

Edenhofer, Ottmar; Franks, Max

Working Paper

Carbon, Natural Capital and the Option Values of Climate Policies

CESifo Working Paper, No. 12426

Provided in Cooperation with:

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Edenhofer, Ottmar; Franks, Max (2026) : Carbon, Natural Capital and the Option Values of Climate Policies, CESifo Working Paper, No. 12426, Munich Society for the Promotion of Economic Research - CESifo GmbH, Munich

This Version is available at:

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CES ifo

**12426
2026**

January 2026

Working Papers

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CES ifo

Imprint:

CESifo Working Papers

ISSN 2364-1428 (digital)

Publisher and distributor: Munich Society for the Promotion
of Economic Research - CESifo GmbH

Poschingerstr. 5, 81679 Munich, Germany
Telephone +49 (0)89 2180-2740

Email office@cesifo.de
<https://www.cesifo.org>

Editor: Clemens Fuest

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Carbon, Natural Capital and the Option Values of Climate Policies *

Ottmar Edenhofer[†] Max Franks[‡]

January 26, 2026

Abstract

We develop a unified cost-benefit framework that allows for a better understanding of nature conservation and climate policies under risk and uncertainty. We derive modified Hotelling rules from a social planner's welfare optimization. They reveal four forces that jointly determine market design for climate and nature conservation: First, discounted marginal climate damages enter the social cost of carbon (SCC) and marginal ecosystem services the social value of nature (SVN). Second, climate and nature are coupled, which raises both prices: degradation of ecosystems increases the SCC, while climate damages raise the SVN. Third, a climate-nature beta quantifies additional hedging components of policies against fat tails, when we consider a stochastic setting with exogenous random shocks. The climate-nature beta summarizes the option values for abatement, adaptation, ecosystem restoration and carbon dioxide removal. Fourth, Markov markups quantify tipping risks, which we capture by extending the model to a constrained Markov decision process with state-contingent transition probabilities. Thereby, we endogenize tipping points: the likelihood of moving into a high-damage regime becomes a function of the atmospheric carbon stock and natural capital, which depend on policy choices. Thus, hazard risks are a policy-sensitive component of the system's dynamics. The model yields state-contingent asset-pricing formulas for carbon prices, restoration subsidies, land charges, and capacity payments. We propose institutions at the level of the European Union that could implement Pigouvian taxes and subsidies as well as new types of SCC- and SVN-indexed bonds to share non-diversifiable risks arising from Earth's changing climate and the degradation of its biosphere.

Keywords CDR · Natural Capital · Biodiversity · Sustainability · Asset pricing · Welfare economics · Tipping points · Option Values

JEL Classification Q51 · Q54 · Q57 · D81 · G12

Declarations of interest: None

*We thank Moritz Drupp, Jacob Edenhofer, Sabine Fuss, Matthias Kalkuhl, Charlotte Plinke, Michael Sureth and Ibrahim Tahri for their valuable and insightful comments. Moreover, we have benefitted from discussions with further colleagues participating in PIK's RD5 PCF Seminar. Ottmar Edenhofer gratefully acknowledges the generous financial support by the Werner Siemens Foundation.

[†]Potsdam Institute for Climate Impact Research, Technische Universität Berlin, Centre for Economic Policy Research (CEPR), CESifo Research Network. Email: edenhofer@pik-potsdam.de

[‡]Potsdam Institute for Climate Impact Research. Email: franks@pik-potsdam.de

1. Introduction

Climate change and the large-scale loss of biodiversity are deeply intertwined systemic risks. Both threaten to trigger catastrophic regime shifts, and both shape the long-run development of the global economy. A coherent risk-management perspective therefore requires a framework for jointly analyzing climate and natural capital dynamics, as well as the policies that influence them. Yet the existing literature still treats these risks largely in isolation and offers no consistent rules for the timing and scale of climate and nature conservation policies.

Building on this perspective, we develop a unified framework in which climate and natural capital are treated as coupled assets. Their shadow prices jointly determine efficient policy and reveal the misallocation of resources across climate and nature interventions aimed at protecting ecosystems, biodiversity and the biosphere. Extending the deterministic core, we formulate a stochastic version of the model that allows us to compute the option values of the central climate and nature conservation policy options — abatement, adaptation, carbon dioxide removal (CDR) and ecosystem restoration. The option values capture the exposure of irreversible capacity investments to payoff risk in an uncertain climate–nature system.

A key result is the derivation of a climate–nature beta, which summarizes the component of climate and biodiversity risk that cannot be diversified by markets and therefore commands a risk premium. The climate-nature beta links the marginal value of policy instruments to their performance in high-damage states and provides an asset-pricing interpretation of climate and nature conservation policies. We further extend the analysis by allowing for regime shifts in both the climate system and the biosphere. In this setting, hazard risks enter the asset-pricing formulas for carbon and nature through endogenous transition probabilities. Crucially, these hazards are not imposed exogenously but depend on policy choices via their effects on atmospheric carbon concentrations and natural capital.

The framework yields implementable policy instruments, including taxes, subsidies, land charges, and capacity payments, that transform climate and nature into risk-bearing assets with state-contingent returns. These results have far-reaching implications for institutional

design. In particular, they highlight the necessity of strong coordination mechanisms, as climate and nature values are jointly determined and cannot be managed efficiently in isolation. It provides a system-risk perspective and rules for how to manage potential catastrophic risks over time. Our contribution proceeds in several steps that build on each other.

Section 2 reviews the literature on integrated assessment frameworks of climate change economics, the social cost of carbon under uncertainty, and risk perspectives on biodiversity loss and nature's values. This review highlights an important gap: while the social cost of carbon (SCC) and the social value of nature (SVN) are the conceptually correct asset prices for guiding climate and nature conservation policy, the literature does not yet provide a coherent framework in which these prices are jointly determined and can guide the allocation across the full set of climate and nature conservation policy instruments.

Section 3 develops such a framework in a deterministic setting, linking a simple representation of the climate system and the biosphere in one dynamic model. Within this environment we derive the coupled SCC and SVN. The expressions we derive for the two prices show how climate and nature as assets interact through emissions, land use, and biodiversity dynamics, where we interpret biodiversity as a characteristic of natural capital, which influences its productivity. We also link land conversion and land use to climate change and natural capital and show how the social value of converted land (SVCL) must take the SVN into account. This provides, for the first time, a joint valuation structure for climate, land and natural capital that can serve as the foundation for climate and nature conservation policy analysis.

Building on the coupled deterministic model as foundation, Section 4 makes two extensions. First, we introduce stochasticity by adding random shocks to the climate system, the biosphere and total factor productivity (TFP). Second, we introduce irreversible investments in capacity stocks for abatement, carbon dioxide removal (CDR), restoration and adaptation. These investments generate option values, but in contrast to conventional real-options theory, they are option values for a portfolio of policy options, not for single assets. Deriving these option values allows us to price the investment risks associated with abatement, carbon dioxide removal, restoration and adaptation. Here, we still assume that policy

choices do not influence the probabilities associated with different climate-, biosphere- and TFP-outcomes.

Section 5 relaxes the assumption by introducing a constrained Markov decision process (CMDP) with climate and biosphere tipping points. In this setting, policy actions affect the transition probabilities between good and bad climate-nature regimes via their impact on atmospheric carbon concentrations and natural capital. The extension fundamentally changes the structure of the problem. The SCC and SVN become hazard-adjusted shadow prices. The CMDP therefore provides guidance for the timing of abatement, CDR, adaptation and restoration as well as their interactions. A key conceptual result emerges: climate and nature become risk-bearing assets. Their shadow prices encode not only scarcity and damages, but also how policy instruments shift the probabilities of systemic transitions. This introduces a new class of policy-relevant risk premia that are absent in the existing literature.

Section 6 synthesizes the results of the two preceding sections. Four components jointly determine the SCC and the SVN: climate damages and ecosystem services, the hedging channel reflects the risks which cannot be diversified; the hazard channel captures how policies affect the probability of tipping events; and the coupling channel describes how the climate state influences the value of nature and vice versa.

Section 7 turns to the institutional implications of the findings. We show how risk- and hazard-adjusted SCC and SVN paths can be implemented by specialized institutions. As an example, we discuss these institutions for the case of the European Union: a European Biodiversity Pricing Authority, which has the goal of internalizing land scarcity and biodiversity value and a Climate Central Bank, which can ensure time-consistent carbon pricing and credible CDR commitments. Together, the institutions form a coherent architecture for managing intertwined large-scale climate and nature risks.

2. Previous Literature

Earth system science has put forward the concept of planetary boundaries and the notion of a safe operating space for human activity (Rockström et al., 2009; Steffen et al., 2015).

One consequence of human economic activity is the emission of greenhouse gases, for which [Nordhaus \(1993, 2024\)](#) in his seminal contributions with the deterministic DICE model computes the intertemporal optimal emissions pathway: climate damages act like a flat tax on GDP, and the optimal carbon price equals the social cost of carbon (SCC). DICE, however, lacks an explicit biodiversity or natural capital lens. More recent work extends the DICE model to include non-market (environmental) goods, as for example in [Drupp and Hänsel \(2021\)](#). [Bastien-Olvera and Moore \(2021\)](#), in their GreenDICE model, include natural capital, which has several functions. It serves as production factor, as the source of non-use value and is affected by climate damages. However, they do not model its role as carbon sink. Compared to Nordhaus, or also to extensions of the DICE model, [Dasgupta \(2021\)](#) embeds the economy within ecological systems far more comprehensively. In his landmark work on biodiversity economics, he frames nature as an asset, emphasizing that its shadow price should evolve with the long-run risks to biodiversity.

Similarly, [Perrings and Kinzig \(2021\)](#) treat nature as an asset with a shadow price; they argue for intertemporal rules to guide conservation — conceptually aligned with our modified Hotelling rules. However, while they provide an analytical expression of the social value of natural capital, they stop short of modeling the two-way coupling between climate and nature within a single dynamic framework. They also do not provide policy guidance to manage large-scale system risks. Our approach fills these gaps by jointly modeling climate and biodiversity, deriving dual shadow prices and modified Hotelling rules, and making the climate–nature link operational for policy when the irreversible activation of tipping points is at risk.

[Martin and Pindyck \(2015\)](#), too, discuss multiple catastrophes. At first glance, their framework appears suited for analyzing the great twin crisis of climate and biodiversity. However, in their framework, catastrophic risks are effectively separable in the state dynamics and are coupled only through preferences (the utility aggregator). Therefore, interactions show up in a society’s willingness-to-pay (WTP) but the underlying hazards are not linked. By contrast, in our framework climate and biodiversity are dynamically coupled states. Because nature and climate mutually affect each other’s future paths and shadow prices (SCC and SVN), there is no policy that allows avoiding only one catastrophe in isolation. Any action

that stabilizes climate (abatement and CDR) also shifts biodiversity risk. Many ways of ecosystem restoration in turn can reduce climate risk. Hence optimal risk management is interdependent: prices, capacities and taxes must be designed for the coupled system, not for single hazards in isolation.

[Giglio et al. \(2025b\)](#) model how climate change and biodiversity loss interact and thus mutually amplify their harmful consequences. They derive a twin-crisis multiplier and show how it systematically affects optimal climate and nature conservation policies. Their two period model, however, omits carbon removal, restoration and adaptation, and remains deterministic, thereby precluding the possibility of analyzing the role of risk and fat tails. We take their model as starting point and extend it to stochastic settings with both exogenous shocks and endogenous tipping risk. The extensions allow us to introduce option values and a detailed discussion of policy instrument portfolios.

[Weitzman \(2007\)](#) convincingly argues that fat tails deserve extra weight and that precaution has real value. However, the dismal theorem does not translate into implementable risk management, because in the presence of sufficiently heavy tails it drives the social cost of carbon toward infinity. That knife-edge result relies on the very strong assumption of infinite damages once consumption falls below a threshold. Thus, the theorem is more a cautionary bound than a policy formula. Without additional structure, such as bounded marginal utility of consumption, it cannot tell us how to set prices or build capacities today (see [Pindyck, 2011](#)). We share Weitzman’s emphasis on risk. In contrast, we go beyond diagnosis by computing the option values of abatement, CDR, restoration and adaptation, enabling operational risk-management plans — including pricing rules and capacity payments.

In the [Dixit and Pindyck \(1994\)](#) tradition, real options measure the value of delaying an irreversible project (“invest now or wait”). We depart from their approach. In our setting, the decision problem is not about the timing of a single project, but about the optimal portfolio and scale of different capacities. We therefore use “option value” to denote the risk-adjusted capacity value of these instruments across future states, rather than the value of waiting per se. Uncertainty raises the value of flexibility: greater volatility, fatter tails, or more persistent shocks increase the benefit of holding dispatchable capacity. Capacity constraints matter, making the timing and scale of investment central. Another central

difference from classic real-options work is scope: we do not price one project's option to invest; we jointly value the entire menu of policies, that is, abatement, adaptation, restoration and multiple CDR technologies, while the underlying prices (SCC and SVN) are endogenous and co-evolve through coupled climate–nature dynamics.

2.1. Climate Change, uncertainty and risk

Others have also used various forms of stochastic analyses to enhance the economics of climate change. There are notable contributions introducing uncertainty and tipping risk to the economics of climate change. [Zhao et al. \(2023\)](#) compute the SCC when the social planner has full-path risk-sensitive (robust) preferences rather than the usual per-stage recursive setup. The method of full-path accumulated robustness based on the theory of stochastic control addresses potential model misspecifications. [Lemoine and Traeger \(2014\)](#) analyze the optimal carbon price considering two types of tipping points that endogenously depend on the atmospheric carbon concentration. Including tipping points increases the SCC. Learning about safe zones in the state space reduces the SCC to a certain extent. [van der Ploeg and de Zeeuw \(2018, 2019\)](#) show that tipping risk changes both optimal carbon pricing and optimal long-run saving. Both depend crucially on how fast a catastrophe unfolds. In their model, the hazard rate is endogenous and depends on the atmospheric carbon stock. [Cai and Lontzek \(2019\)](#) deliver the leading stochastic SCC benchmark: SCC is a random, heavy-tailed process shaped by long-run economic risk and tipping hazards, with lower discounting for catastrophic, persistent damages. [Fillon et al. \(2023\)](#) consider the social planner's aversion to futures where many generations are jointly worse off after a climate tipping point. They show that tipping points make temporal risk aversion normatively relevant. They show that this increases the SCC because society pays to lower both current and future tipping probabilities.

However, these contributions do not include the biodiversity channel and focus mostly on a single-instrument, the carbon tax. Our framework operationalizes the risks that [Lemoine and Traeger \(2014\)](#); [Cai and Lontzek \(2019\)](#); [van der Ploeg and de Zeeuw \(2019\)](#) describe. We add the possibility of investing in different types of capital and show how this requires portfolio decisions under risk and uncertainty leading to different policy strategies. In

particular, we explicitly model biosphere–climate coupling and option values for abatement, CDR, restoration and adaptation. We characterize the optimal policy instrument portfolio with a set of pricing policies and suggest indexed bonds as instruments to share the risks posed by climate change and biosphere degradation. This full market design yields both better pricing of risk and implementable contracts to build and finance the capacities that matter most in tail states.

2.2. Biosphere and nature’s values

To address the current limits to policy guidance with respect to the economic value of the biosphere, [Drupp et al. \(2024\)](#) emphasize the increasing relative scarcity of nature. Their approach is based on consumer preferences and the willingness to pay for ecosystem services. This makes it possible to derive substitution elasticities, for example, from contingent valuation studies (see also [Drupp et al. 2025](#)). Our approach reflects changes in relative scarcity by modeling nature as a factor of production, following [Gollier \(2019\)](#), who shows that uncertainty surrounding the substitutability of natural capital always increases its value. Ongoing work by [Flammer et al. \(2025\)](#) engages with green finance for biodiversity and Knightian uncertainty, contrasting this with standard risk.

[Cantillon et al. \(2025\)](#) propose a new market structure for nature-based solutions. Their proposal would enable buyers who want to green their economic activities to buy shares in projects that yield nature benefits. Instead of purchasing carbon (or nature) credits that are based on a flow, for example emissions reductions, here, buyers would obtain part of a stock that yields a return in the form of a green dividend. Our model takes up the idea that there is a stock. But instead of thinking of several heterogeneous projects and hence several heterogeneous stocks, we take a bird’s eye perspective and rely on one aggregate stock of natural capital. Still, the stock of natural capital yields returns or perhaps dividends. First, it is a factor of production due to ecosystem services. Second, it provides climate adaptation. It reduces the severity of climate damages, for example when mangroves provide protection from floods. Third, the biosphere provides services that mitigate dangerous climate change. It is a carbon sink and thus removes carbon from the atmosphere.

