Balla, Katalin; Köllő, János; Simonovits, András

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Transition with Heterogeneous Labor

Katalin Balla
János Köllo
András Simonovits

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Katalin Balla  
*Hungarian Academy of Sciences*

János Köllő  
*Hungarian Academy of Sciences and IZA Bonn*

András Simonovits  
*Hungarian Academy of Sciences, Budapest University of Technology and CEU*

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IZA  
P.O. Box 7240  
53072 Bonn  
Germany

Phone: +49-228-3894-0  
Fax: +49-228-3894-180  
Email: iza@iza.org

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ABSTRACT

Transition with Heterogeneous Labor*

We extend the benchmark model of Aghion and Blanchard (1994), assuming two segments of the emerging private sector that differ in workers’ productivity. We look at the paths of employment, wages, taxes, labor costs and profits during and after the transition, up until the shock is fully absorbed. Viability is a function of the speed of job destruction and the strength of the initial shock to employment. In the long run, the system asymptotically converges to full employment. If the rate of job destruction is sufficiently low, the unemployment rates can get close to steady-state values during the transition. Within the realm of feasible scenarios, unemployment differentials are simultaneously determined by the speed of destruction, the level of benefits and the cross-subsidization of low-productivity groups. Lower benefits induce higher aggregate employment and inequalities throughout the redeployment process, while higher subsidies are conducive to lower inequalities and higher aggregate employment. The choice between low versus high benefits is a matter of preferences but the systems with subsidies dominate the systems with no subsidies. The subsidy has strongest marginal effect on employment and income when job destruction is fast and benefits are high.

JEL Classification: J64, P31, H53

Keywords: transition, heterogeneous labor, job creation, unemployment benefit, wage subsidy

Corresponding author:

András Simonovits
Institute of Economics, HAS
Budaorsi ut 45
Budapest 1112
Hungary
Email: simonov@econ.core.hu

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1. Introduction

Fifteen years after the collapse of communism the countries of Central and Eastern Europe (CEE) have full-fledged market systems providing people with all the amenities a medium-income capitalist economy can yield, from a never-dreamed-of abundance of goods and services to the freedom of enterprise and consumer choice. Long-term unemployment and permanent exclusion from the labor market, however, prevent millions from enjoying these benefits. People born at the wrong place or endowed with the wrong skills have meager hope of being integrated into the working society and face hard times – harder than most of their counterparts in the West.

There are ‘universal’ causes behind a high degree of persistent regional and skills-related inequalities in the CEEs ranging from de-industrialization and trade reorientation to institutional rigidities, competition with low-wage countries and skill-biased technological change. Much less is known about how the post-communist transition per se contributed to inequality. The paper wants to make a step in this direction by evoking and supplementing what the early ‘optimal speed of transition’ (OST) models taught us some ten years ago.

The OST literature of the labor market emanated from a seminal paper by Aghion and Blanchard (1994, A–B henceforth) albeit further similar models were developed to study the pace of restructuring and the political support of the reforms (Dewatripont and Roland, 1992; Freeman, 1994). The strength of the A–B model, we believe, lay in making non-standard but realistic assumptions about the nature of post-communist transition as opposed to usual economic shocks. First, the model considered that the closing of state-owned enterprises (SOEs) generated a steady stream of inflows to unemployment, one that had little to do with changes in trade and technology: there was a multitude of firms past recovery that had to be liquidated or drastically down-sized. Second, the model took into account that the transition started with severely distorted wages that failed to reflect marginal productivity and scarcities. The process of wage adjustment did not happen overnight: it took many years of trial and error until workers and firms learned what they could ask for and what they could offer. Third, the model put emphasis on the fiscal burden associated with a shrinking tax base and fast-increasing outlays for benefits.\footnote{We call every transfer provided to non-employed benefit (social benefits, early pension, maternity grant) and we call every non-employed unemployed. For a distinction between unemployment and non-employment, see Bruno (2006).}
Summarizing briefly, with inevitable simplifications, the benchmark variant of the A–B model proposed that the speed of closing the state sector should not exceed a maximum rate compatible with successful transition. A too drastic shock at the beginning leads to excessive fiscal burden and insufficient private job creation. The speed of closure should not be very low either: unemployment must be high enough to put pressure on wages and thereby stimulate job creation. Within the realm of feasible scenarios equilibrium unemployment is affected by the speed of closing the SOEs; the level of unemployment benefits; the feed-back from redeployment probabilities to wages; and non-labor costs affecting how expected profits per worker are transmitted to actual job creation.

The A–B model did not address the inequalities that were likely to arise in the transition setting. The extensions and applications of the model (Brixiova and Kyotaki, 1997, Castanheira and Roland, 2000, Boeri 2001, Jurajda and Terrell, 2000, 2003) maintained the assumption of homogeneous labor except for Commander and Tolstopyatenko (2001). With fifteen years of experience in hand, it is certainly desirable to reconsider the model with an eye on heterogeneity.

This paper presents a model assuming that some job–worker matches result in higher productivity yields than others. This, conditional on wages and taxes, can lead to unequal rates of private job creation in different segments of the economy. The model can not provide exhaustive explanation of why regional and skill-based differences are so large and persistent in the CEEs – what it tries to clarify is the possible contribution of the transition process in an imaginary world free of technological renewal, trade shocks and institutional rigidities.

The nature of change was not a priori clear. Some features of the post-communist period were expected to keep inequalities low; some predicted a widening gap between more and less favored groups, while others had uncertain effects. (i) The collapse of the state socialist system was a basically neutral shock affecting regions and skill groups in a relatively balanced way. It is not at all misleading to think of it as an unconditional loss of state-sector employment that was followed

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2 Their paper distinguished formal (full-time) and informal (part-time) jobs, and analyzed how the optimal allocation – shaped by the gains and expected losses from tax evasion – was affected by shocks to demand and non-wage subsidies such as housing and health-care provided in the formal sector. As will become clear, our questions and assumptions are of quite different nature.
by a highly selective process of hiring by private and privatized firms. The time until being
rehired could be very short for some groups and very long for others, however. (ii) The
probability of being re-hired was expected to differ widely across regions and skills. Dramatic
wage decompression accompanying the transition is the key to understand why inequalities were
likely to grow even in the absence of technological and trade-related shocks. In a period when
wages move from almost uniform levels to marginal products, the low-productivity groups tend
to set too high reservation wages because their unemployment benefits are high relative to their
prospective earnings (irrespective of whether benefits are flat or set as a fraction of previous
wages). Low-productivity workers learn this indirectly, from having poor prospects of being re-
rehired. They further adjust their reservation wages under the pressure of unemployment,
presumably with substantial delay. The lengthy adjustment of wages to productivity thus gives
rise to unequal job opportunities, at least temporarily. (iii) Finally, as the demand for labor is
affected by labor costs rather than net wages, much depends on how the fiscal burden is allocated
across regions and social groups. The government can aggravate or mitigate the temporary
growth of inequality by shifting the burden onto low versus high-productivity groups.

