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Network-based allocation of responsibility for greenhouse gas emissions

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Abstract

We provide an axiomatic approach to the allocation of responsibility for GHG emissions in supply chains. Considering a set of axioms standardly used in networks and decision theory, and consistent with legal principles underlying responsibility, we show that responsibility measures shall be based on exponential discounting of upstream and downstream emissions. From a network theory perspective, the proposed responsibility measure corresponds to a convex combination of the Bonacich centralities for the upstream and downstream weighted adjacency matrices. Scope 1 emissions, consumption-based accounting and income-based accounting are obtained as particular cases of our approach, which also gives a precise meaning to scope 3 emissions while avoiding double-counting. We apply our approach to the assessment of country-level responsibility for global GHG emissions and to sector-level responsibility in the USA. We examine how the responsibility of countries/sectors varies with the discounting of indirect emissions. We identify three groups of countries/sectors: producers of emissions whose responsibility decreases with the discounting factor, consumers of emissions whose responsibility increases with the discounting factor, and an intermediary group whose responsibility mostly depends on the network position and varies non-monotonically with the discounting factor. Overall, our axiomatic approach provides strong normative foundations for the definition of reporting requirements for

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indirect emissions and for the allocation of responsibility in claims for climate-related loss and damage.

Keywords Upstream and downstream emission responsibilities · Networks · Responsibility measure · Axiomatization · Bonacich centrality

1 Introduction

Defining responsibility for greenhouse gas (GHG) emissions is a key operational requirement in view of developing efficient climate regulation, monitoring its implementation, and, more broadly, providing incentives to firms to reduce their emissions (Gopalakrishnan et al. 2021a, b). GHG emissions are key metrics in climate-related financial disclosures imposed by investors (TCFD 2017) and regulators (Directive (EU) 2022; U.S. Securities and Exchange Commission 2022). They are the basis for carbon pricing and taxation, including the carbon border adjustment mechanism (Kortum and Weisbach 2017). They are emerging as a liability risk measure due to rising climate mitigation (Burger et al. 2020).¹

Against this background, there exists a range of carbon accounting standards and principles. The GHG protocol, a widely used corporate reporting standard, defines three emission scopes: direct emissions from sources controlled by the company (scope 1), emissions from purchased energy (scope 2), and all other indirect emissions (scope 3) (WRI and WBCSD 2011). The definition of scope 3 emissions is extremely broad. It can lead to inconsistencies at the aggregate level because of double-counting (Caro et al. 2013; Gopalakrishnan 2022) and, in actual company reports, is often found missing (Depoers et al. 2016) or inconsistent (Busch et al. 2022). At the country level, emission targets are the central tenet in international environmental negotiations (Falkner 2016; Schluessner et al. 2016). The UNFCCC² assigns to a country responsibility for the emissions that occur within its borders (see e.g. Harrison 2015). There have been substantial criticisms of this “production-based” approach in particular in terms of fairness and of the incentives it provides for carbon leakage (see e.g. Peters 2008; Steininger et al. 2014, and references therein). An alternative “consumption-based” approach has accordingly been put forward (Lenzen et al. 2007; Davis and Caldeira 2010), as well as the “income-based or revenue-based” approach (Marques et al. 2012; Rodrigues et al. 2006).

Overall, several problems remain to be solved in order to provide a consistent carbon accounting framework: a precise definition of indirect emissions is lacking; the requirements for a consistent measurement of direct and indirect emissions at the individual and at the aggregate levels are unspecified; the relationships between direct emissions, indirect emissions and consumption-based emissions are unclear and last

¹ For example, the Dutch court ordered Shell to cut total emissions, including those of their customers, by 45 per cent in 2030 relative to their 2019 levels (The Hague District Court, 26th of May 2021, ECLI:NL:RBDHA:2021:5339). However, on appeal this was overturned on the grounds that there was insufficient scientific consensus on the exact reduction percentage for an individual company like Shell (The Hague District Court, 12 November 2024, ECLI:NL:GHDHA:2024:2100).

² United Nations Framework Convention on Climate Change.

but not least, responsibility (indirect emissions) measures lack normative foundations if they ought to be used in a legal context.

In this paper, we propose a network-based approach to the measure of GHG emissions that addresses these issues both from theoretical and empirical points of view. From the theoretical point of view, we provide axiomatic foundations for a class of measures that define responsibility for GHG emissions in a supply chain as a convex combination of the discounted sums of upstream and downstream emissions. From a network perspective, these correspond to a convex combination of the Bonacich centralities of adjacency matrices normalized from an upstream and a downstream perspective respectively and weighted according to individual emissions. We further show that this responsibility measure is consistent with both the production-based/direct emission approach, which corresponds to the case where the discount factor tends towards zero, and the consumption-based and income-based approach, which correspond to the case where the discount factor tends towards one. Furthermore, we show that in the no-discounting case, an agent's responsibility depends only on its network position and not on its individual emissions.

From the empirical point of view, we build on the world input–output (WIOD) database (Timmer et al. 2015) and its satellite environmental accounts (Corsatea et al. 2019) to characterize, at the country level and at the sectoral level for the USA, how responsibility for GHG emissions evolves when one shifts from a production-based to a consumption-based approach by accounting progressively for indirect emissions and network effects. We find three broad classes of countries: (i) the most developed and service-oriented economies whose share of responsibility grows as one increasingly accounts for emissions (upstream or downstream) in the supply chains, (ii) manufacturing-oriented economies whose share of responsibility decreases as one increasingly accounts for emissions in the supply chains and (iii) an intermediary class of economies whose responsibility can exhibit non-monotonic relationships with respect to the discounting of indirect emissions because they have a relatively high emission intensity but are more closely connected to the most developed service-oriented economies than to the manufacturing-oriented economies. At the sectoral level, we find that the few sectors that are responsible for most GHG emissions (e.g. energy utilities and transport) see their responsibility decrease very rapidly when indirect emissions are accounted for. The service sector accordingly sees its share of responsibility increase rapidly. The responsibility of the mining and quarrying sector (which embeds fossil-fuel extraction) has an interesting non-monotonic (increasing and decreasing) behaviour: it is closely connected downstream to the most emitting sectors but is not very central in the overall economic system.

From the methodological point of view, our axiomatization builds on the characterization of centrality measures by Bloch et al. (2023). The sequence of indirect emissions (with increasing network distance, upstream and downstream) can be interpreted as a nodal statistic and the discounted emissions measure is then characterized by a series of six axioms. First, anonymity states that the responsibility of an agent is not influenced by their identity. Second, monotonicity sets forth that responsibility is monotonic with respect to the natural order on emission sequences. Additivity amounts to considering that total responsibility is invariant when an agent is split into subunits or when two agents are merged. Tail-convergence ensures consistent measurement of

infinite emission sequences. Recursivity states that the responsibility ratio between two emission patterns is independent of their level of upstreamness/downstreamness. This axiom is closely connected to the notion of stationarity in the axiomatization of discounted expected utility by Koopmans (1960) and induces the exponential discounting aspect of the responsibility measure. Finally, consistency ensures that the responsibility allocates exactly the total amount of emissions in the system and thus prevents double-counting. All these axioms are consistent with legal principles underlying responsibility and can contribute to sustainable operation and carbon accounting management.

Related Literature

This paper considers operational problems from the point of view of environmental (public) decision-making, as did, for example, Alizamir et al. (2016) who evaluate mechanisms in which governments purchase green energy at higher tariffs to accelerate the energy transition or Cruz (2008) who analyzes the dynamics of supply chains with corporate social responsibility. More specifically, we contribute to the research on supply chain emission accounting and the apportioning of shared emissions. For example, Sunar and Plambeck (2016) investigate the allocation of emissions among co-products. Gopalakrishnan et al. (2021b, a) focus on the allocation of responsibility for upstream GHG emissions in supply chains using a cooperative game-theoretic approach. They put forward firm-level emission measures using solution concepts such as the Shapley (1953) value and the nucleolus (Schmeidler 1969).

Game theoretic approaches have been also applied more broadly to supply chain management (see e.g. Nagarajan and Sošič 2008, for a survey), to cost-sharing problems in the context of river management (see e.g. Alcalde-Unzu et al. 2015; van den Brink et al. 2018), to loss allocation in energy transmission networks (Bergañinos et al. 2017; Bergantiños et al. 2019), to allocate forest resources (Rahmoune et al. 2024), to responsibility in the context of hazardous transportation problems (Techer 2022) or to allocate the global carbon budget (Giménez-Gómez et al. 2016) or carbon emission permits (Moretti and Trabelsi 2021). In this spirit, Gutiérrez et al. (2018) adopt a combination of bankruptcy problems and linear production situations to allocate carbon permits to firms and Feichtinger et al. (2022) consider a dynamic oligopoly game to regulate emissions.

These approaches often focus on specific network structures (e.g. trees) and their axiomatization (when available) can hardly be applied in the context of GHG emission allocation. In the present paper, a complementary network-based and normative approach is adopted. We propose a responsibility measure that allows for both upstream and downstream responsibilities and that applies to a generic network structure. The axiomatization of this measure is tailored to the problem of GHG emission allocations. Our approach is also much less computationally intensive than cooperative game solution concepts as it only involves the computation of network centrality measures via matrix multiplication.

Another strand of literature focuses on the allocation of responsibility for GHG emissions across countries using input–output analysis (see e.g. Gallego and Lenzen 2005; Lenzen et al. 2007). This literature has notably introduced the notion of consumption-based (Eder and Narodoslawsky 1999; Munksgaard and Pedersen 2001) and income-based (Marques et al. 2012) emissions. However, it has mostly focused

on country-level analysis and generally lacks axiomatic foundations. Our approach subsumes the different notions of responsibility introduced in this literature and clarifies the relationships they entertain. It also provides axiomatic foundations that were missing or incomplete.

We take a network-based perspective to study the responsibility of GHG emissions. Such a network-based approach is also adopted by King et al. (2019) in the context of carbon tax reforms. They show that taxing specific sectors based on their position in the production network can lead to a larger reduction of aggregate emissions than when targeting sectors based on only their direct emissions. Moreover, Meléndez-Jiménez and Polanski (2020) adopt a network approach to evaluate consumption that generates mostly local pollution, and analyze the effects of transfers among agents on the aggregate pollution levels. They also connect the individual pollution levels to the network centralities. From a more theoretical perspective, our approach relates to the network theory literature that aims to provide axiomatic characterizations of centrality measures (see e.g. Jackson 2008; Newman 2010, for surveys). Garg (2009) axiomatizes the degree, decay and closeness centralities. Palacios-Huerta and Volij (2004); Slutzki and Volij (2006); Dequiedt and Zenou (2017); Kitti (2016) provide characterizations of some prestige and eigenvector-related centrality measures. Boldi and Vigna (2014) axiomatize a variation of closeness centrality. Bloch et al. (2023) characterize the standard centrality measures within a unified framework. They introduce the concept of nodal statistics which captures a node's position in a network and show that each of these centrality measures can be viewed as an additively separable weighted average of some nodal statistic. However, to assign responsibility for direct and indirect emissions, we investigate and characterize axiomatically a more general centrality measure that corresponds to a convex combination of the Bonacich centralities for the upstream and downstream weighted adjacency matrices.

Structure of the paper

The remaining part of the paper is organized as follows. In Sect. 2, we introduce our model and the responsibility measure, and illustrate the new setting by a simple numerical example. Section 3 provides an axiomatic characterization of the responsibility measure. Section 4 investigates relationships with existing measures. In Sect. 5, we apply our approach to the allocation of responsibility for global emissions across countries and to the allocation of sectoral emissions in the USA. Section 6 provides concluding remarks. The paper contains two appendices. In Appendix A, we present all proofs. Appendix B contains supplementary empirical results.

2 Preliminaries and the model

2.1 Basic concepts in network theory

We recall some basic concepts in network theory (see, e.g., Jackson 2008) that are used in the paper. A *directed network* or *directed graph* (N, g) is an ordered pair of a set $N = \{1, \dots, n\}$ of nodes also called agents and a set of edges $g \subseteq \{ij \in N \times N\}$, where ij denotes the directed link between nodes i and j . If $ij \in g$ for $i = j$, the network is said to have *loops*. Equivalently, a directed network is represented by a set

of nodes $N = \{1, \dots, n\}$ and the *adjacency matrix* $\mathbf{G} = [g_{ij}]$ being a real-valued $n \times n$ matrix with $g_{ij} \in [0, 1]$, where $ij \in g$ if and only if $g_{ij} > 0$. If $g_{ij} \in \{0, 1\}$ for all $i, j \in N$, then the network is referred to as an *unweighted* network, otherwise, i.e., if the entries of \mathbf{G} can take on more than two values 0 or 1, the network is *weighted*. When N is fixed or given, one usually refers to g as being a network.

A *walk* in a network g between i and j is a sequence of links $i_1 i_2, \dots, i_{K-1} i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \dots, K - 1\}$, where $i_1 = i$ and $i_K = j$. Note that for a walk, the nodes or the links in the sequence i_1, \dots, i_K may appear more than once.

Let $\mathbf{G}^k = [g_{ij}^k]$ denote the k th power of the adjacency matrix $\mathbf{G} = [g_{ij}]$ with $g_{ij} \in \{0, 1\}$, where \mathbf{G}^0 is the $n \times n$ identity matrix I . Then g_{ij}^k is equal to the number of walks of length k that exist between i and j in the network g .

The issue of the most important or central nodes in a network has been widely studied in the literature since the late 1940s. Consequently, various centrality measures have been developed to capture different aspects of prominence or centrality of a node. One of the well-known and widely used centrality measures is *Bonacich centrality* (Bonacich 1987) defined in matrix terms by:

$$\text{Bon}(\mathbf{G}, c, d) = (I - c\mathbf{G})^{-1} d\mathbf{G}\mathbf{1} \tag{1}$$

where $0 < c < 1$ is an attenuation parameter adjusting the measure for the lower ‘effectiveness’ of longer walks in a network, $d > 0$ is a parameter capturing the base value on each node and is sufficiently small so that the measure (1) is well defined, and $\mathbf{1}$ is the $n \times 1$ vector of 1’s. The Bonacich centrality of a node is therefore equal to a weighted sum of the walks that emanate from it, and a walk of length k is of worth c^k .

Another prominent and widely used centrality measure is *eigenvector centrality* (Bonacich 1972) defined in matrix terms by:

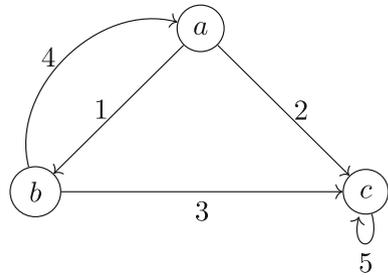
$$\tilde{\lambda}\text{eig}(\mathbf{G}) = \mathbf{G}\text{eig}(\mathbf{G}) \tag{2}$$

and thus $\text{eig}(\mathbf{G})$ is an eigenvector of \mathbf{G} with $\tilde{\lambda}$ being the corresponding largest eigenvalue of matrix \mathbf{G} . The eigenvector centrality of a node is therefore proportional to the sum of the centrality of its neighbours.

2.2 Responsibility measure

We consider a production network formed by economic agents that are linked by supply relationships and whose productive activities generate GHG emissions. Formally, we denote by $N \subset \mathbb{N}$ the set of agents, by $F := (f_i)_{i \in N} \in \mathbb{R}_+^N$ the vector of GHG emissions where f_i is the quantity of direct emissions from agent i , and by $A = (a_{i,j})_{i,j \in N} \in \mathbb{R}_+^{N \times N}$ the matrix of supply relationships where $a_{i,j}$ is the (nominal) value of the output of agent i used as input by agent j . We refer to such a triple (N, F, A) as an emission system. In our setting, the relative strength of supply relationships might be of greater concern than the absolute one. Therefore, we associate to the matrix of supply relationships A , two auxiliary adjacency matrices

Fig. 1 Graphical representation of the emission system (N, F, A)



normalized from the upstream and downstream perspective. Namely, we denote by $\phi_{i,j} := \frac{a_{j,i}}{\sum_{k \in N} a_{j,k}}$ the share of output from agent j used as input by agent i . The matrix $\Phi := (\phi_{i,j})_{i,j \in N}$ is the adjacency matrix of the production network normalized from an upstream perspective: the weighted sum of links upstream towards a supplier equals 1, i.e., $\sum_{i \in N} \phi_{i,j} = 1$, and Φ is column-stochastic. Conversely, we denote by $\psi_{i,j} := \frac{a_{i,j}}{\sum_{k \in N} a_{k,j}}$ the share of agent i in the input basket of agent j . The matrix $\Psi := (\psi_{i,j})_{i,j \in N}$ is the adjacency matrix of the production network normalized from a downstream perspective: the weighted sum of links downstream towards agent j equals 1, i.e., $\sum_{i \in N} \psi_{i,j} = 1$ and Ψ is column-stochastic. With a slight abuse of terminology, we shall refer to (N, F, Φ, Ψ) as an emission system interchangeably with (N, F, A) . We denote by \mathcal{E} the set of all emission systems.

