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Pigou's advice and Sisyphus' warning: Carbon pricing with non-permanent carbon-dioxide removal

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Pigou's Advice and Sisyphus' Warning: Carbon Pricing with Non-Permanent Carbon Dioxide Removal

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Abstract

This paper develops a welfare and public economics perspective on optimal policies for carbon removal and storage (CDR) in permanent and non-permanent sinks. Non-permanent CDR reduces mitigation costs, even though the stored carbon is released into the atmosphere eventually. It may serve as bridge technology until permanent CDR becomes available. In contrast to permanent removals, non-permanent CDR does not reduce the optimal long-run temperature level. Its valuation differs from the social cost of carbon since a social cost of carbon removal arises from marginal damages caused by emissions released from non-permanent storage. We discuss three policy regimes that ensure optimal deployment of non-permanent CDR in terms of their informational and institutional requirements for monitoring, liability, and financing. (JEL: D61, H23, Q54, Q58)

Keywords: Carbon Dioxide Removal · Carbon Capture · Social Cost of Carbon · Climate Policy · Impermanence.

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Aye, and I saw Sisyphus in violent torment, seeking to raise a monstrous stone with both his hands. Verily he would brace himself with hands and feet, and thrust the stone toward the crest of a hill, but as often as he was about to heave it over the top, the weight would turn it back, and then down again to the plain would come rolling the ruthless stone.

— Homer, *Odyssey*

(...) [For] every industry in which the value of the marginal social net product is greater than that of the marginal private net product, there will (...) be one rate of bounty, the granting of which would have the optimum effect (...)

— A. C. Pigou, *The Economics of Welfare*

1. Introduction

By mid-century, most countries aim to become carbon neutral. However, carbon neutrality is not easily achieved on this timescale solely by reducing fossil fuel combustion. While low-carbon substitutes exist for electricity generation from fossil resources, other economic activities, such as aviation or the production of steel, cement, and chemicals, are difficult to decarbonize.

Therefore, scenarios with deep decarbonization and low-temperature targets rely on carbon dioxide removal (CDR) technologies. In some scenarios that limit global warming to 1.5°C, for example, the Intergovernmental Panel on Climate Change estimates that up to 20 GtCO₂, equivalent to about half of the current carbon emissions, have to be removed annually in the second half of the 21st century (Masson-Delmotte et al. 2018).

Despite the significant role projected for CDR in these scenarios, research on the policies that govern CDR is still in its infancy.¹ In contrast, a vast literature has explored policies to reduce fossil emissions, but its applicability to the case of carbon removal is limited.² One key difference is that, in general, the benefit of emission reductions is permanent (since it corresponds to the permanent reduction of climate damage) while the benefit of

1. Research on governance of forest management comes closest, for example Tahvonen (1995) who shows that optimal forest management can be implemented by subsidizing the carbon accumulation at the level of the (exogenously given) carbon tax when simultaneously taxing emissions from using timber. Interestingly, Tahvonen finds that differentiating the carbon intensity of timber use is optimal – similar to non-permanence – though storage time is not explored. Subsequent research on carbon sequestration in forests added consideration of additionality, permanence and leakage, see Sedjo and Sohngen (2012) for an overview. Some methodological similarities can be found in research on carbon capture and storage in combination with fossil fuel use (Rickels and Lontzek 2012; Moreaux and Withagen 2015; Moreaux et al. 2024). Further research on the specifics of CDR policies is found in Franks et al. (2022) who analyze optimal pricing policies for CDR under inter-regional leakage. Furthermore, an emerging literature assesses institutional and governance questions of CDR and its inclusion into carbon markets in particular, see Rickels et al. (2021, 2022); Edenhofer et al. (2024).

2. Foundations go back to the seminal works on taxation of pollution, see e.g. Pigou (1920), Farzin (1996) and Hoel and Kverndokk (1996). See Edenhofer et al. (2021) for a survey of more recent work.

Removal and storage pathway	Storage duration (<i>half-life</i>)
Direct air capture or bioenergy with geological CCS (DACCS/BECCS)	Millennia
Enhanced weathering	Centuries
Biochar	Centuries
Forestry techniques & wood products	Decades to centuries
<i>Single family home</i>	100
<i>Furniture, residential upkeep and improvement</i>	30
<i>Paper</i>	2
Soil carbon sequestration techniques	Years to decades

TABLE 1. Storage time for different CO₂ removal technologies. Based on [Smith \(2006\)](#); [Hiraishi et al. \(2014\)](#); [Hepburn et al. \(2019\)](#); [Smith et al. \(2023\)](#).

carbon removal may only be temporary. The duration of carbon storage varies significantly across removal technologies. For example, storage in geological formations, which is used by technologies such as direct air capture or biomass energy with carbon capture and storage, is close to permanent.³ Land-based storage technologies, such as afforestation and reforestation, or storage options related to carbon capture and utilization are less permanent (Table 1).⁴

The non-permanence of carbon storage raises an important question: If the removed and stored carbon is returned to the atmosphere eventually, what (if anything) can CDR with non-permanent storage contribute to optimal global warming? Without knowledge of the optimal deployment of CDR and the appropriate policy instruments, regulators run the risk

3. The literature on carbon capture and storage (CCS) in leaky geological reservoirs investigates some of the same questions as our study, such as when CCS is part of the economically efficient strategy, and how non-permanence should be accounted for. Our study, however, considers non-permanence of storage more broadly, i.e., much shorter storage durations than in the case of geological storage, and, beyond the question of efficient storage, focusses on policy instrument and their implementation. Closest to our analysis is [Van Der Zwaan and Gerlagh \(2009\)](#), who analyze CCS effectiveness and compare the net present value of CCS with leaky storage with the social cost of carbon (SCC). In contrast to our study, they use exogenously given SCC. Similar characterizations of CCS effectiveness are provided in [Ha-Duong and Keith \(2003\)](#) and by [García and Torvanger \(2019\)](#), who interpret the effectiveness as a trading ratio that could account for leakage of CCS in an emissions permit trading system. [Van Der Zwaan and Gerlagh \(2009\)](#) focus on the implication of leakage on the efficient use of CCS, reporting substantial deployment in simulations with leakage rates that exceed the low rates expected for geological storage. Similarly, [Torvanger et al. \(2012\)](#) find that, despite leakage, a substantial share of fossil reserves can be stored for a 2.5° target, and the vast majority in case of a 3° target. Also, [Ha-Duong and Keith \(2003\)](#) conclude that leakage rates "one order of magnitude below the discount rate" are negligible. Later studies extending the scope to subsurface leakage [Deng et al. \(2017\)](#), or ocean sequestration [Van Der Zwaan and Gerlagh \(2016\)](#) confirm the result of a substantial role for CCS with the low leakage rates expected for geological storage.

4. Depending on the management strategy, half-lives of land-based storage range from decades to centuries. Storage options related to carbon capture and utilization involve storing carbon in the productsphere. Typical examples include the production of bioplastics, harvested wood products and synthetic fuels ([Hepburn et al. 2019](#)). While some of these products are short-lived, others can achieve storage durations comparable to other carbon removal technologies (Table 1). For instance, the use of wood in the construction of new buildings could store carbon for centuries and has the potential to sequester up to 73 GtCO₂ over the next thirty years ([Churkina et al. 2020](#)).

of allowing an over-utilization of CDR or might fail to set incentives for an urgently needed set of technologies.

Several studies have made an effort to differentiate between avoided and removed emissions. As a common theme, the literature finds that despite the non-permanence of storage, carbon removal should be carried out but at a subsidy that is corrected for the non-permanence, that is, lower than for permanent storage.⁵ Such correction factors frequently depend on the discount rate for future payments, the storage duration, and the development of the carbon price. Consequently, assumptions about carbon prices, discount rates, and the growth rate of the economy are crucial.

However, in most studies, carbon prices, storage duration, the time paths of the temperature and the social cost of carbon or emission and removal quantities remain exogenous.⁶⁷ Additionally, previous work has not analyzed the informational, institutional and liability aspects of different policy approaches to incentivize carbon removal in a consistent formal model. The question thus remains how CDR with non-permanent storage should be incentivized when long-term climate stabilization is driven by optimal carbon pricing.

We address this question for the first time by introducing a welfare maximization approach with non-permanent carbon removal technologies in a dynamic partial equilibrium model. In the model, economic activity requires polluting fossil energy inputs. Carbon emissions accumulate in the atmosphere and cause climate change damages. For removal, we introduce a set of CDR technologies, which differ in their marginal cost and permanence.

We use the model to characterize the optimal use of emission abatement, carbon removal technologies and optimal Pigouvian carbon pricing regimes. Our contribution to the literature arises from the use of the dynamic optimization framework, which allows us to study endogenous cost-benefit trade-offs and equilibrium effects, as well as optimal Pigouvian policies.

5. [Brander and Broekhoff \(2023\)](#) stress the need for adequately accounting for future released emissions and call for policies to incentivize temporary removals to buy time until carbon can be permanently stored at lower cost. [Groom and Venmans \(2023\)](#) discuss the social value of carbon offsets. When offsets are generated by removal with non-permanent storage, their value is still positive but falls below the value of avoided or permanently removed emissions. In a similar vein, [Kim et al. \(2008\)](#) and [van Kooten \(2009\)](#) compute price discounts for non-permanent carbon offsets that factor in the additional cost of released emissions. [Rickels et al. \(2018\)](#) recognize a difference between social costs due to atmospheric carbon and ocean carbon cycle feedbacks in a numerical integrated assessment exercise.

6. This is also the case in [Matthews et al. \(2022\)](#) who analyze the climate implications of a perpetual and increasing carbon removal with storage in non-permanent sinks.

7. There are two notable exceptions. [Lemoine \(2023\)](#) suggests a carbon stock tax that emitters pay as a rental charge for storing carbon in the atmosphere until they remove it again. The incentive to remove emissions is impaired when firms can forego the rental charge in case of bankruptcy. To address this moral hazard, Lemoine introduces up-front payment via bonds and tradable carbon shares. [Belfiori \(2017\)](#) considers the case of differing social and private discount rates, which implies two reasons for policy intervention – the climate externality and intertemporal distribution. Thus, while a CDR subsidy should be set at the social cost of carbon, the carbon tax is set above this Pigouvian level to achieve the desired intertemporal wealth distribution. However, neither [Lemoine](#) nor [Belfiori](#) consider where or how carbon is stored after removal or issues of non-permanence.

As a key result of our welfare maximization approach, we find that using both CDR with permanent and non-permanent storage is welfare-enhancing. Permanent CDR technologies are used to offset residual emissions in the long run. Carbon removal with temporary storage reduces mitigation costs along the transition. However, to an extent determined by the discount rate, these costs are shifted to future generation: emissions released from storage need to be removed from the atmosphere in perpetuity. Thus, even though net carbon emissions are zero in the long-run steady state, the emissions released from the storage sites of past removal activities commit the economy to continuous removal in the steady state, that is, to "paying back a carbon debt". The commitment to perpetual carbon removal is evocative of Sisyphus pushing a heavy stone to the top of the mountain only to watch it roll down again.

While the availability of non-permanent CDR is welfare enhancing, the optimal long-term global temperature is not affected by their availability or their costs. Non-permanent CDR creates an additional carbon sink that allows the economy to reach the same optimal temperature target with additional cumulative use of fossil resources and lower cumulative climate damages. By the same token, we find that changes in marginal damages have no effect on the long-term level of non-permanent carbon removal.

As long as permanent CDR technologies are not available, our analysis suggests that non-permanent CDR has a limited role as bridge technology. In this case, its deployment follows an inverted U-shape. The optimal temperature also peaks at a higher level and then returns to a lower steady state level. Thus, our model rationalizes overshoot scenarios (IPCC 2022) in a cost-benefit analysis.

We further find that non-permanent carbon removal introduces a new social cost of carbon metric, the social cost of (non-permanent) carbon removal (SCC_R). Whereas the conventional social cost of carbon emissions (SCC_E) is a measure of the marginal damage associated with emitting an additional unit of emissions into the atmosphere, the SCC_R is a measure of the marginal damage resulting from releasing emissions from storage. Hence, the SCC_R is a measure of the cost of a delayed carbon emission. The cost arises from the time of removal on. It is, thus, an intertemporal cost. In addition, we include in our discussion the intertemporal costs associated with the necessary diligence to maintain a certain level of the release rate. Together, the SCC_R and the intertemporal diligence cost IDC make up the intertemporal cost of carbon removal ICR . These metrics turn out to be central concepts for the design of tax and subsidy policies.

In general, the regulator can incentivize the optimal use of CDR by paying a Pigouvian subsidy on the flow of removed emissions or the stock of stored carbon. If the regulator can tax released emissions, the optimal subsidy for removal is equal to the tax on carbon emissions, which is equal to the SCC_E (*downstream pricing*). Alternatively, the regulator can pay a subsidy on the stock of removed carbon (*storage subsidy*).⁸ Theoretically, the regulator could forgo the effort of monitoring storage size or released emissions by reducing the subsidy on removal relative to the carbon tax on emission (*upstream pricing*).⁹ The

8. The idea of a rental charge on a carbon stock is similar to the rental charge on atmospheric carbon in Lemoine (2023) or the rental payments on carbon sequestered in forests in Sohngen and Brown (2008)

9. Similar to the non-permanence related correction factors discussed above.

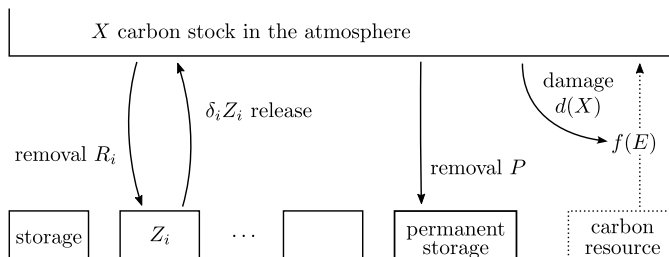


FIGURE 1. Model structure

optimal correction factor for the removal subsidy has to take future released emissions into account and is determined by the SCC_R . It is decreasing in the release rate and increasing in the discount rate. However, if diligence matters for the release rate, then an efficient policy requires knowledge of the release flows. Hence, schemes initially intended as upstream effectively become equivalent to downstream schemes in terms of their informational requirements. We show this in a decentralized version of our model.