Technically, we plant these points into a formal model by assuming that under socialism all
workers were employed in SOEs and received the same wage. During the transition, jobs in the
state sector are destroyed at a rate determined by political decisions. Workers who lose their jobs
in the state sector may or may not be hired in the emerging private sector: their probability of
being hired depends on value added when hired, their wages and taxes. Wages, in turn, depend on
benefits and the probability of being hired. Benefits and other transfers such as wage subsidies
are financed from taxes levied on employees. We study how employment, unemployment, wages
and profits evolve during and after the transition if the private economy is composed of two (low-
and high-productivity) segments, and how the path of the variables are affected by policy
instruments like the pre-determined rate of job destruction, the level of unemployment benefits
and potential subsidies to low-productivity jobs.

To be more precise, we assume that the private sector has two segments differing in their levels
of value added per worker (constant marginal product). Further, we assume that (i) workers are
confined to one or another segment; (ii) job creation rates are determined in each segment by
expected profits per worker in that segment. The segments, however, are connected by a national system of taxation, welfare and business support.

Assumptions (i) and (ii) are not at all restrictive as long as we speak of large regions. On the one hand, in lack of well-performing rental housing markets as well as by virtue of tradition, prime-age adults are effectively locked into their regions in the CEEs. On the other hand, jobs are created in each and every region even though the returns are lower in those farther from the newly emerging trade portals or the booming metropolitan areas providing positive externalities.

The assumption of two, partly isolated segments is somewhat harder to justify if the productivity differentials arise from skill differentials since the various skills can substitute each other. The assumption of no within-sector substitution does not seem very strong as far as we talk about broad occupational groups. In our model, for given productivities, the demand for the two types of labor only depends on the corresponding wages, without any cross effects. This is in harmony with Köllő's (2002, 2004) estimations for 1996–1999 on cross elasticities between –0.03 and –0.2 concerning groups with and without finished high school.³

In Sections 2 and 3 we introduce and analyze the augmented A–B model assuming two segments (in addition to a decaying government sector) and two types of workers attached to one or another segment. The analysis yields a series of meaningful results on viability and the effects of the key variables but proves insufficient to answer a number of important questions. Therefore Section 4 goes on with numerical simulations including robustness checks.

The analysis and the simulations suggest that viability (meaning that all the relevant variables, including profits are positive) is a function of the speed of job destruction and the strength of the initial shock to employment. In the long run, the system asymptotically converges to full employment of all groups, that is, the shock of the transition fades away and then completely disappears. If, and only if, the rate of job destruction is sufficiently low, the unemployment rates can get close to steady-state values during the transition (before the last SOE is closed). Within the domain of feasible scenarios, unemployment differentials are basically determined by the

³ According to the International Reading Tests (Statistics Canada, 2000), in the foregoing period 19 percent of the Czech, Hungarian, Polish and Slovenian workers participated in on-the-job training, in contrast with 38 percent of the other participant countries. These proportions amounted to 7 vs. 18 percents for workers without high school diploma, and 5 versus 14 percent for workers older than 35.
level of benefits and the cross-subsidization of low-productivity groups. Lower benefits imply higher aggregate employment and inequalities (except for the case of very high benefit is combined with very fast job destruction), while employment bonuses are conducive to lower inequalities and higher employment throughout the redeployment process. The evaluation of different scenarios in terms of aggregate income versus inequality reveals that the choice between low versus high benefits can be a matter of preferences but the systems with subsidies dominate the systems with no subsidies.

Our conclusion on the possibly benevolent role of employment tax credits supports a much-debated proposal by Akerlof et al. (1991) to subsidize East German employees after the unification. We discuss the relation of our results to this proposal, and draw conclusions in Section 5. The difficult theorems and proofs are relegated to an Appendix.

Before starting to present the model, we emphasize that this model assumes away optimization and learning: governments set the key parameters at the start of transition and do not modify their policies in view of what happens in the labor market. We opted for a non-optimizing model because it is unclear (even ex post) what kind of social welfare functions were conceived by the post-communist governments: some apparently opted for fast transformation, others tried to minimize the income loss from transition, and some made efforts at keeping inequalities within ‘tolerable’ limits. The assumption of no learning is easy to justify regarding the speed of reforms. Indicators constructed by the EBRD on 27 transition countries (EBRD, 2003) suggested that a country’s large-scale privatization index of 1995 (a proxy running from 0 to 4) was an excellent predictor of its 2003 index: in a univariate regression the 1995 index has a coefficient of 0.67 with a t-value of 5.5, and the model has an R-square of 0.53. Unemployment compensation, another key variable in our model, was substantially tightened in several countries, however (Cazes and Nesporova, 2003). Despite some ‘learning by doing’, the basic differences that characterized the start of transition remained valid for protracted periods, making the Czech Republic (with relatively slow destruction, parsimonious benefits and significant redeployment subsidies) rather different of Hungary and Poland (fast destruction, high benefits and modest expenditures on active labor market policy), not to mention Russia, where unemployment compensation was virtually nonexistent and the reforms proceeded at a slow pace.
2. The model

In this section we present a simple generalization of the A–B-model assuming two segments of the arising private sector. The segments are characterized by different constant marginal product of workers. Workers are confined to one or another segment. The segments are connected by a national system of taxation and welfare. The ingredients of this simple model are the following.

The last days of socialism. Under socialism, all workers are employed in the state sector and assumed to be paid a uniform wage. Due to the sudden collapse of the Soviet-type system, full employment is impaired and the unemployment ratio rises from zero to $u^0$.

Transition – the key variables. Following the initial shock private sector job creation takes off. Workers are dismissed randomly and hired by the private sector from unemployment. While unemployed, they are paid unemployment benefit $b$, which can be thought of as either lump-sum or representing a fraction of the state sector wage. The productivity yields of job–worker matches differ in the two segments of the emerging private sector. We distinguish high-productivity (H-type) and low-productivity (L-type) matches with time-invariant productivity yields $y_H > y_L$. Net earnings $w_i$ change endogenously as given in (4) below. Similar to A–B, we assume that the government charges a poll tax $z$ on every worker. The value of $z$ is determined by the macrobudget equation (5) below. The private sector only employs an $i$-type worker if the per capita profit is positive. To alleviate unemployment, the government can support L-type jobs by paying a transfer $k$. One part of the transfer ($k_1$) is simply a compensation, without which L-type workers would have higher tax rates than their H-type counterparts under a poll tax. Given the other parameters of the model, one can determine a transfer $k = k_1$, so that the tax rates $\tau_H = z/w_H$ and $\tau_L = (z-k)/w_L$ be approximately equal at period $T/2$ and $3T/2$, where $T$ is the duration of transition. The remaining part ($k_2$) serves as an employment subsidy: $k = k_1 + k_2$. Net profits per worker are given by $\pi_H = y_H - w_H - z$ and $\pi_L = y_L - w_L - z + k$, respectively.