Giglio et al. (2025a) use the simple model put forward in Giglio et al. (2025b) to propose a research agenda for how financial economics should incorporate the risks due to biosphere degradation. They emphasize the need for empirical research to accommodate the heterogeneity in the value of natural resources and the heterogeneity in the ways in which nature and financial markets interact. In terms of empirical research, a recent estimate by Bastien-Olvera et al. (2024) quantifies climate impacts on natural capital. They find that under the Shared Socioeconomic Pathway SSP2-6.0, the value of ecosystem services will be reduced by 9.2% in 2100 and global GDP by 1.3%. Moreover, ongoing research by Colesanti Senni et al. (2025) empirically investigates to what extent, and through which channels financial markets already price nature and climate risks. In this paper, we follow Giglio et al.’s call in the sense that we contribute to the development of a new economic theory of the biosphere in terms of asset pricing and corporate finance research.

3. Coupled climate-biosphere system

We now describe a deterministic model that allows us to assign economic value to the scarcities imposed by the earth’s climate system and its biosphere. Assigning such values is necessary for the sustainable management of stocks, or in other words, of assets. An overview over the economic mechanics is presented in Figure 1. All variables are listed at the beginning of Appendix A.

3.1. Model description

To capture how earth’s climate evolves over time, we introduce the stock of atmospheric carbon Z_t . We measure the quality of earth’s biosphere in terms of a stock E_t that we call natural capital (Gollier, 2019; Dasgupta, 2021). We model here the aggregate services of the biosphere. Giglio et al. (2024) assume that these aggregate services are produced by combining several non-substitutable ecosystem functions such as pollination and water filtration, which are each provided by many substitutable species playing similar roles. For the purpose of our model we do not model the link between aggregate ecosystem services and biodiversity explicitly. To interpret our model, we follow Dasgupta (2021) and under-

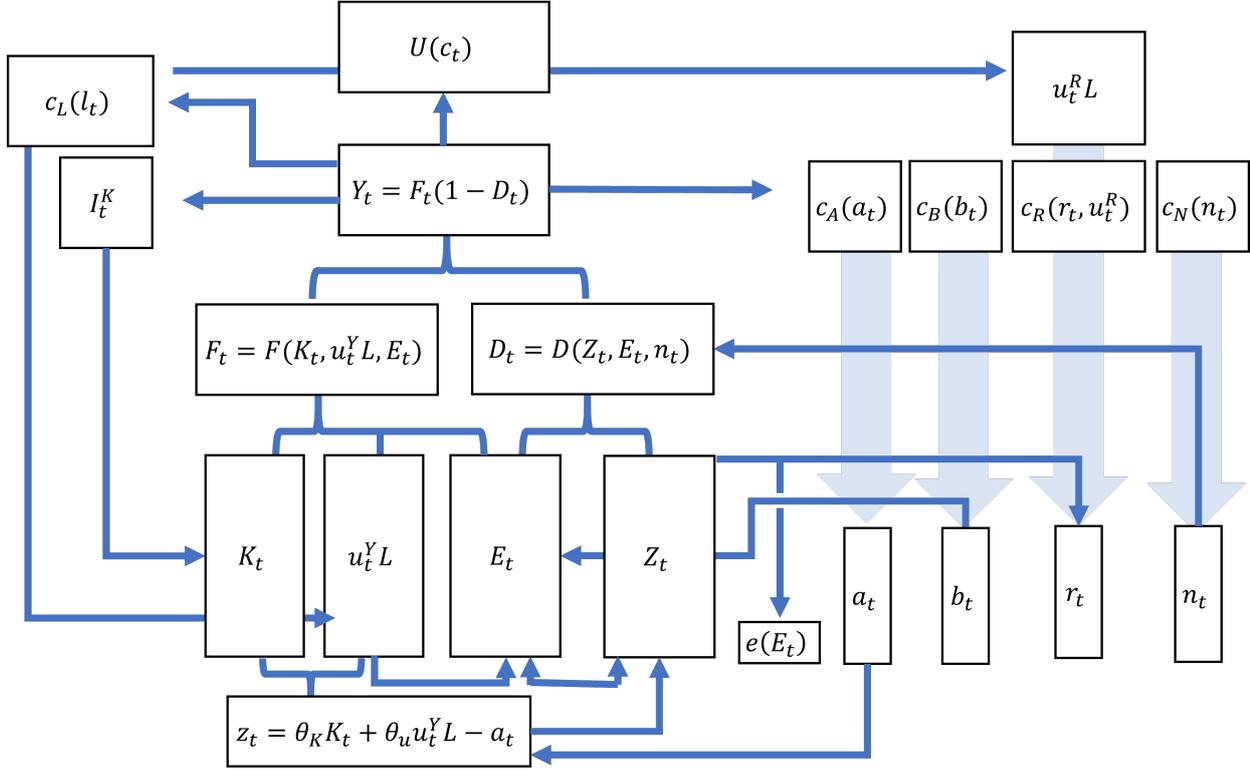


Figure 1: Overview of the deterministic coupled climate–biosphere–economy model. Economic output Y_t is produced using human capital K_t (with investments I_t^K), land $u_t^Y L$, and natural capital E_t , and is reduced by climate damages $D(Z_t, E_t, n_t)$. Emissions $z_t > 0$ increase atmospheric carbon Z_t . Abatement a_t reduces emissions. Carbon dioxide removal r_t reduces atmospheric carbon. Adaptation n reduces climate damages and biosphere restoration b increases natural capital. All four policy options have costs c_X , $X = A, B, R, N$. Removal costs can be reduced by increasing land in this sector u_t^R . Converting land for human use has costs $c_L(l_t)$. Depending on the state of natural capital, the biosphere absorbs atmospheric carbon $e(E_t)$.

stand biodiversity as a characteristic of natural capital, which influences the productivity of natural capital and, more concretely, ecosystems.¹ Consistent with Giglio et al.’s multi-level approach we use a macro-economic production function: Together with human produced capital K_t and the fraction of total land L that is used for production $u_t^Y L$, the levels of these stocks determine how much economic output F_t is produced each period. Land L is assumed to be fixed. One fraction is natural land while the rest is used by humans — either for the production of a generic consumption good c_t or for land-intensive CDR methods (u_t^R). Each period, l_t denotes how much natural land is converted for human use. We assume convex land conversion costs $c_{LC}(l_t)$ where $l_t = u_{t+1} - u_t$ and all changes in the share of converted land u_t , both positive and negative, are associated with positive costs. The stocks evolve as determined by the following equations of motion.

$$E_{t+1} = (1 + \delta_E)E_t - \gamma(E_t, Z_t) - \gamma^u(u_{t+1} - u_t) + b_t \quad (1)$$

$$K_{t+1} = (1 - \delta_K)K_t + I_t^K \quad (2)$$

$$u_{t+1} = u_t + l_t \quad (3)$$

$$Z_{t+1} = Z_t + z_t(K_t, u_t^Y, a_t, l_t) - r_t - e(E_t) \quad (4)$$

By investing I_t^K , the human produced capital stock increases. It depreciates over time at the rate $\delta_K > 0$. The atmosphere and the biosphere co-evolve. Atmospheric carbon increases with current emissions due to capital use in production and land-use and land-conversion emissions $z_t(K_t, u_t^Y, l_t, a_t) = \theta_K K_t + \theta_u u_t^Y L + \theta_{LC}(l_t) - a_t \geq 0$, where $\theta_K, \theta_u > 0$ and $\theta_{LC}(l_t) \geq 0$ is a smooth function with $\theta'_{LC} \geq 0$ and $\theta_{LC}(l_t) = 0$ for all $l_t \leq 0$. Abatement a_t can reduce the flow of emissions and comes at convex costs $c_A(a_t)$. Carbon dioxide removal (CDR) r_t can reduce the carbon stock. The biosphere absorbs atmospheric carbon based on a non-linear function $e(E_t)$, which is assumed to capture saturation effects for higher levels of E_t . We assume very low costs for nature-based CDR options like afforestation or soil carbon management such that they, too, are captured by the biosphere’s stock dependent carbon absorption. Natural capital regenerates by itself at the constant rate $\delta_E > 0$. Humans can

¹Dasgupta (2021) further states that "[biodiversity] is an enabling asset. Which is why environmental and resource economists estimate the accounting prices of items of natural capital – for example ecosystems – not biodiversity. The value of biodiversity is embedded in the accounting prices of natural capital."

restore natural capital by efforts to restore parts of the biosphere b_t , which have convex costs $c_B(b_t)$. Natural capital is affected by climate damages through the impact function $\gamma(E_t, Z_t)$ where $\frac{\partial \gamma}{\partial E_t} = \gamma_{E,t} \leq 0$ and $\frac{\partial \gamma}{\partial Z_t} = \gamma_{Z,t} \geq 0$.² We, thus, assume that positive effects of global warming on natural capital, such as CO₂-fertilization, are overcompensated by adverse impacts. Natural capital also degrades when land is converted, which we capture in the damage term $\gamma_u(u_{t+1} - u_t)$, where $\gamma_u > 0$ is a constant and $u_t = u_t^Y + u_t^R \leq 1$ is the fraction of total land used by humans.

An interaction between atmosphere and biosphere is worth emphasizing here: Loss of natural land reduces natural capital, which in turn lowers the biosphere’s ability to remove and store carbon from the atmosphere. When humans convert land from its natural state, it is used either for generating output or to reduce the costs of technical carbon dioxide removal $c_R(r_t, u_t^R)$ by adding land-based technologies such as bioenergy with carbon capture and storage (BECCS) and enhanced weathering to the CDR portfolio and shifting the convex cost curve downward. Our model thus captures a trade-off that comes with the use of biomass for climate change mitigation. Different land-intensive CDR technologies have different impacts on planetary boundaries. We represent these differences by modeling two channels through which carbon can be removed from the stock of atmospheric carbon in a stylized way. First, we have the ”technical” channel, through which land is converted and thus reduces natural capital. Second, we have the ”natural” channel, which enhances natural capital.

Aggregate economic output net of climate damages D is

$$Y_t = F(K_t, u_t^Y L, E_t) [1 - D(Z_t, E_t, n_t)] \quad (5)$$

The production function can represent “weak” or “strong” sustainability depending on how easily ecosystem services can be substituted by other production factors. This is captured by the assumed elasticity of substitution between ecosystem services and the remaining inputs. Damages are similar to [Giglio et al. \(2025b\)](#), that is, they are convex in the carbon

²By modeling only climate damages to the biosphere, we depart from [Dasgupta \(2021\)](#), where the sources of biosphere degradation are modeled in a more general way, that is, like extraction or harvest of a natural resource. Our framework can easily be extended to include an additional source of non-greenhouse gas pollution as argument in the climate damage function D and the biosphere impacts γ .

stock $\partial D/\partial Z, \partial^2 D/\partial Z^2 > 0$ and also depend on the state of the biosphere. We also include human adaptation efforts n_t such that $\partial D/\partial E, \partial D/\partial n \leq 0$. Allowing equality in the later formulation provides the flexibility to include thresholds at which adaptation can no longer prevent losses. We define $Y_{K,t} = \partial Y_t/\partial K_t$, $Y_{u,t} = \partial Y_t/\partial u_t^Y$, $Y_{E,t} = \partial Y_t/\partial E_t$ and $Y_{Z,t} = \partial Y_t/\partial Z_t$. The budget equation is

$$Y_t = c_t + I_t^K + c_A(a_t) + c_B(b_t) + c_{LC}(l_t) + c_N(n_t) + c_R(r_t, u_t^R) \quad (6)$$

In particular, in the budget, we see four different types of policy options that a social planner can use to mitigate dangerous climate change and biosphere degradation: emissions abatement, carbon dioxide removal, adaptation to climate impacts and the restoration of damaged ecosystems.

3.2. Key mechanisms

To summarize, our model extends the four key mechanisms of climate-nature-economy interactions identified by Giglio et al. (2025b) in several important ways, which we describe in the following.

M1: Nature as a net carbon sink. In our model, natural carbon uptake is endogenously determined by the function $e(E)$, which allows for saturation and collapse effects.

M2: Ecosystem services are a clean factor of production. This is analogous to Giglio et al. (2025b).

M3: Ecosystems provide climate adaptation since $\frac{\partial D}{\partial E} \leq 0$. This is analogous to Giglio et al. (2025b).

M4: Climate-nature feedback. In contrast to the twin-crisis-multiplier as in Giglio et al. (2025b), our key metric that measures climate-nature coupling are the modified Hotelling rules that jointly determine the SCC and SVN.

M5: Land as factor of production causes two externalities. First, land use and

land conversion generate greenhouse gas emissions. Second, land conversion and use degrade natural capital. This interaction is captured in a no-arbitrage condition that involves the social value of nature (SVN) and the social value of converted land (SVCL).

M6: Land can also be used for technical CDR. Carbon dioxide removal has fundamentally different characteristics than emissions abatement. We spell out a detailed representation of this important policy option and endogenize CDR's dependency on land.

3.3. Asset prices and Hotelling rules

The social planner maximizes discounted utility $\sum_t \beta^t U(c_t)$ of a representative household, where U is concave, c_t is consumption and $\beta \in (0, 1)$ is the discount factor, subject to the monetary budget, the physical land budget and the equations of motion of the stocks. The Lagrangian and the first order conditions are listed in Appendix A. We will use the following shadow prices.

$$\frac{\partial U_t}{\partial c_t} = \mu_t \quad (7)$$

$$-\frac{\lambda_t^Z}{\mu_t} =: SCC_t \quad (8)$$

$$\frac{\lambda_t^E}{\mu_t} =: SVN_t \quad (9)$$

$$\frac{\lambda_t^L}{\mu_t} =: SVCL_t \quad (10)$$

The social cost of carbon SCC_t is given as the shadow price associated with the equation of motion of the carbon stock (4) normalized by the marginal utility of consumption. Similarly, we define the social value of nature SVN_t as the normalized shadow price of natural capital. Note the difference in sign due to carbon being unambiguously harmful and the biosphere being beneficial from the social planner's view. We further define the social value of converted land $SVCL_t$ as the normalized shadow price of the stock of converted land.

Optimal land use is determined by the following conditions.

$$SCC_t \theta'_{LC} + c'_L(l_t) = \lambda_t^L / \mu_t = SVCL_t \quad (11)$$

$$-\frac{\partial c_R(r_t, u_t^R)}{\partial u_t^R} = \frac{\partial Y_t}{\partial u_t^Y} \quad (12)$$

First, marginal conversion cost and the SCC should be equal to the social value of converted land. Second, marginal cost savings from using land-based CDR options should equal the marginal product of land in final goods production.

The social planner's optimization problem yields three modified Hotelling rules

$$SCC_t / M_{t,t+1} = -Y_{Z,t+1} + SCC_{t+1} + \gamma_{Z,t+1} SVN_{t+1} \quad (13)$$

$$SVN_t / M_{t,t+1} = Y_{E,t+1} + e' SCC_{t+1} + (1 + \delta_E - \gamma_{E,t+1}) SVN_{t+1} \quad (14)$$

$$SVCL_t / M_{t,t+1} = Y_{u,t+1} + SVCL_{t+1} + \gamma_u (SVN_{t+1} - SVN_t / M_{t,t+1}) \quad (15)$$

where we have used the definition $M_{t,t+1} := \beta \mu_{t+1} / \mu_t$ to rewrite the equations of motion of the shadow prices (A.11) to (A.13), which we derived in the appendix.

Equations (13) to (15) define a portfolio of assets: the equilibrium returns of the climate system, natural capital and man-made assets. Like in the conventional Hotelling-rule, the growth-rate of SCC, SVN and SVCL determine the rate of return. In contrast to the conventional Hotelling-rule, the asset returns to atmospheric carbon and natural capital are simultaneously determined by the SCC and the SVN. The social value of converted land, by contrast, is closely tied to the social value of nature. Therefore, it is important to note that Equations (13) and (14) are fundamental to our dynamic system, while, via Equation (15), the SVCL is determined by the SVN only and not the SCC. Note that SVN and SVCL will typically move in opposing directions. For the SVN to grow, the SVCL has to fall. If the SVN falls faster than some threshold value that depends on the marginal product of land, then the SVCL has to grow. This highlights the trade-off involved in decisions about land conversion.