Therefore, the decentral model reveals the importance of monitoring, reporting and verification (MRV) of carbon removal and storage for the implementation of policies. Besides MRV, moral hazard may also complicate their implementation. Under optimal downstream pricing, for example, firm may run a deficit in the long-run and moral hazard becomes a problem. Since firms are judgement proof, that is, limited in their liability for released emissions, they could go bankrupt strategically and shift the burden of the social cost of carbon removal to taxpayers.

Our paper is structured as follows. The next section puts forward the model. Here, we derive the concept of the social cost of carbon removal and characterize the steady state as well as the transitional dynamics. In Section 3, we describe three policy regimes and discuss optimal carbon prices and subsidies. Here, we also describe how implementation of policies is complicated by informational requirements for the regulator, the financial flows, liability and moral hazard. Section 4 concludes.

2. Model

We first develop a social planner model that serves as a benchmark for an optimal allocation in the economy. The social planner model characterizes optimal removal quantities and stocks during the transition and in the long run. The transition to a carbon neutral economy is driven by increasing climate damage. The social planner solution also describes shadow prices associated with carbon flows. The shadow prices are fundamental to understanding the value of carbon dioxide removal and emissions abatement and constitute the foundation for calculating optimal taxes and subsidies in the decentralized economy. Figure 1 gives an overview of the model structure.

Economic output $\tilde{f}(E(t), t) = f(E(t))\Delta^E(t)$ can be obtained by using fossil-based energy, which causes a flow of emissions $E(t)$. We abstract from the possibility of capturing

emissions at the point of combustion to decouple fossil fuel use from emissions. We assume an aggregate production technology with decreasing returns ($f' > 0$, $f'' < 0$) with productivity growth $\Delta^E(t) \in [1, \infty)$.¹⁰ We further assume that fossil energy is supplied at zero costs.¹¹ The use of fossil energy causes emissions that accumulate in the atmospheric carbon stock $X(t)$. We assume that the average global surface temperature depends monotonically on $X(t)$ and thus that an optimal atmospheric CO₂ concentration is equivalent to an optimal temperature target. The atmospheric carbon concentration leads to climate damage $d(X(t))$ with $d', d'' > 0$. Carbon can be removed from the atmosphere in two different ways. First, it can be removed permanently at rate $P(t)$, which comes at cost $\tilde{h}(P(t), t) = h(P(t)) \Delta^P(t)$, with $h', h'' > 0$ and technological progress $\Delta^P(t) \in (0, 1]$. Second, there are $i \in \{1, \dots, n\}$ removal technologies with non-permanent storage for which removing at rate $R_i(t)$ comes at a cost $\tilde{g}_i(R_i(t), t) = g_i(R_i(t)) \Delta^{R_i}(t)$ with $g'_i, g''_i > 0$. We assume that technological progress $\Delta^{R_i}(t) \in (0, 1]$ leads to cost reductions over time and that $g'_i(0) \geq 0$ is very low or zero.¹² The removed carbon is stored in the technology-specific storage stocks $Z_i(t)$. These non-permanent sinks release carbon back into the atmospheric stock $X(t)$ at rate $\delta_i(t) > 0$.¹³ While our model is deterministic, $\delta_i(t)$ may also be interpreted as the expected release rate at time t and, thus, as an endogenously chosen probability of emission release. In addition, we assume that maintaining a unit of storage at a given (expected) release rate $\delta_i(t)$ requires costly *diligence*, which comes at marginal cost $\tilde{w}_i(\delta_i(t)) = w_i(\delta_i(t)) \Delta^{\delta_i}(t)$ with $w'_i < 0$, $w''_i \geq 0$, $\lim_{\delta \rightarrow 0} w_i(\delta) = \infty$ and technological progress $\Delta^{\delta_i}(t) \in (0, 1]$, which reduces costs over time. In order to avoid unnecessary complexity, we assume no explicit physical limit to storage capacity for permanent and non-permanent CDR. Instead, we follow [Lemoine \(2023\)](#) and include capacity limits conceptually in the assumption of the convexity of the removal costs. The above assumptions on functional forms ensure a solution with $X(t) > 0$ and $Z_i(t) > 0$ for all t .

Where no ambiguity arises, we suppress the time index t to improve readability. The social planner maximizes intertemporal welfare

$$\max_{E, P, R_i, \delta_i} \int_0^{\infty} \left[\tilde{f}(E) - \tilde{h}(P) - \sum_i \tilde{g}_i(R_i) - \sum_i \tilde{w}_i(\delta_i) Z_i - d(X) \right] e^{-rt} dt \quad (1)$$

10. By allowing productivity growth Δ^E , our model can accommodate scenarios with very small marginal benefits of fossil resource use in the future and thus very low long run levels of emissions. This represents an economy that is largely decarbonized without the need for explicitly introducing renewable energy in our model.

11. This assumption implies that fossil fuel demand is only constrained by climate damages, not by extraction costs or the physical availability of fossil energy.

12. Afforestation, reforestation and soil carbon sequestration are examples for technologies with very low (and potentially even negative) costs ([Smith et al. 2023](#)).

13. Note that we focus on CDR, that is, deliberate human and costly interventions to remove carbon from the atmosphere. We abstract from natural mechanisms by which carbon moves from the atmosphere to some other sink, such as the ocean, as modeled, for example, by [Rickels and Lontzek \(2012\)](#) and [Rickels et al. \(2018\)](#).

$$\text{such that } \dot{X} = E - P - \sum_i R_i + \sum_i \delta_i Z_i \quad \perp \mu \quad (2)$$

$$\dot{Z}_i = R_i - \delta_i Z_i \quad \perp \psi_i \quad (3)$$

Initial values are given by $X(0) = X_0$ and $Z_i(0) = Z_{i,0}$. The shadow prices associated with X and Z_i are μ and ψ_i . The optimal solution is characterized by the following first-order conditions.

$$f'(E)\Delta^E = -\mu \quad (4)$$

$$h'(P)\Delta^P = -\mu \quad (5)$$

$$g'_i(R_i)\Delta^{R_i} = \psi_i - \mu \quad (6)$$

$$w'_i(\delta_i)\Delta^{\delta_i} = -(\psi_i - \mu) \quad (7)$$

$$\dot{\mu} = r\mu + d'(X) \quad (8)$$

$$\dot{\psi}_i = (r + \delta_i)\psi_i - \delta_i\mu + \tilde{w}_i(\delta_i) \quad (9)$$

$$0 = \lim_{t \rightarrow \infty} \mu(t)X(t)e^{-rt} \quad (10)$$

$$0 = \lim_{t \rightarrow \infty} \psi_i(t)Z_i(t)e^{-rt} \quad (11)$$

The shadow prices μ and ψ_i each have an important economic intuition. The former measures the marginal damage of emitting one ton of carbon (social cost of carbon emissions). The latter measures two types of intertemporal cost associated with carbon removal. First, it measures the marginal damage associated with the release from initially stored carbon over future periods. The second is the intertemporal cost for diligence due to the initially removed marginal unit of carbon.¹⁴

PROPOSITION 1. *Assume that in the social optimum, $X(t) > 0$ and $Z_i(t) > 0$ for all t . The social cost of a carbon emission (SCC_E) is*

$$SCC_E(t) = -\mu(t) = \left[\int_t^\infty d'(X(s)) e^{-r(s-t)} ds \right] \quad (12)$$

The intertemporal cost of carbon removal (ICR_i) is positive and given as the sum

$$ICR_i(t) = -\psi_i(t) = \int_t^\infty [\delta_i(s)SCC_E(s) + \tilde{w}_i(\delta_i(s))] e^{-\int_t^s (r+\delta_i(v))dv} ds \quad (13)$$

$$= \underbrace{\int_t^\infty \delta_i(s)SCC_E(s) e^{-\int_t^s (r+\delta_i(v))dv} ds}_{\text{Social Cost of Carbon Removal (SCC}_{R,i})} + \underbrace{\int_t^\infty \tilde{w}_i(\delta_i(s)) e^{-\int_t^s (r+\delta_i(v))dv} ds}_{\text{Intertemporal diligence cost (IDC}_i)} \quad (14)$$

Proof. See Appendix A.1. □

14. Our shadow price ψ_i bears some resemblance to the co-state variable π of oceanic storage introduced by Rickels and Lontzek (2012). However, two important differences set them apart. First, we model endogenous diligence. Second, in our model there is no natural flow from the atmosphere to the storage stocks Z_i .

The proposition indicates that considering non-permanent carbon removal creates two distinct social costs of carbon – the conventional measure for the social damage of emitting one ton of carbon, and a new measure accounting for the social damage caused when this ton is released back into the atmosphere over time – the social cost of carbon removal $SCC_{R,i}$. However, next to the SCC_R , there is a second type of intertemporal costs: the diligence cost due to the storage of an impermanently removed ton of carbon in all future periods.

An intuitive reformulation of the optimality condition for non-permanent removals is, thus, $\tilde{g}'_i(R_i) = SCC_E - SCC_{R,i} - IDC_i$. Figure 2 shows how each of the cost components associated with technology i lowers the optimal amount of non-permanent CDR.

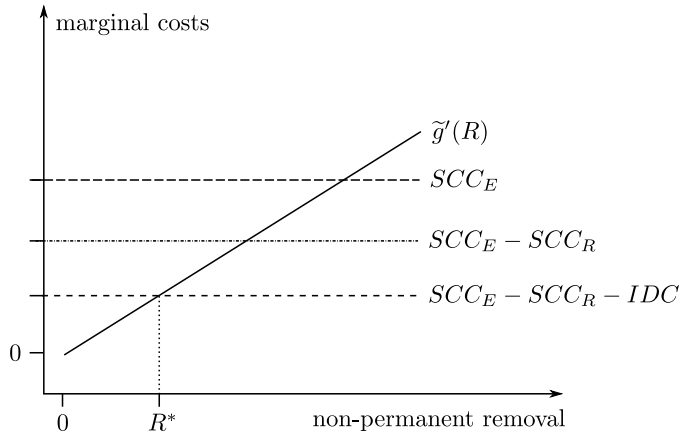


FIGURE 2. Optimality condition for non-permanent removals

Optimal non-permanent removal and optimal diligence are closely related as stated in the following lemma, which will be useful for subsequent analysis.

LEMMA 1. *Optimal non-permanent removal and diligence are complements, that is, optimal release rates decrease as removal increases.*

Proof. From (6) and (7) we find that $\frac{d\delta_i}{dR_i} = -\frac{g''_i(R_i)\Delta^R(t)}{w''(\delta_i)\Delta^{\delta_i}(t)} < 0$. □

Thus, the optimal quantity of non-permanent removal and the optimal level of diligence move in tandem. If, for example, the optimal level of non-permanent removal is initially low and then ramped up over time, then so is the optimal level of diligence.

2.1. Steady state

A steady state in both stocks and co-state variables is feasible if we assume that for some \hat{t} , technological progress comes to a halt. That is, $\dot{\Delta}^E(t) = \dot{\Delta}^P(t) = \dot{\Delta}^{R_i}(t) = 0$ for all $t \geq \hat{t}$.¹⁵ Then, we have $0 = \dot{X} = \dot{\mu} = \dot{Z} = \dot{\psi}$, which implies

$$E^s = P^s \quad (15)$$

$$R_i^s = \delta_i^s Z_i^s \quad (16)$$

$$SCC_E^s = -\mu^s = \frac{d'(X^s)}{r} \quad (17)$$

$$ICR_i^s = -\psi_i^s = \frac{\delta_i^s d'(X^s)}{r(r + \delta_i^s)} + \frac{\tilde{w}_i(\delta_i^s)}{r + \delta_i^s} \quad (18)$$

Assuming sufficiently low marginal removal costs $\tilde{g}_i'(0)$ for the first unit, we further obtain from the FOCs

$$\tilde{f}'(E^s) = \frac{d'(X^s)}{r} = \tilde{h}'(P^s) \quad (19)$$

$$\tilde{g}_i'(R_i^s) = \frac{d'(X^s) - \tilde{w}_i(\delta_i^s)}{r + \delta_i^s} \quad (20)$$

$$-\tilde{w}_i'(\delta_i^s) = \frac{d'(X^s) - \tilde{w}_i(\delta_i^s)}{r + \delta_i^s} \quad (21)$$

Equation (15) states that there is a constant level of residual emissions E^s that is exactly offset by permanent removals P^s . The level of residual emissions depends only on marginal benefits \tilde{f}' and marginal removal costs \tilde{h}' . The functional forms of \tilde{f} and \tilde{h} , thus, determine $E^s = P^s$ entirely. Residual emissions are independent of the release rate δ_i , non-permanent removal costs \tilde{g}_i , climate damages, and the discount rate r . Figure 3 illustrates that higher marginal benefits of emissions or lower marginal costs of permanent removal lead to an increase in residual emissions – and vice versa. Even if marginal permanent removal costs become almost prohibitively high, that is, the marginal cost curve \tilde{h}' is shifted up substantially, the qualitative results remains the same. It also illustrates the requirement that the optimal steady state level of carbon in the atmosphere X^s must be such that the social cost of carbon emissions SCC_E^s equals the marginal benefits of emissions and the marginal cost of permanent removal.

Importantly, equation (16) states that in the long-run, the flow of non-permanent carbon removal is equal to the flow of released emissions $\delta_i^s Z_i^s$ from the non-permanent stock. Thus, non-permanent carbon removal does not provide any net-removals from the atmosphere in the steady state. Non-permanent carbon removal has to offset the emissions released from

15. In Appendix A.3, we show for simple functional forms and zero diligence costs that the steady state exhibits saddle path stability.