Job destruction. We must break down state-sector jobs to potential H- and L-types: let $E_H$ and $E_L$ be the number of H-type and L-type workers, respectively, and $E = E_H + E_L$ be their sum. We shall assume that the initial drop in employment from $E^* = 1$ to $E^0 < 1$ is followed by a
continuous decline in state-sector employment. The initial drop was from \((E^*_H, E^*_L)\) to \((E^0_H, E^0_L) = E^0(E^*_H, E^*_L)\). Denoting the state-sector employment rates with \(e_i = E_i / E^*_i\), \(e^0 = E^0\) and the constant rate of job destruction with \(s > 0\) we have:

\[
\begin{align*}
\dot{e}_H &= -s, \\
\dot{e}_L &= -s,
\end{align*}
\]

where \(e^0\) is given. Obviously, \(e_H = e_L = e = e^0 - st\). The elimination of the state sector is completed at date \(T = E^0 / s\). After this date, \(e = 0\).

**Job creation.** Let \(N_i\) be the number of \(i\)-type workers employed in the private sector. Denote the private employment ratio of \(i\)-type workers with \(n_i = N_i / E^*_i\). We assume that the employment rate increases in proportion to the firm's profit per worker:

\[
\begin{align*}
\dot{n}_H &= a(y_H - w_H - z), \quad \text{where} \quad n_H = N_H / E^*_H, \\
\dot{n}_L &= a(y_L - w_L - z + k), \quad \text{where} \quad n_L = N_L / E^*_L.
\end{align*}
\]

**Unemployment.** The creation of private jobs cannot keep up with state-sector job destruction, therefore unemployment emerges. The unemployment rates are

\[
\begin{align*}
u_H &= 1 - e - n_H = \Delta e - n_H, \quad U_H = u_H E^*_H, \\
u_L &= 1 - e - n_L = \Delta e - n_L, \quad U_L = u_L E^*_L,
\end{align*}
\]

where \(\Delta e = 1 - e\) is the absolute value of change in the state-sector employment rate (equal for both H- and L-types). We also need measures of total unemployment \(U = U_H + U_L\), total change in state-sector jobs \(\Delta E = \Delta E_H + \Delta E_L\), where \(\Delta E_H = E^*_H \Delta e\) and \(\Delta E_L = E^*_L \Delta e\).

**Wages.** We retain the wage equation of A--B derived from arbitrage equations under the assumption that wages respond to the probability of exit from unemployment rather than the unemployment rate itself. Assuming a uniform rate of interest \(r\) and a uniform rate \(c\) denoting the surplus value of being employed over being unemployed we have:

\[
w_H = b + c(r + \dot{n}_H / u_H)
\]

(4–H)

8
Following A–B, we assume that the productivity of the state-sector workers is \( x \), the surplus absorbed by a worker is \( \alpha \) and the net wage is \( v = (1 + \alpha)x - z \). In contrast to the private sector, the wage in the state sector is irresponsive to market pressures and contains a rent.\(^4\)

**Taxes and transfers.** Benefits and transfers are financed from taxes: \( Ub + N_Lk = (1-U)z \). The macro budget equation thus takes the form

\[
(E_H^*u_H + E_L^*u_L)b + E_L^*n_Lk = (1-E_H^*u_H - E_L^*u_L)z .
\]

**System of differential equations.** Note that we have a simultaneous system of equations: the wage depends on changes of employment and *vice versa*. As we will show in the Appendix, substituting (4) into (2) and using (5) yields

\[
\begin{align*}
\dot{n}_H &= a \frac{u_H}{u_H + ca} \left( y_H - cr - \frac{b + kE_L^*n_L}{1-E_H^*u_H - E_L^*u_L} \right) , \quad n_H^0 = 0 , \\
\dot{n}_L &= a \frac{u_L}{u_L + ca} \left( y_L - cr - \frac{b - k(e + E_H^*n_H)}{1-E_H^*u_H - E_L^*u_L} \right) , \quad n_L^0 = 0 .
\end{align*}
\]

Although we have six equations, (1) can be solved directly and (3) can be eliminated. In fact, we have a system of nonlinear differential equations containing only two interdependent equations. Introducing the notation \( \bar{y}_i = y_i - cr \) for what we shall call the *reduced productivity* of type \( i \) labor, we have

\[
\begin{align*}
\dot{n}_H &= a \frac{\Delta e - n_H}{\Delta e - n_H + ca} \left( \bar{y}_H - \frac{b + kE_L^*n_L}{e + E_H^*n_H + E_L^*n_L} \right) , \quad n_H^0 = 0 , \\
\dot{n}_L &= a \frac{\Delta e - n_L}{\Delta e - n_L + ca} \left( \bar{y}_L - \frac{b - k(e + E_H^*n_H)}{e + E_H^*n_H + E_L^*n_L} \right) , \quad n_L^0 = 0 .
\end{align*}
\]

We shall call the system *viable* if the profit rates, the employment rates and the unemployment rates are all nonnegative: \( \pi_i \geq 0 \), (i.e. \( \dot{n}_i \geq 0 \)), and \( 0 \leq n_i \leq \Delta e \), \( i = H, L \).

\[\text{---}\]

\(^4\) As we shall see in the simulations, the employment of L-type in the government sector at wage level \( w_L \) could be profitable in principle, but due to the lack of suitable owner's control, the appropriation of rent survives, making the government sector permanently loss-maker. This is the reason that all SOEs should be closed down.
3. Analytical results

In this section we analyze our two-segment model. Under simple and plausible assumptions to be introduced below, the two-segment generalization of A–B yields a well-functioning system, which preserves the basic properties of the original. The time period considered consists of three parts. There is a short initial stage when the hardening of the budget constraint and the CMEA-shock generates some unemployment. The transition stage lasts from the start of private job creation to the closing of the last SOE. This is followed by the mature stage lasting until (nearly) all unemployed workers are redeployed.

We start with a set of assumptions supplementing the one-segment A–B model and a brief discussion of the resulting properties. Then the basic features of the mature stage are discussed. If the mature stage starts from full employment, the system stays in the full-employment steady state. If the end-of-transition employment rates are high enough, the system is able to approximate the full employment state. Having analyzed the properties of the mature stage, we turn to the conditions of viability of the transitional stage. We demonstrate that if the destruction of the state sector is sufficiently slow, then an important property of the A–B model is preserved: the transition stage is characterized by a stable unemployment equilibrium. Finally, we present some analytical results on the impacts of benefits and subsidies and their interactions.

Assumptions and basic properties. We make three natural assumptions. A1. The composition of the labor force is fairly balanced, say $1/2 < E_H^* / E_L^* < 2$. A2. The transfer is lower than the difference between the two productivities: $0 < k < y_H - y_L$. A3. The product of the reduced L-productivity and the initial employment rate is greater than the unemployment benefit: $\bar{y}_L (1 - u^0) > b$. This assumption is only slightly stronger than $\bar{y}_L > b$, which is self-evident.