Reformulating the equations of motion of the asset prices further, we obtain

$$\frac{1}{M_{t,t+1}} = \underbrace{\frac{SCC_{t+1}}{SCC_t}}_{\text{growth factor}} - \underbrace{\frac{Y_{Z,t+1} - SVN_{t+1}\gamma_{Z,t+1}}{SCC_t}}_{\text{climate rent}} \quad (16)$$

$$\frac{1}{M_{t,t+1}} = (1 + \delta_E - \gamma_{E,t+1}) \underbrace{\frac{SVN_{t+1}}{SVN_t}}_{\text{growth factor}} + \underbrace{\frac{Y_{E,t+1} + SCC_{t+1}e'}{SVN_t}}_{\text{nature rent}} \quad (17)$$

$$\underbrace{\frac{1}{M_{t,t+1}}}_{\text{risk-free interest rate}} = \underbrace{\frac{SVCL_{t+1}}{SVCL_t}}_{\text{growth factor}} + \underbrace{\frac{Y_{u,t+1} + \gamma_u(SVN_{t+1} - SVN_t/M_{t,t+1})}{SVCL_t}}_{\text{social land rent}} \quad (18)$$

In equation (16), the growth of the optimal carbon price is pinned down by three components. First, there is the benchmark return, the risk-free rate, that is, what the SCC must earn overall to be consistent with optimal intertemporal allocation. Second, the “capital gain” term captures that if future climate damages are expected to be higher, the SCC must rise faster to reflect that worsening future impact. Third, there is a negative dividend term. Current carbon generates contemporaneous climate damages and erodes natural capital. Holding carbon today, thus, pays a negative dividend. If current damages are large, less future price growth is required; if current damages are small but future damages loom, the SCC must grow more rapidly. The coupling term captures that carbon also reduces future ecosystem services, so carbon is costly not only because of direct climate damages but also because it undermines nature’s buffering role.

The growth of the social value of nature described by Equation (17) follows the same logic. It must cover the carrying costs (ecological depreciation, climate stress) plus pay a dividend equal to ecosystem services and climate buffering such that the total return equals the risk-free rate. If nature’s services are large, the required SVN growth falls; if ecosystems depreciate fast or climate stress is high, the required growth rises.

Equation (18) is the no-arbitrage condition for a land market with an environmental externality. The interest rate on the left hand side is equal to the growth factor of the land price, that is, the SVCL and the land rent, which in this case includes not only the marginal product of land, but also the impact on natural capital via the change in the SVN. Hence,

Equation (18) is also the rule how to optimally balance converted and natural land. The SCC enter only indirectly via their coupling with the SVN.

Finally, from the recursive equations for the SCC, the SVN and the SVCL, it becomes clear that one can formulate the prices as discounted sums of marginal climate damages $Y_{Z,t} < 0$, marginal ecosystem services $Y_{E,t} > 0$ and the marginal product of converted land — weighted according to their impact as captured in the functions $\phi^{X,E}$, $\phi^{X,Z}$ and $\phi^{X,u}$, where $X \in \{SCC, SVN, SVCL\}$. The expressions have the form

$$SCC_t = \sum_{k=1}^{\infty} M_{t,t+k} \left[-Y_{Z,t+k} \phi_{t+k}^{SCC,Z} (\{E_s, Z_s\}_{s=1,\dots,t+k}) \right. \\ \left. + Y_{E_{t+k}} \phi_{t+k}^{SCC,E} (\{E_s, Z_s\}_{s=1,\dots,t+k}) \right] \quad (19)$$

$$SVN_t = \sum_{k=1}^{\infty} M_{t,t+k} \left[-Y_{Z,t+k} \phi_{t+k}^{SVN,Z} (\{E_s, Z_s\}_{s=1,\dots,t+k}) \right. \\ \left. + Y_{E_{t+k}} \phi_{t+k}^{SVN,E} (\{E_s, Z_s\}_{s=1,\dots,t+k}) \right] \quad (20)$$

$$SVCL_t = \sum_{k=1}^{\infty} M_{t,t+k} \left[Y_{u,t+k} \phi_{t+k}^{SVCL,u} (\{E_s, u_s\}_{s=1,\dots,t+k}) \right. \\ \left. + Y_{E_{t+k}} \phi_{t+k}^{SVCL,E} (\{E_s, u_s\}_{s=1,\dots,t+k}) \right] \quad (21)$$

where $M_{t,t+k} = \prod_{s=t}^{t+k} M_{s,s+1} = \beta^k \mu_{t+k} / \mu_t$ and the functions $\phi^{X,Z}, \phi^{X,E}, \phi^{X,u}$, where $X \in \{SCC, SVN, SVCL\}$ are combinations of the functions $\gamma(E_t, Z_t)$ and $e(E_t)$ and the parameter δ_E .³

Natural capital, converted land and climate are, thus, interlinked assets. Their required price growth is determined by a risk-free benchmark (the return of human-made assets), current rents, and adjusted for payouts (carbon), carrying costs (nature) and land degradation. This gives a clear, operational rule for how fast optimal carbon price, land rent and social value of nature should grow and which levers move them. The three equations clearly indicate that the current practice to calculate the SCC, SVN and SVCL (if calculated at all) is deeply flawed because it ignores buffer service of nature destroyed by climate change and

³We do not further specify these functions because they would add complexity but yield no further insights for the purpose of this paper.

land degradation; it also neglects that the SVN has to grow faster when climate damages spike.

Equations (19) to (21) show that the prices are present values of future flows: marginal climate damages, the marginal product of converted land and marginal ecosystem services. They capture propagation and coupling and show how an extra unit of carbon, land and natural capital shifts future damages and services, and how persistent those shifts are. The SCC internalizes biosphere degradation because emissions are more expensive not only for climate damage, but also for the future lost ecosystem services they trigger. The SVN internalizes climate damages: conserving and restoring nature is more valuable because it reduces future climate damages and delivers services. The SVCL internalizes the economic benefits of using land and the costs in terms of biosphere degradation.

Additionally, persistence and substitutability matter: A more durable ecosystem, captured by a higher regeneration rate δ_E , makes nature's benefits and buffering last. Thus, an increase in the regeneration rate allows the social planner to have both a lower SCC and more carbon emissions. Moreover, it leads to lower growth rates of the SVN and consumption (see Equations 14 and 17). A lower elasticity of substitution between natural capital and produced capital implies a steeper welfare loss from reductions in natural capital (see Equation A.7 and A.12), resulting in a higher SVN and, via nature-climate linkages, a higher SCC. In other words, the concept of strong sustainability increases the value of both assets - nature and climate.

3.4. Policy instruments

The discussion of the social planner's optimal management of assets points at a set of externalities that private agents cannot internalize. It is worth noting that we assume complete markets and that all households face the same discount factor. Later, in the stochastic model, we also assume a single stochastic discount factor. Under these assumptions, a representative household can be used to represent the social planner's problem. When markets become incomplete and not all risks can be traded, the (stochastic) discount factor becomes household-specific. Without targeted transfers, markets will no longer deliver a Pareto-optimal allocation. For the purpose of this paper, we refrain from introducing this

complication, as our aim is to formulate a benchmark for the subsequent analysis. In order to achieve the first best outcome, a regulator can implement price signals for each policy lever. Then, the SCC and the SVN are the key metrics for the optimal policy instrument portfolio.

The following taxes and subsidies implement the first-best allocation. The derivations can be found in Appendix A.3. First, the optimal carbon tax is determined by the SCC, which also determines the optimal marginal abatement costs and thereby the abatement effort.

$$\tau_t^Z = -\lambda_t^Z / \mu_t^H = SCC_t = c'_A(a_t) \quad (22)$$

Second, land conversion generates emissions in proportion to the function $\theta_{LC}(l_t)$ and must also be taxed according to the social cost of carbon and under evaluation of the emissions function at the optimal level of land conversion l_t^* .

$$\tau_t^{LC} = SCC_t \theta'_{LC}(l_t^*) \quad (23)$$

Third, the land-use tax τ^L corrects firms' decisions by making them face the net social user cost of land. Using land in production affects the change in the value of converted land and in the value of natural land — typically in opposing directions, as discussed above. If land use reduces their total value, the net user cost is positive. The optimal tax equals the sum of the user cost of converted land and the user cost of natural land, net of emissions damages already internalized through the carbon tax, and net of the private land rent q_t . This ensures that firms equate the marginal product of land to its true social opportunity

cost.⁴

$$\tau_t^L = \frac{1}{L} \left[\underbrace{\frac{SVCL_{t-1}}{M_{t-1,t}} - SVCL_t}_{\text{user cost of converted land}} + \gamma_u \underbrace{\left(\frac{SVN_{t-1}}{M_{t-1,t}} - SVN_t \right)}_{\text{user cost of natural land}} \right] - \tau_t^Z \theta_u - q_t \quad (24)$$

Fourth, ecosystem restoration should be subsidized at the rate $s_t^E = SVN_t$, reflecting the social value of nature.

Fifth, firms that produce technical carbon dioxide removal should be subsidized for each ton of CO₂ at the SCC, that is $s_t^r = SCC_t$. However, if they use land to bring down their costs, then they must also pay the land tax $\tau_t^{LR} = \tau^L - \tau^Z \theta_u$ ensuring a correction for any land degradation due to land based CDR methods — only without the carbon tax component.

Sixth, adaptation comprises a broad spectrum of private, club, and public good characteristics. For the purpose of the present paper, we abstract from this complexity. Accordingly, we assume that the government ensures that the efficient level of adaptation is implemented in line with equation (A.5), which expresses the requirement that the marginal costs of adaptation are equal to the current marginal damages.

4. Option Values and Uncertainty

Linking the economics of climate change and nature conservation improves the SCC and SVN metrics conceptually. However, the deterministic nature of the preceding analysis masks an important source of value of climate and nature conservation policies. The twin crises of climate change and biodiversity loss involve substantial risks and uncertainties. Hence, abatement, removal, adaptation and nature restoration are not only valuable for their immediate effect on the economy. Investing in the capacity to abate, remove, restore and adapt adds extra value due to the insurance against unlikely but catastrophic outcomes in terms of climate damages or biosphere degradation. Investments in capacity, hence, have

⁴Absent environmental externalities and associated taxes, firms would retain the entire land rent. In the present model, firms belong to the households and no further efficiency losses or distributional issues arise. However, disaggregation into several households and different generations would have strong implications for growth, the distribution of wealth and social welfare (Stiglitz, 2016; Franks et al., 2018; Franks and Edenhofer, 2023).

an option value (Weisbrod, 1964). Option values in our sense differ from real options analysis (Dixit and Pindyck, 1994), which measures the value of delaying an irreversible project (“invest now or wait”). In our setting, the decision problem is not about the timing of a single project, but about the optimal portfolio and scale of different capacities. Our concept of option value describes the risk-adjusted capacity value of abatement, removal, adaptation and restoration across future states, rather than the value of waiting per se. Uncertainty raises the value of flexibility: greater volatility, fatter tails, or more persistent shocks increase the benefit of holding dispatchable capacity.

Carbon removal, for example, is an interesting case. There is large uncertainty about the needed quantity of the annual removal stream in the future. Smith et al. (2024) report a range from -2 to -11 GtCO₂ for the year 2040 and up to -15GtCO₂ in 2050. To be able to reduce this much CO₂ from the atmosphere each year by mid-century requires large and costly investments in CDR capacity in the near term. Investing, thus, provides strategic flexibility to respond to overshoot or underperformance in abatement and therefore reduces future regret or lock-in risks.

The deterministic model we described in the preceding section is a useful benchmark but misses critical aspects of climate and nature conservation policy. It cannot capture extreme outcomes, the irreversibility of capacity investments, or the hedging value of flexible responses under uncertainty. That is why we extend the deterministic model in the following two ways. First, we add random shocks to productivity growth (TFP), to the climate system and to the biosphere dynamics to account for risk and uncertainty. Second, we assume that abatement, removal, restoration and adaptation require the costly accumulation of capacity stocks. We assume for these sectors a higher degree of inertia and irreversibility compared to the rest of the economy. Figure 2 gives an overview. The full set of mathematical formulas describing the stochastic model is in Appendix B. Here, we limit our attention to the most important model aspects.

4.1. Model description

We introduce stochastic processes \mathcal{E}_t^Z and \mathcal{E}_t^E , which represent random shocks to the equations of motion of the stocks of atmospheric carbon and natural capital, and a stochastic

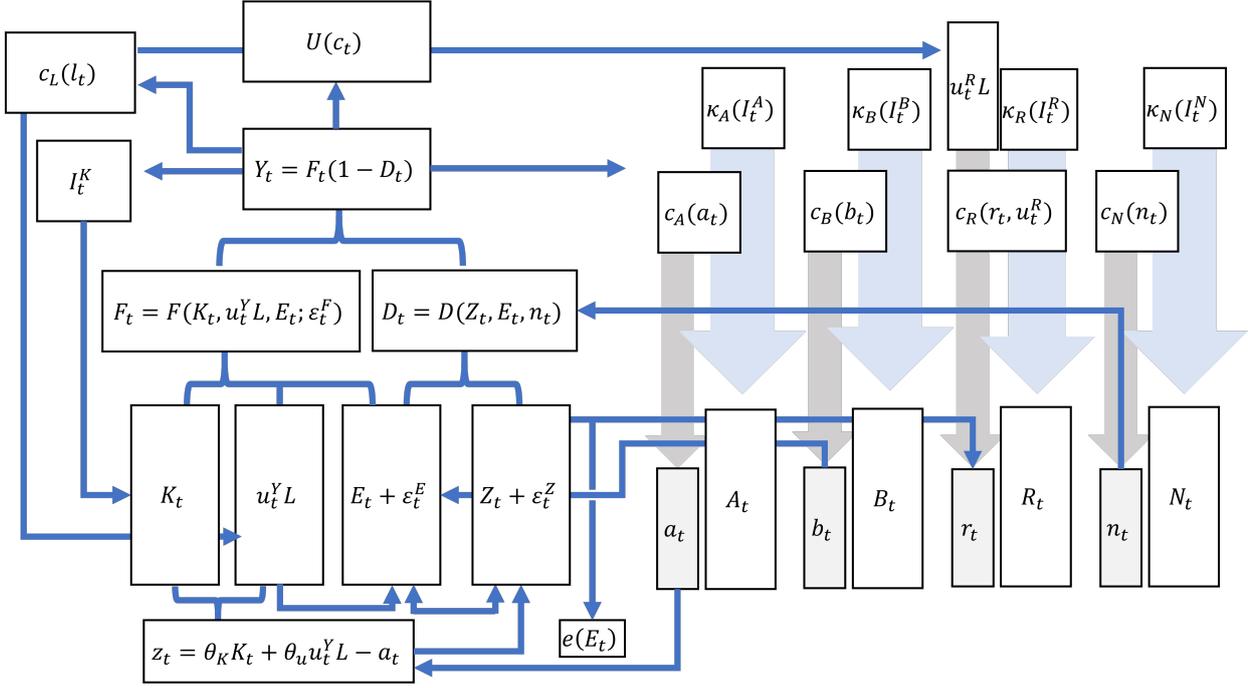


Figure 2: Model overview of the stochastic extension with capacity stocks. The model augments the deterministic framework by introducing shocks \mathcal{E}_t^F , \mathcal{E}_t^Z , and \mathcal{E}_t^E , and by distinguishing between capacity stocks A_t, B_t, R_t, N_t and their utilization a_t, b_t, r_t, n_t for abatement, restoration, removal, and adaptation. Costly investments I_t^X with $X \in \{A, B, R, N\}$ build capacities that constrain current actions and generate option values reflected in distinct shadow prices.

process \mathcal{E}_t^F which represents shocks to the production technology's total factor productivity. All three processes are uncorrelated and are i.i.d.. The shocks to natural capital and the atmospheric carbon stock may have fat tails. The shocks to atmospheric carbon, for example, may represent tipping points such as the dissociation of ocean methane hydrates and thawing permafrost, which [Dietz et al. \(2021\)](#) find to be among those tipping elements with the largest economic impacts.

Moreover, we extend our model to distinguish between capacity stocks for abatement, biosphere restoration, adaptation and removal A_t, B_t, N_t, R_t and actual deployment of abatement, restoration, adaptation and removal a_t, b_t, n_t, r_t . We require $a_t \leq A_t, b_t \leq B_t, n_t \leq N_t$ and $r_t \leq R_t$. With $X \in \{A, B, N, R\}$, the KKT multipliers are ν_t^X . Capacities are stocks with associated shadow prices λ_t^X . Investments I_t^X into their accumulation are assumed to be costly according to convex cost functions κ_X and the stocks depreciate at rates δ_X .

Assuming convex investment costs for the capacity stocks gives the model the interesting property that $\lambda^X \neq \mu$ for $X = A, B, N, R$, that is, they differ from the shadow price of human produced capital K (see [Appendix B](#)) reflecting the higher degree of inertia and irreversibility compared to the rest of the economy. In the following interpretation of the stochastic model, we will use these shadow prices to discuss the option values of the capacity stocks for abatement, removal, adaptation and restoration.