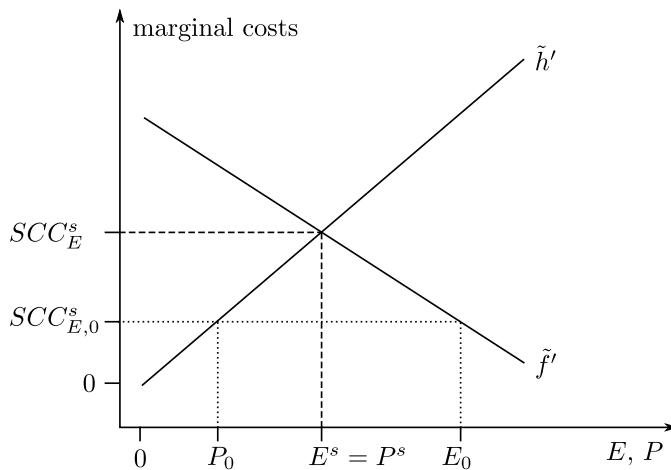


FIGURE 3. In an optimal steady state, the atmospheric carbon stock adjusts so that the SCC_E are at the unique level at which permanent removals exactly offset residual emissions.

the storage stock in perpetuity. This result is evocative of Sisyphus' task of pushing a heavy stone to the top of a mountain just to watch it roll down and having to start all over again.

Non-permanent CDR appears, thus, as double-edged sword: it facilitates short-term welfare gains during the transition but commits future generations to continuously return released emissions back to their reservoirs. Similar to Sisyphus's task, non-permanent CDR creates a perpetual "carbon debt" to future generations that consists of undertaking removal into leaky reservoirs.

Performing further comparative statics analyses of the steady state reveals several key insights. We summarize the most relevant findings in the following propositions and present a comprehensive and systematic overview in Appendix B. The first result that merits attention concerns the role of non-permanent CDR for the optimal temperature target and the social cost of carbon.

PROPOSITION 2 (Independence of optimal temperature target from non-permanent CDR). *In the cost-benefit analysis, the properties of non-permanent CDR technologies do not influence the optimal temperature and the social cost of carbon in the steady state. The steady state level of the carbon stock in the atmosphere X^s remains invariant under changes in the characteristics of the non-permanent removal option, that is, the costs of diligence \tilde{w}_i and the removal costs \tilde{g}_i .*

Proof. Follows directly from (19) and (17), which imply that the functional forms of \tilde{f} , \tilde{h} and the damage function d determine the steady state level of the carbon stock. There is only one level $d'(X^s)/r$ at which marginal benefits and marginal costs of permanent removal intersect. Due to the monotonicity of d' , the final results is obtained. \square

Intuitively, non-permanent CDR is not equivalent to an abatement technology that can be used to lower emissions once and forever relative to a baseline. The availability of non-permanent CDR introduces a trade-off along a different margin. It creates an additional carbon sink that gives the economy the possibility to reach a given temperature target with additional cumulative use of emissions and lower cumulative climate damages. We show this in the following corollary formally.

COROLLARY 1. *Assume that for the non-permanent removal technology i in the steady state $Z_i^s > 0$. Then the use of this technology yields higher cumulative emissions and lower cumulative climate damages than if it is not used.*

Proof. See Appendix A.2. □

By the same token, changes in marginal climate damages have no effect on the steady state level of non-permanent removal and storage.

PROPOSITION 3 (Comparative statics for non-permanent removal and storage with definite sign). *The steady-state levels of non-permanent carbon removal R_i^s and storage Z_i^s remain constant under a shift of the marginal climate damage function d' .*

Proof. Any increase in marginal damages results in a reduction of X^s such that $d'(X^s)$ remains constant due to (19). Equation (21) can be rearranged so that the term $d'(X^s)$ is on one side and the other side is a function of δ_i^s only. Hence, the increase in marginal damages also leave δ_i^s unchanged. Finally, using (20), we conclude that R^s and via (16) also Z^s have to remain constant. □

As we have seen, properties of non-permanent removal do not affect the optimal temperature target. Due to (19), the optimal residual emissions and permanent CDR levels are also unaffected. However, if the marginal costs of permanent CDR shift, then this does lead to changes in the usage of non-permanent CDR. The direction of the change is technology-dependent. In particular, it depends on the functional forms for the costs associated with non-permanent CDR \tilde{g}_i and \tilde{w}_i , as summarized in the next proposition.

PROPOSITION 4 (Comparative statics for non-permanent removal and storage with ambiguous sign). *Shifts of marginal removal costs for permanent and non-permanent CDR \tilde{h}' and \tilde{g}'_i , marginal benefits of emissions \tilde{f}' and marginal diligence costs \tilde{w}_i lead to an ambiguous change in non-permanent removal R_i^s , storage Z_i^s and diligence δ_i^s . The direction of the change depends on the marginal costs associated with non-permanent removal.*

Proof. Due to (19), a higher marginal cost function \tilde{h}' or a higher marginal benefit function \tilde{f}' translate to a higher level X^s . Now, reformulate (20) to see that

$$d'(X^s) = \tilde{g}'_i(R_i^s)r + \tilde{g}'_i(R_i^s)\delta_i^s + \tilde{w}_i(\delta_i^s). \quad (20')$$

Diligence and R_i^s are complements (Lemma 1). Thus, a change in R_i^s leads to a change of the opposite direction in δ_i^s . Whether an increase or a decrease of R_i^s is required to match an increase of the left hand side depends on the functional forms of \tilde{g}_i and \tilde{w}_i .

Moreover, changes in \tilde{g}_i' or \tilde{w}_i leave X^s constant, thus, the left hand side of the above equation, too. The right hand side, however, changes. The direction of change for R_i^s depends on the specific functional forms of \tilde{g}_i and \tilde{w}_i .

The ambiguity also hold for Z_i^s due to (16). □

To illustrate the intuition behind the latter result, consider a non-permanent CDR technology that does not depend on diligence. Then an increase in immediate technical removal costs \tilde{g}_i' has the unambiguous effect of reducing the steady state level of removal activity R_i^s . However, if diligence matters, then the social planner will either reduce removal and diligence or increase the two since they are complements (Lemma 1). Reducing removal, for example, reduces immediate costs. However, it also means less diligence and thereby a higher steady-state release rate. Thus, the intertemporal cost of carbon removal increases. Without specifying functional forms it remains unclear whether an adjustment in one direction yields a cost reduction or an increase of costs. The effects of an upward shift in marginal diligence costs can be described analogously.

In the cases of an upward shift of \tilde{h}' and \tilde{f}' , there is an increase of X^s and hence an increase of the left side of (20'). If the immediate technical costs of removal $\tilde{g}'(R^s)$ are relatively steep at R^s and the marginal diligence costs $\tilde{w}(\delta^s)$ are relatively steep at δ^s , then an increase in marginal costs of permanent removal leads to an increase of non-permanent CDR R^s and of diligence. The reason is that in this case the increases in the first and the third term in (20') overcompensate the decrease of δ^s in the second term. If the costs are rather flat, then the opposite would hold.

Finally, we can use the above results to compare how the relative amounts of permanent and non-permanent CDR in the steady state change under variation of the model parameters. We, thus, consider the quotient $\frac{R_i^s}{P^s}$ as indicator for the CDR portfolio.

COROLLARY 2 (CDR portfolio).

1. *When the release rate increases, the fraction of permanent CDR in the portfolio increases.*
2. *A shift of the marginal climate damage function has no effect on the composition of the CDR portfolio.*
3. *Shifts in the functions for marginal removal costs \tilde{g}_i' and \tilde{h}' , marginal diligence costs \tilde{w}_i and marginal benefits \tilde{f}' as well as changes of the discount rate r have an ambiguous effect on the portfolio composition.*

Proof.

1. Due to Equation (19), only the functional forms of \tilde{f} and \tilde{h} determine P^s . The release rate does not affect it. Increases in the latter only decrease R_i^s (Lemma 1). This proves the first statement.

2. The second statement directly follows from the above and Proposition 3.
3. The results on ambiguity follow from (19) and Propositions 4 and B.3.

□

2.2. Transitional dynamics

Transitional dynamics can be described by the growth rates of the two shadow prices μ and ψ_i for the carbon stock in the atmosphere and the removal stocks, respectively. They follow from the optimality conditions (8) and (9). We denote the growth rates by hats:

$$\hat{\mu} = r + \frac{d'(X)}{\mu} \quad (22)$$

$$\hat{\psi}_i = r + \delta_i \left(1 - \frac{\mu}{\psi_i} + \frac{\tilde{w}_i(\delta_i)}{\delta_i \psi_i} \right) \quad (23)$$

Together, the arbitrage conditions (22) and (23) determine the optimal use of the atmosphere, permanent storage reservoirs, and non-permanent storage sites for carbon storage. Their optimal valuations ($-\mu$ and $-\psi_i$) increase in the discount rate r such that (all else equal) an increase of the discount rate leads to an accelerated deployment of CDR and a more rapid use of fossil fuels. With the additional terms in (22) and (23) it follows that in the optimum, the social cost of carbon emissions, $-\mu$, grows at a rate lower than the discount rate r because climate change damages make a slower increase of emissions in the atmosphere more desirable (Goulder and Mathai 2000). Similarly, the intertemporal cost of carbon removal $ICR_i = -\psi_i$ grows at a rate lower than $r + \delta_i$ due to climate damages and diligence costs.

Transitional dynamics also depend crucially on technological progress Δ^k , where $k \in \{E, P, R, \delta\}$, which leads to cost reductions of emissions abatement, permanent and non-permanent CDR as well as diligence. In particular, permanent removal grows relatively fast when technological progress in removal technologies is relatively large compared to progress in mitigation technologies – and vice versa.¹⁶ Similarly, for non-permanent removal, the time path of the optimal allocation of resources toward removal $\tilde{g}(R_i)$ and diligence $\tilde{w}_i(\delta_i)$ depends on cost reductions of each activity. We already know that the optimal values of the two variables are negatively correlated (Lemma 1). If costs for diligence fall faster than those due to the removal process, then the release rate falls relatively fast and the removal rate

16. We can also illustrate this statement mathematically. In particular, from (4) and (5) we find that $f'(E) = h'(P) \frac{\Delta^P(t)}{\Delta^E(t)}$. Convenient forms for conveying the intuition are, for instance, $f(E) = \alpha \ln(E)$ and $h(P) = \eta P^2$. Then, we get $\hat{E} + \hat{P} = \hat{\Delta}^E \frac{\alpha \Delta^E}{E P \Delta^P} - \hat{\Delta}^P$, where $\frac{\alpha \Delta^E}{E P \Delta^P} > 0$. Assuming that $\hat{E} < 0$, the left-hand side denotes the difference between the growth rates of permanent removal and emission abatement. It is negative if emissions decrease faster than removals increase. When technological progress in permanent removals accelerates, the right-hand side increases since $\hat{\Delta}^P < 0$, thereby shifting mitigation efforts more toward permanent CDR.

increases relatively slow. The optimal allocation of expenditure on these two activities, thus, shifts toward diligence.¹⁷

Non-permanent CDR as bridge technology and temperature overshoot. An interesting question to consider is the role of non-permanent CDR during the transition. At the time of writing this article, virtually all of the approximately 2 GtCO₂ that are removed from the atmosphere annually are due to nature based solutions, that is, non-permanent CDR. It is highly uncertain when novel technological methods will become available at scale (Smith et al. 2024). To characterize the role of non-permanent CDR, we implement a simplified version of the analytical model in GAMS to obtain numerical results. In particular, we abstract from technological progress. There is only one generic non-permanent CDR technology with quadratic costs. Climate damages and permanent removal costs are quadratic as well, while benefits of emissions follow a fractional power function. For diligence costs, we assume that $w(\delta) \sim \frac{1}{\delta}$. In Appendix C, we list functional forms and parameter values. The full code is available in the electronic supplementary material.

In Figure 4, we compare two scenarios. The two panels on the left show emission flows and global temperature change under the assumption that permanent CDR is already available at the beginning of the time horizon. The two right panels show, ceteris paribus, a simulation under the constraint that permanent CDR only becomes available in the year 2100, which is perfectly anticipated by the social planner. In Figure D.1 in Appendix D, we also present the trajectories of the optimal diligence level and the shadow prices.

The time at which permanent CDR becomes available does not affect steady state levels. However, the paths along the transition differ. The planner increases deployment of non-permanent CDR when the availability of permanent CDR is delayed to the year 2100. Once permanent CDR is available, non-permanent removal peaks and fossil emissions reach a minimum. Contrary to the early availability case, emissions converge to their steady state level from below and non-permanent removal from above. Also, with late arrival of permanent CDR, the optimal temperature level overshoots its steady state level, peaking in 2100, before returning to the optimal long-term level. Thus, our model rationalizes overshoot scenarios (IPCC 2022) in a cost-benefit analysis.

In the analytical model, the rationalization of overshoot scenarios can also be seen. In the steady state, permanent removal exactly offsets fossil emissions. A cost-reducing technology shock in permanent removal would lead to an increase in removal, resulting in a new steady state with lower atmospheric carbon concentration, lower marginal damage and higher residual emissions. In Figure 3, this would correspond to a downward shift of the \tilde{h}' curve. In the model, an overshoot of the long-run steady-state temperature level along the transition can occur for (at least) two different reasons. First, if there is bad news about climate change, that is, an unforeseen increase in marginal climate damages occurs, then the SCC_E are higher and the horizontal SCC_E curve in Figure 3 shifts upward. Second, when

17. Analogous to the previous footnote, assume that $g(R) = \gamma R^2$ and $w(\delta) = \frac{\omega}{\delta}$. Then using the FOCs one can obtain that $\frac{4\gamma\delta^2\Delta^R}{\omega\Delta^{\delta}}(\hat{R} + 2\hat{\delta}) = \hat{\Delta}^{\delta} - \hat{\Delta}^R$ where $\frac{4\gamma\delta^2\Delta^R}{\omega\Delta^{\delta}} > 0$.