Example 1 Let us consider a three-firm production network with the set of nodes $N = \{a, b, c\}$ and supply relationships given by the matrix A such that:

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 4 & 0 & 3 \\ 0 & 0 & 5 \end{pmatrix}$$

For any vector of emissions $F = (f_a, f_b, f_c)^T \in \mathbb{R}_+^3$, the triple (N, F, A) defines an emission system. A graphical representation of the emission system is provided in Fig. 1.

In the setting of Example 1, the upstream adjacency matrix Φ is obtained by transposing links and ensuring that the sum of incoming links equals to 1 while the downstream adjacency matrix Ψ is obtained by normalizing links such that the sum of incoming links towards each node equals 1. The resulting matrices and their corresponding graphical representations are shown in Fig. 2.

In an emission system, agent i uses a vector of input $a_{\cdot,i} \in \mathbb{R}_+^N$ to produce a quantity of output $\sum_{j \in N} a_{i,j}$ that generates, as a by-product, a quantity f_i of emissions. If all inputs are assumed to be necessary for production, the suppliers of agent i are, at least indirectly, necessary for the emissions f_i to occur. Accordingly, if the existence of a demand is a necessary condition for production, customers of agent i are, at least indirectly, necessary for the emissions f_i to occur. These causal relationships suggest

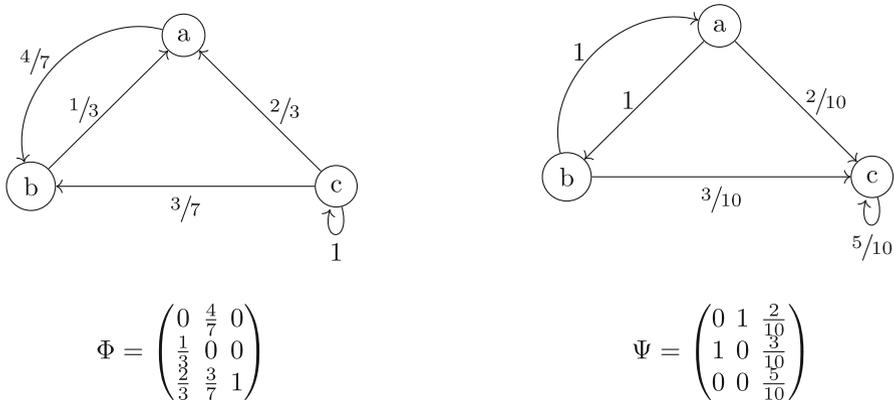


Fig. 2 The graphical representations of emission system (N, F, A) normalized from the upstream perspective (left) and downstream perspective (right) and the corresponding adjacency matrices Φ and Ψ

that agents upstream and downstream from i in the production network shall carry a responsibility for the emissions f_i . In the following, we are concerned with the measure of this indirect responsibility, i.e., we aim to define a measure that associates with each node/agent in an emission system, the amount of emissions it shall be considered responsible for and/or that it shall declare in its annual report statement, given its position in the production network and the distribution of emissions. More formally, we define an emission responsibility measure as follows.

Definition 1 An emission responsibility measure is a mapping ρ that associates to every emission system $E := (N, F, \Phi, \Psi) \in \mathcal{E}$, a vector $\rho(E) \in \mathbb{R}^N$ of responsibilities such that, for all $i \in N$, $\rho_i(E)$ is the amount of emissions for which agent i is deemed responsible.

From a network theory perspective, defining the responsibility of agent i amounts to defining the centrality of agent i in the network A , considering that each node i is weighted by its emissions f_i . A standard approach in this context is to define centrality in terms of the discounted sum of walks in the network (see e.g. Jackson 2008). In our setting, this amounts to defining the responsibility for GHG emissions in a supply chain through the discounted sum of indirect emissions. Namely, we shall assign to agent i in the emission system $E = (N, F, \Phi, \Psi)$, a measure of responsibility of the form:

$$\bar{\rho}_{(\lambda, \gamma, \delta), i}(E) = \lambda(1 - \gamma) \sum_{v=0}^{+\infty} \gamma^v \sum_{j \in N} \Phi_{i,j}^v f_j + (1 - \lambda)(1 - \delta) \sum_{v=0}^{+\infty} \delta^v \sum_{j \in N} \Psi_{i,j}^v f_j \quad (3)$$

where $\gamma, \delta \in [0, 1)$ are discount factors, $\lambda \in [0, 1]$ measures the relative share of responsibility for upstream and downstream emissions, $\Phi_{i,j}^v$ is the sum of walks of length v upstream from i to j (given by the (i, j) coefficient of the v th power of Φ) and $\Psi_{i,j}^v$ is the sum of walks of length v downstream from i to j (given by the

(i, j) coefficient of the ν th power of Ψ). Hence, $\bar{\rho}_{(\lambda,\gamma,\delta),i}(E)$ associates to agent i a share of responsibility in the emissions generated by its upstream and downstream connections in the network that is proportional to the strength of their supply/demand relationships, as measured by the sum of walks, and discounted according to the network distance. Upstream responsibility is conceptually similar to consumption-based accounting (Lenzen et al. 2007; Davis and Caldeira 2010) while downstream responsibility shares similarities with income-based accounting (Marques et al. 2012; Rodrigues et al. 2006). We will explore these connections further in Sect. 4. Formally, the measure is similar to discounted intertemporal expected utility: the sum of weighted walks of length ν between i and j corresponds to the probability of reaching node j from node i in ν steps. It is also closely related to Bonacich centrality (Bonacich 1987). Equation (3) can indeed be written as:

$$\bar{\rho}_{(\lambda,\gamma,\delta)}(E) = \lambda(1 - \gamma)(I - \gamma\Phi)^{-1}F + (1 - \lambda)(1 - \delta)(I - \delta\Psi)^{-1}F \tag{4}$$

where $\bar{\rho} := (\bar{\rho}_i)_{i \in N}$. Hence, $\bar{\rho}_{(\lambda,\gamma,\delta)}$ corresponds to a convex combination of the Bonacich centralities for the upstream and downstream adjacency matrices weighted by the vector of emissions.

Example 1 (continued) In the case of Example 1 considered above, we find for respectively $(1 - \gamma)(I - \gamma\Phi)^{-1}$ and $(1 - \delta)(I - \delta\Psi)^{-1}$ the following matrices:

$$(1 - \gamma)(I - \gamma\Phi)^{-1} = \begin{pmatrix} \frac{21(1-\gamma)}{21-4\gamma^2} & \frac{12\gamma(1-\gamma)}{21-4\gamma^2} & 0 \\ \frac{7\gamma(1-\gamma)}{21-4\gamma^2} & \frac{21(1-\gamma)}{21-4\gamma^2} & 0 \\ \frac{3\gamma^2+14\gamma}{21-4\gamma^2} & \frac{9\gamma+8\gamma^2}{21-4\gamma^2} & 1 \end{pmatrix} \tag{5}$$

$$(1 - \delta)(I - \delta\Psi)^{-1} = \begin{pmatrix} \frac{1}{1+\delta} & \frac{\delta}{1+\delta} & \frac{\delta(3\delta+2)}{5(2-\delta)(1+\delta)} \\ \frac{\delta}{1+\delta} & \frac{1}{1+\delta} & \frac{\delta(2\delta+3)}{5(2-\delta)(1+\delta)} \\ 0 & 0 & \frac{2(1-\delta)}{2-\delta} \end{pmatrix} \tag{6}$$

Then, if we let $\lambda = \gamma = \delta = 0.5$, we find the following responsibilities:

$$\begin{aligned} \bar{\rho}_{(0.5,0.5,0.5),a}(E) &= 0.5 \left[\frac{21}{40} f_a + \frac{3}{20} f_b \right] + 0.5 \left[\frac{2}{3} f_a + \frac{1}{3} f_b + \frac{7}{45} f_c \right] \\ &\simeq 0.60 f_a + 0.24 f_b + 0.08 f_c \end{aligned}$$

$$\begin{aligned} \bar{\rho}_{(0.5,0.5,0.5),b}(E) &= 0.5 \left[\frac{7}{80} f_a + \frac{21}{40} f_b \right] + 0.5 \left[\frac{1}{3} f_a + \frac{2}{3} f_b + \frac{8}{45} f_c \right] \\ &\simeq 0.21 f_a + 0.60 f_b + 0.09 f_c \end{aligned}$$

$$\begin{aligned} \bar{\rho}_{(0.5,0.5,0.5),c}(E) &= 0.5\left[\frac{31}{80}f_a + \frac{13}{40}f_b + f_c\right] + 0.5\left[\frac{2}{3}f_c\right] \\ &\simeq 0.19f_a + 0.16f_b + 0.83f_c \end{aligned}$$

Note that if $\gamma = \delta = 0$, then $\bar{\rho}_{(\lambda,\gamma,\delta)}(E) = \begin{pmatrix} f_a \\ f_b \\ f_c \end{pmatrix} = F$ for any value of $\lambda \in [0, 1]$.

Moreover, if $\gamma \rightarrow 1$ and $\delta \rightarrow 1$, then:

$$\bar{\rho}_{(\lambda,\gamma,\delta)}(E) \rightarrow \lambda \begin{pmatrix} 0 \\ 0 \\ f_a + f_b + f_c \end{pmatrix} + (1 - \lambda) \begin{pmatrix} \frac{1}{2}(f_a + f_b + f_c) \\ \frac{1}{2}(f_a + f_b + f_c) \\ 0 \end{pmatrix}$$

These last two results correspond to the general results for the particular cases ($\gamma = \delta = 0, \gamma \rightarrow 1$ and $\delta \rightarrow 1$), as will be discussed in Sect. 4.

3 Axiomatic foundations of the responsibility measure

Normative foundations are crucial if the discounted emission responsibility measure is to be used in a legal context or included in standardised corporate disclosures. In order to provide such normative foundations, we shall show that $\bar{\rho}$ is characterized by a set of axioms that are consistent with legal principles underlying responsibility (Burger et al. 2020). These axioms are closely related to these used for the axiomatization of network centrality measures (in particular Bloch et al. 2023) and discounted expected utility.

In this respect, a first standard requirement is to assume that the responsibility measure is anonymous, i.e., that the agent’s responsibility is not influenced by its identity. Namely, one has:

Axiom 1 (Anonymity) *The responsibility measure ρ is anonymous if for all emission systems $E = (N, F, \Phi, \Psi) \in \mathcal{E}$, for all permutations π of N and for all $i \in N$ one has:*

$$\rho_i(E) = \rho_{\pi(i)}(E_\pi)$$

where $E_\pi := (N, F_\pi, \Phi_\pi, \Psi_\pi)$, $F_\pi := (F_{\pi(i)})_{i \in N}$, $\Phi_\pi := (\Phi_{\pi(i),\pi(j)})_{i,j \in N}$ and $\Psi_\pi := (\Psi_{\pi(i),\pi(j)})_{i,j \in N}$.

From a more fundamental perspective, our second axiom defines the drivers of responsibility. It shall increase with (direct and indirect) emissions and with the

strength of relationships with upstream/downstream emitters. Formally, let us associate to agent i in the emission system (N, F, Φ, Ψ) , the (generalized) nodal statistic $(u_i(F, \Phi), v_i(F, \Psi)) \in \mathbb{R}_+^{\mathbb{N}} \times \mathbb{R}_+^{\mathbb{N}}$ defined by:

$$u_i(F, \Phi) := \left(\sum_{j \in N} \Phi_{i,j}^v f_j \right)_{v \in \mathbb{N}} = \left(f_i, \sum_{j \in N} \Phi_{i,j} f_j, \dots, \sum_{j \in N} \Phi_{i,j}^v f_j, \dots \right)$$

$$v_i(F, \Psi) := \left(\sum_{j \in N} \Psi_{i,j}^v f_j \right)_{v \in \mathbb{N}} = \left(f_i, \sum_{j \in N} \Psi_{i,j} f_j, \dots, \sum_{j \in N} \Psi_{i,j}^v f_j, \dots \right)$$

The statistic $u_i(F, \Phi)$ corresponds to the sequence of upstream emissions for agent i , i.e., for each $v \in \mathbb{N}$, the sum of emissions occurring v steps upstream from i weighted by the (indirect) share of i in the consumption of the corresponding output. Respectively, $v_i(F, \Psi)$ corresponds to the sequence of downstream emissions for agent i , i.e., for each $v \in \mathbb{N}$, the sum of emissions occurring v steps downstream from i weighted by the (indirect) share of i in the corresponding input mix. Our second axiom assumes that the responsibility measure is monotonic with respect to the Euclidian partial order on the sequences of upstream and downstream emissions, namely³:

Axiom 2 (Monotonicity) *The responsibility measure ρ is monotonic if for all $i \in N$, and all (F, Φ, Ψ) and $(\tilde{F}, \tilde{\Phi}, \tilde{\Psi})$ such that for all $v \in \mathbb{N}$:*

$$\sum_{j \in N} \Phi_{i,j}^v f_j \geq \sum_{j \in N} \tilde{\Phi}_{i,j}^v \tilde{f}_j \text{ and } \sum_{j \in N} \Psi_{i,j}^v f_j \geq \sum_{j \in N} \tilde{\Psi}_{i,j}^v \tilde{f}_j,$$

one has:

$$\rho_i(E) \geq \rho_i(\tilde{E}),$$

where $E = (N, F, \Phi, \Psi)$ and $\tilde{E} = (N, \tilde{F}, \tilde{\Phi}, \tilde{\Psi})$.

Hence, ρ satisfies monotonicity if the responsibility of an agent (weakly) increases when its weighted share in upstream and downstream emissions increases. In particular, ceteris paribus, the responsibility of an agent (weakly) increases (i) if emissions increase upstream or downstream in its supply chain or (ii) if the strengths of its supply or demand relationships increase.

Following Bloch et al. (2023), anonymity and monotonicity imply that the responsibility measure is a function of the sequence of upstream and downstream emissions. Namely, one has:

Lemma 1 *If the responsibility measure ρ satisfies anonymity and monotonicity, then there exists a function $\sigma : \mathbb{R}_+^{\mathbb{N}} \times \mathbb{R}_+^{\mathbb{N}} \rightarrow \mathbb{R}$ such that for all emission systems $E = (N, F, \Phi, \Psi)$, for all $i \in N$, one has:*

$$\rho_i(E) = \sigma(u_i(F, \Phi), v_i(F, \Psi)).$$

³ We denote by \geq the partial Euclidian order on $\mathbb{R}^{\mathbb{N}}$ so that $u \geq v$ if and only if $u_n \geq v_n$ for all $n \in \mathbb{N}$.

Furthermore, σ is monotonic with respect to the Euclidian partial order on $\mathbb{R}_+^N \times \mathbb{R}_+^N$.

In view of Lemma 1, assuming in the following that anonymity and monotonicity hold, further axioms can equivalently be stated in terms of ρ or σ . In particular, our third axiom assumes the additivity of the responsibility measure.

