reduced. It can be demonstrated that the return on the optimal invested capital is only dependent on the cost of capital and output elasticity as

$$1 + \text{ROIC}_t^* = \frac{\text{CF}_{t+1}^*}{I_t^*} = \frac{\beta_t \cdot \left(\left(\frac{\alpha \cdot \beta_t}{1+r} \right)^{\frac{1}{1-\alpha}} \right)^\alpha}{\left(\frac{\alpha \cdot \beta_t}{1+r} \right)^{\frac{1}{1-\alpha}}} \quad (\text{A5})$$

can be simplified to

$$1 + \text{ROIC}_{t+1}^* = \frac{1+r}{\alpha}. \quad (\text{A6})$$

Equation (A6) suggests that the ROIC under value-optimizing investing is higher for firms or industries facing an investment technology with a lower output elasticity of invested capital.

7.3 Necessary Technological Improvement

We can show the relationship between the required technology change and a sustainable growth rate in a steady state by defining that the productivity factor β_t of our investment technology changes at a rate of g_β . We can then transform the steady-state growth at the optimal investment scale as follows:

$$1 + g = \frac{I_{t+1}^*}{I_t^*} = \frac{\left(\frac{\alpha \cdot \beta_{t+1}}{1+r} \right)^{\frac{1}{1-\alpha}}}{\left(\frac{\alpha \cdot \beta_t}{1+r} \right)^{\frac{1}{1-\alpha}}} = \left(\frac{\beta_{t+1}}{\beta_t} \right)^{\frac{1}{1-\alpha}} = \left(\frac{\beta_t (1 + g_\beta)}{\beta_t} \right)^{\frac{1}{1-\alpha}}, \quad (\text{A7})$$

reader, we thus directly define the cash flow with equation (A1) but could derive the same results for a classic microeconomic function.

which leads to

$$1 + g = (1 + g_\beta)^{\frac{1}{1-\alpha}} \tag{A8}$$

The only way to reconcile value-maximizing investment behavior and a steady state with a sustainable growth rate is a constant technology-driven growth g_β of the productivity factor, which then translates directly into a single possible growth rate g of all financial figures through an exponent that depends solely on input factor elasticity.⁴⁶ In this relationship, higher technological improvements of g_β increase the possible steady-state growth and a higher elasticity of α provides an additional positive leverage effect. For example, a constant efficiency improvement of $g_\beta = 1\%$ under an output elasticity α of 0.4 translates into over 1.6% as the only possible perpetual growth rate g . Formula (A8) reveals that the required technological growth is always below the grow rate of financial figures in a realistic setup.

7.4 Value-neutral Investment Scale

For deriving the exact investment level needed for value-neutral net investments, we start with the RONIC definition of Eq. 30 in combination with property (16) and consider that for the cash flow without net investments ($NI_t = 0$), the invested capital I_t and following D&A equals the previously invested capital of I_{t-1} :

$$RONIC_{t+1} = \frac{(CF_{t+1} - I_t) - (CF_{t+1}^{NI=0} - I_{t-1})}{NI_t} = \frac{CF_{t+1} - CF_{t+1}^{NI=0} - NI_t}{NI_t} \tag{A9}$$

Adding the implied condition of $RONIC_{t+1} = r$ and inserting equation (A1) of the investment technology ultimately leads to

$$NI_t (1 + r) = \beta_t \cdot I_t^\alpha - \beta_t \cdot I_{t-1}^\alpha \tag{A10}$$

To further solve for I_t , we must consider an endogenously determined constant growth rate $g(I)$ of the invested capital, which can be characterized by $NI_t = I_{t-1} \cdot g(I)$ and $I_{t-1} = I_t / (1 + g(I))$. Inserting those definitions and further transforming yields the needed investment scale I_t^{VN} for a value-neutral net investment:

$$I_t^{VN} = \left(\frac{(1 + g(I)) - (1 + g(I))^{1-\alpha}}{g(I)} \cdot \frac{\beta_t}{(1 + r)} \right)^{\frac{1}{1-\alpha}} \tag{A11}$$

⁴⁶ It is evident that—even with several input factors like labor input in a Cobb-Douglas function—this formula remains a pure play of the productivity factor and all input elasticities defined in the production function.

Dividing this result by equation (A4) of the optimal I_t^* and further simplifying the resulting equation reveals that avoiding excess returns on net investments requires an overinvestment factor⁴⁷ of

$$\frac{I_t^{VN}}{I_t^*} = \left(\frac{(1 + g(I)) - (1 + g(I))^{1-\alpha}}{\alpha \cdot g(I)} \right)^{\frac{1}{1-\alpha}}. \quad (\text{A12})$$

For this relationship, simulations show that the amount of overinvestment is almost immune to changes in the elasticity α but substantially increases with $g(I)$. A desired growth $g(I) = 2\%$ of the invested capital results in a necessary overinvestment close to 1% for the defined technology with $\alpha = 0.4$.

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Availability of data and material The data underlying the findings of this study is available upon request from the corresponding author, Bernhard Schwetzler.

Code availability n. a.

Conflict of interest B. Schwetzler and F. Bogateck declare that they have no competing interests.

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⁴⁷ Besides optimal investment scale and value-neutral net investments, we could also solve for value-neutral total invested capital with $ROIC_{t+1} = r$. This would lead to $I_t^{VN} = (\beta_t / (1 + k))^{1-\alpha}$, which is higher than the optimum in equation (A4) by a simple factor of $\alpha^{\frac{1}{\alpha-1}}$ for any decreasing returns to scale with $\alpha \in (0, 1)$.

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