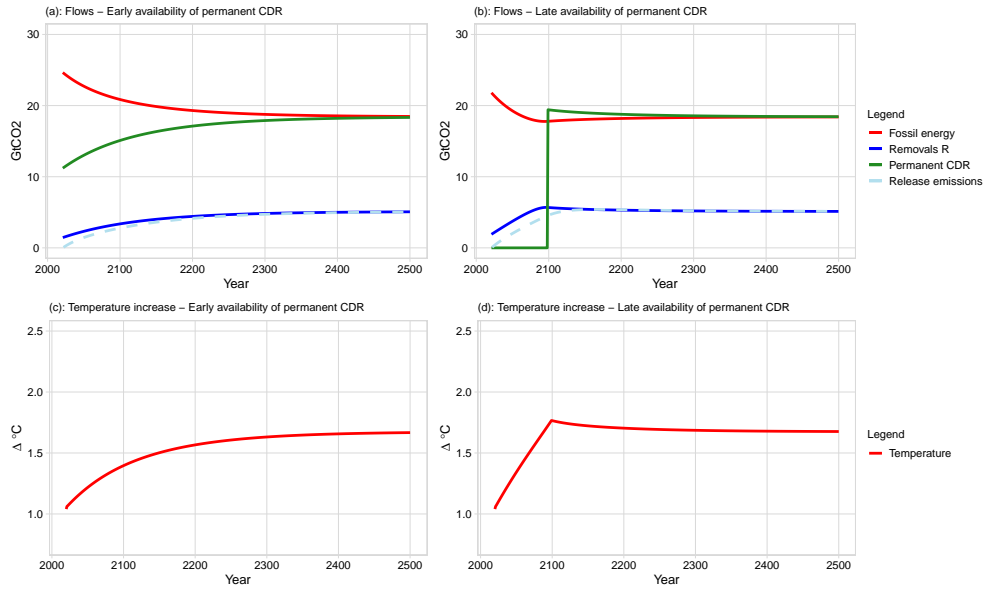


FIGURE 4. Numerical simulations for early (left panels) and late (right panels) availability of permanent CDR. Upper panels show flows over time, bottom panels show the temperature.

there is good news about marginal permanent removal cost, the \tilde{h}' curve shifts downward. Either of the two shocks induces $P > E$, thereby leading to net-negative emissions.

To summarize, our numerical simulation suggests an additional but limited role for non-permanent CDR as bridge technology as long as permanent CDR is not available. Notably, the increase in the deployment of non-permanent CDR shown in Figure 4 is accompanied by a considerably larger reduction of fossil emissions. Moreover, in our cost-benefit analysis, we can also rationalize the overshoot scenarios in IPCC (2022), which are rooted in a cost-effectiveness analysis.

3. Policy instruments

We now discuss different policy regimes, the informational requirements for the government and how the policies affect firms' profits over time. Thus, we take the perspective of a representative firm in a market setting that differs from the first-best social planner economy as described in Section 2 in two regards. First, the firm ignores its effect on cumulative emissions, and therefore we drop the emission accumulation equation (2) and the climate damage term $-d(X)$ from (1). Second, the firm is subject to regulation in the form of various taxes and subsidies, which take the place of the climate damage term. The policy instruments put prices on stocks and flows. Figure 5 shows at which point in the carbon cycle the prices apply. We assume that the social and the private discount rates are

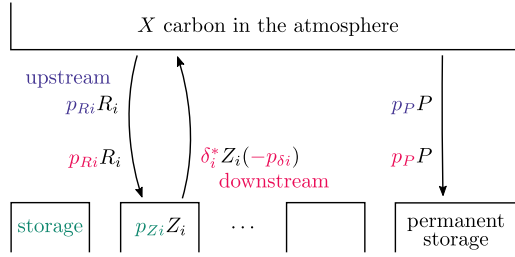


FIGURE 5. Overview of pricing instruments for removal. All policy regimes include a carbon tax p_E on emissions (not shown).

equal, abstracting from further market failures or welfare aspects that would require further and potentially complex micro foundations. The problem of the firm reads

$$\begin{aligned} \max \pi = \int_0^\infty e^{-rt} \left[\tilde{f}(E) - p_E E - \tilde{h}(P) + p_P P \right. \\ \left. + \sum_i \left(p_{Ri} R_i + p_{Zi} Z_i - \tilde{g}_i(R_i) - \tilde{w}_i(\delta_i) Z_i - p_{\delta_i} \delta_i Z_i \right) \right] dt \end{aligned} \quad (24)$$

$$\text{such that } \dot{Z}_i = R_i - \delta_i Z_i \quad \perp \lambda_i \quad (25)$$

We consider three different policy regimes. First, under downstream pricing, the government applies a price to all emission flows: it taxes emissions from production (p_E), subsidizes removal to permanent and non-permanent storage (p_P and p_{Ri} , respectively), and taxes released emissions (p_{δ_i}). Second, under upstream pricing, there is no tax on released emissions. Instead, the subsidy for non-permanent removal p_{Ri} is adjusted to take eventual released emissions into account. Third, we consider a subsidy p_{Zi} on the stocks of stored carbon instead of the flows of removal. We find that all three policy schemes can implement the socially efficient solution of the planner economy in Section 2 as captured in the following proposition.

PROPOSITION 5. *Three pricing schemes for non-permanent removal with $p_E = p_P = SCC_E$ achieve the first-best allocation of the economy:*

- a downstream pricing scheme with $p_{Ri} = p_{\delta_i} = SCC_E$
- an upstream pricing scheme with a reduced removal subsidy $p_{Ri} = \tilde{g}'_i + IDC_i = SCC_E - SCC_R$ and regulation that ensures optimal diligence such that $\delta_i(t) \equiv \delta_i^*(t)$ with $\tilde{g}'_i(R_i^*) \Delta^{Ri} = \tilde{w}'_i(\delta_i^*) \Delta^{\delta i}$
- a stock subsidy with $p_{Zi} = p_Z = d'(X)$

where the government implements either a, b or c and in each case all other instruments are set to zero.

TABLE 2. Pricing schemes overview

Informational requirements		
	pricing	quantities
Downstream	$p_{Ri} = SCC_E$	$\{R_{it}, \delta_{it}Z_{it}\}, \{R_{it}, \delta_{it}\}$ or $\{\Delta Z_i\}$
Upstream	$p_{Ri} = SCC_E - SCC_R$	R_{it}, δ_{it} for all t
Stock subsidy	$p_Z = d'(X)$	Z_{it}
Firm's financial flows		
	near-term	steady-state
Downstream	profit	deficit
Upstream ^a	profit	profit
Stock subsidy	deficit	profit ^b
Government's payments		
	near-term	steady-state
Downstream	$p_E(R_i - \delta_i Z_i)$	0
Upstream	$p_{Ri}R_i$	$p_{Ri}R_i^S$
Stock subsidy	$p_Z Z_i$	$p_Z Z_i^S > p_{Ri}R_i^S$

^a Per-period profits are positive when diligence costs are “sufficiently small”, i.e., equation (29) holds.

^b Steady-state profits are positive when initial removal costs $g_i(0)$ are “sufficiently small”, see equation (32) for details.

Proof. Proof for parts a.-c. are in Appendices E.1.1, E.1.2 and E.1.3. □

By Proposition 5, the three pricing schemes are equally efficient. However, the policy regimes differ substantially from the perspectives of (a) informational requirements, (b) agency problems, and (c) private financial flows, that is, the induced trajectories of firms' profitability over time. The following subsections 3.1, 3.2 and 3.3 are devoted to each of three policy regimes. For each policy regime, we discuss the three perspectives (a), (b) and (c). In subsection 3.4, we compare all three policy regimes in terms of their implications for public finance, that is, the government's payments. We collect the insights in Table 2. In Figure D.2 in Appendix D, we illustrate the financial flows of firms and governments as characterized by the numerical model presented in Section 2.2.

3.1. Downstream pricing

The downstream pricing scheme achieves the socially efficient allocation by uniformly pricing all emission fluxes at the level of the social cost of carbon (Proposition 5a). As the pricing scheme includes the released emissions, downstream pricing simultaneously sets the optimal incentive for the endogenous choice of diligence by the firm, that is, no additional instrument for diligence is required.

Informational requirements of downstream pricing. The level of both the subsidy for removal and the tax on released emissions in the downstream pricing scheme is at the social

cost of carbon SCC_E . Since the SCC_E also determines the optimal tax on carbon emissions p_E , no additional information is required, and the regulator can simply set $p_{R_i} = p_{\delta_i} = p_E = SCC_E$.

For the total payments $p_{R_i}R_i$ and $p_{\delta_i}\delta_i Z_i$, the regulator needs to know the volumes of removal and released emissions. Depending on the specific removal technology, different approaches may be preferable: (i) When harvested wood is stored in wood vaults, additions R_i to the vault as well as the release rate δ_i may be known with reasonable certainty; it is then only a matter of bookkeeping to track the carbon in the vault Z_i according to (3) in order to know $\delta_i Z_i$. (ii) When δ_i is difficult to know, monitoring all flows may be preferable. For example, when utilizing a forest for carbon storage, monitoring removal R_i and released emissions ($\delta_i Z_i$) amounts to tracking the planting and growth of trees, as well as the emissions from wildfires and the dying off and decay of trees. (iii) Since downstream pricing imposes a uniform price on removal and released emissions, the latter approach can be made simpler still by subsidizing the net emission removal at $p_{R_i} = p_{\delta_i} = p_E$, which implies a subsidy payment of $p_E(R_i - \delta_i Z_i)$. It is then enough to monitor and periodically verify Z_i to pay the subsidy for the net addition to the storage $\Delta Z_i := Z_i(t) - Z_i(t - \Delta t)$.

Transaction costs and administrative costs for raising the information $\{R_{it}, \delta_{it} Z_{it}\}$, $\{R_{it}, \delta_{it}\}$ or $\{\Delta Z_i\}$ will differ across technologies, and regulation could be flexible. Nevertheless, the cost of monitoring and verification may be substantial. For example, for the wood products in Table 1 the life-cycle of houses, furniture and paper would need to be tracked.

Liability risk of downstream pricing. Downstream pricing depends on the continued operation of the firm. If the firm goes out of business, for example in case of bankruptcy, diligence would drop to zero. The release rate would be maximal but no tax on released emissions could be levied from the insolvent firm. The problem of the firm being “judgment-proof”, that is, being limited in its liability for released emissions, is well-known (Shavell 1986; Shogren et al. 1993). Bonds posted by the firm as collateral to be claimed by the regulator solve this problem in principle, but imply transaction costs and constrain the firm’s liquidity (Gerard and Wilson 2009; White et al. 2012) unless backed up by further financial derivatives. Held and Edenhofer (2009), for example, suggest that storage of captured carbon can be ensured by demanding that storage operators must hold state-issued bonds. These “CCS bonds” work as follows: for each ton of CO₂ sequestered, the operator purchases a bond from the government at a price reflecting the social cost of carbon. The bond earns interest over time (typically 30 years) and is returned with accumulated interest if the operator can prove that the stored carbon has not leaked above acceptable thresholds. If the operator can convince capital markets of safe operation, they can sell the bond early, providing immediate liquidity while maintaining the incentive for secure storage. Lemoine (2023) also suggests that emitters post a bond. In his scheme this is done instead of paying a per-period charge on the stock. Here, liquidity is created by providing tradable “carbon shares” in exchange for the bond. A carbon share represents a claim on a unit of atmospheric carbon. Its face value starts equal to the bond but is reduced each period to account for realized climate damages, while the shareholder receives periodic dividends that reflect the difference

between the worst-case and the (continuously updated) expected social cost of carbon. If the shareholder removes the associated emissions from the atmosphere, they receive the remaining face value as a payoff. This creates a valuable asset that investors want to hold, ensuring that even if the original emitter goes bankrupt, someone else has an incentive to remove the emissions. The carbon share essentially converts past emissions into a tradable asset that provides ongoing incentives for carbon removal.

Financial flows of downstream pricing. A firm that manages a stock of stored carbon faces dynamic decision problems. It will only operate when the net present value of operation is positive. But a positive net present value does not guarantee a positive cash flow at every point in time. To identify periods of deficit with bankruptcy risk for the firm, and to study the incentive of the firm to continue its business, we consider the per-period cash flow of the firm. Here and for the other pricing schemes we focus on the operation of one non-permanent removal technology.¹⁸

Under downstream pricing, the non-permanent removal (“nr”) firm receives a subsidy of p_E for the difference in emissions removal and release. Its expenditures are removal and diligence costs.

$$\pi_{it}^{\text{nr}} = \underbrace{p_E(R_i - \delta_i Z_i)}_{\text{revenues}} - \underbrace{\tilde{g}_i(R_i) - \tilde{w}(\delta_i)Z_i}_{\text{costs}}$$

We know that these subsidy revenues vanish in the optimal steady state due to (16), leaving the firm without income due to the provision of R_i but costs of $\tilde{g}_i(R_i^S) + \tilde{w}_i(\delta_i^S)$. The cash flow under the downstream pricing is, therefore, negative in the long term. Since we assume that the firm only starts operation for non-negative net-present value $\pi_t^{\text{nr}} \geq 0$ it follows

$$\int_0^{\infty} e^{-rt} p_E(R_i - \delta_i Z_i) dt \geq \int_0^{\infty} e^{-rt} (\tilde{g}_i(R_i) + \tilde{w}(\delta_i)Z_i) dt$$

As the net-present value of the revenues outweigh the costs, we can conclude that there are enough strictly positive revenues in the near term to cover all costs in present value terms. That is, if the revenues are invested to earn a rate of return equal to the discount rate r , all future costs can be financed. In short, for downstream pricing, storage becomes a financial burden for the firm in the long-term, which requires the storage branch to establish a fund, sourced from the revenues in early years.¹⁹

18. The production net of the emission tax, as well as permanent removal will always generate a positive producer surplus due to our assumptions about the functional forms. While funds from these “branches” could be used to “cross-subsidize” a branch that is in deficit, a rational firm would close down a loss-making branch.