Under A1–A3 the system is well-behaved in the sense that H-workers are paid higher wages than L-workers and H-labor also has higher employment rate. Due to its utmost simplicity, it is worth looking at the starting point of the transition process first. Recall that before the private sector

---

5 In the excluded limit case, when the transfer is $k = y_H - y_L$, the differences between the employment rates and the wages of the two types disappear: $n_H = n_L$ and $w_H = w_L$. In that case both factors of the RHS of (7–H) are equal to those of the RHS of (7–L). By (4), the earning paths are also identical.
takes off, employment in the state sector suddenly falls \((E^0 < E^*)\), and unemployment arises: \(u^0 = 1 - e^0 > 0\). The initial changes in the private employment rates of the two types are given by

\[
\dot{n}_H(0) = \frac{au^0}{ca+u^0} \left( \bar{y}_H - \frac{b}{1-u^0} \right) \quad \text{and} \quad \dot{n}_L(0) = \frac{au^0}{ca+u^0} \left( \bar{y}_L + k - \frac{b}{1-u^0} \right).
\]

Due to A3, low-skilled employment takes off: \(\dot{n}_L(0) > 0\). In addition, A2 implies that H-employment grows faster than L-employment: \(\dot{n}_H(0) > \dot{n}_L(0)\). The initial earnings are as follows:

\[
w_H(0) = b + c \left( r + \frac{\dot{n}_H(0)}{u^0} \right) \quad \text{and} \quad w_L(0) = b + c \left( r + \frac{\dot{n}_L(0)}{u^0} \right).
\]

Hence the private sector pays higher wages to H-workers than to L-workers: \(w_H(0) > w_L(0)\).\(^6\)

Having examined the start of the transition we now show that the H-workers are always paid better than the L-workers. The difference in earnings, however, is not so large that profits in the H-segment were lower than in the L-segment. Therefore the H-employment rate is always higher than the L-one. This is formulated as a theorem.

**Theorem 1.** Under A2, (and apart from the start) the employment as well as the wage is respectively greater in the H-segment than in the L-segment: \(n_H > n_L, \ w_H > w_L\).

**Proof.** Appendix.

**Remarks.** Since earnings depend on employment and employment depends on earnings, the direct comparisons of \((2-H)\) with \((2-L)\) or \((4-H)\) with \((4-L)\) are insufficient. The essence of the proof of \(n_H > n_L\) can be simply summarized: it is the last factor what is decisive in (7), and by \(\bar{y}_H > \bar{y}_L + k\), it is larger for \(H\) than for \(L\). In turn, \(w_H > w_L\) is a straightforward consequence of the wage equations (4) and \(n_H > n_L\).

**Analysis of the mature stage.** The system enters the mature stage when the government sector completely disappears: \(e(T)=0\). From that point on (7) simplifies to a time-invariant system:

\[\]
\[^6\text{Although we always assume } u^0 > 0, \text{ it is worth examining for a moment what happens in the inadmissible limit case } u^0 = 0. \text{ If } k = 0, \text{ then the initial earnings are just identical to the productivities: } w_H(0) = y_H, \ w_L(0) = y_L.\]
\[ \dot{n}_H = a \frac{1-n_H}{1-n_H + ca} \left( \bar{y}_H - \frac{b + kE^*_H n_H}{E^*_H n_H + E^*_L n_L} \right), \quad n_H(T) = n_H^r, \quad (8-H) \]

\[ \dot{n}_L = a \frac{1-n_L}{1-n_L + ca} \left( \bar{y}_L - \frac{b + kE^*_L n_H}{E^*_H n_H + E^*_L n_L} \right), \quad n_L(T) = n_L^\tau \leq n_H(T). \quad (8-L) \]

Evidently, full employment is a steady state: \( n_H^o = 1 \) and \( n_L^o = 1 \) because the first factors of (8) are zero. In addition to full employment, there exist other steady states but they are not viable. Full employment is a locally asymptotic stable steady state of the post-transition system if the end-of-transition employment levels are high enough. We present this statement as a theorem.

**Theorem 2.** Under A1–A2, in the mature stage, full employment is a locally asymptotically stable steady state. Both types' employment rates rise monotonically to 1 if the pair of end-of-transition employment rates \((n_H(T), n_L(T))\) are high enough, i.e. condition (9) holds:

\[ (\bar{y}_L + k)E^*_H n_H(T) + \bar{y}_L E^*_L n_L(T) > b. \quad (9) \]

**Remarks.**
1. Let \( n = E^*_H n_H + E^*_L n_L \) be the aggregate employment rate. In the transfer-free case, (9) reduces to \( \bar{y}_L n(T) > b \), i.e. the constraint \( e^o > b / \bar{y}_L \) determined by A2 is now replaced by \( n(T) > b / \bar{y}_L \). 2. If the aggregate unemployment rate reaches its maximum before the end of transition, say at \( t^o \), then the viability condition of the whole process is

\[ (\bar{y}_L + k)(e^o + E^*_H n_H^o) + \bar{y}_L E^*_L n_L^o > b. \quad (9^o) \]

**Proof.** Since viability requires positive profits in our model, both employment variables must increase. By (8–H), \( \dot{n}_H > 0 \) if and only if

\[ \bar{y}_H E^*_H n_H + (\bar{y}_H - k)E^*_L n_L > b. \quad (10-H) \]

By (8–L), \( \dot{n}_L > 0 \) if and only if

\[ (\bar{y}_L + k)E^*_H n_H + \bar{y}_L E^*_L n_L > b \quad (10-L) \]
In view of \( Y_H - Y_L = y_H - y_L \) and A2, (10–L) implies (10–H). Because the variables are strictly increasing in time, (9) implies (10–L) for any \( t > T \). In this region the only stable steady state is full employment. Therefore, any path starting in this region converges to it.

When employment is full, the taxes are fully used to finance the subsidy to L-workers: \( z^* = E_L k \). Therefore the wages are equal to the corresponding marginal productivities corrected by weighted subventions: \( w^*_H = y_H - E_L^* k \) and \( w^*_L = y_L + E_L^* k \). The subsidy is fully financed by the H-workers.

**Analysis of the transition stage.** To prove viability of the transition we need stricter conditions than those of Theorem 2. However, we cannot give them analytically. Instead, we shall rely on the concept of unemployment equilibrium introduced in the A–B model. In their one-segment model A–B proved (pp. 297-300) that under suitable assumptions (slow destruction) there exist two unemployment equilibria during the transition: the lower one is stable, while the higher one is unstable. Moreover, any initial unemployment below the higher equilibrium generates a viable path. In this part, we discuss whether these properties are preserved in the two-segment model. For simplicity, we neglect the transfers here.

Following the idea of A–B, we work with unemployment rather than employment rates. In this case, the process of the transition is described by a time-invariant (autonomous) system of differential equations composed of \( \dot{u}_H = s - F_H(u_H, u_L) \) and \( \dot{u}_L = s - F_L(u_H, u_L) \), where \( F_H(u_H, u_L) = f_H(1 - e_H - u_H, 1 - e_L - u_L) \), and \( (f_H, f_L) \) is the vector–vector function on the RHS of (7). Any vector of the unemployment equilibrium is determined by the system of equations \( F_H(u_H^0, u_L^0) = s \) and \( F_L(u_H^0, u_L^0) = s \).

In the Appendix (Theorem 3), we demonstrate that for slow enough rates of job destruction and zero transfer, there exists at least one unemployment equilibrium (a pair of equilibrium unemployment rates), which is locally asymptotically stable: paths starting sufficiently close to the stable unemployment equilibrium not only remain sufficiently close to it but converge to it. We shall see in the simulations, however, that in the numerical A–B-model as well as in its present generalization (with zero transfer) a too low rate of job destruction should be assumed in
order to have stable unemployment equilibrium in the transitional stage. Therefore we renounce
this assumption from now on.