Emissions are as in the deterministic model's [Equation \(4\)](#) and net output as in [\(5\)](#) but with the random shocks added.

$$Y_t = F(K_t, E_t, n_t; \mathcal{E}_t^F)[1 - D(E_t, Z_t, n_t)] \quad (25)$$

The budget equation is now

$$\begin{aligned} Y_t = c_t + I_t^K + c_A(a_t) + c_B(b_t) + c_{LC}(l_t) + c_N(n_t) + c_R(r_t, u_t^R) \\ + \kappa^A(I_t^A) + \kappa^B(I_t^B) + \kappa^N(I_t^N) + \kappa^R(I_t^R) \end{aligned} \quad (26)$$

We derive the social cost of carbon and the social values of nature and converted land in [Appendix B](#). To highlight the difference between the deterministic and the stochastic

setting, we use an overline in the notation.

$$\overline{SCC}_t = \mathbb{E}_t [M_{t,t+1} (\overline{SCC}_{t+1} - Y_{Z,t+1} + \gamma_{Z,t+1} \overline{SVN}_{t+1})] \quad (27)$$

$$\overline{SVN}_t = \mathbb{E}_t [M_{t,t+1} ((1 + \delta_E - \gamma_{E,t+1}) \overline{SVN}_{t+1} + Y_{E,t+1} + e' \overline{SCC}_{t+1})] \quad (28)$$

$$\overline{SVCL}_t = \mathbb{E}_t [M_{t,t+1} (\overline{SVL}_{t+1} + \gamma_u (\overline{SVN}_{t+1} - \overline{SVN}_t / M_{t,t+1}))] \quad (29)$$

4.2. Interpretation

We normalize the shadow prices as $\Theta_t^X := \frac{\lambda_t^X}{\mu_t}$, $X = A, B, N, R$ for capacity buildup for abatement, biosphere restoration, adaptation and CDR. We define the normalized shadow prices as the option value of each type of capacity. Moreover, the first order conditions for abatement and CDR (30) and (33) relate the marginal costs of actual deployment with the social cost of carbon and shadow prices of the capacity constraint. Similarly, for biosphere restoration, the first order condition (31) relates marginal restoration costs with the social value of nature. For adaptation, Equation (32) is similar but shows one crucial difference. Instead of the SCC or SVN that sum over all future periods, only the instantaneous marginal avoided output loss in period t due to one unit of adaptation in that same period $\partial Y_t / \partial n_t$ appears. Adaptation investments are irreversible and therefore possess option value. However, unlike abatement, removal, or restoration, adaptation cannot prevent or repair future damages stemming from changes in the stocks of E and Z .

$$\frac{\nu_t^A}{\mu_t} = \overline{SCC}_t - c'_A(a_t) \quad (30)$$

$$\frac{\nu_t^B}{\mu_t} = \overline{SVN}_t - c'_B(b_t) \quad (31)$$

$$\frac{\nu_t^N}{\mu_t} = \partial Y_t / \partial n_t - c'_N(n_t) \quad (32)$$

$$\frac{\nu_t^R}{\mu_t} = \overline{SCC}_t - \frac{\partial c_R(r_t, u_t^R)}{\partial r_t} \quad (33)$$

The first order conditions (30) to (33) describe the gap between the social value of an option and the costs of its actual deployment. Figure 3 illustrates this relation.

From the equation of motion of the shadow prices of the capacity stocks $X \in \{A, B, N, R\}$,

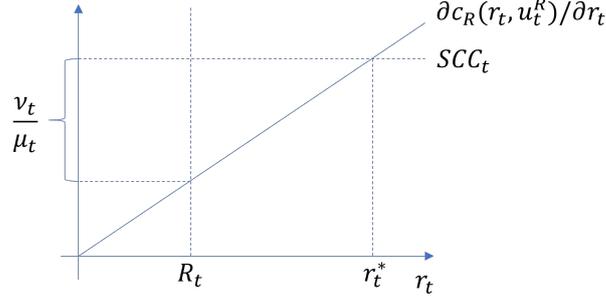


Figure 3: Illustration of the capacity constraint using CDR as example. Removal quantity is on the horizontal, marginal costs on the vertical axis. We denote by r_t^* the optimal unconstrained removal quantity such that marginal removal costs equal the SCC. If the capacity constraint of an option such as CDR bites and $r_t = R_t < r_t^*$, the KKT multiplier ν_t^R is non-zero.

we obtain

$$\Theta_t^X = \mathbb{E}_t \left[M_{t,t+1} \left(\frac{\nu_{t+1}^X}{\mu_{t+1}} + (1 - \delta_X) \Theta_{t+1}^X \right) \right] \quad (34)$$

and via forward iteration

$$\Theta_t^X = \sum_{k \geq 1} \mathbb{E}_t \left[M_{t,t+k} (1 - \delta_X)^{k-1} \frac{\nu_{t+k}^X}{\mu_{t+k}} \right] \quad (35)$$

Focusing on the summands for period $t+k$ and using (30)-(33), we obtain for removal and abatement

$$\mathbb{E}_t \left[M_{t,t+k} \frac{\nu_{t+k}^R}{\mu_{t+k}} \right] = \mathbb{E}_t [M_{t,t+k}] \mathbb{E}_t \left[\overline{SCC}_{t+k} - \frac{\partial c_R}{\partial r_{t+k}} \right] + Cov_t \left[M_{t,t+k}, \overline{SCC}_{t+k} - \frac{\partial c_R}{\partial r_{t+k}} \right] \quad (36)$$

$$\mathbb{E}_t \left[M_{t,t+k} \frac{\nu_{t+k}^A}{\mu_{t+k}} \right] = \mathbb{E}_t [M_{t,t+k}] \mathbb{E}_t [\overline{SCC}_{t+k} - c'_A] + Cov_t [M_{t,t+k}, \overline{SCC}_{t+k} - c'_A] \quad (37)$$

for restoration

$$\mathbb{E}_t \left[M_{t,t+k} \frac{\nu_{t+k}^B}{\mu_{t+k}} \right] = \mathbb{E}_t [M_{t,t+k}] \mathbb{E}_t [\overline{SVN}_{t+k} - c'_B] + Cov_t [M_{t,t+k}, \overline{SVN}_{t+k} - c'_B] \quad (38)$$

and for adaptation

$$\mathbb{E}_t \left[M_{t,t+k} \frac{\nu_{t+k}^N}{\mu_{t+k}} \right] = \mathbb{E}_t [M_{t,t+k}] \mathbb{E}_t [Y_{n,t+k} - c'_N] + Cov_t [M_{t,t+k}, Y_{n,t+k} - c'_N] \quad (39)$$

A useful way to interpret the option values of CDR, abatement, restoration and adaptation is through the lens of asset pricing. In our model, each capacity investment behaves like a risky asset. In the case of CDR and abatement, the return of the capacity investment depends on the difference between the marginal cost of deployment and the SCC; for restoration on the difference between marginal cost and the SVN. In the case of adaptation the return increases when the marginal cost is below the current level of marginal damages. It is important to recall that SCC and SVN are determined simultaneously implying that the strength of the coupling between climate and nature impacts the option values of climate policy. The option value evolves by the co-movement of the return with the stochastic discount factor (SDF) and with climate–nature risks. The key statistic summarizing the co-movement is the climate-nature beta that is defined as the covariance between an option’s marginal return and the SDF normalized by the variance of the SDF.^{5,6}

$$\beta_{t,t+k}^R = Cov_t \left[M_{t,t+k}, \overline{SCC}_{t+k} - \frac{\partial c_R}{\partial r_{t+k}} \right] / Var(M_{t,t+k}) \quad (40)$$

$$\beta_{t,t+k}^A = Cov_t [M_{t,t+k}, \overline{SCC}_{t+k} - c'_A] / Var(M_{t,t+k}) \quad (41)$$

$$\beta_{t,t+k}^B = Cov_t [M_{t,t+k}, \overline{SVN}_{t+k} - c'_B] / Var(M_{t,t+k}) \quad (42)$$

$$\beta_{t,t+k}^N = Cov_t [M_{t,t+k}, Y_{n,t+k} - c'_N] / Var(M_{t,t+k}) \quad (43)$$

The climate-nature betas measure how the marginal social value of the capacity to deploy a policy option varies across climate-nature states. A positive climate-nature beta means that the asset pays out more in high-damage states with high SCC and high SVN. Assets with a positive climate-nature beta have the role of hedging against the worst climate outcomes.⁷

⁵See Edenhofer et al. (2024) and Gollier (2024), who discuss the concept of the climate beta.

⁶The difference between the discount factor β and the climate-nature betas is reflected in the notation that the latter have sub- and superscripts, while the former does not.

⁷In a capital asset pricing model (CAPM) a hedge normally carries a negative market beta, because the covariance is considered between the return of the asset and the consumption growth rate. In our case, we consider the covariance of the asset with the stochastic discount factor, which is, under the assumption of

In the following, we characterize the difference between the climate-nature betas of the four policy options.

Abatement reduces the probability of high-damage states by lowering emissions today. Its payoff is front-loaded: it stabilizes the SCC path before risks materialize. When the SCC evolves smoothly with limited tail risk, abatement has a low or negative climate nature beta, because it delivers value mainly in moderate states. The option value of abatement is largest when the SCC path is smooth (limited tails) and the marginal abatement costs are low and scalable. Its option value is therefore dominated by early certainty rather than hedging against catastrophic outcomes. However, the option value declines over time when the marginal costs approach infinity as the flow of emissions approaches zero.

CDR has a fundamentally different payoff structure: it does not suffer from potentially infinite marginal costs like abatement (Edenhofer et al., 2025a). Therefore, in scenarios where all cheap abatement methods are already exhausted, CDR can reduce the stock of atmospheric carbon at lower costs. CDR and abatement are asymmetric because abatement only reduces the increment to the stock while removal reduces the stock itself. Removal's payoff is largest when the SCC spikes, because that is exactly when the marginal abatement costs rise much faster than marginal removal costs. CDR therefore has a large positive climate nature beta: it pays out precisely when society faces high climate damages. We illustrate this in Figure 4.

decreasing marginal utility of consumption, proportional to the inverse of the consumption growth rate.

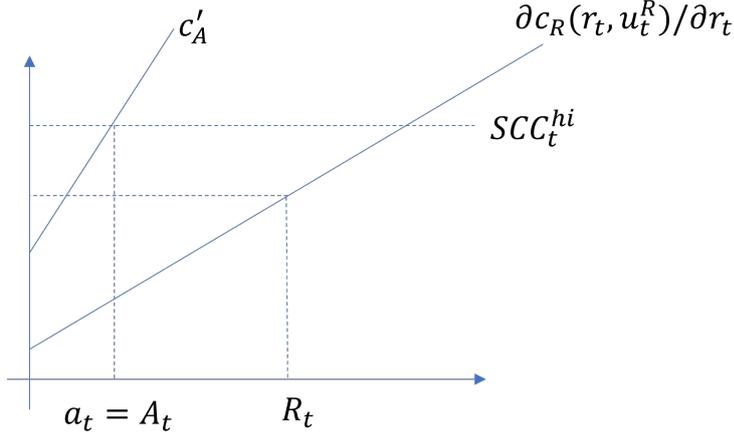


Figure 4: We compare abatement with CDR when the SCC are high and, thus, also the stochastic discount factor is high. Due to the asymmetric costs, it is more likely that the capacity constraint for abatement is binding, that is, $\nu^A/\mu = 0$ while the one for CDR is not, that is, $\nu^R/\mu \neq 0$. In such a situation, CDR's climate nature beta is positive while that of abatement is zero.

Land-intensive CDR faces a covariance drag via land degradation. Technical CDR methods that are more robust against climate damages or biosphere degradation can avoid it. If technical CDR costs do not rise much when the SCC increases sharply, then the covariance is positive. The difference between SCC and the marginal costs, that is, $\nu^R/\mu = \overline{SCC} - \frac{\partial c_R}{\partial r}$, increases with the SCC. Thus, it co-moves with the discount rate because $M_{t,t+k}$ is high when the SCC increases sharply. Land-based CDR provides lower hedging value than technical CDR because its covariance is dampened or even negative in years with high damages, when land prices increase because of droughts, floods, and wildfires. In asset-pricing language, CDR is a high-beta climate hedge asset, which explains why its option value can exceed the option value of abatement or adaptation.

Biosphere restoration is closer to CDR than to abatement because it directly repairs a degraded stock by increasing natural capital E just like carbon removal would increase a carbon budget. Both options, thus, reduce climate damages $D(Z, E, n)$, albeit to different degrees. However, restoration differs in two ways. First, it also enhances carbon uptake, partially improving the atmospheric carbon stock Z . More importantly, though, by rebuilding ecosystems, restoration raises output via the production function $F(K, E, u^Y L)$, where natural capital is a complementary input. Thus, restoration has a payoff profile that

is more state-contingent than abatement's: its value increases in high-damage states when natural systems have eroded and the marginal effect of rebuilding natural capital becomes particularly large. However, restoration's climate-nature beta is typically lower than that of technical CDR because its effectiveness deteriorates when ecosystems are heavily damaged or near collapse, causing marginal costs to increase and limiting scalability in extreme states. Consequently, restoration behaves as a moderate-beta hedge. It exploits both the damage and the production channel, but its option value is constrained by ecological limits that prevent it from matching the high-beta, tail-state payoff structure of scalable technical CDR.

Adaptation reduces realized damages and investments in adaptation capacity also help reducing future damages. However, adaptation investments and deployment do not affect the atmospheric carbon stock. Therefore, adaptation does not shift the future SCC path as much as abatement or removal. It insures primarily against realized damages: As a result, adaptation has a moderate climate nature beta—it lacks the stock effect that gives CDR high option value. Its option value is therefore typically lower than that of CDR and abatement.

Thinking in terms of climate-nature beta clarifies why option values differ across policy options: A high climate-nature beta implies a high hedging value and a high option value. The asset-pricing perspective explains how real options help to hedge against systemic risks. The private market will neither finance the capacity investments nor bear the collective or systemic risks properly. Public capacity payments and risk sharing between government and financial markets are needed to provide sufficient revenue streams. It is worth noting that without capital adjustment costs the climate-nature beta for capacity investments collapses to zero because these investments would no longer be irreversible. Capacity has no longer an insurance or hedging value because it can be adjusted instantly.⁸

In the stochastic model, option values reflect exposure to payoff risk only, as the transition probabilities across states are exogenous. This is a significant limitation for the problem at hand, because a central motivation for climate and biodiversity policy is precisely to reduce

⁸Mathematically, assuming $\kappa^X(I^X) = I^X$ would lead to $\Theta_t^X = 1$ for $X = A, B, N, R$ and for all t .

the risk of catastrophic transitions. Thus, in the next section, we assume that the transition probabilities become endogenous functions of the stock variables and, thus, policy choices. This enables us to link option values not only to payoff risk, but also to the control of systemic climate–nature risk itself.

5. Endogenous Tipping Points

We model climate- and biodiversity-tipping as policy-dependent Markov chains. The system can move from a good to a bad (post-tipping) regime with a transition probability (hazard) that is endogenous to the state of earth’s climate system and its biosphere integrity (Lenton et al., 2025; Rockström and Grandsoult, 2025). That is, the Markov transition matrix $P(E_t, Z_t)$ is a function of the stock of atmospheric carbon and natural capital.⁹

5.1. Assets prices and Markovian mark-ups

From the model we get the two asset price equations (see Appendix C). The SCC and SVN in period t now receive Markovian mark-ups \mathcal{M}^Z and \mathcal{M}^E . Marginal climate damages and output change due to changes in natural capital at time t are now weighted by the probabilities $\pi_t(i)$, $i \in \{G, B\}$ of being in the good, pre-tipping regime or the bad, post-tipping regime. To indicate the difference to the deterministic and the other stochastic version of the model, we use a hat in the notation.

$$\frac{\widehat{SCC}_t}{M_{t,t+1}} = \left[\left(\sum_{i=G,B} -\pi_{t+1}(i) Y_{Z,t+1}^i \right) + \widehat{SCC}_{t+1} + \gamma_{Z,t+1} \widehat{SVN}_{t+1} + \mathcal{M}_{t+1}^Z \right] \quad (44)$$

$$\frac{\widehat{SVN}_t}{M_{t,t+1}} = \left[\left(\sum_{i=G,B} \pi_{t+1}(i) Y_{E,t+1}^i \right) + (1 + \delta_E - \gamma_{E,t+1}) \widehat{SVN}_{t+1} + e' \widehat{SCC}_{t+1} + \mathcal{M}_{t+1}^E \right] \quad (45)$$

The Markovian mark-ups \mathcal{M} turn the classic Hotelling rules into “hazard-adjusted” Hotelling rules, where the growth of SCC and SVN reflects not only rents and carrying costs, but also how today’s actions reshape the probability of landing in high-damage, high-SCC,

⁹By explicitly modeling tipping points as probabilistic thresholds that policy can influence, we go beyond Dasgupta (2021), where the tipping point is defined as a deterministic collapse point of an ecosystem that an optimizing agent would never choose to cross. Dasgupta acknowledges risk and argues for precaution, but does not include it in his formal model.

high-land-price futures. The SCC and SVN are the expected returns given the endogenous tipping probabilities. The “Hotelling” rule survives, but is amended by the Markovian mark-up: the marginal hazard rate multiplied with probability of the regime shift from a “good” to a “bad” state and the relevant Lagrange-multiplier (see Appendix C). In the Markovian markup, risks are priced into the SCC and the SVN, such that the costs of reducing tipping risks are weighed against their expected benefits.