19. A similar approach is taken in Germany's Nuclear Waste Disposal Fund which manages a share of the profits of nuclear power companies to finance long-term storage.

3.2. Upstream pricing

Under upstream pricing, the government does not tax released emissions. Instead, firms receive a reduced subsidy for carbon removal with non-permanent storage. The subsidy is reduced compared to the downstream pricing by the social costs of removal (SCC_R) to take the non-permanence into account (Proposition 5b). The per-period cash flow of the non-permanent removal firm becomes

$$\pi_i^{\text{nr}} = p_{Ri}R_i - \tilde{g}_i(R_i) - \tilde{w}(\delta_i)Z_i \quad (26)$$

Remunerating removal up-front provides no incentive for diligence. Therefore, an additional command-and-control instrument ensures optimal diligence. Without such an instrument, diligence would drop to its minimum level.²⁰

Proposition 5b sets the removal subsidy in the upstream pricing scheme at $p_{Ri} = SCC_E - SCC_{R,i}$ to internalize the social benefit of the non-permanent storage. We can interpret $\Omega_i(t) := p_{Ri}/p_P$ as an optimal correction factor. When applied to the carbon price for fossil emissions and permanent removal, it yields the correct subsidy for incentivizing the optimal use of removal technology i . Note that it resembles the correction factors derived by [Groom and Venmans \(2023\)](#) with exogenously given temperature trajectory. Moreover, for quantity based policy instruments [Edenhofer et al. \(2024\)](#) have derived similar correction factors. Ω_i has a simple form, in particular in the steady state:

PROPOSITION 6. *The optimal correction factor for subsidizing non-permanent carbon removal is*

$$\Omega_i(t) = 1 - \frac{SCC_{R,i}(t)}{SCC_E(t)} \quad \text{in general} \quad (27)$$

$$\Omega_i^s = \frac{r}{r + \delta_i^s} \quad \text{in steady state} \quad (28)$$

Proof. (27) follows directly from Proposition 5b. In the steady state, $\dot{\lambda} = 0$ and hence $\lambda^s = \tilde{w}_i(\delta_i^s)/(r + \delta_i^s)$. Recalling μ^* and ψ^* from (17)–(18) and plugging the three expressions into Ω_i yields (28). \square

Table 3 shows Ω_i^s for a range of half-lives and discount rates. Removal technologies with no emission release ($\delta_i = 0$) are permanent and receive the same subsidy as P . For $\delta_i \approx r$, a non-permanent removal receives only half of the subsidy for permanent removal. In the

20. Instead of using command and control regulation here, additional market-based instruments could also be an option. For example a subsidy on diligence or a subsidy on the stock to reward its maintenance. To model this, we would add $p_{\delta_i} \delta_i Z_i$ for the former and $p_{Z_i} Z_i$ for the latter to the per-period cash flow (26). We find, however, that in equilibrium, including the subsidy on diligence p_{δ_i} is equivalent to downstream pricing as described in Section 3.1. In a similar vein, including p_{Z_i} here is equivalent to the policy regime of a stock subsidy as described in Section 3.3. Thus, when a stock subsidy is introduced to complement upstream pricing, the optimal upstream price is zero. We show this in the appendix (Lemma E.1).

Half-life (years)	δ_i	Discount rate r				
		0.01	0.02	0.03	0.05	0.07
1	0.69	0.01	0.03	0.04	0.07	0.09
5	0.14	0.07	0.13	0.18	0.27	0.34
10	0.07	0.13	0.22	0.30	0.42	0.50
50	0.014	0.42	0.59	0.68	0.78	0.83
100	0.007	0.59	0.74	0.81	0.88	0.91
500	0.0014	0.88	0.94	0.96	0.97	0.98
1000	0.0007	0.94	0.97	0.98	0.99	0.99

TABLE 3. Optimal correction factor for the subsidy for carbon removal with non-permanent storage, Ω^s .

extreme case of $\delta_i = 1$ the removed carbon is retained for only one period, and the subsidy would be two orders of magnitude smaller than the SCC_E for typical values of the discount rate r .

Informational requirements of upstream pricing. At first glance, the upstream system is at an informational advantage, as it requires the regulator only to quantify removal but not the released emissions. Monitoring only the removal quantity might be cheaper than monitoring the released quantity and thereby lower the administrative costs.²¹ If in addition uncertainty about technology-specific release rates is low, the upstream system could have a cost advantage over the downstream system. This advantage comes at the cost of raising information on the release rate δ_i . As shown in Proposition 6, the level of the subsidy on removal p_{R_i} now needs to reflect the non-permanence by reducing the subsidy according to the social cost of removal SCC_R . Knowing the SCC_R , however, requires knowledge of all future $\delta_i(t)$. Knowing the optimal level δ_i^* is also required to set and enforce appropriate standards for diligence.

We expect that the cost of knowing optimal release rates varies across technologies, and could be substantial where the release rate is not narrowly determined by observables as the technology used but is sensitive to the extent of diligence and, thus, a decision variable. As we have shown in Proposition 5b, CDR technologies for which diligence matters for the release rate require regulators to know the release flows in order for them to design the optimal upstream subsidy. This, however, makes the upstream pricing system effectively a downstream pricing system again.

Liability risk of upstream pricing. Like downstream pricing, upstream pricing depends on the continued operation of the firm, albeit to a lesser degree. In contrast to downstream pricing, the social cost of carbon removal is already accounted for by the correction factor

21. The reason for this cost asymmetry is twofold: (1) removal quantities are often spatially concentrated while later emission releases from stored carbon can be very dispersed, (2) removal can take place at a short period of time while emission release from storage might occur in small rates over large time horizons. Both aspects imply large monitoring costs relative to the size of the emission flows.

Ω_i . However, upstream pricing does not address the fact that firms are judgment-proof and if they go out of business, diligence drops to its minimum.

To maintain the incentive to apply the optimal level of diligence, a staggered payment of a subsidy on diligence could be implemented. That is, the upfront removal subsidy could be corrected not only for the SCC_R , but also the IDC . Then, subsidies equivalent to the IDC would be paid out over time conditional on the government actually observing optimal diligence by the firm. However, in terms of informational requirements, such staggered payments are the most challenging among all instrument we consider in this paper since the regulator has to monitor the release rate and has to know the firms' diligence costs. We provide the formal analysis in Appendix E.3.

Financial flows of upstream pricing. In the upstream pricing system, firms receive a revenue of $p_{Ri}R_i$ and do not need to pay for released emissions. Since R_i is produced with increasing marginal costs (recall that g is convex), the firm earns a producer surplus – a profit if we assume zero fixed costs and if we abstract from diligence costs. The per-period cash flow is therefore positive, such that non-permanent removal is a continuous stream of income for the firm, paid by the government, that is, the tax payer via the subsidy on removal.

When the firm needs to take non-zero diligence costs into account, it will choose removal R_i such that their marginal costs fall below the subsidy $(p_{Ri} - \tilde{g}'_i(R_i)) = IDC > 0$, this way earning an additional profit $IDC \cdot R_i$ to cover the cost of diligence.

Whether the cash flow at time t remains positive depends on whether diligence costs per ton of CO_2 in the storage are less than producer surplus and additional profits.

$$\underbrace{w(\delta_i^*(t))\Delta^{\delta_i}(t)}_{\text{diligence costs}} < \underbrace{\left[(g'_i(R_i(t))R_i(t) - g_i(R_i(t)))\Delta^{R_i}(t) \right]}_{\text{producer surplus}} + \underbrace{IDC(t)R_i(t)}_{\text{additional profit}} / Z_i(t) \quad (29)$$

Technological progress ($\dot{\Delta}^k < 0$ for $k \in \{\delta_i, R_i\}$) makes this more likely to hold when it applies for diligence costs, but less likely when it reduces the producer surplus.

3.3. Stock subsidy

Rather than subsidizing removal flows, the regulator could subsidize the storage stock Z_i . Firms would have to prove the amount of retained carbon in every period to obtain the subsidy $p_Z = d'(X)$ (Proposition 5c).

Informational requirements of the stock subsidy. The removal stock subsidy only requires the regulator to observe the *contemporaneous* marginal climate damages $d'(X)$ to be able to raise the stock subsidy to the optimal level. The contemporaneous marginal damages are a subset of the social cost of carbon that need to be estimated anyway for the emissions tax p_E . Note, that the same subsidy is sufficient for all technologies i . In addition, the regulator needs to observe the amount of carbon Z_i stored in a particular reservoir at any point in time but does not need to observe any carbon fluxes. Therefore, if monitoring stocks is cheaper

than monitoring flows, the removal stock subsidy has the lowest informational requirement of the policy instruments considered.

Liability risk of the stock subsidy. The stock subsidy also has a favorable incentive structure. Since the subsidy is always paid for the stock at that time, there is no commitment problem and firms cannot benefit from lack of diligence or strategic bankruptcy. No additional instrument is needed to set the optimal incentive for the firm's diligence decision (as evident from the proof of Proposition 5c).

Financial flows of the removal stock subsidy. The per-period cash flow of non-permanent removal is

$$\pi_i^{\text{nr}} = p_Z Z_i - \tilde{g}_i(R_i) - \tilde{w}_i(\delta_i) Z_i \quad (30)$$

When the regulator applies the removal stock subsidy, the firm earns $p_Z Z_i$. Assuming that we start with empty storage, removal costs will initially exceed the storage subsidy since $Z_i(0)$ is close to zero (unless the subsidy $p_Z = d'(X)$ is very large). In the steady state, we can use $p_Z = d'(X^S) = (\delta_i^S + r)\tilde{g}'_i(R_i^S) + \tilde{w}_i(\delta_i^S)$ from (20) and $R^S = \delta_i^S Z_i^S$ to rewrite the per-period cash flow as

$$p_Z Z_i^S - \tilde{g}_i(R_i^S) - \tilde{w}_i(\delta_i^S) Z_i^S = \frac{\delta_i^S + r}{\delta_i^S} \tilde{g}'_i(R_i^S) R_i^S - \tilde{g}_i(R_i^S) \quad (31)$$

which is positive when

$$\frac{\delta_i^S + r}{\delta_i^S} \tilde{g}'_i(R_i^S) R_i^S > \tilde{g}_i(R_i^S) \quad (32)$$

Due to convexity of \tilde{g}_i , the above inequality is true if $\tilde{g}_i(0)$ is sufficiently small.²² In aggregate, the net present value of optimal removal can only be positive (as in the downstream pricing case). Thus if the firm can borrow at an interest rate less than or equal to r it can cover the early deficits and the potential deficit in the long run from the periods with positive profits. Firms, thus, may be hesitant to commit to removal under the removal stock subsidy, as they will initially run a deficit and possibly in the long run, too.

A second problem arises from the regulators need to define for each storage site j utilizing technology i what it means that $Z_{ij} = 0$, that is, the baseline. Some carbon sinks such as peatlands or forests can turn to sources. This property might lead to rent-seeking behavior and non-additionality of CDR projects. Owners of large natural sinks could be tempted to purposefully release large volumes of carbon from their sinks to have their baseline set as low as possible. To prevent the problem, the correct sequencing of policies appears crucial. For example, taking stock of all natural carbon sinks within the jurisdiction of the regulator should be done as early as possible.

22. Convexity implies that $\tilde{g}'_i(R_i)(0 - R_i) = -\tilde{g}'_i(R_i)R_i < \tilde{g}_i(0) - \tilde{g}_i(R_i)$ for any R_i at which \tilde{g}_i is differentiable.

3.4. Public finance

For all of the above pricing schemes, the regulator needs to finance subsidies. Financing options include taxation, public debt and selling public assets – each with distinct costs and distributional implications that would affect the optimal policies, were they included in the model. For example, raising revenue with additional taxes may cause distortions. In that case, the theory of second-best taxation provides guidance. Incurring debt or selling public assets have obvious implications for inter-generational burden sharing. We abstract from revenue raising to keep the model simple. However, the regulator's flow of present and future expenditures for subsidies, as well as their net present value (NPV) are distinctly different across the pricing schemes. We include a comparison of the volumes of their flows in the near term and the steady state in Table 2.

The NPV of public expenditures depends crucially on the initial level of the stock that is subsidized $Z_i(0) = 0$. The NPV due to downstream pricing is less or equal than that due to the stock subsidy with strict equality if $Z_i(0) = 0$ and strict inequality if $Z_i(0) > 0$. The ranking between downstream and upstream pricing is ambiguous and depends on functional forms as well as on the initial stock size $Z_i(0)$. We show this in Appendix E.4.

Intuitively, the more the released emissions of initially existing storage stocks are subject to pricing, the more an asymmetry is introduced and the more the public expenditures will differ between the three schemes. With downstream pricing, for example, the release flow $\delta_i(0)Z_i(0)$ will be taxed while the preceding removal activity is not accounted for. With upstream pricing, the government misses the tax revenues due to $\delta_i(0)Z_i(0)$. With the stock subsidy, non-zero initial storage stocks even turn into a financial burden.

In the long run steady state the differences are pronounced, and there is an unambiguous ranking of the pricing schemes which increasingly shifts the burden of the policy toward future generations: In the downstream pricing scheme, as removal equals emissions release the subsidy payment and the tax revenue cancel out. The net payment for the regulator is zero. In both other pricing schemes, the regulator has strictly positive expenditures in the steady state. Payments in case of the stock subsidy are higher than those in the upstream pricing scheme.²³

In the near term, the ranking is less clear. If the storage is initially empty, then the subsidy on the stored stock of carbon, too, is initially zero and thus lower than the upstream and downstream subsidies on the flow of carbon removal. The ranking of the latter two is indeterminate.²⁴

23. To see this, compare $p_{zi}Z_i^s$ and $p_{Ri}R^s$ using $p_{zi} = d'(X)$ and $p_{Ri} = SCC_E - SCC_R$ together with steady-state condition (17) and (18). The stock subsidy payments are greater than the upstream subsidy payments if

$$\frac{d'(X^s)}{\delta_i^s} > \frac{d'(X^s) - \tilde{w}_i(\delta_i^s)}{(r + \delta_i^s)}$$

This is true for any $w_i(\delta_i^s) \geq 0$, as \tilde{w}_i enters negatively on the right-hand side.