We can obtain, however, viable transition paths without having stable unemployment
equilibrium. We need only assume the speed of destruction is sufficiently slow: \(0 < s < s_M\) and
the initial employment rate \(e^0\) is sufficiently high: \(e^0(s) < e^0 < 1\), the lower employment bound \(e^0\)
\((s)\) depends on the speed of closure. While the assumption of very slow destruction is attractive as
it generates a steady state during the transition stage, the assumptions of the model lose their
validity during a prolonged traction. Therefore we shall make a fourth assumption: \(A4\). The rate
of closing is higher than the initial growth rate of job creation: \(\dot{n}_L(0) < s\), i.e. \(s_m < s (< s_M)\).

**Impacts of benefits and subsidies.** The impact of benefits on employment is quite obvious
within the model. Under low benefits, the job seekers accept lower wage offers, profits rise and
job creation proceeds faster. The effect of the subsidy is far less evident. One of the most
important statements of our paper is as follows: *with a well-chosen subsidy, the unemployment
rate of L-type labor can significantly be reduced without much affecting the employment of H-
type labor.* In this section we summarize some analytical results supporting this statement.

We have already seen that at the start of the transition an increase in \(k\) increases L-type
employment without affecting H-type employment. Because of continuity, our favorable result
approximately holds for a while. Here this result will be extended for the *whole period.* The basic
idea is that the introduction of a subsidy does not diminish the average productivity of the system.
We start from the assumption that the average productivity of the private sector
\(y = E^*_H y_H + E^*_L y_L\) is given. It can be shown that if the difference between the productivities of the
two types of labor diminishes, then L-employment and total employment increase, while H-
employment decreases. The intuition behind this observation is as follows: given state-sector and
private-sector productivities, a reduction of \(y_H\) increases the RHS of \((7–L)\) and decreases that of
\((7–H)\). Because of the usual concavity, the aggregate impact is positive.

To see the details, let us increase transfer \(k\) by a very small \(\Delta k\), and add the resulting changes in
\((7)\) to the productivities:
A simple calculation yields $\Delta \bar{y}_H = -\frac{\Delta k E^*_H n_L}{e + E^*_H n_H + E^*_L n_L}$ and $\Delta \bar{y}_L = \frac{\Delta k(e + E^*_H n_H + E^*_L n_L)}{e + E^*_H n_H + E^*_L n_L}$. Our new productivities become endogenous but this presumably does not destroy the validity of the above observation.

Strictly speaking, the statement that a subsidy increases aggregate employment can only be proved for the mature stage and ‘high levels’ of the benefit. To make clear what is exactly meant by ‘high levels’, we introduce the concept of separating benefit. For given parameters, the unemployment benefit $b^*$ is called separating if H-employment (at the end of the transition) is insensitive to a change in $k$. Measuring the sensitivity to subsidy by indexes $m_H(t) = \partial n_H(t)/\partial k$ and $m_L(t) = \partial n_L(t)/\partial k$ we shall speak of a separating value if $m_H(T) = 0$.

Theorem 4 of the Appendix determines implicit but precise relations between the dynamics of L-type and H-type employment: in the mature stage, the two employment rates (more precisely, their increasing transformations) converge to each other, when the subsidy is raised. Intuitively, as will be supported by the simulations, this is explained by the higher sensitivity of H- than L-employment to changes in $k$. This observation is formulated in Theorem 5 of the Appendix, and proved for benefit levels above the separating value, i.e., $b > b^*$ and therefore $m_H > 0$.

Can H-employment be an increasing function of the subsidy, at all? Our numerical experiences suggest the answer is yes: this seemingly perverse case can occur when the employment situation is so depressed that H-employment gains more from an increase in L-employment (and the resulting drop in the tax burden) than it loses from an increase in $k$. This holds with good approximation for $t \neq T$ as well.

Since we cannot prove that $k$ has benign effect on aggregate employment in the more common case of $m_H < 0$, we recourse to numerical simulations, which will show that this effect is positive for a broad range of parameters. In addition, the simulations will hopefully make perceptible how the full system works.
4. Numerical results

Choosing parameters. Apart from dividing the emerging private economy into two segments we follow A–B in choosing parameters for the numerical simulations. Productivity in the state sector equals \( x = 1 \). State-sector workers appropriate rents \((\alpha = 0.3)\) so their net wage is \( v = (1 + \alpha)x - z \). Furthermore, the unemployment benefit is \( b = 0.5 \); the proportionality factor between the per capita profit and the speed of job creation is \( a = 0.1 \); the subjective value of employment is \( c = 2 \) and the discount rate is \( r = 0.1 \). Let the size of H-type and L-type labor be \( E_H^* = 0.5 \) and \( E_L^* = 0.5 \) respectively and the initial unemployment \( u^0 = 0.04 \). The productivity of the private sector \((y=1.8)\) is also symmetrically broken down: \( y_H = 2.2 \) and \( y_L = 1.4 \). Finally, \( s = 0.08 \) is the speed of job destruction implying that the duration of the transition is \( T = 12 \) years. For the time being, we exclude subsidies, i.e. \( k_2 = 0 \) and \( k_i = 0.08 \).

First we display the stability domain of Theorem 2. It can be seen in Figure 1 that for end-of-transition employment levels \( N_H(T) = 0.2 \) and \( N_L(T) = 0.2 \), close to the demarcation line (9), the system still converges.

Figure 1

Turning to the simulation of the full period, first we retain the zero subsidy assumption: \( k_2 = 0 \). Using the same data as in Figure 1, the system converges to full employment, although the employment rate is very low at the end of the transition with \( N_H(T) = 0.36 \) and \( N_L(T) = 0.194 \). Unemployment is high and disproportionate: \( U_H(T) = 0.14 \) and \( U_L(T) = 0.31 \) (Figure 2a). Wages first decrease, then they increase (Figure 2b). The poll tax first increases, then it decreases. The tax rates do not diverge too much (Figure 2c) and they are equal on average.

Figure 2 a-c

The paths of employment, wages, profits and taxes are similar to those in Figures 2 a-c for other parameter sets. The way in which the paths are shifted by changes in the parameters will be discussed below. If the speed of destruction is sufficiently low, the unemployment rates get close to equilibrium levels during the transition stage. For \( s = 0.02 \), the unemployment equilibrium is
With initial unemployment rates $u_H^0 = 0.05$ and $u_L^0 = 0.05$, for instance, we get a path which approximates an equilibrium within 5-10 years. When the transition is over, the path starting from this equilibrium converges to a full employment steady state (Figure 3a).

**Figure 3 a–b**

The effect of changes in $s$ is examined in Figure 3b. The system is started from $u^0 = 0.1$ and $s$ is changed between 1 and 9 per cent. (There exists a viable path even for $s = 0.1$. ) Figure 3b suggests that $u_H(T)$ and $u_H(T−1)$ hardly differ from each other as $s$ grows, while the gap between $u_L(T)$ and $u_L(T−1)$ opens up for $s > 0.04$. The choice of $s$ also has implications on how strong the initial shock can be. It is visible that the higher the rate of job destruction, the higher initial employment rate is needed. Indeed, for $s = 0.05$, the critical value is $e^0 = 0.514$, while for $s = 0.10$, $e^0 = 0.839$.