More generally, one could impose an additional safety markup on the SCC and the SVN that rules out risks exceeding a given threshold. We do not deny the existence of such thresholds. However, planetary boundaries have typically been modeled as lexicographic constraints (Sureth et al., 2023). In an optimization framework, this assumption implies that the planner pushes the system up to the threshold. In contrast, our framework emphasizes that a planner who manages risk optimally maintains a distance from these thresholds that depends on the underlying cost–benefit trade-off. Planetary boundaries therefore already matter within a stochastic cost–benefit framework, even when safety markups are not yet binding.¹⁰

With tipping dynamics, the SCC becomes the return to a state-dependent asset. It now also includes the effect of transition risks between “good” and “bad” regimes. Emissions today raise the probability of entering high-damage/high-SCC regimes in the future. The hazard term therefore raises the SCC’s required return. Optimal abatement and CDR face a hazard-adjusted carbon price. The tipping-risk mark-ups reduce the probability of entering catastrophic regimes.

In the Markov version of our model, nature also becomes a risk-bearing asset. Improvements in biosphere integrity measured as increases in natural capital also reduce tipping risks. Thus, optimal restoration and conservation captures the value of reducing transition risks — especially the probability of entering low-ecosystem-function regimes. The hazard term raises the required return on nature and steepens the optimal growth path of the social value of nature.

This is the dynamic core of the risk management mechanism: carbon and nature prices

¹⁰There are also deterministic settings, in which the social planner’s optimization implies keeping a safe distance from thresholds, as for example in Drupp and Hänsel (2021), who include a subsistence level of non-market goods in DICE.

rise not only because they are scarce, but because they insure the economy against tipping-induced regime shifts.

In addition, including tipping risks has implications for the role of land that are worth noting. In the deterministic model, land is allocated by equalizing marginal product and marginal environmental costs: land for production trades off output against land-use emissions and biodiversity loss, that is, both the SCC and the SVN. Land for land-intensive CDR methods trades off cheaper removals against ecosystem degradation, that is, only the SVN. In the Markov extension, these conditions lead to a shift in the optimal allocation of land that captures how land use affects tipping risks. Formally, this is due to the Markovian mark-ups in the SCC and the SVN. If extra cropland or land-intensive CDR raises the hazard of tipping, each hectare carries an additional risk cost on top of emissions and biodiversity damages. Conversely, land uses that stabilize ecosystems and reduce tipping risk increases the SVN. Thus, in the Markov regime the shadow price of land reflects not only its role for output, emissions and biodiversity, but also its role as a driver of regime shifts in the coupled climate–nature system. As a result, the shadow price of land in the Markov regime is systematically higher in states where the climate–nature system is close to a tipping boundary. Land becomes a risk-bearing asset: its marginal social value reflects not only its role in production and ecosystem services, but also its contribution to stabilizing (or destabilizing) the long-run regime mix of the economy–biosphere system. Given the proximity to multiple planetary boundaries, especially biosphere integrity, this risk-sensitive land valuation becomes quantitatively more important.

To sum up: The SCC and the SVN are tipping risk-sensitive asset prices. The SCC prices climate-catastrophe risk, while the SVN prices biodiversity-scarcity risk. The SCC is the price of an Arrow–Debreu security that pays off exactly in states of the world with high climate damages. The SVN is the price of a security that pays off exactly when ecosystem services are scarce. However, SCC and SVN become risk-bearing financial assets only once a government converts the social shadow prices into actual payment streams. Without government-created carbon prices or biodiversity payments, these assets are not trade on private markets.

5.2. Option Values and Tipping Points

In the Markov setting, the option-value equations retain their formal structure: As shown in Appendix C.2.2, starting from the primal (Lagrangian) planner problem we derive recursive Bellman-like equations for the option values. However, their economic content changes fundamentally because the SCC and the SVN become hazard-adjusted asset prices. As a result, each capacity option generates value by shifting the probabilities of moving into the bad regime. These transition effects dominate the option value whenever policy options or policy instruments meaningfully influence tipping hazards.

Formally, all policy instruments that reduce the climate stock or increase the biosphere stock contribute equivalently to lowering the hazard, as shown in the Markov mark-ups (C.79) and (C.80). However, the magnitude and timing of these effects can differ across options.

Abatement capacity is readily available and emissions can be reduced immediately. Building up CDR capacity may turn out to be more difficult and costly, but does not suffer from sharply rising costs that abatement faces once the emission level approaches zero (Edenhofer et al., 2025a). For this reason, although the derivative structure in (C.79) and (C.80) is symmetric, the effective impact of abatement on the hazard rate is largest, followed by CDR, while adaptation has essentially none since adaptation neither influences the stock carbon nor natural capital.

The role of restoration is less straightforward to characterize since it operates through several channels. It stabilizes and strengthens natural carbon sinks, buffers against certain climate damages, increase output through ecosystem services. However, its potential to avert tipping appears limited for high carbon concentrations due to high climate damages to ecosystems. Moreover, restoration is only possible for certain types of tipping points. For example, it is hard to conceive of feasible methods to restore melting glaciers in an economic way.

Our concept of option values provides the theoretical underpinning for ranking climate and nature conservation policies. Abatement exerts a strong downward pressure on tipping risks, CDR provides substantial value mostly after tipping and through stock repair, while

adaptation contributes little to hazard reduction because it does not influence transition probabilities. Adaptation offers valuable protection within regimes but does not meaningfully reduce tipping risk. The Markov approach makes it possible, for the first time, to clearly distinguish between prevention (the ex-ante hazard channel) and insurance/repair (the payoff channel operating within bad regimes). This distinction, in turn, generates the qualitative ranking across policy options.

6. Synthesis of the models

In the preceding sections we have shone a light on different aspects of the coupled SCC and SVN and the option values of abatement, removal, adaptation and restoration. We now combine the models from Sections 4 and 5.¹¹ We see that the SCC and the SVN take the following form, where we use the bar and the hat to make the difference to the other model version clear:

$$\widehat{SCC}_t = \mathbb{E}_t \left[M_{t,t+1} \left(\left(\sum_{i=G,B} -\pi_{t+1}(i) Y_{Z,t+1}^i \right) + \widehat{SCC}_{t+1} + \gamma_{Z,t+1} \widehat{SVN}_{t+1} + \mathcal{M}_{t+1}^Z \right) \right] \quad (46)$$

$$\widehat{SVN}_t = \mathbb{E}_t \left[M_{t,t+1} \left(\left(\sum_{i=G,B} \pi_{t+1}(i) Y_{E,t+1}^i \right) + (1 + \delta_E - \gamma_{E,t+1}) \widehat{SVN}_{t+1} + e' \widehat{SCC}_{t+1} + \mathcal{M}_{t+1}^E \right) \right] \quad (47)$$

The above equations reveal that the asset values, and thereby also the option values, are jointly determined by four forces: First, there is the intuitive way in which discounted climate damages enter the SCC and biosphere services the SVN. Second, climate and nature are coupled, as argued in Section 3, meaning that a higher stock of CO₂ reduces the value of nature and increases its riskiness, while a larger biodiversity stock dampens climate risk and therefore lowers the SCC. Third, the hedging component, which we have developed in Section 4 and which is represented by the expectation operators, leads to the definition of

¹¹See Appendix D for mathematical details.

the climate-nature beta. Fourth, the Markov markups \mathcal{M}_t^X , $X = E, Z$ and the probabilities $\pi_t(i)$, $i = G, B$ introduced in Section 5 quantify the tipping risk through the hazard channel. However, due to the stochastic shocks to the carbon stock and the biosphere, these become random variables. Importantly, we see that the combination of the two stochastic models is not merely additional. Instead, the two sources of uncertainty become intertwined. Our model’s asset-pricing equations imply two parallel requirements for policy instrument design. First, a regulator should impose carbon prices that reflect the state-contingent SCC that captures not only marginal climate damages but also their interaction with the degradation of natural capital, their stochastic behavior due to the random exogenous shocks \mathcal{E}^X , $X = E, F, Z$ and the state-contingent tipping risk. Second, the regulator must remunerate farmers and landowners for biosphere services at rates linked to the stochastic and state-contingent SVN.

In addition, subsidies for technical carbon sinks, payments for ecosystem restoration, and land taxes that reflect scarcity of natural capital are required to decentralize the planner’s allocation. The instruments generate cash-flow claims whose value depends on the realized climate–nature state. Such claims could be used as collateral, securitized, and traded.¹² Farmers would initially hold the nature-related claims on their land, but financial intermediaries including banks, insurers, and institutional investors would gradually transform carbon- and nature-related claims into distinct asset classes whose prices and risk profiles are governed by the same stochastic discount factor and by the coupled SCC–SVN dynamics derived in the planner model.

Strong institutions are required to implement such a policy instrument portfolio in the real world. In the next section, we focus on the case of the European Union and outline a set of new institutions that put in practice what we have theorized in the above.

¹²Such financial instruments are discussed in the case of only CDR in a working paper by Lemoine (2022). Little is known about pricing financial risks related to biodiversity loss (Karolyi and Tobin-de la Puente, 2023).

7. Institutional Options

Implementing the shadow prices derived in the preceding model sections—namely, the risk- and hazard-adjusted social cost of carbon (SCC), the risk- and hazard-adjusted social value of nature (SVN), and the associated land-use tax schedule—requires dedicated institutional design. Because climate and nature are tightly coupled, neither frontier can be managed efficiently in isolation. In other words, coordination can increase the welfare significantly. Moreover, hazard-adjusted shadow prices become highly volatile as the system approaches tipping boundaries. Credible, independent institutions are therefore needed to commit to the state-contingent price paths implied by the Markov Hotelling conditions. These prices define state-contingent assets: they depend on the prevailing regime (good versus bad), on future transition hazards, and on how policy choices shape the transition matrix through their effects on atmospheric carbon and natural capital. In addition, the option values associated with abatement, ecosystem restoration, carbon dioxide removal (CDR), and adaptation must be actively managed.

Markets and governments acting alone are unable to implement these price paths or to finance the large-scale, irreversible capacities required for abatement, adaptation, restoration, and CDR. Doing so requires credible, independent institutions capable of committing to hazard-adjusted, regime-dependent price signals over long horizons.

Multiple institutional design options are conceivable. In what follows, we focus on the European Union, where a substantial institutional architecture for climate and nature governance already exists through the European Green Deal and its associated regulatory framework. Building on this foundation, we outline how existing institutions could be extended and adapted to implement the risk- and hazard-adjusted shadow prices derived above, yielding a coherent framework for managing the coupled climate–nature system under risk. While the discussion is framed in the EU context, the underlying principles are more general, and the EU’s regulatory reach makes it a particularly informative and policy-relevant case.

We therefore translate the model’s theoretical shadow prices into two specialized policy authorities:

1. A European Climate Central Bank (ECCB) — guardian of the carbon budget and

the SCC path.

2. A European Biodiversity Pricing Authority (EBPA) — guardian of land, biodiversity, and the nature price.

Figure 5 lays out an overview over their responsibilities, their coordination and their relation to complementary institutions in the EU context. We describe the institutions' roles in detail in the following.

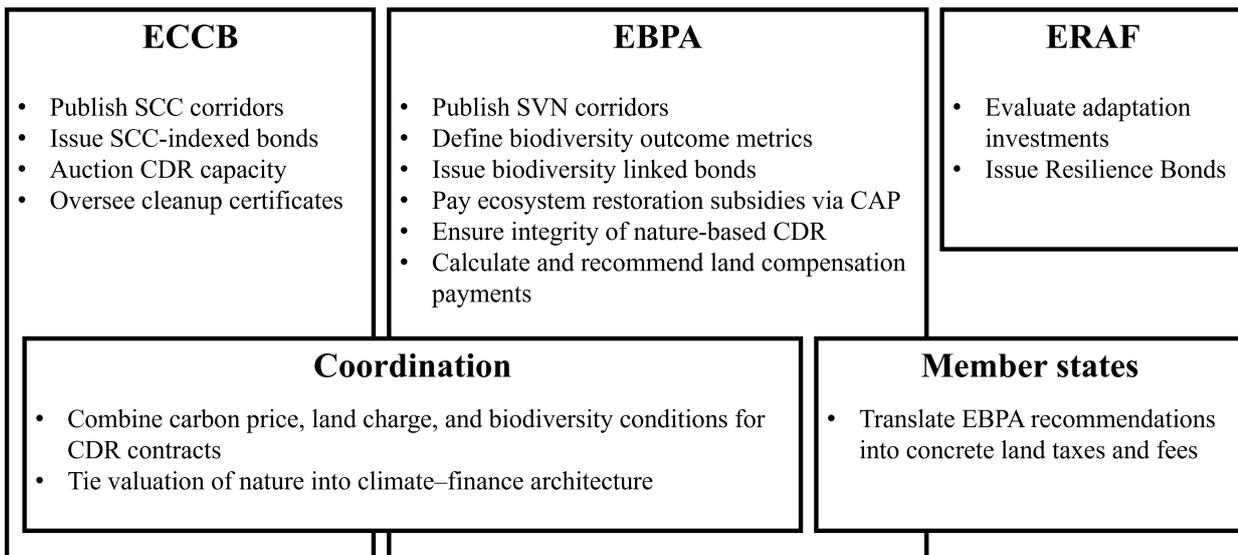


Figure 5: Overview of institutions.

7.1. The Role of a European Climate Central Bank (ECCB)

The ECCB's mandate is to anchor the SCC path that emerges from the model's risk- and hazard-adjusted Hotelling condition. The SCC determines how strongly emissions today worsen future climate and biosphere states and influence transition risks. The SCC is not a single number but a price path that includes coupling, hedging and hazard effects.

The ECCB publishes SCC corridors consistent with the model's risk- and hazard-adjusted Hotelling rule, stabilizing expectations and ensuring time-consistent carbon pricing.¹³ To guarantee that CDR capacity is available when SCC is high, the ECCB organizes CDR

¹³The EU's current climate policy architecture relies on budgets and certificates, that is on quantity instruments instead of pricing instruments. Therefore, it seems appropriate to set a lower and an upper bound for the carbon price, which may then still form flexibly and based on the currently existing structure of the EU ETS.

capacity auctions and issues CDR capacity bonds. The ECCB ensures that enough CDR “fire brigade” is built and kept ready to meet climate targets and manage tail risks. Because the value of CDR has a strong hedging component — it delivers benefits precisely in bad climate states — the ECCB issues SCC-indexed bonds with state-contingent coupons, financed from ETS auction revenues, carbon levies, and clean-up charges. For the latter, the ECCB oversees the clean-up certificate system, backed by a collateral (Lessmann et al., 2024): every ton emitted today must be removed later by CDR. Our proposal to create tradeable, state-contingent claims on CDR capacities is closely related to Shiller’s idea of “macro markets” (Shiller, 2009, Ch.9), that is, financial markets designed to share aggregate, non-diversifiable risks. Whereas Shiller emphasized income, housing and macroeconomic indices, we apply the same logic in this to SCC linked instruments which generate asset classes for carbon and nature. As we will see below, the same ideas can be applied to SVN-linked assets like biodiversity bonds.

These instruments mobilize long-term capital, share climate risk with private investors, and internalize the hedging value and hazard risks that appear in the model’s option-value equations (Appendix C.2.2). By turning climate commitments into tradable assets, the ECCB reduces fiscal fragility and strengthens long-term policy credibility, since reversing these contracts would require explicit and costly default.

7.2. The Role of a European Biodiversity Pricing Authority (EBPA)

The European Biodiversity Pricing Authority (EBPA) has to set and maintain a price for land and natural capital. In our model, land and natural capital are scarce assets, and the social value of nature and the social cost of carbon are determined simultaneously. The EBPA has to implement the hazard-adjusted SVN and land prices. Without a clear price signal, land is systematically overused for farming, construction and land-based CDR, because nature is effectively undervalued.

The EBPA therefore has to construct a nature and land price index based on land scarcity, ecosystem quality and restoration costs. Like the hazard-adjusted SCC, the hazard-adjusted SVN must be published regularly and provide a transparent reference for policy and markets. On this basis, member states can implement land conversion charges, and the EBPA can

calculate compensation payments for land used by CDR projects, as well as the social value of restoration and nature-based solutions. In doing so, the EBPA sends a clear signal that land and nature are scarce and that anyone who uses land heavily has to pay for that scarcity. Through these functions, the EBPA implements the model’s insight that land becomes a risk-bearing asset whose value rises with scarcity and hazard exposure.