Axiom 3 (Additivity) *A responsibility measure that is monotonic and anonymous is additive if for all $(u, v), (u', v') \in \mathbb{R}_+^N \times \mathbb{R}_+^N$, one has:*

$$\sigma((u, v) + (u', v')) = \sigma(u, v) + \sigma(u', v').$$

Additivity plays in our setting a similar role to the independence axiom in the Von Neumann-Morgenstern expected utility theory. It assumes that the change in responsibility induced by additional upstream or downstream emissions is independent of the preexisting level of responsibility. It can also be interpreted as a form of invariance with respect to corporate structure: total responsibility shall not be modified if an agent is split into subunits or if two agents are merged. The opposite could lead to strategic behaviour in the choice of organisational/corporate structure to reduce total responsibility. This interpretation is related to the invariance to mitosis axiom as used by Csóka and Herings (2021) in the context of bankruptcy rules in financial networks, that states that merging a set of identical agents does not affect the payment to and from the agents outside this merge. Additivity implies in particular that one can separate upstream and downstream responsibility of an agent. Namely:

Lemma 2 *If the responsibility measure ρ satisfies anonymity, monotonicity and additivity, then there exist functions $\alpha : \mathbb{R}_+^N \rightarrow \mathbb{R}$ and $\beta : \mathbb{R}_+^N \rightarrow \mathbb{R}$ such that for all emission systems $E = (N, F, \Phi, \Psi)$ and for all $i \in N$, one has:*

$$\rho_i(E) = \alpha(u_i(F, \Phi)) + \beta(v_i(F, \Psi)).$$

Furthermore, α and β are additive and monotone.

In view of Lemma 2, assuming that anonymity, monotonicity and additivity hold, further axioms can be stated in terms of the functions α and β . In this setting, our fourth axiom characterizes the tail behaviour of the responsibility measure.

Axiom 4 (Tail-convergence) *A responsibility measure that is monotonic, anonymous and additive is tail-convergent if for every $(u, v) \in \mathbb{R}_+^N \times \mathbb{R}_+^N$, one has:*

$$\lim_{\nu \rightarrow +\infty} \alpha(u_1, \dots, u_\nu, 0, \dots) = \alpha(u) \text{ and } \lim_{\nu \rightarrow +\infty} \beta(v_1, \dots, v_\nu, 0, \dots) = \beta(v).$$

The tail-convergence axiom implies that distant upstream and downstream emissions eventually become negligible so that the responsibility for an emission sequence

can be approximated by truncated emission sequences of sufficient length. Tail-convergence ensures continuity and consistent measurement of infinite emission sequences. Our fifth axiom assumes the recursivity of the responsibility measure in the sense of Bloch et al. (2023). Namely:

Axiom 5 (Recursivity) *A responsibility measure that is monotonic, anonymous and additive is recursive if for all $(u, v), (u', v') \in \mathbb{R}_+^{\mathbb{N}} \times \mathbb{R}_+^{\mathbb{N}}$ and all $v \in \mathbb{N}$, one has:*

$$\frac{\alpha(0, \dots, 0, u_{v+1}, u_{v+2}, \dots)}{\alpha(0, \dots, 0, u'_{v+1}, u'_{v+2}, \dots)} = \frac{\alpha(u_{v+1}, u_{v+2}, \dots)}{\alpha(u'_{v+1}, u'_{v+2}, \dots)}$$

and

$$\frac{\beta(0, \dots, 0, v_{v+1}, v_{v+2}, \dots)}{\beta(0, \dots, 0, v'_{v+1}, v'_{v+2}, \dots)} = \frac{\beta(v_{v+1}, v_{v+2}, \dots)}{\beta(v'_{v+1}, v'_{v+2}, \dots)}.$$

The recursivity axiom is closely related to the stationarity axioms introduced by Koopmans (1960) in its axiomatization of discounting. In our setting, the axiom implies that the responsibility ratio between two emission patterns is independent of their level of upstreamness or downstreamness. In other words, the relative importance of an emission pattern does not depend on its level of upstreamness/downstreamness. Consistently with Koopmans (1960) and Bloch et al. (2023), recursivity implies that the responsibility measure is given by the discounted sum of emissions. Namely:

Lemma 3 *If the responsibility measure ρ satisfies anonymity, monotonicity, additivity, tail-convergence and recursivity, then there exist $\gamma, \delta \in [0, 1]$ and $a, b \in \mathbb{R}_+$ such that for all emission systems $E = (N, F, \Phi, \Psi)$ and for all $i \in N$, one has:*

$$\rho_i(E) = a \left(\sum_{v=0}^{+\infty} \gamma^v \sum_{j \in N} \Phi_{i,j}^v f_j \right) + b \left(\sum_{v=0}^{+\infty} \delta^v \sum_{j \in N} \Psi_{i,j}^v f_j \right).$$

Our final axiom ensures the consistency of the responsibility measure by requiring that it covers all emissions in the system. Namely:

Axiom 6 (Consistency) *The responsibility measure ρ is consistent if for all emission systems $E = (N, F, \Phi, \Psi) \in \mathcal{E}$, one has:*

$$\sum_{i \in N} \rho_i(E) = \sum_{i \in N} f_i.$$

Consistency ensures that responsibility is allocated for all emissions in the system and that there is no double-counting of emissions. In other words, the responsibility for every unit of emission is allocated once and only once. This property is required

for a range of empirical applications, in particular, to define liability in an accounting or legal setting. It also allows us to compare the responsibility measure with direct emissions for each agent. Formally, the consistency axiom leads to a normalization of the responsibility measure and its characterization. Namely, one has:

Theorem 1 *A responsibility measure satisfies anonymity, monotonicity, additivity, tail-convergence, recursivity and consistency if and only if, for every emission system $E = (N, F, \Phi, \Psi)$, it is of the form of Equation (3) with $\gamma, \delta \in [0, 1)$ and $\lambda \in [0, 1]$.*

Theorem 1 thus provides the desired normative foundations for the discounted emission responsibility measures.

4 Relation with existing measures

After illustrating the limit cases of the discount factors in Example 1 (continued) of Subsection 2.2, now we shall show that a number of responsibility measures currently in use or put forward in the literature are in fact particular cases of the discounted emission metric $\bar{\rho}$.

4.1 Upstream vs downstream responsibility and direct emissions

First, if $\lambda = 1$, the responsibility measure is of the form:

$$\bar{\rho}_{(\lambda, \gamma, \delta), i}(E) = (1 - \gamma) \sum_{v=0}^{+\infty} \gamma^v \sum_{j \in N} \Phi_{i, j}^v f_j$$

and thus only accounts for upstream emissions. Conversely, if $\lambda = 0$, the responsibility measure is of the form:

$$\bar{\rho}_{(\lambda, \gamma, \delta), i}(E) = (1 - \delta) \sum_{v=0}^{+\infty} \delta^v \sum_{j \in N} \Psi_{i, j}^v f_j$$

and thus only accounts for downstream emissions.

Second, if $\gamma = \delta = 0$, one has $\bar{\rho}_{(\lambda, \gamma, \delta), i}(E) = f_i$ and the responsibility measure only accounts for direct emissions. It thus coincides with Scope 1 emissions in the sense of the GHG protocol (WRI and WBCSD 2011). In such a setting, discounting is “infinitely strong” and indirect emissions are discarded.

4.2 No-discounting limit and independence from own emissions

We next consider the case when γ and δ tend towards 1 so that the effect of discounting becomes negligible and indirect emissions become as important as direct ones.

The limit of the responsibility measure can be characterized through the eigenvector centrality of the adjacency matrices, as defined in Equation (2). More precisely, let $\text{Eig}(M)$ denote a matrix for which all rows are equal to the normalized vector of eigenvector centralities $\text{eig}(M)$ in M with the sum of coordinates equal to 1. Let $\text{Eig}^T(M)$ be the transpose of $\text{Eig}(M)$ with its i -row denoted by $\text{Eig}_i^T(M)$. Then one has:

Proposition 1 *If the adjacency matrix A is aperiodic and irreducible, then for every $i \in N$:*

$$\lim_{(\gamma, \delta) \rightarrow (1, 1)} \lambda(1 - \gamma) \sum_{v=0}^{+\infty} \gamma^v \sum_{j \in N} \Phi_{i,j}^v f_j + (1 - \lambda)(1 - \delta) \sum_{v=0}^{+\infty} \delta^v \sum_{j \in N} \Psi_{i,j}^v f_j = \lambda \text{Eig}_i^T(\Phi)F + (1 - \lambda) \text{Eig}_i^T(\Psi)F.$$

Hence, in the “no-discounting” limit, the responsibility of an agent is independent of its level of direct or indirect emissions but only depends on its network position. The more central an agent is, the larger its responsibility for GHG emissions.

Example 1 (continued) Applying Proposition 1 to the emission system introduced in Example 1, one has:

$$\text{Eig}^T(\Phi) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{Eig}^T(\Psi) = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

consistently with the computations of $\bar{\rho}(E, \lambda, \gamma, \delta)$ performed in Example 1.

4.3 Consumption-based accounting

Consumption-based accounting corresponds to a particular case of the responsibility where only upstream responsibility is taken into account (i.e., $\lambda = 1$) and one considers the no-discounting limit ($\gamma \rightarrow 1$ as considered in Proposition 1). Indeed, in its standard formulation (see Yamano and Guilhoto 2020; Rodrigues and Domingos 2008; Gallego and Lenzen 2005), consumption-based accounting considers a set of L countries and M industries per country. Let us denote by \mathcal{L} the set of countries, by \mathcal{M} the set of industries and by $\mathcal{J} = \mathcal{L} \times \mathcal{M}$ the set of country-industry pairs. In the following, let $i, j \in \mathcal{J}$ and $k \in \mathcal{L}$. Production relationships are defined by an intermediary consumption matrix $C = (c_{i,j}) \in \mathbb{R}_+^{\mathcal{J} \times \mathcal{J}}$ where $c_{i,j}$ corresponds to the amount of input that country-industry pair j requires from i in order to produce one unit of output. The final demand to each country-industry pair from each country is given by a matrix $D = (d_{i,k}) \in \mathbb{R}_+^{\mathcal{J} \times \mathcal{L}}$ where $d_{i,k}$ is the final demand from country k to the country-industry pair i . Emission intensities are given by a vector $g \in \mathbb{R}_+^{\mathcal{J}}$ where g_i

is the amount of GHG emissions per unit of production for the country-industry pair i .

In this setting, the vector of country-level consumption-based emissions $\chi \in \mathbb{R}^{\mathcal{L}}$ is defined as:

$$\chi = e^T \text{diag}(g)(I - C)^{-1}D = \left(\sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{J}} g_j (I - C)^{-1}_{j,i} D_{i,k} \right)_{k \in \mathcal{L}}$$

where $e = (1, \dots, 1) \in \mathbb{R}^{\mathcal{J}}$ is the column vector of all ones, $\text{diag}(g)$ denotes the diagonal matrix whose diagonal is given by g . Moreover, $(I - C)^{-1}D$ is the $\mathcal{J} \times \mathcal{L}$ matrix whose coefficients correspond to the total production in each country-sector pair required to fulfil the final demand. Namely, $(I - C)^{-1}$ is the Leontief inverse (Leontief 1970) that we will denote by $Q = (q_{i,j})_{i,j \in \mathcal{J}}$, an element $q_{i,j}$ of which represents the output from i that is directly and indirectly required to meet the demand of j . We denote by $q_i = \sum_{j \in \mathcal{J}} q_{i,j}$ the total output of country-industry pair $i \in \mathcal{J}$. Accordingly, the total direct emissions f_i of country-industry pair i are given by $f_i = g_i q_i$ and we denote by $v_{i,j} = c_{i,j} q_j$ the total amount of output of i used by j .

We then consider the network with nodes in $\mathcal{N} := \mathcal{J} \cup \mathcal{L}$, i.e., one node per country-industry pair and one node to represent final demand in each country. As emphasized above, we focus on the upstream adjacency matrix. Hence, for $i, j \in \mathcal{J}$, the value of the adjacency coefficient is given by the share of output from j used by i as input, $\phi_{i,j} = \frac{v_{j,i}}{q_j}$. For $k \in \mathcal{L}$ and $j \in \mathcal{J}$, the value of the adjacency coefficient is given

by the share of output from j supplied to k as final demand $\phi_{k,j} = \frac{D_{j,k}}{q_j}$. Finally, by

convention, we consider that $k \in \mathcal{L}$ only has supply relationships towards itself so that $\phi_{k,k} = 1$ and $\phi_{j,k} = 0$ for $j \neq k$; see Fig. 3. Hence, the upstream adjacency matrix $\Phi \in \mathbb{R}^{\mathcal{N} \times \mathcal{N}}$ is column-stochastic and consumption-based accounting corresponds to the upstream responsibility in the network in the no-discounting limit. Namely, one has:

Proposition 2 *For the emission system defined above, one has for all $k \in \mathcal{L}$:*

$$\chi_k = \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{J}} g_j (I - C)^{-1}_{j,i} D_{i,k} = \lim_{\gamma \rightarrow 1} \sum_{j \in \mathcal{J}} (1 - \gamma)(I - \gamma\Phi)^{-1}_{k,j} f_j.$$

4.4 Income-based accounting

Besides considering the emissions embodied in the final demand, one can also consider emissions embodied in the primary inputs. This approach is referred to as income-based or revenue-based accounting in the literature (Marques et al. 2012; Liang et al. 2017; Rodrigues et al. 2006; Pinero et al. 2019). It corresponds to the particular case

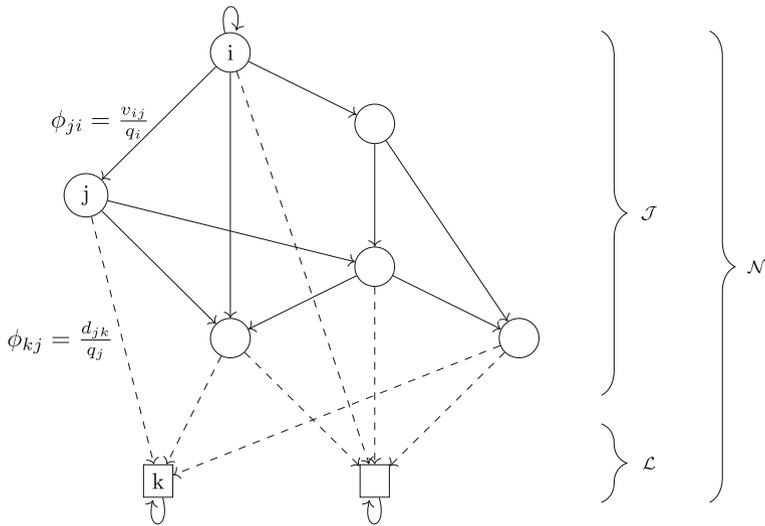


Fig. 3 Consumption-based accounting. Six country-industry pairs are represented by the circle nodes and the final demand of the countries is represented by the square nodes. The solid arrows represent the supply from one country-industry pair to another, whereas the dashed arrows represent the final demand from a country to a country-industry pair

of the discounted emission metric where we only take downstream responsibility into account ($\lambda = 0$) and consider the no-discounting limit ($\delta \rightarrow 1$).

Using the same notation as in Sect. 4.3 unless stated otherwise, we let $\bar{C} \in \mathbb{R}_+^{\mathcal{J} \times \mathcal{J}}$ be the intermediary output matrix, an element $\bar{c}_{i,j}$ of which is the share of output of country-industry pair i that is used by country-industry pair j . That is, $\bar{c}_{i,j} = \frac{v_{i,j}}{\bar{q}_i}$ with \bar{q}_i the total output of i from its direct and indirect inputs. We denote the primary input from every country $k \in \mathcal{L}$ to every country-industry pair $i \in \mathcal{J}$ by the matrix $P = (p_{k,i}) \in \mathbb{R}_+^{\mathcal{L} \times \mathcal{J}}$. Then, the vector of country-level income-based emissions $\xi \in \mathbb{R}^{\mathcal{L}}$ is given by:

$$\xi = P(I - \bar{C})^{-1} \text{diag}(g)e = \left(\sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{J}} P_{k,i} (I - \bar{C})_{i,j}^{-1} g_j \right)_{k \in \mathcal{L}}$$

where $(I - \bar{C})^{-1} = (\bar{q}_{i,j})_{i,j \in \mathcal{J}}$ is the Ghosh inverse (Ghosh 1958) with $\bar{q}_{i,j}$ representing the value of production in country-industry j per unit of input in country-industry pair i . Accordingly, $P(I - \bar{C})^{-1} \in \mathbb{R}^{\mathcal{L} \times \mathcal{J}}$ represents the production in every country-industry pair that is required for the primary inputs to be attainable. Again, we consider the network with nodes $\mathcal{N} := \mathcal{L} \cup \mathcal{J}$ where the nodes $k \in \mathcal{L}$ now represent the primary input in country k . Considering the downstream matrix, we obtain for $k \in \mathcal{L}$ and $i, j \in \mathcal{J}$ the following values of the adjacency coefficients: $\psi_{k,j} = \frac{P_{k,j}}{\bar{q}_j}$, and

$\psi_{i,j} = \frac{v_{i,j}}{\bar{q}_j}$. As before, we have by convention $\psi_{k,k} = 1$ and $\psi_{j,k} = 0$. This leads to column-stochastic matrix $\Psi \in \mathbb{R}_+^{\mathcal{N} \times \mathcal{N}}$, for which we then have:

Proposition 3 *For the emission system defined above, one has for all $k \in \mathcal{L}$:*

$$\xi_k = \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{J}} P_{k,i} (I - \bar{C})_{i,j}^{-1} g_j = \lim_{\delta \rightarrow 1} \sum_{j \in \mathcal{J}} (1 - \delta) (I - \delta \Psi)_{k,j}^{-1} f_j.$$

4.5 Regulatory specification of the responsibility measure

The examples introduced above highlight that a range of responsibility metrics (e.g., income-based and consumption-based accounting) are compatible with the axioms put forward in Theorem 1. Additional regulatory choices can nevertheless be made to further specify the metric. For example, a regulator could specify that a responsibility metric could focus solely on downstream and/or upstream emissions. From an axiomatic perspective, this is equivalent to a requirement of invariance with respect to upstream (resp. downstream) emissions, which will focus the metric on downstream (resp. upstream) emissions. Namely, let us consider the corresponding axioms.