**Impacts of benefits and subsidies.** In Figure 4 we compare four scenarios differing in the level of the benefit ($b = 0.3$ versus $b = 0.5$) and existence of a subsidy ($k = 0.3$ versus $k = 0.08$ meaning that $k_2 = 0$). The introduction of a subsidy improves the employment of L-type labor, but tightening the unemployment benefit is even more effective. H-employment is only marginally affected by the subsidy and increased by tightening of the benefit. Aggregate employment, shown in Figure 4, is responsive to both benefits and subsidies.

**Figure 4**

Turning to wages, the emerging picture is similar. The introduction of a subsidy raises the wages of L-type workers, but unemployment benefit has a stronger impact. For the employment of H-type workers, the effect of $k$ is negligible. Figure 5 summarizes these effects showing the evolution of a measure of wage inequality $\ln(w_H/w_L)$. The introduction of a subsidy diminishes inequality but lower benefits have a similar effect. Initially, the effect of the subsidy seems stronger, while the case is reversed at later stages of the transition.

**Figure 5**
Subsidy effects: checking robustness. The marginal effect of the subsidy at different values of \( s \) and \( b \) is examined in Table 1. Each cell of the table contains a pair of numbers with the first relating to H-type and the second to L-type labor. The figures show the percentage increase in H and L employment, respectively, in response to a one per cent increase of the subsidy.

Table 1

The positive impact on L-employment is always higher than the negative impact on H-employment. The higher the unemployment benefit and the faster the destruction of state-sector jobs (i.e., the farther we are from the Northwest corner), the larger is the difference between the two impacts. For a given benefit and rate of closure, the higher the benefit, the weaker is its marginal impact. At the entries \( b = 0.5 \) and \( b = 0.7 \), the benefit is above the separating value: the marginal impact on H-employment becomes positive.

We note that the assumption of time-invariant transfers can be relaxed. The calculation would be more precise if the transfers were at least cleared ex-post. Let us denote the length of an elementary interval by \( h \), and choose \( k(t+h) \) to be determined such a way as to ensure the equality of the tax rates at \( t \):

\[
\frac{z(t)}{w_H(t)} = \frac{z(t)-k(t+h)}{w_L(t)}.
\]

Numerical simulations attest that using an averaging constant transfer is conducive to almost identical results.

Average income and inequality. To obtain an approximate evaluation of the welfare impacts we calculate the intertemporal mean of average income and its coefficient of variation. Net average income at \( t \) and its coefficient of variation are calculated as

\[
I = E v + N_H w_H + N_L w_L + Ub \quad \text{and} \quad \sigma = \frac{\sqrt{E(v-I)^2 + N_H (w_H-I)^2 + N_L (w_L-I)^2 + Ub (b-I)^2}}{I}.
\]

We treat sigma as a measure of inequality admitting that it only captures the cross-section variance in incomes and ignores the part of inequality, which stems from fluctuations over time of income for both types of labor. Average income and its standard deviation in \([0,2T]\) are
\[ \bar{I} = \frac{1}{2T} \int_0^{2T} I(t) dt \quad \text{and} \quad \bar{\sigma} = \frac{1}{2T} \int_0^{2T} \sigma(t) dt. \]

We expect that the introduction of a subsidy raises average income and decreases its variance, while the reduction of unemployment benefits raises average income as well as its variance.\(^7\) In Figure 6 we present a summary chart, which compares the implications of choosing high versus low benefits and transfers, respectively, under fast and slow pace of closing the state sector. To fix the length of the period, we choose \(T\) as the length of the slowest transformation. The total income accumulated during time \(2T\) – depending on employment, wages and the benefit – is depicted on the vertical axis, while the relative income variance is shown on the horizontal axis.

The chart suggests that at low speed of transition \((s=0.02)\) higher benefits enhance equity but reduce aggregate employment and thus average income during the transition, while a subsidy increases both income and equity. Assuming well-behaved indifference curves in the \((I, 1-\sigma)\) space we can conclude that the choice between high/low benefits can be a matter of preferences (average income rises at the cost of equity), while a subsidy is likely to generate welfare surplus in the sense that both average income and equity rise. The price for that is paid by H-workers, who are worse off than they would be in an intervention-free economy (apart from the special case \(b > b^*\)).

The case is somewhat different at high speed of transition \((s = 0.08)\). The above-mentioned implications continue to hold when benefits are low. However, when the state sector is closed fast and benefits are high, further increases in the benefits reduce both income and equity. Both the income effect and the equity effect of the subsidy are particularly strong when benefits are high, while the income effect fades away when the benefits are tight.

In brief, the effects of subsidies and benefits are interdependent and strongly influenced by the pace of closing the state sector. When job destruction is slow, benefit levels do not matter near as much as they do in the case of fast transition, when there are huge exogenous inflows to unemployment. In case of slow destruction, \(k\) has relatively strong impact on equity but not on income (since the fiscal channel is not so important in determining employment), and its effect is

\[ I_0 = E_0v + U_0b \approx 1.3, \quad \text{while the final one is} \quad \bar{I}^* = N^*_H y^*_H + N^*_L y^*_L = 1.8. \]

\(^7\) Note that initial average income is \(I_0 = E_0v + U_0b \approx 1.3\), while the final one is \(I^* = N^*_H y^*_H + N^*_L y^*_L = 1.8\).
Insensitive to $b$. In case of fast destruction and low benefits, $k$ is equity-enhancing and exerts some influence on income. When destruction is fast and benefits are high, the income effect of $k$ is very strong because subsidization has crucial impact on both L-type and aggregate employment, as was discussed earlier.

**Figure 6**

**Phasing out.** Our model did not address the question of how subsidies could be phased out. Once all the displaced workers are absorbed, the program automatically stops but the distortions in wages remain, as the corresponding wages converge to $w_h^* = y_h - E_k^i k$ and $w_l^* = y_l + E_k^i k$, respectively. Since the model ignored the age of workers, it would be unfair to argue that aging of the existing labor force solves the problem. Within the framework of the model the subsidy can only be phased out at will, after the shock of the transition is over. Fortunately, since at the end of the transition shock $s=0$ and the fiscal burden of unemployment ($Ub$) is negligible, the elimination of $k$ exerts weak influence on employment.

5. Discussion and conclusions

In the preceding sections we analyzed and numerically simulated a model of the transition where marginal products differed in the two segments of the emerging private sector, and wages did not immediately adjust. We found that the shock of the transition was absorbed in the long run irrespective of how the key policy variables were set. The temporary loss to society in terms of aggregate income and equity, however, largely varied with the speed of job destruction, level of the benefits, distribution of the tax burden and the way these instruments were combined. We found that subsidies to $L$-type labor may enhance equity and raise aggregate employment.