24. The difference between the net public expenditures under upstream pricing $-SCC_E E + (SCC_E - SCC_R)R_i$ and those under downstream pricing $-SCC_E E + SCC_E R_i - SCC_E \delta_i Z_i$ is given by $SCC_R R_i - SCC_E \delta_i Z_i$. This can

4. Conclusion

Governments around the world have announced their plans to reach carbon neutrality by mid-century. Large-scale deployment of carbon dioxide removal (CDR) appears as a *conditio sine qua non* to achieve this goal. CDR has already become a part of private corporations' strategy to achieve carbon neutrality. However, the role of technologies that cannot store carbon permanently is not self-evident. This applies in particular to various 'nature-based' solutions to increase carbon stocks in forests and soils, and to carbon capture and usage approaches, where carbon is stored in produced goods for the short- or medium-term.

The present paper sheds light on whether and how carbon dioxide removal should be deployed when storage is non-permanent. Our findings lay out some fundamental principles on which regulators should design policy instruments to incentivize CDR. First, even if a specific technology cannot store carbon permanently, it is still a valuable option because it lowers the cost of mitigation along the transition. Lower release rates imply higher levels of carbon stored in non-permanent sinks in the long run. Second, valuation of non-permanent CDR is based on the conventional social cost of carbon emissions SCC_E and two additional intertemporal costs. The *social cost of carbon removal* SCC_R expresses the marginal additional damage of carbon that was stored impermanently and is then released over time. The *intertemporal diligence cost* IDC expresses the cost of limiting the release rate to a certain level. Third, the optimal long-run temperature level is independent of the characteristics of non-permanent storage technologies and the volume of stored carbon. Increasing availability or decreasing costs of non-permanent CDR will not help to increase the ambition of climate protection. Fourth, governments need to ensure monitoring, reporting and verification (MRV). While Pigouvian carbon pricing can be extended to non-permanent carbon removal, new instruments with additional informational requirements are necessary. Moreover, optimal policy instruments must prevent moral hazard, for example, when firms have an incentive to go bankrupt strategically to evade their liability to remove carbon in perpetuity.

There are open questions that remain. Our modeling, for example, abstracts from uncertain storage duration or release rates that are stochastic. We have identified intertemporal financing as a prerequisite for two of the proposed policy instruments. If financial constraints were considered, supplementary policies could be necessary to achieve efficient use of removal; we leave these considerations for future research. Moreover, our analysis is based on a cost-benefit framework. When regulators need to implement climate policy in the context of carbon budgets, as, for example in the EU ETS or with respect to the Paris Accord, the question arises how to translate our findings on price based instruments to quantity based instruments. There is a nascent literature discussing institutional and governance aspects²⁵, but the question would merit further formal analysis. Additional governance challenges arise beyond the closed economy setting we have chosen when one

be positive or negative since we have $SCC_E > SCC_R$, but also during the transition the released emissions are less than removal, $\delta_i Z_i < R_i$ (until the two flows balance in the steady state).

25. See, for example, Rickels et al. (2022, 2021); Edenhofer et al. (2024)

considers an international context. Asymmetric information about the permanence of national removal projects may incentivize countries to rely more heavily on cheap non-permanent removal, undermining the climate integrity of nationally determined contributions (NDCs). In anticipation of this market failure, a global certification scheme may further adjust the value of non-permanent removals downward. Our analysis furthermore suggests that the volume of carbon dioxide to be removed and stored may be large, raising questions about the physical and sustainability limits of storage capacity as well as the scale of finance necessary to fund the removal subsidies. If revenues for subsidy payments need to be raised via distortionary taxation, the cost of public funds should be factored into the policy design. There are additional market failures related to the non-permanence of storage, such as the risk of accidental release or the default risk of firms in charge of maintaining the storage, that pose additional challenges for the design of optimal removal policies.²⁶ The model developed in this paper could be extended to include a risk premium to the discount rate along the lines of [Reed and Heras \(1992\)](#) to account for the sudden collapse of a carbon sink.²⁷ In accordance with [Groom and Venmans \(2023\)](#) we expect that the inclusion of that risk would lead to an increase in the social cost of carbon removal SCC_R . Finally, distinguishing between capital-, energy- and land-intensive CDR technologies would allow to generate further quantitative insights on the impact of non-permanence on optimal CDR portfolios. First steps in this direction including deep uncertainties have been made by [Mendez et al. \(2025\)](#).

To conclude, our analysis shows that carbon removal technologies present policy makers with a double-edged sword: non-permanent CDR facilitates short-term welfare gains during the transition but commits future generations to continuously return released emissions back to their reservoirs. Similar to Sisyphus's task, non-permanent CDR creates a perpetual "carbon debt" to future generations that consists of undertaking removal into leaky reservoirs.

26. See, [Groom and Venmans \(2023\)](#) for an overview.

27. We thank an anonymous reviewer for making us aware of this possibility.

References

- Belfiori, Maria Elisa (2017). "Carbon pricing, carbon sequestration and social discounting." *European Economic Review*, 96, 1–17.
- Brander, Matthew and Derik Broekhoff (2023). "Methods that equate temporary carbon storage with permanent CO₂ emission reductions lead to false claims on temperature alignment." *Carbon Management*, 14(1), 2284714.
- Churkina, Galina, Alan Organschi, Christopher PO Reyer, Andrew Ruff, Kira Vinke, Zhu Liu, Barbara K Reck, TE Graedel, and Hans Joachim Schellnhuber (2020). "Buildings as a global carbon sink." *Nature Sustainability*, 3(4), 269–276.
- Deng, Hang, Jeffrey M. Bielicki, Michael Oppenheimer, Jeffrey P. Fitts, and Catherine A. Peters (2017). "Leakage risks of geologic CO₂ storage and the impacts on the global energy system and climate change mitigation." *Climatic Change*, 144(2), 151–163.
- Edenhofer, Ottmar, Max Franks, and Matthias Kalkuhl (2021). "Pigou in the 21st Century: a tribute on the occasion of the 100th anniversary of the publication of *The Economics of Welfare*." *International Tax and Public Finance*, pp. 1–32.
- Edenhofer, Ottmar, Max Franks, Matthias Kalkuhl, and Artur Runge-Metzger (2024). "On the Governance of Carbon Dioxide Removal—A Public Economics Perspective." *FinanzArchiv*.
- Farzin, Y. H. (1996). "Optimal pricing of environmental and natural resource use with stock externalities." *Journal of Public Economics*, 62(1-2), 31–57.
- Franks, Max, Matthias Kalkuhl, and Kai Lessmann (2022). "Optimal pricing for carbon dioxide removal under inter-regional leakage." *Journal of Environmental Economics and Management*, p. 102769.
- García, Jorge H. and Asbjørn Torvanger (2019). "Carbon leakage from geological storage sites: Implications for carbon trading." *Energy Policy*, 127, 320–329. Publisher: Elsevier Ltd.
- Gerard, David and Elizabeth J Wilson (2009). "Environmental bonds and the challenge of long-term carbon sequestration." *Journal of Environmental Management*, 90(2), 1097–1105.
- Goulder, Lawrence H. and Koshy Mathai (2000). "Optimal CO₂ Abatement in the Presence of Induced Technological Change." *Journal of Environmental Economics and Management*, 39(1), 1–38.
- Groom, Ben and Frank Venmans (2023). "The social value of offsets." *Nature*, pp. 1–6.
- Ha-Duong, Minh and David W. Keith (2003). "Carbon storage: the economic efficiency of storing CO₂ in leaky reservoirs." *Clean Technologies and Environmental Policy*, 5(3-4), 181–189.
- Held, Hermann and Ottmar Edenhofer (2009). "CCS-Bonds as a superior instrument to incentivize secure carbon sequestration." *Energy Procedia*, 1(1), 4559–4566.
- Hepburn, Cameron, Ella Adlen, John Beddington, Emily A Carter, Sabine Fuss, Niall Mac Dowell, Jan C Minx, Pete Smith, and Charlotte K Williams (2019). "The technological and economic prospects for CO₂ utilization and removal." *Nature*, 575(7781), 87–97.
- Hiraishi, Takahiko, Thelma Krug, Kiyoto Tanabe, Nalin Srivastava, Baasansuren Jamsranjav, Maya Fukuda, and Tiffany Troxler (2014). *2013 Revised supplementary methods and good practice guidance arising from the Kyoto Protocol*. Intergovernmental Panel on Climate Change Switzerland.
- Hoel, Michael and Snorre Kverndokk (1996). "Depletion of fossil fuels and the impacts of global warming." *Resource and energy economics*, 18(2), 115–136.
- IPCC (2022). *Climate Change 2022: Mitigation of Climate Change. Contribution of Working Group III to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*. Cambridge University Press, Cambridge, UK and New York, NY, USA.
- Kim, Man-Keun, Bruce A. McCarl, and Brian C. Murray (2008). "Permanence discounting for land-based carbon sequestration." *Ecological Economics*, 64(4), 763–769.
- Lemoine, Derek (2023). "Informationally Efficient Climate Policy: Designing Markets to Measure and Price Externalities." Tech. rep., National Bureau of Economic Research.
- Masson-Delmotte, Valérie, Panmao Zhai, Hans-Otto Pörtner, Debra Roberts, Jim Skea, Priyadarshi R Shukla, Anna Pirani, Wilfran Moufouma-Okia, Clotilde Péan, Roz Pidcock, Sarah Connors, J B Robin Matthews, Yang Chen, Xiao Zhou, Melissa I Gomis, Elisabeth Lonnoy, Tom Maycock, Melinda Tignor,

- Tim Waterfield, Anna Pirani, Wilfran Moufouma-Okia, Clotilde Péan, Roz Pidcock, Sarah Connors, J B Robin Matthews, Yang Chen, Xiao Zhou, Melissa I Gomis, Elisabeth Lonnoy, Tom Maycock, Melinda Tignor, and Tim Waterfield (eds.) (2018). *Global Warming of 1.5°C: An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty*. World Meteorological Organization, Geneva, Switzerland, URL <https://www.ipcc.ch/sr15/>.
- Matthews, H Damon, Kirsten Zickfeld, Alexander Koch, and Amy Luers (2022). "Reimagining tonne-year accounting to capture the climate benefit of temporary carbon storage." *PREPRINT (Version 1) available at Research Square*.
- Mendez, Quirina Rodriguez, Felix Creutzig, and Sabine Fuss (2025). "Deep uncertainty in carbon dioxide removal portfolios." *Environmental Research Letters*, 20(5), 054013.
- Moreaux, Michel, Jean-Pierre Amigues, Gerard van der Meijden, and Cees Withagen (2024). "Carbon capture: Storage vs. utilization." *Journal of Environmental Economics and Management*, 125, 102976.
- Moreaux, Michel and Cees Withagen (2015). "Optimal abatement of carbon emission flows." *Journal of Environmental Economics and management*, 74, 55–70.
- Pigou, Arthur C. (1920). *The Economics of Welfare*. Macmillan and Co. Limited.
- Reed, William J and Hector Echavarría Heras (1992). "The conservation and exploitation of vulnerable resources." *Bulletin of Mathematical Biology*, 54(2-3), 185–207.
- Rickels, Wilfried and Thomas S Lontzek (2012). "Optimal global carbon management with ocean sequestration." *Oxford Economic Papers*, 64(2), 323–349.
- Rickels, Wilfried, Alexander Proelß, Oliver Geden, Julian Burhenne, and Mathias Fridahl (2021). "Integrating carbon dioxide removal into European emissions trading." *Frontiers in Climate*, 3, 690023.
- Rickels, Wilfried, Fabian Reith, David Keller, Andreas Oschlies, and Martin F Quaas (2018). "Integrated assessment of carbon dioxide removal." *Earth's future*, 6(3), 565–582.
- Rickels, Wilfried, Roland Rothenstein, Felix Schenuit, and Mathias Fridahl (2022). "Procure, Bank, Release: Carbon Removal Certificate Reserves to Manage Carbon Prices on the Path to Net-Zero." *Energy Research & Social Science*, 94, 102858.
- Sedjo, Roger and Brent Sohngen (2012). "Carbon sequestration in forests and soils." *Annual Review of Resource Economics*, 4, 127–144.
- Shavell, S. (1986). "The judgment proof problem." *International Review of Law and Economics*, 6(1), 45–58.
- Shogren, Jason F., Joseph A. Herriges, and Ramu Govindasamy (1993). "Limits to environmental bonds." *Ecological Economics*, 8(2), 109–133.
- Smith, James E (2006). *Methods for calculating forest ecosystem and harvested carbon with standard estimates for forest types of the United States*. 343, United States Department of Agriculture, Forest Service, Northeastern
- Smith, S.M., O. Geden, M.J. Gidden, et al. (2024). "The state of carbon dioxide removal report 2024 – 2nd edition." Available online at <https://www.stateofcdr.org/resources>.
- Smith, Steve, Oliver Geden, Gregory Nemet, Matthew Gidden, William Lamb, Carter Powis, Robert Bellamy, Max Callaghan, Annette Cowie, Emily Cox, Sabine Fuss, T. Gasser, G. Grassi, J. Greene, S. Lück, A. Mohan, F. Müller-Hansen, G. Peters, Y. Pratama, T. Repke, K. Riahi, F. Schenuit, J. Steinhauser, J. Strefler, J. M. Valenzuela, and J. C Minx (2023). "The State of Carbon Dioxide Removal—1st Edition."
- Sohngen, Brent and Sandra Brown (2008). "Extending timber rotations : carbon and cost implications." 8, 435–451.
- Tahvonen, Olli (1995). "Net national emissions, CO₂ taxation and the role of forestry." *Resource and energy economics*, 17(4), 307–315.
- Torvanger, Asbjørn, Alv Arne Grimstad, Erik Lindeberg, Nathan Rive, Kristin Rypdal, Ragnhild Bieltvedt Skeie, Jan Fuglestedt, and Petter Tollefsen (2012). "Quality of geological CO₂ storage to avoid jeopardizing climate targets." *Climatic Change*, 114(2), 245–260.