Axiom 7 (Upstream Invariance) *The responsibility measure ρ is upstream invariant if for all emission systems $E = (N, F, \Phi, \Psi) \in \mathcal{E}$ and $E' = (N, F, \Phi', \Psi) \in \mathcal{E}$, one has:*

$$\rho(E) = \rho(E').$$

Axiom 8 (Downstream Invariance) *The responsibility measure ρ is downstream invariant if for all emission systems $E = (N, F, \Phi, \Psi) \in \mathcal{E}$ and $E' = (N, F, \Phi, \Psi') \in \mathcal{E}$, one has:*

$$\rho(E) = \rho(E').$$

One then has an immediate corollary of Theorem 1:

Corollary 1 *A responsibility measure satisfies anonymity, monotonicity, additivity, tail-convergence, recursivity, consistency and upstream (resp. downstream) invariance if and only if, for every emission system $E = (N, F, \Phi, \Psi)$, it is of the form*

$$\bar{\rho}_{(\lambda, \gamma, \delta), i}(E) = (1 - \delta) \sum_{v=0}^{+\infty} \delta^v \sum_{j \in N} \Psi_{i,j}^v f_j \tag{7}$$

with $\delta \in [0, 1)$

(resp. $\bar{\rho}_{(\lambda, \gamma, \delta), i}(E) = (1 - \gamma) \sum_{v=0}^{+\infty} \gamma^v \sum_{j \in N} \Phi_{i,j}^v f_j$ with $\gamma \in [0, 1)$).

More broadly, a regulator could specify the relative importance of upstream and downstream emissions. It suffices in fact to specify the relative importance of emissions from direct clients and suppliers. Namely, let us consider the following axiom.

Axiom 9 ($\bar{\lambda}$ -upstreamness) *A responsibility measure that is monotonic, anonymous, additive and recursive satisfies $\bar{\lambda}$ -upstreamness if one has*

$$\alpha(0, 1, 0, \dots) = \frac{\bar{\lambda}}{1 - \bar{\lambda}} \beta(0, 1, 0, \dots) \quad (8)$$

with $\lambda \in [0, 1)$.

Accordingly, a regulator could specify the relative importance of direct and indirect emissions. To this effect, it suffices to specify the relative importance of direct emissions and of these of direct clients and suppliers. Namely, let us consider the following refinement of the recursivity axiom.

Axiom 10 ($(\bar{\gamma}, \bar{\delta})$ -recursivity) *A responsibility measure that is monotonic, anonymous and additive is $(\bar{\gamma}, \bar{\delta})$ -recursive if it is recursive and one has*

$$\alpha(0, 1, 0, \dots) = \bar{\gamma}\alpha(1, 0, \dots) \text{ and } \beta(0, 1, 0, \dots) = \bar{\delta}\beta(1, 0, \dots) \quad (9)$$

for $\bar{\gamma}, \bar{\delta} \in [0, 1)$.

The axioms of $\bar{\lambda}$ -upstreamness and $(\bar{\delta}, \bar{\gamma})$ -recursivity then suffice to characterize completely the responsibility measure. Namely, one has the following corollary of Theorem 1.

Corollary 2 *The only responsibility measure that satisfies anonymity, monotonicity, additivity, tail-convergence, $(\bar{\gamma}, \bar{\delta})$ -recursivity, consistency and $\bar{\lambda}$ -upstreamness is $\bar{\rho}_{(\bar{\lambda}, \bar{\gamma}, \bar{\delta})}$.*

Hence, a regulator can fully specify the responsibility metric by fixing the relative importance of own emissions and these of direct clients and suppliers. The axiomatic characterization then unambiguously determines how responsibility extends along the supply chain.

5 Applications

In this section we illustrate the application of the discounted emission responsibility measure to the allocation of global emissions across countries and to the allocation of sectoral responsibility in the USA economy.⁴

⁴ The Python code used for this application is publicly available on <https://github.com/rosavandenende/Network-based-allocation-of-responsibility-for-GHG-emissions>.

Table 1 Responsibility for Switzerland (CHE), China (CHN), France (FRA), United States of America (USA), Russia (RUS) with $\gamma = 0.7$ and $\delta = 0.7$, in kton CO_2

Country	Emissions	$\lambda = 1$ (upstream)	$\lambda = 0$ (downstream)	$\lambda = \frac{1}{2}$
CHE	44,067	99,565	145,574	122,570
CHN	10,528,550	10,516,086	10,234,564	10,375,325
FRA	345,586	504,557	478,519	491,538
USA	5,257,452	5,433,719	5,198,942	5,3163,31
RUS	1,723,086	1,208,205	1,777.928	1,493,067

5.1 Data

We use the World Input–Output Database (WIOD) (2016 release, see Timmer et al. 2015) and the associated Environmental Accounts (Corsatea et al. 2019).⁵ A world input–output table provides information about all the transactions between industries and final users in the global economy. The data concern the year 2014 and contain the trade of all 56 sectors of all EU-countries plus seventeen large economies. With 56 sectors and 44 countries, we thus obtain a matrix with 2464 rows and 2464 columns. The columns regard the production processes, the corresponding products of which can be either used as intermediates for other sectors, or as final products by firms, governments or households. The rows provide information on how the output of industries is distributed over those user categories. Denote by A the world input–output matrix, then, an element $a_{i,j}$ is thus the value of the output of agent i used as input by agent j . The Environmental Account data consists of the carbon emissions of all sectors of these countries.⁶

5.2 Allocation of responsibility for global emissions

In Table 1, we can see some examples of assigned responsibility in kton CO_2 for Switzerland (CHE), China (CHN), France (FRA), United States of America (USA) and Russia (RUS) when we set both $\gamma = 0.7$ and $\delta = 0.7$. All the upstream and downstream responsibilities for all countries and for various levels of discounting are given in Table 2 and Table 3 in Appendix B. In Fig. 4 we see both the upstream and downstream responsibility for Brazil, Canada, Indonesia, South Korea and France, with $v_{max} = 1000$, for various values of γ and δ .

There are a few interesting remarks we can make based on this responsibility data. First, we observe that the responsibility of the most developed and service-oriented economies increases rapidly when one accounts for indirect emissions upstream or downstream in the supply chain. For those countries, the value that γ and δ take makes a large difference in what responsibility they are assigned. For example, for

⁵ The WIOD 2016 Release and the corresponding environmental accounts are publicly available and can be found at <https://www.rug.nl/ggdc/valuechain/wiod/wiod-2016-release?lang=en>.

⁶ The data set does not include the carbon emissions of the Netherlands, which have therefore been added manually with the data from Olivier et al. (2015).

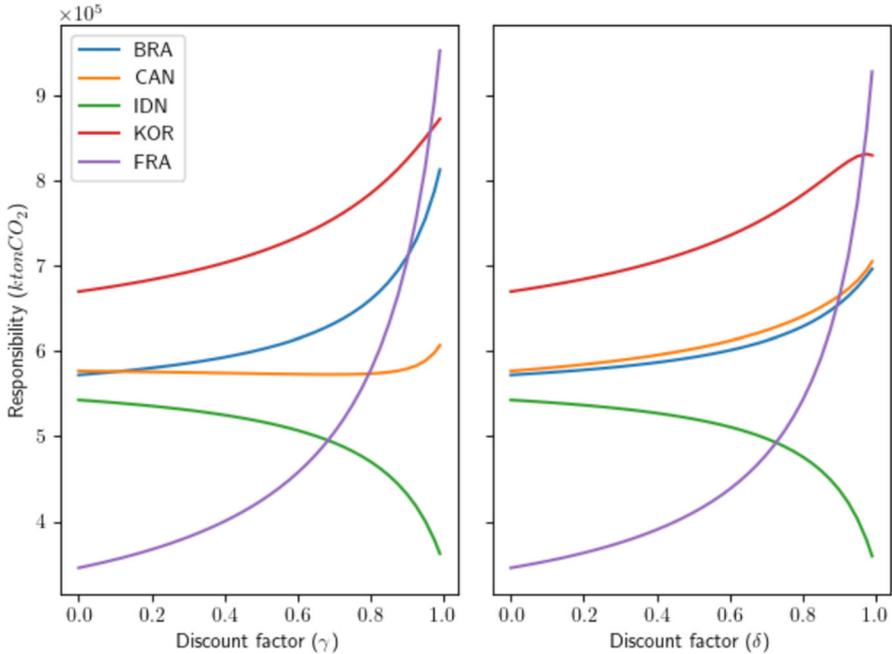


Fig. 4 Upstream (left) and downstream responsibility for Brazil, Canada, Indonesia, Korea and France in kton CO_2 , with $v_{max} = 1000$, for γ and δ ranging from 0.0 to 0.99

Switzerland, when the discounting factor δ is high, for example $\delta = 0.9$ and $\lambda = 0$, its responsibility is almost 6 times its direct emissions. The same effect occurs when we take $\lambda = 1$ and evaluate the upstream responsibility. In this case, for $\gamma = 0.9$ the responsibility is approximately 3.5 times as large as the direct emissions. We can see this in Fig. 5. Other examples of such countries are The Netherlands and Luxembourg.

Secondly, we can distinguish a class of countries with manufacturing-oriented economies that have high direct emissions for whom the share of responsibility decreases as one increasingly accounts for upstream or downstream emissions in the supply chain. A clear example of such a country is China, which has a high input as well as output, but due to its high direct emissions, the direct responsibility is very large compared to the indirect responsibility. Accounting from a production-based point of view yields a high responsibility for China, while moving towards a consumption-based point of view by increasingly accounting for the emissions in the supply chain results in a lower responsibility, as we can see in Fig. 6. Other countries in this class are for example India and Indonesia. These patterns indeed confirm that in the “no-discounting” limit, the responsibility of an agent depends only on its network position and not on its direct and indirect emissions.

Thirdly, we find an intermediary class of economies for whom the responsibility does not always increase or decrease monotonically when increasingly accounting for emissions in the supply chain. An example of this is given by the downstream responsibility of Poland, whose responsibility decreases when δ increases until $\delta \approx 0.85$,

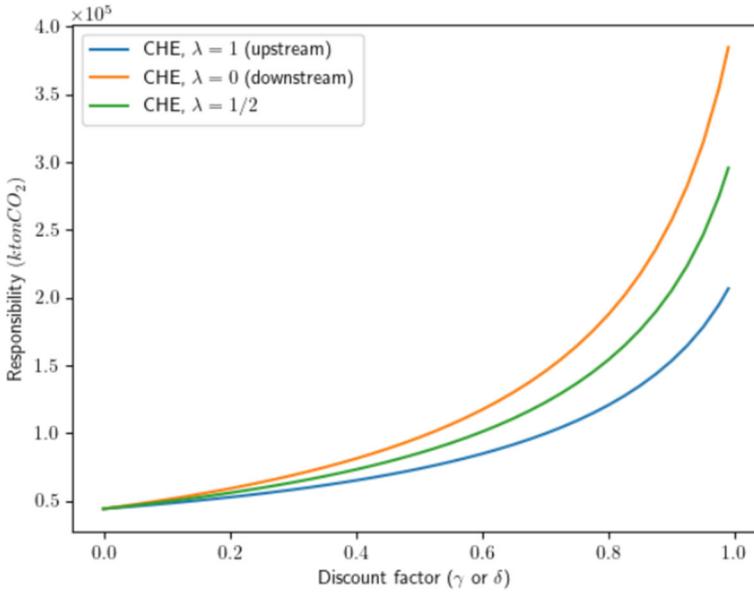


Fig. 5 Upstream and downstream responsibility for Switzerland in kton CO_2 , with $v_{max} = 1000$, for various values of γ and δ

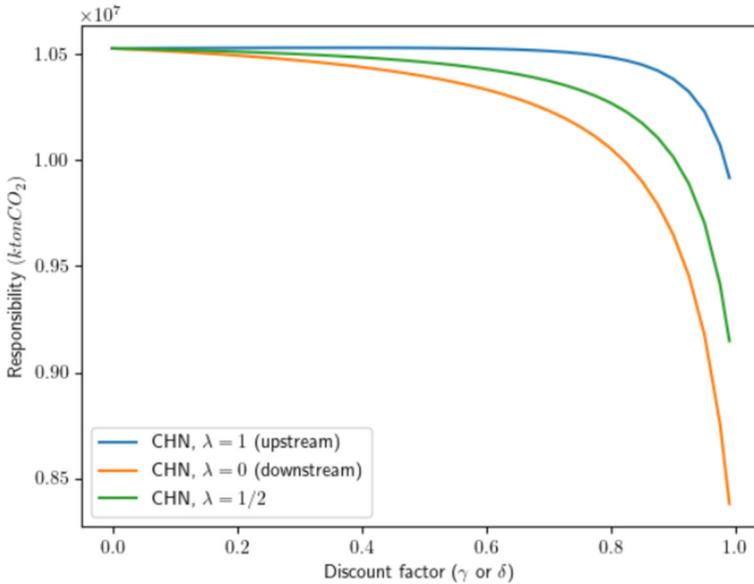


Fig. 6 Upstream and downstream responsibility for China in kton CO_2 , with $v_{max} = 1000$, for various values of γ and δ

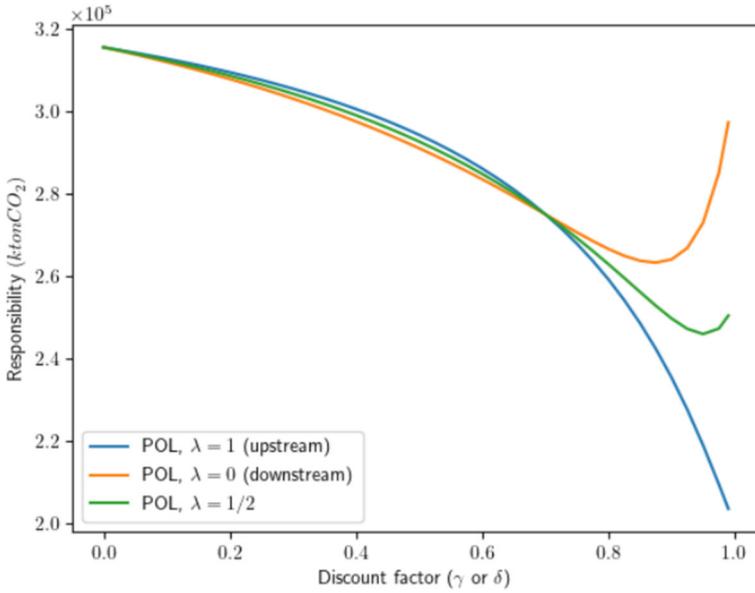


Fig. 7 Upstream and downstream responsibility for Poland, $v_{max} = 1000$, for various values of γ and δ

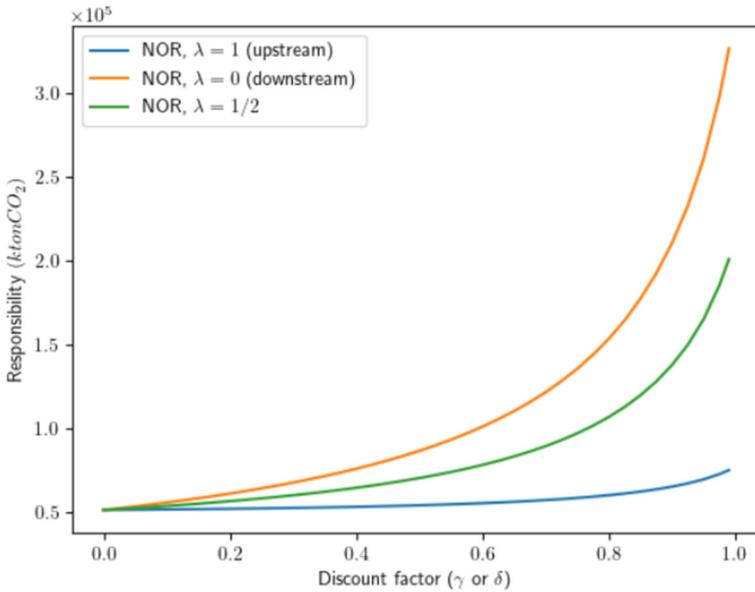


Fig. 8 Upstream and downstream responsibility for Norway, $v_{max} = 1000$, for various values of γ and δ

but increases when δ continues to increase, as depicted in Fig. 7. From the downstream perspective, we see similar behaviour for example for Hungary and Romania. The same behaviour but from the upstream perspective is exhibited for example by Mexico, Turkey or Canada. The countries in this class have a relatively high emission intensity but are more closely connected to service-oriented countries than to emission-intensive manufacturing countries. Then, as we can interpret δ and γ as a factor that determines to what extent we take the walks that are sufficiently far away into account, the centrality of the agent in the network can differ for various values of δ and γ which results in different responsibilities and non-monotonic behaviour with respect to the discount factor.