Land becomes a risk-bearing asset because its marginal social value depends not only on output, emissions, and biodiversity loss, but also on how land use affects the transition probabilities between the good and bad climate–nature regimes. Land-converting activities that increase ecosystem fragility or accelerate carbon accumulation raise the hazard of moving into the high-damage regime and therefore enter the first-order conditions with a negative hazard markup. Conversely, land uses that strengthen ecosystem resilience (such as restoration or rewetting) reduce the hazard and carry a positive hazard markup. Formally, the shadow price of land incorporates both a payoff channel (production, emissions, biodiversity impacts) and a hazard channel, which captures the effect of land use on the probability of tipping. This makes land a state-contingent asset whose value increases when the system approaches tipping boundaries.

The fact that land is a risk-bearing asset has potentially profound implications for private investors. If policymakers implement the hazard-adjusted shadow prices—via land-use charges, biodiversity payments, and land-linked financial instruments private land returns become risk-adjusted returns. Land uses that exacerbate tipping risks face higher implicit or explicit charges and become exposed to transition risk, while land uses that enhance ecosystem stability benefit from subsidies, lower risk premia, or favorable financing conditions. For investors, land thus ceases to be a “neutral” real asset: its expected return depends on its contribution to systemic climate and biodiversity risk. Portfolios that hold land-intensive, degradation-prone assets carry higher climate-nature transition risk, whereas portfolios including restoration, rewetting, or conservation projects acquire a natural hedge against adverse regime shifts. If private investors expect the credible implementation of these policy instruments, a substantive adjustment of their portfolios can be expected. However, the potential portfolio adjustments of private investors could also mobilize significant lobby power against these policies.

In addition, the EBPA defines biodiversity outcome metrics¹⁴ and pays subsidies for verified improvements in ecosystems—for example through restoration, rewetting and habitat protection. It ensures that nature-based CDR (forests, soils, wetlands) is only rewarded if it genuinely improves both the atmospheric carbon concentration and natural capital, and not just for delivering cheap tonnes with poor nature outcomes. In simple terms, the EBPA pays for nature’s recovery and makes sure that climate projects do not destroy ecosystems as a side effect. By creating robust indices and standards, the EBPA can also help turn land and natural capital into a recognized asset class, for example, via biodiversity-linked bonds, without losing sight of ecological integrity.

7.3. Coordination between ECCB and EBPA

The key insight of our model is that climate and natural capital are tightly coupled. Eroding natural capital implies a higher social cost of carbon (SCC), because degraded ecosystems amplify climate damages. More climate stress, in turn, leads to greater natural capital losses and increasing land scarcity. You cannot manage one frontier efficiently while ignoring the other.

In this setting, the EBPA feeds nature and land scarcity directly into climate decisions of the ECCB. Land-intensive CDR projects must pay land charges that reflect the scarcity of ecosystems as discussed in Section 3.4 on policy instruments. Restoration activities are rewarded because they improve resilience and reduce climate risk. The ECCB then ensures that the SCC and the optimal mix between abatement and CDR are chosen given that land and nature are scarce and already priced by the EBPA.

Together, the EBPA and the ECCB design coherent policy contracts: CDR contracts that combine a carbon price, a land charge, and biodiversity conditions. The EBPA creates biodiversity- and land-linked bonds, whose coupons depend on biodiversity and land indices. These bonds finance ecosystem restoration and protection, and their payoffs move with biodiversity and land conditions in a transparent, rule-based way.

The implementing of carbon prices, biodiversity payments, land taxes and such state-

¹⁴Burgess et al. (2024) review 573 biodiversity-related metrics, indicators, indices, and layers and make recommendations for using a small number of metrics focused on those that are most pertinent for use in decision-making.

contingent contracts requires a rich information and data infrastructure because SCC- and SVN-based instruments are only as good as the measurement of climate and biodiversity states, hazards, and performance on the ground. Monitoring, reporting and verification (MRV) of emissions, ecosystem services, and tipping risks is a data-production activity. It requires public–private arrangements that invest in high-quality ecological and climate data, govern access and ownership, and mitigate information asymmetries between states, landowners, and financial intermediaries. This data economy perspective highlights the information infrastructure and governance needed to make such a system both implementable and welfare-enhancing (Baley and Veldkamp, 2025).

7.4. Complementary institutions and coordination

The institutional setting we propose already internalizes the option values of abatement, CDR and restoration. By contrast, the option value of adaptation currently falls between the cracks. To address this, finance ministries, infrastructure planners, development banks and public investment banks such as the EIB and KfW could establish a European Resilience and Adaptation Facility (ERAF).

The ERAF’s role would be to evaluate adaptation investments, for example dykes, drainage systems, health capacity, cooling infrastructure, fire prevention and resilient electricity grids. It should use option values that are based on a stochastic discount factor and not just on static net present value. Consequently, the investments pay off precisely in bad climate states, and their value lies in the protection and flexibility they provide under extreme conditions.

To finance this, the ERAF could issue resilience bonds, whose coupons rise in bad physical-climate states, for instance when a heatwave index, flood index or catastrophe loss index exceeds certain thresholds. In this way, public authorities can share climate-disaster risks with private markets, while investors gain a transparent, state-contingent instrument that finances concrete adaptation and resilience projects.

Regarding land pricing, a division of labour between the EBPA and the Member States is required. The EBPA sets the price signal, not the tax. It defines and regularly updates the land and nature price index, and recommends a reference land charge, publishing this

much like a “shadow rate” for land use.

The Member States then implement the actual land taxes and charges within their own fiscal and planning systems. Under the principle of subsidiarity, it is natural that Member States (i) integrate land charges into property taxes, (ii) introduce land conversion levies, for example, when farmland is converted into built-up land, and/or (iii) add land-use fees for specific projects including land-based CDR and other types of large infrastructure.

The Common Agricultural Policy (CAP) can serve as a main funding channel for biodiversity payments. The CAP already transfers large sums to farmers and land managers and has instruments for eco-schemes, agri-environmental measures and rural development. In our design, the EBPA’s biodiversity index and land price become the reference metrics for CAP payments and the criteria for eco-schemes and result-based payments.

The CAP could be reoriented so that payments are explicitly linked to measurable biodiversity outcomes and to respecting land-scarcity signals, for example, by not converting high-value habitats. Part of the CAP budget would thus finance restoration subsidies, permanent grassland protection and rewetting.

7.5. Challenges

The decentralization of the social planner model developed above is less demanding from a computational perspective, but it suffers from several potential institutional shortcomings that challenge its real-world applicability.

Biosphere metrics

While carbon emissions can be measured in tons of CO₂eq, there is no standardized unit for measuring natural capital, biodiversity and ecosystem restoration. While there is a multitude of indicators (Burgess et al., 2024), the complexity on the “book keeping” side poses serious challenges for the practical implementation of the proposed policy instruments that should incentivize the socially optimal treatment of the biosphere.

Multidimensionality, locality, and non-substitutability of biodiversity.

A related challenge arises from the inherently multidimensional, spatially localized, and only imperfectly substitutable nature of biodiversity. Biodiversity losses are often irreversible at

local or ecosystem-specific thresholds, with limited scope for substitution across space or ecosystem types. Calculating the social value of nature therefore risks implicitly assuming compensability across heterogeneous ecological dimensions, locations, and functions. This assumption fails in cases where ecosystems provide unique services or host endemic species. Moreover, many components of biodiversity generate substantial non-use values, such as existence, bequest, and cultural values, which add an additional layer of complexity to finding good restoration metrics and to the design of market-based instruments. These features complicate aggregation, challenge the interpretation of a scalar social value of nature, and limit the extent to which biodiversity losses in one region can be offset by gains elsewhere, even under state-contingent pricing schemes.

International Cooperation

The models presented in this paper serve as social-planner benchmarks. At first glance, the prospects for global cooperation may appear to deteriorate as policy design becomes more complex, with multiple investment margins and state-contingent asset prices. However, greater complexity can also expand the scope for issue linkage, thereby facilitating international cooperation. For instance, carbon dioxide removal (CDR) mitigates carbon leakage and can thus strengthen incentives to cooperate (Edenhofer et al., 2025a; Franks et al., 2023). Similarly, the Paris Agreement could be complemented by international macro-markets. Additionally, tax-importer clubs for oil and gas could support the implementation of incentive-compatible carbon pricing schemes (Edenhofer et al., 2025b). Moreover, restoration investments, biodiversity payments (Cantillon et al., 2025), and tropical forest funds can be structured as conditional financial transfers, enabling higher and more stable investments in global public goods. In essence, although greater institutional complexity increases cooperation challenges, it also expands the scope for credible commitments, side payments, and mutually beneficial forms of smaller coalitions which be expanded over time.

Time-consistency and commitment

Delegating policy implementation to two independent authorities may, in principle, help a political system commit to long-term SCC and SVN paths. However, in practice it is highly unlikely that real-world policymaking can generate credible long-term commitments

to the taxes, charges, and state-contingent payments required to sustain capacity markets, macro-bonds, and other risk-sharing instruments. The absence of commitment undermines the intertemporal incentives that the decentralized solution relies on. Modern democracies might face structural commitment problems in climate policy: short electoral cycles, distributional conflicts and uncertainty make it hard to credibly lock in ambitious long-term decarbonization paths. Specific climate laws, independent climate councils, budget rules and monitoring mechanisms can partially solve this by shifting expectations and creating constituencies with a stake in stable climate policy as [Zwar et al. \(2023\)](#) discuss. However, [Gratton and Edenhofer \(2025\)](#) highlight the limits of technocratic delegation: strong delegation to experts can increase credibility, but if it bypasses democratic contestation over distribution, it risks backlash and erosion of legitimacy.

Moral hazard

A central moral-hazard problem might arise because biodiversity and capacity payments are performance-based: landowners can influence future cash flows by degrading or maintaining ecosystems, so financial intermediaries purchasing these claims must manage the risk that the underlying environmental performance falls short of contractual requirements. The moral-hazard problem can be mitigated by combining robust monitoring, reporting and verification (MRV) of biodiversity outcomes, partial risk retention by landowners (so they lose payments if they underperform), portfolio diversification on the side of intermediaries, and, where appropriate, public guarantees or insurance schemes that absorb systemic but not idiosyncratic performance risk.

Distributional challenges

The model relies on the second welfare theorem, assuming that distributional concerns can be fully separated from allocative decisions ([Kaplow and Shavell, 1994, 2002](#)). In political reality, finance ministries are unlikely to adjust the tax–transfer system in response to large increases in carbon prices, biodiversity payments, or carbon-sink rewards. When distributional instruments are sticky or politically constrained, allocative instruments become distributionally loaded, thereby distorting the theoretical decentralized optimum.

Political economy

While the social planner solution can incorporate heterogeneous welfare weights across households or countries, real-world policies cannot generally replicate such transfers. In principle, household- or group-specific carbon prices, nature payments, or carbon-sink rewards could sustain the first best when transfers are unavailable. Yet such differentiated instruments are politically complicate to implement. The deployment of risk-sensitive assets such land, CDR and abatement infrastructure, carbon sinks and ecosystems will affect wealth, income, and asset positions, mobilizing resistance from powerful rent-seeking groups. Ultimately, the political economy of rent redistribution may become the dominant determinant of policy feasibility.

Institutional capacity and implementation constraints

The decentralized system presupposes a set of institutions capable of measuring, monitoring, and enforcing state-contingent prices and payments—such as biodiversity metrics, carbon stocks, land degradation indices, and hazard-relevant ecological variables. In practice, measurement uncertainty, weak monitoring systems, fragmented jurisdictions, and limited administrative capacity reduce the implementability of such instruments. Moreover, creating credible capacity markets or biodiversity payment schemes requires regulatory stability, standardized contracts, and long-term monitoring infrastructures that many countries currently lack. Without these institutional foundations, it may not be possible to implement the elegant CMDP-based policy architecture.

However, even if the welfare optimum is not directly implementable, it remains indispensable as a normative and analytical benchmark: it defines the efficient risk-sharing and pricing structure, against which institutional, political, and distributional constraints can be systematically understood, quantified, and — where possible — relaxed.

8. Conclusion - Pigou and Hotelling meet Merton

The twin crises of large-scale biodiversity loss and dangerous climate change require new metrics for designing policy instruments: the social cost of carbon (SCC) and the social value of nature (SVN). Carbon prices, land taxes, biodiversity payments, subsidies for

ecosystem restoration, and charges on land-induced emissions should evolve in line with these state-contingent asset prices.

At the same time, addressing the twin crises requires large-scale investment in abatement, adaptation, carbon dioxide removal (CDR), and nature restoration. These investments must be evaluated within a framework that accounts for non-diversifiable aggregate risks, and the possibility of crossing tipping points — risks which have been emphasized very strongly by Earth-System science. In other words: Pigou and Hotelling must meet Merton. Climate economics needs the corrective-pricing logic of [Pigou \(1920\)](#); ¹⁵ the [Hotelling \(1931\)](#) framework provides the relevant asset prices. Financial economics contributes the tools to understand risk, asset pricing, and intertemporal allocation ([Merton, 1973](#)). Earth system science emphasizes the hazard-risks which have to be priced properly ([Lenton et al., 2025](#); [Rockström et al., 2009](#); [Rockström and Grandsoult, 2025](#)).

Climate and nature conservation policy inevitably follow political cycles. When the SCC and the SVN should rise, abatement and CDR may lose political support while adaptation becomes more attractive. Political economy helps to rationalize these shifts. Welfare economics, however, offers a countercyclical benchmark: it identifies the focal points for socially optimal investment. The concept of option values developed in this paper provides a coherent framework for determining which of the capacities for abatement, adaptation, CDR and restoration should be expanded, and when. The ultimate objective is sustainable wealth ensured by long-term investments. When SCC and SVN are increasing, adaptation has low option value. Under such circumstances the shift away from abatement and CDR towards adaptation is not based on prudent advice for investors.

Future research may address the following issues: Extending the model with empirically grounded parameters for biodiversity dynamics, restoration responses, and climate–nature interactions would help to quantify SCC–SVN trajectories and option values. Heterogeneous agents and political economy allows to model distributional conflicts, heterogeneous risk preferences, and political constraints. For example, in an incomplete-market environment, agents are exposed to heterogeneous stochastic discount factors, and the classic

¹⁵See also [Edenhofer et al. \(2021\)](#) for a discussion of Pigou’s relevance for the challenges posed by climate change.

separation between efficiency and distribution breaks down. The option values of abatement, carbon removal, ecosystem restoration, and adaptation then depend not only on their physical impacts on the climate-nature system, but also on who bears the residual risks and how these risks covary with the consumption prospects of different households, investors, or countries. A promising direction for future work is therefore to study how targeted expansions of financial instruments with high exposure to adverse climate-nature states — so-called high-beta hedges — can partially complete markets and attenuate welfare losses arising from incomplete risk sharing. The analysis of financial instruments when markets are incomplete could enable the design of biodiversity-linked securities, resilience bonds, and state-contingent contracts under real-world conditions. The extension of the CMDP framework to several interacting ecological or climatic tipping points would provide insights into systemic risk and portfolio effects across environmental domains. The investigation of Pigouvian and Merton-type mechanisms can be jointly embedded in long-term regulatory institutions—carbon banks, biodiversity authorities, or capacity markets—could help make the model’s prescriptions operational. Finally, in our stochastic models we assume that probabilities are known. However, some components may carry Knightian uncertainty, which would require other frameworks.

Without Pigouvian pricing, financial markets would not internalize the SCC or the SVN. Without the Hotelling framework, social costs could not be translated into asset prices. Without the Merton framework, societies would ignore the collective risks and would therefore forgo the potential for risk sharing between the public and private sector. Without earth system science, society would be ignorant about hazard risks. The purpose of this paper is to provide a unified framework that identifies the principles needed to navigate a world exposed to multiple, interacting ecological catastrophes – by aligning corrective prices with risk-sensitive asset-pricing disciplines.

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A. Deterministic model

Here, we describe the deterministic model used in Section 3. We first list all variables, then present the social planner solution and finally a decentralized version of the model.

A.1. List of variable and parameters

Table 1: List of variables and parameters in the deterministic coupled climate–biosphere model. Time subscripts are omitted for readability.