Our result on the benign effect of employment subsidies may remind the reader of a 'breathtaking proposal', as Dornbusch (1991) labeled it, by Akerlof at al. (1991) to subsidize East German jobs after the unification.\(^8\) The sudden jump of wages and welfare payments to nearly Western levels as well as the diversion of consumer spending toward Western products led to exceptionally severe price-cost squeeze in the former GDR. This menaced with an immediate collapse of the state sector and seemed to paralyze the Treuhandanstalt’s efforts to privatize the outdated

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\(^8\) It may also recall a brand of the literature marked, among others, by Phelps (1994), Snower (1994), Katz (1996) or Nickell and Bell (1996) advocating subsidies in addition to unemployment compensation in the US and the UK.
kombinuten. Accordingly, a large part of the proposed subsidies would have gone to the SOEs but the program would have covered newly created jobs, too. The Akerlof et al. proposal was an operational one supported by empirical arguments, which suggested that the program could have been fiscally viable, while the risk of excessive union wage claims (encouraged by improved employment prospects) could have been reduced by proper design.

The model presented in this paper is clearly not an operational one. As Katz (1996) notes, “the extent to which [subsidies] raise the wages and employment of the targeted group and has impacts on non-subsidized workers is an empirical question that depends on the relevant labor demand and supply parameters as well as administrative aspects of the design of the subsidy program” (p. 2). We rather tried to draw attention to the interactions between different policy instruments in the special settings of the post-communist transformation.

We argued, first of all, that the kind of factor price distortion motivating the Akerlof et al. proposal was also inherent in the transition process of other transition countries although in milder forms. In the GDR, “the high level of wages relative to productivity (…) result[ed] in too little current employment and too slow a pace of investment and new job creation” (op.cit. p. 70). In fact, this statement could have been applied to the remote regions and low-skilled groups of other transition countries unaffected by the type of wage and demand shocks hitting East Germany.

We found that for a broad range of parameters, subsidies could have been welfare-enhancing not only in the ‘East’ (remote regions, low-skilled groups) but also in the entire economy, in the sense that the gains acquired by the disadvantaged exceeded the losses of the better endowed.

The study of interactions between various policy instruments in a closed model, however, showed that the impact of the employment subsidy varied with the government’s choice of other policy variables. At low speed of closing the state sector and parsimonious benefits (upper left block of Table 1) the introduction of a subsidy amounting to ten per cent of low-productivity workers’ wage increased the employment of low-productivity workers by about 2.8 per cent. This effect is modest by any standard, and accidentally falls close to what is expected in competitive labor
markets free of exogenous disturbances characterizing the transition. At high benefits and high speed of closing the SOEs the subsidy appeared to have much stronger effect because the fiscal channel became more important. In the baseline scenario ($s = 0.08$, $b = 0.5$) the effect on $L$-employment of a ten per cent proportional subsidy exceeded 15 per cent.

Generally, it seems that there was a particularly wide scope for bettering (or worsening) labor market outcomes by policy instruments in the transition setting. There are several ways of getting to the bottom left corner of Figure 6. It seems that the worst conceivable policy was combining fast destruction with high benefits and no support for the low-productivity groups. In countries opting for fast elimination of the state sector and generous compensation for the job losers there was a strong case for assisting low-skilled workers and badly affected regions. Failure to do so threatened with substantial inequality and an unnecessarily sharp decline in aggregate employment during the transition. Slow transition implied income loss, while it helped to keep inequalities low. A gradualist policy combined with subsidization could nevertheless achieve income levels similar to that achieved in a ‘fast and generous’ regime. The best conceivable scenario arising in our model is one where the state sector is closed fast, benefits are parsimonious (but not very low) and low-productivity groups are assisted in entering the emerging private sector.

We believe that these predictions help to ‘map’ the mixed policies pursued in vivo by post-communist governments. The Czech Republic adopted a gradualist policy of job destruction combined with parsimonious benefits and significant expenditures on active labor market policy (ALMP) as discussed in detail in Jurajda and Terrell (2003). In the early stage of the transition the country managed to keep unemployment at exceptionally low levels compared to other CEEs, while it may have lost income in the long run. Hungary, a fast reformer operating an initially generous unemployment compensation system combined with modest ALMP expenditures is a good example of the other extreme. Our model suggested that once the government had chosen high levels of $s$, it is advised to choose low levels of $b$ and high levels of $k$ in order to avoid

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9 As shown in Katz (1996), a proportional subsidy for low-wage workers starting from zero subsidy is expected to increase low-wage employment by $\eta \varepsilon / (\eta + \varepsilon)$ where $\eta$ is the absolute value of the labor demand elasticity and $\varepsilon$ is the effective labor supply elasticity. Katz argued that the best estimate for the US, assuming $\eta = -0.5$ and $\varepsilon = 0.3$, was 0.19 a decade ago – a ten per cent subsidy was expected to increase low-wage employment by about 2 per cent.
serious damages to aggregate employment and equity.\textsuperscript{10} Hungary’s failure to do so have most probably contributed to its post-transition diseases: the country has one of the lowest aggregate employment rates in the OECD; there is an unprecedented, nearly 50-percentage point’s difference in the employment ratios of primary school and college graduates and the economy continues to be struck by severe regional inequalities.\textsuperscript{11} Our results also confirmed that setting tight benefits, as in Russia, may not help if the state sector is closed very slowly and/or the losers of the transition are not assisted in getting a job in the private economy.

We do know that this is a retrospective ‘toy model’, which comes far too late to have relevance for policy-making: the transition is over and cannot be played again. We hope, however, that the framework suggested here can promote the discussion of why employment opportunities are more balanced, and aggregate employment persistently higher, in some post-communist countries than others.

\textbf{Appendix}

\textbf{Derivation of (6)}

For the simultaneous handling of the equations, let us introduce $k_H = 0$, $k_L = k$. Substituting (4) into (2), yields for $i = H, L$,

$$\dot{n}_i = a \left( y_i - b - c \left( r + \frac{\dot{n}_i}{u_i} \right) - z + k_i \right).$$

Expressing $\dot{n}_i$:

$$\dot{n}_i = \frac{au_i}{u_i + ca} (\bar{y}_i - b - z + k_i).$$

Inserting $z = (Ub + N_i k)/(1-U)$ and rearranging, yields (6–H) and (6–L).

\textsuperscript{10} In our baseline scenario $s = 0.08$, $b = 0.5$ and $k = 0.08$. This yields $I = 1.148$ and $\sigma = 0.37$. In order to maintain these levels of $I$ and $\sigma$, while $s$ is increased to 0.1 benefits should be tightened ($b = 0.3$) and ALMP should be substantially expanded ($k = 0.34$).