Remark that for most countries, the upstream responsibility and downstream responsibility show the same kind of behaviour. However, when a country either exports or imports more, the upstream responsibility can show different behaviour. Norway, known for its export of emission-intensive petroleum gases and oils, is an insightful example: its downstream responsibility is substantially higher than its upstream responsibility, as depicted in Figure 8.

5.3 Sectoral allocation of responsibility for US emissions

In order to allocate responsibility within one country, we assign responsibility on a sectoral level of the USA. Instead of aggregating over all sectors with the data of Timmer et al. (2015) and Corsatea et al. (2019) as in the previous section, we now just evaluate the input and output for every sector of the USA and evaluate the responsibility. For the 56 sectors, we use the NACE classification codes, which can be found in Appendix B. The total amount of emissions of the USA that we want to allocate is 4,343,481 kton CO_2 . In Appendix B, one can find the responsibility of every (sub-)sector for various values of γ and δ . In the following, we evaluate the responsibility of a few sectors.

According to the Environmental Accounts (Corsatea et al. 2019), about 48% of the emissions of the USA in 2014 comes from the sector *D35: Electricity, gas, steam and air conditioning supply*. For this sector, there is a large discrepancy between the share of emissions of this sector and the share of total input and output, which are respectively only 0.9% and 1.6%. As we increasingly account for indirect emissions, the responsibility of this sector decreases rapidly, as can be seen in Fig. 9. The same behaviour is exhibited by for example the Transport sector *H*, which consists of the subsectors *H49 Land transport and transport via pipelines; H50 Water transport; H51 Air transport; H52 Warehousing and support activities for transportation and H53 Postal and courier activities*, and that emits 29% of the total USA emissions. For this sector, the responsibility also decreases very quickly when the discount factors γ or δ increase.

The sector *B: Mining and Quarrying*, which embeds fossil fuel extraction, exhibits non-monotonic behaviour. We see that the responsibility decreases when we increasingly account for the upstream emissions in the supply chain. On the other hand, accounting for downstream emissions in the supply chain, results in a non-monotonic increase in responsibility, see Fig. 10. From the downstream perspective, this sector

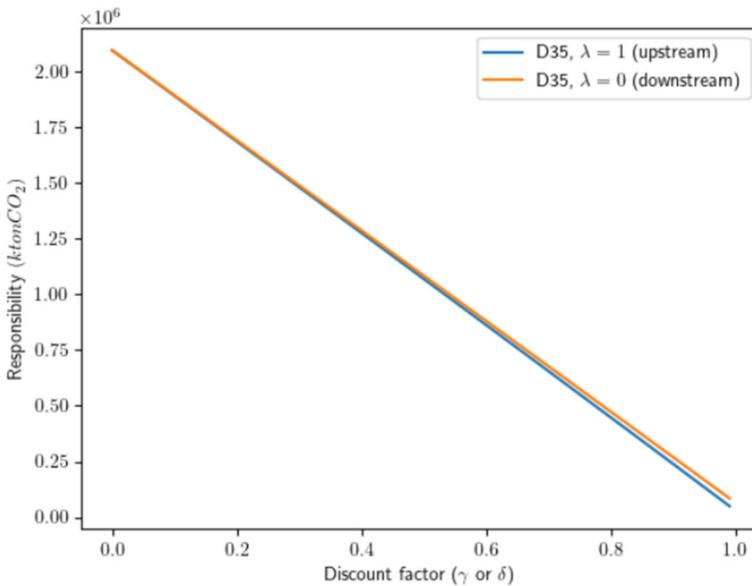


Fig. 9 Upstream and downstream responsibility for sector *D35: Electricity, gas, steam and air conditioning supply* in kton CO₂, with $v_{max} = 1000$, for γ and δ ranging from 0.0 to 0.99

is closely connected to the emission-intensive sectors such as the energy utilities and transport sector, but in the overall economic system, this sector of fossil fuel extraction does not have a central position.

Lastly, the manufacturing sector *C*, which consists of nineteen subsectors amongst which the manufacture of food and beverages, textiles, and machinery, emits 15% of the total emissions of the USA, and has a 25% share of output and a share of input of 27%. The upstream and downstream responsibility of this sector as a whole are depicted in Fig. 11. Within this sector, though, there can be large differences. For example, in the upstream perspective, the manufacture of food products, beverages and tobacco products (C10-C12), as well as the manufacture of motor vehicles and trailers (C29) has a high centrality and therefore a high responsibility compared to its direct emissions in the non-discounting limit, whereas the manufacture of chemicals (C20) has a fairly low centrality and therefore low responsibility in this limit. We can see these differences in the upstream responsibility in Fig. 12.

Figures 9 to 12 show how responsibility for sectors changes when we move from responsibility for direct emissions only, i.e., when γ and δ are zero at the far left of each figure, to responsibility for all indirect emissions as γ and δ increase. With our responsibility measure, we have given a more precise meaning to scope 3 emissions that avoids double counting. We will extend the empirical application at the US sector level by comparing these scope 3 emissions with scope 2 emissions. Scope 2 emissions are defined as direct emissions plus emissions from purchased electricity (WRI and WBCSD 2011). To model scope 2 responsibility at the US sectoral level, we allocate the emissions of sector *D35 Electricity, gas, steam and air conditioning supply* to all

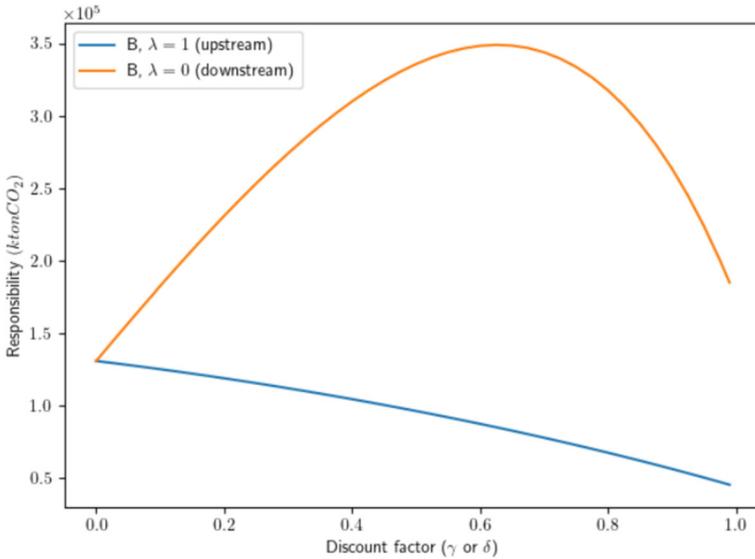


Fig. 10 Upstream and downstream responsibility for the *Mining and Quarrying* sector *B* in $\text{kton } CO_2$, with $v_{max} = 1000$, for γ and δ ranging from 0.0 to 0.99

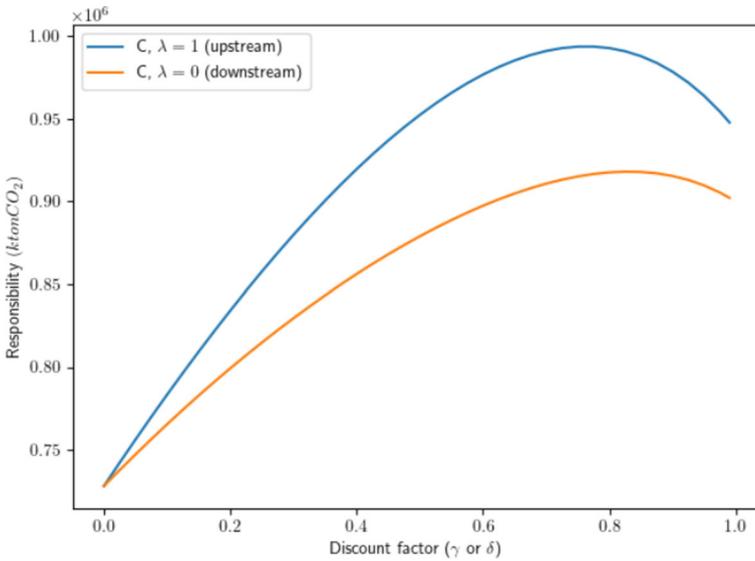


Fig. 11 Upstream and downstream responsibility for the *Manufacturing* sector *C*, in $\text{kton } CO_2$, with $v_{max} = 1000$, for γ and δ ranging from 0.0 to 0.99

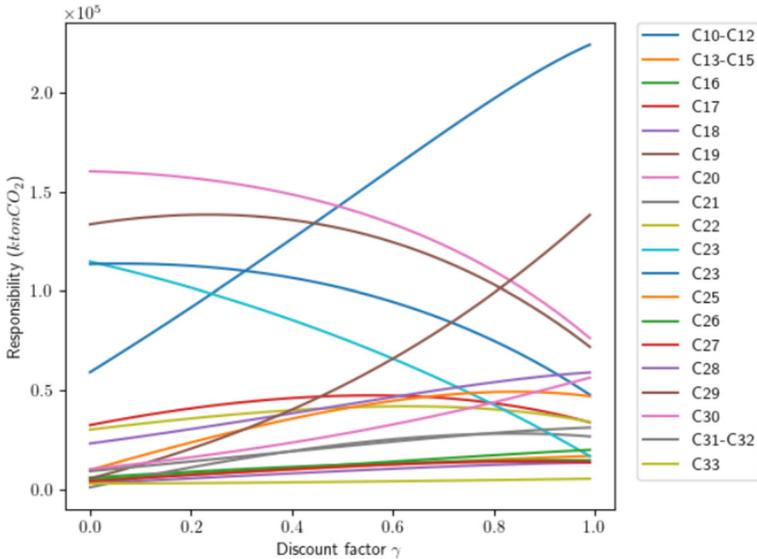


Fig. 12 Upstream responsibility for all subsectors of the *Manufacturing* sector *C*, in kton CO₂, with $v_{max} = 1000$, for γ ranging from 0.0 to 0.99

sectors. Each sector is thus responsible for their own direct emissions, plus its share of responsibility for the emissions of sector D35 according to its position in the network.

As expected, by comparing Figs. 9 and 14, we see that the responsibility for the sector D35 itself is the same in the case of scope 2 and scope 3 responsibility: they have large direct emissions, the responsibility for which is transferred to the other actors as the discount factor increases. For the Mining and Quarrying Sector B, we observe that the downstream responsibility behaves similarly when moving from scope 2 to scope 3, but while the upstream responsibility decreases with increasing γ in the case of scope 3 (Fig. 10), it increases for scope 2 (Fig. 15). This confirms that this sector does not have a central position in the overall economic system, but it is closely linked to the energy sector. The responsibility of Manufacturing sector B also increases in the scope 2 responsibility as γ and δ increase. Notably, the responsibility for this sector is significantly higher in the scope 2 case, due to its strong connection to the energy sector.

6 Concluding remarks

In this paper, we consider the problem of allocating responsibility for emissions of greenhouse gases in production networks. We introduce a class of responsibility measures based on the discounted emissions across paths in the production network. We show that this measure has a strong normative foundation as it is characterized by a set of axioms consistent with legal principles underlying responsibility. A policy maker can adjust the measure to its use case (e.g. allocation of responsibility across emitters in

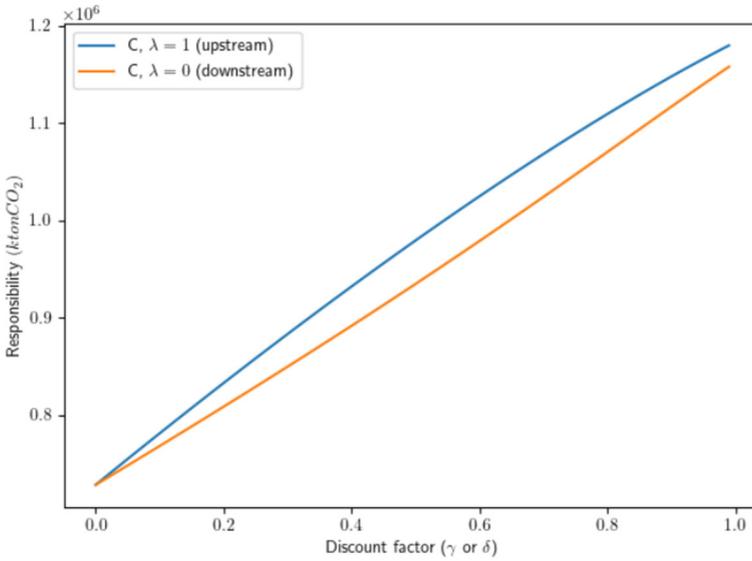


Fig. 13 “Scope 2” upstream and downstream responsibility for the *Manufacturing sector C*, in kton CO_2 , with $v_{max} = 1000$, for γ and δ ranging from 0.0 to 0.99

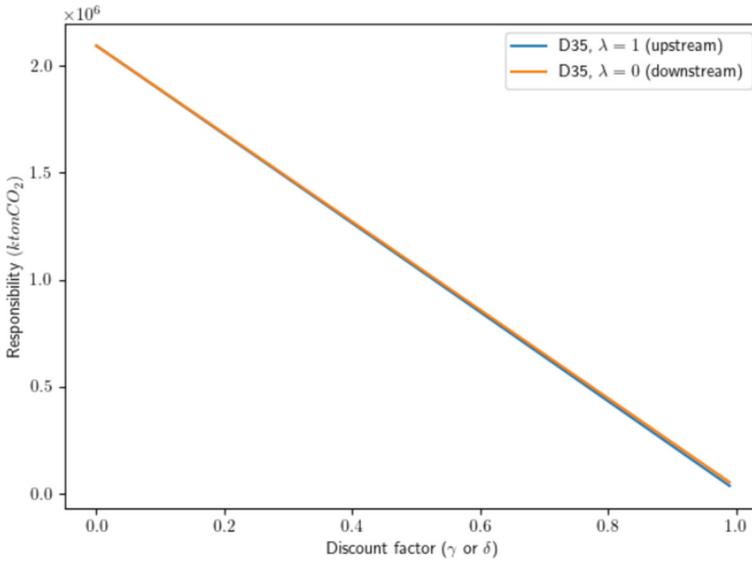


Fig. 14 “Scope 2” upstream and downstream responsibility for sector D35: *Electricity, gas, steam and air conditioning supply*, in kton CO_2 , with $v_{max} = 1000$, for γ ranging from 0.0 to 0.99