- Van Der Zwaan, Bob and Reyer Gerlagh (2009). "Economics of geological CO₂ storage and leakage." *Climatic Change*, 93(3-4), 285–309. ISBN: 1058400995586.
- Van Der Zwaan, Bob and Reyer Gerlagh (2016). "Offshore CCS and ocean acidification: a global long-term probabilistic cost-benefit analysis of climate change mitigation." *Climatic Change*, 137(1-2), 157–170. Publisher: Springer Science and Business Media LLC.
- van Kooten, G. Cornelis (2009). "Biological carbon sequestration and carbon trading re-visited." *Climatic Change*, 95(3-4), 449–463.
- White, Ben, Graeme J. Doole, David J. Pannell, and Veronique Florec (2012). "Optimal environmental policy design for mine rehabilitation and pollution with a risk of non-compliance owing to firm insolvency." *Australian Journal of Agricultural and Resource Economics*, 56(2), 280–301.

Appendix A: Proofs for the social planner model

A.1. Proof of Proposition 1

Proof. To derive the expression for the social cost of carbon emission (SCC_E), solve (8) to obtain

$$\mu(t) = \left(\mu_0 + \int_0^t d'(X(s)) e^{-rs} ds \right) e^{rt} \quad (\text{A.1})$$

Plugging (A.1) into the transversality condition (10) yields $\lim_{t \rightarrow \infty} (\mu_0 + \int_0^t d'(X(s)) e^{-rs} ds) X(t) = 0$. Thus, either $X \rightarrow 0$, or we have an interior solution with $X > 0$

$$\lim_{t \rightarrow \infty} \int_0^t d'(X(s)) e^{-rs} ds = -\mu_0 \quad (\text{A.2})$$

We can split up the integral in (A.1) as follows

$$\begin{aligned} \mu(t) &= \left(\mu_0 + \int_0^t d'(X(s)) e^{-rs} ds \right) e^{rt} \\ &= \mu_0 e^{rt} + e^{rt} \int_0^{\infty} d'(X(s)) e^{-rs} ds - e^{rt} \int_t^{\infty} d'(X(s)) e^{-rs} ds \\ \stackrel{(\text{A.2})}{\implies} \mu(t) &= -e^{rt} \underbrace{\int_t^{\infty} d'(X(s)) e^{-rs} ds}_{SCC_E(t)} \end{aligned}$$

This completes the proof for the first part of the proposition. Analogously, to derive the expression for the ICR_i , solve (9) to obtain

$$\psi_i(t) = e^{\int_0^t (r + \delta_i(v)) dv} \left(\psi_{i,0} + \int_0^t e^{-\int_0^s (r + \delta_i(v)) dv} (\tilde{w}_i(\delta_i(s)) - \delta_i(s) \mu(s)) ds \right) \quad (\text{A.3})$$

Plug (A.3) into the transversality condition (11). Observe that either $\lim_{t \rightarrow \infty} e^{\int_0^t (\delta_i(v)) dv} Z_i(t) \rightarrow 0$, or we have an interior solution with $Z_i > 0$ and

$$\psi_{i,0} = - \int_0^{\infty} e^{-\int_0^s (r + \delta_i(v)) dv} (\tilde{w}_i(\delta_i(s)) - \delta_i(s) \mu(s)) ds \quad (\text{A.4})$$

Plugging the expression for $\psi_{i,0}$ back into (A.3) and simplifying yields:

$$\psi_i(t) = e^{\int_0^t (r + \delta_i(v)) dv} \left(- \int_t^{\infty} e^{-\int_0^s (r + \delta_i(v)) dv} (\tilde{w}_i(\delta_i(s)) - \delta_i(s) \mu(s)) ds \right) \quad (\text{A.5})$$

Finally, note that $s \geq t$, and use the first part of the proposition to obtain the final expression:

$$\begin{aligned} -\psi(t) &= \int_t^{\infty} (\tilde{w}_i(\delta_i(s)) - \delta_i(s) \mu(s)) e^{-\int_t^s (r + \delta_i(v)) dv} ds \\ &= \int_t^{\infty} \delta_i(s) SCC_E(s) e^{-\int_t^s (r + \delta_i(v)) dv} ds + \int_t^{\infty} \tilde{w}_i(\delta_i(s)) e^{-\int_t^s (r + \delta_i(v)) dv} ds \end{aligned}$$

□

A.2. Proof of Corollary 1

Assume that the optimal use of a non-permanent removal technology i is such that in the steady state $Z_i^s > 0$, Then the optimal use of this technology yields higher cumulative emissions and lower cumulative damages than if it is not used.

Proof. Denote T^s as time when the economy reaches a steady state. The subscript *noR* labels the scenario without non-permanent CDR. We need to show that

$$\int_0^{T_{noR}^s} [E_{noR}(t) - P_{noR}(t)] dt < \int_0^{T^s} [E(t) - P(t)] dt$$

Due to Proposition 2, we know that

$$\int_0^{T_{noR}^s} [E_{noR}(t) - P_{noR}(t)] dt = X^s = \int_0^{T^s} [E(t) - P(t) - R(t) + \delta(t)Z(t)] dt$$

assuming without loss of generality that $X_0 = 0$. We can rewrite the above as follows.

$$\int_0^{T_{noR}^s} [E_{noR}(t) - P_{noR}(t)] dt = \underbrace{\int_0^{T^s} [E(t) - P(t)] dt}_{> LHS} - \underbrace{\int_0^{T^s} [R(t) - \delta(t)Z(t)] dt}_{= Z^s > 0 \text{ by assumption}}$$

The optimal use of non-permanent removal leads to higher cumulative emissions along the transition. Thus, on average, the carbon price $-\mu$ is lower when non-permanent carbon removal is used. Due to the mean value theorem, cumulative damages are then also lower. This result is independent from the time when the economy reaches the steady state:

- $T_{noR}^s \geq T^s$: Economy with non-permanent removal emits more carbon in the same or a shorter time span. This is only possible if on average $-\mu < -\mu_{noR}$ because of $\tilde{f}'(E) = \tilde{h}'(P) = -\mu$
- $T_{noR}^s < T^s$: Economy with non-permanent removal reaches μ^s later, implying that $-\mu(t) < -\mu_{noR}(t)$ before that.

□

A.3. Stability analysis of the steady state

We perform a stability analysis around the steady state for the case of only one generic non-permanent CDR technology with zero diligence costs. We have a system of four ODEs in $v := (X, Z, \mu, \psi) \in \mathbb{R}^4$. Let $\dot{v} = F(v)$. From above, we obtain

$$\dot{X} = (f')^{-1}(-\mu) - (h')^{-1}(-\mu) - (g')^{-1}(\psi - \mu) + \delta Z \quad (\text{A.6})$$

$$\dot{Z} = (g')^{-1}(\psi - \mu) - \delta Z \quad (\text{A.7})$$

$$\dot{\mu} = r\mu + d'(X) \quad (\text{A.8})$$

$$\dot{\psi} = (\delta + r)\psi - \delta\mu \quad (\text{A.9})$$

We linearize the system around the steady-state v^s , for which $F(v^s) = 0$. The Jacobian is

$$J = \begin{pmatrix} 0 & \delta & -(f')^{-1'} + (h')^{-1'} + (g')^{-1'} & -(g')^{-1'} \\ 0 & -\delta & -(g')^{-1'} & (g')^{-1'} \\ d''(X) & 0 & r & 0 \\ 0 & 0 & -\delta & \delta + r \end{pmatrix}$$

Now, we assume simple functional forms $h(P) = \eta P^2$, $f(R) = \alpha \log(E)$, $g(R) = \gamma R^2$, $d(X) = dX^2$. Even with these simple functions, the calculation of Eigenvalues becomes quite involved. Therefore, we have used Wolfram Mathematica to numerically prove that the real parts of two EVs are positive and those of the other two negative. This implies saddle path stability along a two-dimensional submanifold. In the supplementary material, we provide the Mathematica script.

Appendix B: Additional comparative statics

Here, we present comparative statics results for how the variables $X^s, R^s, Z^s, P^s, \delta^s, P^s/R^s$ change when the following quantities are varied: $g', w, \delta^s, r, h', f', d'$.

PROPOSITION B.1 (Comparative statics for the atmospheric carbon stock). *The optimal atmospheric carbon stock in the steady state X^s*

- increases when the discount rate r or marginal benefits of emissions f' increase,
- is independent of the release rate δ^s , marginal removal costs g' and marginal diligence cost w .
- decreases if marginal damages d' or marginal permanent removal costs h' increase.

Proof.

$X_r^s > 0$ can be seen by differentiating (19) and recalling from above that $E_r^s = 0$.

Higher marginal benefits of emissions lead to higher atmospheric carbon X^s and higher marginal costs of permanent removal lead to lower atmospheric carbon X^s since $f'(E^s) = h'(P^s)$ increases in the former and decreases in the latter case and due to (19) $d'(X^s)$ and thus also X^s must react in the same way.

The independence of X^s from g', δ^s, w follows directly from (19) and (17).

Higher climate damages lead to lower atmospheric carbon X^s since $f'(E^s) = h'(P^s)$ remain constant in (19). Thus, when climate damages are higher (the convex function d shifts up), X^s has to adjust downward. \square

PROPOSITION B.2 (Comparative statics for release rate). *The optimal release rate δ_i^s in the steady state*

- remain constant under changes in marginal damages
- changes in an ambiguous way under changes in g', w, r, h' and f' .

Proof. For the first part, consider that a change in d' is compensated for by a change in X^s so that $d'(X^s)$ remains constant. This term can be isolated in Equation (21), so that the other side of the equals sign, which depends only on δ_i^s , has to remain constant, too. This is only possible in general, if δ_i^s remains constant under variation of d' .

Proposition 4 entails the proof for the results with ambiguous outcome except for $(\delta_i^s)_r$. To see the latter, differentiate (21) with respect to r to obtain $\delta_r = -\frac{d'X_r+w}{(r+\delta)^2w''} + \frac{d'}{(r+\delta)^2w''}$. The first term is negative, the second positive. Thus, the sign of δ_r depends on functional choices. \square

PROPOSITION B.3 (Comparative statics for removal rates). *The optimal removal rate R_i^s in the steady state*

- decrease if the release rate δ_i^s increases,
- remains constant under changes in marginal damages
- changes in an ambiguous way under changes in g', w, r, h' and f' .

Proof. $(R_i^s)_\delta < 0$ and constancy under changes in marginal damages follow from Proposition 3 and Lemma 1. Proposition 4 entails the proof for the results with ambiguous outcome except for $(R_i^s)_r$. To see the latter, differentiate (20) with respect to r and note that $(R_i^s)_r$ inherits its ambiguity from $(\delta_i^s)_r$. \square

COROLLARY B.B.1 (Comparative statics for removal stocks). *The optimal removal stock Z_i^s in the steady state behaves just like R_i^s under changes in parameters and marginal costs.*

Proof. Use the proof of Proposition B.3 in combination with (16). \square

PROPOSITION B.4 (Comparative statics for residual emissions and permanent removal). *There is a constant level of residual emissions E^s that is exactly offset by permanent removals P^s . The level of residual emissions depends only on marginal benefits f' and marginal removal costs h' . The functional forms of f and h , thus, determine $E^s = P^s$ entirely. Residual emissions are independent of the release rate δ_i , non-permanent removal costs g_i , diligence costs w_i , climate damages and the discount rate r .*

Proof. Follows directly from (15). \square

Appendix C: GAMS model assumptions

For the numerical simulations, we make the following assumptions. There is only one non-permanent CDR technology and there is no technological progress ($\Delta_j(t) = 1 \quad \forall t$). Functional forms and parameters are as follows.

$$f(E(t)) = f_0 \left(\frac{E(t)}{e_0} \right)^{1-\alpha} \tag{C.1}$$

$$g(R(t)) = g_1 \cdot R(t) + g_2 \cdot (R(t))^2 \tag{C.2}$$

$$w(\delta(t)) = \frac{w_0}{\delta(t)} - w_0 \tag{C.3}$$

$$h(P(t)) = h_1 \cdot P(t) + h_2 \cdot (P(t))^2 \tag{C.4}$$

$$d(X(t)) = d_0(X(t))^2 \tag{C.5}$$

Table C.1 he

Parameter	Description	Value
ρ	Discount rate	0.02
f_0	Initial output (trillion \$)	100
α	Production function coefficient	0.95
e_0	Initial fossil energy use (GtCO ₂)	40
g_1	Linear CDR cost coefficient	0.02
g_2	Quadratic CDR cost coefficient	0.00365854
w_0	Diligence cost coefficient	0.000075
h_1	Linear permanent removal cost	0.1
h_2	Quadratic permanent removal cost	0.004375
d_0	Damage parameter	0.0000008112
Z_0	Initial storage (GtCO ₂)	0.0
X_0	Initial atmospheric carbon (GtCO ₂)	2000

TABLE C.1. Model parameters, their descriptions, and assigned values

Appendix D: Additional numerical results

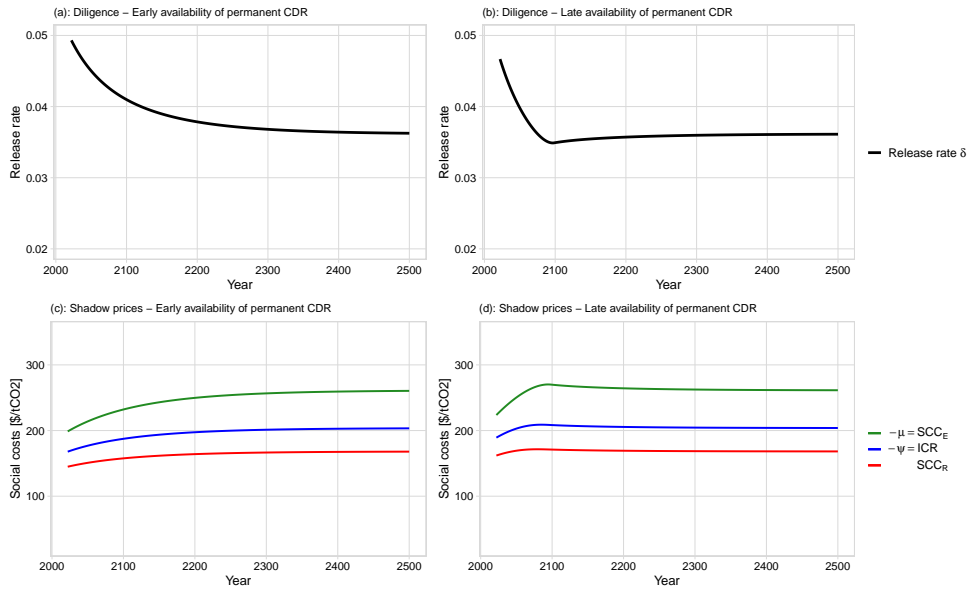


FIGURE D.1. Time paths of optimal diligence and the shadow prices $-\mu$ and $-\psi$ for the model runs in Figure 4.