Symbol	Meaning
c_t	Consumption
Y_t	Net economic output after climate damages
F_t	Gross economic output (production function)
K_t	Stock of human-produced (physical) capital
I_t^K	Investment in human-produced capital
Z_t	Stock of atmospheric carbon
E_t	Stock of natural capital / biosphere quality
a_t	Emissions abatement effort
r_t	Carbon dioxide removal (CDR)
b_t	Investment in biosphere restoration
n_t	Adaptation effort reducing climate damages
u_t^Y	Fraction of land used for economic production
u_t^R	Fraction of land used for CDR technologies
u_t	Total fraction of land used by humans ($u_t^Y + u_t^R$)
l_t	Cost of converting land
$D(Z_t, E_t, n_t)$	Climate damage function
η	Lagrange multiplier of the land budget

Continued on next page

Table 1 continued from previous page

Symbol	Meaning
μ_t	Lagrange multiplier of the resource constraint (marginal utility of consumption)
λ_t^E	Shadow value of natural capital
λ_t^K	Shadow value of physical capital
λ_t^L	Shadow value of converted land
λ_t^Z	Shadow value of atmospheric carbon stock
SCC_t	Social cost of carbon (λ_t^Z/μ_t)
SVN_t	Social value of nature ($-\lambda_t^E/\mu_t$)
$M_{t,t+1}$	Intertemporal discount factor ($\beta\mu_{t+1}/\mu_t$)
$z_t(K_t, u_t^Y, a_t)$	Flow of carbon emissions
$c_A(a_t)$	Cost of emissions abatement
$c_B(b_t)$	Cost of biosphere restoration
$c_{LC}(l_t)$	Cost of converting land for human use
$c_N(n_t)$	Cost of adaptation
$c_R(r_t, u_t^R)$	Cost of carbon dioxide removal
$Y_{K,t}$	Marginal product of physical capital
$Y_{u,t}$	Marginal product of land in production
$Y_{E,t}$	Marginal ecosystem services (output effect of E_t)
$Y_{Z,t}$	Marginal climate damages (output effect of Z_t)
β	Social discount factor
β_t^X	Climate-nature beta for period t and policy option $X \in \{A, B, N, R\}$
δ_K	Depreciation rate of physical capital
δ_E	Natural regeneration rate of the biosphere
θ_K	Emissions intensity of physical capital
θ_u	Emissions intensity of land used in production
$\theta_{LC}(l_t)$	Emissions due to land conversion

Continued on next page

Table 1 continued from previous page

Symbol	Meaning
$e(E_t)$	Carbon absorption efficiency of the biosphere
$\gamma(E_t, Z_t)$	Climate damages leading to biosphere degradation
γ_u	Biodiversity loss from land conversion
L	Total land endowment (fixed)
ϕ^Z, ϕ^E	Propagation weight functions in the SCC
ζ^Z, ζ^E	Propagation weight functions in the SVN
τ_t^Z	Carbon tax on emissions
τ_t^L	Land tax on production land
τ_t^{LR}	Land tax on land used for CDR
s_t^b	Subsidy to ecosystem restoration investment
s_t^R	Subsidy to carbon dioxide removal
q_t	Land rent paid by firms
Γ	Lump-sum transfer between government and households

A.2. Social planner model

The social planner's Lagrangian is as follows. In the following we assume a solution that never converts all land, that is, $u_t < 1$ for all t .

$$\begin{aligned}
 \mathcal{L} = \sum_t \beta^t & \left\{ U(c_t) + \mu_t [Y_t - c_t - I_t^K - c_A(a_t) - c_B(b_t) - c_{LC}(l_t) - c_N(n_t) - c_R(r_t, u_t^R)] \right. \\
 & + \eta_t (u_t - u_t^Y - u_t^R) \\
 & + \lambda_t^K [(1 - \delta_K)K_t + I_t^K - K_{t+1}] \\
 & + \lambda_t^Z [Z_t + z_t(K_t, u_t^Y, l_t, a_t) - r_t - e(E_t) - Z_{t+1}] \\
 & + \lambda_t^E [(1 + \delta_E)E_t - \gamma(E_t, Z_t) - \gamma_u(u_{t+1} - u_t) + b_t - E_{t+1}] \\
 & \left. + \lambda_t^L [u_t + l_t - u_{t+1}] \right\} \tag{A.1}
 \end{aligned}$$

The first order conditions are as follows. First, we have consumption and capital.

$$\frac{\partial U_t}{\partial c_t} = \mu_t \quad (\text{A.2})$$

$$\mu_t + \lambda_t^K = 0 \Rightarrow \lambda_t^K = \mu_t \quad (\text{A.3})$$

Abatement, adaptation, novel technical CDR and restoration

$$-\mu_t c'_A(a_t) - \lambda_t^Z = 0 \quad \Rightarrow \quad c'_A(a_t) = -\frac{\lambda_t^Z}{\mu_t} =: \text{SCC}_t \quad (\text{A.4})$$

$$-c'_N(n_t) + Y_{n,t} = 0 \quad \Rightarrow \quad c'_N(n_t) = -F \frac{\partial D}{\partial n_t} \quad (\text{A.5})$$

$$-\mu_t \frac{\partial c_R(r_t, u_t^R)}{\partial r_t} - \lambda_t^Z = 0 \quad \Rightarrow \quad \frac{\partial c_R(r_t, u_t^R)}{\partial r_t} = \text{SCC}_t \quad (\text{A.6})$$

$$-\mu_t c'_B(b_t) + \lambda_t^E = 0 \quad \Rightarrow \quad c'_B(b_t) = \frac{\lambda_t^E}{\mu_t} =: \text{SVN}_t \quad (\text{A.7})$$

Optimal land conversion and land allocation

$$\text{SCC}_t \theta'_{LC}(l_t) + c'_{LC}(l_t) = \lambda_t^L / \mu_t =: \text{SVCL}_t \quad (\text{A.8})$$

$$-\frac{\partial c_R(r_t, u_t^R)}{\partial u_t^R} = \eta_t / \mu_t = \frac{\partial Y_t}{\partial u_t^Y} \quad (\text{A.9})$$

Hence, optimal land conversion is governed by the condition that the social value of converted land (SVCL_t) must equal the conversion cost and the contribution of land conversion to climate damages. Converted land must be allocated such that the benefits in terms of lower removal costs and increases in macroeconomic production are equal.

The co-state variables evolve as follows.

$$\lambda_t^K = \beta [\mu_{t+1} Y_{K,t+1} + \lambda_{t+1}^K (1 - \delta_K) + \lambda_{t+1}^Z \theta_K] \quad (\text{A.10})$$

$$\lambda_t^Z = \beta [\mu_{t+1} Y_{Z,t+1} + \lambda_{t+1}^Z - \lambda_{t+1}^E \gamma_{Z,t+1}] \quad (\text{A.11})$$

$$\lambda_t^E = \beta [\mu_{t+1} Y_{E,t+1} - \lambda_{t+1}^Z e' + \lambda_{t+1}^E (1 + \delta_E - \gamma_{E,t+1})] \quad (\text{A.12})$$

$$\lambda_t^L = \beta [\lambda_{t+1}^L + \eta_{t+1} + \lambda_{t+1}^E \gamma_u] - \gamma_u \lambda_t^E \quad (\text{A.13})$$

Transversality conditions

$$\lim_{t \rightarrow \infty} \beta^t \lambda_t^K K_t = 0 \quad (\text{A.14})$$

$$\lim_{t \rightarrow \infty} \beta^t \lambda_t^Z Z_t = 0 \quad (\text{A.15})$$

$$\lim_{t \rightarrow \infty} \beta^t \lambda_t^E E_t = 0 \quad (\text{A.16})$$

A.3. Decentral market solution

A representative firm rents human produced capital at the interest rate r_t and land at the land rent q_t from a representative household to produce and sell a single final good. It takes the level of natural capital as given and fixed. Neither the firm nor the household consider climate damages $D_t = D(Z_t, E_t, n_t)$. The firm can engage in abatement activity a_t at cost $c_A(a_t)$. The firm must pay a tax τ^Z on net carbon emissions due to production $z^Y(K_t, u_t^Y, l_t, a_t) = \theta_K K_t + \theta_u u_t^Y L - a_t$ that is intended to internalize the climate externality and a tax on land τ^L to internalize the social costs due to land use emissions. In each period, the firm maximizes profits π .

$$\begin{aligned} \pi_t = & F(K_t, E_t, u_t^Y L) (1 - D(Z_t, E_t, n_t)) \\ & - r_t K_t - (q_t + \tau_t^L) u_t^Y L - \tau_t^Z z^Y(K_t, u_t^Y, a_t) - c_A(a_t) \end{aligned} \quad (\text{A.17})$$

The first order conditions are

$$c'_A(a_t) = \tau_t^Z \quad (\text{A.18})$$

$$Y_{K,t} = F_{K,t}(1 - D_t) = r_t + \tau_t^Z \theta_K \quad (\text{A.19})$$

$$Y_{u,t}/L = F_{u^Y L,t}(1 - D_t) = q_t + \tau_t^L + \tau_t^Z \theta_u \quad (\text{A.20})$$

The household's budget is

$$c_t + I_t^K + c_{LC}(l_t) + \tau_t^{LC}(l_t) = r_t K_t + q_t u_t L + \Gamma \quad (\text{A.21})$$

where $c_{LC}(l_t)$ is the cost of land conversion, $\tau_t^{LC}(l_t)$ is a non-linear land conversion tax, Γ is a lump-sum transfer from or tax paid to the government. This leads to the Lagrangian

$$\begin{aligned} L^H = \sum_t \beta^t \left\{ U(c_t) + \mu_t^H [r_t K_t + q_t u_t L + \Gamma - c_t - I_t^K - c_{LC}(l_t) - \tau_t^{LC}(l_t)] \right. \\ \left. + \lambda_t^{K,H} (K_t(1 - \delta_K) + I_t^K - K_{t+1}) \right. \\ \left. + \lambda_t^{L,H} (u_t + l_t - u_{t+1}) \right\} \quad (\text{A.22}) \end{aligned}$$

and the first order conditions

$$U'(c_t) = \mu_t^H = \lambda_t^{K,H} \quad (\text{A.23})$$

$$\lambda_t^{K,H} / \beta = r_{t+1} \mu_{t+1}^H + \lambda_{t+1}^{K,H} (1 - \delta_K) \quad (\text{A.24})$$

$$\lambda_t^{L,H} / \beta = q_{t+1} L \mu_{t+1}^H + \lambda_{t+1}^{L,H} \quad (\text{A.25})$$

$$\lambda_t^{L,H} = \mu_t^H \left(\tau_t^{LC'} + c'_{LC} \right) \quad (\text{A.26})$$

Using (A.19) and (A.24) from the decentral equilibrium and comparing with the social planner's (A.10) we obtain that in order to achieve the first best, the carbon tax should be

$$\tau_t^Z = \lambda_t^Z / \mu_t^H = SCC_t \quad (\text{22})$$

Comparing (A.26) with the social planner's condition (A.8), we find the optimal land conversion tax to be

$$\tau_t^{LC} = SCC_t \theta'_{LC}(l_t^*) \quad (\text{A.27})$$

where l_t^* is the level of land conversion in the social planner optimum. Further, comparing the firm's FOC for profit maximization with respect to the share of land to use in production (A.20) with the social planner's (A.9) and (A.13) shows the optimal land tax to be

$$\tau_t^L = \frac{1}{L} \left[\underbrace{\frac{SVCL_{t-1}}{M_{t-1,t}} - SVCL_t}_{\text{user cost of converted land}} + \gamma_u \underbrace{\left(\frac{SVN_{t-1}}{M_{t-1,t}} - SVN_t \right)}_{\text{user cost of natural land}} \right] - \tau_t^Z \theta_u - q_t \quad (24)$$

The optimal land tax thus makes the firm pay the user cost of switching land from nature to cultivation net of the private scarcity already captured by the land rent q_t and the carbon tax $\tau_t^Z \theta_u$.

Moreover, if we add representative profit maximizing firms for ecosystem restoration, it is straightforward to see that they should be subsidized at their respective marginal social values, that is

$$s_t^b = SVN_t \quad (\text{A.28})$$

For CDR, we assume that there is a representative firm that incurs convex deployment costs $c_R(r_t, u_t^R)$ with $\frac{\partial c_R}{\partial r} > 0$, $\frac{\partial c_R}{\partial u^R} < 0$. It rents land at the rate q_t , receives a per-unit subsidy s_t^r but must also pay a tax based on the land used τ^{LR} .

$$\pi^r = s_t^r r_t - c_R(r_t, u_t^R) - (q_t + \tau_t^{LR}) u_t^R L \quad (\text{A.29})$$

Then, profit maximizing leads to the FOCs

$$s_t^R = \frac{\partial c_R}{\partial r} \quad (\text{A.30})$$

$$(q_t + \tau_t^{LR})L = -\frac{\partial c_R}{\partial u^R} \quad (\text{A.31})$$

Comparing these with (A.6), (A.9) and (A.13), we see that CDR firms should receive a subsidy based on the SCC and pay the same land tax as the final goods producing firm for net losses in social value of converted land and nature due to land use degradation, albeit without the carbon tax component:

$$s^R = SCC_t \quad (\text{A.32})$$

$$\tau_t^{LR} = \tau_t^L - \tau_t^Z \theta_u = \frac{1}{L} \left[\frac{SVCL_{t-1}}{M_{t-1,t}} - SVCL_t + \gamma_u \left(\frac{SVN_{t-1}}{M_{t-1,t}} - SVN_t \right) \right] - q_t \quad (\text{A.33})$$

B. Stochastic model

To describe the role of abatement, biosphere restoration, adaptation and removal as hedging tools (Section 4), we introduce stochastic processes \mathcal{E}_t^Z and \mathcal{E}_t^E , which represent random shocks to the equations of motion of the stocks of atmospheric carbon and natural capital, and a stochastic process \mathcal{E}_t^F which represents shocks to the production technology. All three processes are uncorrelated, have zero mean and are i.i.d.. The shocks to the biosphere and the atmospheric carbon stock may have fat tails. Moreover, we extend our model to distinguish between capacity stocks for abatement, biosphere restoration, adaptation and removal A_t, B_t, N_t, R_t and actual abatement, biosphere restoration, adaptation and removal a_t, b_t, n_t, r_t . We require $a_t \leq A_t, b_t \leq B_t, n_t \leq N_t$ and $r_t \leq R_t$. With $X \in \{A, B, N, R\}$, the KKT multipliers are ν_t^X . Capacities are stocks with associated shadow prices λ_t^X . Investing into their accumulation is assumed to be costly according to some cost functions κ_X and

the stocks depreciate at rates δ_X . Hence, the stocks evolve according to the following.

$$K_{t+1} = (1 - \delta_K)K_t + I_t^K \quad (\text{B.34})$$

$$Z_{t+1} = Z_t + z_t - r_t - e(E_t) + \mathcal{E}_{t+1}^Z \quad (\text{B.35})$$

$$E_{t+1} = (1 + \delta_E)E_t - \gamma(E_t, Z_t) - \gamma_u(u_{t+1} - u_t) + b_t + \mathcal{E}_{t+1}^E \quad (\text{B.36})$$

$$u_{t+1} = u_t + l_t \quad (\text{B.37})$$

$$A_{t+1} = (1 - \delta_A)A_t + I_t^A \quad (\text{B.38})$$

$$B_{t+1} = (1 - \delta_B)B_t + I_t^B \quad (\text{B.39})$$

$$N_{t+1} = (1 - \delta_N)N_t + I_t^N \quad (\text{B.40})$$

$$R_{t+1} = (1 - \delta_R)R_t + I_t^R \quad (\text{B.41})$$

Emissions are as in the deterministic model's Equation (4) and net output similar to (5) but with the random shocks.

$$Y_t = F(K_t, E_t, n_t; \mathcal{E}_t^F)[1 - D(E_t, Z_t, n_t)] \quad (\text{B.42})$$

The budget equation is now

$$\begin{aligned} Y_t = c_t + I_t^K + c^A(a_t) + c^B(b_t) + c^L(l_t) + c^N(n_t) + c^R(r_t, u_t^R) \\ + \kappa^A(I_t^A) + \kappa^B(I_t^B) + \kappa^N(I_t^N) + \kappa^R(I_t^R) \end{aligned} \quad (\text{B.43})$$

Then, the Lagrangian is

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_0 & \left[\sum_t^{\infty} \beta^t \{ U(c_t) \right. \\
& + \mu_t \left(Y_t - c_t - c_A(a_t) - c_b(b_t) - c_N(n_t) - c_R(r_t, u_t^R) - I_t^K - \sum_{X=A,B,N,R} \kappa^X(I_t^X) \right) \\
& + \sum_{X=A,B,K,N,R} \lambda_t^X ((1 - \delta_X)X_t + I_t^X - X_{t+1}) \\
& + \lambda_t^L (u_{t+1} - l_t - u_t) \\
& + \eta_t (u_t - u_t^Y - u_t^R) \\
& + \lambda_t^E (E_{t+1} - (1 + \delta_E)E_t - \gamma(E_t, Z_t) + \gamma_u(u_{t+1} - u_t) - b_t - \mathcal{E}_{t+1}^E) \\
& + \lambda_t^Z (Z_{t+1} - Z_t - z(K_t, u_t^Y, a_t) + r_t + e(E_t) - \mathcal{E}_{t+1}^Z) \\
& \left. + \nu_t^A (A_t - a_t) + \nu_t^B (B_t - b_t) + \nu_t^N (N_t - n_t) + \nu_t^R (R_t - r_t) \} \right] \tag{B.44}
\end{aligned}$$