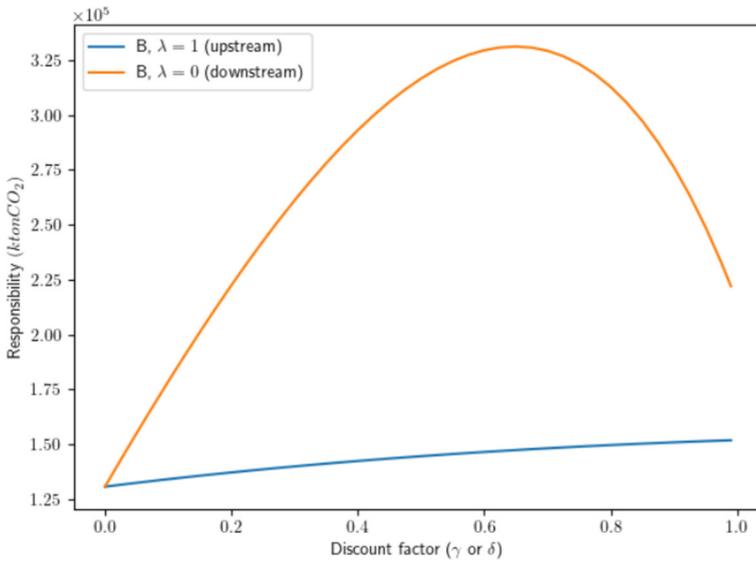


Fig. 15 “Scope 2” upstream and downstream responsibility for the *Mining and Quarrying* sector *B*, in kton *C*O₂, with $v_{max} = 1000$, for γ and δ ranging from 0.0 to 0.99

a system, definition of individual liability in a legal context) by adjusting the discount factors and relative share of responsibility for upstream and downstream emissions. In particular, when indirect emissions are infinitely discounted, the responsibility of each agent coincides with its direct emissions. Oppositely, in the “no-discounting” limit, the responsibility of an agent depends entirely on its position in the network and is independent of its level of emissions. Furthermore, several responsibility measures currently in use or put forward in the literature, including consumption-based and income-based accounting, are obtained as a particular case of the discounted emission measure. The introduced responsibility measure can have great policy implications. As this measure is shown to be consistent, we overcome the issue of double counting, which could facilitate setting and meeting GHG emission reduction goals. Moreover, by offering a clear definition of ‘scope 3 emissions’, the measure could play an important role in the (corporate) reporting requirements that are being put forward. The normative foundation of the responsibility measure enables it to guide climate litigation cases. Finally, the responsibility measure can also serve as a basis for a carbon taxing scheme. In this case, as in the general equilibrium framework of King et al. (2019), the tax determines the total level of emissions and the allocation of responsibility across economic actors. More precisely, for each actor, the marginal cost of reducing emission responsibility ought to be equal to the tax rate and thus efforts towards emission reduction are proportional to responsibility. Furthermore, the production network evolves endogenously up to the point where the marginal contribution of a purchase or of a sale to responsibility is accounted for, proportionally to the tax, when deciding about the formation of a commercial relationship. In this sense, the

production network eventually adjusts to the responsibility metric, thereby achieving ‘fair’ allocation of responsibility.

Appendix A Proofs

Proof of Lemma 1 Assume $E = (N, F, \Phi, \Psi)$ and $E' = (N, F', \Phi', \Psi')$ are such that $(u_i(F, \Phi), v_i(F, \Psi)) = (u_i(F', \Phi'), v_i(F', \Psi'))$. By reflexivity of the Euclidian partial order, one has

$(u_i(F, \Phi), v_i(F, \Psi)) \geq (u_i(F', \Phi'), v_i(F', \Psi'))$ and $(u_i(F', \Phi'), v_i(F', \Psi')) \geq (u_i(F, \Phi), v_i(F, \Psi))$. Thus monotonicity implies that $\rho_i(E, \lambda, \gamma, \delta) \geq \rho_i(E', \lambda, \gamma, \delta)$ and $\rho_i(E', \lambda, \gamma, \delta) \geq \rho_i(E, \lambda, \gamma, \delta)$. Hence,

$\rho_i(E, \lambda, \gamma, \delta) = \rho_i(E', \lambda, \gamma, \delta)$ and there exists a function $\sigma_i : \mathbb{R}_+^N \times \mathbb{R}_+^N \rightarrow \mathbb{R}$ such that for all $E = (N, F, \Phi, \Psi) \in \mathcal{E}$, $\rho_i(E, \lambda, \gamma, \delta) = \sigma_i(u_i(F, \Phi), v_i(F, \Psi))$. The domain of σ_i is $\mathbb{R}_+^N \times \mathbb{R}_+^N$ as this is the range of (u_i, v_i) on \mathcal{E} . The fact that σ_i is independent of i is a direct consequence of anonymity and the monotonicity of σ is a direct consequence of the monotonicity axiom. \square

Proof of Lemma 2 Following Lemma 1, one has for all emission systems $E = (N, F, \Phi, \Psi)$ and for all $i \in N$:

$$\rho_i(E, \lambda, \gamma, \delta) = \sigma(u_i(F, \Phi), v_i(F, \Psi)).$$

Additivity then implies that

$$\rho_i(E, \lambda, \gamma, \delta) = \sigma(u_i(F, \Phi), 0) + \sigma(0, v_i(F, \Psi)).$$

Letting for all $(u, v) \in \mathbb{R}_+^N \times \mathbb{R}_+^N$, $\alpha(u) = \sigma(u, 0)$ and $\beta(v) = \sigma(0, v)$ yields the required decomposition. The additivity and the monotonicity of α and β then are direct consequences of the additivity and monotonicity of σ . \square

Proof of Lemma 3 Following Lemma 2, it is straightforward that for all emission systems $E = (N, F, \Phi, \Psi)$ and for all $i \in N$, one has:

$$\rho_i(E, \lambda, \gamma, \delta) = \alpha(u_i(F, \Phi)) + \beta(v_i(F, \Psi)). \tag{10}$$

Then it suffices to show that there exist $\gamma \geq 0$ and $a \geq 0$ such that for all $u \in \mathbb{R}_+^N$:

$$\alpha(u) = a \sum_{v=0}^{+\infty} \gamma^v u_v.$$

Indeed, the proof that there exist $\delta \geq 0$ and $b \geq 0$ such that for all $v \in \mathbb{R}_+^N$:

$$\beta(v) = b \sum_{v=0}^{+\infty} \delta^v v_v.$$

follows by a symmetric argument. Let us first remark that additivity implies that $\alpha(0) = 0$. Let us then consider $u \in \mathbb{R}_+^{\mathbb{N}}$ and $v \in \mathbb{N}$. Considering the truncation of u at rank v , one has using additivity that:

$$\alpha(u_1, \dots, u_v, 0, \dots) = \sum_{\tau=1}^v \alpha(0, \dots, 0, u_\tau, 0, \dots)$$

which can equivalently be written as

$$\alpha(u_1, \dots, u_v, 0, \dots) = \sum_{\tau=1}^v \alpha_\tau(u_\tau) \tag{11}$$

where for $x \in \mathbb{R}_+$, one sets $\alpha_\tau(x) := \alpha(0, \dots, 0, x, 0, \dots)$.

Using recursivity, one has that for all $x \geq 0$, there exists $\delta(x) \geq 0$ such that for all $\tau \leq v$, one has:

$$\frac{\alpha_{\tau+1}(x)}{\alpha_\tau(x)} = \frac{\alpha_2(x)}{\alpha_1(x)} = \gamma(x).$$

Recursivity also implies that for all $x, x' \geq 0$:

$$\frac{\alpha_1(x')}{\alpha_1(x)} = \frac{\alpha_2(x')}{\alpha_2(x)},$$

so that

$$\frac{\alpha_2(x)}{\alpha_1(x)} = \frac{\alpha_2(x')}{\alpha_1(x')}$$

and $\gamma(x)$ is in fact independent of x and can be denoted by $\gamma \geq 0$. Thus Equation (11) can in fact be written as

$$\alpha(u_1, \dots, u_v, 0, \dots) = \sum_{\tau=1}^v \gamma^{\tau-1} \alpha_1(u_\tau).$$

The additivity of α implies the additivity of α_1 , α_1 is thus a group morphism of \mathbb{R} and consequently linear according to standard arguments. This implies that there exists $a \geq 0$ (non-negativity is implied by monotonicity) such that

$$\alpha(u_1, \dots, u_v, 0, \dots) = a \sum_{\tau=1}^v \gamma^{\tau-1} u_\tau.$$

As this holds for all $v \geq 0$, tail-convergence eventually yields:

$$\alpha(u) = a \sum_{\tau=1}^{+\infty} \gamma^{\tau-1} u_\tau. \tag{12}$$

As mentioned above, a symmetric argument yields that there exist $\delta \geq 0$ and $b \geq 0$ such that for all $v \in \mathbb{R}_+^N$

$$\beta(v) = b \sum_{\tau=1}^{+\infty} \gamma^{\tau-1} v_\tau. \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (10) eventually yields

$$\rho_i(E, \lambda, \gamma, \delta) = a \left(\sum_{v=0}^{+\infty} \gamma^v \sum_{j \in N} \Phi_{i,j}^v f_j \right) + b \left(\sum_{v=0}^{+\infty} \delta^v \sum_{j \in N} \Psi_{i,j}^v f_j \right).$$

□

Proof of Theorem 1 It is straightforward to show that if ρ_i is of the required form, it satisfies all the axioms. We thus focus on the converse.

Using Lemma 3, one knows that $\bar{\rho}_i$ is of the form

$$\bar{\rho}_i(E) = a \left(\sum_{v=0}^{+\infty} \gamma^v \sum_{j \in N} \Phi_{i,j}^v f_j \right) + b \left(\sum_{v=0}^{+\infty} \delta^v \sum_{j \in N} \Psi_{i,j}^v f_j \right). \tag{14}$$

Tail-convergence further implies that $\gamma, \delta \in [0, 1)$.

Summing over $i \in N$, Equation (14) yields:

$$\sum_{i \in N} \bar{\rho}_i(E) = a \left(\sum_{v=0}^{+\infty} \gamma^v \sum_{j \in N} \sum_{i \in N} \Phi_{i,j}^v f_j \right) + b \left(\sum_{v=0}^{+\infty} \delta^v \sum_{j \in N} \sum_{i \in N} \Psi_{i,j}^v f_j \right).$$

Using consistency, this yields

$$\sum_{i \in N} f_i = a \left(\sum_{v=0}^{+\infty} \gamma^v \sum_{j \in N} \sum_{i \in N} \Phi_{i,j}^v f_j \right) + b \left(\sum_{v=0}^{+\infty} \delta^v \sum_{j \in N} \sum_{i \in N} \Psi_{i,j}^v f_j \right).$$

Now, Φ and Ψ are column-stochastic. This implies that for all $v \in \mathbb{N}$, Φ^v and Ψ^v are also column-stochastic so that

$$\sum_{i \in N} f_i = a \left(\sum_{v=0}^{+\infty} \gamma^v \sum_{j \in N} f_j \right) + b \left(\sum_{v=0}^{+\infty} \delta^v \sum_{j \in N} f_j \right)$$

and thus:

$$1 = a \sum_{v=0}^{+\infty} \gamma^v + b \sum_{v=0}^{+\infty} \delta^v.$$

As $\gamma, \delta \in [0, 1)$, this yields

$$1 = \frac{a}{1 - \gamma} + \frac{b}{1 - \delta}.$$

Letting $\lambda = \frac{a}{1 - \gamma}$, one obtains $\lambda \in [0, 1]$, $a = \lambda(1 - \gamma)$ and $b = (1 - \lambda)(1 - \delta)$, which eventually yields:

$$\bar{\rho}_i(E) = (1 - \gamma)\lambda \left(\sum_{\nu=0}^{+\infty} \gamma^\nu \sum_{j \in N} \Phi_{i,j}^\nu f_j \right) + (1 - \delta)(1 - \lambda) \left(\sum_{\nu=0}^{+\infty} \delta^\nu \sum_{j \in N} \Psi_{i,j}^\nu f_j \right).$$

□

Proof of Proposition 1 If A is aperiodic and irreducible, so are Φ and Ψ since they describe the same graph. Then it is a consequence of the linearity of the limit and the Perron-Frobenius theorem that for a column-stochastic matrix M , as δ tends towards 1, the matrix $(I - \delta M)^{-1}$ converges to $Eig^T(M)$. The proof follows. □

Proof of Proposition 2 In the following, given a walk $w := (i_1, \dots, i_\tau) \in \mathcal{N}^\tau$, we denote, with a slight abuse of notation its weight as $\phi_w := \prod_{t=1}^{\tau-1} \phi_{i_t, i_{t+1}}$. We shall also refer more specifically to the set of walks of length τ between i and j : $\mathcal{W}_{i,j}^\tau := \{(i_1, \dots, i_\tau) \in \mathcal{N}^\tau \mid i_1 = i \wedge i_\tau = j\}$, and the set of direct walks of length τ between i and j : $\mathcal{D}_{i,j}^\tau := \{(i_1, \dots, i_\tau) \in \mathcal{W}_{i,j}^\tau \mid \forall t < \tau \mid i_t \neq j\}$. Let us then consider a walk of length τ , (i_1, \dots, i_τ) . One has

$$g_{i_1} \prod_{t=1}^{\tau-1} c_{i_t, i_{t+1}} = \frac{f_{i_1}}{q_{i_1}} \prod_{t=1}^{\tau-1} \frac{v_{i_t, i_{t+1}}}{q_{i_{t+1}}} = f_{i_1} \left(\prod_{t=1}^{\tau-1} \frac{v_{i_t, i_{t+1}}}{q_{i_t}} \right) \frac{1}{q_{i_\tau}} = f_{i_1} \left(\prod_{t=1}^{\tau-1} \phi_{i_{t+1}, i_t} \right) \frac{1}{q_{i_\tau}}. \tag{15}$$

As for $j \neq k$, one has $\phi_{j,k} = 0$, there is no walk between $i \in \mathcal{J}$ and $j \in \mathcal{J}$ that can go through $k \in \mathcal{L}$ and thus one has for all $i, j \in \mathcal{J}$ and $k \in \mathcal{L}$:

$$g_j C_{j,i}^\tau D_{i,k} = \frac{D_{i,k}}{q_i} \left(\sum_{w \in \mathcal{W}_{i,j}^\tau} \phi_w \right) f_j. \tag{16}$$

This yields

$$g_j C_{j,i}^\tau D_{i,k} = \phi_{k,i} \left(\sum_{w \in \mathcal{W}_{i,j}^\tau} \phi_w \right) f_j. \tag{17}$$

Now, a walk of the form (k, w) with $w \in \mathcal{W}_{i,j}^\tau$ and $i, j \in \mathcal{J}$ is a direct walk of length $\tau + 1$ from k to j . Furthermore all direct walks are of this form because no walk from k to j can go through $\ell \in \mathcal{L} \setminus \{k\}$. Thus one has:

$$\sum_{i=1}^{ML} g_j C_{j,i}^\tau D_{i,k} = \left(\sum_{w \in \mathcal{D}_{k,j}^{\tau+1}} \phi_w \right) f_j. \tag{18}$$

Accordingly, for all $\gamma \in [0, 1]$, one has:

$$\gamma \sum_{i=1}^{ML} g_j \gamma^\tau C_{j,i}^\tau D_{i,k} = \left(\sum_{w \in \mathcal{D}_{k,j}^{\tau+1}} \gamma^{\tau+1} \phi_w \right) f_j. \tag{19}$$

Summing over τ this yields:

$$\gamma \sum_{i=1}^{ML} g_j (I - \gamma C)_{j,i}^{-1} D_{i,k} = \gamma \sum_{\tau=0}^{+\infty} \sum_{i=1}^{ML} g_j \gamma^\tau C_{j,i}^\tau D_{i,k} = \left(\sum_{\tau=0}^{+\infty} \sum_{w \in \mathcal{D}_{k,j}^{\tau+1}} \gamma^{\tau+1} \phi_w \right) f_j. \tag{20}$$

Now $(\sum_{\tau=0}^{+\infty} \sum_{w \in \mathcal{D}_{k,j}^{\tau+1}} \gamma^{\tau+1} \phi_w)$ is the discounted sum of all direct walks from k to j . Any walk from k to j consists in an arbitrary number of cycles around k followed by a direct walk from k to j . In our setting, all cycles around k consist in the self-loop $\phi_{k,k} = 1$ and thus a walk from k to j consists in an arbitrary number of self-loops around k followed by a direct walk from k to j . Therefrom, one gets that

$$\sum_{\tau=0}^{+\infty} \sum_{w \in \mathcal{W}_{k,j}^{\tau+1}} \gamma^{\tau+1} \phi_w = \sum_{k \in \mathbb{N}} \gamma^k \sum_{\tau=0}^{+\infty} \sum_{w \in \mathcal{D}_{k,j}^{\tau+1}} \gamma^{\tau+1} \phi_w = \frac{1}{1 - \gamma} \sum_{\tau=0}^{+\infty} \sum_{w \in \mathcal{D}_{k,j}^{\tau+1}} \gamma^{\tau+1} \phi_w. \tag{21}$$

In turn this yields that for all $\tau \geq 0$, for all $k \in \mathcal{L}$ and all $j \in \mathcal{N}$, one gets:

$$(1 - \gamma)(\gamma \Phi)_{k,j}^{\tau+1} = \sum_{\tau=0}^{+\infty} \sum_{w \in \mathcal{D}_{k,j}^{\tau+1}} \gamma^{\tau+1} \phi_w. \tag{22}$$

Substituting Eq. (22) into Eq. (20), one gets

$$\gamma \sum_{i \in \mathcal{J}} g_j (I - \gamma C)_{j,i}^{-1} D_{i,k} = \sum_{\tau=0}^{+\infty} (1 - \gamma)(\gamma \Phi)_{k,j}^{\tau+1} f_j = (1 - \gamma)(I - \gamma \Phi)_{k,j}^{-1} f_j \tag{23}$$

and then

$$\gamma \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{J}} g_j (I - \gamma C)_{j,i}^{-1} D_{i,k} = \sum_{j \in \mathcal{J}} (1 - \gamma)(I - \gamma \Phi)_{k,j}^{-1} f_j. \tag{24}$$

So that, taking the limit as γ tends towards 1, one gets

$$\sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{J}} g_j (I - C)_{j,i}^{-1} D_{i,k} = \lim_{\gamma \rightarrow 1} \sum_{j \in \mathcal{J}} (1 - \gamma)(I - \gamma \Phi)_{k,j}^{-1} f_j. \tag{25}$$

□

Proof of Proposition 3 Proof follows from the proof of Proposition 2. □

Appendix B Empirical results

See Tables 2, 3, 4, 5 and 6.