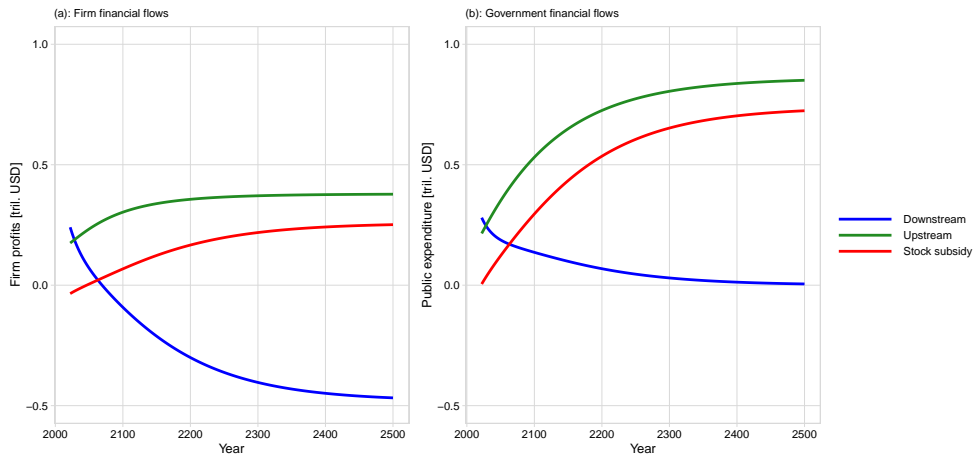


FIGURE D.2. Time paths of the financial flows of firms and governments in the model run with early availability in Figure 4.

Appendix E: Proofs for policy instrument analysis

E.1. Proofs for Proposition 5

E.1.1. Proof for Proposition 5a (downstream pricing). We have to set-up a Lagrangian with $\tilde{\psi}_i$ as the co-state variable for Z_i . The resulting first-order conditions are:

$$f'(E)\Delta^E(t) = p_E \quad (\text{E.1})$$

$$h'(P)\Delta^P(t) = p_P \quad (\text{E.2})$$

$$g'_i(R_i)\Delta^{R_i}(t) = p_E + \tilde{\psi}_i \quad (\text{E.3})$$

$$w'_i(\delta_i) = -(p_E + \tilde{\psi}_i) \quad (\text{E.4})$$

$$\dot{\tilde{\psi}}_i = (r + \delta_i)\tilde{\psi}_i + \delta_i p_E + \tilde{w}_i(\delta_i) \quad (\text{E.5})$$

$$0 = \lim_{t \rightarrow \infty} \tilde{\psi}_i Z_i(t) e^{-rt} \quad (\text{E.6})$$

When we set $p_E = -\mu^*$ with μ^* the optimal carbon price from the social planner, we have $\tilde{\psi} = \psi$, the optimality conditions of the firm are equivalent to those of the social planner (4–11) and, thus, imply the same allocation.

E.1.2. Proof for Proposition 5b (upstream pricing). When diligence is enforced via command-and-control, we find the following first-order conditions by setting up a Lagrangian with λ_i as the co-state variable for Z_i :

$$f'(E)\Delta^E(t) = p_E \quad (\text{E.7})$$

$$h'(P)\Delta^P(t) = p_P \quad (\text{E.8})$$

$$g'_i(R_i)\Delta^{R_i}(t) = p_{R_i} + \lambda_i \quad (\text{E.9})$$

$$\dot{\lambda}_i = (r + \delta_i)\lambda_i + \tilde{w}_i(\delta_i) \quad (\text{E.10})$$

$$0 = \lim_{t \rightarrow \infty} \lambda_i(t) Z_i(t) e^{-rt} \quad (\text{E.11})$$

From (E.7) and (E.8) we find that setting $p_E = p_P = -\mu^*$ from the social planner solution implements the optimal quantities for emission and permanent removal. To derive the optimal level of the upstream subsidy p_{R_i} , we use (E.9) and (E.10) to pin down an expression for λ_i . Using an analogous derivation as in Proposition 1, we find that

$$-\lambda_i(t) = \int_t^\infty \tilde{w}_i(\delta_i(s)) e^{-\int_t^s (r + \delta_i(v)) dv} ds = IDC_i \quad (\text{E.12})$$

Consequently, setting $p_{R_i} = SCC_E - SCC_{R_i}$ achieves the social planner solution with optimal diligence enforced via command-and-control.

E.1.3. Proof for Proposition 5c (stock subsidy). For a pricing scheme using only p_E , p_P and p_{Z_i} , the first-order condition for R_i , δ_i and Z_i imply $g'_i(R_i)\Delta^R = \lambda_i = -w'_i(\delta_i)\Delta^{\delta_i}$ and

$$\dot{\lambda}_i = (r + \delta_i)\lambda_i + \tilde{w}_i(\delta_i) - p_{Z_i} \quad (\text{E.13})$$

When we rewrite the first-order conditions of the social planner by taking the difference (8) and (9) using $\varphi_i = \psi_i - \mu$ we have

$$\begin{aligned}\dot{\psi}_i - \dot{\mu} &= (\delta + r)(\psi_i - \mu) + \tilde{w}'(\delta_i) - d'(X) = \frac{d}{dt}(\psi_i - \mu) \\ \dot{\varphi}_i &= (\delta + r)\varphi_i + \tilde{w}'(\delta_i) - d'(X)\end{aligned}\quad (\text{E.14})$$

We know from the first-order conditions of R_i and δ_i , (6) and (7), that an optimal solution for the firm's problem requires $\lambda_i = \varphi_i$. With this, the optimal subsidy on storage $p_{Z_i} = p_Z = d'(X)$ follows by taking the difference of (E.13) and (E.14).

E.2. Pricing instruments to incentives optimal diligence

With upstream pricing, the per-period cash flow of the non-permanent removal firm becomes

$$\begin{aligned}\pi_{it}^{\text{nr}} &= p_{R_i}R_i - \tilde{g}_i(R_i) - \tilde{w}(\delta_i)Z_i \\ &\quad - p_{\delta_i}\delta_i Z_i + p_{Z_i}Z_i\end{aligned}\quad (\text{E.15})$$

Remunerating removal up-front provides no incentive for diligence, and diligence would drop to its minimum level. The additional instruments on the second line of (E.15) can address this by setting incentives by subsidizing diligence δ_i (p_{δ_i}) or subsidizing the stock Z_i (p_{Z_i}), but they are degenerate.

LEMMA E.1. *The market instruments (i) subsidy on diligence (p_{δ_i}) and (ii) subsidy on non-permanent storage (p_{Z_i}) are degenerate, that is:*

- *With the additional instrument p_{δ_i} upstream pricing is equivalent to downstream pricing, and $p_{R_i} = p_{\delta_i} = SCC_E$.*
- *With the additional instrument p_{Z_i} , upstream pricing is equivalent to the stock subsidy, and $p_{R_i} = 0$, $p_{Z_i} = d'(X)$.*

Proof.

Subsidy on diligence. The tax on diligence ($p_{\delta_i}\delta_i Z_i$) enters the firm problem in the same way released emissions are priced in the downstream regime. Consequently, (E.10) is augmented by the term $p_{\delta_i}\delta_i$ the same way $p_E\delta_i$ enters (E.5), and similar to (7) we have

$$\tilde{w}'_i(\delta_i) = -(p_{\delta_i} + \lambda_i)\quad (\text{E.16})$$

It follows that $p_{\delta_i} = p_E$.

Subsidy on storage. The first-order conditions with respect to R_i and δ_i can be combined to read $g'_i(R_i)\Delta^{R_i} - p_{R_i} = \tilde{w}'_i(\delta_i)\Delta^{\delta_i}$. A zero subsidy of $p_{R_i} = 0$ is necessary to restore the balance in marginal costs as given (6) and (7). The remaining first-order conditions are the same as in Appendix E.1.3, and thus as for the case of storage subsidy $p_{Z_i} = d'(X)$ follows. \square

E.3. Upstream subsidy with staggered payment

The upstream subsidy fails to set an incentive for diligence after the upfront subsidy payment for removal has been received. A staggered payout of the upstream subsidy, conditional on the observed diligence, can address this. In such a scheme, the firm receives a reduced upfront payment $p_R^0(t)$ for a unit of removal in t , and a stream of staggered payments $p_{si}(t, s)$ at each subsequent time period s for each unit of carbon removed in period t with a net present value of P_{Si} .

With staggered payments, the profits of the firm become

$$\pi_i(\tau) = \begin{cases} \int_{\tau}^{\infty} \sum_i [p_{R_i}^0 R_i - \tilde{g}_i(R_i) - \tilde{w}_i(\delta_i) Z_i + P_{Si}(t)] e^{-rt} dt & \text{if } \delta_i(t) = \delta_i^* \\ \int_{\tau}^{\infty} \sum_i [p_{R_i}^0 R_i - \tilde{g}_i(R_i)] e^{-rt} dt & \text{else} \end{cases}$$

where $P_{Si}(t) = \int_0^t (R_i(s) p_{si}(s, t)) ds$ is the staggered payment.

A sufficient incentive to conform to the optimal effort in diligence $\delta_i^*(t)$ in any period τ is given when staggered payments are no less than the cost of diligence.

$$\tilde{w}_i(\delta_i(\tau)) Z_i(\tau) \leq P_{Si}(\tau) \quad (\text{E.17})$$

The firm's optimization problem is

$$\max_{R_i} \int_0^{\infty} \sum_i \left[p_{R_i}^0 R_i - \tilde{g}_i(R_i) - \tilde{w}_i(\delta_i) Z_i + \int_0^t (R_i(s) p_{si}(s, t)) ds \right] e^{-rt} dt$$

$$\dot{Z}_i = R_i - \delta_i Z_i \perp \lambda_i$$

The first-order condition with respect to R_i is

$$\tilde{g}'_i(R_i) = p_{R_i}^0 + \lambda_i + \int_t^{\infty} p_{si}(t, s) e^{-(s-t)r} ds \quad (\text{E.18})$$

where the last term represents the net present value of the future revenue stream of staggered payments. We can now derive the minimal value of the staggered payment that is required to ensure first-best removal quantities. First, note that we know that from E.1.2 that $\lambda_i = -IDC_i$. Furthermore, note that if we set the staggered payment in period s for a removal that happened in period t equal to the diligence cost, $p_{si}(t, s) = e^{-\int_t^s \delta_i(k) dk} w_i(\delta_i(s))$, the last term is also equal to the IDC . It follows that setting $p_R^0 = SCC_E - SCC_R - IDC$ achieves the first-best removal quantities.

This is intuitive: while in the standard upstream system with diligence command-and-control the firm pays for the diligence, in this case the government pays for the diligence. The upstream subsidy is reduced by the IDC , and this part is paid out in a staggered fashion.

E.4. Remarks on public finance

To see the results on the NPV stated in Section 3.4, note that the NPVs of total public expenditures for the three schemes share the component

$$G_{fix}^{NPV} = \int_0^{\infty} e^{-rt} (Pp_P - Ep_E) dt \quad (E.19)$$

and differ by a second component:

$$G^{up} = \int_0^{\infty} e^{-rt} R_i p_{R_i} dt \quad (E.20)$$

$$G^{down} = \int_0^{\infty} e^{-rt} p_E (R_i - \delta_i Z_i) dt \quad (E.21)$$

$$G^{stksbs} = \int_0^{\infty} e^{-rt} Z_i p_{Z_i} dt \quad (E.22)$$

First, we compare G^{down} with G^{stksbs} . Using the expressions derived in Proposition 5 and then applying integration by parts we obtain

$$G^{down} = \int_0^{\infty} e^{-rt} \dot{Z}_i SCC_E dt \quad (E.23)$$

$$= -SCC_E(0)Z_0 + \int_0^{\infty} e^{-rt} Z_i d'(X) dt \quad (E.24)$$

$$= \underbrace{-SCC_E(0)Z_0}_{<0} + G^{stksbs} \quad (E.25)$$

from which follows that the NPV of public expenditures due to downstream pricing is less or equal to that due to the stock subsidy.

To compare G^{down} with G^{up} , observe that

$$G^{up} = \int_0^{\infty} e^{-rt} R_i (SCC_E - SCC_R) dt \quad (E.26)$$

and

$$G^{down} = \int_0^{\infty} e^{-rt} SCC_E (R_i - \delta_i Z_i) dt \quad (E.27)$$

It follows that

$$G^{up} - G^{down} = \int_0^{\infty} e^{-rt} SCC_E \delta_i Z_i - R_i SCC_R dt \quad (E.28)$$

From the social planners first order conditions, it follows that $SCC_E > SCC_R$ for all t . However, the relation between removal rate R_i and release flows $\delta_i Z_i$ depends on functional forms and the initial stock size. The comparison between G^{down} and G^{up} , thus, remains ambiguous.