Proof of Theorem 1

a) First we prove the employment inequality \( n_H(t) > n_L(t) \). We shall formulate system (7) in a more general form:

\[
\begin{align*}
\dot{n}_H &= g(t,n_H)h_H(t,n_H,n_L), & n_H^0 &= 0, & (A-1-H) \\
\dot{n}_L &= g(t,n_L)h_L(t,n_H,n_L), & n_L^0 &= 0. & (A-1-L)
\end{align*}
\]

According to the analysis of the start, the H-employment rate increases faster than does the L-employment, both starting from 0. We shall prove the inequality for an arbitrary \( t > 0 \), assuming the contrary. Let us assume that it is at date \( t^* > 0 \), when the inequality is first upset: \( n_H(t) \) intersects \( n_L(t) \) from above. Substituting \( n_H(t^*) = n_L(t^*) = n^0 \) into (A-1), the first factors are equal to each other, and for the second factors, \( h_H(t^*,n^0,n^0) \geq h_L(t^*,n^0,n^0) \) holds, i.e. by (A-1), \( \dot{n}_H(t^*) \geq \dot{n}_L(t^*) \), contradicting the intersection condition.

b) Now we are able to prove the inequality between earnings in H and in L: \( w_H(t) > w_L(t) \). Assume the contrary: there exists an instant \( t^* > 0 \) such at which \( w_H(t^*) \leq w_L(t^*) \). Since \( w_H(0) > w_L(0) \), and the wage–time functions are continuous, there exists an earliest date \( 0 < \bar{t} \leq t^* \) at which \( w_H(\bar{t}) = w_L(\bar{t}) \). Consider the difference between (2–H) and (2–L) at this instant: \( \dot{n}_H(\bar{t}) - \dot{n}_L(\bar{t}) = a(y_H - y_L - k) \). By (A2), \( \dot{n}_H(\bar{t}) > \dot{n}_L(\bar{t}) \). Due to the employment inequality, \( u_H(\bar{t}) < u_L(\bar{t}) \). By comparing (4–H) and (4–L) yields \( w_H(\bar{t}) > w_L(\bar{t}) \), a contradiction.

Unemployment equilibrium

We shall formulate the role of the unemployment equilibrium in the transition for the case \( k = 0 \). According to Theorem 1, for \( s = 0 \) the system has a unique unemployment steady state: \( (u_H^0, u_L^0) = 0 \), which is asymptotically stable. Using continuity, one can show that for small enough closing rates \( s \), there also exists an unemployment steady state, close to zero, which is also asymptotically stable.

Theorem 3. In a transfer-free economy, if the closing rate \( s \) is small enough, and the vector of initial unemployment rates is close enough to its equilibrium value, then during the transition,
this vector stays always close to the equilibrium vector, and after the transition is over, it quickly converges to the new equilibrium, (0,0).

Remarks. 1. Note that without simulation, we cannot tell what sufficiently close is. 2. If the closing rate \( s \) were very small, then there would not be transitional recession and the difference between the two employment rates would also be small. 3. One can easily demonstrate that for excessively fast closing, the system becomes unviable: for example, if \( s > a(\bar{y} - b) \), then even with \( c \approx 0 \), much more employment is destroyed than created during the transition. But there exists an interval of closing rates, such that there is already no unemployment equilibrium but there are still viable paths. In Section 4, we mainly discuss such paths.

Proof. If the initial unemployment rate vector \((u^0_H, u^0_L)\) is sufficiently close to the equilibrium vector \((u^s_H(s), u^s_L(s))\), then – due to the smallness of \( s \) and the positivity of convergence speed, the unemployment vector \((u_H, u_L)\) stays close to the equilibrium vector during the entire transition. After the transition is over, the new system of differential equations,

\[
\dot{u}_H = -F_H(u_H, u_L) \quad \text{and} \quad \dot{u}_L = -F_L(u_H, u_L)
\]

ensures an even faster convergence to the new steady state, (0,0).

Sensitivity to subsidy
We shall characterize the quantitative relation between the two employment rates during the mature stage. To do this, we need the following notations. Let function \( \gamma \), mapping interval \([0, 1]\) onto interval \([0, \infty)\), be defined by \( \gamma(x) = x - ca \ln(1 - x) \). Derivation yields \( \gamma'(x) = 1 + ca/(1 - x) > 0 \), therefore \( \gamma \) has an inverse, it is strictly increasing. Let \( \Gamma_i(t) = \gamma(n_i(t)) \), \( i = H, L \). Then we have

Theorem 4. In the post-transition period, the two employment rates satisfy the equation

\[
\Gamma_H(t) - \Gamma_L(t) = a(y_H - y_L - k)t + \Gamma_H(T) - \Gamma_L(T). \quad (A-2)
\]

Proof. Rearrange (8) so that only the expression in ( ) remain on the RHS:
\[
\frac{1 - n_H + ca}{a(1 - n_H)} \dot{n}_H = \bar{y}_H - \frac{b + kE^*_n n_L}{E^*_n n_H + E^*_n n_L}, \quad n^r_H = n_H(T) \quad (8'–H)
\]

and
\[
\frac{1 - n_L + ca}{a(1 - n_L)} \dot{n}_L = \bar{y}_L - \frac{b - kE^*_n n_H}{E^*_n n_H + E^*_n n_L}, \quad n^r_L = n_L(T) \leq n_H(T) \quad . \quad (8'–L)
\]

Taking into account \((1 - n_i + ca)/(1 - n_i) = \gamma'_i(n_i)\) and deducing \((8'–L)\) from \((8'–L)\), yields
\[
\dot{\Gamma}_H - \dot{\Gamma}_L = a(y_H - y_L - k).
\]

Integration yields \((A–2)\).

With the help of our sensitivity indicators, we can deepen our knowledge about the employment rates of the two types.

**Theorem 5.** In the post-transition period, the two employment rates and sensitivity rates satisfy
\[
m_L = \frac{\gamma'(n_H)m_H + at - c_k}{\gamma'(n_L)} \quad (A–4)
\]

where \(c_k = \gamma'(n_H(T))m_H(T) - \gamma'(n_L(T))m_L(T)\) is a constant.

**Remark.** Similarly to Theorem 4, \((A–4)\) only provides a relative information: if \(m_H \geq 0\) is known (i.e. \(b > b^*\)), then \(n_H > n_L\) (Theorem 1) and \((A–4)\) imply \(m_L > 0\), moreover, \(m_L > m_H\). However, if \(m_H < 0\) (i.e. \(b < b^*\)), then we cannot deduct anything.

**Proof.** Take the derivative of both sides of \((A–2)\) with respect to \(k\):
\[
\gamma'(n_H)m_H - \gamma'(n_L)m_L = -at + c_k.
\]

This already implies \((A–4)\).

**References**


Nickell, Stephen and Brian Bell (1996): ‘Would cutting payroll taxes on the unskilled have a significant impact on unemployment?’, CEPR Discussion Paper No. 276.


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Figure 1. Viable initial states

Figure 2a. High benefit, no subsidy: employment paths
Figure 2b. High benefit, no subsidy: wages and profits

Figure 2c. High benefit, no subsidy: taxes and subsidies
Figure 3: Unemployment equilibrium and the speed of job destruction

(a) Example of transitional unemployment equilibrium at \( s = 0.02 \) and \( k = 0 \)

(b) End-of-transition unemployment as a function of \( s \)
Figure 4: Aggregate employment under different scenarios

Figure 5: Wage inequality under different scenarios $[\ln(w^H/w^L)]$
The curves depict combinations of income and equity generated by different levels of benefits, subsidies and speed of closing the state sector in the following way:

**Benefits** rise by 0.02 stepwise from 0.1 to 0.54 along each curve, as indicated

**Subsidy** is increased by 0.1 stepwise from 0 to 0.4. Each step shifts the curve outward to the north-east

**Fast** transition (s=0.08) : long curves on the top

**Slow** transition (s=0.02): shorter curves in the bottom