Table 2 Upstream responsibility ($\lambda = 1, \nu_{max} = 1000$) for various values of γ , in kton CO_2

Country	$\gamma = 0.0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$	$\gamma = 0.99$
AUS	404,864	405,276	405,749	409,283	409,854	408,961	402,119
AUT	57,324	59,466	61,956	85,015	94,475	108,549	131,216
BEL	97,560	101,616	106,305	148,168	164,595	188,352	225,096
BGR	45,142	44,577	43,920	37,876	35,500	32,257	28,445
BRA	572,372	576,092	580,588	632,584	660,570	710,911	813,335
CAN	576,979	576,458	575,878	572,901	573,842	579,583	607,279
CHE	44,067	47,977	52,615	99,565	120,607	153,222	206,574
CHN	10,528,550	10,529,692	10,530,744	10,516,086	10,485,845	10,383,828	9,917,066
CYP	7,029	6,982	6,927	6,358	6,095	5,690	5,125
CZE	95,800	96,071	96,386	99,343	100,637	102,852	108,062
DEU	855,312	857,768	861,000	910,545	941,553	999,772	1,123,097
DNK	71,497	72,369	73,409	84,238	89,247	97,177	110,740
ESP	270,385	274,773	280,081	341,534	374,529	433,571	551,804
EST	19,565	19,138	18,649	14,563	13,107	11,193	8,896
FIN	51,307	52,434	53,749	65,952	70,850	77,846	88,227
FRA	345,586	355,340	367,164	504,557	577,692	706,210	953,274
GBR	488,332	493,499	499,836	577,171	620,063	697,299	851,647
GRC	78,655	78,349	77,981	73,919	71,902	68,651	63,887
HRV	17,850	17,774	17,683	16,817	16,468	16,014	15,707
HUN	47,587	47,919	48,312	52,238	54,022	56,909	62,284
IDN	542,875	539,667	535,808	492,612	470,398	432,212	362,284
IND	2,195,307	2,182,361	2,166,548	1,971,004	1,856,618	1,636,829	1,155,133
IRL	44,079	47,364	50,986	77,138	85,297	95,895	111,164
ITA	356,596	363,367	371,574	467,310	518,909	611,226	795,770

Table 2 continued

Country	$\gamma = 0.0$	$\gamma = 0.1$	$\gamma = 0.2$..	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$	$\gamma = 0.99$
JPN	1,251,980	1,267,237	1,285,496	..	1,483,183	1,580,098	1,739,147	2,006,186
KOR	670,005	676,433	683,978	..	755,965	785,231	825,470	872,898
LTU	17,832	17,810	17,766	..	16,489	15,590	13,997	11,376
LUX	8,270	10,307	12,587	..	30,406	36,611	45,228	58,297
LVA	8,695	8,840	9,002	..	10,133	10,343	10,342	9,749
MEX	488,677	487,749	486,694	..	479,055	477,828	479,692	497,593
MLT	3,634	3,728	3,843	..	5,139	5,738	6,665	8,190
NLD	160,000	163,472	167,494	..	203,835	218,337	239,540	272,755
NOR	51,116	51,430	51,826	..	57,037	59,957	65,079	74,947
POL	315,600	312,842	309,552	..	275,111	259,321	235,524	203,693
PRT	49,364	49,780	50,291	..	56,632	60,265	67,029	81,271
NLD	160,000	163,472	167,494	..	203,835	218,337	239,540	272,755
NOR	51,116	51,430	51,826	..	57,037	59,957	65,079	74,947
POL	315,600	312,842	309,552	..	275,111	259,321	235,524	203,693
PRT	49,364	49,780	50,291	..	56,632	60,265	67,029	81,271
ROU	80,216	80,220	80,217	..	79,727	79,322	78,691	78,792
RUS	1,723,086	1,686,183	1,642,597	..	1,208,205	1,017,945	733,706	324,820
SVK	33,564	34,265	35,068	..	41,889	44,370	47,763	52,824
SVN	14,409	14,368	14,325	..	14,220	14,345	14,733	15,894
SWE	51,175	53,690	56,641	..	85,041	97,081	115,053	142,976
TUR	357,560	357,881	358,221	..	358,945	357,509	352,969	341,016
TWN	314,702	312,608	310,224	..	290,580	283,623	274,228	261,170
USA	5,257,452	5,267,728	5,280,252	..	5,433,719	5,524,109	5,705,225	6,181,470
ROW	7,835,123	7,804,173	7,767,151	..	7,364,989	7,166,778	6,841,984	6,311,360

Table 3 Downstream responsibility ($\lambda = 0, v_{max} = 1000$) for various values of δ , in kton $C O_2$

Country	$\delta = 0.0$	$\delta = 0.1$	$\delta = 0.2$	$\delta = 0.7$	$\delta = 0.8$	$\delta = 0.9$	$\delta = 0.99$
AUS	404,864	411,880	420,335	515,477	563,960	644,076	762,315
AUT	57,324	59,962	63,093	96,276	112,855	142,036	202,937
BEL	97,560	101,232	105,560	149,328	169,997	205,077	275,319
BGR	45,142	44,502	43,759	37,159	34,740	31,771	29,634
BRA	572,372	575,045	578,240	612,501	629,017	655,493	696,607
CAN	576,979	580,520	584,676	624,490	641,041	665,166	705,285
CHE	44,067	50,873	58,998	145,574	187,465	256,963	384,610
CHN	10,528,550	10,513,650	10,495,100	10,234,564	10,055,027	9,647,863	8,379,388
CYP	7,029	7,070	7,118	7,573	7,792	8,207	9,280
CZE	95,800	95,447	95,097	95,819	98,905	107,811	136,042
DEU	855,312	868,318	884,102	1,071,930	1,177,611	1,377,584	1,831,071
DNK	71,497	72,323	73,319	84,843	91,170	102,942	128,855
ESP	270,385	272,622	275,336	308,032	326,998	364,305	455,507
EST	19,565	19,230	18,855	16,276	15,775	15,784	17,989
FIN	51,307	52,442	53,817	69,895	78,734	95,072	130,534
FRA	345,586	353,539	363,207	478,519	542,944	663,246	928,458
GBR	488,332	497,181	508,044	641,555	716,906	856,633	1,157,654
GRC	78,655	78,464	78,221	74,901	72,974	69,687	64,922
HRV	17,850	17,842	17,833	17,874	18,021	18,524	20,503
HUN	47,587	47,175	46,752	45,604	46,656	50,168	61,590

Table 3 continued

Country	$\delta = 0.0$	$\delta = 0.1$	$\delta = 0.2$	$\delta = 0.7$	$\delta = 0.8$	$\delta = 0.9$	$\delta = 0.99$
IDN	542,875	540,128	536,796	497,582	476,110	437,241	359,385
IND	2,195,307	2,167,262	2,133,501	1,754,716	1,560,511	1,230,246	647,479
IRL	44,079	46,820	49,849	72,393	79,982	90,686	108,435
ITA	356,596	361,738	368,060	449,635	500,063	602,617	859,185
JPN	1,251,980	1,256,531	1,261,963	1,318,327	1,343,077	1,376,584	1,397,754
KOR	670,005	676,793	684,718	757,123	783,928	815,592	830,156
LTU	17,832	17,894	17,969	18,855	19,414	20,621	23,953
LUX	8,270	10,713	13,403	32,839	39,000	47,224	59,717
LVA	8,695	8,875	9,091	11,523	12,840	15,307	20,908
MEX	488,677	481,923	474,025	399,657	369,316	326,338	271,565
MLT	3,634	3,602	3,572	3,621	3,797	4,231	5,391
NLD	160,000	166,658	174,569	257,964	299,012	370,172	515,257
NOR	51,116	55,572	60,937	121,659	153,661	210,731	327,186
POL	315,600	312,021	307,910	275,098	266,693	264,160	297,402
PRT	49,364	49,827	50,380	56,407	59,530	65,232	78,076
ROU	80,216	80,038	79,842	79,021	79,624	82,445	94,673
RUS	1,723,086	1,726,585	1,730,773	1,777,928	1,804,321	1,858,641	2,024,597
SVK	33,564	33,752	33,981	37,032	39,073	43,497	55,418
SVN	14,409	14,499	14,612	16,181	17,211	19,370	24,940
SWE	51,175	55,027	59,661	111,974	139,461	188,434	288,571
TUR	357,560	355,676	353,429	330,224	320,049	306,116	296,115
TWN	314,702	317,454	320,561	343,663	349,507	352,997	341,826
USA	5,257,452	5,253,461	5,248,737	5,198,942	5,175,710	5,144,894	5,173,991
ROW	7,835,123	7,794,910	7,747,274	7,256,524	7,026,595	6,655,291	6,025,021

Table 4 Sectoral upstream responsibility ($\lambda = 1, v_{max} = 1000$) in kton CO₂

Sector	$\gamma = 0.0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$	$\gamma = 0.99$
A01	53,153	56,532	59,653	69,931	70,385	69,955	68,553
A02	2,756	2,659	2,569	2,211	2,145	2,073	2,000
A03	1,099	1,098	1,102	1,176	1,195	1,211	1,220
B	130,773	125,099	118,818	77,741	67,377	56,165	45,269
C10 - C12	59,146	75,239	91,830	179,980	197,122	212,765	224,402
C13 - C15	4,523	5,898	7,269	13,833	14,989	16,028	16,820
C16	5,907	7,454	8,883	13,920	14,399	14,644	14,628
C17	32,447	37,028	40,879	46,135	43,525	39,273	33,812
C18	2,949	4,252	5,546	11,445	12,357	13,092	13,549
C19	133,686	136,925	138,458	115,358	103,336	88,304	71,928
C20	160,432	159,329	157,107	123,244	110,130	93,981	76,327
C21	1,003	6,242	11,082	27,148	28,095	27,958	26,719
C22	30,125	33,102	35,784	41,675	40,402	37,819	34,092
C23	114,851	108,649	101,711	54,819	42,706	29,554	16,770
C24	113,632	113,760	112,719	85,823	74,987	61,823	47,728
C25	9,751	16,893	23,667	47,610	49,142	48,967	47,024
C26	4,900	6,271	7,714	15,644	17,225	18,716	19,928
C27	4,072	5,667	7,236	13,399	13,918	14,011	13,630
C28	23,151	26,690	30,432	50,494	54,081	57,090	59,017
C29	5,498	12,511	20,547	81,579	99,180	118,905	138,412
C30	10,213	13,052	16,205	38,083	43,932	50,291	56,383

Table 4 continued

Sector	$\gamma = 0.0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$	$\gamma = 0.99$
C31,C32	9,171	11,594	14,083	26,393	28,433	30,110	31,175
C33	2,858	2,977	3,133	4,375	4,695	5,032	5,346
D35	2,094,483	1,890,303	1,685,676	655,035	447,187	238,660	50,348
E36	18,548	16,873	15,183	6,477	4,676	2,853	1,190
E37 - E39	7,867	8,269	8,738	12,194	13,122	14,127	15,093
F	62,943	72,908	83,811	148,804	162,144	174,574	184,398
G45	12,770	14,728	16,960	31,928	35,664	39,656	43,470
G46	26,891	35,622	45,328	109,818	126,474	144,744	162,798
G47	61,415	70,059	79,491	132,332	142,814	152,865	161,389
H49	246,868	229,431	212,205	128,027	111,347	94,661	79,637
H50	64,157	59,595	54,928	29,714	24,234	18,591	13,372
H51	194,809	177,368	160,062	74,006	56,521	38,793	22,563
H52	21,217	22,814	24,332	30,049	30,684	31,102	31,277
H53	7,109	8,167	9,259	15,138	16,382	17,643	18,790
I	61,085	68,220	75,910	122,050	132,410	142,760	151,700
J58	52	1,361	2,930	16,618	21,098	26,443	32,146
J59,J60	174	1,819	3,739	19,093	23,770	29,193	34,814
J61	3,673	7,814	12,610	50,293	61,634	74,730	88,244
J62,J63	14,379	17,591	21,169	45,145	51,319	58,023	64,538
K64	11,505	14,052	17,004	41,832	49,934	59,634	70,029
K65	10,217	12,630	16,039	63,148	84,453	113,625	149,435
K66	4,377	7,427	11,129	42,018	51,565	62,740	74,498

Table 4 continued

Sector	$\gamma = 0.0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.7$	$\gamma = 0.8$	$\gamma = 0.9$	$\gamma = 0.99$
L68	12,913	73,164	126,572	287,080	296,706	298,188	292,178
M69-M70	20,294	28,298	36,927	91,087	104,670	119,465	133,977
M71	12,801	15,647	18,848	40,298	45,734	51,584	57,217
M72	6,897	8,430	10,155	21,712	24,641	27,793	30,828
M73	6,945	8,489	10,226	21,864	24,813	27,987	31,043
M74,M75	2,267	2,771	3,338	7,135	8,097	9,131	10,127
N	59,695	62,498	66,025	96,635	105,946	116,608	127,505
O84	242,418	267,882	294,056	448,490	487,028	529,282	570,996
P85	21,931	30,851	38,665	58,181	57,511	55,038	51,123
Q	84,405	101,134	120,701	269,046	311,567	360,051	409,919
R,S	32,280	38,096	44,545	84,256	93,285	102,531	110,966
T	0	222	465	1,934	2,267	2,609	2,924
U	0	0	0	0	0	0	0

Note that sector U, activities of extraterritorial organizations and bodies, does emissions nor nonzero supply relationships, and carries therefore zero responsibility. Neither does sector T. Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use have any emissions, but the nonzero supply relationships of this sector result in a nonzero responsibility

Table 5 Sectoral downstream responsibility ($\lambda = 0$, $v_{max} = 1000$) in kton CO_2