The FOCs are, for expenditures on consumption, investment, land conversion and options to address climate change and biosphere degradation,

$$\mathcal{L}_{c_t} = 0 = \frac{\partial U_t}{\partial c_t} - \mu_t \tag{B.45}$$

$$\mathcal{L}_{I_t^A} = 0 = -\mu_t \kappa'_A - \lambda_t^A \tag{B.46}$$

$$\mathcal{L}_{I_t^B} = 0 = -\mu_t \kappa'_B - \lambda_t^B \tag{B.47}$$

$$\mathcal{L}_{I_t^K} = 0 = -\mu_t - \lambda_t^K \tag{B.48}$$

$$\mathcal{L}_{I_t^N} = 0 = -\mu_t \kappa'_N - \lambda_t^N \tag{B.49}$$

$$\mathcal{L}_{I_t^R} = 0 = -\mu_t \kappa'_R - \lambda_t^R \tag{B.50}$$

$$\mathcal{L}_{l_t} = 0 = -\mu_t c'_{LC} + \lambda_t^L \tag{B.51}$$

$$\mathcal{L}_{a_t} = 0 = -\mu_t c'_A + \lambda_t^Z - \nu_t^A \tag{B.52}$$

$$\mathcal{L}_{b_t} = 0 = -\mu_t c'_B - \lambda_t^E - \nu_t^R \tag{B.53}$$

$$\mathcal{L}_{n_t} = 0 = \mu_t (Y_{n,t} - c'_N) - \nu_t^N \tag{B.54}$$

$$\mathcal{L}_{r_t} = 0 = -\mu_t c'_R + \lambda_t^Z - \nu_t^R \tag{B.55}$$

$$\nu^X \geq 0 \quad \text{for } X = A, B, N, R \tag{B.56}$$

for the allocation of land

$$\mathcal{L}_{u_t^R} = 0 = \mathbb{E}_{t-1} [-\mu_t c_{u,t}^R - \eta_t] \quad (\text{B.57})$$

$$\mathcal{L}_{u_t^Y} = 0 = \mathbb{E}_{t-1} [\mu_t Y_{u,t} - \eta_t] \quad (\text{B.58})$$

for the evolution of converted land, producible capital, natural capital and atmospheric carbon

$$\mathcal{L}_{u_t} = 0 = \mathbb{E}_{t-1} [\lambda_{t-1}^L - \beta(\lambda_t^L + \lambda_t^E \gamma_u - \eta_t) + \gamma_u \lambda_{t-1}^E] \quad (\text{B.59})$$

$$\mathcal{L}_{K_t} = 0 = \mathbb{E}_{t-1} [\mu_t \partial Y_t / \partial K_t - \lambda_{t-1}^K / \beta + \lambda_t^K (1 - \delta_K) - \theta_K] \quad (\text{B.60})$$

$$\mathcal{L}_{E_t} = 0 = \mathbb{E}_{t-1} [\mu_t \partial Y_t / \partial E_t + \lambda_{t-1}^E / \beta - \lambda_t^E (1 + \delta_E - \gamma_{E,t}) + \lambda_t^Z e'] \quad (\text{B.61})$$

$$\mathcal{L}_{Z_t} = 0 = \mathbb{E}_{t-1} [\mu_t \partial Y_t / \partial Z_t + \lambda_{t-1}^Z / \beta - \lambda_t^Z + \lambda_t^E \gamma_{Z,t}] \quad (\text{B.62})$$

and for the capacities for abatement, restoration, adaptation and removal

$$\mathcal{L}_{A_t} = 0 = \mathbb{E}_{t-1} [\nu_t^A + \lambda_t^A (1 - \delta_A)] - \lambda_{t-1}^A / \beta \quad (\text{B.63})$$

$$\mathcal{L}_{B_t} = 0 = \mathbb{E}_{t-1} [\nu_t^R + \lambda_t^B (1 - \delta_B)] - \lambda_{t-1}^B / \beta \quad (\text{B.64})$$

$$\mathcal{L}_{N_t} = 0 = \mathbb{E}_{t-1} [\nu_t^N + \lambda_t^N (1 - \delta_N)] - \lambda_{t-1}^N / \beta \quad (\text{B.65})$$

$$\mathcal{L}_{R_t} = 0 = \mathbb{E}_{t-1} [\nu_t^R + \lambda_t^R (1 - \delta_R)] - \lambda_{t-1}^R / \beta \quad (\text{B.66})$$

Recursive formulas for the SCC and SVN are obtained from (B.59), (B.61) and (B.62). To mark the difference to the deterministic model, we use an upper bar as notation.

$$\overline{SCC}_t = \mathbb{E}_t [M_{t,t+1} (SCC_{t+1} - Y_{Z,t+1} - \gamma_{Z,t+1} SVN_{t+1})] \quad (27)$$

$$\overline{SVN}_t = \mathbb{E}_t [M_{t,t+1} ((1 + \delta_E - \gamma_{E,t+1}) SVN_{t+1} + Y_{E,t+1} - e' SCC_{t+1})] \quad (28)$$

$$\overline{SVCL}_t = \mathbb{E}_t [M_{t,t+1} (SVCL_{t+1} + Y_{u,t+1} + \gamma_u (SVN_{t+1} - SVN_t / M_{t,t+1}))] \quad (29)$$

C. Discrete Regimes and Markov Transitions

Unlike standard Markov decision models in macroeconomics (Sargent and Stachurski, 2025), where transition probabilities are taken as exogenous, our constrained Markov decision process (CMDP) framework allows policy-dependent transition hazards (Altman, 1999). Abatement, CDR, adaptation and ecosystem restoration not only respond to risk; by changing the atmospheric carbon concentration and natural capital, they reshape the probability of entering high-damage regimes. This turns carbon and nature into fully fledged risk-bearing assets and yields hazard-adjusted Hotelling rules and regime-dependent option values. In this appendix we solve a CMDP as a primal–dual Lagrange problem. The CMDP allows for endogenous risk control because transitions (tipping hazards) depend on policy options. Abatement, adaptation, CDR and restoration affect both damages and the chance of falling into a bad regime. As a result we derive Bellman-like equations for the option values for abatement, adaptation and CDR.

C.1. Endogenous transition probabilities

We extend the deterministic model by two discrete regimes, "good" and "bad", in which the world can be at each point in time t , $r_t \in \{G, B\}$. The regime determines the severity of climate damages $D_i(Z_t, E_t, n_t)$ with $D_G < D_B$. The probability of being in regime i at time t is $\pi_t(i)$. The transition matrix is $P_{ij}(E_t, Z_t) = \Pr(r_{t+1} = j \mid r_t = i, E_t, Z_t)$, which depends on the state of the climate system and the biosphere. We, thus, work with the distribution $\pi_t(i)$:

$$\pi_{t+1}(j) = \sum_{i=G,B} \pi_t(i) P_{ij}(E_t, Z_t), \quad j = G, B \quad (\text{C.67})$$

To define the transition matrix, we introduce the hazard rate $h(E_t, Z_t) \in [0, 1]$ as the state-dependent probability that the regime changes from good to bad. We assume that with a stronger biosphere integrity and with lower carbon concentrations, the hazard rate is lower,

that is, $h_E(E_t, Z_t) < 0$ and $h_Z(E_t, Z_t) > 0$. Then, we define

$$P_{GG}(E_t, Z_t) = 1 - h(E_t, Z_t) \tag{C.68}$$

$$P_{GB}(E_t, Z_t) = h(E_t, Z_t) \tag{C.69}$$

$$P_{BB} = 1 \tag{C.70}$$

$$P_{BG} = 0. \tag{C.71}$$

The regime probabilities evolve as

$$\pi_{t+1}(G) = (1 - h(E_t, Z_t)) \pi_t(G), \tag{C.72}$$

$$\pi_{t+1}(B) = \pi_t(B) + h(E_t, Z_t) \pi_t(G), \tag{C.73}$$

C.2. Lagrangian Setup: Markov Constraints and Multipliers

We now introduce the endogenous transition probabilities in the social planner's optimization, which will replace the random shock we had introduced in Section 4. That is, we continue to use the model with capacities for abatement, removal and adaptation from Section 4 but set the random shocks \mathcal{E}^X , $X = E, F, Z$ to zero. Now, $\pi_t(G)$ and $\pi_t(B)$ are state variables and their transition rules (C.72) and (C.73) act as constraints with multipliers $\eta_t(G)$ and $\eta_t(B)$.

Then, the Lagrangian is

$$\begin{aligned}
\mathcal{L} = & \sum_t^{\infty} \beta^t \left\{ U(c_t) \right. \\
& + \mu_t \left[\pi_t(G)Y_t^G + \pi_t(B)Y_t^B \right. \\
& \quad \left. - c_t - c_A(a_t) - c_B(b_t) - c_{LC}(l_t) - c_N(n_t) - c_R(r_t, u_t^R) - I_t^K - \sum_{X=A,B,N,R} \kappa^X(I_t^X) \right] \\
& + \sum_{X=A,B,K,N,R} \lambda_t^X (X_{t+1} - (1 - \delta_X)X_t - I_t^X) \\
& + \lambda_t^L (u_t + l_t - u_{t+1}) \\
& + \eta_t (u_t - u_t^R - u_t^Y) \\
& + \lambda_t^E (E_{t+1} - (1 + \delta_E)E_t - \gamma(E_t, Z_t) + \gamma_u(u_{t+1} - u_t) - b_t) \\
& + \lambda_t^Z (Z_{t+1} - Z_t - z(K_t, u_t^Y, a_t) + r_t + e(E_t)) \\
& + \nu_t^A (A_t - a_t) + \nu_t^R (B_t - b_t) + \nu_t^N (N_t - n_t) + \nu_t^R (R_t - r_t) \\
& + \eta_t(G) [-\pi_{t+1}(G) + (1 - h(E_t, Z_t))\pi_t(G)] \\
& \left. + \eta_t(B) [-\pi_{t+1}(B) + \pi_t(B) + h(E_t, Z_t)\pi_t(G)] \right\} \tag{C.74}
\end{aligned}$$

Hence, compared to the first order conditions for E_t and Z_t from the previous section, Equations (B.61) and (B.62), we lose the expectation operator and obtain additional terms. The extra terms result from differentiating the last two lines with respect to E_t and Z_t . For the G -terms, we have

$$\frac{\partial}{\partial E_t} \eta_t(G) (1 - h(E_t, Z_t)) \pi_t(G) = -\eta_t(G) \frac{\partial h(E_t, Z_t)}{\partial E_t} \pi_t(G) \tag{C.75}$$

$$\frac{\partial}{\partial Z_t} \eta_t(G) (1 - h(E_t, Z_t)) \pi_t(G) = -\eta_t(G) \frac{\partial h(E_t, Z_t)}{\partial Z_t} \pi_t(G) \tag{C.76}$$

For the B -terms:

$$\frac{\partial}{\partial E_t} \eta_t(B) h(E_t, Z_t) \pi_t(G) = \eta_t(B) \frac{\partial h(E_t, Z_t)}{\partial E_t} \pi_t(G) \quad (\text{C.77})$$

$$\frac{\partial}{\partial Z_t} \eta_t(B) h(E_t, Z_t) \pi_t(G) = \eta_t(B) \frac{\partial h(E_t, Z_t)}{\partial Z_t} \pi_t(G) \quad (\text{C.78})$$

Adding the contributions and normalizing by the shadow price of the budget equation μ_t gives the Markov or tipping term in the first order conditions

$$\mathcal{M}_t^E = \pi_t(G) \frac{\partial h}{\partial E_t} (\eta_t(B) - \eta_t(G)) / \mu_t \quad (\text{C.79})$$

$$\mathcal{M}_t^Z = \pi_t(G) \frac{\partial h}{\partial Z_t} (\eta_t(B) - \eta_t(G)) / \mu_t \quad (\text{C.80})$$

C.2.1. Hotelling rules

With the tipping term in the equation of motion for the shadow prices of natural capital and carbon, λ^E and λ^Z , we obtain recursive formulas for the SCC and the SVN and add hats to emphasize the difference to the prices derived from the other model versions.

$$\widehat{SCC}_t = M_{t,t+1} \left[\left(\sum_{i=G,B} -\pi_{t+1}(i) Y_{Z,t+1}^i \right) + \widehat{SCC}_{t+1} + \gamma_{Z,t+1} \widehat{SVN}_{t+1} + \mathcal{M}_{t+1}^Z \right] \quad (44)$$

$$\widehat{SVN}_t = M_{t,t+1} \left[\left(\sum_{i=G,B} \pi_{t+1}(i) Y_{E,t+1}^i \right) + (1 + \delta_E - \gamma_{E,t}) \widehat{SVN}_{t+1} + e' \widehat{SCC}_{t+1} + \mathcal{M}_{t+1}^E \right] \quad (45)$$

The last term in the two equations captures the additional ‘‘hazard’’ or ‘‘tipping risk’’ effect through $\frac{\partial h}{\partial E}$ and $\frac{\partial h}{\partial Z}$. The formula for the social value of converted land does not change formally compared to the deterministic model, but since it depends on \widehat{SVN}_t , it also differs from the $SVCL$ in deterministic model.

$$\widehat{SVCL}_t = M_{t,t+1} \left[+ \left(\sum_{i=G,B} \pi_{t+1}(i) Y_{u,t+1}^i \right) + \widehat{SVCL}_{t+1} + \gamma_u (\widehat{SVN}_{t+1} - \widehat{SVN}_t / M_{t,t+1}) \right] \quad (\text{C.81})$$

C.2.2. Option values

As in the model version with random shocks, the options values of abatement, adaptation and CDR capacity are obtained from the first order conditions $\frac{\partial \mathcal{L}}{\partial X_t} = 0$ where $X = A, B, N, R$.

$$\Theta_t^X = M_{t,t+1} \left(\frac{\nu_{t+1}^X}{\mu_{t+1}} + (1 - \delta_X) \Theta_{t+1}^X \right) \quad (\text{C.82})$$

and via forward iteration

$$\Theta_t^A = \sum_{k \geq 1} M_{t,t+k} (1 - \delta_A)^{k-1} (SCC_{t+k} - c'_A(a_{t+k})) \quad (\text{C.83})$$

$$\Theta_t^B = \sum_{k \geq 1} M_{t,t+k} (1 - \delta_B)^{k-1} (SVN_{t+k} - c'_B(b_{t+k})) \quad (\text{C.84})$$

$$\Theta_t^N = \sum_{k \geq 1} M_{t,t+k} (1 - \delta_N)^{k-1} (Y_{Z,t+k} - c'_N(n_{t+k})) \quad (\text{C.85})$$

$$\Theta_t^R = \sum_{k \geq 1} M_{t,t+k} (1 - \delta_R)^{k-1} \left(SCC_{t+k} - \frac{\partial c_R}{\partial r_{t+k}} \right) \quad (\text{C.86})$$

The formulas appear identical up to the fact that the expectation operator is dropped. The more important difference, however, lies in the SCC and the SVN that are adjusted for the tipping-risk – as in Equation (45) and (44).

D. Model Synthesis

Combining the stochastic model with exogenously given random processes \mathcal{E}^X , $X = E, F, Z$ and the Markov model yields the stochastic Lagrangian with the endogenous tipping risk.

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \left[\sum_t^{\infty} \beta^t \left\{ U(c_t) \right. \right. \\
+ \mu_t \left(\pi_t(G)Y_t^G + \pi_t(B)Y_t^B - c_t - I_t^K - c_{LC}(l_t) - \sum_{X=A,B,N,R} (\kappa^X + c_X) \right) \\
+ \sum_{X=A,B,K,N,R} \lambda_t^X (X_{t+1} - (1 - \delta_X)X_t - I_t^X) \\
+ \lambda_t^L (u_{t+1} - l_t - u_t) \\
+ \eta_t (u_t - u_t^R - u_t^Y) \\
+ \lambda_t^E (E_{t+1} - (1 + \delta_E)E_t + \gamma(E_t, Z_t) + \gamma_u(u_{t+1} - u_t) - b_t - \mathcal{E}_{t+1}^E) \\
+ \lambda_t^Z (Z_{t+1} - Z_t - z(K_t, u_t^Y, a_t) + r_t + e(E_t) - \mathcal{E}_{t+1}^Z) \\
+ \nu_t^A (A_t - a_t) + \nu_t^B (B_t - b_t) + \nu_t^N (N_t - n_t) + \nu_t^R (R_t - r_t) \\
+ \eta_t(G) [-\pi_{t+1}(G) + (1 - h(E_t, Z_t))\pi_t(G)] \\
\left. \left. + \eta_t(B) [-\pi_{t+1}(B) + \pi_t(B) + h(E_t, Z_t)\pi_t(G)] \right\} \right] \tag{D.87}
\end{aligned}$$

Deriving the first order conditions is analogous to the above and yields the SCC and SVN as described in Section 6