Sector	$\delta = 0.0$	$\delta = 0.1$	$\delta = 0.2$	$\delta = 0.7$	$\delta = 0.8$	$\delta = 0.9$	$\delta = 0.99$
A01	53,153	52,094	51,002	44,980	43,906	43,181	43,123
A02	2,756	3,000	3,264	5,263	5,952	6,847	7,908
A03	1,099	1,298	1,509	2,968	3,433	4,019	4,700
B	130,773	182,410	230,805	343,865	317,582	263,509	184,740
C10 - C12	59,146	59,151	59,033	57,758	58,043	59,024	60,882
C13 - C15	4,523	4,765	5,053	8,061	9,279	10,870	12,717
C16	5,907	6,620	7,513	15,640	18,332	21,565	25,043
C17	32,447	32,105	31,937	35,847	38,273	41,598	45,531
C18	2,949	3,734	4,716	14,254	17,613	21,741	26,283
C19	133,686	173,060	205,530	245,175	222,898	187,625	143,069
C20	160,432	154,144	147,943	117,812	111,296	103,943	96,001
C21	1,003	5,814	10,658	35,120	39,707	43,843	46,864
C22	30,125	30,049	30,314	37,115	39,515	42,145	44,579
C23	114,851	106,710	98,463	55,039	45,718	36,060	26,987
C24	113,632	108,149	103,064	88,643	88,708	89,550	90,395
C25	9,751	14,557	20,059	59,490	69,679	80,281	89,775
C26	4,900	6,686	8,849	29,127	36,212	44,869	54,293
C27	4,072	4,669	5,424	12,361	14,568	17,100	19,653
C28	23,151	24,878	27,075	41,636	43,557	43,996	42,353
C29	5,498	7,281	9,374	25,080	29,281	33,735	37,833
C30	10,213	10,974	11,763	16,140	17,067	17,968	18,717

Table 5 continued

Sector	$\delta = 0.0$	$\delta = 0.1$	$\delta = 0.2$	$\delta = 0.7$	$\delta = 0.8$	$\delta = 0.9$	$\delta = 0.99$
C31,C32	9,171	9,514	9,916	13,346	14,456	15,759	17,110
C33	2,858	2,793	2,757	3,160	3,404	3,728	4,101
D35	2,094,483	1,892,898	1,690,987	676,866	473,170	269,160	85,230
E36	18,548	16,962	15,365	7,223	5,564	3,894	2,380
E37 - E39	7,867	10,340	12,831	24,944	27,030	28,803	29,963
F	62,943	67,608	71,566	80,543	80,168	79,117	77,647
G45	12,770	13,181	13,772	20,353	22,688	25,521	28,582
G46	26,891	47,796	68,543	167,618	185,485	201,921	214,831
G47	61,415	58,932	56,537	45,074	42,815	40,593	38,666
H49	246,868	248,125	246,664	189,845	165,572	135,273	101,772
H50	64,157	58,906	53,602	26,129	20,376	14,487	9,038
H51	194,809	177,243	159,746	74,029	57,487	41,278	27,049
H52	21,217	30,885	39,917	70,357	72,037	71,515	68,738
H53	7,109	16,032	24,129	48,276	48,763	47,279	43,966
I	61,085	61,765	62,546	72,600	77,290	83,643	91,229
J58	52	2,141	4,571	25,508	32,610	41,293	50,806
J59,J60	174	1,630	3,537	26,209	35,652	48,103	62,765
J61	3,673	7,996	13,211	61,736	78,939	100,347	124,255
J62,J63	14,379	21,155	28,661	81,912	96,747	113,373	130,043
K64	11,505	26,618	41,763	126,389	148,202	173,587	200,813
K65	10,217	14,704	20,550	84,994	110,596	144,260	184,332
K66	4,377	8,732	13,953	61,247	78,143	99,699	124,725

Table 5 continued

Sector	$\delta = 0.0$	$\delta = 0.1$	$\delta = 0.2$	$\delta = 0.7$	$\delta = 0.8$	$\delta = 0.9$	$\delta = 0.99$
L68	12,913	26,752	41,918	152,510	186,364	226,722	270,116
M69-M70	20,294	43,682	69,764	238,138	277,578	317,542	352,940
M71	12,801	19,309	26,211	71,879	84,698	99,429	114,676
M72	6,897	10,516	14,352	39,707	46,816	54,983	63,433
M73	6,945	10,588	14,450	39,980	47,138	55,362	63,872
M74,M75	2,267	3,433	4,670	12,849	15,144	17,780	20,509
N	59,695	80,675	103,200	252,017	292,759	339,010	386,346
O84	242,418	229,014	215,821	153,721	142,373	131,570	122,434
P85	21,931	20,980	20,011	15,441	14,741	14,193	13,874
Q	84,405	77,136	69,784	32,024	24,341	16,649	9,740
R,S	32,280	33,141	34,541	54,193	62,136	72,225	83,614
T	0	121	259	1,261	1,551	1,884	2,227
U	0	0	0	0	0	0	0

Table 6 Sector codes

Code	Sector
A01	Crop and animal production, hunting and related service activities
A02	Forestry and logging
A03	Fishing and aquaculture
B	Mining and quarrying
C10-C12	Manufacture of food products, beverages and tobacco products
C13-C15	Manufacture of textiles, wearing apparel and leather products
C16	Manufacture of wood and of products of wood and cork, except furniture manufacture of articles of straw and plaiting materials
C17	Manufacture of paper and paper products
C18	Printing and reproduction of recorded media
C19	Manufacture of coke and refined petroleum products
C20	Manufacture of chemicals and chemical products
C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
C22	Manufacture of rubber and plastic products
C23	Manufacture of other non-metallic mineral products
C24	Manufacture of basic metals
C25	Manufacture of fabricated metal products, except machinery and equipment
C26	Manufacture of computer, electronic and optical products
C27	Manufacture of electrical equipment
C28	Manufacture of machinery and equipment n.e.c
C29	Manufacture of motor vehicles, trailers and semi-trailers
C30	Manufacture of other transport equipment
C31- C32	Manufacture of furniture; other manufacturing
C33	Repair and installation of machinery and equipment

Table 6 continued

Code	Sector
D35	Electricity, gas, steam and air conditioning supply
E36	Water collection, treatment and supply
E37-E39	Sewerage; waste collection, treatment and disposal activities; materials recovery remediation activities and other waste management services
F	Construction
G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
G46	Wholesale trade, except of motor vehicles and motorcycles
G47	Retail trade, except of motor vehicles and motorcycles
H49	Land transport and transport via pipelines
H50	Water transport
H51	Air transport
H52	Warehousing and support activities for transportation
H53	Postal and courier activities
I	Accommodation and food service activities
J58	Publishing activities
J59-J60	Motion picture, video and television programme production, sound recording and music publishing activities; programming and broadcasting activities
J61	Telecommunications
J62-J63	Computer programming, consultancy and related activities; information service activities
K64	Financial service activities, except insurance and pension funding

Table 6 continued

Code	Sector
K65	Insurance, reinsurance and pension funding, except compulsory social security
K66	Activities auxiliary to financial services and insurance activities
L68	Real estate activities
M69-M70	Legal and accounting activities; activities of head offices; management consultancy activities
M71	Architectural and engineering activities; technical testing and analysis
M72	Scientific research and development
M73	Advertising and market research
M74-M75	Other professional, scientific and technical activities; veterinary activities
N	Administrative and support service activities
O84	Public administration and defence; compulsory social security
P85	Education
Q	Human health and social work activities
R-S	Other service activities
T	Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use
U	Activities of extraterritorial organizations and bodies

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References

- Alcalde-Unzu J, Gómez-Rúa M, Molis E (2015) Sharing the costs of cleaning a river: The upstream responsibility rule. *Games Econom Behav* 90:134–150
- Alizamir S, de Véricourt F, Sun P (2016) Efficient feed-in-tariff policies for renewable energy technologies. *Oper Res* 64(1):52–66
- Bergañinos G, González-Díaz J, González-Rueda ÁM et al (2017) Loss allocation in energy transmission networks. *Games Econom Behav* 102:69–97
- Bergañinos G, González-Díaz J, González-Rueda ÁM (2019) The Shapley rule for loss allocation in energy transmission networks. In: *Handbook of the Shapley Value*. CRC Press, p 369–392
- Bloch F, Jackson MO, Tebaldi P (2023) Centrality measures in networks. *Soc Choice Welf* 61(2):413–453
- Boldi P, Vigna S (2014) Game-theoretic analysis of cooperation among supply chain agents: review and extensions. *Internet Math* 10(3–4):222–262
- Bonacich P (1972) Factoring and weighting approaches to status scores and clique identification. *J Math Sociol* 2(1):113–120
- Bonacich P (1987) Power and centrality: A family of measures. *Am J Sociol* 92(5):1170–1182
- Burger M, Wentz J, Horton R (2020) The law and science of climate change attribution. *Columbia J Environ Law* 45:57
- Busch T, Johnson M, Pioch T (2022) Corporate carbon performance data: Quo vadis? *J Ind Ecol* 26(1):350–363
- Caro F, Corbett CJ, Tan T et al (2013) Double counting in supply chain carbon footprinting. *Manuf Serv Oper Manag* 15(4):545–558
- Corsatea T, Lindner S, Arto I et al (2019) World Input-Output Database Environmental Accounts. Update 2000–2016. Publications Office of the European Union, Luxembourg. https://doi.org/10.2760/024036_JRC116234
- Cruz JM (2008) Dynamics of supply chain networks with corporate social responsibility through integrated environmental decision-making. *Eur J Oper Res* 184(3):1005–1031
- Csóka P, Herings P (2021) An axiomatization of the proportional rule in financial networks. *Manag Sci* 67(5):2799–2812
- Davis SJ, Caldeira K (2010) Consumption-based accounting of CO₂ emissions. *Proc Natl Acad Sci* 107(12):5687–5692
- Depoers F, Jeanjean T, Jérôme T (2016) Voluntary disclosure of greenhouse gas emissions: contrasting the carbon disclosure project and corporate reports. *J Bus Ethics* 134:445–461
- Dequiedt V, Zenou Y (2017) Local and consistent centrality measures in parameterized networks. *Math Soc Sci* 88:28–36

- Directive (EU) 2022/2464 (2022) Corporate sustainability reporting directive (csrd)
- Eder P, Narodoslawsky M (1999) What environmental pressures are a region's industries responsible for? A method of analysis with descriptive indices and input-output models. *Ecol Econ* 29(3):359–374
- Falkner R (2016) The Paris Agreement and the new logic of international climate politics. *Int Aff* 92(5):1107–1125
- Feichtinger G, Lambertini L, Leitmann G et al (2022) Managing the tragedy of commons and polluting emissions: a unified view. *Eur J Oper Res* 303(1):487–499
- Gallego B, Lenzen M (2005) A consistent input-output formulation of shared consumer and producer responsibility. *Econ Syst Res* 17(4):365–391
- Garg M (2009) Axiomatic foundations of centrality in networks. Tech. rep, Mimeo
- Ghosh A (1958) Input-output approach in an allocation system. *Economica* 25(97):58–64
- Giménez-Gómez JM, Teixidó-Figueras J, Vilella C (2016) The global carbon budget: a conflicting claims problem. *Clim Chang* 136:693–703
- Gopalakrishnan S (2022) The why and how of assigning responsibility for supply chain emissions. *Nat Clim Chang* 12:1075–1077
- Gopalakrishnan S, Granot D, Granot F (2021) Consistent allocation of emission responsibility in fossil fuel supply chains. *Manag Sci* 67(12):7637–7668
- Gopalakrishnan S, Granot D, Granot F et al (2021) Incentives and emission responsibility allocation in supply chains. *Manag Sci* 67(7):4172–4190
- Gutiérrez E, Llorca N, Sánchez-Soriano J et al (2018) Sustainable allocation of greenhouse gas emission permits for firms with Leontief technologies. *Eur J Oper Res* 269(1):5–15
- Harrison K (2015) International carbon trade and domestic climate politics. *Global Environ Polit* 15(3):27–48
- Jackson MO (2008) Social and economic networks. Princeton University Press, Princeton
- King M, Tarbush B, Teytelboym A (2019) Targeted carbon tax reforms. *Eur Econ Rev* 119:526–547
- Kitti M (2016) Axioms for centrality scoring with principal eigenvectors. *Soc Choice Welf* 46(3):639–653
- Koopmans TC (1960) Stationary ordinal utility and impatience. *Econometrica* 28(2):287–309
- Kortum S, Weisbach D (2017) The design of border adjustments for carbon prices. *Natl Tax J* 70(2):421–446
- Lenzen M, Murray J, Sack F et al (2007) Shared producer and consumer responsibility - theory and practice. *Ecol Econ* 61(1):27–42
- Leontief W (1970) Environmental repercussions and the economic structure: an input–output approach. *Rev Econ Stat* pp 262–271
- Liang S, Qu S, Zhu Z et al (2017) Income-based greenhouse gas emissions of nations. *Environ Sci Technol* 51(1):346–355
- Marques A, Rodrigues J, Lenzen M et al (2012) Income-based environmental responsibility. *Ecol Econ* 84:57–65
- Meléndez-Jiménez MA, Polanski A (2020) Dirty neighbors-pollution in an interlinked world. *Energy Econ* 86:104636
- Moretti S, Trabelsi R (2021) A double-weighted bankruptcy method to allocate CO₂ emissions permits. *Games* 12(4):78
- Munksgaard J, Pedersen KA (2001) CO₂ accounts for open economies: Producer or consumer responsibility? *Energy Policy* 29(4):327–334
- Nagarajan M, Sošič G (2008) Game-theoretic analysis of cooperation among supply chain agents: Review and extensions. *Eur J Oper Res* 187(3):719–745
- Newman M (2010) Networks: an introduction. Oxford University Press, Oxford
- Olivier J, Janssens-Maenhout G, Muntean M et al (2015) Trends in global CO₂ emissions: 2015 report. 2015. PBL Netherlands Environmental Assessment Agency
- Palacios-Huerta I, Volij O (2004) The measurement of intellectual influence. *Econometrica* 72(3):963–977
- Peters GP (2008) From production-based to consumption-based national emission inventories. *Ecol Econ* 65(1):13–23
- Pinero P, Bruckner M, Wieland H et al (2019) The raw material basis of global value chains: allocating environmental responsibility based on value generation. *Econ Syst Res* 31(2):206–227
- Rahmoune M, Radjef MS, Boukherroub T et al (2024) A new integrated cooperative game and optimization model for the allocation of forest resources. *Eur J Oper Res* 316(1):329–340
- Rodrigues J, Domingos T (2008) Consumer and producer environmental responsibility: comparing two approaches. *Ecol Econ* 66(2–3):533–546

- Rodrigues J, Domingos T, Giljum S et al (2006) Designing an indicator of environmental responsibility. *Ecol Econ* 59(3):256–266
- Schleussner CF, Rogelj J, Schaeffer M et al (2016) Science and policy characteristics of the Paris Agreement temperature goal. *Nat Clim Chang* 6(9):827–835
- Schmeidler D (1969) The nucleolus of a characteristic function game. *SIAM J Appl Math* 17(6):1163–1170
- U.S. Securities and Exchange Commission (2022) SEC news: press release. <https://www.sec.gov/news/press-release/2022-46>. Accessed 30 Jan 2023
- Shapley LS (1953) A value for n-person games. *Contribution to the Theory of Games*, vol 2. Princeton University Press, Princeton, NJ, pp 307–317
- Slutzki G, Volij O (2006) Scoring of web pages and tournaments - axiomatizations. *Soc Choice Welf* 26(1):75–92
- Steininger K, Lininger C, Droege S et al (2014) Justice and cost effectiveness of consumption-based versus production-based approaches in the case of unilateral climate policies. *Glob Environ Chang* 24:75–87
- Sunar N, Plambeck E (2016) Allocating emissions among co-products: implications for procurement and climate policy. *Manuf Serv Oper Manag* 18(3):414–428
- TCCFD (2017) Recommendations of the task force on climate-related financial disclosures
- Techer K (2022) Cost allocation of hazardous wastes transportation. Tech. rep., Working Paper
- Timmer MP, Dietzenbacher E, Los B et al (2015) An illustrated user guide to the world input-output database: the case of global automotive production. *Rev Int Econ* 23(3):575–605
- van den Brink R, He S, Huang JP (2018) Polluted river problems and games with a permission structure. *Games Econ Behav* 108:182–205
- WRI, WBCSD (2011) GHG Protocol Corporate Accounting and Reporting Standard, revised edition. World Resources Institute and World Business Council for Sustainable Development
- Yamano N, Guilhoto J (2020) CO2 emissions embodied in international trade and domestic final demand: Methodology and results using the OECD Inter-Country Input-Output Database. Documents de travail de l'OCDE sur la science, la technologie et l'industrie 2020/11

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